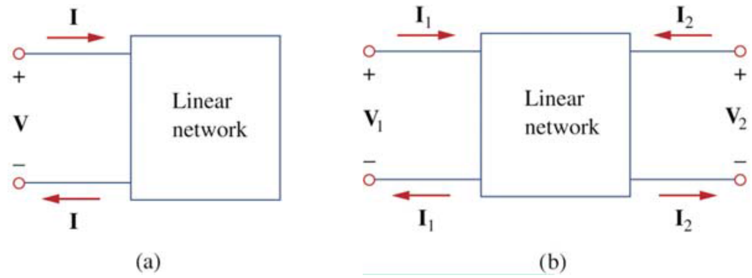
5.1 Introduction

• A two-port network is an electrical network with two separate ports for input and output.

Fig(a) –Single Port Network

Fig(b) –Two Port Network



There are several reasons why we should study two-ports and the parameters that describe them.

Most of the circuits which we come across have two ports. Usually an input signal is connected in one port and an output signal is obtained from the other port.

The parameters of a two-port network completely describes its behaviour in terms of the voltage and current at each port. Thus knowing the parameters of a two port network permits us to describe its operation when it is connected into a larger network.

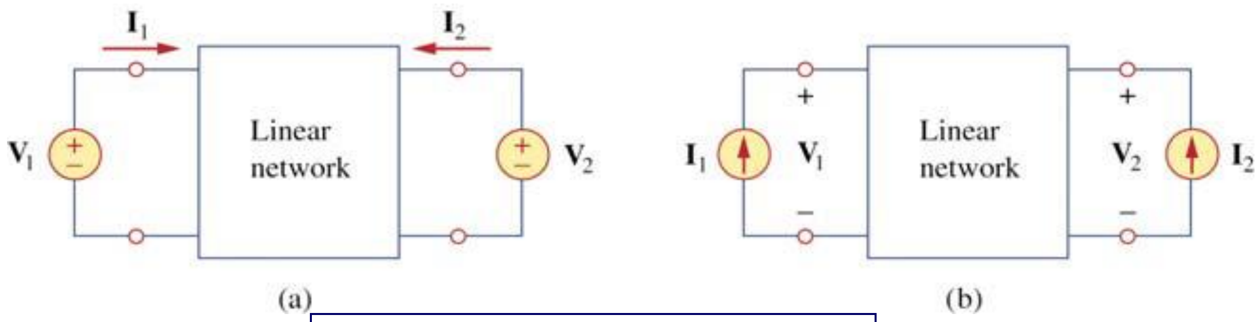
- **Two-port networks are also important in modeling electronic devices and system components.**
- **In electronics, two-port networks are employed to model transistors and op-amps.,**
- **Electrical circuits are modeled by two-ports are transformers and transmission lines.**

Four popular types of two-ports parameters are examined:

- **Impedance**
- **Admittance**
- **Hybrid**
- **Transmission.**

In the later part of discussion, usefulness of each set of parameters, demonstration of how they are related to each other and in the end how two-port networks can be interconnected (parallel, series, series-parallel and cascade).

Impedence parameters:



$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Impedance Parameters, z

$$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \quad \mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

$$\mathbf{z}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \quad \mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

$\mathbf{z}_{11}$ = Open-circuit input impedance

$\mathbf{z}_{12}$ = Open-circuit transfer impedance from port 1 to port 2

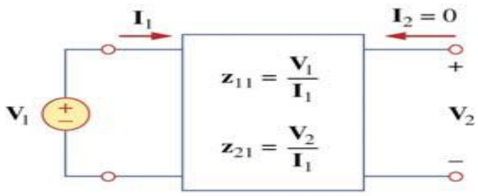
$\mathbf{z}_{21}$ = Open-circuit transfer impedance from port 2 to port 1

$\mathbf{z}_{22}$ = Open-circuit output impedance

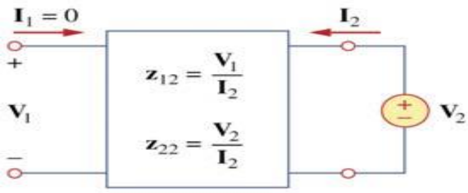
Impedance Parameters, z

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1}, \mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1}$$

$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2}, \mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2}$$

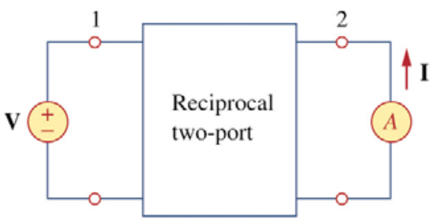


(a)

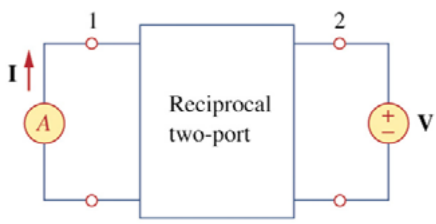


(b)

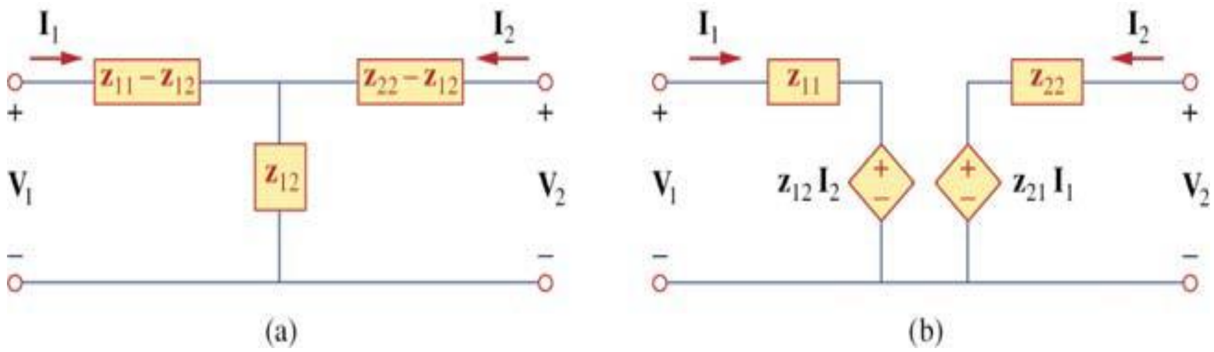
Impedance Parameters, z



(a)



(b)



Example

Determine the z parameters for the circuit in Fig.

Solution:.

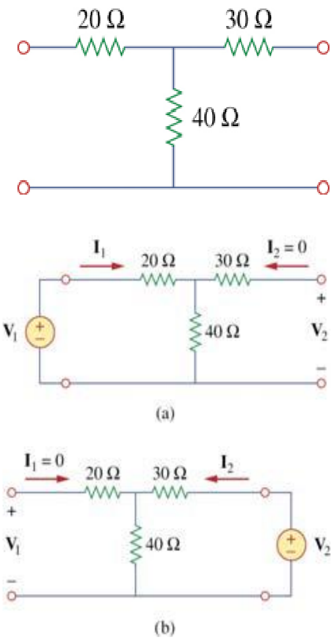
$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{(20+40)\,I_1}{I_1} = 60\,\Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{40\,I_1}{I_1} = 40\,\Omega$$

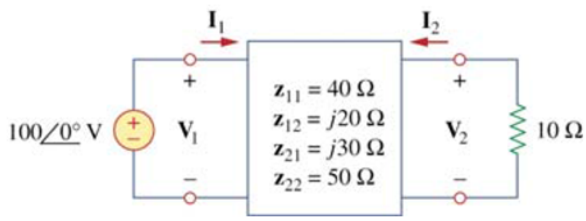
$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{40\,I_2}{I_2} = 40\,\Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{(30+40)\,I_2}{I_2} = 70\,\Omega$$

$$\text{Thus } [z]=\begin{bmatrix} 60\Omega & 40\Omega \\ 40\Omega & 70\Omega \end{bmatrix}$$



- Find I_1 and I_2 in the circuit in Fig.
- Solution:



$$V_1 = 40I_1 + j20I_2$$

$$V_2 = j30I_1 + 50I_2$$

$$\text{Since } V_1 = 100\angle 0^\circ, \quad V_2 = -10I_2$$

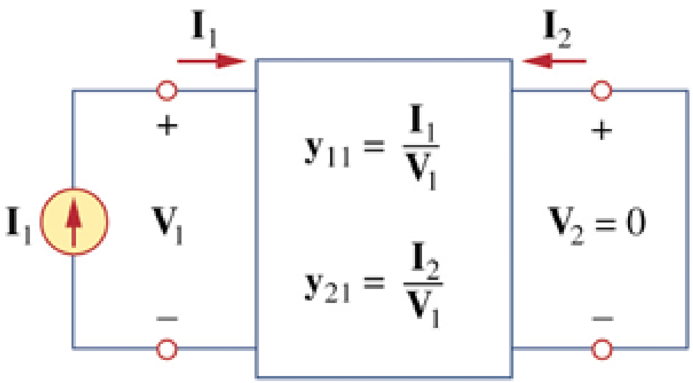
$$\Rightarrow 100 = 40I_1 + j20I_2$$

$$\Rightarrow -10I_2 = j30I_1 + 50I_2 \Rightarrow I_1 = j2I_2$$

$$\Rightarrow 100 = j80I_2 + k20I_2 \Rightarrow I_2 = \frac{100}{j100} = -j$$

$$\text{Since } I_1 = j2(-j) = 2, \text{ thus } I_1 = 2\angle 0^0\ \text{A}, \quad I_2 = 1\angle -90^\circ\ \text{A}$$

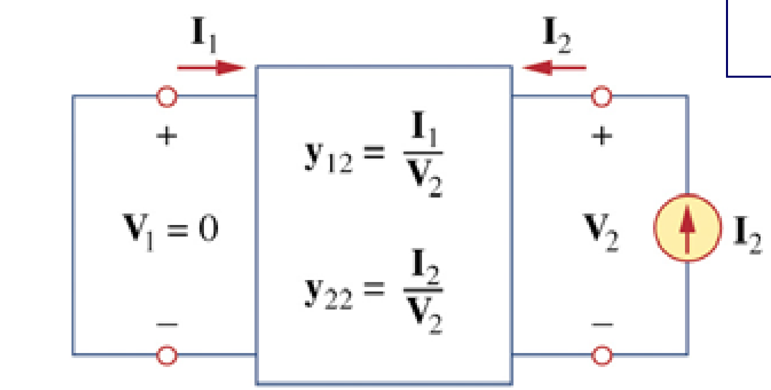
5.3 Admittance Parameters, y



(a)

$I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$



(b)

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Admittance Parameters Y

$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \quad \mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

$$\mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \quad \mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

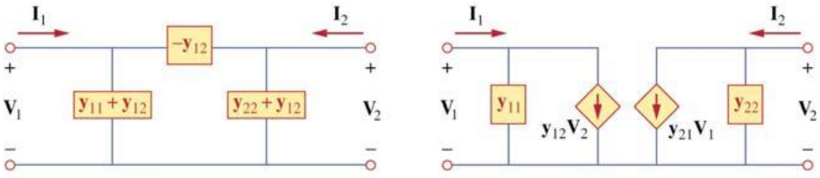
$\mathbf{y}_{11}$ = Short-circuit input admittance

$\mathbf{y}_{12}$ = Short-circuit transfer admittance from port 1 to port 2

y$_{21}$ = Short-circuit transfer admittance from port 2 to port 1

y$_{22}$ = Short-circuit output admittance

Fig.



(a) (b)

$$y_{11} = \frac{V_1}{I_1}, \quad y_{21} = \frac{V_2}{I_1}$$

$$y_{12} = \frac{V_1}{I_2}, \quad y_{22} = \frac{V_2}{I_2}$$

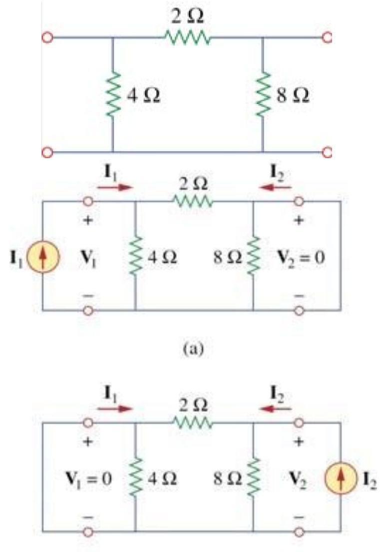
Obtain the y parameters for the Π network shown in Fig.

$$V_1 = I_1(4\|2) = \tfrac{4}{3}I_1 \Rightarrow y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{I_1}{\tfrac{4}{3}I_1} = 0.75S$$

$$-I_2 = \frac{4}{4+2}\,I_1 = \tfrac{2}{3}I_1 \Rightarrow y_{12} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-\tfrac{2}{3}I_1}{\tfrac{4}{3}I_1} = 0.5S$$

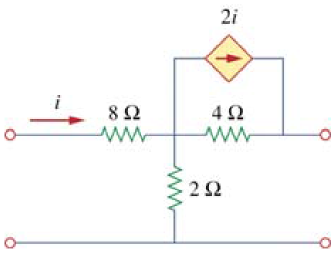
$$V_2 = I_2(8\|2) = \tfrac{8}{5}I_2 \Rightarrow y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{I_2}{\tfrac{8}{5}I_2} = 0.625S$$

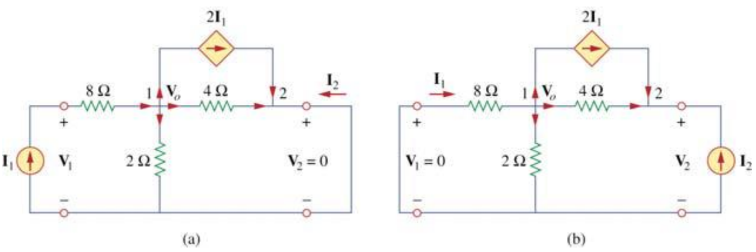
$$-I_1 = \frac{8}{8+2}\,I_2 = \tfrac{4}{5}I_2 \Rightarrow y_{21} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{-\tfrac{4}{5}I_1}{\tfrac{8}{5}I_1} = -0.5S$$



(b)

Determine the *y* parameters for the *T* network shown in Fig.





Solution :

At node 1, $\frac{V_1-V_0}{8} = 2I_1 + \frac{V_0}{2} + \frac{V_0-0}{4}$

But $I_1 = \frac{V_1-V_0}{8} \rightarrow 0 = \frac{V_1-V_0}{8} + \frac{3V_0}{4}$

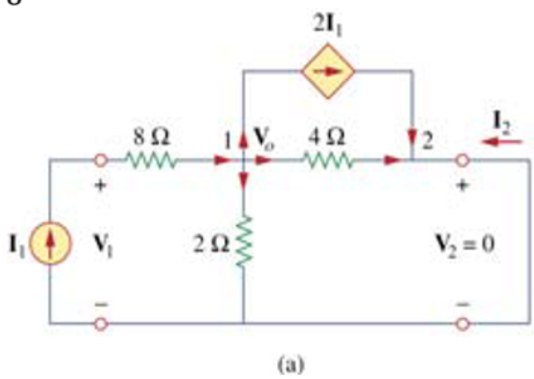
$0 = V_1 - V_0 = 6V_0 \rightarrow V_1 = -5V_0 \rightarrow I_1 = \frac{-5V_0 - V_0}{8} = -0.75V_0$

$\Rightarrow y_{11} = \frac{I1}{V1} = \frac{-0.75V_0}{-V_0} = 0.15S$

At node 2, $\frac{V0-0}{4} + 2I_1 + I_2 = 0$

$\rightarrow -I_2 = 0.25V_0 - 1.5V_0 = -1.25V_0$

$\Rightarrow y_{21} = \frac{I_2}{V_1} = \frac{1.25V_0}{-5V_0} = -0.25S$



At node 1, $\frac{0-V_0}{8} = 2I_1 + \frac{V_0}{2} + \frac{V_0-0}{4}$

But $I_1 = \frac{0-V_0}{8} \rightarrow 0 = -\frac{V_0}{8} + \frac{V_0}{2} + \frac{V_0-V_2}{4}$

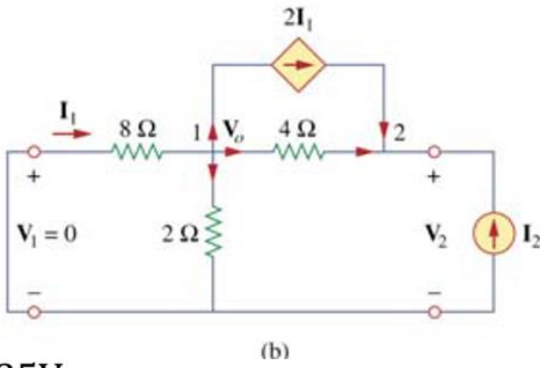
$\rightarrow 0 = -V_0 + 4V_0 + 2V_0 - 2V_2 \rightarrow V_2 = 2.5V_0$

$\Rightarrow y_{12} = \frac{I_1}{V_2} = \frac{-V_0/8}{2.5\,V_0} = -0.05S$

At node 2, $\frac{V_0-V_2}{4} + 2I_1 + I_2 = 0$

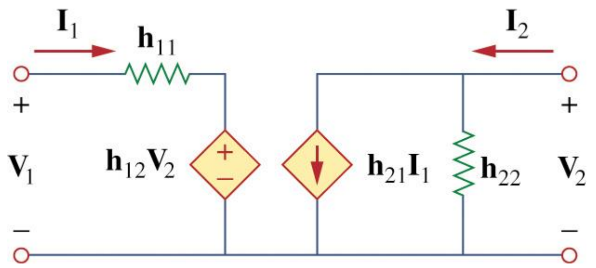
$-I_2 = 0.25V_0 - \frac{1}{4}(2.5)V_0 - \frac{2V_0}{8} = -0.625V_0$

$\Rightarrow y_{22} = \frac{I_2}{V_2} = \frac{0.625V_0}{2.5\,V_0} = 0.25$



Notice that y12 ≠y21 in this case, since the network isn't reciprocal.

5.4 Hybrid Parameters, h



$$V_1 = h_{11} I_1 + h_{12} I_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$\begin{bmatrix} \mathrm{V}_1 \\ \mathrm{I}_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [h] \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Hybrid Parameters, h

$$\mathbf{h}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \quad \mathbf{h}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$

$$\mathbf{h}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \quad \mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$

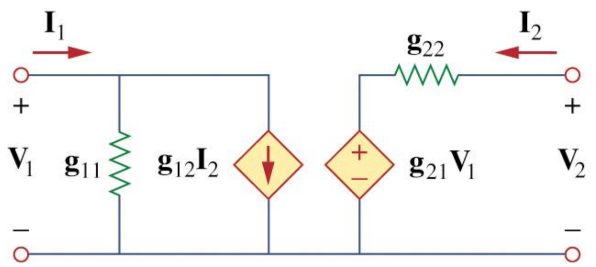
$\mathbf{h}_{11}$= Short-circuit input impedance

$\mathbf{h}_{12}$ = Open-circuit reverse voltage gain

$\mathbf{h}_{21}$ = Short-circuit forward current gain

$\mathbf{h}_{22}$ = Open-circuit output admittance

Inverse Hybrid Parameters, g



$$I_1 = g_{11} V_1 + g_{12} I_2$$

$$V_2 = g_{21} V_1 + g_{22} I_2$$

$$\begin{bmatrix} \mathrm{I}_1 \\ \mathrm{V}_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = [g] \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

Inverse Hybrid Parameters, g

$$\mathbf{g}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{g}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_1=0}$$

$$\mathbf{g}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{V}_1}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{g}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{V}_1=0}$$

$\mathbf{g}_{11}$ = Open-circuit input impedance

$\mathbf{g}_{12}$ = Short-circuit reverse voltage gain

$\mathbf{g}_{21}$ = Open-circuit forward current gain

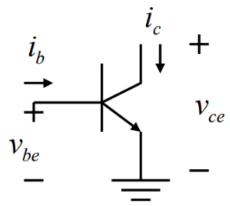
$\mathbf{g}_{22}$ = Short-circuit output admittance

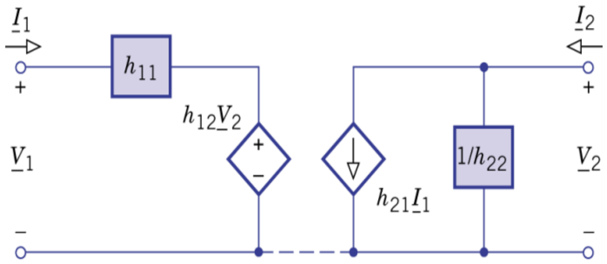
H-Parameters applied to Common Emitter Transistor Configuration

$$\text{sources}: i_1, v_2, \text{responses}: v_1, i_2$$

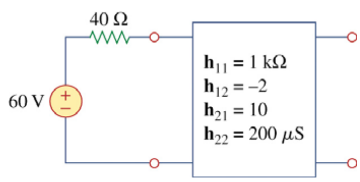
$$\begin{cases} v_1 = h_{11} i_1 + h_{12} v_2 \\ i_2 = h_{21} i_1 + h_{22} v_2 \end{cases} \rightarrow \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

$$CE \rightarrow \begin{bmatrix} v_{be} \\ i_c \end{bmatrix} = \begin{bmatrix} h_{ie} & h_{re} \\ h_{fe} & h_{oe} \end{bmatrix} \begin{bmatrix} i_b \\ v_{ce} \end{bmatrix}$$





• Determine the Thevenin equivalent at the output port of the circuit in Fig.



Solution: To find Z_{TH} and V_{TH}

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\,\mathbf{V}_2$$

$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$$

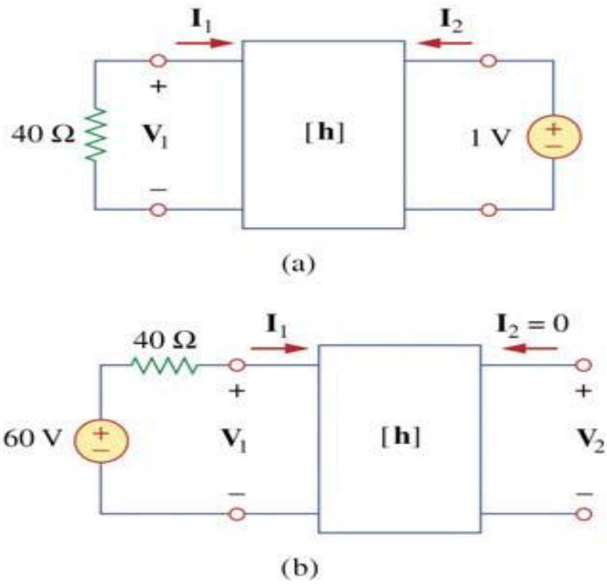
But $\mathbf{V}_2 = 1$, and $\mathbf{V}_1 = -40\mathrm{I}_1$

we get

$$-40\mathbf{I}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}$$

$$\Longrightarrow \mathbf{I}_1 = \frac{\mathbf{h}_{12}}{40+\mathbf{h}_{11}}$$

$$\Longrightarrow \mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}$$



$$\mathrm{I}_2 = \mathrm{h}_{22} - \frac{\mathbf{h}_{21}\mathbf{h}_{12}}{\mathbf{h}_{11}+40} = \frac{\mathbf{h}_{11}\mathbf{h}_{22}-\mathbf{h}_{21}\mathbf{h}_{12}+\mathbf{h}_{22}40}{\mathbf{h}_{11}+40}$$

Therefore

$$\mathrm{Z}_{\mathrm{TH}} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{\mathbf{I}_2} = \frac{\mathbf{h}_{11}+40}{\mathbf{h}_{11}\mathbf{h}_{22}-\mathbf{h}_{21}\mathbf{h}_{12}+\mathbf{h}_{22}40}$$

$$= \frac{1000+40}{10^3\times200\times10^{-6}+20+40\times200\times10^{-6}}$$

$$= \frac{1040}{20.21} = 51.46\Omega$$

From Fig.

$$-60 + 40\mathrm{I}_1 + \mathrm{V}_1 = 0 \Rightarrow \mathrm{V}_1 = 60-40\mathrm{I}_1$$

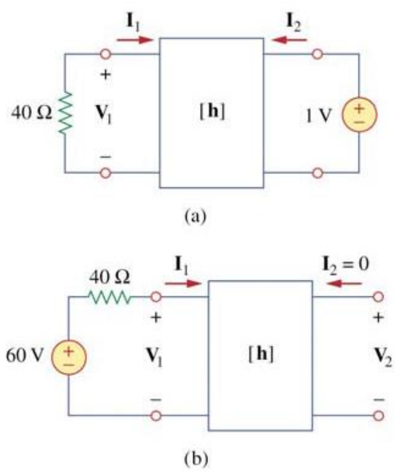
At the output, $\mathrm{I}_2 = 0$

$$\Rightarrow 60-40\mathrm{I}_1 = \mathrm{h}_{11}\mathrm{I}_1 + \mathrm{h}_{12}\mathrm{V}_2$$

or $60 = (\mathrm{h}_{11} + 40)\mathrm{I}_1 + \mathrm{h}_{12}\mathrm{V}_2$

and $0 = \mathrm{h}_{21}\mathrm{I}_1 + \mathrm{h}_{22}\mathrm{V}_2 \Rightarrow \mathrm{I}_1 = -\frac{\boldsymbol{h}_{22}}{\boldsymbol{h}_{21}}\mathrm{V}_2$

$$\Rightarrow 60 = \left[-(\boldsymbol{h}_{11} + 40)\frac{\boldsymbol{h}_{22}}{\boldsymbol{h}_{21}} + \boldsymbol{h}_{12}\right]\mathrm{V}_2$$

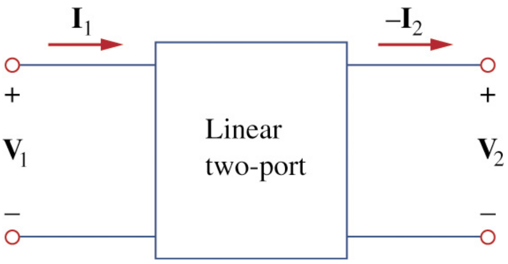


$$\Rightarrow V_{TH}=V_2=\frac{60}{-(h_{11}+40)h_{22}/h_{21}+h_{12}}$$

$$=\frac{60h_{21}}{h_{12}h_{21}-h_{11}h_{22}-40h_{22}}$$

$$=\frac{60\times 10}{-20.21}=-29.69V$$

5.5 Transmission Parameters,T (ABCD Parameters)



$$V_1=AV_2-BI_2$$
$$I_1=CV_2-DI_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = [T] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Transmission Parameters, T

$$\mathbf{A} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0}, \quad \mathbf{B} = -\left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0}$$
$$\mathbf{C} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0}, \quad \mathbf{D} = -\left.\frac{\mathbf{I}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0}$$

A = Open-circuit voltage ratio

B = Negative short-circuit transfer impedance

C = Open-circuit transfer admittance

D = Negative short-circuit current ratio

Inverse Transmission Parameters, t

$$\mathbf{a} = \left.\frac{\mathbf{V}_2}{\mathbf{V}_1}\right|_{\mathbf{I}_1=0}, \quad \mathbf{b} = -\left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_1=0}$$

$$\mathbf{c} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{I}_1=0}, \quad \mathbf{d} = -\left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_1=0}$$

a = Open-circuit voltage gain

b = Negative short-circuit transfer impedance

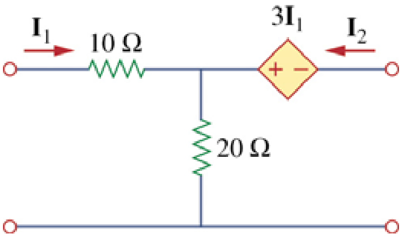
c = Open-circuit transfer admittance

d = Negative short-circuit current gain

AD–BC=1, **ad–bc**=1

Example 5.7

- Find the transmission parameters for the two-port network in Fig.



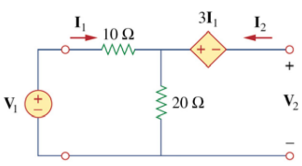
Solution:

From Fig. (a)

$\mathbf{V}_1 = (10+20)\mathbf{I}_1 = 30\mathbf{I}_1$ and $\mathbf{V}_2 = 20\mathbf{I}_1 - 3\mathbf{I}_1 = 17\mathbf{I}_1$

Thus

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{30I_1}{17I_1} = 1.765,\ C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{I_1}{17I_1} = 0.0588\text{S}$$



(a)

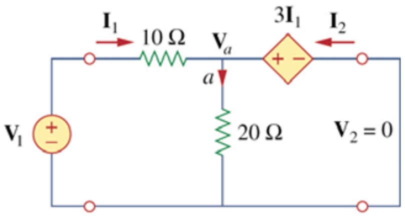
From Fig. (b)

$$\frac{V_1 - V_a}{10} - \frac{V_a}{20} + I_2 = 0$$

But, $V_a=3I_1$ and $I_1=(V_1-V_a)/10$,

$V_a=3I_1$, $V_1=13I_1$

$I_1 - \frac{3I_1}{20} + I_2=0 \Longrightarrow \frac{17}{20}I_1= -I_2$



(b)

Therefore,

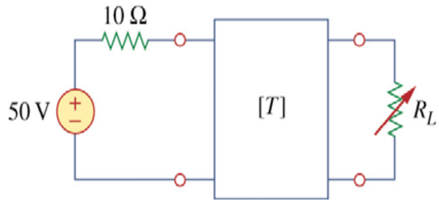
$$A=\frac{I_1}{I_2}\Bigg|_{V_2=0} = \frac{20}{17} =1.176,\ B= - \frac{V_1}{I_2}\Bigg|_{V_2=0} = \frac{-13I_1}{-(17/20)I_1} = 15.29\Omega$$

Example 5.8

- The **ABCD** parameters of the two-port network in Fig. are

$$\begin{bmatrix} 4 & 20\ \Omega \\ 0.1\ \text{S} & 2 \end{bmatrix}$$

The output port is connected to a variable load for maximum power transfer. Find RL and the maximum power transferred.



Solution:

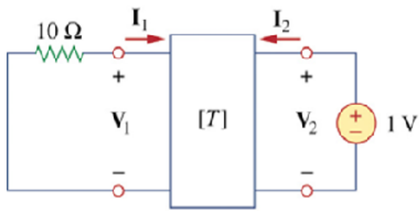
$\mathbf{V}_1 = 4\mathbf{V}_2 - 20\mathbf{I}_2$

$\mathbf{I}_1 = 0.1\mathbf{V}_2 - 2\mathbf{I}_2$

At the input port, $\mathbf{V}_1 = -10\mathbf{I}_1$

$-10\mathbf{I}_1 = 4\mathbf{V}_2 - 20\mathbf{I}_2$

or $\mathbf{I}_1 = -0.4\mathbf{V}_2 + 2\mathbf{I}_2$



(a)

$\Rightarrow 0.1\mathbf{V}_2 - 2\mathbf{I}_2 = -0.4\mathbf{V}_2 + 2\mathbf{I}_2 \Rightarrow 0.5\mathbf{V}_2 = 4\mathbf{I}_2$

Hence,

$$\mathbf{Z}_{TH} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{4}{0.5} = 8\Omega$$

To find $\mathbf{V}_{TH}$, we use the circuit in Fig. (b)

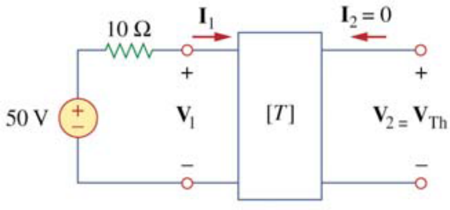
At the output port $\mathbf{I}_2 = 0$,

and the input port $\mathbf{V}_1 = 50 - 10\mathbf{I}_1$

$\Rightarrow 50 - 10\mathbf{I}_1 = 4\mathbf{V}_2 \Rightarrow \mathbf{I}_1 = 0.1\mathbf{V}_2$

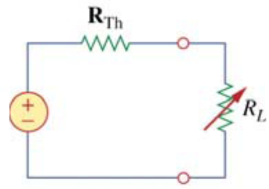
$\Rightarrow 50 - \mathbf{V}_2 = 4\mathbf{V}_2 \Rightarrow \mathbf{V}_2 = 10$

Thus, $\mathbf{V}_{TH} = \mathbf{V}_2 = 10$ V



(b)

The equivalent circuit is shown in Fig. (c)



(c)

$\mathrm{R_L} = \mathbf{Z}_{TH} = 8\ \Omega$

$$\Rightarrow P = I^2 R_L = \left(\frac{V_{TH}}{2R_L}\right)^2 R_L = \frac{V_{TH}^2}{4R_L} = \frac{100}{4\times 8} = 3.125\ \mathrm{W}$$

Condition for Reciprocity and Symmetry

- A network is said to be reciprocal , if the ratio of the response transform to the excitation transform is invariant to an interchange of the positions of the excitation and response in the network
- A two-port network is said to be symmetrical if the ports of the two-port network can be interchanged without changing the port voltages and currents

Parameter	*Condition of Reciprocity*	*Condition of Symmetry*
z	$z_{12} = z_{21}$	$z_{11} = z_{22}$
y	$y_{12} = y_{21}$	$y_{11} = y_{22}$
T (ABCD)	$(AD - BC) = 1$	$A = D$
h	$h_{12} = -h_{21}$	$(h_{11}h_{22} - h_{12}h_{21}) = 1$

Relationship Between Two-Port Network Parameters:

Any two-port network parameter can be expressed in terms of other. For example, if the Z-parameter for given two port network is computed by solving the network, the remaining Y, h and ABCD can be established without solving the network again. This provides an opportunity for the designer to choose the required combination in interconnecting more than one two port network and forming larger systems.

$$\begin{cases} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \\ \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z]^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \end{cases} \Rightarrow [y] = [z]^{-1}$$

The adjoint of the [z] matrix and its determinant are

$$\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}, \Delta_z = z_{11}z_{22} - z_{12}z_{21}$$

$$[y] = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \frac{\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}}{\Delta_z}$$

$$y_{11} = \frac{z_{22}}{\Delta_z}, y_{12} = -\frac{z_{12}}{\Delta_z}, y_{21} = -\frac{z_{21}}{\Delta_z}, y_{22} = \frac{z_{11}}{\Delta_z}$$

Relationship Between Two-Port Network Parameters

$$\boxed{\begin{array}{l} [g]=[h]^{-1} \\ [t] \neq [T]^{-1} \end{array}}$$

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2 \rightarrow I_2 = -\frac{z_{21}}{z_{22}} I_1 + \frac{1}{z_{22}} V_2$$

$$\rightarrow V_1 = \frac{z_{11}z_{22} - z_{12}z_{21}}{z_{22}} I_1 + \frac{z_{12}}{z_{22}} V_2$$

$$\rightarrow \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{z_{11}z_{22} - z_{12}z_{21}}{z_{22}} & \frac{z_{12}}{z_{22}} \\ -\frac{z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\Rightarrow h_{11} = \frac{\Delta_z}{z_{22}}, h_{12} = \frac{z_{12}}{z_{22}}, h_{21} = -\frac{z_{21}}{z_{22}}, h_{22} = \frac{1}{z_{22}}$$

Example Problem: Find [z] and [g] parameters of a two-port network if

$$[T]=\begin{bmatrix} 10 & 1.5\Omega \\ 2S & 4 \end{bmatrix}$$

Solution:

If **A**=10, **B**=1.5, **C**=2, **D**=4 4, the determinant of the matrix is
Δ_T=**AD**−**BC**= 40−3 = 37. From Table 19.1,

$$\mathbf{Z}_{11}=\frac{\mathbf{A}}{\mathbf{C}}=\frac{10}{2}=5 \qquad \mathbf{Z}_{12}=\frac{\Delta_T}{\mathbf{C}}=\frac{37}{2}=18.5$$

$$\mathbf{Z}_{21}=\frac{\mathbf{1}}{\mathbf{C}}=\frac{1}{2}=0.5 \qquad \mathbf{Z}_{22}=\frac{\mathbf{D}}{\mathbf{C}}=\frac{4}{2}=2$$

$$\mathbf{g}_{11}=\frac{\mathbf{C}}{\mathbf{A}}=\frac{2}{10}=0.2 \qquad \mathbf{g}_{12}=-\frac{\Delta_T}{\mathbf{A}}=\frac{37}{10}=-37$$

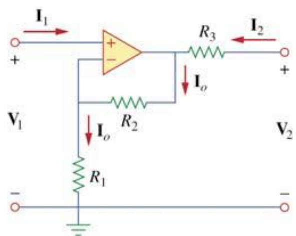
$$\mathbf{g}_{21}=\frac{\mathbf{1}}{\mathbf{A}}=\frac{1}{10}=0.1 \qquad \mathbf{g}_{22}=\frac{\mathbf{B}}{\mathbf{A}}=\frac{1.5}{10}=0.15$$

Thus, $[\mathbf{Z}]=\begin{bmatrix} 5 & 18.5 \\ 0.5 & 2 \end{bmatrix}\Omega$, $[\mathbf{g}]=\begin{bmatrix} 0.2\,S & -37 \\ 0.1 & 0.15\Omega \end{bmatrix}$

Example Problem: • Obtain the *y* parameters of the op amp circuit in Fig. Show that the circuit has no *z* parameters.

Solution:

Since no current can enter the input terminals of the op amps, $\mathbf{I}_1 = 0$, which can be expressed in terms of $\mathbf{V}_1$ and $\mathbf{V}_2$ as



$\mathbf{I}_1 = 0\mathbf{V}_1 + 0\mathbf{V}_2, \quad \mathbf{y}_{11} = 0 = \mathbf{y}_{12}$

Also, $\mathbf{V}_2 = R_3\mathbf{I}_2 + \mathbf{I}_o(R_1 + R_2)$,

But $\mathbf{I}_o = \mathbf{V}_1/R_1$. Hence,

$$\mathbf{V}_2 = R_3\mathbf{I}_2 + \frac{\mathbf{V}_1(R_1+R_2)}{R_1} \Longrightarrow \mathbf{I}_2 = -\frac{(R_1+R_2)}{R_1R_3}\mathbf{V}_1 + \frac{\mathbf{V}_2}{R_3}$$

$$\Longrightarrow y_{21} = -\frac{(R_1+R_2)}{R_1R_3}, \; y_{22} = \frac{1}{R_3}$$

The determinant of the [**y**] matrix is

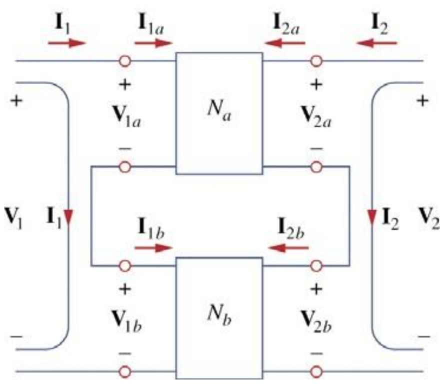
$\Delta_y = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21} = 0$

Since $\Delta_y = 0$, the [**y**] matrix has no inverse.

Desired Parameters	Given Parameters			
	[z]	[y]	[h]	[t]
[z]	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta_y} & \frac{-y_{12}}{\Delta_y} \\ \frac{-y_{21}}{\Delta_y} & \frac{y_{11}}{\Delta_y} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta_h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta_t}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$
[y]	$\begin{bmatrix} \frac{z_{22}}{\Delta_z} & \frac{-z_{12}}{\Delta_z} \\ \frac{-z_{21}}{\Delta_z} & \frac{z_{11}}{\Delta_z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_h}{h_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta_t}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$
[h]	$\begin{bmatrix} \frac{\Delta_z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta_t}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{bmatrix}$
[t]	$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta_y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta_h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$
$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$ $\Delta_y = y_{11}y_{22} - y_{12}y_{21}$ $\Delta_h = h_{11}h_{22} - h_{12}h_{21}$ $\Delta_t = AD - BC$				

Interconnection of Networks

- The series connection



$V_{1a}=z_{11}I_{1a}+z_{12}I_{2a}$

$V_{2a}=z_{21}I_{1a}+z_{22}I_{2a}$

$V_{1b}=z_{11}I_{1b}+z_{12}I_{2b}$

$V_{2b}=z_{21}I_{1b}+z_{22}I_{2b}$

$I_1 = I_{1a}=I_{1b}$, $I_2=I_{2a}=I_{2b}$

$V_1=V_{1a}+V_{1b}$

$= (z_{11a}+z_{11b})I_1 +(z_{12a}+z_{12b})I_2$

$V_2 = V_{2a}+V_{2b}$

$= (z_{21a}+z_{21b})I_1+(z_{22a}+z_{22b})I_2$

$$\Rightarrow \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} z_{11a}+z_{11b} & z_{12a}+z_{12b} \\ z_{21a}+z_{21b} & z_{22a}+z_{22b} \end{bmatrix}$$

$$\boxed{[Z]=[Z_a]+[Z_b]}$$

$$I_{1a} = y_{11}V_{1a} + y_{12}V_{2a}$$

$$I_{2a} = y_{21}V_{1a} + y_{22}V_{2a}$$

$$I_{1b} = y_{11}V_{1b} + y_{12}V_{2b}$$

$$I_{2b} = y_{21}V_{1b} + y_{22}V_{2b}$$

$$V_1 = V_{1a} = V_{1b}, V_2 = V_{2a} = V_{2b}$$

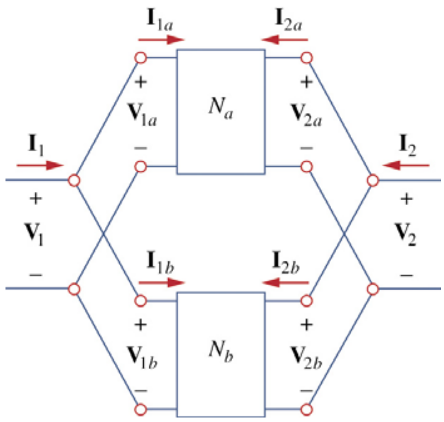
$$I_1 = I_{1a} + I_{1b}$$

$$=(y_{11a} + y_{11b})V_1 + (y_{12a} + y_{12b})V_2$$

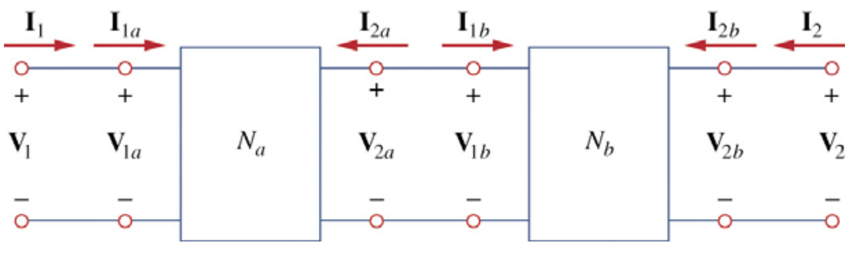
$$I2 = I2a + I2b$$

$$= (y_{21a} + y_{21b})V_1 + (y_{22a} + y_{22b})V_2$$

$$\Rightarrow \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} y_{11a} + y_{11b} & y_{12a} + y_{12b} \\ y_{21a} + y_{21b} & y_{22a} + y_{22b} \end{bmatrix}$$



$$\boxed{[y]=[y_a]+[y_b]}$$



$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \bullet \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}, \quad \begin{bmatrix} V_{1b} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \bullet \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix}, \begin{bmatrix} V_{2a} \\ -I_{2b} \end{bmatrix} = \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix}, \begin{bmatrix} V_{2b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} V_{2b} \\ -I_2 \end{bmatrix},$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \cdot \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \cdot \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \Rightarrow \boxed{[T]=[T_a]\cdot[T_b]}$$

Example Problem: Evaluate $\mathbf{V}_2/\mathbf{V}_s$ in the circuit in Fig.

This may be regarded as two-ports in series.

For N_b,

$\mathbf{z}_{12b} = \mathbf{z}_{21b} = 10 = \mathbf{z}_{11b} = \mathbf{z}_{22b}$

Thus,

$[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b]$

$$= \begin{bmatrix} 12 & 8 \\ 8 & 20 \end{bmatrix} + \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix} = \begin{bmatrix} 22 & 18 \\ 18 & 30 \end{bmatrix}$$

But

$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 = 22\mathbf{I}_1 + 18\mathbf{I}_2$

$\mathbf{V}_2 = \mathbf{z}_{32}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 = 18\mathbf{I}_1 + 30\mathbf{I}_2$

Also, at the input port $\mathbf{V}_1 = \mathbf{V}_s - 5\mathbf{I}_1$

and at the output port $\mathbf{V}_2 = -20\mathbf{I}_2 \Rightarrow \mathbf{I}_2 = -\frac{\mathbf{V}_2}{20}$

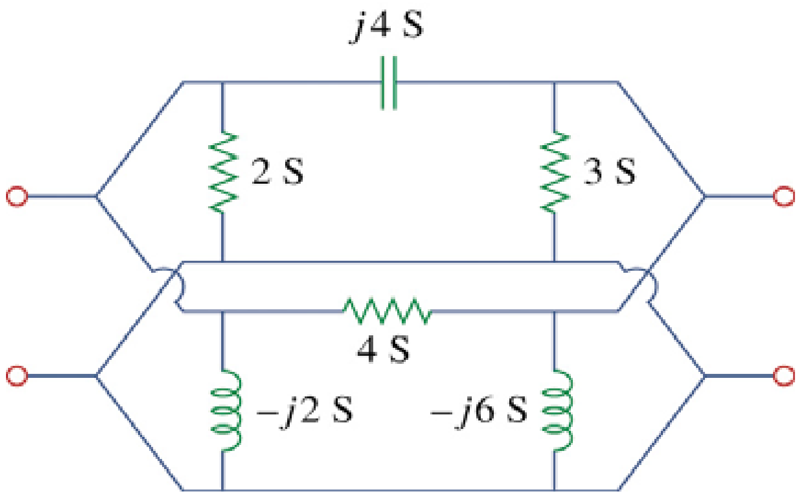
$\Rightarrow \mathbf{V}_s - 5\mathbf{I}_1 = 22\mathbf{I}_1 - \frac{18}{20}\mathbf{V}_2 \Rightarrow \mathbf{V}_s = 27\mathbf{I}_1 - 0.9\mathbf{V}_2$

$\Rightarrow \mathbf{V2} = 18\mathbf{I1} - \frac{30}{20}\mathbf{V2} \Rightarrow \mathbf{I}_{1=} \frac{2.5}{18}\mathbf{V}_2$

$\Rightarrow \mathbf{V}_s = 27 \times \frac{2.5}{18}\mathbf{V}_2 - 0.9\mathbf{V}_2 = 2.85\mathbf{V}_2$

And also, $\frac{V_2}{V_s} = \frac{1}{2.85} = 0.3509$

Example Problem: : Find the *y* parameters of the two-port in Fig.



Solution:

$\mathbf{y}_{12a} = -j4 = \mathbf{y}_{21a}$, $\mathbf{y}_{11a} = 2 + j4$, $\mathbf{y}_{22a} = 3 + j4$

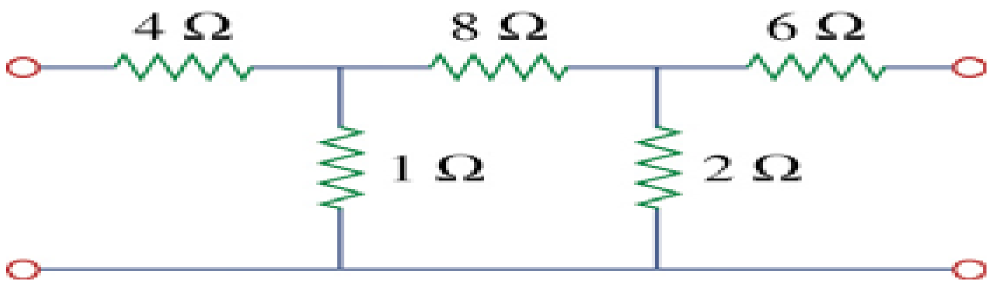
$[\mathbf{y_a}] = \begin{bmatrix} 2+j4 & -j4 \\ -j4 & 3+j4 \end{bmatrix} \text{S}$

and $\mathbf{y}_{12b} = -4 = \mathbf{y}_{21a}$, $\mathbf{y}_{11a} = 4 - j2$, $\mathbf{y}_{22b} = 4 - j6$

$[\mathbf{y_b}] = \begin{bmatrix} 4-\text{j}2 & -4 \\ -4 & 4-\text{j}6 \end{bmatrix} \text{S}$

$\Rightarrow [\mathbf{y}] = [\mathbf{y_a}] + [\mathbf{y_b}] = \begin{bmatrix} 6+j2 & -4-j4 \\ -4-j4 & 7-j2 \end{bmatrix} \text{S}$

Example Problem: Find the transmission parameters for the circuit in Fig.



$\mathbf{A} = 1 + \frac{R_1}{R_2}$, $\mathbf{B} = R_3 + \frac{R(R_2+R_3)}{R_2}$, $\text{C} = \frac{1}{R_2}$, $\text{D} = 1 + \frac{R_3}{R_2}$

$\mathbf{A}_a = 1 + 4 = 5$, $\mathbf{B}_a = 8 + 4\times 9 = 44\ \Omega$,

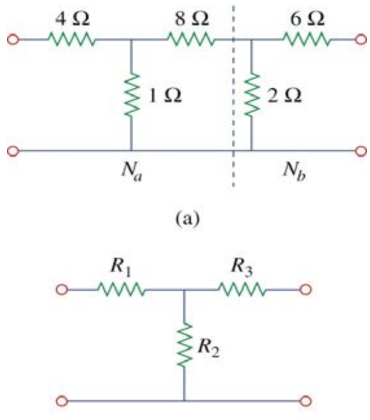
$\mathbf{C}_a = 1$ S, $\mathbf{D}_a = 1+8 = 9$

$\Rightarrow [\mathbf{T}_a] = \begin{bmatrix} 5 & 44\Omega \\ 1\text{S} & 9 \end{bmatrix}$

and $\mathbf{A}_b = 1 + 4 = 5$, $\mathbf{B}_b = 8 + 4\times 9 = 44\ \Omega$, $\mathbf{C}_b = 1$ S, $\mathbf{D}_b = 1+8 = 9$

$\Rightarrow [\mathbf{T}_b] = \begin{bmatrix} 1 & 6\Omega \\ 0.5\text{S} & 4 \end{bmatrix}$

Thus, for the total network in Fig. ,



(b)

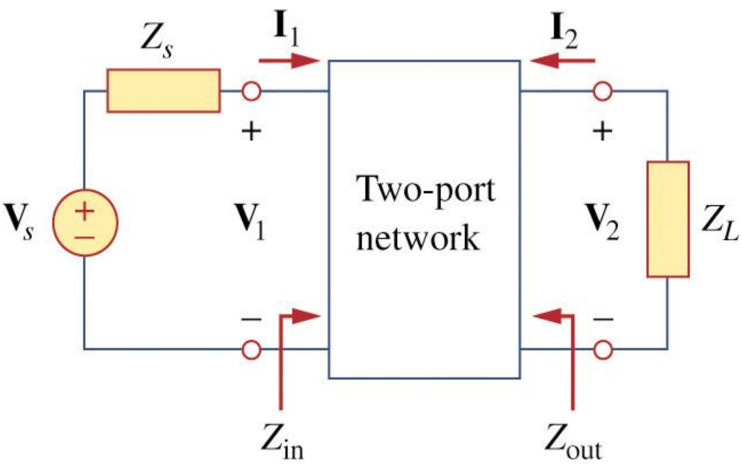
[**T**]=[**T**$_a$] [**T**$_b$]=$\begin{bmatrix} 5 & 44 \\ 1 & 9 \end{bmatrix}\begin{bmatrix} 1 & 6 \\ 0.5 & 4 \end{bmatrix}$

=$\begin{bmatrix} 5\times 1+44\times 0.5 & 5\times 6+44\times 4 \\ 1\times 1+9\times 0.5 & 1\times 6+9\times 4 \end{bmatrix}$

= $\begin{bmatrix} 27 & 206\Omega \\ 5.5S & 42 \end{bmatrix}$

Notice that $\Delta_{Ta} = \Delta_{Tb} = \Delta_T = 1$.

ApplicationsTransistor Circuits



$A_v=\frac{V_2(S)}{V_1(S)}$

$A_i=\frac{I_2(S)}{I_1(S)}$

$Z_{in}=\frac{V_1(S)}{I_1(S)}$

$Z_{out}=\left.\frac{V_2(S)}{I_2(S)}\right|_{V_s=0}$

Transistor Circuits

$hi = h11$, $hr = h12$, $hf = h21$, $ho = h22$

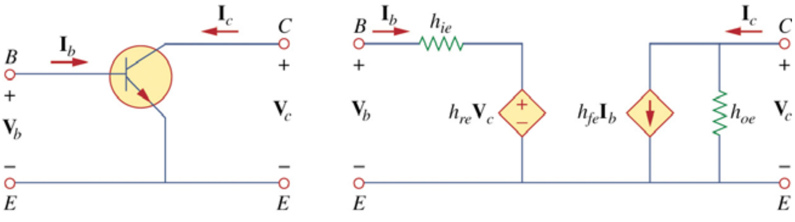
h_{ie} = Basic input impedance

h_{re} = Reverse voltage feedback ratio

h_{fe} = Basic-collector current gain

h_{oe} = Output admittance

Fig:

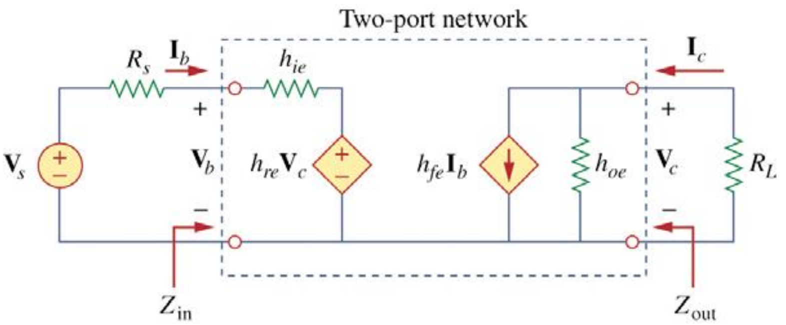


(a) (b)

$$\mathbf{V}_b = h_{ie}\mathbf{I}_b + h_{re}\mathbf{V}_c$$

$$\mathbf{I}_c = h_{fe}\mathbf{I}_b + h_{oe}\mathbf{V}_c$$

Fig.



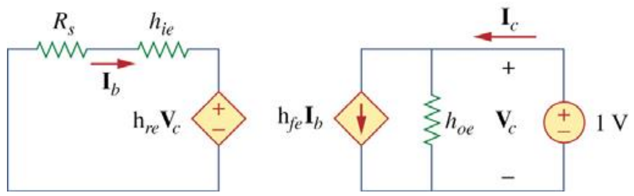
$$\boxed{A_i = \frac{\mathbf{I}_c}{\mathbf{I}_b} = \frac{h_{fe}}{1 + h_{oe}R_L}}$$

$$\boxed{A_v = \frac{\mathbf{V}_c}{\mathbf{V}_b} = \frac{-h_{fe}R_L}{h_{ie} + (h_{ie}h_{oe} - h_{re}h_{fe})R_L}}$$

Transistor Amplifiers

$$\boxed{Z_{\text{in}} = \frac{\mathbf{V}_b}{\mathbf{I}_b} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1 + h_{oe}R_l}}$$

$$\boxed{Z_{\text{out}} = \frac{R_s + h_{ie}}{(R_s + h_{ie})h_{oe} - h_{re}h_{fe}}}$$



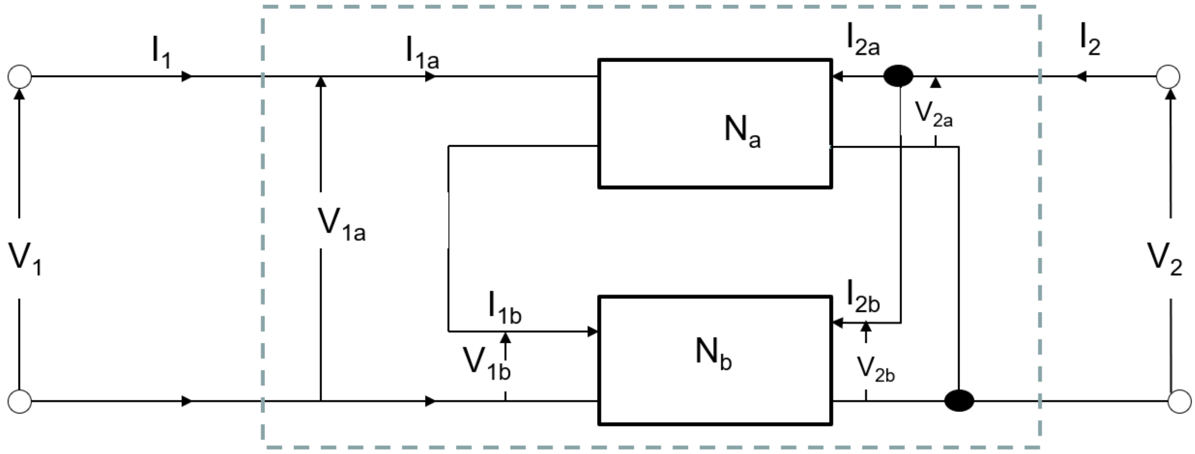
Series-parallel Connection: Useful for H-parameter interconnection

For network N_a

$$V_{1a} = h_{11a}I_{1a} + h_{12a}V_{2a}$$
$$I_{2a} = h_{21a}I_{1a} + h_{22a}V_{2a}$$

Similarly, for Network N_b

$$V_{1b} = h_{11b}I_{1b} + h_{12b}V_{2b}$$
$$I_{2b} = h_{21b}I_{1b} + h_{22b}V_{2b}$$

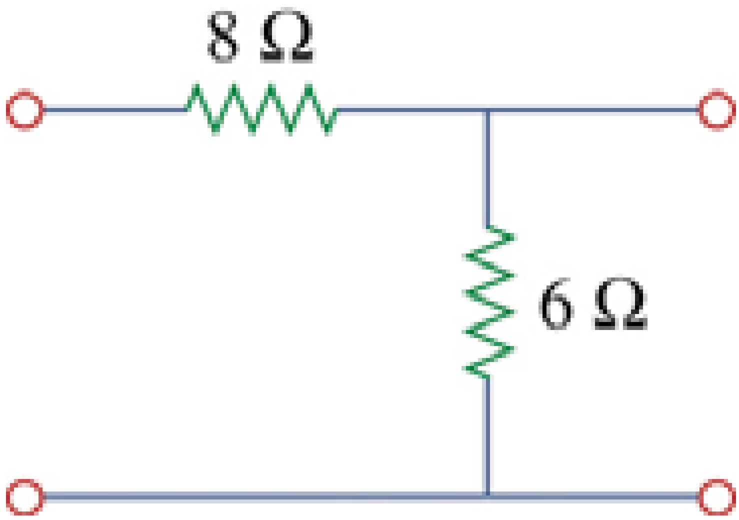


$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

For the combined network connected in series -parallel model resultant matrix is:

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} h_{11a} + h_{11b} & h_{12a} + h_{12b} \\ h_{21a} + h_{21b} & h_{22a} + h_{22b} \end{bmatrix}$$

Practice problem: Evaluate the Z parameters verify symmetric and reciprocity property



Solution:

$V_1=Z_{11}I_1+Z_{12}I_2$

$V_2=Z_{21}+Z_{12}I_2$

with $I_2=0 \Rightarrow Z_{11}=\frac{V1}{I1}$ and $Z_{21}=\frac{V2}{I1}$

with $I_1=0 \Rightarrow Z_{11}=\frac{V1}{I1}$ and $Z_{21}=\frac{V2}{I1}$

On applying KVL at the input port

$V_1=I_1(8+6) \therefore Z_{11}=\frac{V1}{I1}=14\ \Omega$

At output port $V_2=I_1\times 6 \therefore Z_{21}=\frac{V2}{I1}=6\ \Omega$

On writing circuit with $I_1=0$

$V_2=6\times I_1 \therefore Z_{22}=\frac{V2}{I2}=6\ \Omega$

$V_1=6\times I_2 \therefore Z_{12}=\frac{V1}{I2}=6\ \Omega$

$Z_{12}=Z_{21}$, the given circuit is reciprocal

$Z_{11}\neq Z_{22}$ the given circuit is not symmetrical

Find the H parameter for the same network and Verify the reciprocity and symmetry property

$V_1=h_{11}I_1+h_{12}V_2$

$I_2=h_{21}I_1+h_{22}V_2$

Short Circuit the output port $V_2=0$

$h_{11}=\frac{V1}{I1}$ and $h_{21}=\frac{I2}{I1}$

$V_1=8\ I_1 \Rightarrow h_{11}=\frac{V1}{I1} \Rightarrow 8\ \Omega$

$I_2=-I_1$, $\therefore h_{21}=\frac{I2}{I1} = -I_1$

Open circuit the input port

$I_1= 0.7, h_{21}=\frac{V1}{V2}$, $h_{22}=\frac{I2}{V2}$

$V_1=V_2=6I_2 \therefore h_{12}=\frac{6I2}{6I2}= 1$ and

$h_{22}=\frac{I2}{V2}=\frac{6}{1}$

reciprocal $h_{12}=-h_{21}$

Symmetric $h_{11}h_{12}- h_{12}h_{21} = 1$

$$8\times\frac{6}{1}-(1)(-1) \neq 0$$

[Hence the given circuit is not symmetric]

Expressing h – parameters in terms of Z parameters

$h_{11}=\frac{\Delta Z}{Z_{22}} = \frac{14\times 6-36}{6} = (14-6) = 8\ \Omega$

$h_{12}=\frac{Z_{12}}{Z_{22}} = \frac{6}{6}= 1$

$h_{21}=\frac{Z_{21}}{Z_{22}} = \frac{-6}{6} = -1$

$h_{22}=\frac{1}{Z_{22}}=\frac{1}{6}$ S

<u>ABCD Parameters expressed in terms of Z parameters:</u>

ABCD parameters eq.

$V_1= AV_2 - BI_2$-------------- (1)

$I_1= CV_2 - DI_2$-------------- (2)

Z parameters eq.

$V_1= Z_{11}\ I_1+Z_{12}\ I_2$---------(3)

$V_2= Z_{21}\ I_1+Z_{22}\ I_2$---------(4)

Comparing equation (4) with (2) the variables also remain same

Rearranging equation (4) in such a way that I_1 is on one side

$Z_{21}I_1 = V_2 - Z_{22}I_2$

$I_1=\frac{1}{Z_{21}}V_2-\frac{Z_{22}}{Z_{21}}I_2$--------(5)

On comparing eq(5) and eq(2)

$C=\frac{1}{Z_{22}}$ and $D = \frac{Z_{22}}{Z_{21}}$

Now, on substituting the value of I_1, from eq(5) in eq(3)

$V_1= Z_{11}\left[\frac{1}{Z_{22}}V_2 - \frac{Z_{22}}{Z_{21}}I_2\right]+Z_{12}I_2$

$V_1=\frac{Z_{11}}{Z_{21}}V_2-\left[Z_{12} - \frac{Z_{11}\,Z_{22}}{Z_{21}}\right]I_2$

$V_1=\frac{Z_{11}}{Z_{21}}V_2-\frac{\Delta Z}{Z_{21}}\ I_2$

$A=\frac{Z_{11}}{Z_{21}}$, $B = \frac{\Delta Z}{Z_{21}}$

Now we can also express Z- parameters in terms of ABCD parameters.

$CV_2= I_1+ DI_2$

$V_2=\frac{1}{C}I_1+\frac{D}{C}I_2$---------(6)

On comparing eq(6) with eq(4)

$Z_{21} =\frac{1}{C}$ and $Z_{22} =\frac{D}{C}$

By substituting for V_2 in eq(1)

$V_1 = A\left[\frac{1}{C}I_1 + \frac{D}{C}I_2\right]- BI_2$

$= \frac{A}{C}I_1+ \left[\frac{AD}{C} - \ \mathrm{B}\right]I_2$

$= \frac{A}{C}I_1+ \left[\frac{AD-BC}{C}\right]I_2$

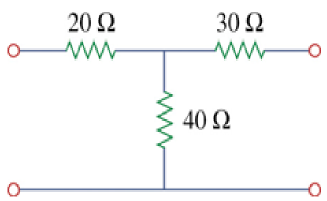
$= \frac{A}{C}I_1+ \frac{\Delta T}{C}I_2$------------(7)

Comparing eq(7) with eq(3)

$Z_{11}=\frac{A}{C}$

$Z_{12}=\frac{\Delta T}{C}$

<u>ABCD Parameters in terms of Z ;</u> considering the following example network with evaluated values for Z_{11}=60Ω, Z_{12}=60Ω, Z_{21}=60Ω and Z_{22}=60Ω



Expressing ABCD parameters in terms of Z

A=$\frac{Z_{11}}{Z_{21}}$, B = $\frac{\Delta Z}{Z_{21}}$

C = $\frac{1}{Z_{21}}$, D = $\frac{Z_{22}}{Z_{21}}$

We know that,

V_1= AV_2- BI_2

I_1= CV_2- DI_2

<u>With output port open circuit,</u>

I_2= 0 , A = $\frac{V_1}{V_2}$, $\mathrm{B} = \frac{I_1}{I_2}$

Applying KVL at input port

V_1= (20+40)I_1

V_2 = 40 I_1

A = $\frac{60}{40}$,AC= $\frac{1}{40}$

Now,

A = $\frac{Z_{11}}{Z_{21}}$= $\frac{60}{40}$

C = $\frac{1}{Z_{21}}$= $\frac{1}{40}$

B = - $\frac{V_1}{I_2}$

D = - $\frac{I_1}{I_2}$

Now with output SC V_2=0

V_1= 20 I_1+ I $\times 40$

= 20 I_1+40 $\left[I_1 \times \frac{30}{30+40}\right]$

= $\frac{2600}{70}$ I_1

I_2 = - $I_1 \times \frac{30}{30+40}$ = $\frac{30}{70}$ I_1 = c

B= $\frac{\frac{2600}{70} I_1}{-\frac{40}{70} I_1}$ = $\frac{2600}{40}$ $\Longrightarrow$B = $\frac{\Delta Z}{Z_{21}}$ = $\frac{2600}{40}$

From ?

D = - $\frac{I_1}{I_2} = \frac{70}{40}$ $\Longrightarrow$D = $\frac{Z_{22}}{Z_{21}} = \frac{70}{40}$

H – parameters in terms of Z

Express h parameters in terms of Z-parameters

$h_{11} = \frac{\Delta Z}{Z_{22}}$

$h_{12} = \frac{Z_{12}}{Z_{22}}$

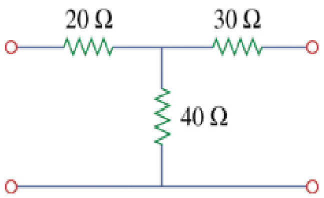
$h_{21} = \frac{Z_{21}}{Z_{22}}$

$h_{22} = \frac{1}{Z_{22}}$

we know that

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$



[ ]

We have already computed Z-parameters : $\begin{bmatrix} 60 & 40 \\ 40 & 70 \end{bmatrix}$

With output port short circuit $V_2 = 0$

$h_{11} = \frac{V_1}{I_1}$

$h_{21} = \frac{I_2}{I_1}$

$V_1 = 20\, I_1 + 40\ I_\lambda$

$V_1 = 20\, I_1 + 40\ \left[\frac{I_1 \times 30}{30+40}\right]$

$V_1 = \left[\frac{1400\ I_1 + 1200 I_1}{70}\right]$

$h_{11} = \frac{V_1}{I_1} = \frac{2600}{70}$

Now, $I_2 = -I_1 \times \frac{40}{40+30} = -I_1 \frac{40}{70}$

$h_{21} = \frac{I_2}{I_1} = -\frac{40}{70} \Longrightarrow \frac{Z_{21}}{Z_{22}} = -\frac{40}{70}$

with input port open circuit, $I_1 = 0$

$h_{12} = \frac{V_1}{V_2}$ and $h_{22} = \frac{I_2}{V_2}$

on applying KVL at output port,

$V_2 = (40+30) I_2$

$h_{22}=\frac{I_2}{V_2}=\frac{1}{70}$ S

but,

$h_{22}=\frac{1}{Z_{22}}=\frac{1}{70}$ S

on__________

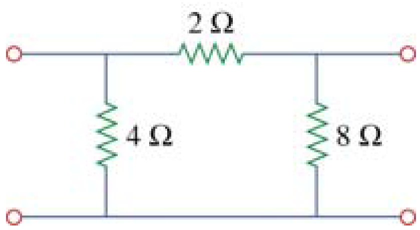
$V_1= 40I_2$

Now

$h_{12}=\frac{V_1}{V_2}=\frac{40I_2}{70I_2}=\frac{40}{70}$

$h_{21}=\frac{Z_{12}}{Z_{22}}=\frac{40}{70}$

Considering - Π network which is solved as example problem earlier to determine Y-Parameter:



$[Y]=\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}=\begin{bmatrix} 0.75S & -0.5S \\ -0.5S & 0.625S \end{bmatrix}$

Let us find the Z parameters for the same circuit

With I_2=0, i.e. open circuit the output port

$V_1= Z_{11} I_1+Z_{12} I_2$

$V_2= Z_{21} I_1+Z_{22} I_2$

At the input port

$V_1= I_x \times 4$

$$\boxed{\begin{gathered} Z_{11}=\frac{Y_{22}}{\Delta Y}=\frac{0.625}{0.2187} \\ Z_{11}=2.8571 \end{gathered}}$$

$I_x = I_1 \times \frac{10}{4}$

$V_1 = I_1 \frac{10}{4} \times 4 \Longrightarrow \frac{V_1}{I_1} = \frac{20}{7}\ \Omega = Z_{11}$

$Z_{11} = 2.8571$

Similarly

$V_2 = I_y \times 8$

$I_y = I_1 \times \frac{4}{14}$

$V_2 = I_1 \frac{10}{4} \times 4 \Longrightarrow \frac{V_2}{I_1} = \frac{6}{7}\ \Omega$

$Z_{11} = 2.285\Omega$

$$\boxed{Z_{21} = \frac{Y_{12}}{\Delta\, Y} = \frac{0.5}{0.21875} \qquad Z_{11} = 2.2833\ \Omega}$$

With I_1=0, open circuit the input port

$V_2 = I_x \times 8$

$I_x = I_2 \times \frac{6}{14}$

$V_2 = I_2 \frac{6}{14} \times 8 = \frac{24}{7} I_2 \Longrightarrow \frac{V_2}{I_2} = \frac{24}{7}\ \Omega$

$Z_{22} = 3.428\ \Omega$

$$\boxed{Z_{22} = \frac{Y_{11}}{\Delta\, Y} = \frac{0.75}{0.21875} \qquad Z_{22} = 3.4285\ \Omega}$$

Similarly

$V_1 = I_x \times 4$

$I_y = I_2 \times \frac{8}{14}$

$V_1 = I_2 \frac{8}{14} \times 4 \Longrightarrow I_1 \frac{16}{7}\ \Omega$

$\frac{V_1}{I_2} = Z_{12} = \frac{16}{7} = 2.285\Omega$

$$\boxed{Z_{12} = -\frac{Y_{11}}{\Delta\, Y} = \frac{0.5}{0.21875} \qquad Z_{11} = 2.28533\ \Omega}$$