

MODULE - 1 **Static Force Analysis**

CONTENTS

- 1.1. Introduction:
- 1.2. Static equilibrium.
- 1.3. Equilibrium of two and three force members.
- 1.4. Members with two forces and torque.
- 1.5. Free body diagrams.
- 1.6. Static force analysis of four bar and single slider mechanism
- 1.7. Slider-crank mechanism with and without friction.

Objectives

- To analyze static force analysis of four bar chain mechanism
- To analyze static force analysis of slider crank mechanism

1.1. Introduction:

Relation between motion and forces causing is a fascinating subject. This study is a generally referred as dynamic. Modern Engineering aims at analysing and predicting dynamics behavior of physical systems

Theory of Mechanisms & Machines is used to understand the relationships between the geometry and motions of the parts of a machine or mechanism and forces which produce motion.

TOM (M&M theory) is divided into two parts:-

Kinematics of Machinery: Study of motion of the components and basic geometry of the mechanism and is not concerned with the forces which cause or affect motion. Study includes the determination of velocity and acceleration of the machine members

Dynamics of Machinery: Analyses the forces and couples on the members of the machine due to external forces (static force analysis) also analyses the forces and couples due to accelerations of machine members (Dynamic force analysis)

Deflections of the machine members are neglected in general by treating machine members as rigidbodies (also called rigid body dynamics). In other words the link must be properly designed to withstand the forces without undue deformation to facilitate proper functioning of the system.

In order to design the parts of a machine or mechanism for strength, it is necessary to determine the forces and torques acting on individual links. Each component however small, should be carefully analysed for its role in transmitting force.

The forces associated with the principal function of the machine are usually known or assumed.

Ex:

- a) Piston type of engine: gas force on the piston is known or assumed
- b) QRM – Resistance of the cutting tool is assumed.

a & b are called static forces.

Example of other static forces are:

- i. Energy transmitted
- ii. Forces due to assembly
- iii. Forces due to applied loads
- iv. Forces due to changes in temperature
- v. Impact forces
- vi. Spring forces
- vii. Belt and pulley
- viii. Weights of different parts

Apart from static forces, mechanism also experiences inertia forces when subjected to acceleration, called dynamic forces.

Static forces are predominant at lower speeds and dynamic forces are predominant at higher speeds.

Force analysis:

The analysis is aimed at determining the forces transmitted from one point to another, essentially from input point to out put point. This would be the starting point for strength design of a component/ system, basically to decide the dimensions of the components

Force analysis is essential to avoid either overestimation or under estimation of forces on machine member.

Under estimation: leads to design of insufficient strength and to early failure.
Overestimation: machine component would have more strength than required. Over design leads to heavier machines, costlier and becomes not competitive

Graphical analysis of machine forces will be used here because of the simplification it offers to a problem, especially in cases of complex machines. Moreover, the graphical analysis of forces is a direct application of the equations of equilibrium.

General Principle of force analysis:

A machine / mechanism is a three dimensional object, with forces acting in three dimensions. For a complete force analysis, all the forces are projected on to three mutually perpendicular planes. Then, for each reference plane, it is necessary that, the vector sum of the applied forces in zero and that, the moment of the forces about any axis perpendicular to the reference plane or about any point in the plane is zero for equilibrium.

That is $\sum F = 0$ & $\sum M = 0$ or

$\sum F_x=0$ & $\sum F_y=0$ and $\sum M=0$

A force is a vector quantity and three in properties define a force completely;

Magnitude

Direction

Point of application

1.2. Static equilibrium.

Equilibrium

For a rigid body to be in Equilibrium

- i) Sum of all the forces must be zero
- ii) Sum of all the moments of all the forces about any axis must be zero

i.e, (i) $\sum F = 0$ (ii) $\sum M = 0$

or $\begin{matrix} \sum F_x = 0 \\ \sum F_y = 0 \end{matrix}$ $\begin{matrix} \sum TM = 0 \\ \sum My = 0 \end{matrix}$

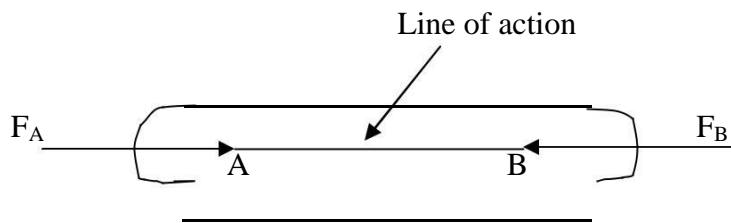
$\sum F_z = 0$ $\sum T_z = 0$ (For a planar system represented by 2D vectors)

F_x, F_y, F_z force Components along X, Y & Z axis

Similarly moments

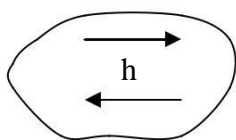
1.3 Equilibrium of two and three force members.

(i) **Equilibrium of a body under the action of two forces only (no torque)**



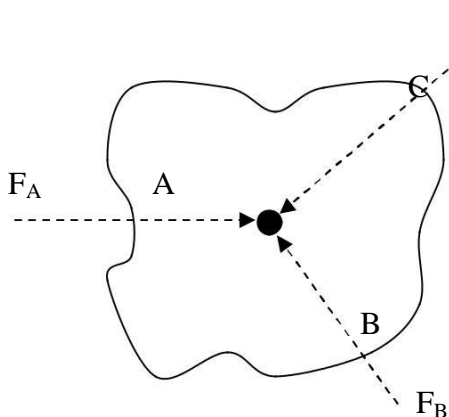
For body to be in Equilibrium under the action of 2 forces (only), the two forces must be equal, opposite and collinear. The forces must be acting along the line joining A&B.

That is,
 $F_A = - F_B$ (for equilibrium)



If this body is to be under equilibrium 'h' should tend to zero

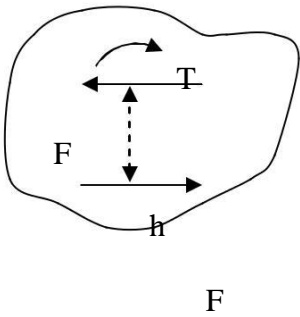
(ii) **Equilibrium of a body under the action of three forces only (no torque / couple)**



For equilibrium, the 3 forces must be concurrent and the force polygon will be a triangle.

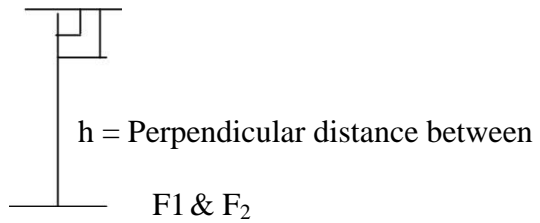
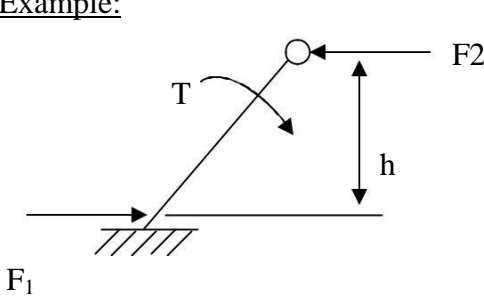
1.4 Members with two forces and torque.

(iii) *Equilibrium of a body acted upon by 2 forces and a torque.*



For equilibrium, the two forces must form a counter couple. Therefore the forces must be equal, opposite and parallel and their senses must be so as to oppose the couple acting on the body

Example:



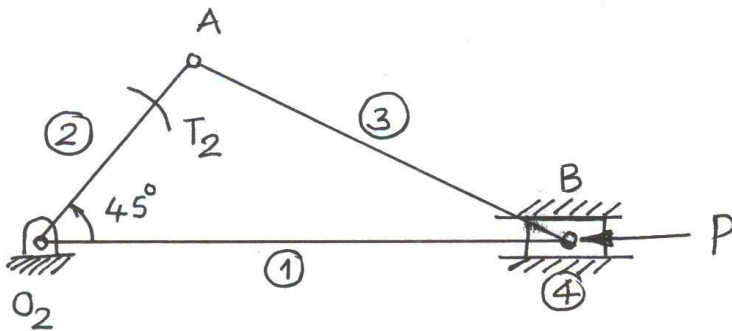
Free body diagram

The mass is separated from the system and all the forces acting on the mass are represented.

slider-crank mechanism with and without friction.

Problem No.1: Slider crank mechanism

Figure shows a slider crank mechanism in which the resultant gas pressure $8 \times 10^4 \text{ Nm}^{-2}$ acts on the piston of cross sectional area 0.1 m^2 . The system is kept in equilibrium as a result of the couple applied to the crank 2, through the shaft at O_2 . Determine forces acting on all the links (including the pins) and the couple on 2.

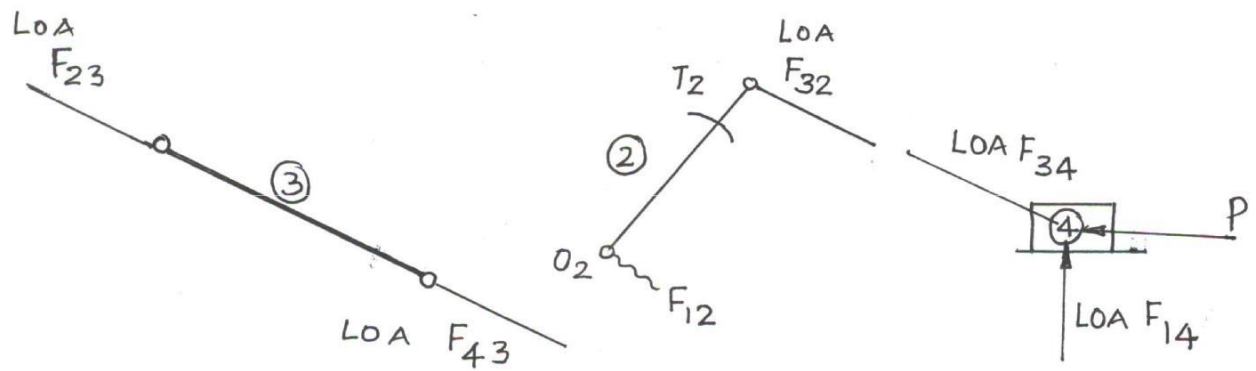


$$P = (8 \times 10^4) \times (0.1)$$

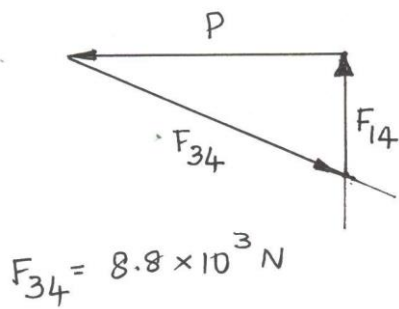
$$= 8 \times 10^3 \text{ N}$$

1.5 Free body diagrams.

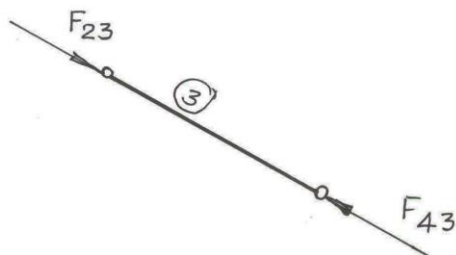
Free body diagram



Force triangle for the forces acting on (4) is drawn to some suitable scale. Magnitude and direction of P known and lines of action of F_{34} & F_{14} known.



Measure the lengths of vectors and multiply by the scale factor to get the magnitudes of F_{14} & F_{34} . Directions are also fixed.

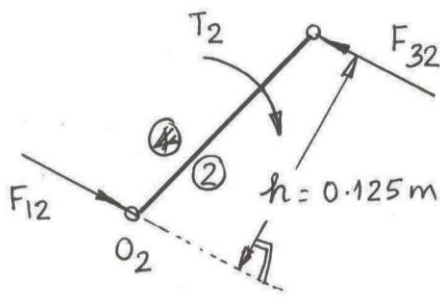


$i.e., F_{23} = -F_{32}$

Since link 3 is acted upon by only two forces, F_{43} and F_{23} are collinear, equal in magnitude and opposite in direction

$i.e., F_{43} = -F_{23} = 8.8 \times 10^3 \text{ N}$

Also, $F_{23} = -F_{32}$ (equal in magnitude and opposite in direction).



Link 2 is acted upon by 2 forces and a torque (stated in the problem), for equilibrium the two forces must be equal, parallel and opposite and their sense must oppose T_2 .

Therefore,

$$F_{32} = -F_{12} = 8.8 \times 10^3 \text{ N}$$

F_{32} & F_{12} form a counter clock wise couple of magnitude,

$$F_{23} \times h = F_{12} \times h = 8.8 \times 10^3 \times 0.125 = 1100 \text{ Nm.}$$

To keep 2 in equilibrium, T_2 should act clockwise and magnitude is 1100 Nm. Important to note;

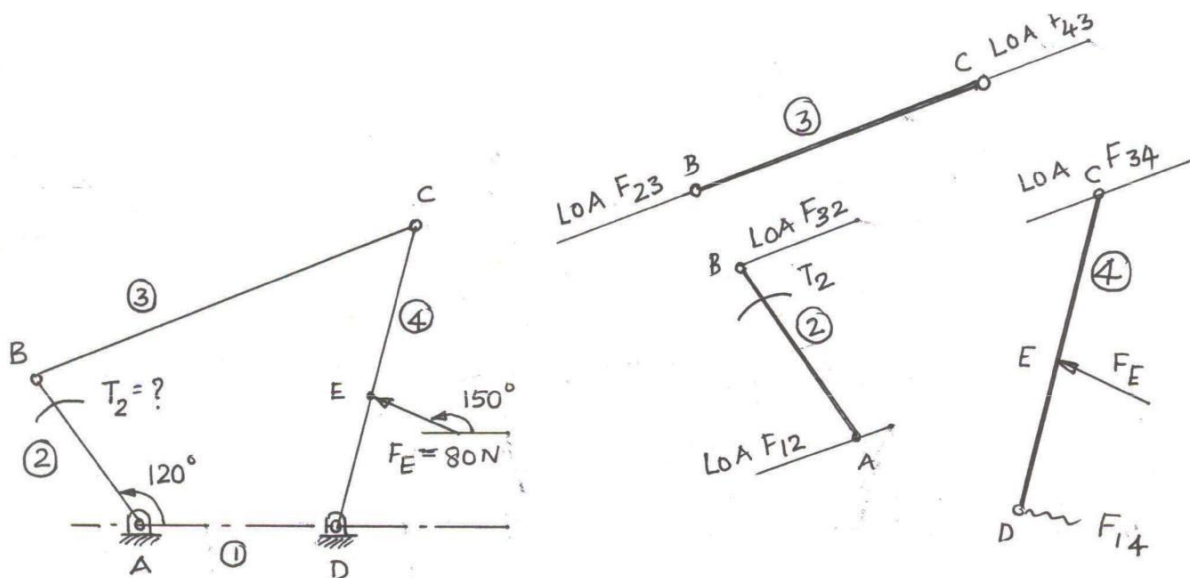
- i) h is measured perpendicular to F_{32} & F_{12} ;
- ii) always multiply back by scale factors.

1.6 Static force analysis of four bar and single slider mechanism

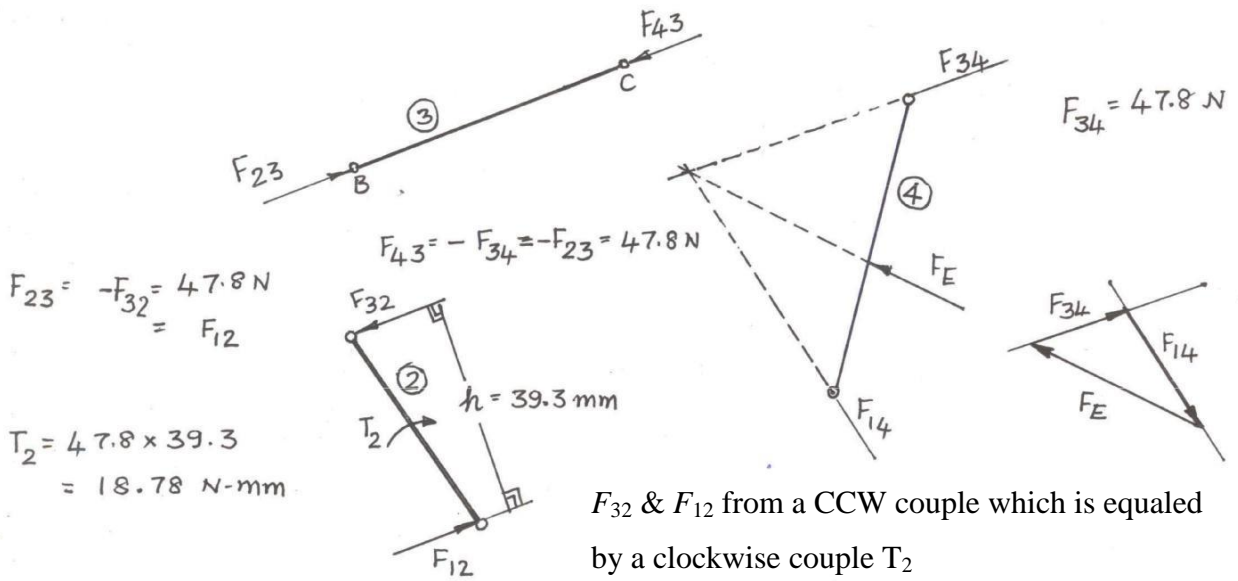
Problem No 2. Four link mechanism.

A four link mechanism is acted upon by forces as shown in the figure. Determine the torque T_2 to be applied on link 2 to keep the mechanism in equilibrium.

$AD=50\text{mm}$, $AB=40\text{mm}$, $BC=100\text{mm}$, $Dc=75\text{mm}$, $DE= 35\text{mm}$,

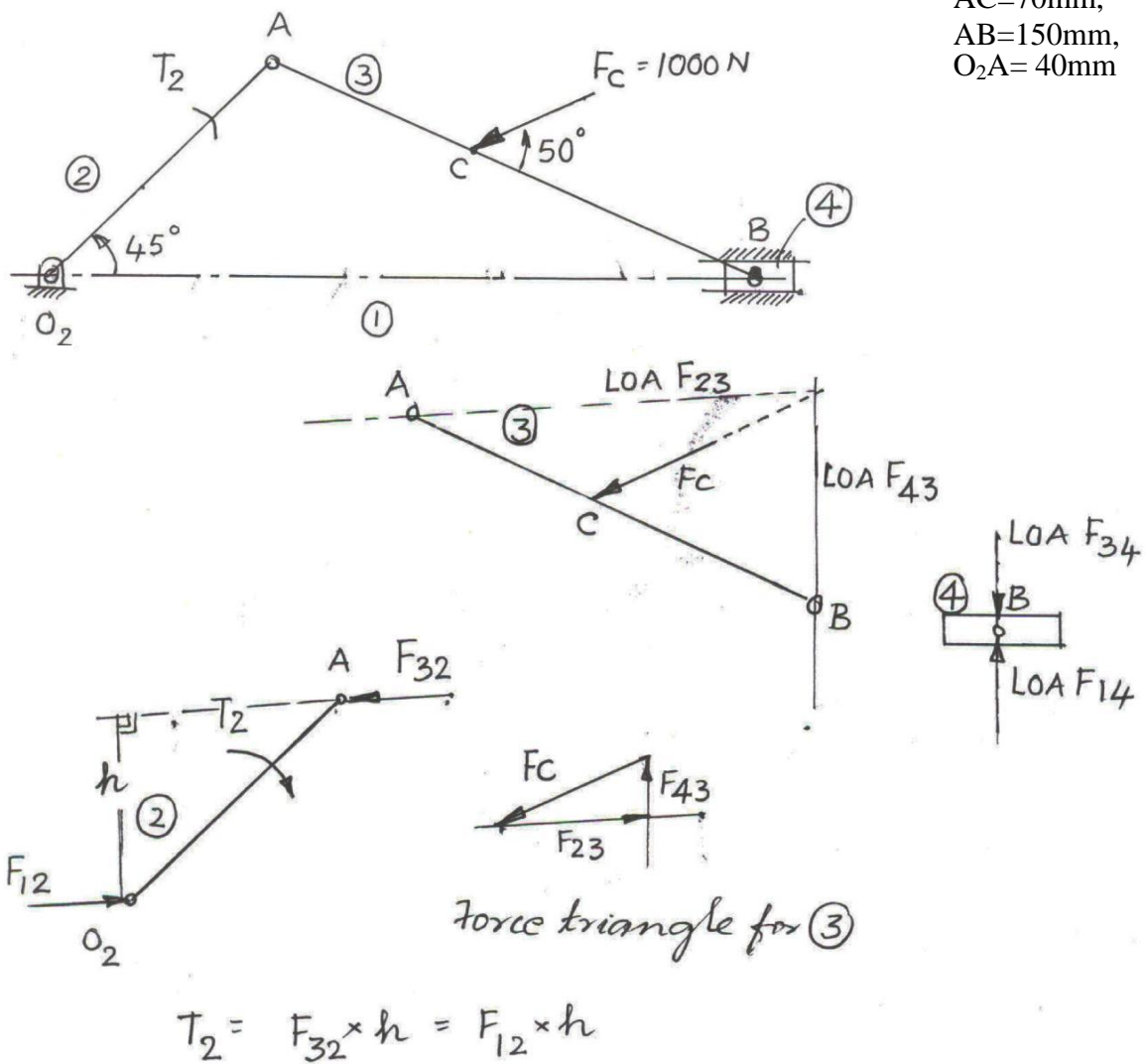


Link 3 is acted upon by only two forces F_{23} & F_{43} and they must be collinear & along BC. Link 4 is acted upon by three forces F_{14} , F_{34} & F_4 and they must be concurrent. LOA F_{34} is known and F_E completely given.



Problem No 3.

Determine T_2 to keep the mechanism in equilibrium

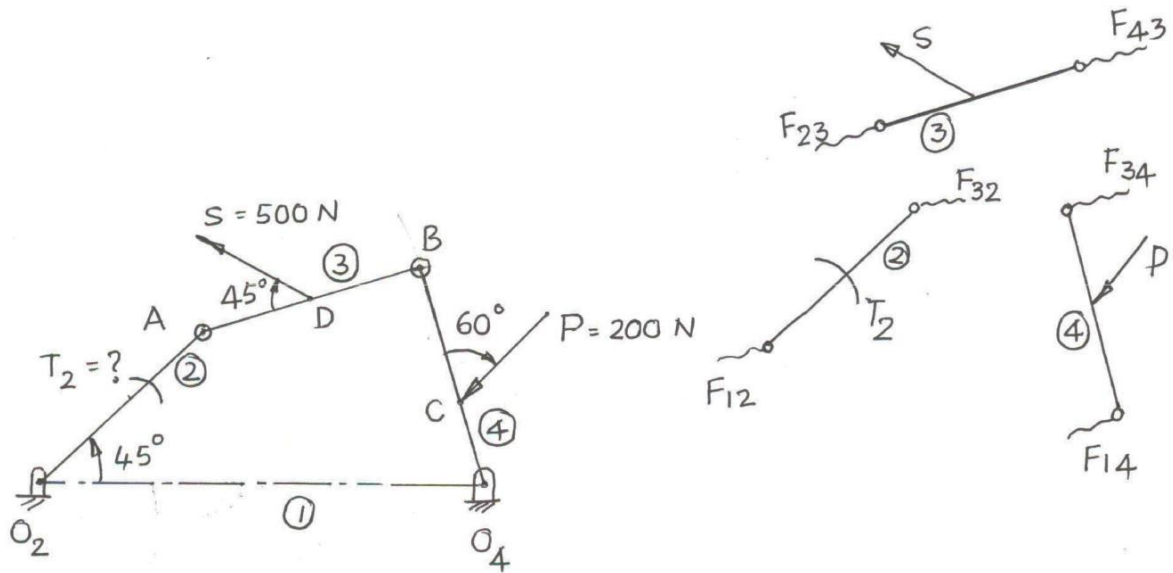


F_{32} and F_{12} form a CCW couple and hence T_2 acts clock wise.

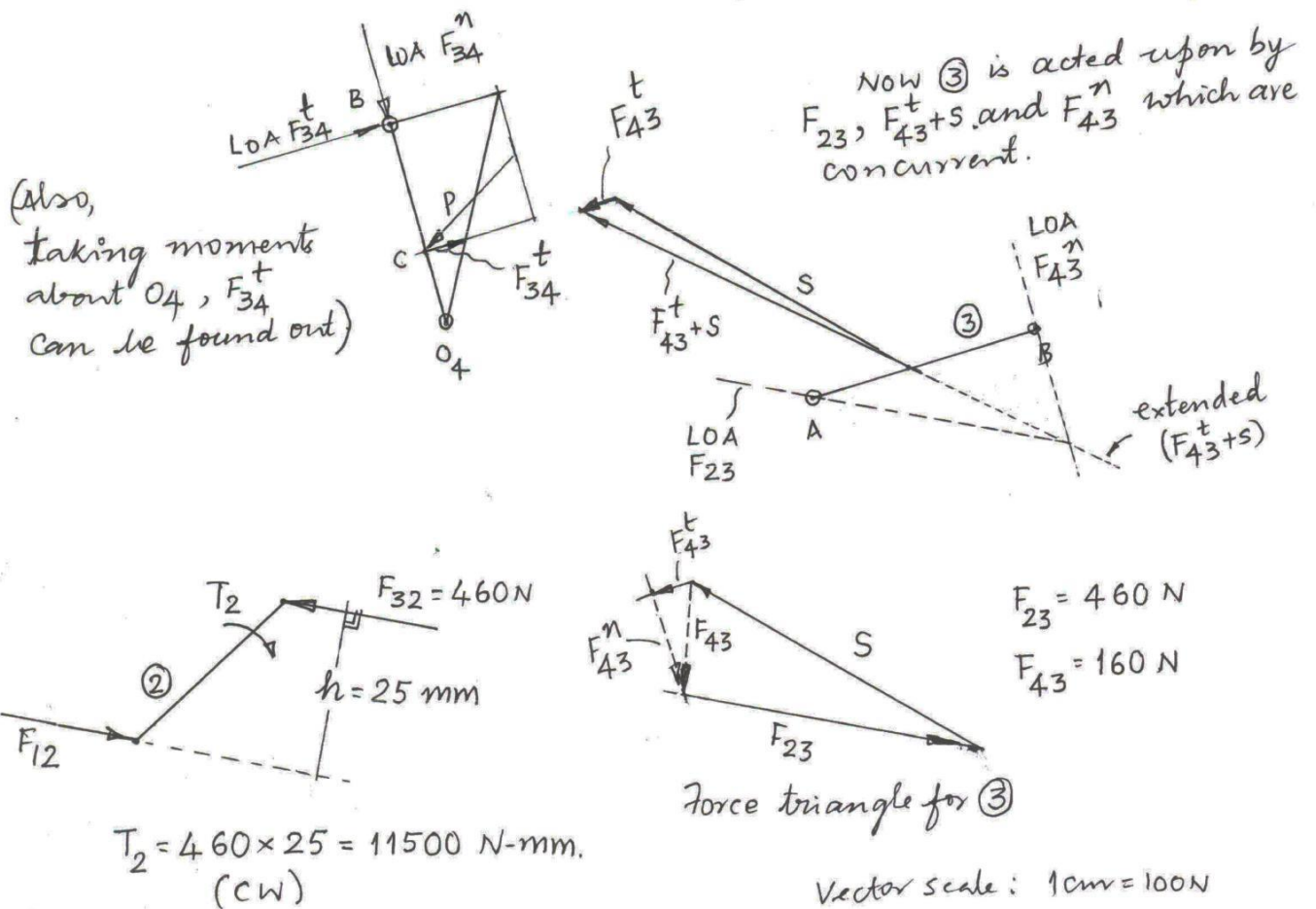
Problem No 4.

Determine the torque T_2 required to keep the given mechanism in equilibrium.

$O_2A = 30\text{mm}$, $AB = O_4B$, $O_2O_4 = 60\text{mm}$, $\angle A O_2 O_4 = 60^\circ$, $BC = 19\text{mm}$, $AD = 15\text{mm}$.



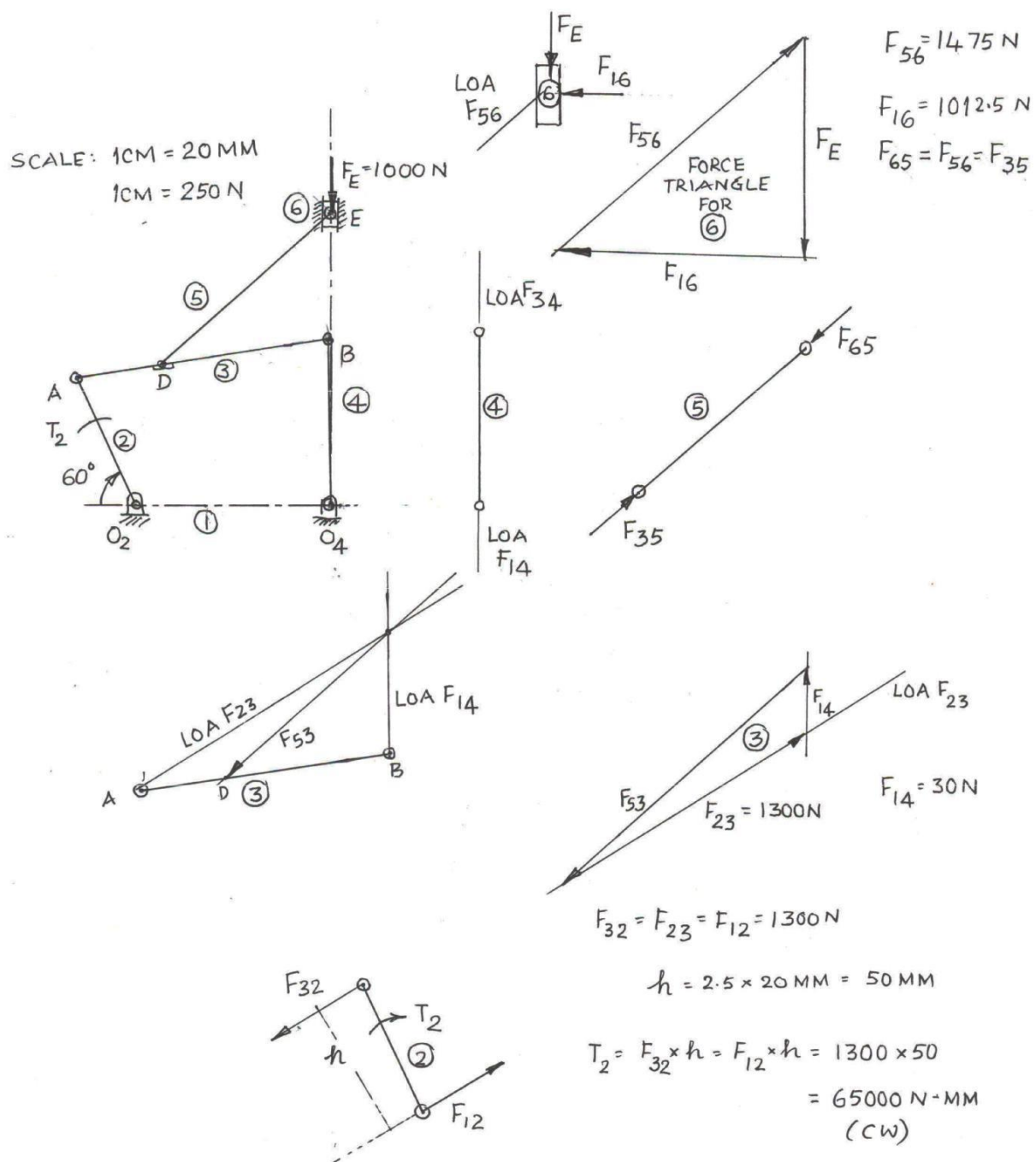
None of the links are acted upon by only 2 forces. Therefore links can't be analyzed individually.



Problem No 5.

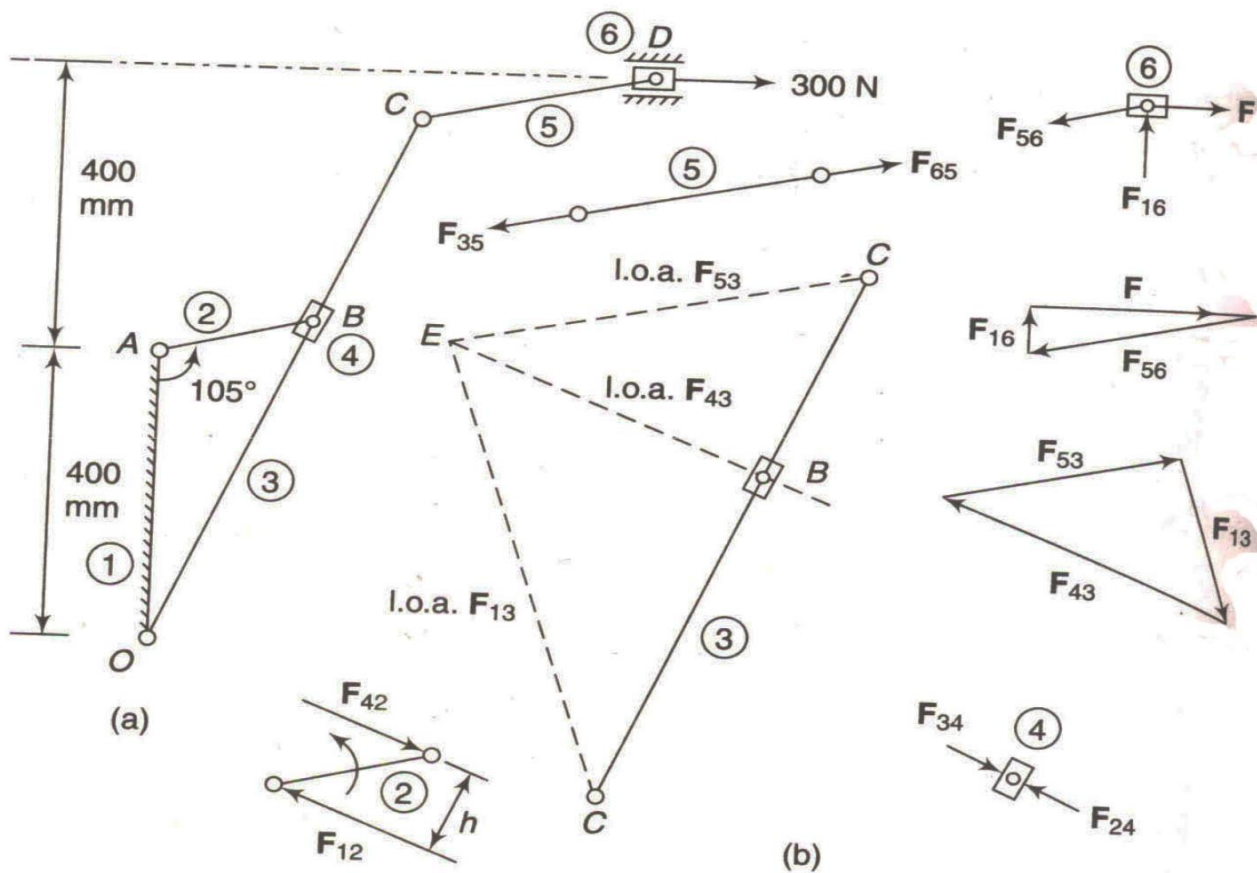
Determine the torque T_2 required to overcome the force F_E along the link 6.

$AD=30\text{mm}$, $AB=90\text{mm}$, $O_4 B=60\text{mm}$, $DE=80\text{mm}$, $O_2 A=50\text{mm}$, $O_2 O_4 =70\text{mm}$



Problem No 6

For the static equilibrium of the quick return mechanism shown in fig. 12.11 (a), determine the input torque T_2 to be applied on link AB for a force of 300N on the slider D. The dimensions of the various links are $OA=400\text{mm}$, $AB=200\text{mm}$, $OC=800\text{mm}$, $CD=300\text{mm}$

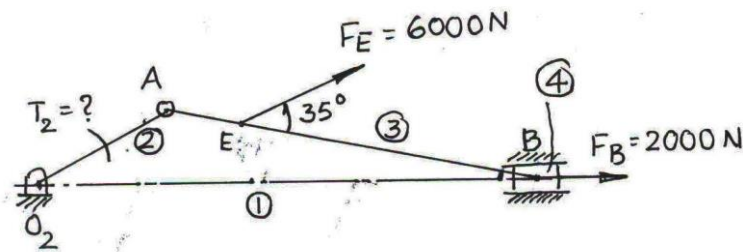


Then, torque on link 2,

$$T_2 = F_{42} \times h = 403 \times 120 = 48\,360 \text{ N counter-clockwise}$$

Problem No 7. Determine T_2 to keep the body in equilibrium. $O_2A = 100\text{MM}$, $AB = 250\text{MM}$, $AE = 50\text{MM}$, $\angle AOB = 30^\circ$

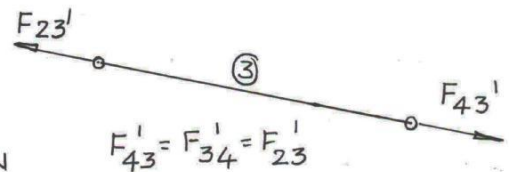
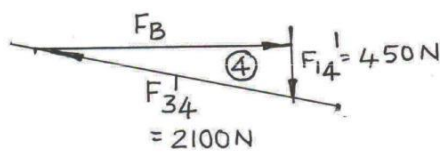
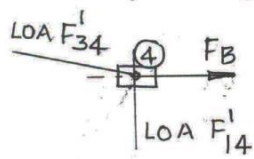
1CM = 50 MM



The problem is solved as two sub problems:

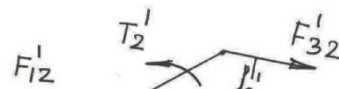
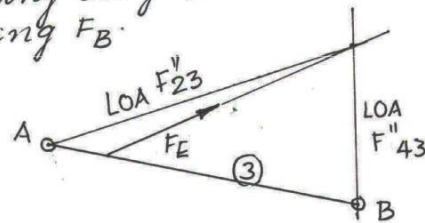
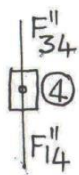
- i) Considering only F_B
- ii) Considering only F_E

(i) Considering only F_B & neglecting F_E ; scale 1CM = 500N



$$F_{43}^I = F_{34}^I = F_{23}^I$$

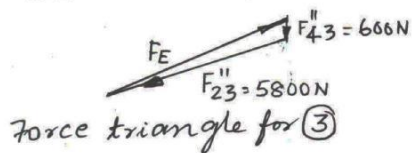
(ii) Considering only F_E & neglecting F_B



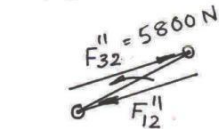
$$T_2^I = F_{32}^I \times h^I = 2100 \times 70$$

$$147000 \text{ N-MM (ccw)}$$

1CM = 2000N



Force triangle for ③



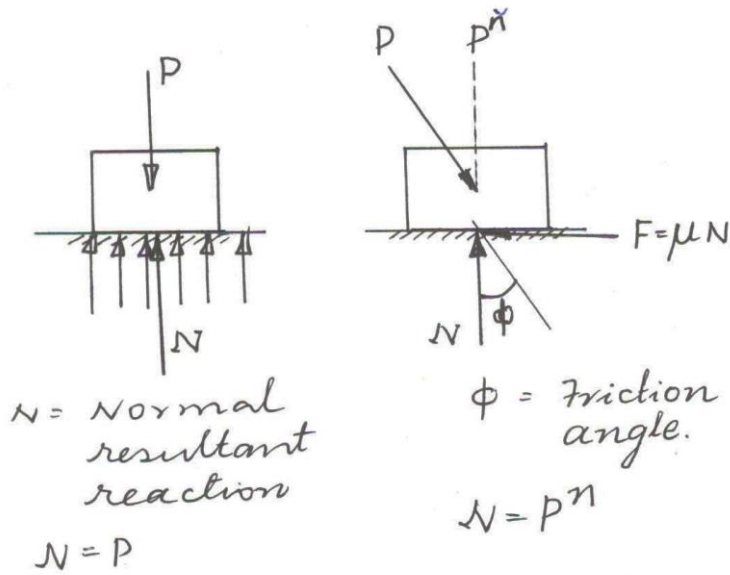
$$T_2^{II} = F_{32}^{II} \times h^{II} = 5800 \times 20 = 116000 \text{ N-MM (ccw)}$$

$$T_2 = T_2^I + T_2^{II} = 263000 \text{ N-MM (ccw)}$$

1.7 Force Analysis considering friction.

If friction is considered in the analysis, the resultant force on a pin doesn't pass through the centre of the pin. Coefficient of friction μ is assumed to be known and is independent of load and speed.

Friction in sliding member.

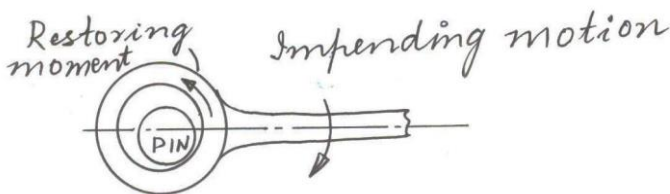


F = Frictional force

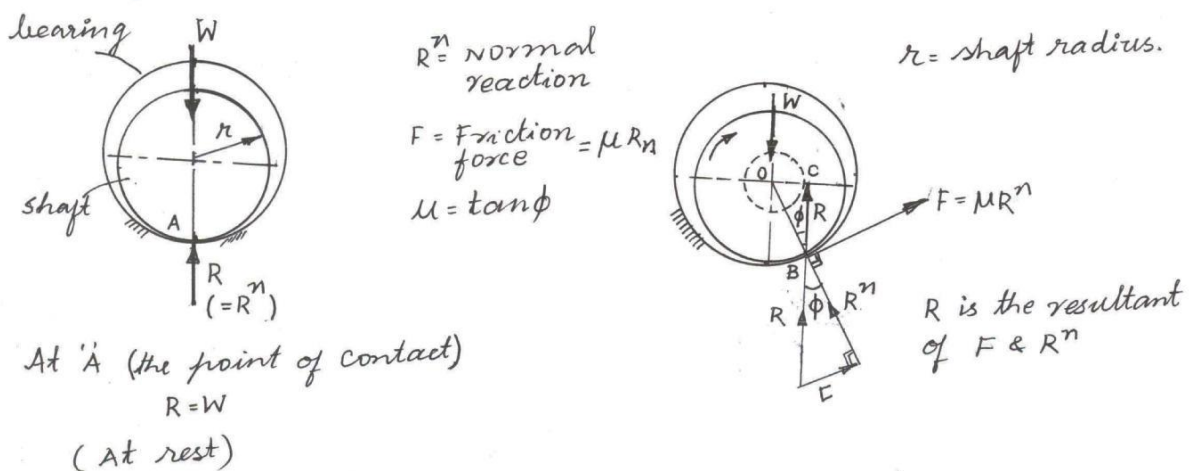
μ = coefficient of friction

$$\tan \phi = \mu = \frac{\mu N}{N}$$

Friction at pin points (bearings) & friction circle.



When a shaft revolves in a bearing, some power is lost due to friction between surfaces.



While rotating, the point of contact shifts to B; R^n passes through B.

The resultant 'R' is in a direction opposite to ω .

The circle drawn at O, with OC as radius is called 'FRICTION CIRCLE'

For the shaft to be in equilibrium; $W = R$

$$\begin{aligned} \text{Frictional moment } M &= R \times OC \\ &= W \times OC \\ &= W \times r \sin \phi \\ &= W \times r \tan \phi \\ &(\sin \phi \approx \tan \phi, \text{ for small } \phi) \\ &\text{i.e, } M = w \times r \times \mu \end{aligned}$$

$$\therefore \text{Radius of the friction circle (OC)} = \mu r.$$

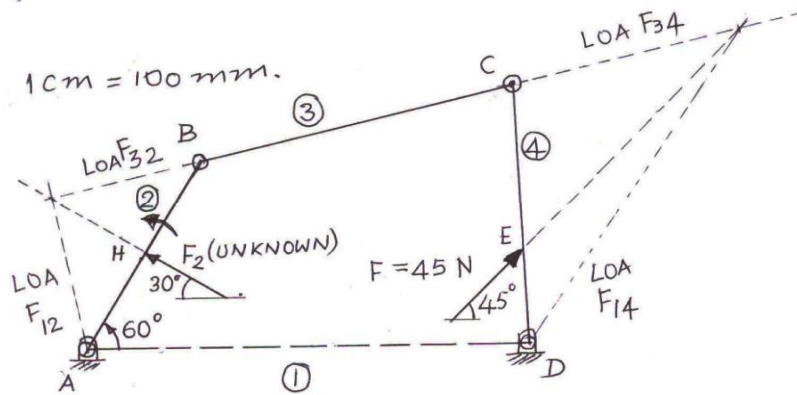
The friction circle is used to locate the line of action of the force between the shaft (pin) and the bearing or a pin joint. The direction of the force is always be tangent to it (friction axis) Friction axis: the new axis along which the thrust acts.

Problem No 8.

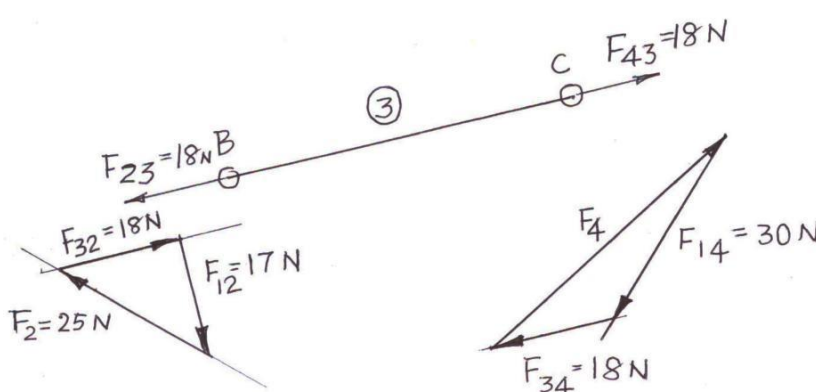
In a four bar mechanism ABCD, AB=350mm, BC=50mm, CD=400mm, AD=700mm, DE=150mm, $\angle DAB = 60^\circ$, AD is fixed. Determine the force on link AB required at the mid point, in the direction shown, for static equilibrium. $\mu=0.4$ for each revolving pair. Assume CCW impending motion of AB. Radius of each journal is 50mm.

Also find the torque on AB for its impending CW motion.

Analysis for CCW motion



Solve the problem neglecting friction to know the magnitudes and directions of forces



$$\text{Radius of the friction circle} = \mu \times \text{journal radius} = 0.4 \times 50 = 20 \text{ mm}$$

Analysis with Friction considered---

AB rotates CCW, DC rotates CCW

ABC decreasing, LBCD increasing

At C:

BCD increases & 3 rotates CW w.r.t 4

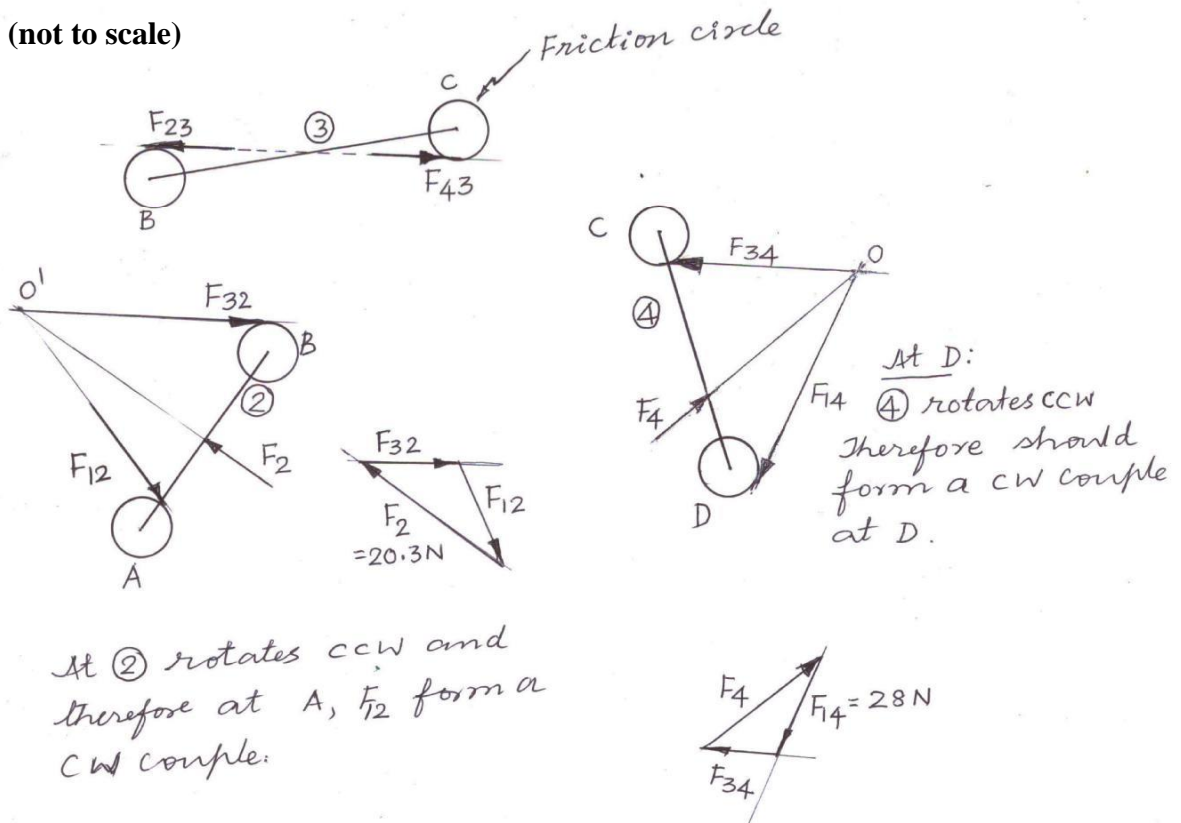
Therefore, F_{43} opposes the rotation of 4 by generating a CCW friction couple at C

At B:

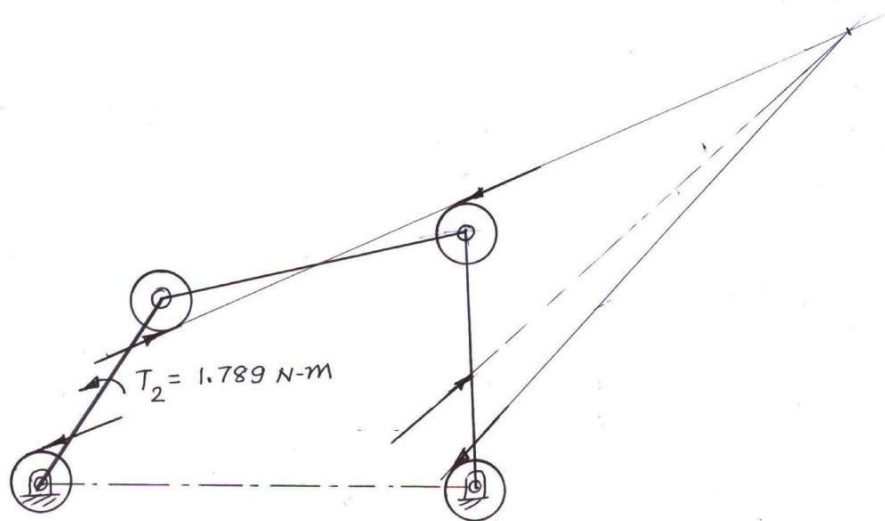
BCD decreases & 3 rotates CW w.r.t 2

Therefore, F_{23} forms a CCW friction couple at B

(not to scale)



For CW rotation of AB



OUT COMES

- 1) Students will be able to analyze mechanism for a given force system by graphical method.
- 2) Students will be able to analyze mechanism considering and not considering friction.
- 3) Student will be able to analyze 4 bar and single slider mechanism for the given static force.

Exercise

1. State D'Alembert's principle.
2. Define static force analysis
3. What is free body diagram?
4. The lengths of crank and connecting rod of horizontal steam engine are 300 mm and 1.2 m respectively. When the crank has moved 30° from the inner dead center, the acceleration of piston is 35 m/s^2 . The average frictional resistance to the motion of piston is equivalent to a force of 550 N and net effective steam pressure on piston is 500 kN/m^2 . The diameter of piston is 0.3 m and mass of reciprocating parts is 160 kg. Determine (i) Reaction on the cross-head guides; (ii) Thrust on the crankshaft bearings; and (iii) Torque on the crank shaft.

FURTHER READING

- 1) Theory of machines and mechanisms by Dr.Jagdishlal, Metropolitan Book co. Pvt. Ltd., New Delhi
- 2) Mechanisms and Dynamics of machinery by Hamiton H.Mabie and Fred W.Ocvirk, John Wiley & sons, Newyork.
- 3) Machine Dynamics (DOM), Vol ii, G.Bapaiah, Mechanical Engineering, Monograph Series, IIT, Madras.
- 4) Theory of Machines, by S.S Rathan, Tata McGraw-hill.
- 5) Mechanism & Machine Theory by Ashok G.Ambekar, Prentice Hall of India Pvt. Limited, New Delhi – 110001, 2007.