Effect of Boiler Pressure (Using Molliar Diagram i.e., h-s diagram)

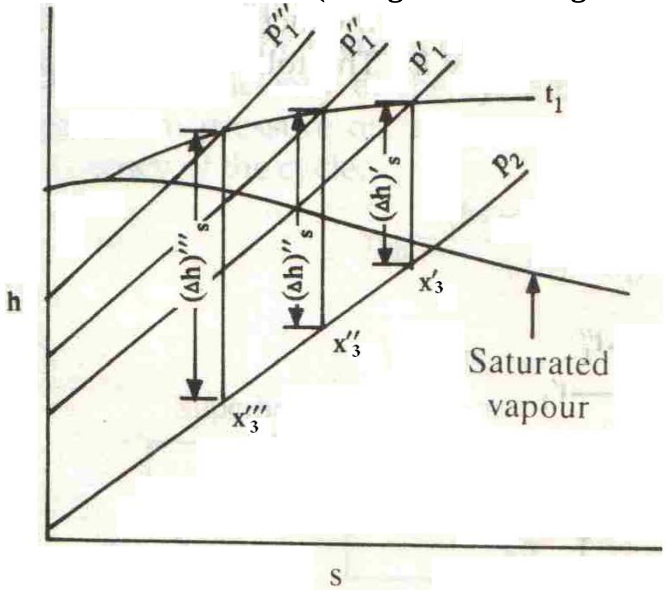


Fig. 13.5 Effect of boiler pressure

We have,

$$\eta_{th} = \frac{(h_2 - h_3) - (h_1 - h_4)}{h_2 - h_1} \text{ but } W_P << W_T$$

$$\therefore \eta_{th} = \frac{h_2 - h_3}{h_2 - h_1} = \frac{(\Delta h)_S}{(h_2 - h_1)}$$

i.e., Rankine cycle η depends on h_2, h_1 and Δh_S. From figure as $P_1''' > P_1'' > P_1'$ for the fixed maximum temperature of the steam t_1 and condenser pressure P_2, Isentropic heat drops increases with boiler pressure i.e., from the figure therefore it is evident that as boiler pressure increases, the isentropic heat drop $(\Delta h)_S$ increases, but the enthalpy of the steam entering the turbine decreases, with the result that the Rankine η increases. But quality of the steam at the exit of the turbine suffers i.e., $x_3''' < x_3'' < x_3'$, which leads to serious wear of the turbine blades.

Effect of Super Heating (Using Molliar Diagram i.e., h-s diagram)

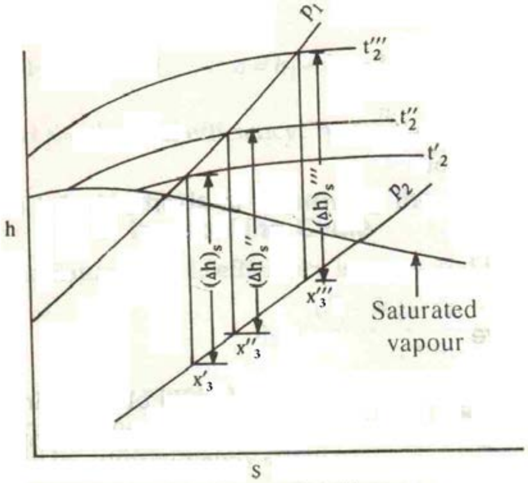
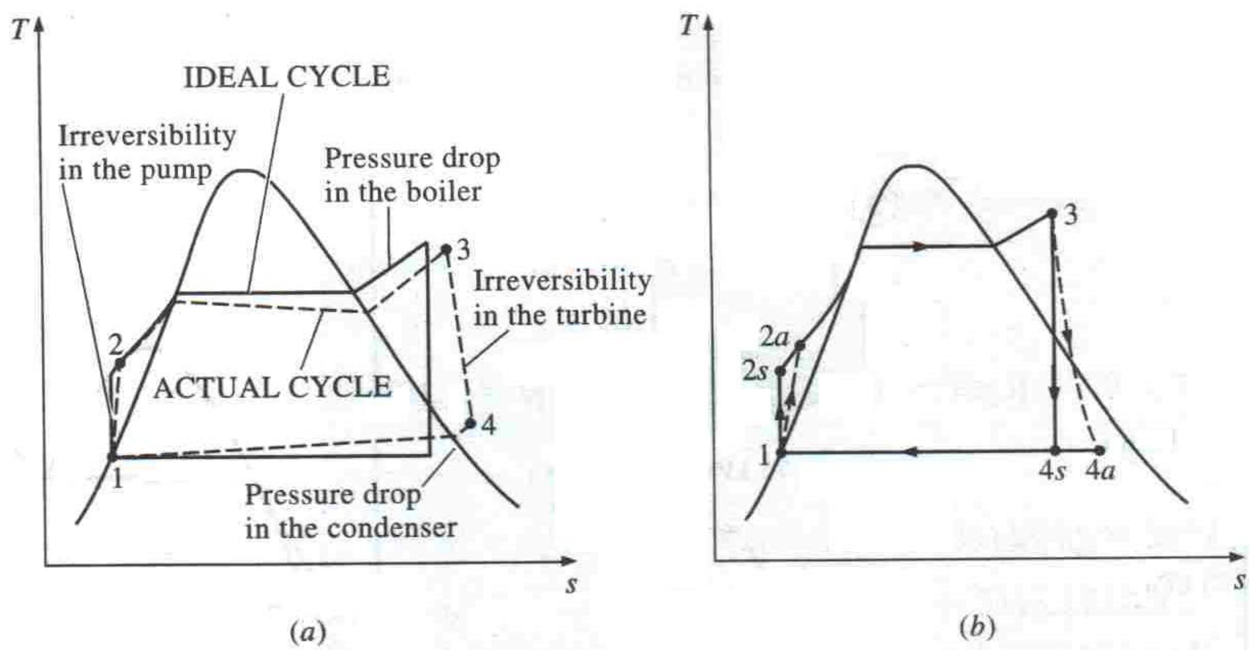


Fig. Effect of superheating

The moisture in the steam at the end of the expansion may be reduced by increasing the super heated temperature of steam t_1. This can be seen in figure where $t_1''' > t_1'' > t_1'$, but $x_3' < x_3'' < x_3'''$. It is, therefore, natural that to avoid erosion of the turbine blades, an increase in the boiler pressure must be accompanied by super heating at a higher temperature and since this raises the mean average temperature at which heat is transferred to the steam, the Rankine η increases.

Deviation of Actual Vapour Power cycles from Ideal cycle



The actual Vapour power cycle differs from the ideal Rankine cycle, as shown in figure, as a result of irreversibilities in various components mainly because of fluid friction and heat loss to the surroundings.

Fluid friction causes pressure drops in the boiler, the condenser, and the piping between various components. As a result, steam leaves the boiler at a lower pressure. Also the pressure at the turbine inlet is lower than that at the boiler exit due to pressure drop in the connecting pipes. The pressure drop in the condenser is usually very small. To compensate these pressure drops, the water must be pumped to sufficiently higher pressure which requires the larger pump and larger work input to the pump.

The other major source of irreversibility is the heat loss from the steam to the surroundings as the steam flows through various components. To maintain the same level of net work output, more heat needs to be transferred to the steam in the boiler to compensate for these undesired heat losses. As a result, cycle efficiency decreases.

As a result of irreversibilities, a pump requires a greater work input, and a turbine produces a smaller work output. Under the ideal conditions, the flow through these devices are isentropic. The deviation of actual pumps and turbines from the isentropic ones can be accounted for by utilizing isentropic efficiencies, defined as

$$\eta_P = \frac{W_S}{W_a} = \frac{h_{1S} - h_4}{h_1 - h_4}$$

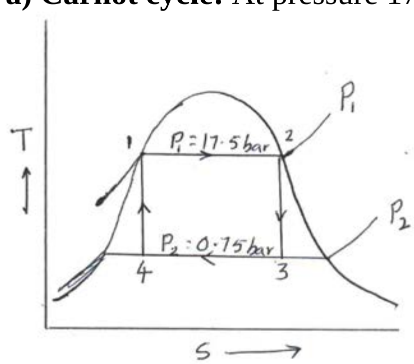
And $\eta_t = \dfrac{W_a}{W_S} = \dfrac{h_2 - h_3}{h_2 - h_{3S}}$

Numerical Problems:

1. **Dry saturated steam at 17.5 bar enters the turbine of a steam power plant and expands to the condenser pressure of 0.75 bar. Determine the Carnot and Rankine cycle efficiencies. Also find the work ratio of the Rankine cycle.**

Solution: P_1 = 17.5 bar P_2 = 0.75 bar η_{Carnot} = ? $\eta_{Rankine}$ = ?

a) Carnot cycle: At pressure 17.5 bar from steam tables,



P	t_S	h_f	h_{fg}	h_g	S_f	S_{fg}	S_g
17	204.3	871.8	1921.6	2793.4	2.3712	4.0246	6.3958
18	207.11	884.5	1910.3	2794.8	2.3976	3.9776	6.3751

For P = 17.5 bar, using linear interpolation

For t_S, $204.3+\frac{207.11-204.3}{1}x0.5=205.71^{0}C$

$= 478.71$ K

Similarly, $h_f = 878.15$ kJ/kg $h_{fg} = 1915.95$ kJ/kg $h_g = 2794.1$ kJ/kg

$S_f = 2.3844$ kJ/kg^{0}K $S_{fg} = 4.0011$ kJ/kg^{0}K $S_g = 6.3855$ kJ/kg K

Also at pressure 0.75 bar from steam tables

P	t_S	h_f	h_{fg}	h_g	S_f	S_{fg}	S_g
0.8	93.51	391.7	2274.0	2665.8	1.233	6.2022	7.4352
0.7	89.96	376.8	2283.3	2660.1	1.1921	6.2883	7.4804

$\therefore$ For 0.75 bar, using linear interpolation,

$t_S = 91.74^0$C $h_f = 384.25$ $h_{fg} = 2278.65$ $h_g = 2662.95$

$S_f = 1.2126$ $S_{fg} = 6.2453$ $S_g = 7.4578$

The Carnot cycle η, $\eta_C = \frac{T_1-T_2}{T_1}=\frac{478.71-364.74}{478.71}=0.2381$

Steam rate or SSC = $\frac{1}{\oint \delta W}=\frac{1}{W_T-W_P}$

Since the expansion work is isentropic, $S_2 = S_3$

But $S_2 = S_g = 6.3855$ and $S_3 = S_{f3} + x_3 S_{fg3}$

i.e., $6.3855 = 1.2126 + x_3 (6.2453)$ $\therefore x_3 = 0.828$

$\therefore$ Enthalpy at state 3, $h_3 = h_{f3} + x_3 h_{fg3}$

$= 384.25 + 0.828 (2278.65) = 2271.63$ kJ/kg

$\therefore$ Turbine work or expansion work or positive work = $h_2 - h_3$

$= 2794.1 - 2271.63 = 522.47$ kJ/kg

Again since the compression process is isentropic

i.e., $S_4 = S_1 = S_{f1} = 2.3844$

Hence $2.3844 = S_{f4} + x_4 S_{fg4}$

$= 1.2126 + x_4 (6.2453)$ $\therefore x_4 = 0.188$

$\therefore$ Enthalpy at state 4 is $h_4 = h_{f4} + x_4 h_{fg4}$

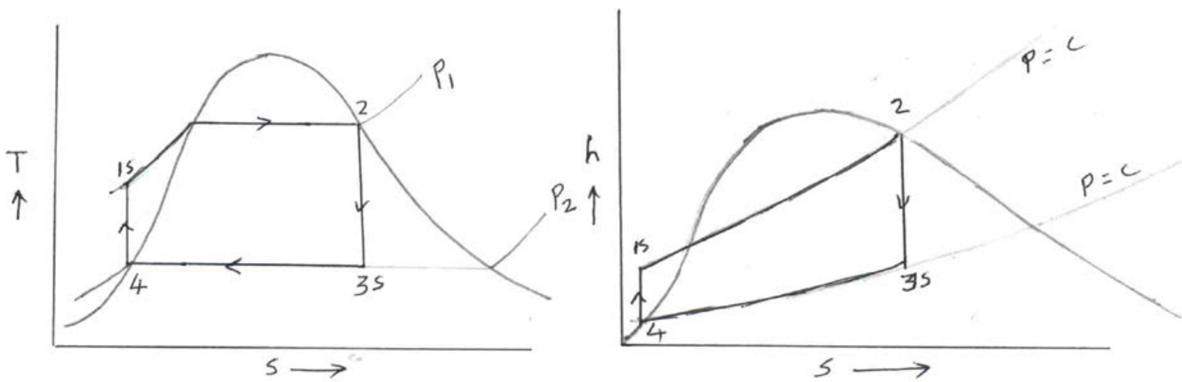
= 384.25 + 0.188 (2278.65)
= 811.79 kJ/kg

$\therefore$ Compression work, = $h_1 - h_4$ = 878.15 – 811.79
W_P = 66.36 kJ/kg

$$\therefore SSC = \frac{1}{522.47 - 66.36} = 2.192\ x\,10^{-3}\,kg/kJ$$

$$\text{work ratio} = r_w = \frac{\oint \delta w}{+ve\,work} = \frac{W_T - W_P}{W_T} = \frac{456.11}{522.47} = 0.873$$

b) Rankine cycle:



$$\eta_R = \frac{W_T - W_P}{Q_H} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$$

Since the change in volume of the saturated liquid water during compression from state 4 to state 1 is very small, v_4 may be taken as constant. In a steady flow process, work W = $-v\int dp$

$\therefore W_P = h_{1S} - h_4 = v_{fP2}\,(P_1 - P_2)$
= 0.001037 (17.5 – 0.75) x 10^5 x (1/1000)
= 1.737 kJ/kg

$\therefore h_{1S}$ = 1.737 + 384.25 = 385.99 kJ/kg

Hence, turbine work = $W_T = h_2 - h_3$ = 522.47kJ/kg
Heat supplied = $Q_H = h_2 - h_{1S}$ = 2.794.1 – 385.99 = 2408.11 kJ/kg

$$\therefore \eta_R = \frac{522.47 - 1.737}{2408.11} = 0.2162$$

$$\therefore SSC = \frac{1}{522.47 - 1.737} = 19204\ x\,10^{-3}\,kg/kJ$$

Work ratio, $r_w = \frac{522.47 - 1737}{522.47} = 0.9967$

2. If in problem (1), the turbine and the pump have each 85% efficiency, find the % reduction in the net work and cycle efficiency for Rankine cycle.

Solution: If $\eta_P = 0.85$, $\eta_T = 0.85$

$$W_P = \frac{W_P}{0.85} = \frac{1.737}{0.85} = 2.0435 kJ/kg$$

$W_T = \eta_T W_T = 0.85 (522.47) = 444.09$ kJ/kg

$\therefore W_{net} = W_T – W_P = 442.06$ kJ/kg

$\therefore$ % reduction in work output = $\frac{520.73 - 442.06}{520.73} = 15.11\%$

$W_P = h_{1S} – h_4$ $\therefore h_{1S} = 2.0435 + 384.25 = 386.29$ kJ/kg

$\therefore Q_H – h_2 – h_{1S} = 2794.1 – 386.29 = 2407.81$ kJ/kg

$\therefore \eta_{cycle} = \frac{442.06}{2407.81} = 0.1836$

$\therefore$ % reduction in cycle efficiency $= \frac{0.2162 - 0.1836}{0.2162} = 15.08\%$

Note: Alternative method for problem 1 using h-s diagram (Mollier diagram) though the result may not be as accurate as the analytical solution. The method is as follows

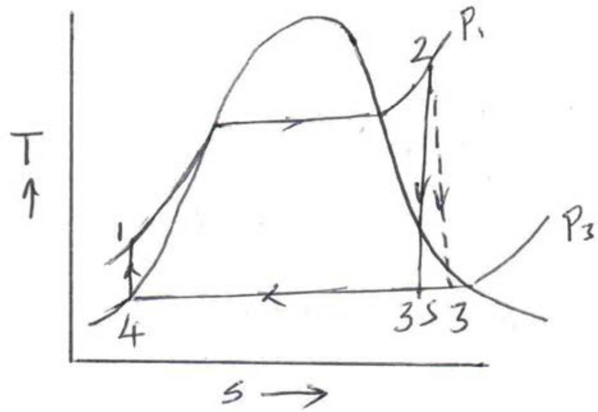
Since steam is dry saturated at state 2, locate this state at the pressure P_2 = 17.5 bar on the saturation line and read the enthalpy at this state. This will give the value of h_2.

As the expansion process 2-3 is isentropic, draw a vertical line through the state 2 to meet the pressure line, P = 0.75 bar. The intersection of the vertical line with the pressure line will fix state 3. From the chart, find the value of h_3.

The value of h_4 can be found from the steam tables at pressure, P = 0.75 bar, as $h_4 = h_{f4}$. After finding the values of h_2, h_3 and h_4, apply the equation used in the analytical solution for determining the Rankine cycle η and SSC.

3. **Steam enters the turbine of a steam power plant, operating on Rankine cycle, at 10 bar, 300^0C. The condenser pressure is 0.1 bar. Steam leaving the turbine is 90% dry. Calculate the adiabatic efficiency of the turbine and also the cycle η, neglecting pump work.**

Solution:



P_1 = 10 bar $t_2 = 300^0$C P_3 = 0.1 bar

$x_3 = 0.9$ η_t = ? η_{cycle} = ? Neglect W_P

From superheated steam tables,

For P_2 = 10 bar and $t_2 = 300^0$C, h_2 = 3052.1 kJ/kg, s_2 = 7.1251 kJ/kg

From table A – 1, For P_3 = 0.1 bar

$t_S = 45.83^0$C h_f = 191.8 h_{fg} = 2392.9

S_f = 0.6493 S_{fg} = 7.5018

Since $x_3 = 0.9$, $h_3 = h_{f4} + x_3\, h_{fg3}$

$= 191.8 + 0.9\,(2392.9)$

$= 2345.4$ kJ/kg

Also, since process 2-3s is isentropic, $S_2 = S_{3S}$

i.e., $7.1251 = S_{fg4} + x_{3S}\, S_{fg3}$

$= 0.6493 + x_{3S}\,(7.5018)$

$\therefore x_{3S} = 0.863$

$\therefore h_{3S} = 191.8 + 0.863\,(2392.9) = 2257.43$ kJ/kg

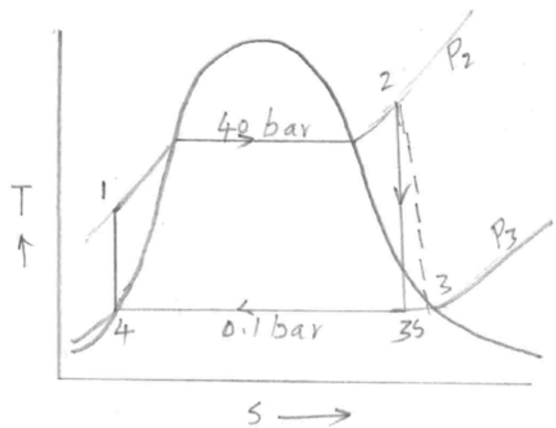
$$\therefore \textit{Turbine efficiency}, \eta_t = \frac{h_2 - h_3}{h_2 - h_{3S}} = \frac{3052.1 - 2345.4}{3052.1 - 2257.43} = 0.89$$

$$\eta_{cycle} = \frac{W_T}{Q_H} = \frac{h_2 - h_3}{h_2 - h_1} \quad but \quad h_1 = 191.8 \text{ kJ/kg}$$

$$= \frac{3052.1 - 2345.4}{3052.1 - 191.8} = 0.25 \quad i.e., \quad 25\%$$

4. **A 40 mW steam plant working on Rankine cycle operates between boiler pressure of 4 MPa and condenser pressure of 10 KPa. The steam leaves the boiler and enters the steam turbine at 400^{0}C. The isentropic η of the steam turbine is 85%. Determine (i) the cycle η (ii) the quality of steam from the turbine and (iii) the steam flow rate in kg per hour. Consider pump work.**

Solution:



P_2 = 4 MPa = 40 bar P_3 = 10 KPa = 0.1 bar

P = 40000kW $t_2 = 400^0$C $\eta_t = 0.85$ $\eta_{cycle} = ?$ $x_3 = ?$

$\dot{m} = ?$

$h_2 = h\big|_{40\,bar,\,400^0 C} = 3215.7 kJ/kg$ and s_2 = 6.7733 kJ/kg-K

$h_4 = h_f\big|_{0.1 bar} = 191.8 kJ/kg$

Process 2-3s is isentropic i.e., $S_2 = S_{3S}$

$$6.7733 = 0.6493 + x_{3S}\,(7.5018)$$

$$\therefore x_{3S} = 0.816$$

$\therefore h_{3S} = h_{f3} + x_{3S}\, h_{fg3} = 191.8 + 0.816\ (2392.9)$

$= 2145.2$ kJ/kg

$$But \quad \eta_t = \frac{h_2 - h_3}{h_2 - h_{3S}} \quad i.e., \quad 0.85 = \frac{3215.7 - h_3}{3215.7 - 2145.2}$$

$\therefore h_3 = 2305.8$ kJ/kg

$\therefore W_T = h_2 - h_3 = 3215.7 - 2305.8 = 909.9$ kJ/kg

$W_P = v\int dP \qquad = 0.0010102\ (40 - 0.1)\ 10^5/10^2$

$= 4.031$ kJ/kg

$= h_1 - h_4 \qquad \therefore h_1 = 195.8$ kJ/kg

$$(i)\ \eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{909.9 - 4.031}{(3215.7 - 195.8)} = 29.9\%$$

(ii) $x_3 = ?$ we have $2305.8 = 191.8 + x_3\ (2392.9)$ $\therefore x_3 = 0.88$

(iii) $P = \dot{m}\, W_{net}$ i.e., $40000 = \dot{m}\ (905.87)$

$\therefore \dot{m} = 44.2$ kg/s

$= 159120$ kg/hr

Ideal Reheat cycle: We know that, the efficiency of the Rankine cycle could be increased by increasing steam pressure in the boiler and superheating the steam. But this increases the moisture content of the steam in the lower pressure stages in the turbine, which may lead to erosion of the turbine blade. $\therefore$ The reheat cycle has been developed to take advantage of the increased pressure of the boiler, avoiding the excessive moisture of the steam in the low pressure stages. In the reheat cycle, steam after partial expansion in the turbine is brought back to the boiler, reheated by combustion gases and then fed back to the turbine for further expansion.

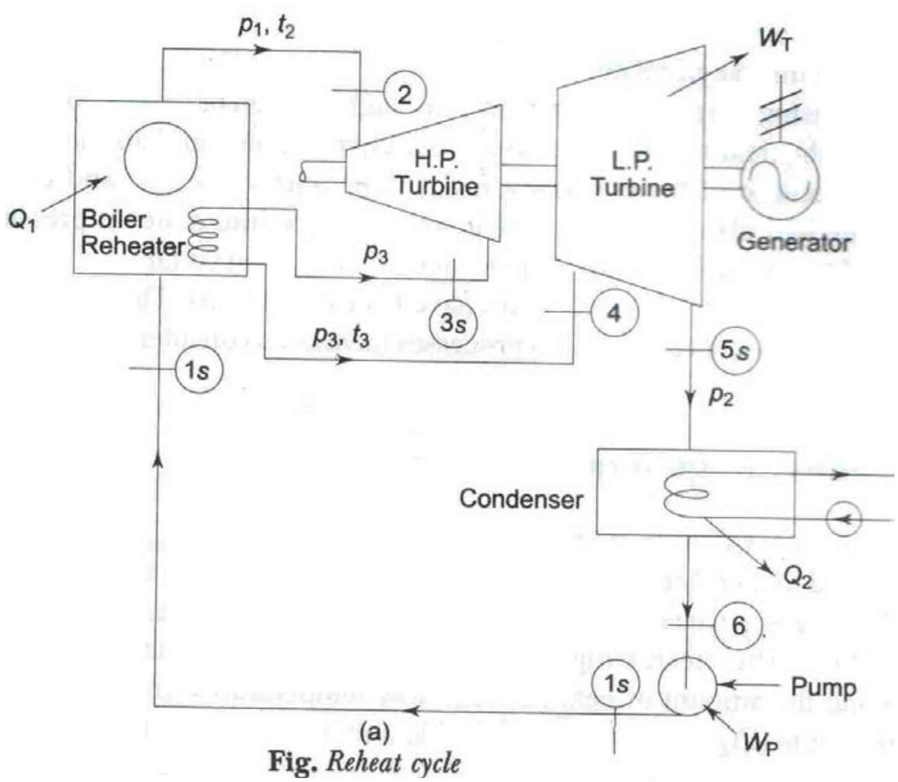
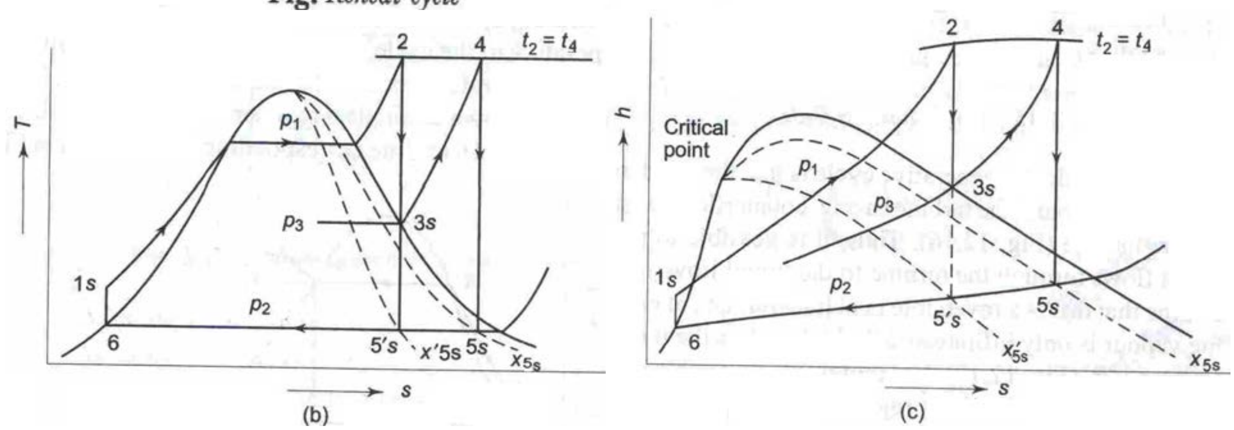


Fig. Reheat cycle



In the reheat cycle the expansion of steam from the initial state (2) to the condenser pressure is carried out in two or more steps, depending upon the number of reheats used.

In the first step, steam expands in HP turbine from state 2 to approximate the saturated vapour line (process 2-3s). The steam is then reheated (or resuperheated) at constant pressure in the boiler (or in a reheater) process 3s-4 and the remaining expansion process 4s-5 is carried out in the LP turbine.

Note: 1) To protect the reheater tubes, steam is not allowed to expand deep into the two-phase region before it is taken for reheating, because in that case the moisture particles in steam while evaporating would leave behind solid deposits in the form of scale which is difficult to remove. Also a low reheat pressure may bring down T_{m1} and hence cycle η. Again a high reheat pressure increases the moisture content at turbine exhaust. Thus reheat pressure is optimized. Optimum reheat pressure is about 0.2 to 0.25 of initial pressure.

We have for 1 kg of steam

$Q_H = (h_2 - h_{1S}) + (h_4 - h_{3S})$; $Q_L = h_{5S} - h_6$
$W_T = (h_2 - h_{3S}) + (h_4 - h_{5S})$; $W_P = h_{1S} - h_6$

$$\therefore \quad \eta_R = \frac{W_T - W_P}{Q_H} \ ;$$

Steam rate $= \dfrac{3600}{(W_T - W_P)} kg / kWh$

Since higher reheat pressure is used, W_P work is appreciable.

2) In practice, the use of reheat gives a marginal increase in cycle η, but it increases the net work output by making possible the use of higher pressures, keeping the quality of steam at turbine exhaust within a permissible limit. The quality improves from $x_{5^1 S}$ to x_{5S} by the use of reheat.

Ideal Regenerative cycle: The mean temperature of heat addition can also be increased by decreasing the amount of heat added at low temperatures. In a simple Rankine cycle (saturated steam entering the turbine), a considerable part of the total heat supplied is in the liquid phase when heating up water from 1 to 1^1, at a temperature lower than T_2, the maximum temperature of the cycle. For maximum η, all heat should be supplied at T_2, and feed water should enter the boiler at 1^1. This may be accomplished in what is known as an ideal regenerative cycle as shown in figures (a) and (b).

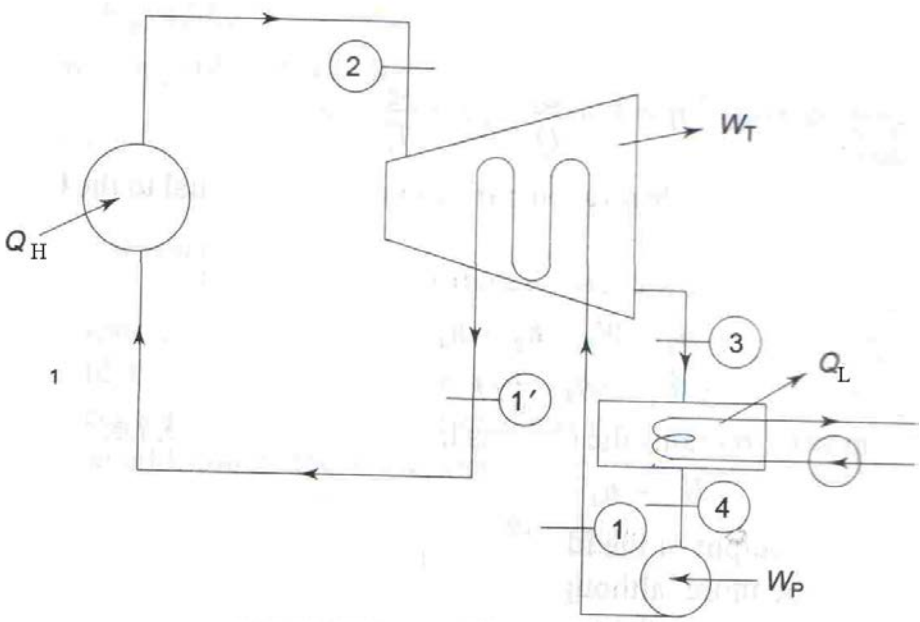


Fig. *Ideal regenerative cycle*

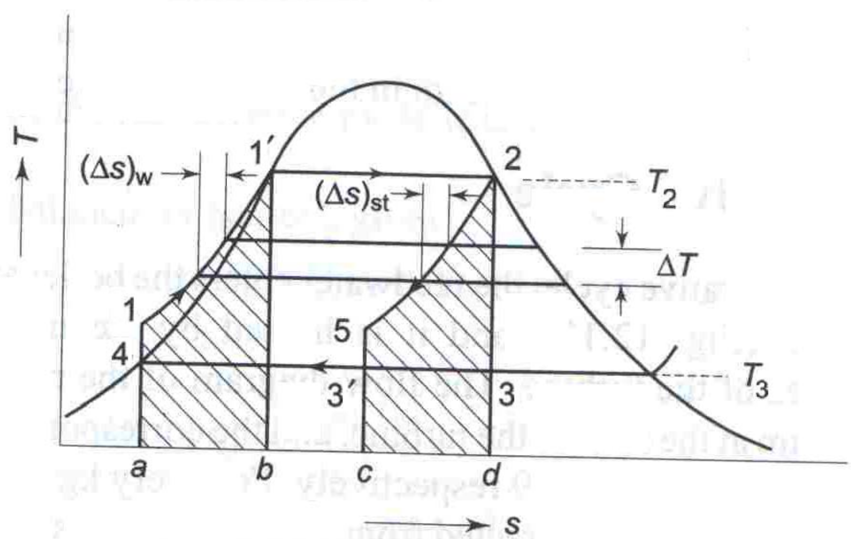


Fig. *Ideal regenerative cycle on T-s plot*

ΔT(water) = – ΔT (steam)

The unique feature of the ideal regenerative cycle is that the condensate, after leaving the pump circulates around the turbine casing, counter-flow to the direction of vapour flow in the turbine. Thus it is possible to transfer heat from the vapour as it flows through the turbine to the liquid flowing around the turbine.

Let us assume that this is a reversible heat transfer i.e., at each point, the temperature of the vapour is only infinitesimally higher than the temperature of the liquid. $\therefore$The process $2\text{-}3^1$ represents reversible expansion of steam in the turbine with reversible heat rejection. i.e., for any small step in the process of heating the water $\Delta T_{(water)} = -\Delta T_{(steam)}$ and $(\Delta S)_{water} = (\Delta S)_{steam}$. Then the slopes of lines $2\text{-}3^1$ and $1^1\text{-}4$ will be identical at every temperature and the lines will be identical in contour. Areas $1\text{-}1^1\text{-b-a-}1$ and $3^1\text{-}2\text{-d-c-}3^1$ are not only equal but congruous. $\therefore$, all heat added from external source (Q_H) is at constant temperature T_2 and all heat rejected (Q_L) is at constant temperature T_3, both being reversible.

Then $Q_H = h_2 - h_1^1 = T_2 (S_2 - S_1^1)$

$Q_L = h_3^1 - h_4 = T_3 (S_3^1 - S_4)$

Since $S_1^1 - S_4 = S_2 - S_3^1$ or $S_2 - S_1^1 = S_3^1 - S_4$

$\therefore \quad \eta_{\text{Reg}} = 1 - \dfrac{Q_L}{Q_H} = 1 - \dfrac{T_3}{T_2}$ i.e., the η of ideal regenerative cycle is thus equal to the Carnot cycle η.

Writing SFEE to turbine,

$h_2 + h_1 = W_T + h_1^1 + h_3^1$

i.e., $W_T = (h_2 - h_3^1) - (h_1^1 - h_1)$

or $W_T = (h_2 - h_3^1) - (h_1^1 - h_1)$ --- (1)

and the W_P is same as simple rankine cycle i.e., $W_P = (h_1 - h_4)$

$\therefore$The net work output of the ideal regenerative cycle is less and hence its steam rate will be more. Although it is more efficient when compared to rankine cycle, this cycle is not practicable for the following reasons.

1) Reversible heat transfer cannot be obtained in finite time.
2) Heat exchanger in the turbine is mechanically impracticable.
3) The moisture content of the steam in the turbine is high.

Regenerative cycle:

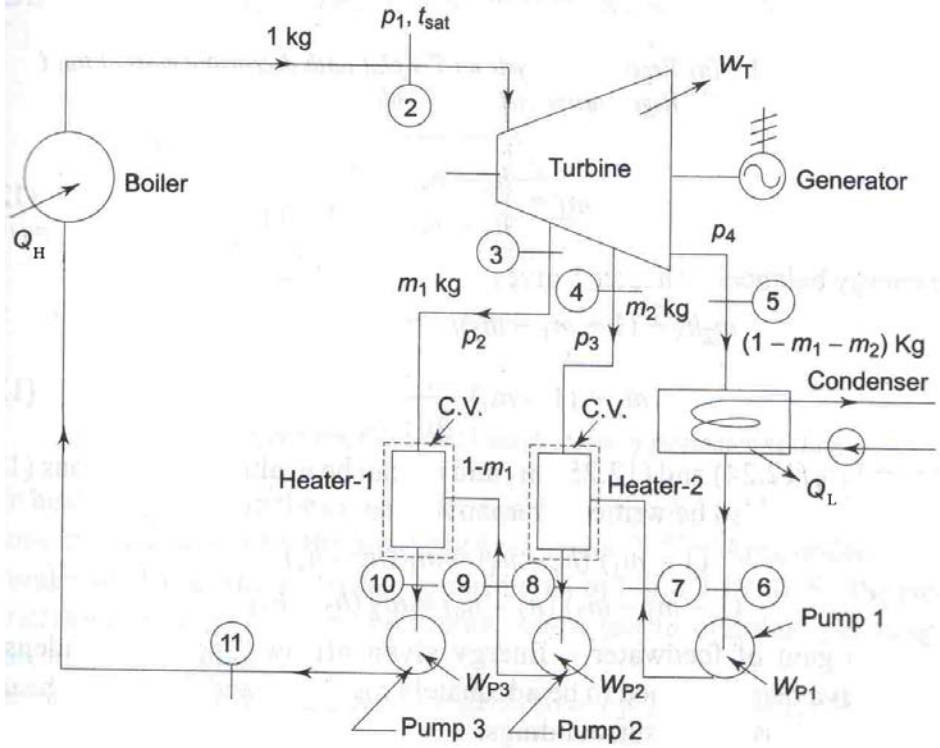
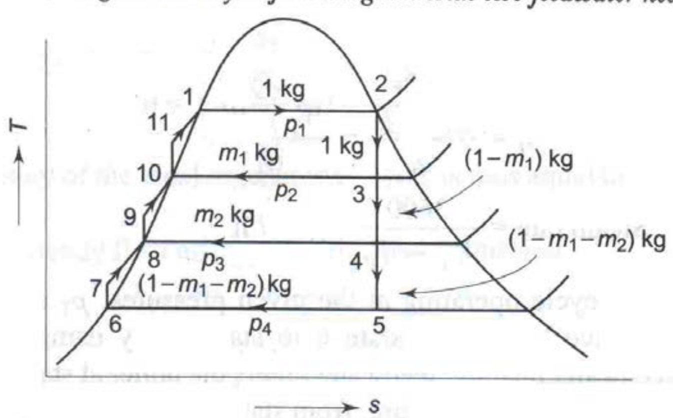
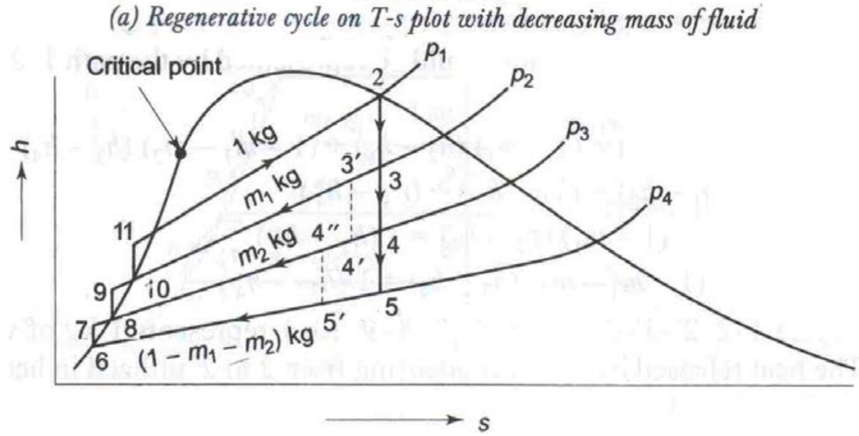


Fig. *Regenerative cycle flow diagram with two feedwater heaters*



(a) Regenerative cycle on T-s plot with decreasing mass of fluid



In a practical regenerative cycle, the feed water enters the boiler at a temperature between 1 and 1^1 (previous article figure), and it is heated by steam extracted from intermediate stages of the turbine. The flow diagram of the regenerative cycle with saturated steam at the inlet to the turbine and the corresponding T-S diagram are shown in figure.

For every kg of steam entering the turbine, let m_1 kg steam be extracted from an intermediate stage of the turbine where the pressure is P_2, and it is used to heat up feed water [$(1 – m_1)$ kg at state 9] by mixing in heater (1). The remaining $(1-m_1)$ kg of steam then expands in the turbine from pressure P_2 (state 3) to pressure P_3 (state 4) when m_2 kg of steam is extracted for heating feed water in heater (2). So $(1 – m_1 – m_2)$kg of steam then expands in the remaining stages of the turbine to pressure P_4, gets condensed into water in the condenser, and then pumped to heater (2), where it mixes with m_2 kg of steam extracted at pressure P_3. Then $(1-m_1)$ kg of water is pumped to heater (1) where it mixes with m_1 kg of steam extracted at pressure P_2. The resulting 1kg of steam is then pumped to the boiler where heat from an external source is supplied. Heaters 1 and 2 thus operate at pressure P_2 and P_3 respectively. The amounts of steam m_1 and m_2 extracted from the turbine are such that at the exit from each of the heaters, the state is saturated liquid at the respective pressures.

$\therefore$ Turbine work, $W_T = 1(h_2 – h_3) + (1 – m_1)(h_3 – h_4) + (1 – m_1 – m_2)(h_4 – h_5)$
Pump work, $W_P = W_{P1} + W_{P2} + W_{P3}$
$= (1 – m_1 – m_2)(h_7 – h_6) + (1 – m_1)(h_9 – h_8) + 1(h_{11} – h_{10})$

$Q_H = (h_2 – h_{11})$; $\quad Q_L = (1 – m_1 – m_2)(h_5 – h_6)$

$\therefore$ Cycle efficiency, $\eta = \dfrac{Q_H - Q_L}{Q_H} = \dfrac{W_T - W_P}{Q_H}$

$$SSC = \frac{3600}{W_T - W_P} kg/kWh$$

In the Rankine cycle operating at the given pressure P_1 and P_4, the heat addition would have been from state 7 to state 2. By using two stages of regenerative feed water heating., feed water enters the boiler at state 11, instead of state 7, and heat addition is, therefore from state 11 to state 2. Therefore $\left(T_{m1}\right)_{with\ regeneration} = \dfrac{h_2 - h_{11}}{S_2 - S_{11}}$

And $\left(T_{m1}\right)_{without\ regeneration} = \dfrac{h_2 - h_7}{S_2 - S_7}$

Since $(T_{m1})_{\text{with regenerative}} > (T_{m1})_{\text{without regenerative}}$, the η of the regenerative cycle will be higher than that of the Rankine cycle.

The energy balance for heater 1,

$$m_1 h_3 + (1 – m_1) h_9 = 1\ h_{10}$$

$$\therefore m_1 = \frac{h_{10} - h_9}{h_3 - h_9} \qquad \text{--- (1)}$$

The energy balance for heater 2,

$m_2 h_4 + (1 - m_1 - m_2) h_7 = (1 - m_1) h_8$

Or $m_2 = (1 - m_1)\dfrac{(h_8 - h_7)}{(h_4 - h_7)}$ --- (2)

Above equations (1) and (2) can also be written alternatively as

$(1 - m_1)(h_{10} - h_9) = m_1 (h_3 - h_{10})$

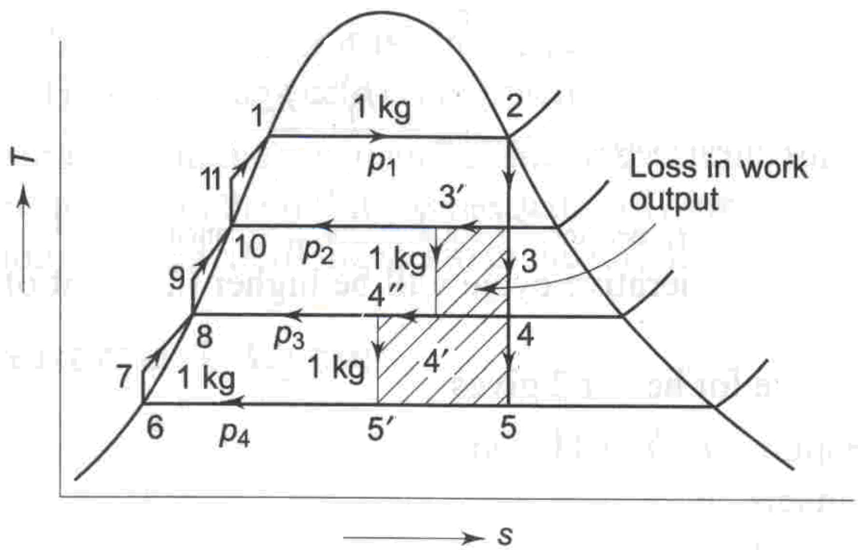
and $(1 - m_1 - m_2)(h_8 - h_7) = m_2 (h_4 - h_8)$

Energy gain of feed water = energy given off by vapour in condensation.

Heaters have been assumed to be adequately insulated and there is no heat gain from, or heat loss to, the surroundings.

In figure (a) path 2-3-4-5 represents the states of a decreasing mass of fluid.

For 1kg of steam, the states would be represented by the path $2\text{-}3^1\text{-}4^{11}\text{-}5^1$. [Figure (b)].



(b) Regenerative cycle on T-s plot for unit mass of fluid

We have $W_T = (h_2 - h_3) + (1 - m_1)(h_3 - h_4) + (1 - m_1 - m_2)(h_4 - h_5)$

$= (h_2 - h_3) + (h_3^1 - h_4^1) + (h_4^{11} - h_5^1)$ [From Figure b] --- (3)

The cycle $2 - 3 - 3^1 - 4^1 - 4^{11} - 5^1 - 6 - 7 - 8 - 9 - 10 - 11 - 2$ represents 1kg of working fluid. The heat released by steam condensing from 3 to 3^1 is utilized in heating up the water from 9 to 10.

$\therefore \quad 1 (h_3 - h_3^1) = 1 (h_{10} - h_9)$ --- (4)

Similarly, $1 (h_4^1 - h_4^{11}) = 1 (h_8 - h_7)$ --- (5)

From equation (3), (4) and (5),

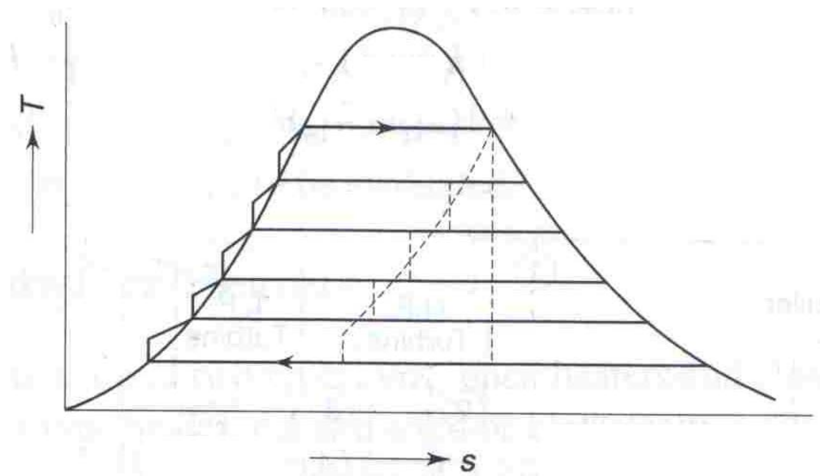
$W_T = (h_2 - h_5^1) - (h_3 - h_3^1) - (h_4^1 - h_4^{11})$

$$= (h_2 - h_5^1) - (h_{10} - h_9) - (h_8 - h_7) \qquad \text{--- (6)}$$

Also from Ideal regenerative cycle, [Previous article]

$$W_T = (h_2 - h_3^1) - (h_1^1 - h_1) \qquad \text{--- (1)}$$

The similarity of equations (6) and equation (1) from previous article is notices. It is seen that the stepped cycle $2 - 3^1 - 4^1 - 4^{11} - 5^1 - 6 - 7 - 8 - 9 - 10 - 11$ approximates the ideal regenerative cycle in Figure (1) [previous article] and that a greater no. of stages would give a closer approximation. Thus the heating of feed water by steam ‘bled’ from the turbine, known as regeneration, “Carnotizes” the Rankine cycle.



Regenerative cycle with many stages of feedwater heating

The heat rejected Q_L in the cycle decreases from $(h_5 - h_6)$ to $(h_5^1 - h_6)$. There is also loss in work output by the amount (area under $3 - 3^1$ + area under $4^1 - 4^{11}$ – area under $5 - 5^1$) as shown by the hatched area in Figure (b). So the steam rate increases by regeneration i.e., more steam has to circulate per hour to produce unit shaft output.

Reheat – Regenerative cycle:

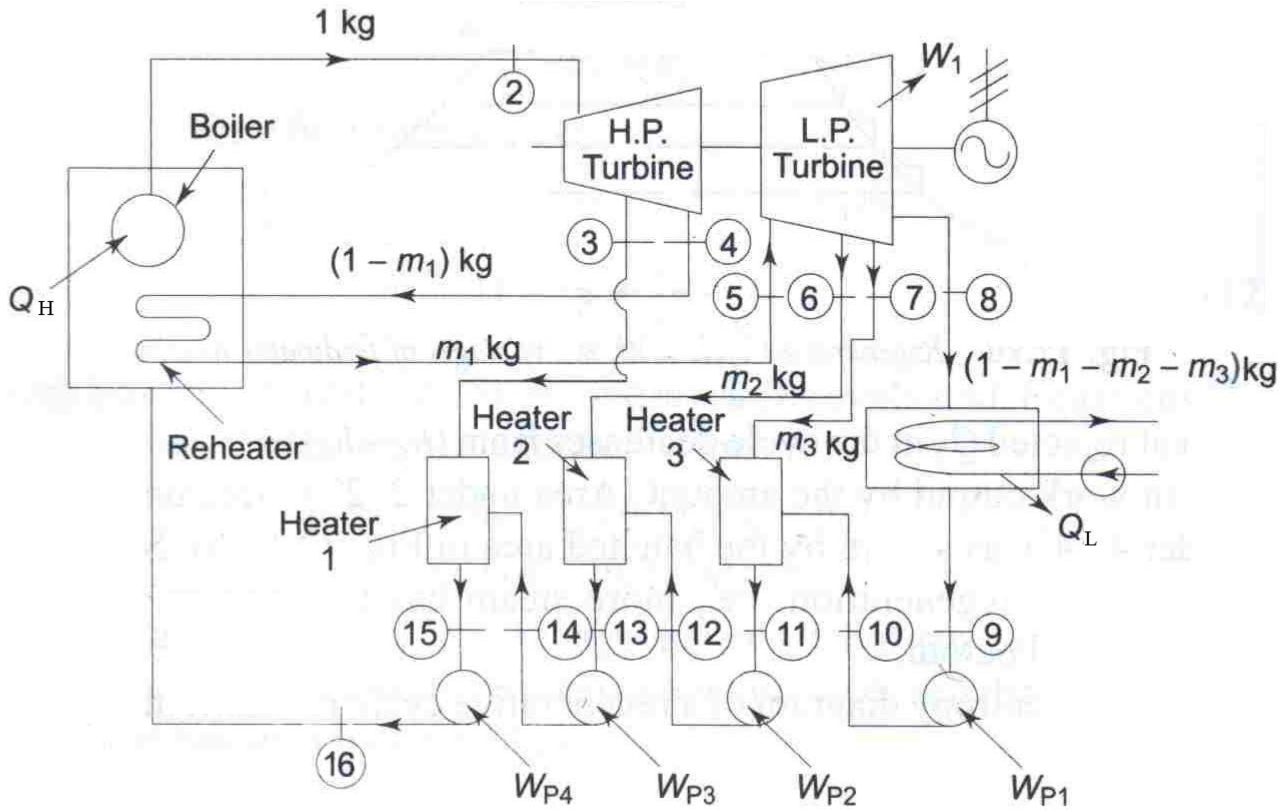


Fig. *Reheat-regenerative cycle flow diagram*

Reheat – regenerative cycle flow diagram (Three-stages of feed water heating)

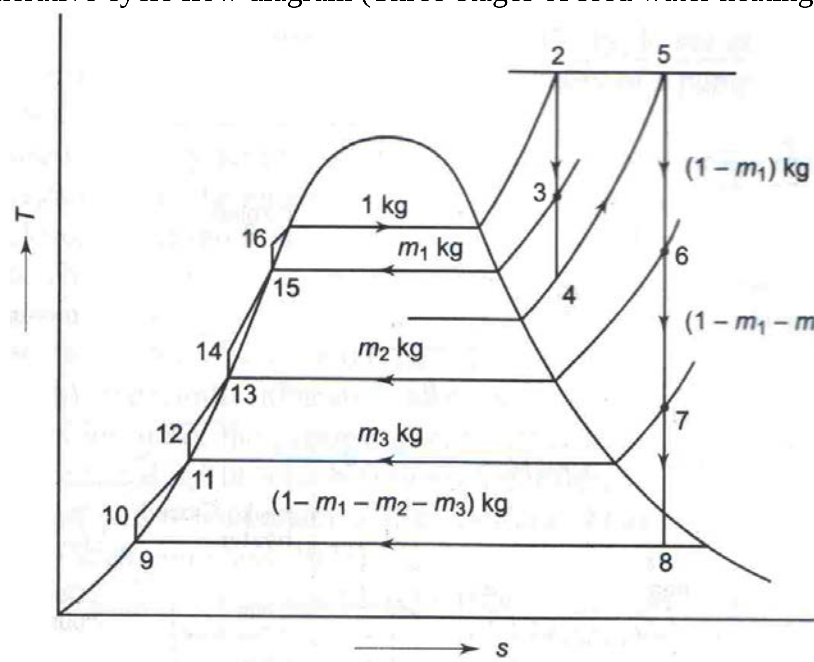


Fig. *T–s diagram of reheat-regenerative cycle*

The reheating of steam is employed when the vapourization pressure is high reheat alone on the thermal η is very small. $\therefore$ Regeneration or the heating up of feed water by steam extracted from the turbine will effect in more increasing in the η_{th}.

Turbine work, $W_T = (h_1 - h_2) + (1 - m_1)(h_2 - h_3) + (1 - m_1)(h_4 - h_5) + (1 - m_1 - m_2)(h_5 - h_6)$
$+ (1 - m_1 - m_2 - m_3)(h_6 - h_7)$ kJ/kg

Pump work, $W_P = (1 - m_1 - m_2 - m_3)(h_9 - h_8) + (1 - m_1 - m_2)(h_{11} - h_{10})$
$+ (1 - m_1)(h_{13} - h_{12}) + 1\,(h_{15} - h_{14})$ kJ/kg

Heat added, $Q_H = (h_1 - h_{15}) + (1 - m_1)(h_4 - h_3)$ kJ/kg

Heat rejected, $Q_L = (1 - m_1 - m_2 - m_3)(h_7 - h_8)$ kJ/kg

The energy balance of heaters 1, 2 and 3 gives

$m_1 h_2 + (1 - m_1) h_{13} = 1 \times h_{14}$

$m_2 h_5 + (1 - m_1 - m_2) h_{11} = (1 - m_1) h_{12}$

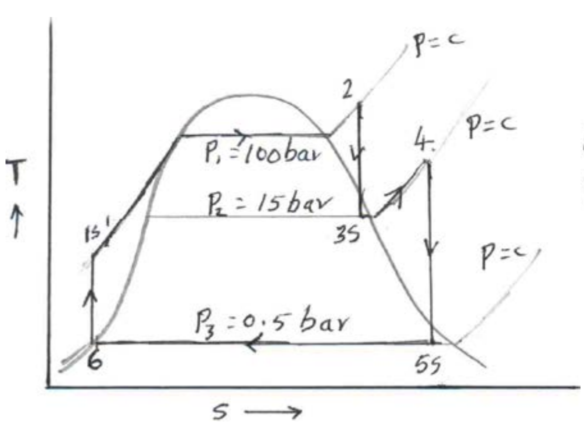
$m_3 h_6 + (1 - m_1 - m_2 - m_3) h_9 = (1 - m_1 - m_2) h_{10}$

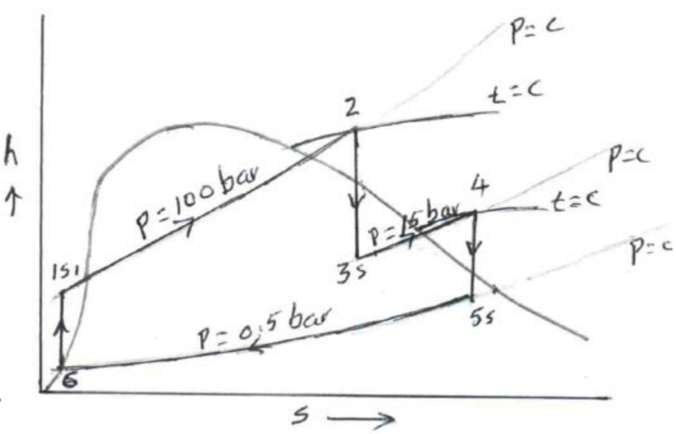
From which m_1, m_2 and m_3 can be evaluated

Numerical Problems:

1. **An ideal reheat cycle utilizes steam as the working fluid. Steam at 100 bar, 400^{0}C is expanded in the HP turbine to 15 bar. After this, it is reheated to 350^{0}C at 15 bar and is then expanded in the LP turbine to the condenser pressure of 0.5 bar. Determine the thermal η and steam rate.**

Solution:





From steam tables

P = 100 bar t = 400^{0}C = v = 0.026408m^3/kg

h = 3099.9 kJ/kg

S = 6.2182 kJ/kg-K

P = 15 bar t_S = 192.28^{0}C, v_f = 0.0011538m^3/kg, v_g = 0.13167m^3/kg

h_f = 844.6 kJ/kg, h_{fg} = 1845.3 kJ/kg, h_g = 2789.9 kJ/kg

s_f = 2.3144kJ/kg-K, s_{fg} = 4.1262 kJ/kg-K, s_g = 6.4406 kJ/kg-K

P = 0.5 bar t_S = 81.35^{0}C, v_f = 0.0010301 m^3/kg, v_g = 3.2401 m^3/kg

h_f = 340.6 kJ/kg, h_{fg} = 2305.4 kJ/kg h_g = 2646.0 kJ/kg
s_f = 1.0912 kJ/kg-K s_{fg} = 6.5035 kJ/kg-K, s_g = 7.5947 kJ/kg-K

h_2 = 3099.9 kJ/kg,
Process 2-3s is isentropic, i.e., $S_2 = S_{3S}$
$6.2182 = 2.3144 + x_{3S} (4.1262)$
$\therefore x_{3S} = 0.946$
$\therefore h_{3S} = 844.6 + x_{3S} (1845.3)$
= 2590.44 kJ/kg

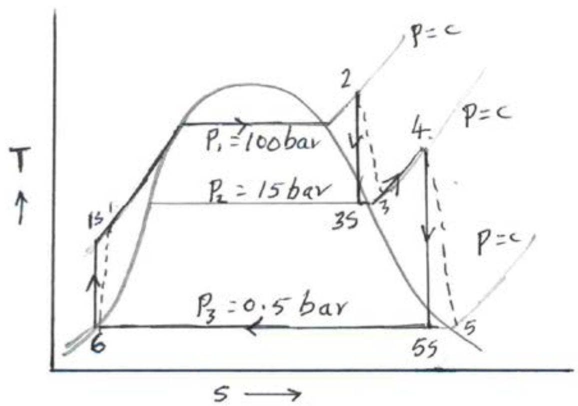
$\therefore$ Expansion of steam in the HP turbine $= h_2 - h_{3S}$
= 3099.9 – 2590.44
= 509.46 kJ/kg
P = 15 bar, t = 350^{0}C = v = 0.18653
h = 3148.7
s = 7.1044
Expansion of steam in the LP cylinder = $h_4 - h_{5s}$
h_4 = 3148.7 kJ/kg
To find h_{5s}:
We have $S_4 = S_{5s}$
$7.1044 = S_{f5} + x_{5S} S_{fg5}$
$= 1.0912 + x_{5S} (6.5035)$
$\therefore x_{5S} = 0.925$
$\therefore h_{5s}$ = 340.6 + 0.925 (2305.4) = 2473.09 kJ/kg
$\therefore$ Expansion of steam in the LP turbine = 3148.7 – 2473.09
= 675.61 kJ/kg
$h_6 = h_f$ for P_3 =0.5 bar i.e., h_6 = 340.6 kJ/kg
Pump work, $W_P = h_{1s} - h_6$ $= v_{f5} (P_3 - P_1) = 0.0010301 (100 - 0.501 \times 10^5)$
= 10.249 kJ/kg
$\therefore h_{1s}$ = 350.85 kJ/kg
$\therefore$ Heat supplied, Q_H $= (h_2 - h_{1S}) + (h_4 - h_{3S})$
= (3099.9 – 350.85) + (3148.7 – 2590.44)
= 2749.05 kJ/kg + 558.26
= 3307.31 kJ/kg

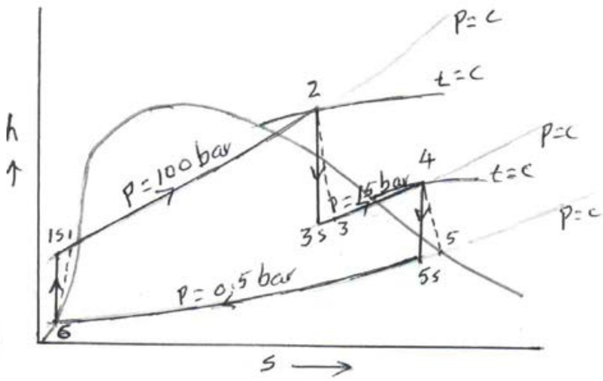
$$\therefore \quad \eta_{th} = \frac{W_{net}}{Q_H} = \frac{(W)_{HP} + (W)_{LP} - W_P}{Q_H}$$

$$= \frac{509.46 + 675.61 - 10.25}{3307.31} = 0.355$$

Steam rate, SSC $= \dfrac{3600}{W_{net}} = 3.064 kg/kWh$

1. b) When η of the HP turbine, LP turbine and feed pump are 80%, 85% and 90% respectively.





$$\eta_{tHP} = \frac{h_2 - h_3}{h_2 - h_{3s}} = 0.8 = \frac{3099.9 - h_3}{3099.9 - 2590.44}$$

$\therefore h_3 = 2692.33$ kJ/kg

$$\eta_{tLP} = \frac{h_4 - h_5}{h_4 - h_{5s}} = 0.85 = \frac{3148.7 - h_5}{3148.7 - 2473.09}$$

$\therefore h_5 = 2574.43$ kJ/kg

$$\eta_P = \frac{h_{15} - h_6}{h_1 - h_6} = 0.9 = \frac{350.85 - 340.6}{h_1 - 340.6}$$

$\therefore h_1 = 351.99$ kJ/kg

$$\therefore \eta_{th} = \frac{(h_2 - h_3) + (h_4 - h_5) - (h_1 - h_6)}{(h_2 - h_1) + (h_4 - h_3)}$$

$$= \frac{(3099.9 - 2692.33) + (3148.7 - 2574.43) - (351.99 - 340.6)}{(3099.9 - 351.99) + (3148.7 - 2692.33)}$$

$= 0.303$ or 30.3%

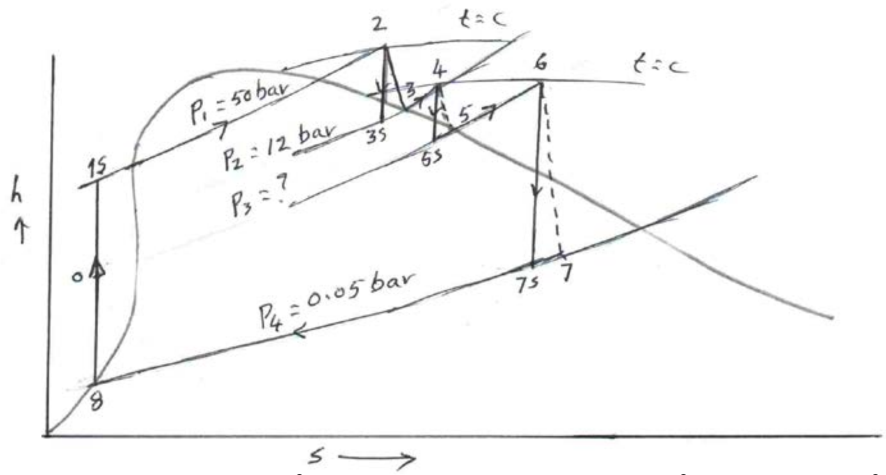
$\therefore$ SSC = 3.71 kg/kWh

Using Mollier-chart: $h_2 = 3095$ kJ/kg, $h_{3s} = 2680$ kJ/kg, $h_4 = 3145$ kJ/kg
$h_5 = 2475$ kJ/kg, $h_6 = 340.6$ kJ/kg (from steam tables)
$W_P = 10.249$ kJ/kg

2. **Steam at 50 bar, 350^{0}C expands to 12 bar in a HP stage, and is dry saturated at the stage exit. This is now reheated to 280^{0}C without any pressure drop. The reheat steam expands in an intermediate stage and again emerges dry and saturated at a low pressure, to be reheated a second time to 280^{0}C. Finally, the steam expands in a LP stage to 0.05 bar. Assuming the work output is the same for the high and intermediate stages, and the efficiencies of the high and low pressure stages are equal, find: (a) η of the HP stage (b) Pressure of steam at the exit of the intermediate stage, (c) Total power output from the three stages for a flow of 1kg/s of steam, (d) Condition of steam at exit**

of LP stage and (e) Then η of the reheat cycle. Also calculate the thermodynamic mean temperature of energy addition for the cycle.

Solution:



P_1 = 50 bar $t_2 = 350^0$C P_2 = 12 bar $t_4 = 280^0$C, $t_6 = 280^0$C

P_3 = ? P_4 = 0.05 bar

From Mollier diagram

h_2 = 3070kJ/kg h_{3s} = 2755 kJ/kg h_3 = 2780 kJ/kg h_4 = 3008 kJ/kg

(a) η_t for HP stage $= \dfrac{h_2 - h_3}{h_2 - h_{3s}} = \dfrac{3070 - 2780}{3070 - 2755}$

= 0.921

(b) Since the power output in the intermediate stage equals that of the HP stage, we have

$h_2 - h_3 = h_4 - h_5$

i.e., 3070 – 2780 = 3008 – h_5

$\therefore h_5$ = 2718 kJ/kg

Since state 5 is on the saturation line, we find from Mollier chart, P_3 = 2.6 bar,

Also from Mollier chart, h_{5s} = 2708 kJ/kg, h_6 = 3038 kJ/kg, h_{7s} = 2368 kJ/kg

Since η_t is same for HP and LP stages,

$$\eta_t = \frac{h_6 - h_7}{h_6 - h_{7s}} = 0.921 = \frac{3038 - h_7}{3038 - 2368} \quad \therefore h_7 = 2420.93\text{kJ/kg}$$

$\therefore$ At a pressure 0.05 bar, $h_7 = h_{f7} + x_7 h_{fg7}$

2420.93 = 137.8 + x_7 (2423.8)

$\therefore x_7$ = 0.941

Total power output $= (h_2 - h_3) + (h_4 - h_5) + (h_6 - h_7)$

= (3070 – 2780) + (3008 – 2718) + (3038 – 2420.93)

= 1197.07 kJ/kg

$\therefore$ Total power output /kg of steam = 1197.07 kW

For P_4 = 0.05 bar from steam tables, h_8 = 137.8 kJ/kg;

$W_P = 0.0010052\ (50 - 0.05)\ 10^2$ = 5.021 kJ/kg

$= h_8 - h_{1s}$

$\therefore h_{1s} = 142.82$ kJ/kg

Heat supplied, Q_H $= (h_2 - h_{1s}) + (h_4 - h_3) + (h_6 - h_5)$
$= (3070 - 142.82) + (3008 - 2780) + (3038 - 2718)$
$= 3475.18$ kJ/kg

$W_{net} = W_T - W_P = 1197.07 - 5.021 = 1192.05$ kJ/kg

$$\therefore \eta_{th} = \frac{W_{net}}{Q_H} = \frac{1192.05}{3475.18} = 0.343$$

$$\eta_{th} = 1 - \frac{T_o}{T_m} = 1 - \frac{(273 + 32.9)}{T_m} = 0.343,$$

$$0.657 = \frac{305.9}{T_m}$$

$\therefore T_m = 465.6$ K

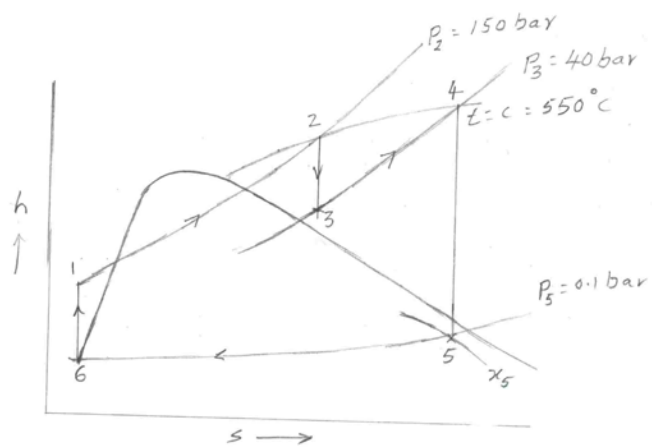
Or

$$T_m = \frac{h_2 - h_{1s}}{S_2 - S_{1s}} = \frac{3070 - 142.82}{6.425 - 0.4763} = 492K$$

$$SSC = \frac{3600}{1192.05} = 3.02 kg / kWh$$

3. A steam power station uses the following cycle: Steam at boiler outlet – 150 bar; reheat at 40 bar, 550^{0}C; condenser at 0.1 bar. Using Mollier chart and assuming that all processes are ideal, find (i) quality at turbine exhaust (ii) cycle η (iii) steam rate.

Solution:



$P_2 = 150$ bar $t_2 = 550^0$C $P_3 = 40$ bar $t_3 = 550^0$C

$P_5 = 0.1$ bar

From Mollier diagram i.e., h-s diagram

$$h_2 = h\Big|_{150\,bar,550^0 C} = 3450 kJ/kg$$

$$h_4 = h\Big|_{40\,bar,550^0 C} = 3562 kJ/kg$$

h_3 = 3050 kJ/kg

h_5 = 2290 kJ/kg

x_5 = 0.876 kJ/kg

h_6 can not determined from h-s diagram, hence steam tables are used.

$$h_6 = h_f\Big|_{0.1\,bar} = 191.8 kJ/kg$$

Process 6-1 is isentropic pump work i.e., $W_{P1} = v\int dP$

$= 0.0010102\ (40 – 01)\ 10^5/10^3 = 4.031$ kJ/kg

$= (h_1 – h_6)$

$\therefore h_1 = 195.8$ kJ/kg

(i) Quality of steam at turbine exhaust = x_5 = 0.876

$$(ii)\ \eta_{cycle} = \frac{W_T - W_P}{Q_H}$$

Turbine work, $W_T = W_{T1} + W_{T2}$

$= (h_2 – h_3) + (h_4 – h_5)$

$= (3450 – 3050) + (3562 – 2290)$

$= 1672$ kJ/kg

$Q_H = Q_1 + Q_R = (h_2 - h_1) + (h_4 – h_3)$

$= (3450 – 195.8) + (3562 – 3050)$

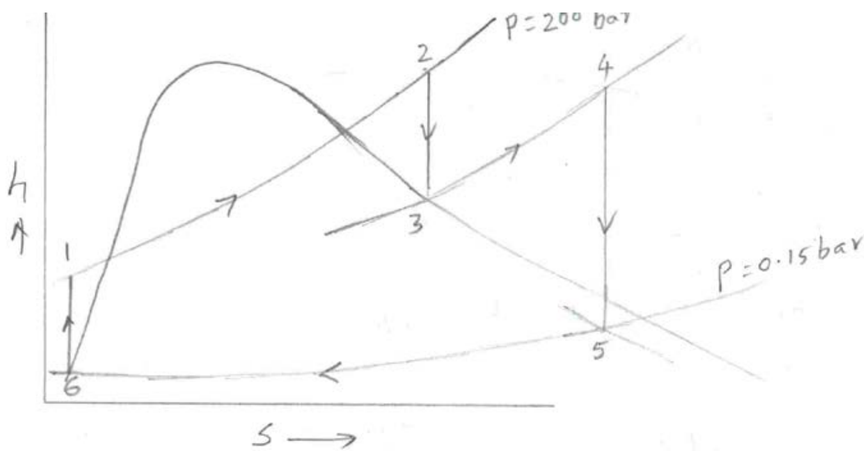
$= 3766.2$ kJ/kg

$$\therefore \eta_{cycle} = \frac{1672 - 4.031}{3766.2} = \frac{1667.97}{3766.2} = 0.443$$

$$(iii) \quad Steam\ rate = \frac{3600}{1667.97} = 2.16 kg / kWh$$

4. An ideal Rankine cycle with reheat is designed to operate according to the following specification. Pressure of steam at high pressure turbine = 20 MPa, Temperature of steam at high pressure turbine inlet = 550^{0}C, Temperature of steam at the end of reheat = 550^{0}C, Pressure of steam at the turbine exhaust = 15 KPa. Quality of steam at turbine exhaust = 90%. Determine (i) the pressure of steam in the reheater (ii) ratio of pump work to turbine work, (iii) ratio of heat rejection to heat addition, (iv) cycle η.

Solution:



P_2 = 200 bar t_2 = 550^{0}C t_4 = 550^{0}C P_5 = 0.15 bar x_5 = 0.9

From Mollier diagram,

h_2 = 3370 kJ/kg

h_3 = 2800 kJ/kg

h_4 = 3580 kJ/kg

h_5 = 2410 kJ/kg

x_5 = 0.915

$P_3 = P_4$ = 28 bar

But given in the data x_5 = 0.9

From steam tables h_6 = 226 kJ/kg

Pump work $W_P = v\int dP$

$= 0.001014 (200 - 0.15) 10^5/10^3$

$= 20.26$ kJ/kg

But $W_P = h_1 - h_6 \quad \therefore h_1 = 246.26$ kJ/kg

(i) Pressure of steam in the reheater = 28 bar

(ii) Turbine work $W_T = (h_2 - h_3) + (h_4 - h_5)$

$= (3370 - 2800) + (3580 - 2410)$

$= 1740$ kJ/kg

$\therefore$ Ratio of $\frac{W_P}{W_T} = 0.0116 \quad i.e., \quad 1.2\%$

(iii) $Q_L = (h_5 - h_6) = (2410 - 226) = 2184$ kJ/kg

$Q_H = (h_2 - h_1) + (h_4 - h_3)$

$= (3370 - 226) + (3580 - 2800)$

$= 3924$ kJ/kg

$\therefore \frac{Q_L}{Q_H} = 0.5565 \quad i.e., \quad 55.65\%$

$(iv)\ \eta_{cycle} = \frac{W_{net}}{Q_{Total}} = \frac{(1740 - 20.26)}{3924} = 0.4383 \quad i.e., \quad 43.8\%$

Feedwater Heaters (FWH)

A practical Regeneration process in steam power plants is accomplished by extracting or bleeding, steam from the turbine at various points. This steam, which could have produced more work by expanding further in the turbine, is used to heat the feed water instead. The device where the feedwater heated by regeneration is called a Regenerator or a Feedwater Heater (FWH).

A feedwater heater is basically a heat exchanger where heat is transferred from the steam to the feedwater either by mixing the two streams (open feedwater heaters) or without mixing them (closed feedwater heaters).

Open Feedwater Heaters

An open (or direct-contact) feedwater heater is basically a mixing chamber, where the steam extracted from the turbine mixes with the feedwater exiting the pump. Ideally, the mixture leaves the heater as a saturated liquid at the heater pressure.

The advantages of open heater are simplicity, lower cost, and high heat transfer capacity. The disadvantage is the necessity of a pump at each heater to handle the large feedwater stream.

Closed Feedwater Heaters

In closed feedwater heater, the heat is transferred from the extracted steam to the feedwater without mixing taking place. The feedwater flows through the tubes in the heater and extracted steam condenses on the outside of the tubes in the shell. The heat released from the condensation is transferred to the feedwater through the walls of the tubes. The condensate (saturated water at the steam extraction pressure), some times called the heater-drip, then passes through a trap into the next lower pressure heater. This, to some extent, reduces the steam required by that heater. The trap passes only liquid and no vapour. The drip from the lowest pressure heater could similarly be trapped to the condenser, but this would be throwing away energy to the condenser cooling water. The avoid this waste, the drip pump feed the drip directly into the feedwater stream.

A closed heaters system requires only a single pump for the main feedwater stream regardless of the number of heaters. The drip pump, if used is relatively small. Closed heaters are costly and may not give as high a feedwater temperature as do open heaters.

In most steam power plants, closed heaters are favoured, but atleast one open heater is used, primarily for the purpose of feedwater deaeration. The open heater in such a system is called deaerator.

Note: The higher the number of heater used, the higher will be the cycle efficiency. The number of heater is fixed up by the energy balance of the whole plant when it is found that the cost of adding another does not justify the saving in Q_H or the marginal increase in cycle efficiency. An increase in feedwater temperature may, in some cases, cause a reduction in boiler efficiency. So the number of heaters get optimized. Five feedwater heaters are often used in practice.

Characteristics of an Ideal working fluid

The maximum temperature that can be used in steam cycles consistent with the best available material is about 600^0C, while the critical temperature of steam is 375^0C, which necessitates large superheating and permits the addition of only an infinitesimal amount of heat at the highest temperature.

The desirable characteristics of the working fluid in a vapour power cycle to obtain best thermal η are as follows:

a) The fluid should have a high critical temperature so that the saturation pressure at the maximum permissible temperature (metallurgical limit) is relatively low. It should have a large enthalpy of evaporation at that pressure.
b) The saturation pressure at the temperature of heat rejection should be above atmosphere pressure so as to avoid the necessity of maintaining vacuum in the condenser.
c) The specific heat of liquid should be small so that little heat transfer is required to raise the liquid to the boiling point.
d) The saturation vapour line of the T-S diagram should be steep, very close to the turbine expansion process so that excessive moisture does not appear during expansion.
e) The freezing point of the fluid should be below room temperature, so that it does not get solidified while flowing through the pipe lines.
f) The fluid should be chemically stable and should not contaminate the materials of construction at any temperature.
g) The fluid should be nontoxic, non corrosive, not excessively viscous, and low in cost.

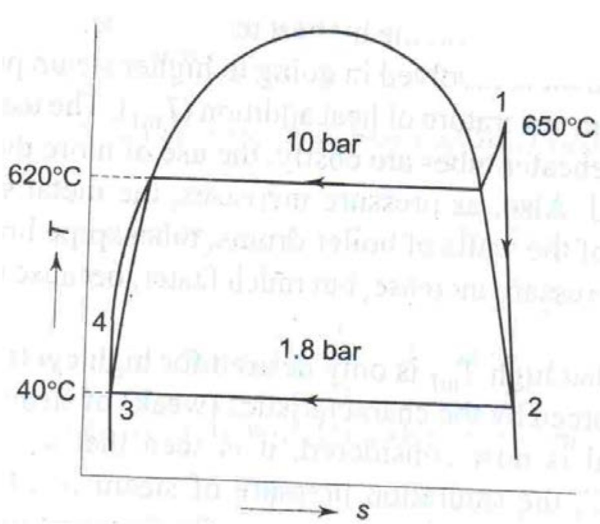
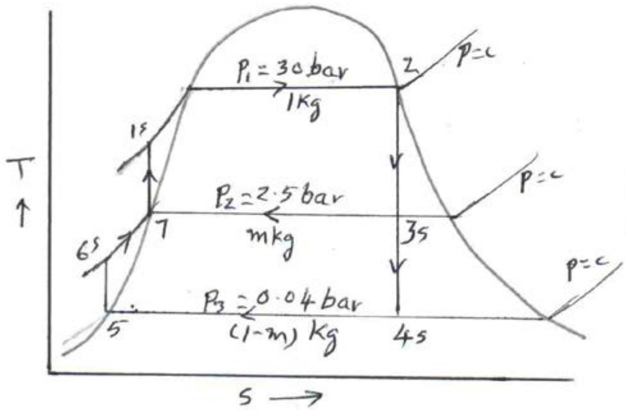


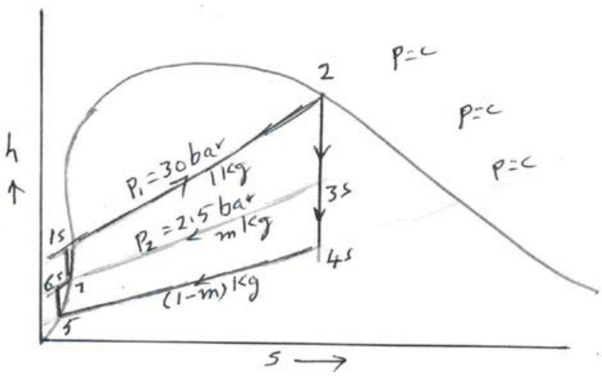
Fig. T-S diagram for an Ideal Working Fluid for a Vapour Power Cycle

Numerical Problems:

1. **An ideal regenerative cycle operates with dry saturated steam, the maximum and minimum pressures being 30 bar and 0.04 bar respectively. The plant is installed with a single mixing type feed water heater. The bled steam pressure is 2.5 bar. Determine (a) the mass of the bled steam, (b) the thermal η of the cycle, and (c) SSC in kg/kWh.**

Solution:





$P_1 = 30$ bar $\quad P_2 = 2.5$ bar $\quad P_3 = 0.04$ bar

From steam tables, For $P_1 = 30$ bar, $\quad h_2 = 2802.3$ kJ/kg, $\quad S_2 = 6.1838$ kJ/kg-^{0}k

But $S_2 = S_{3s}$ i.e., $6.1838 = 1.6072 + x_3 (5.4448)$

$\therefore x_3 = 0.841$

$\therefore h_3 = 535.4 + 0.841 (2281.0)$
$= 2452.68$ kJ/kg

Also $S_2 = S_{4s}$ i.e., $6.1838 = 0.4225 + x_4 (8.053)$

$\therefore x_4 = 0.715$

$\therefore h_4 = 121.4 + 0.715 (2433.1)$
$= 1862.1$ kJ/kg

At $P_3 = 0.04$ bar, $\quad h_5 = 121.4$ kJ/kg, $\quad v_5 = 0.001004$ m^3/kg

$\therefore$ Condensate pump work $= (h_6 - h_5) = v_5 (P_2 - P_3)$
$= 0.001004 (2.5 - 0.04) (10^5/10^3)$
$= 0.247$ kJ/kg

$\therefore h_6 = 0.247 + 121.4 = 121.65$ kJ/kg

Similarly, $h_1 = h_7 + v_7 (P_1 - P_2) (10^5/10^3)$
$= 535.4 + 0.0010676 (30 - 2.5) 10^2$
$= 538.34$ kJ/kg

a) **Mass of the bled steam:**

Applying the energy balance to the feed water heater

$mh_3 + (1 - m)\, h_6 = 1\, (h_7)$

$$\therefore m = \frac{(h_7 - h_6)}{(h_3 - h_6)} = \frac{(535.4 - 121.65)}{(2452.68 - 121.65)} = 0.177 kg/kg \text{ of steam}$$

b) **Thermal η:**

Turbine work, W_T $= 1\,(h_2 - h_{3s}) + (1 - m)\,(h_3 - h_{4s})$
$= 1\,(2802.3 - 2452.65) + (1 - 0.177)\,(2452.68 - 1862.1)$
$= 835.67$ kJ/kg

Pump work, W_P $= (1 - m)\,(h_{6s} - h_5) + 1\,(h_{1s} - h_7)$
$= (1 - 0.177)\,(121.65 - 121.4) + 1\,(538.34 - 535.4)$
$= 3.146$ kJ/kg

$\therefore W_{net} = W_T - W_P = 832.52$ kJ/kg

Heat supplied, Q_H $= 1\,(h_2 - h_{1s})$
$= 1\,(2802.3 - 538.34)$
$= 2263.96$ kJ/kg

$$\therefore \eta_{th} = \frac{W_{net}}{Q_H} = \frac{832.52}{2263.96} = 0.368 \text{ or } \mathbf{36.8\%}$$

c) **SSC:**

$$SSC = \frac{3600}{W_{net}} = 4.324 kg/kWh$$

2. In problem (3), also calculate the increase in mean temperature of heat addition, efficiency and steam rate as compared to the Rankine cycle (without regeneration)

Solution: Tm_1 (with regeneration) $= \dfrac{h_2 - h_1}{S_2 - S_1} = \dfrac{2263.96}{(6.1838 - 1.6072)} = 494.68k$

Tm_1 (without regeneration) $= \dfrac{h_2 - h_6}{S_2 - S_6} = \dfrac{2802.3 - 121.65}{(6.1838 - 0.4225)} = 465.29k$

$\therefore$ Increase in Tm_1 due to regeneration = 494.68 – 465.29 = $\mathbf{29.39^0K}$

W_T (without regeneration) = $h_2 - h_4$ = 2802.3 – 1862.1 = 940.2 kJ/kg

W_P (without regeneration) = $(h_1 - h_5) = v_5\,(30 - 0.04)\,10^2$
$= 0.001004\,(29.96)\,10^2 = 3.01$ kJ/kg

$\therefore h_1 = 3.01 + 121.4 = 124.41$ kJ/kg

$\therefore \eta_{th}$ (without regeneration) $= \dfrac{W_{net}}{Q_H} = \dfrac{(940.2 - 3.01)}{2802.3 - 124.41} = 0.349$

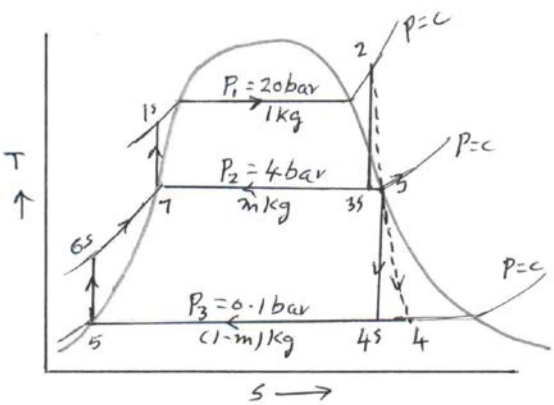
$\therefore$ Increase in η_{th} due to regeneration = 0.368 – 0.349 = 0.018   i.e., 1.8%

Steam rate (without regeneration) = 3.84 kg/kWh

$\therefore$ Increase in steam rate due to regeneration = 4.324 – 3.84
= 0.484 kg/kWh

3. Steam at 20 bar and 300^{0}C is supplied to a turbine in a cycle and is bled at 4 bar. The bled-steam just comes out saturated. This steam heats water in an open heater to its saturation state. The rest of the steam in the turbine expands to a condenser pressure of 0.1 bar. Assuming the turbine efficiency to be the same before and after bleeding, find: a) the turbine η and the steam quality at the exit of the last stage; b) the mass flow rate of bled steam 1kg of steam flow at the turbine inlet; c) power output / (kg/s) of steam flow; and d) overall cycle η.

Solution:



P_1 = 20 bar $t_1 = 300^0$C P_2 = 4 bar P_3 = 0.1 bar

From steam tables,

For P_1 = 20 bar and $t_1 = 300^0$C

$v_2 = 0.12550$ $h_2 = 3025.0$ $S_2 = 6.7696$

For P_2 = 4 bar, $h_3 = 2737.6$, $t_s = 143.63$

$h_f = 604.7$, $h_{fg} = 2132.9$, $S_f = 1.7764$, $S_{fg} = 5.1179$, $S_g = 6.8943$

For P_2 = 0.1 bar, 45.83, 191.8, 2392.9, 2584.8, 0.6493, 7.5018, 8.1511

We have, $S_2 = S_{3s}$ i.e., 6.7696 = 1.7764 + x_3 (5.1179)

$\therefore x_3 = 0.976$

$\therefore h_{3s}$ = 604.7 + 0.976 (2132.9) = 2685.63 kJ/kg

$$\therefore \eta_t = \frac{h_2 - h_3}{h_2 - h_{3s}} = \frac{3025 - 2737.6}{3025 - 2685.63} = 0.847$$

$S_3 = S_{4s}$ i.e., 6.8943 = 0.6493 + x_4 (7.5018)

$\therefore x_{4s} = 0.832$

$\therefore h_{4s}$ = 191.8 + 0.832 (2392.9) = 2183.81kJ/kg

But η_t is same before and after bleeding i.e., $\eta_t = \dfrac{h_3 - h_4}{h_3 - h_{4s}}$

i.e., $0.847 = \dfrac{2737.6 - h_4}{2737.6 - 2183.81}$

$\therefore h_4 = 2268.54$ kJ/kg

$\therefore h_4 = h_{f4} + x_4 h_{fg4}$ $\quad\therefore x_4 = 0.868$

b) Applying energy balance to open heater, $mh_3 + (1 - m) h_{6s} = 1 (h_7)$

$$\therefore m = \frac{h_7 - h_6}{h_3 - h_6}$$

Condensate pump work, $(h_{6s} - h_5) = v_5 (P_3 - P_2)$

$= 0.0010102 (3.9) 10^2 = 0.394$ kJ/kg

$\therefore h_{6s} = 191.8 + 0.394 = 192.19$ kJ/kg

Similarly, $h_{1s} = h_7 + v_7 (P_1 - P_2)$

$= 604.7 + -.0010839 (16) 10^2 = 606.43$ kJ/kg

$$\therefore m = \frac{604.7 - 192.19}{2737.6 - 192.19} = 0.162$$

c) Power output or $W_T = (h_2 - h_3) + (1 - m) (h_3 - h_4)$

$= (3025 - 2737.6) + (1 - 0.162) (2737.6 - 2268.54)$

$= 680.44$ kJ/kg

For 1kg/s of steam, $\quad W_T = 680.44$ kW

d) Overall thermal efficiency, $\eta_0 = \dfrac{W_{net}}{Q_H}$

$W_P = (1 - m) (h_{6s} - h_5) + 1 (h_{1s} - h_7)$

$= (1 - 0162) (192.19 - 191.8) + 1 (606.43 - 604.7)$

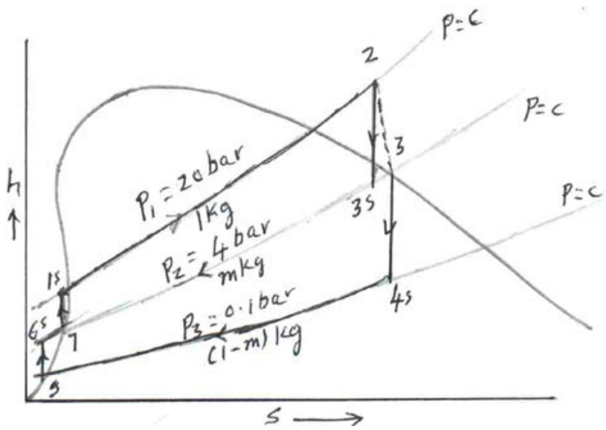
$= 2.057$ kJ/kg

$W_{net} = 680.44 - 2.057 = 678.38$ kJ/kg

$Q_H = 1 (h_2 - h_{1s}) = (3025 - 606.43) = 2418.57$ kJ/kg

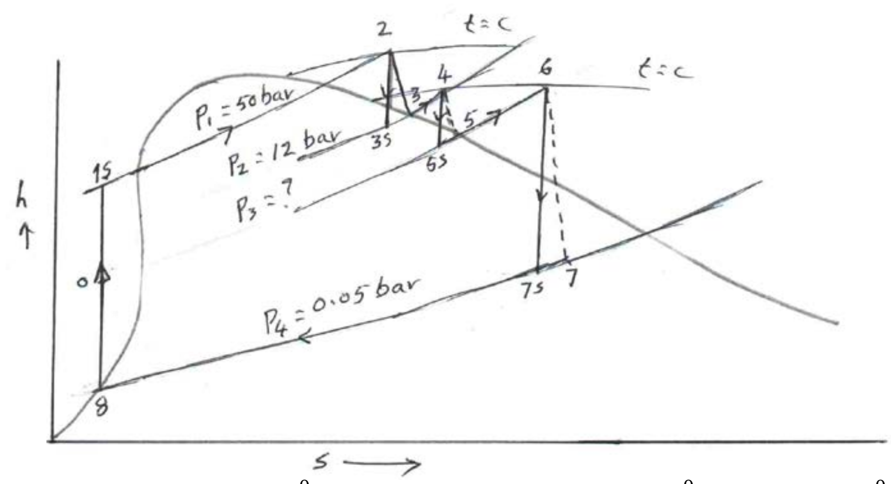
$$\therefore \eta_0 = \frac{678.38}{2418.57} = 0.2805$$

<u>Using Moiller Diagram</u>



4. **Steam at 50 bar, 350^{0}C expands to 12 bar in a HP stage, and is dry saturated at the stage exit. This is now reheated to 280^{0}C without any pressure drop. The reheat steam expands in an intermediate stage and again emerges dry and saturated at a low pressure, to be reheated a second time to 280^{0}C. Finally, the steam expands in a LP stage to 0.05 bar. Assuming the work output is the same for the high and intermediate stages, and the efficiencies of the high and low pressure stages are equal, find: (a) η of the HP stage (b) Pressure of steam at the exit of the intermediate stage, (c) Total power output from the three stages for a flow of 1kg/s of steam, (d) Condition of steam at exit of LP stage and (e) Then η of the reheat cycle. Also calculate the thermodynamic mean temperature of energy addition for the cycle.**

Solution:



$P_1 = 50$ bar $\quad t_2 = 350^0$C $\quad P_2 = 12$ bar $\quad t_4 = 280^0$C, $\quad t_6 = 280^0$C

$P_3 = ?$ $\quad P_4 = 0.05$ bar

From Mollier diagram

$h_2 = 3070$kJ/kg $\quad h_{3s} = 2755$ kJ/kg $\quad h_3 = 2780$ kJ/kg $\quad h_4 = 3008$ kJ/kg

(a) η_t for HP stage $= \dfrac{h_2 - h_3}{h_2 - h_{3s}} = \dfrac{3070 - 2780}{3070 - 2755}$

$= 0.921$

(b) Since the power output in the intermediate stage equals that of the HP stage, we have

$h_2 - h_3 = h_4 - h_5$

i.e., $3070 - 2780 = 3008 - h_5$

$\therefore h_5 = 2718$ kJ/kg

Since state 5 is on the saturation line, we find from Mollier chart, $P_3 = 2.6$ bar,
Also from Mollier chart, $h_{5s} = 2708$ kJ/kg, $h_6 = 3038$ kJ/kg, $h_{7s} = 2368$ kJ/kg

Since η_t is same for HP and LP stages,

$$\eta_t = \frac{h_6 - h_7}{h_6 - h_{7s}} = 0.921 = \frac{3038 - h_7}{3038 - 2368} \quad \therefore h_7 = 2420.93\text{kJ/kg}$$

$\therefore$ At a pressure 0.05 bar, $h_7 = h_{f7} + x_7 h_{fg7}$

$2420.93 = 137.8 + x_7 (2423.8)$

$\therefore x_7 = 0.941$

Total power output $= (h_2 - h_3) + (h_4 - h_5) + (h_6 - h_7)$
$= (3070 - 2780) + (3008 - 2718) + (3038 - 2420.93)$
$= 1197.07$ kJ/kg

$\therefore$ Total power output /kg of steam = 1197.07 kW

For $P_4 = 0.05$ bar from steam tables, $h_8 = 137.8$ kJ/kg;

$W_P = 0.0010052\ (50 - 0.05)\ 10^2 = 5.021$ kJ/kg
$= h_8 - h_{1s}$

$\therefore h_{1s} = 142.82$ kJ/kg

Heat supplied, $Q_H = (h_2 - h_{1s}) + (h_4 - h_3) + (h_6 - h_5)$
$= (3070 - 142.82) + (3008 - 2780) + (3038 - 2718)$
$= 3475.18$ kJ/kg

$W_{net} = W_T - W_P = 1197.07 - 5.021 = 1192.05$ kJ/kg

$$\therefore \eta_{th} = \frac{W_{net}}{Q_H} = \frac{1192.05}{3475.18} = 0.343$$

$$\eta_{th} = 1 - \frac{T_o}{T_m} = 1 - \frac{(273 + 32.9)}{T_m} = 0.343,$$

$$0.657 = \frac{305.9}{T_m}$$

$\therefore T_m = 465.6$ K

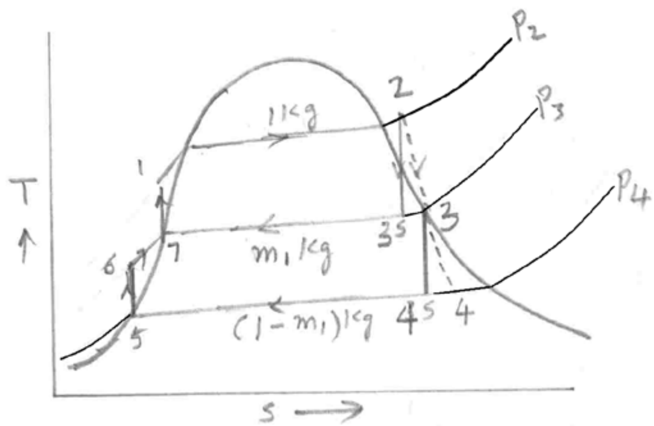
Or

$$T_m = \frac{h_2 - h_{1s}}{S_2 - S_{1s}} = \frac{3070 - 142.82}{6.425 - 0.4763} = 492K$$

$$SSC = \frac{3600}{1192.05} = 3.02 kg / kWh$$

5. Steam at 30 bar and 350^{0}C is supplied to a steam turbine in a practical regenerative cycle and the steam is bled at 4 bar. The bled steam comes out as dry saturated steam and heats the feed water in an open feed water heater to its saturated liquid state. The rest of the steam in the turbine expands to condenser pressure of 0.1 bar. Assuming the turbine η to be same before and after bleeding determine (i) the turbine η, (ii) steam quality at inlet to condenser, (iii) mass flow rate of bled steam per unit mass rate at turbine inlet and (iv) the cycle η.

Solution:



P_2 = 30 bar t_2 = 350^{0}C P_3 = 4 bar P_4 = 0.1 bar

$h_3 = h_g$ at P_3 = 4 bar, = 2737.6 kJ/kg

From superheated steam tables,

$$h_2 = h_3 = h_g\Big|_{P_3=4bar} = 2737.6\ kJ/kg$$

$$h_2 = h\Big|_{P_2=30bar\ \&\ t_2=350^0C} = 3117.5\ kJ/kg \text{ and } \quad S_2 = 6.7471 \text{ kJ/kg-K}$$

$$h_5 = h_f\Big|_{P_5=0.1bar} = 191.8\ kJ/kg$$

$$h_7 = h_f\Big|_{P_7=4bar} = 604.7\ kJ/kg$$

Process 2-3s is isentropic, i.e., $S_2 = S_{3S}$

$$6.7471 = 1.7764 + x_{3S}\,(5.1179)$$

$\therefore x_{3S} = 0.971$

$\therefore h_{3S} = h_{f3} + x_{3S}\, h_{fg3}$

$= 604.7 + 0.971\ (2132.9)$

$= 2676.25$ kJ/kg

Process 3-4s is isentropic i.e., $S_3 = S_{4S}$

i.e., $6.8943 = 0.6493 + x_{4S}\ (7.5018)$

$\therefore x_{4S} = 0.832$

$\therefore h_{4S} = 191.8 + 0.832\ (2392.9) = 2183.8$ kJ/kg

Given, η_t (before bleeding) = η_t (after bleeding)

We have, η_t (before bleeding) $= \dfrac{h_2 - h_3}{h_2 - h_{3S}} = \dfrac{3117.5 - 2737.6}{3117.5 - 2676.25} = 0.86$

$$\therefore 0.86 = \frac{h_3 - h_4}{h_3 - h_{4S}} = \frac{2737.6 - h_4}{2737.6 - 2183.8} \quad \therefore h_4 = 2261.33 kJ/kg$$

But $h_4 = h_{f4} + x_4\, h_{fg4}$

$2261.33 = 191.8 + x_4\ (2392.9)$

$\therefore x_4 = 0.865$

i.e., Dryness fraction at entry to condenser = x_4 = **0.865**

iii) Let m kg of steam is bled. Applying energy balance to FWH,

$mh_3 + (1 - m)\, h_6 = h_7$

We have $W_{P1} = (h_6 - h_5)$ $= v\int dP$

$= 0.0010102\ (4 - 0.1)\ 10^5/10^3$

$= 0.394$ kJ/kg

$\therefore h_6 = 0.394 + 191.8 = 192.19$ kJ/kg

Substituting,

$$m(2737.6) + (1 - m)\,192.19 = 604.7$$

$$\therefore m = 0.162 \text{ kg}$$

Also, $W_{P2} = (h_1 - h_7) = v\int dP$

$$= 0.0010839 \times (30 - 4)\,10^2$$

$$= 2.82 \text{ kJ/kg}$$

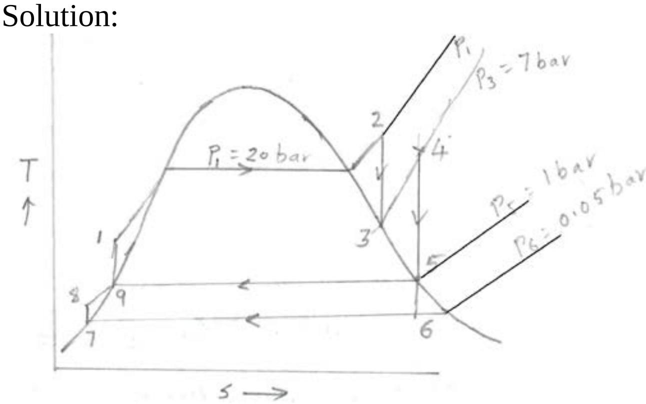
$$\therefore h_1 = 2.82 + 604.7 = 607.52 \text{ kJ/kg}$$

$$\therefore \eta_{cycle} = \frac{W_T = W_P}{Q_H} = \frac{\left[(h_2 - h_3) + (1-m)(h_3 - h_4)\right] - \left[(1-m)(h_6 - h_5) + (h_1 - h_2)\right]}{(h_2 - h_1)}$$

η_{cycle} = 0.31

6. **In an ideal reheat regenerative cycle, the high pressure turbine receives steam at 20 bar, 300^{0}C. After expansion to 7 bar, the steam is reheated to 300^{0}C and expands in an intermediate pressure turbine to 1 bar. A fraction of steam is now extracted for feed water heating in an open type FWH. The remaining steam expands in a low pressure turbine to a final pressure of 0.05 bar. Determine (i) cycle thermal η, (ii) specific steam consumption, (iii) quality of steam entering condenser.**

Solution:



$$h_2 = h\Big|_{20\,bar,\,300^0 C} = 3025\,kJ/kg \text{ and } s_2 = 6.7696 \text{ kJ/kg-K}$$

Process 2-3 is isentropic

i.e., $S_2 = S_3$

$6.7696 = 1.9918 + x_3\,(4.7134)$

$\therefore x_3 = 1.014$

i.e., state 3 can be approximated as dry saturated.

$\therefore h_3 = h\Big|_{7\,bar,\,dry\,sat.} = 2762$kJ/kg

$\therefore h_4 = h\Big|_{7\,bar,\,300^0 C} = 3059.8kJ/kg$ and s_4 = 7.2997 kJ/kg-K

Process 4-5 is isentropic i.e., $S_4 = S_5$

$7.2997 = 1.3027 + x_5\,(6.0571)$

$\therefore x_5 = 0.99$

$\therefore h_5 = h_{f5} + x_5\, h_{fg5} = 417.5 + 0.99\,(2257.9) = 2652.9$ kJ/kg

Process 5-6 is isentropic i.e., $S_5 = S_6$

$7.2997 = 0.4763 + x_6\,(7.9197)$

$\therefore x_6 = 0.862$

$\therefore h_6 = 137.8 + 0.862\,(2423.8) = 2226.1$ kJ/kg

$h_7 = h_7 = h_f\Big|_{0.05\,bar} = 137.8$ kJ/kg

Neglecting W_{P1}, $h_8 = h_7$, Also neglecting W_{P2}, $h_9 = h_1$

$\therefore h_9 = h_f\Big|_{1\,bar} = 417.5$ kJ/kg

Applying energy balance to FWH

$mh_5 + (1 - m)\,h_8 = h_9$

i.e., m (2652.9) + (1 – m) 137.8 = 417.5 $\therefore$ m = 0.111 kg/kg of steam

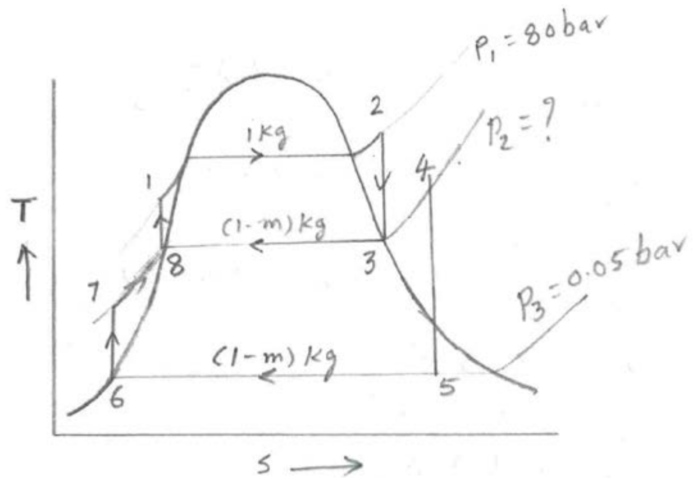
$$(i) \quad \eta_C = \frac{(h_2 - h_3) + (h_4 - h_5) + (1 - m)(h_5 - h_6)}{(h_2 - h_1) + (h_4 - h_3)} = 0.35$$

$$(ii) \quad SSC = \frac{3600}{W_{net}} = 3.57kg/kWh$$

(iii) Quality of steam entering condenser, $x_6 = 0.862$

7. **The net power output of a regenerative – reheat cycle power plant is 80mW. Steam enters the high pressure turbine at 80 bar, 500^{0}C and expands to a pressure P_2 and emerges as dry vapour. Some of the steam goes to an open feed water heater and the balance is reheated at 400^{0}C at constant pressure P_2 and then expanded in the low pressure turbine to 0.05 bar. Determine (i) the reheat pressure P_2, (ii) the mass of bled steam per kg boiler steam, (iii) the steam flow rate in HP turbine, (iv) cycle η. Neglect pump work. Sketch the relevant lines on h-s diagram. Assume expansion in the turbines as isentropic.**

Solution:



P = 80000 kW P_1 = 80 bar t_2 = 500^{0}C P_2 = ? t_3 = 400^{0}C

P_3 = 0.05 bar m = ? $\dot{m}_S = ?$ η_{cycle} = ?

$h_2 = h\big|_{80\,bar,\,500^0C} = 3398.8 kJ/kg$ and s_2 = 6.7262

Process 2-3 is isentropic i.e., $S_2 = S_3$ = 6.7262 kJ/kg-K

Given state 3 is dry saturated i.e., $S_3 = 6.7262 = S_g\big|_{P_2}$

From table A – 1, for dry saturated steam, at P = 6.0 bar, S_g = 6.7575 and at P = 7.0 bar, $S_g = 6.7052$

Using linear interpolation,

$$\Delta P = \frac{6.0 - 7.0}{(6.7575 - 6.7052)} x(6.7262 - 6.7052) = 0.402\ bar$$

$\therefore$ (i) $P_2 = 6 + 0.402 = 6.402$ bar

$$\therefore h_3 = h\Big|_{P_2 = 6.4 bar}$$

From table A – 1, For P = 6 bar $h_g = 2755.5$ $S_g = 6.7575$

For P = 7 bar, $h_g = 2762.0$ $S_g = 6.7052$

$$\therefore For\ P = 6.4\ bar \Rightarrow \frac{2762 - 2755.5}{1} x(0.4) + 2755.5 = 2758.1 kJ/kg$$

$\therefore h_3 = 2758.1$ kJ/kg

$$h_4 = h\Big|_{6.4 bar, 400^0 C}$$

From superheated steam tables, For P = 6.0 bar, h = 3270.6 s = 7.709

P = 7.0 bar, h = 3269.0 s = 7.6362

$\therefore$ For 6.4 bar, $h_4 \Rightarrow 3269.96$ kJ/kg

$S_4 \Rightarrow 7.6798$ kJ/kg-K

Process 4-5 is isentropic, $S_4 = S_5$

i.e., $7.6798 = 0.4763 + x_5 (7.9197)$

$\therefore x_5 = 0.909$

$\therefore h_5 = 137.8 + 0.909 (2423.8) = 2342.41$ kJ/kg

$$h_6 = h_f\Big|_{0.05 bar} = 137.8 kJ/kg$$

$h_7 = h_6$ (since W_{P1} is neglected)

$$h_8 = h_f\Big|_{6.4 bar} = 681.1 kJ/kg$$

$h_1 = h_8$ (since W_{P2} is neglected)

(ii) Applying energy balance to FWH,

$mh_3 + (1 - m)\, h_7 = h_8$

$m\,(2758.1) + (1 - m)\, 137.8 = 681.1$

$\therefore m = 0.313$ kg/kg of steam

(iii) $W_1 = W_{HP} = (h_2 - h_3) \quad = (3398.8 - 2758.1)$

$= 640.7$ kJ/kg

$W_2 = W_{LP} \quad = (1 - m)\,(h_4 - h_5)$

$= (1 - 0.313)\,(3269.96 - 2342.41)$

$= 637.2$ kJ/kg

$\therefore W_{net} = W_1 + W_2 = 1277.9$ kJ/kg

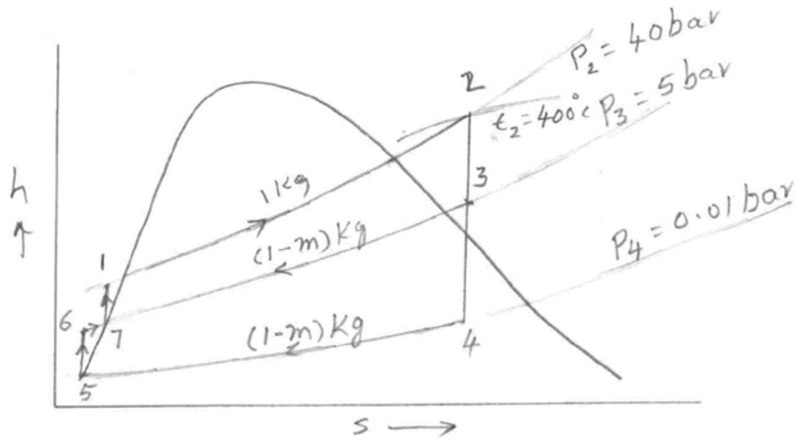
$\therefore$ Steam flow rate through HP turbine $= \dfrac{Power}{W_{net}} = \dfrac{80000}{1277.9} = 62.6\,kg/s$

(iv) $\eta_{cycle} = ?\quad Q_H = (h_2 - h_1) + (1 - m)\,(h_4 - h_3) = 3069.35$ kJ/kg

$$\therefore \eta_{cycle} = \frac{W_{net}}{Q_H} = \frac{1277.9}{3069.35} = 0.42$$

8. In a single heater regenerative cycle, the steam enters the turbine at 30 bar, 400^0C and the exhaust pressure is 0.01 bar. The feed water heater is a direct contact type which operates at 5 bar. Find (i) thermal η and the steam rate of the cycle, (ii) the increase in mean temperature of heat addition, η and steam rate as compared to the Rankine cycle without regeneration. Pump work may neglected.

Solution:



P_2 = 40 bar $t_2 = 400^0$C P_4 = 0.01 bar P_3 = 5 bar

From h-s diagram,

$$h_2 = h\big|_{30\,bar,\,400^0 C} = 3230 kJ/kg$$

h_3 = 2790 kJ/kg

h_4 = 1930 kJ/kg

h_5 = 29.3 kJ/kg

h_7 = 640.1 kJ/kg

Since pump work may neglect, $h_6 = h_5$ & $h_1 = h_7$

(i) η_{cycle} = ?

Let m = mass of steam bled per kg boiler steam

Applying SFEE to FWH,

$mh_3 + (1 – m) h_6 = h_7$

m (2790) + (1 – m) 29.3 = 640.1

$\therefore$ m = 0.221 kg/kg of boiler steam

W_T = $(h_2 – h_3) + (1 – m) (h_3 – h_4)$

= (3230 – 2790) + (1 – 0.221) (2790 – 1930)

= 1109.73 kJ/kg

$Q_H = (h_2 - h_1) = (3230 - 640.1)$

$= 2589.9$ kJ/kg

$$\therefore \eta_{cycle} = \frac{W_T}{Q_H} = 0.428 \qquad \text{Since } W_P \text{ is neglected}$$

$$(ii)\ steam\ rate = \frac{3600}{W_T} = 3.24 kg/kWh$$

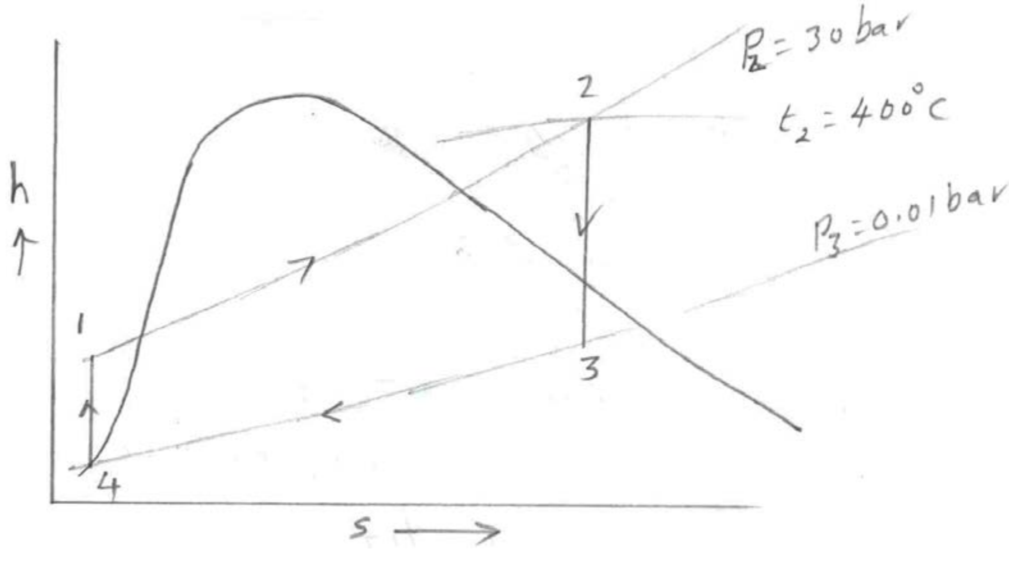
(iii) Mean temperature of heat addition, $\Delta T_m = \dfrac{Q_H}{s_2 - s_5}$

From h-s diagram, $s_2 = 6.83$ kJ/kg-K

From steam tables, $s_5 = 0.1060$ kJ/kg-K

$$\therefore \Delta T_m = \frac{2589.9}{(6.83 - 0.106)} = 385.2^0 K$$

Case (ii) Rankine cycle without Regeneration:



From h-s diagram,

$h_2 = 3230$ kJ/kg

$h_3 = 1930$ kJ/kg

$h_4 = 29.3$ kJ/kg

$h_1 = h_4$

$S_2 = 6.83$ kJ/kg-K

$S_4 = 0.1060$ kJ/kg-K

$$(i) \quad \eta_{cycle} = \frac{W_T}{Q_H} = \frac{(h_2 - h_3)}{(h_2 - h_1)}$$

$$= \frac{1300}{3200.7} = 0.41$$

$$(ii) \quad Steam\ rate = \frac{3600}{W_T} = 2.76 kg / kWh$$

(iii) Mean temperature of heat addition, $\Delta T_m = \dfrac{3200.7}{(6.83 - 0.106)} = 476^0 K$

Comparison	ΔT_m	η_{cycle}	Steam rate
Rankine cycle with regeneration	385.2 K	0.428	3.24 kg/kWh
Rankine cycle without regeneration	476^0K	0.41	2.76kg/kWh
$\therefore$ Increase w.r.t Rankine cycle	- 0.19 i.e., (-19%)	0.044 (4.4%)	0.174 (17.4%)