

MODULE-4

RESONANCE CIRCUITS

Introduction:

Resonance is a condition in an RLC circuit in which the capacitive and inductive Reactance's are equal in magnitude, there by resulting in purely resistive impedance. If a sinusoidal signal is applied to the network, the current is out of phase with the applied voltage. Under special condition, impedance offered by the network is purely resistive and frequency at which the net reactance of the circuit is zero is called resonant frequency. It is denoted by f_0 . At resonance, the power factor is unity and energy released by one reactive element is equal to the energy released by the other reactive element in the circuit and the total power in the circuit is the average power dissipated by the resistive element.

At resonance, the impedance Z offered by the circuit is equal to resistance of the circuit. Net reactance is equal to zero.

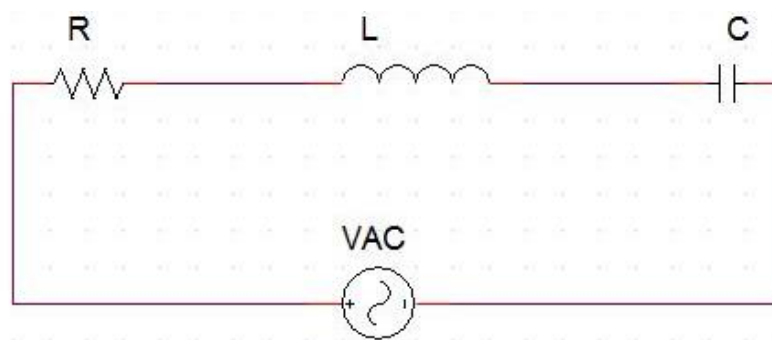
There are two types of resonance:

- Series resonance
- Parallel resonance

Resonance Parameters:

1. $Z = R$
2. $\omega L - \frac{1}{\omega C} = 0 \Rightarrow X_L - X_C = 0$
3. $I_{max} = \frac{V}{R}$
4. Power factor = 1
5. Quality factor $Q = \frac{\omega_0 L}{R}$

Series Resonance



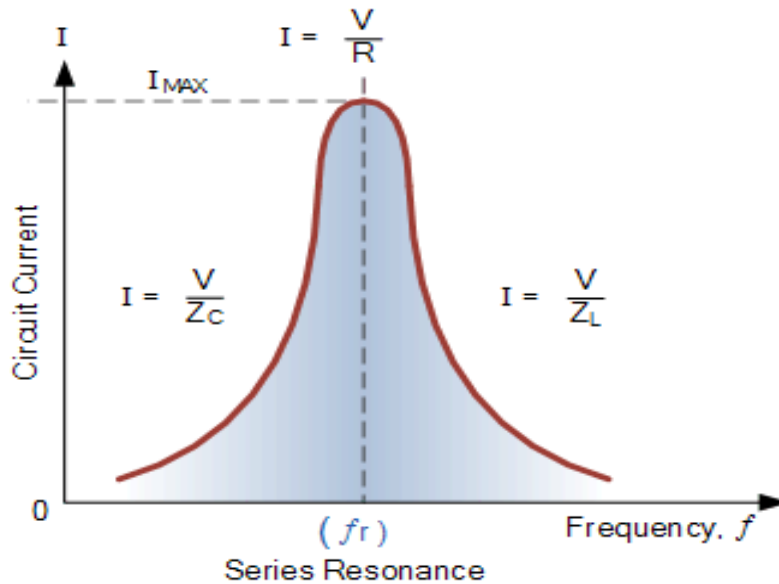
A series resonance circuit consists of an inductance L , resistance R and capacitance C , the RLC circuit is supplied with a sinusoidal voltage from an AC source. The resonance condition in AC circuits can be achieved by varying frequency of the Source. The current flowing through circuit is I , impedance is Z .

$$Z = R + j(X_L - X_C)$$

$$Z = R + j(\omega L - \frac{1}{\omega C}) \dots\dots\dots (1)$$

The current flowing through the circuit is $I = \frac{V}{Z}$ (2)

The circuit is said to be at resonance when the net reactance of the circuit is zero or inductive reactance is equal to capacitive reactance. By varying frequency of source, X_L is made equal to X_C . From (1), reactance part will become zero and $Z=R$.



To Derive Expression for Resonant Frequency:

At Resonance, $X_L = X_C$, $f = f_0$, $\omega = \omega_0$

$$\begin{aligned} \omega_0 L &= \frac{1}{\omega_0 C} \\ 2\pi f_0 L &= \frac{1}{2\pi f_0 C} \\ 4\pi^2 f_0^2 &= \frac{1}{LC} \\ f_0^2 &= \frac{1}{4\pi^2 LC} \\ f_0 &= \frac{1}{2\pi\sqrt{LC}} \end{aligned}$$

Current at Resonance

$$I_0 = I_{max} = \frac{V}{R}$$

Power factor = 1 at resonance

Expressions for f_L and f_C :

f_c . Frequencies at which voltage across inductor and capacitor are maximum are f_L and

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) \dots \dots \dots (1)$$

$$I = \frac{V}{Z} = \frac{V}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \dots \dots \dots (2)$$

Voltage across the capacitor (V_C) is:

$$V_C = IX_C = \frac{I}{j\omega C} \dots \dots \dots (3)$$

Substituting I from equation 2,

$$V_C = \frac{1}{j\omega C} \left[\frac{V}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \right] \dots \dots \dots (4)$$

$$|V_C| = \frac{1}{\omega C} \left[\frac{|V|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \right] \dots \dots \dots (5)$$

The voltage across the capacitance (V_C) is maximum at frequency (f_c). We calculate the frequency by taking the derivative of equation 5. We take square of the equation to make the computation easier.

$$\frac{d|V_C|^2}{d\omega} = 0$$

$$f_c = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$f_c = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{R^2 C}{2L}}$$

$$f_c = f_0 \sqrt{1 - \frac{R^2 C}{2L}} \dots \dots \dots (6)$$

Voltage across the inductor (V_L) is:

$$V_L = IX_L = I(j\omega L) \dots \dots \dots (7)$$

$$V_L = j\omega L \left[\frac{V}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \right] \dots \dots \dots (8)$$

$$|V_L| = j\omega L \left[\frac{|V|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \right] \dots \dots \dots (9)$$

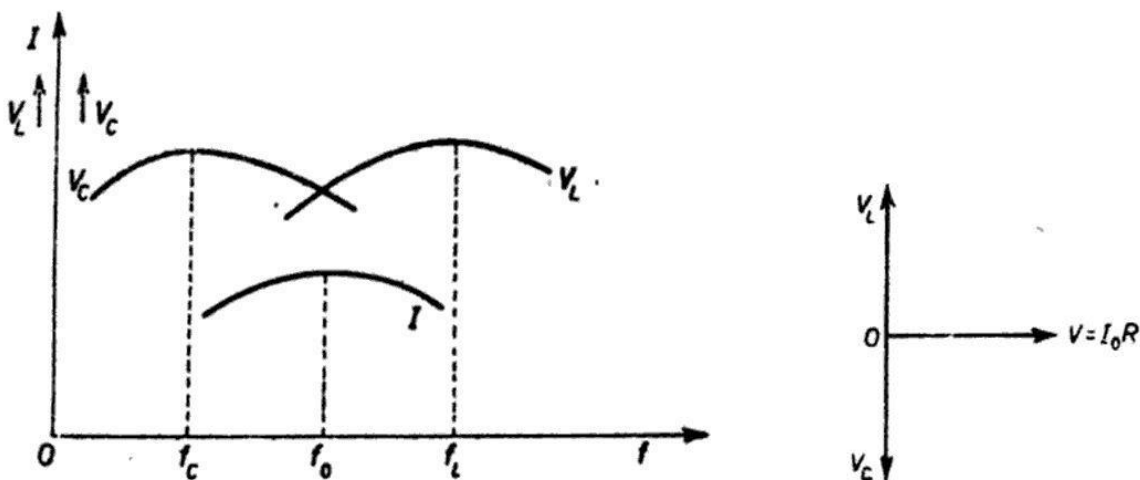
f_L can be computed by equating the derivative of equation 9. We take square of the equation to make the computation easier.

$$\frac{d|V_L|^2}{d\omega} = 0$$

$$f_L = \frac{1}{2\pi \sqrt{LC - \frac{R^2 C^2}{2}}}$$

$$f_L = \frac{f_0}{\sqrt{1 - \frac{R^2 C}{2L}}}$$

At resonance, $V_L = V_C$ in magnitude, but are out of phase. If R is extremely small, frequencies f_L and f_C are $\approx f_0$. The below figure shows voltage variation with frequency:



Problem:

Q: A series RLC circuit has $R = 25\Omega$, $L = 0.04H$, $C=0.01\mu F$. If 1V sine signal of same frequency as the resonance frequency is to be applied to the circuit, calculate frequency at which voltage across inductor and capacitor are maximum. Also calculate voltage across L and C, at resonant frequency.

Ans: Resonant frequency of RLC series circuit is,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 7.95775kHz$$

The frequency at which voltage across inductor is maximum is,

$$f_L = \frac{f_0}{\sqrt{1 - \frac{R^2 C}{2L}}} = \frac{f_0}{0.999} = 7.965kHz$$

The frequency at which voltage across capacitor is maximum is,

$$f_c = f_0 \sqrt{1 - \frac{R^2 C}{2L}} = f_0 * 0.999 = 7.949 \text{ kHz}$$

At resonance, $Z = R = 25 \Omega$.

$$I_0 = \frac{V}{R} = \frac{1}{25} = 0.04 \text{ A}$$

$$V_L = I_0 \omega_0 L = 0.04 \times 2\pi \times 7.95775 \times 10^3 \times 0.04$$

$$V_L = 80 \text{ V}$$

$$V_C = \frac{I_0}{\omega_0 C} = \frac{0.04}{2\pi \times 7.95775 \times 10^3 \times 0.01 \times 10^{-6}}$$

$$V_C = 80 \text{ V}$$

$$Q = \frac{\omega_0 L}{R} = 80$$

Quality Factor of an Inductor:

$$Q = 2\pi \left[\frac{\text{max. energy stored in inductor}}{\text{energy dissipated by resistor}} \right]$$

Let V_m be the peak voltage of applied signal, I_m be the peak current through the circuit.

The maximum energy stored in L is $E = \frac{1}{2} L I_m^2 \dots \dots \dots (1)$

The RMS power dissipated in R is $P = I_{rms}^2 R = \left(\frac{I_m}{\sqrt{2}} \right)^2 R \dots \dots \dots (2)$

The energy dissipated in resistor per cycle is power \times time period of one cycle.

$$E = P.T \text{ but } T = \frac{1}{f}$$

Therefore, energy dissipated in R is $\left(\frac{I_m}{\sqrt{2}} \right)^2 R \cdot \frac{1}{f} \dots \dots \dots (3)$

Substitute (1) and (3) in Q,

$$Q = 2\pi f \cdot \frac{L}{R}$$

$$Q = \frac{\omega L}{R}$$

Quality Factor of Capacitor:

$$Q = \frac{1}{\omega RC}$$

Quality Factor at Resonance Q_0

The quality factor of a series resonance circuit is quality factor of an inductor or quality factor of capacitor at resonant frequency.

$$f = f_0 \quad \omega = \omega_0 \quad Q = Q_0$$

$$Q_0 = \frac{\omega_0 L}{R} \text{ or } \frac{1}{\omega_0 C R}$$

$$\text{w.k.t } \omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q_0 = \frac{1}{\sqrt{LC}} \times \frac{L}{R}$$

$$= \frac{1}{\sqrt{LC}} \times \frac{\sqrt{L} \times \sqrt{L}}{R}$$

$$Q_0 = \frac{1}{R} \sqrt{\frac{L}{C}}$$

Voltage Magnification Factor:

At resonance, the magnitude of voltage across the inductor and capacitor will be same and magnitude will get amplified by the quality factor (Q)

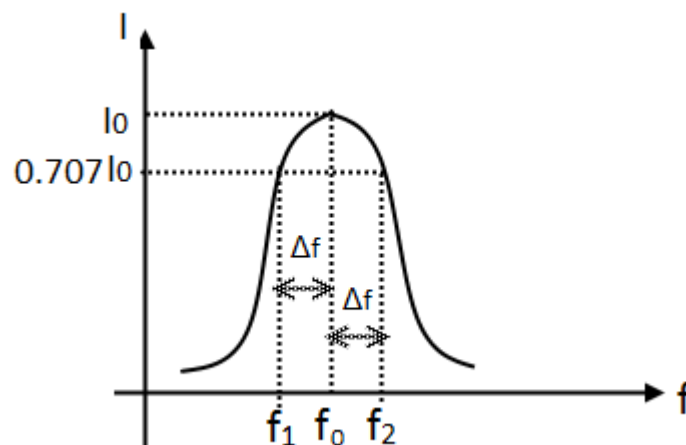
$$V_C = I_0 X_C = I_0 \cdot \frac{1}{\omega_0 C} = \frac{V}{R} \cdot \frac{1}{\omega_0 C} = Q_0 V$$

Where V is applied voltage.

$$V_L = I_0 \times X_L = I_0 \times \omega_0 L = \frac{V}{R} \cdot \omega_0 L = Q_0 V$$

Hence, voltage across capacitor and inductor gets amplified by quality factor.

Cut-Off Frequencies:



In series resonance, the current falls to $\frac{1}{\sqrt{2}}$ times the maximum current I_0 . The corresponding frequencies are called cut-off frequencies f_1 and f_2 .

At resonance, $Z = R$, $I_0 = \frac{V}{R}$ which is the maximum current and maximum power is $P_{max} = I_0^2 R$ when current falls to $\frac{1}{\sqrt{2}}$ times maximum value, there are 2 frequencies at $\frac{I_0}{\sqrt{2}}$.

At lower cut-off frequency, power is half the maximum power

$$P_{max} = \left[\frac{I_0}{\sqrt{2}} \right]^2 \cdot R = \frac{I_0^2 R}{2} = \frac{P_{max}}{2}$$

At upper cut-off frequency too, power is half the maximum power

$$P_{max} = \left[\frac{I_0}{\sqrt{2}} \right]^2 \cdot R = \frac{I_0^2 R}{2} = \frac{P_{max}}{2}$$

Cut-off frequencies are also called Half Power Frequencies.

Bandwidth:

The frequency range between the cut-off frequencies f_1 and f_2 is called bandwidth.

$$BW = f_2 - f_1$$

$$BW = 2 \cdot \Delta f \text{ Hz}$$

Where $\Delta f = f_2 - f_0$ or $f_0 - f_1$

Selectivity:

At resonance frequency, impedance is minimum, current is maximum. Impedance varies with frequency. Thus, a series RLC circuit possesses selectivity.

Selectivity of the circuit is defined as the ability of the circuit to distinguish between desired and undesired frequency. It is ratio of resonant frequency to the bandwidth.

$$\text{Selectivity} = \frac{f_0}{f_2 - f_1} = \frac{f_0}{BW}$$

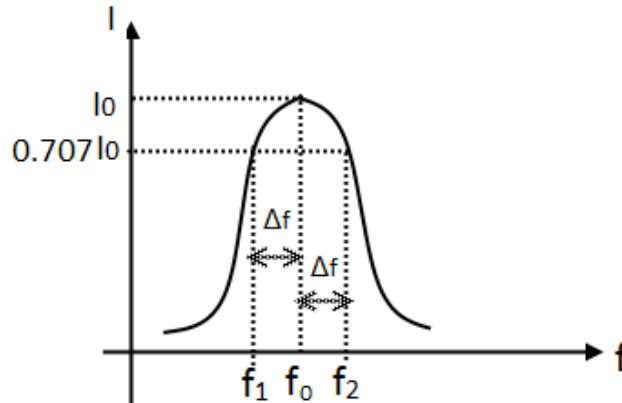
$$= \frac{f_0}{R/2\pi L}$$

$$\text{Selectivity} = \frac{2\pi f_0 L}{R}$$

$$\text{Selectivity} = \frac{\omega_0 L}{R} = Q_0$$

If Q_0 is very high, frequency response becomes sharper or narrower and BW reduces.

Q) Derive Expression for cut-off Frequencies of a Series RLC Circuit



The current flowing through circuit is $I = \frac{V}{Z}$

$$I = \frac{V}{Z} = \frac{V}{R + j(\omega L - \frac{1}{\omega C})}$$

$$|I| = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \dots \dots \dots (1)$$

At resonance, $Z = R$

$$I_0 = \frac{V}{R}$$

At cut-off points,

$$|I| = \frac{I_0}{\sqrt{2}} = \frac{V/R}{\sqrt{2}} = \frac{V}{R\sqrt{2}}$$

Substitute in (1)

$$\frac{V}{R\sqrt{2}} = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

Squaring on both sides,

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\omega L - \frac{1}{\omega C} = \pm R \dots \dots \dots (2)$$

The equation shows that at half power or cut-off frequencies, the reactive part = resistive part. This equation is quadratic in ' ω ' which gives 2 values of ω_1 and ω_2 .

At upper cut-off frequency, $f = f_2$, $\omega = \omega_2$.

$$\omega_2 L - \frac{1}{\omega_2 C} = R \dots \dots \dots (3)$$

At lower cut-off frequency, $f = f_1$, $\omega = \omega_1$.

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \dots \dots \dots (4)$$

From (3),

$$\omega_2 L - \frac{1}{\omega_2 C} = R$$

$$\omega_2^2 L - \frac{1}{C} = \omega_2 R$$

$$L \left[\omega_2^2 - \frac{1}{LC} - \frac{\omega_2 R}{L} \right] = 0$$

$$\omega_2^2 - \frac{\omega_2 R}{L} - \frac{1}{LC} = 0$$

$$\omega_2 = \frac{\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \left(\frac{4}{LC}\right)}}{2}$$

$$\omega_2 = \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

$$f_2 = \frac{1}{2\pi} \left[\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \right] \dots \dots \dots (5)$$

From (4),

$$\omega_1 L - \frac{1}{\omega_1 C} = -R$$

$$\omega_1^2 L - \frac{1}{C} + \omega_1 R = 0$$

$$L \left[\omega_1^2 - \frac{1}{LC} + \frac{\omega_1 R}{L} \right] = 0$$

$$\omega_1^2 - \frac{1}{LC} + \frac{\omega_1 R}{L} = 0$$

$$\omega_1^2 + \frac{\omega_1 R}{L} - \frac{1}{LC} = 0$$

$$\omega_1 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \left(\frac{4}{LC}\right)}}{2}$$

$$\omega_1 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

$$f_1 = \frac{1}{2\pi} \left[-\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \right] \dots \dots (6)$$

Q) Show that the resonance frequency f_o of a series resonance circuit is equal to geometric mean (GM) of the two cut-off frequencies.

$$|I| = \frac{V}{|Z|} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \dots \dots (1)$$

At resonance, $Z = R$

$$I_o = \frac{V}{R}$$

At cut-off frequencies,

$$|I| = \frac{I_o}{\sqrt{2}} = \frac{V}{R\sqrt{2}} \dots \dots (2)$$

$$\frac{V}{R\sqrt{2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\omega L - \frac{1}{\omega C} = \pm R \dots \dots (3)$$

At upper cut-off frequency $f = f_2$, $\omega = \omega_2$

$$\omega_2 L - \frac{1}{\omega_2 C} = R \dots \dots (4)$$

At lower cut-off frequency, $f = f_1$, $\omega = \omega_1$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \dots \dots (5)$$

Add equations (4) and (5)

$$\omega_2 L - \frac{1}{\omega_2 C} + \omega_1 L - \frac{1}{\omega_1 C} = 0$$

$$(\omega_2 + \omega_1)L = \frac{1}{\omega_1 C} + \frac{1}{\omega_2 C}$$

$$(\omega_2 + \omega_1)L = \frac{1}{C} \left[\frac{1}{\omega_1} + \frac{1}{\omega_2} \right]$$

$$(\omega_2 + \omega_1)L = \frac{(\omega_2 + \omega_1)}{(\omega_1\omega_2 C)}$$

$$\omega_1\omega_2 = \frac{1}{LC} \dots \dots \dots (6)$$

$$2\pi f_1 \cdot 2\pi f_2 = \frac{1}{LC}$$

$$f_1 f_2 = \frac{1}{(2\pi)^2 LC}$$

Taking square root on both sides,

$$\sqrt{f_1 f_2} = \frac{1}{2\pi\sqrt{LC}}$$

W.k.t, resonant frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$. Therefore,

$$f_o = \sqrt{f_1 f_2}$$

This shows that f_o is the GM of the cut-off frequencies.

Establish the relation between Quality Factor and Bandwidth in a Series Resonance Circuit and Thereby Prove That $Q_o = \frac{f_o}{BW}$

$$|I| = \frac{V}{|Z|} = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

At resonance, $|I| = \frac{V}{R}$ because $Z = R$

At cut-off frequencies,

$$|I| = \frac{I_o}{\sqrt{2}} = \frac{V}{R\sqrt{2}}$$

$$\frac{V}{R\sqrt{2}} = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

$$R^2 + (\omega L - \frac{1}{\omega C})^2 = 2R^2$$

$$(\omega L - \frac{1}{\omega C})^2 = R^2$$

$$\omega L - \frac{1}{\omega C} = \pm R$$

At upper cut-off frequency $f = f_2$, $\omega = \omega_2$

$$\omega_2 L - \frac{1}{\omega_2 C} = R \dots \dots \dots (1)$$

At lower cut-off frequency, $f = f_1$, $\omega = \omega_1$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \dots \dots \dots (2)$$

(1) - (2) gives

$$\omega_2 L - \frac{1}{\omega_2 C} - \omega_1 L + \frac{1}{\omega_1 C} = 2R$$

$$(\omega_2 - \omega_1)L + \frac{1}{C} \left[\frac{1}{\omega_1} - \frac{1}{\omega_2} \right] = 2R$$

$$(\omega_2 - \omega_1)L + \frac{(\omega_2 - \omega_1)}{(\omega_1 \omega_2 C)} = 2R$$

$$L \left[(\omega_2 - \omega_1) + \frac{(\omega_2 - \omega_1)}{(\omega_1 \omega_2) LC} \right] = 2R$$

$$\omega_1 \omega_2 = \frac{1}{LC}$$

$$(\omega_2 - \omega_1) + (\omega_2 - \omega_1) = \frac{2R}{L}$$

$$2(\omega_2 - \omega_1) = \frac{2R}{L}$$

$$(\omega_2 - \omega_1) = \frac{R}{L}$$

$$B.W = (\omega_2 - \omega_1) = \frac{R}{L}$$

$$B.W = (f_2 - f_1) = \frac{R}{2\pi L}$$

$$\frac{L}{R} = \frac{1}{\omega_2 - \omega_1} \dots \dots \dots (3)$$

Substitute (3) in $Q_o = \frac{\omega_o L}{R}$,

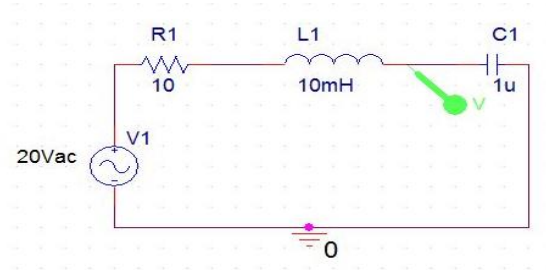
$$Q_o = \omega_o \cdot \frac{1}{\omega_2 - \omega_1}$$

$$= \frac{f_o}{f_2 - f_1}$$

$$Q_o = \frac{f_o}{BW}$$

Problems:

1. A series RLC circuit with $R=10\Omega$, $L=10\text{mH}$, $C=1\mu\text{F}$ has an applied voltage of 200V at resonance frequency f_0 . Calculate f_0 , V_L , V_R , V_C at resonance and also find Q and BW .



Solution:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10 \times 10^{-3} \times 10^{-6}}} = 1.591\text{kHz}$$

$$Q_o = \frac{\omega_o L}{R} = \frac{2\pi f_o L}{R} = \frac{2\pi \times 1.591 \times 10^3 \times 10 \times 10^{-3}}{10}$$

$$Q_o = 10$$

$$\text{w.k.t } Q_o = \frac{f_o}{BW}$$

$$BW = \frac{f_o}{Q_o} = \frac{1.591\text{k}}{10} = 159.1\text{Hz}$$

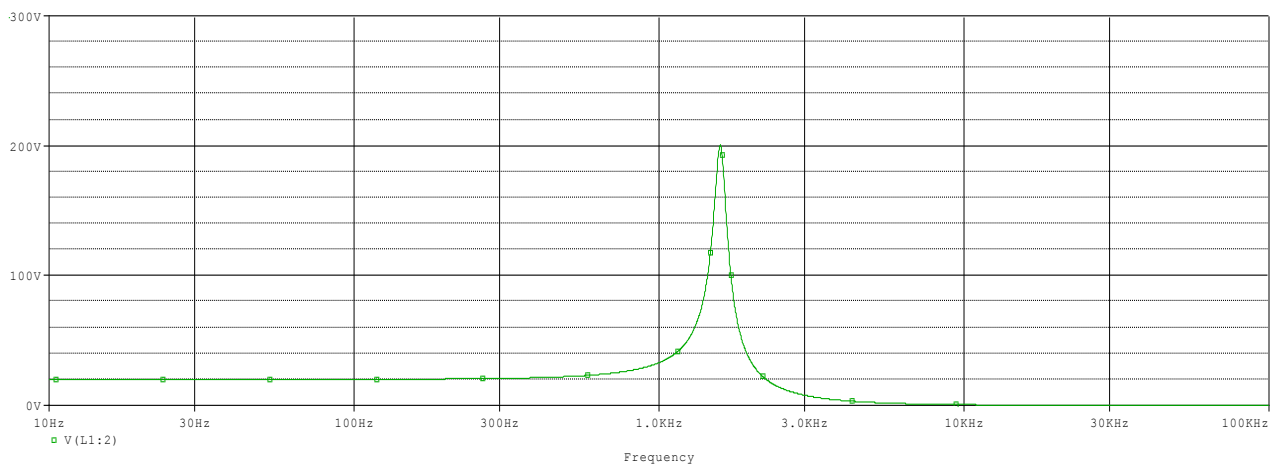
At resonance, $Z = R$. Current (I) is maximum.

$$I = \frac{V}{R} = \frac{200}{10} = 20\text{A}$$

At resonance, $|V_L| = |V_C| = Q_o V$

Voltage across L and C is, $|V_L| = |V_C| = 2000\text{V}$

$V_R = 200\text{V}$ [$X_L = X_C$]



2. A series RLC circuit has $R=10\Omega$, $L=0.01H$, $C=0.01\mu F$ and is connected across $10mV$ supply. Calculate f_o , Q_o , BW , f_1 , f_2 , I_o .

Solution: At resonance,

$$I_o = \frac{V}{R}$$

$$I_o = \frac{10 \times 10^{-3}}{10} = 1mA$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01 \times 0.01 \times 10^{-6}}} = 15.91kHz$$

$$Q_o = \frac{\omega_o L}{R} = \frac{2\pi f_o L}{R} = \frac{2\pi \times 15.91 \times 10^3 \times 0.01}{10} = 100$$

$$BW = \frac{f_o}{Q_o} = \frac{15.91k}{100} = 159.1Hz$$

For high quality factor [$Q_o > 5$]

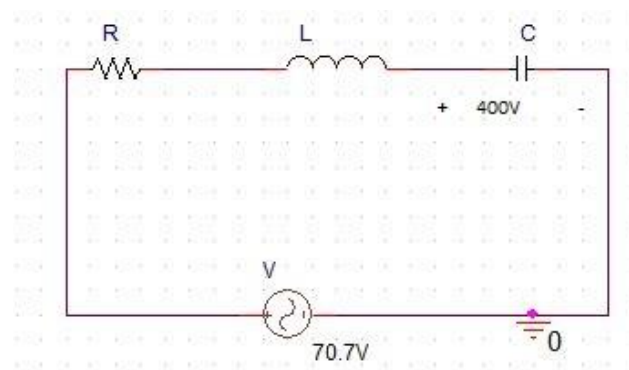
$$BW = 2\Delta F$$

$$f_2 = f_o + \Delta f$$

$$f_2 = f_o + \frac{BW}{2} = 15.91kHz + \frac{159.1}{2} = 15.99kHz$$

$$f_1 = f_o - \Delta f = f_o - \frac{BW}{2} = 15.91kHz - \frac{159.1}{2} = 15.83kHz$$

3. In a series RLC network at resonance, $V_c=400V$, impedance $Z=100\Omega$, $BW=75Hz$ with an applied voltage of $70.7V$. Find R , L , C .



Solution:

At resonance, $Z = R = 100\Omega$

$$V_R = 70.7V$$

At resonance, $|V_C| = Q_o V$

$$Q_o = \frac{|V_C|}{V} = \frac{400}{70.7} = 5.658$$

$$BW = 75\text{Hz} = \frac{R}{2\pi L}$$

$$L = \frac{R}{2\pi \times 75} = \frac{100}{2\pi \times 75} = \mathbf{0.21H}$$

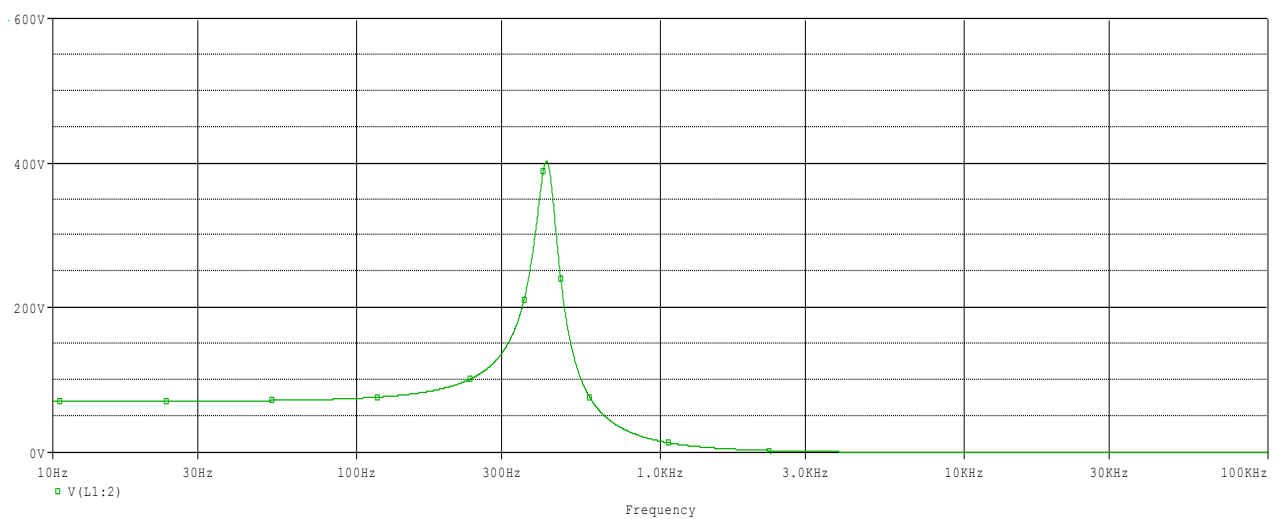
$$Q_o = \frac{1}{\omega CR}$$

$$C = \frac{1}{Q_o \omega R}$$

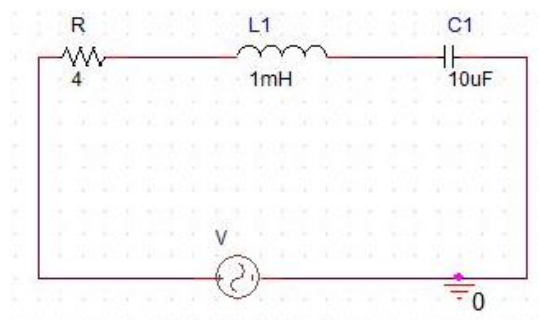
To calculate ω ,

$$f_o = Q_o \cdot BW = 5.658 \times 75 = 424.35\text{Hz}$$

$$C = \frac{1}{5.658 \times 2\pi \times 424.35 \times 100} = \mathbf{663.35nF}$$



4. A series RLC circuit has $R=4\Omega$, $L=1\text{mH}$, $C=10\mu\text{F}$. Find Q , BW , f_o , f_1 , f_2 .



Solution:

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{1 \times 10^{-3} \times 10 \times 10^{-6}}} = 1.591\text{kHz}$$

$$Q_o = \frac{\omega_o L}{R} = \frac{2\pi f_o L}{R} = 2.5$$

$$BW = \frac{R}{2\pi L} = \frac{4}{2\pi \times 1 \times 10^{-3}} = 636.62\text{Hz}$$

Since $Q < 5$

$$f_1 = \frac{1}{2\pi} \left[-\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \right] \dots\dots (6)$$

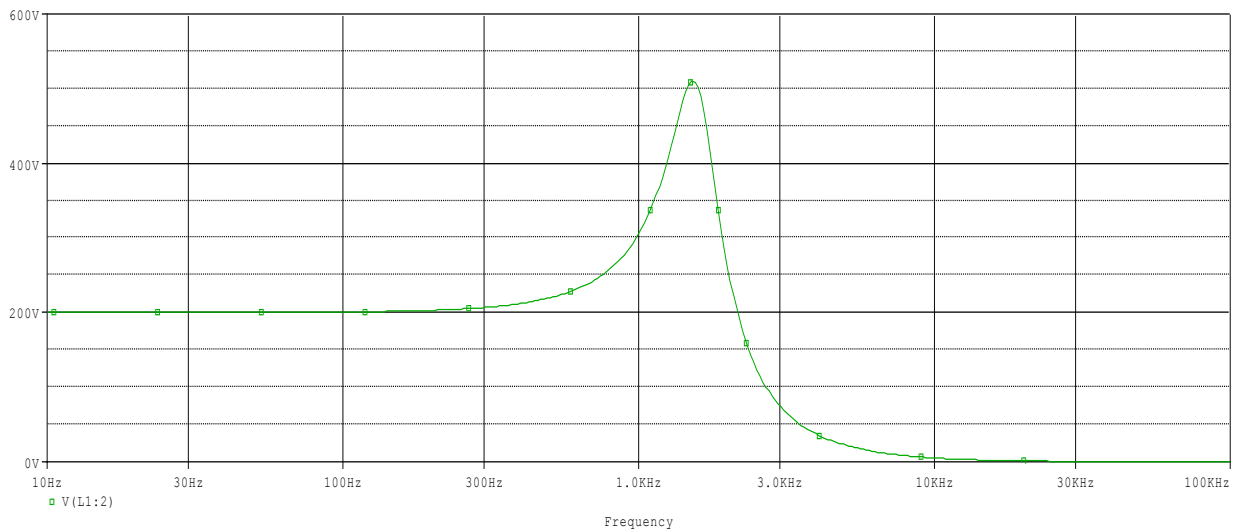
$$= \frac{1}{2\pi} [-2000 + 10.198 \times 10^3]$$

$$f_1 = 1303.49 \text{ Hz}$$

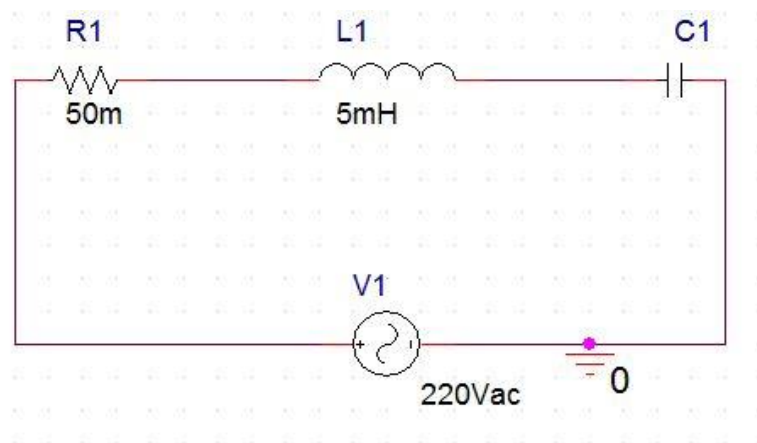
$$BW = f_2 - f_1 \Rightarrow f_2 = f_1 + BW = 1940.11 \text{ Hz}$$

As $Q_0 < 5$,

$$f_2 - f_0 \neq f_0 - f_1$$



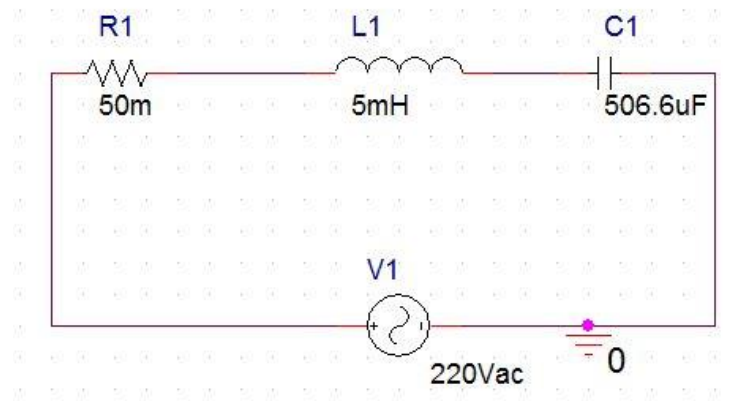
5. A 220V, 100Hz AC source supplies a series RLC circuit with a capacitor and a coil. If the coil has 50mΩ and 5mH inductance, find at $f_0=100\text{Hz}$, the value of capacitor. Also calculate Q_0 , f_1 , f_2 .



Solution:

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$C = \frac{1}{4\pi^2 L f_o^2} = 506.6\mu F$$



$$Q_o = \frac{\omega_o L}{R} = \frac{2\pi f_o L}{R} = 62.83$$

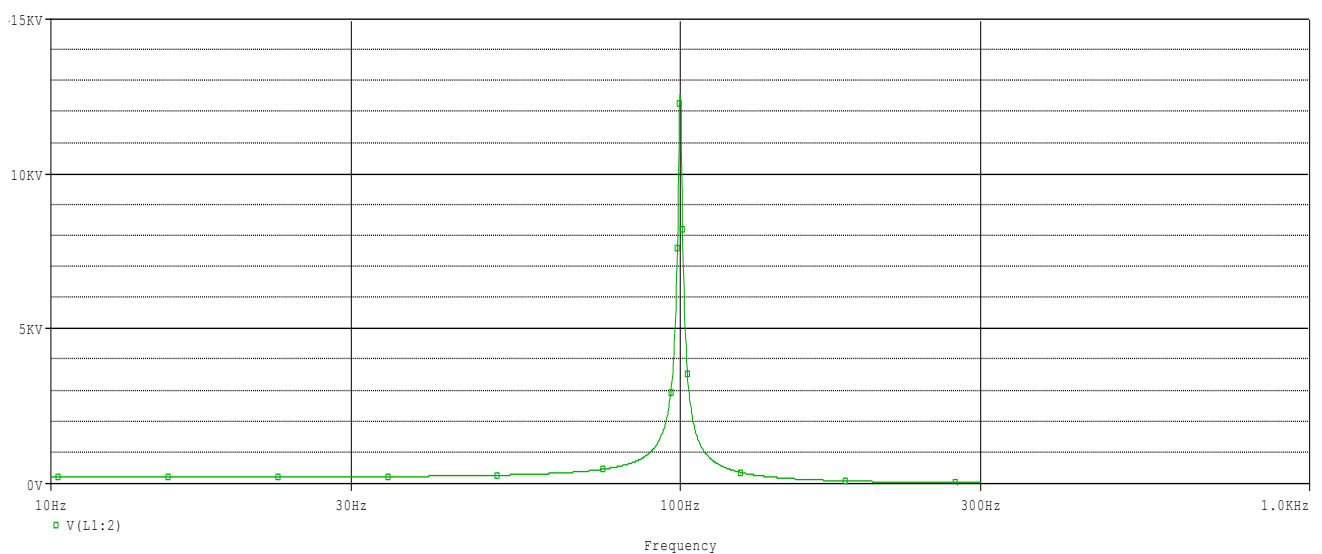
$$BW = \frac{f_o}{Q_o} = 1.59Hz$$

Since $Q_o > 5$

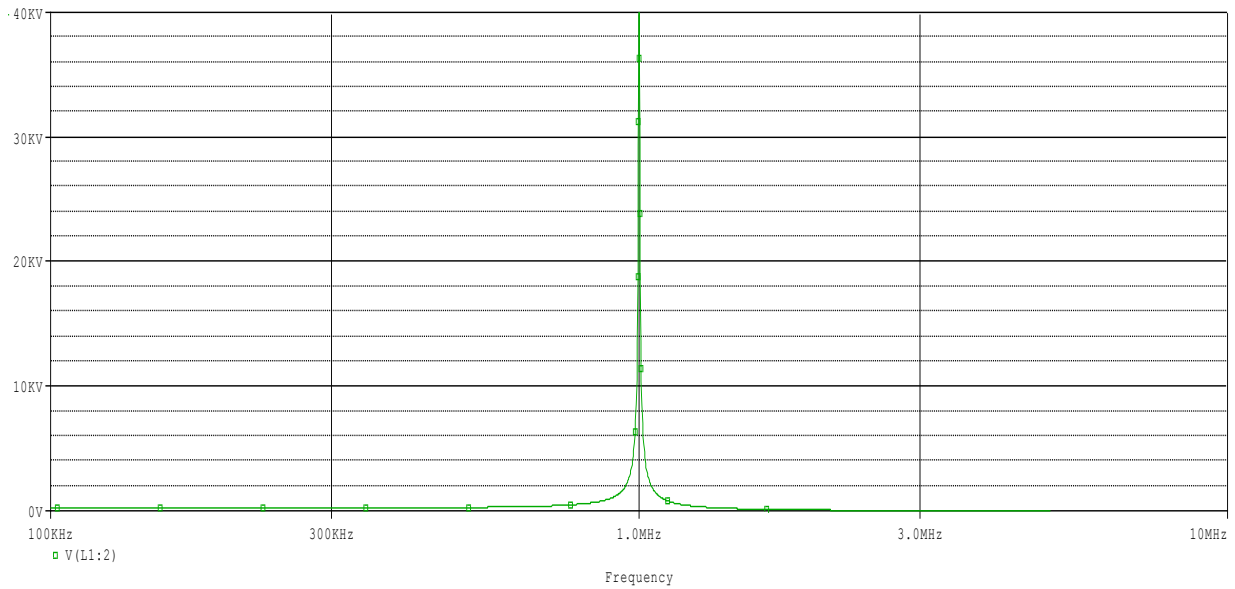
$$BW = 2\Delta F$$

$$f_2 = f_o + \Delta f = f_o + \frac{BW}{2} = 100 + \frac{1.59}{2} = 100.795Hz$$

$$f_1 = f_o - \Delta f = f_o - \frac{BW}{2} = 100 - \frac{1.59}{2} = 99.205Hz$$



6. It is required that a series RLC circuit should resonate a 1Mhz. Determine the values of R, L, C if BW=5kHz, Z=50Ω at resonance. $f_o=1\text{MHz}$, BW=5kHz



Solution:

$$Q_o = \frac{f_o}{BW} = 200$$

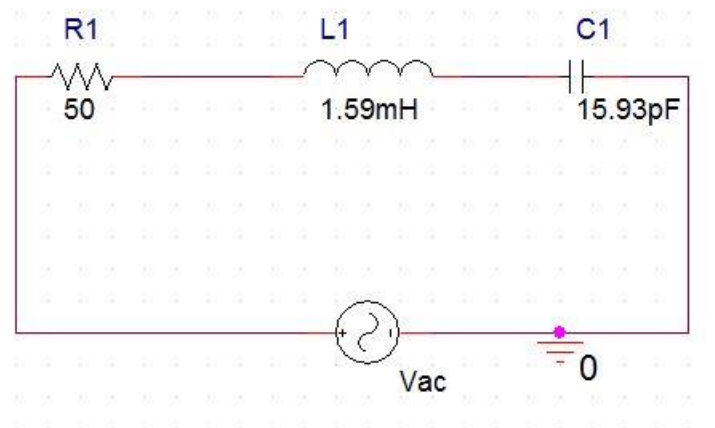
At resonance, $Z = R = 50\Omega$

$$Q_o = \frac{\omega_o L}{R}$$

$$L = \frac{Q_o R}{\omega_o} = \frac{200 \times 50}{2\pi \times 1 \times 10^6} = 1.59\text{mH}$$

$$Q_o = \frac{1}{\omega CR}$$

$$C = \frac{1}{Q_o \omega R} = 15.91\text{pF}$$



7. An RLC circuit has $R=1k$, $L=100mH$, $C=10\mu F$. If a voltage of $100V$ is applied across the circuit, find f_o , Q factor, half power frequencies.

Solution:

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100 \times 10^{-3} \times 10 \times 10^{-6}}} = \mathbf{159.15Hz}$$

$$Q_o = \frac{\omega_o L}{R} = \frac{2\pi \times 159.15 \times 100 \times 10^{-3}}{10^3} = \mathbf{0.1}$$

For $Q < 5$

$$f_1 = \frac{1}{2\pi} \left[-\frac{R}{2L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \left(\frac{1}{LC}\right)} \right] = \mathbf{1519.9Hz}$$

$$f_2 = \frac{1}{2\pi} \left[\frac{R}{2L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \left(\frac{1}{LC}\right)} \right] = \mathbf{2395.26Hz}$$

8. A series RLC circuit has $R=10\Omega$, $L=0.01H$, $C=0.01\mu F$ and it is connected across $10mV$ supply. Calculate f_o , Q_o , BW , f_1 , f_2 , I_o .

Solution:

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01 \times 0.01 \times 10^{-6}}} = \mathbf{15915.49Hz}$$

$$Q_o = \frac{\omega_o L}{R} = \frac{2\pi \times 15915.49 \times 0.01}{10} = \mathbf{100}$$

$$BW = \frac{f_o}{Q_o} = \mathbf{159.15Hz}$$

Since $Q_o > 5$

$$BW = 2\Delta F$$

$$f_2 = f_o + \Delta f = f_o + \frac{BW}{2} = 15915 + \frac{159}{2} = \mathbf{15994.5Hz}$$

$$f_1 = f_o - \Delta f = f_o - \frac{BW}{2} = 15915 - \frac{159}{2} = \mathbf{15835.425Hz}$$

Current at resonance,

$$I_o = \frac{V}{R} = \frac{10 \times 10^{-3}}{10} = \mathbf{10mA}$$

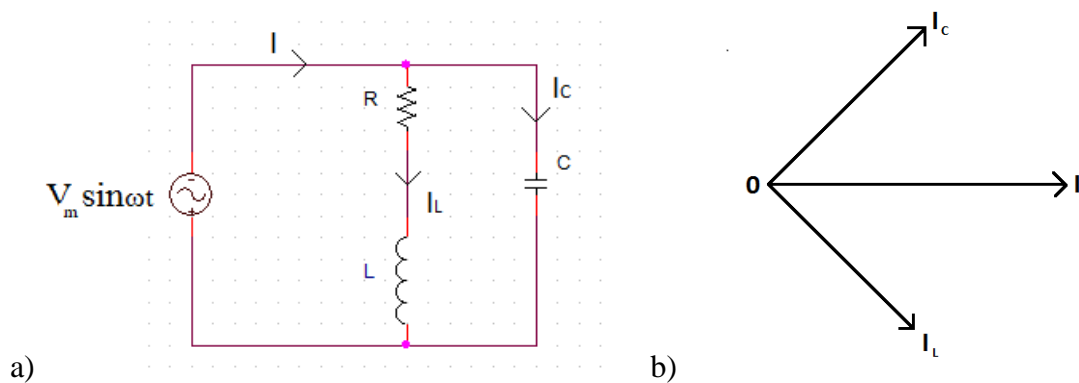
Parallel Resonance (Anti-Resonance)

Loss-Less Capacitor:

Q 1) Derive the expression for Resonance frequency of a circuit with loss less capacitor in parallel with a coil of Inductance ‘L’ and Resistance ‘R’.

A Parallel Resonant or Anti resonant circuit consists of an inductance ‘L’ in parallel with a capacitor ‘C’. A small Resistance ‘R’ is associated with ‘L’.

‘C’ is assumed to be loss less. The tuned circuit is driven by a voltage source. I is the current flowing through the circuit. I_C & I_L are the currents through the Capacitor and Inductor branch respectively.



a) Parallel resonance circuit with lossless capacitor b) phase diagram

Admittance of (inductance) branch containing R & L is given by (On rationalization)

$$Y_L \frac{1}{R + j\omega L} = \frac{R - j\omega L}{R^2 + \omega^2 L^2} \dots \dots \dots (1)$$

The admittance of branch containing capacitor ‘C’ is

$$Y_C = j\omega C \dots \dots \dots (2)$$

Total admittance of the circuit is

$$\begin{aligned}
 Y_T &= Y_L + Y_C = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C \\
 Y_T &= \frac{R}{R^2 + \omega^2 L^2} - \frac{j\omega L}{R^2 + \omega^2 L^2} + j\omega C \\
 Y_T &= \frac{R}{R^2 + \omega^2 L^2} + j \left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2} \right) \\
 &= G + j(B_C - B_L)
 \end{aligned}$$

At Anti Resonance, the circuit must have unity power factor i.e. at resonance $f = f_{ar}$ and the imaginary part of the admittance or the susceptance will be zero (Inductive Susceptance = Capacitive Susceptance at anti resonance).

At resonance $f = f_{ar}$ or $\omega = \omega_{ar}$.

$$\omega_{ar} C - \frac{\omega_{ar} L}{R^2 + \omega_{ar}^2 L^2} \dots \dots \dots (4)$$

On simplifying we get,

$$C(R^2 + \omega_{ar}^2 L^2) = L$$

$$(R^2 + \omega_{ar}^2 L^2) = \frac{L}{C}$$

$$\omega_{ar}^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_{ar}^2 = \frac{1}{L^2} \left(\frac{L}{C} - R^2 \right) = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\therefore \omega_{ar} = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Or Anti resonant frequency or parallel resonance frequency is

$$f_{ar} = \frac{1}{2\pi} * \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \dots \dots \dots (5)$$

This parallel resonance is possible iff $\frac{1}{LC} > \frac{R^2}{L^2}$ otherwise “ f_{ar} ” will be imaginary. It can be expressed by:

$$f_{ar} = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{CR^2}{L}} \dots \dots \dots (6)$$

f_{ar} can be expressed in another form (In terms of Q_o)

For series resonance,

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega_o = \frac{1}{\sqrt{LC}} = 2\pi f_o$$

Quality factor at resonance is

$$Q_o = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

$$\therefore Q_o^2 = \frac{\omega_o L}{R} \times \frac{1}{\omega_o CR} = \frac{L}{CR^2} \dots \dots \dots (7)$$

\therefore From equation (6)

$$f_{ar} = \frac{1}{2\pi\sqrt{LC}} \times \sqrt{1 - \frac{1}{Q_o^2}} \dots \dots \dots (8)$$

In term of series resonant frequency ‘ f_o ’ where $f_o = \frac{1}{2\pi\sqrt{LC}}$,

$$f_{ar} = f_o \times \sqrt{1 - \frac{1}{Q_o^2}} \dots \dots \dots (9)$$

For $Q_o > 10$, $f_{ar} \cong f_o$ as $\sqrt{1 - \frac{1}{Q_o^2}} \cong 1$.

The equation shows the anti-resonant frequency differs from that of series resonant circuit with the same circuit elements by the factor $\sqrt{1 - \frac{1}{Q_o^2}}$. This factor shows that if $Q_o < 1$ then, the **frequency of parallel resonance because imaginary**.

Admittance of anti-resonance circuit at Anti-resonance ‘ f_{ar} ’:

Admittance of parallel resonance circuit is

$$Y_T = \frac{R}{R^2 + \omega^2 L^2} + j \left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2} \right) \dots \dots \dots (1)$$

To get the admittance at anti resonance the susceptance part is considered as zero and consider only real part at anti resonance $B_L = B_C$ or $B_L - B_C = 0$.

\therefore At anti-resonance,

$$Y = Y_{ar} = \frac{R}{R^2 + \omega^2 L^2} = \frac{1}{Z_{ar}}$$

$$\frac{1}{Y_{ar}} = Z_{ar} = \frac{R^2 + \omega^2 L^2}{R}$$

We have,

$$R^2 + \omega^2 L^2 = \frac{L}{C}$$

$$\therefore Z_{ar} = \frac{L}{RC} = R_{ar} \dots \dots \dots (2)$$

At resonance f_{ar} , this is called **dynamic resistance** of the parallel resonant circuit at resonance.

Z_{ar} terms of Q:

At anti-resonance $\omega = \omega_{ar}$

We have,

$$R^2 + \omega^2 L^2 = \frac{L}{C}$$

$$R^2 \left[1 + \frac{\omega^2 L^2}{R^2} \right] = \frac{L}{C}$$

$$R^2 [1 + Q_o^2] = \frac{L}{C}$$

$$R [1 + Q_o^2] = \frac{L}{RC} = Z_{ar} = R_{ar}$$

$$\therefore Z_{ar} = R_{ar} = R (1 + Q_0^2) = \frac{L}{RC} \dots \dots \dots (3)$$

This is the impedance at anti-resonance and is also called dynamic resistance. For high 'Q', $R_{ar} = RQ_0^2$. When the coil resistance is small, Z_{ar} becomes high with the current will be minima at resonance frequency. Parallel tuned circuit is a rejecter circuit. Current at Anti resonance is:

$$I_{ar} = \frac{V}{Z_{ar}}$$

From equation (3), $R_{ar} = \frac{L}{RC}$, the impedance at anti resonance is a pure resistance; which indicates that R_{ar} is a function of the $\frac{L}{C}$ ratio and R_{ar} can be large if inductors of low resistance are employed. In terms of 'Q' and Resonance frequency ω_{ar} .

$$R_{ar} = \frac{L}{RC}, \text{ For } Q > 10.$$

Multiply and divide by ω_{ar} ,

$$R_{ar} = \frac{\omega_{ar}L}{R} \times \frac{1}{\omega_{ar} \cdot C} = \frac{Q_0}{\omega_{ar} \cdot C} \dots \dots \dots (4)$$

Similarly,

$$R_{ar} = \omega_{ar}L \times \frac{1}{\omega_{ar} \cdot RC} = \omega_{ar} \times L \times Q_0 \dots \dots (5)$$

$$\therefore R_{ar} = \frac{Q}{\omega_{ar}C} = \omega_{ar}LQ, \text{ at resonance}$$

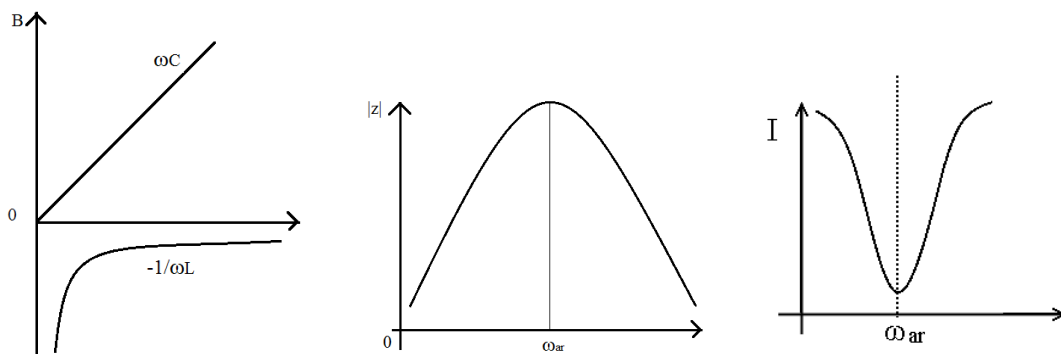
Quality Factor:

In a parallel - tuned circuit, the quality factor at resonance

$$Q = \frac{\omega_{ar}L}{R} = \frac{1}{\omega_{ar} \cdot R \cdot C}$$

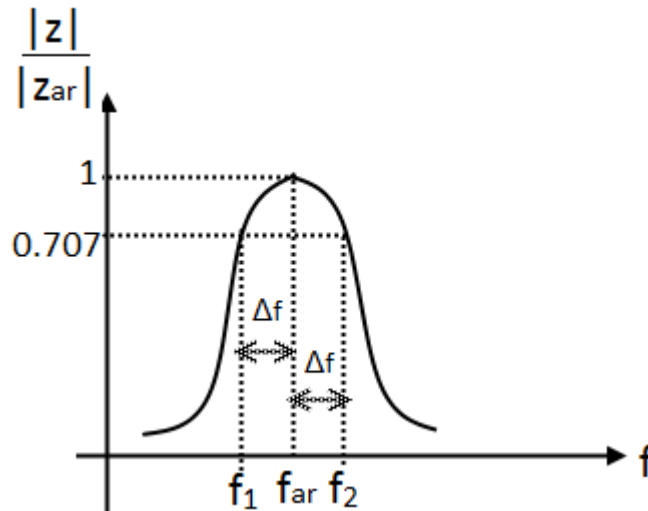
Bandwidth:

$$B.W = f_2 - f_1$$



(Variation of reactance with frequency)

SELECTIVITY AND BANDWIDTH:



The half power frequency points f_1 & f_2 for a parallel resonant circuit are obtained when the impedance ‘Z’ of the circuit becomes equal to 0.707 times the value of maximum impedance “ Z_{ar} ” at resonance.

At anti resonance

$$R_{ar} = Z_{ar} = \frac{L}{CR} \dots \dots \dots (1)$$

In terms of ‘ Q_0 ’:

$$R_{ar} = Z_{ar} = Q_0^2 R \dots \dots \dots (2)$$

Impedance of parallel resonance circuit in terms of ‘ Q_0 ’:

$$Z = \frac{R_{ar}}{1 + j2\delta Q} \dots \dots \dots (3)$$

Fractional frequency deviation, $\delta = \frac{\omega - \omega_{ar}}{\omega_{ar}}$

The condition for half power frequencies is given as $|Z| = 0.707|Z_{ar}|$

Hence, $\frac{|Z|}{|Z_{ar}|} = \frac{1}{\sqrt{2}}$

From equations (1) & (3) $\frac{Z}{R_{ar}} = \frac{1}{1 + j2\delta Q}$

$$|1 + j2\delta Q| = |1 + j| = \sqrt{2}$$

Comparing imaginary terms we get,

$$2\delta Q = \pm 1$$

At upper half power frequency $f = f_2$, ($f > f_{ar}$ & $\delta = \frac{1}{2Q}$)

$$\delta = \frac{1}{2Q}$$

$$\delta = \frac{f_2 - f_{ar}}{f_{ar}} = \frac{1}{2Q} \dots \dots \dots (4)$$

Similarly, at lower half power frequency $f = f_1$, ($f < f_{ar}$ & $\delta = -\frac{1}{2Q}$)

$$\delta = \frac{f_{ar} - f_1}{f_{ar}} = \frac{1}{2Q} \dots \dots \dots (5)$$

Adding equations (4) and (5),

$$BW = f_2 - f_1 = \frac{f_{ar}}{Q}$$

$$Q = \frac{f_{ar}}{f_2 - f_1} = \frac{f_{ar}}{BW}$$

$$BW = \frac{f_{ar}}{Q} \dots \dots \dots (6)$$

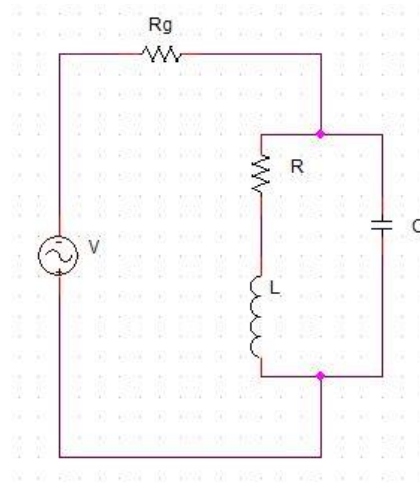
The higher the value of 'Q', the more selective will the circuit be and lesser will be the BW. At resonance, quality factor:

$$Q = \frac{\omega_{ar} L}{R} = \frac{1}{\omega_{ar} \cdot R \cdot C} = \frac{1}{R} \sqrt{\frac{L}{C}} \dots \dots \dots (7)$$

Selectivity:

$$\text{Selectivity} = \frac{\text{Resonant frequency}}{BW} = \frac{f_{ar}}{f_2 - f_1} = Q_{ar}$$

EFFECT OF GENERATOR RESISTANCE ON B.W AND SELECTIVITY:



At resonance Impedance $Z = R_{ar}$, with generator resistance ' R_g '. At anti resonance then total impedance is $R_{ar} \parallel R_g$

$$\therefore Q'(\text{with } R_g) = \frac{Q_{ar}}{1 + \frac{R_{ar}}{R_g}} \dots \dots (1)$$

Q factor is decreased by a factor $1 + \frac{R_{ar}}{R_g}$.

$$BW' \text{ with } R_g = \frac{f_{ar}}{Q'_{ar} \text{ with } R_g} BW' \text{ with } \frac{f_{ar}}{\frac{Q_o}{1 + \frac{R_{ar}}{R_g}}}$$

$$BW' = BW \left(1 + \frac{R_{ar}}{R_g} \right) \dots \dots \dots (2)$$

For matched condition and to get maximum power transfer condition R_g is selected as R_{ar} .

CURRENT AMPLIFICATION FACTOR:

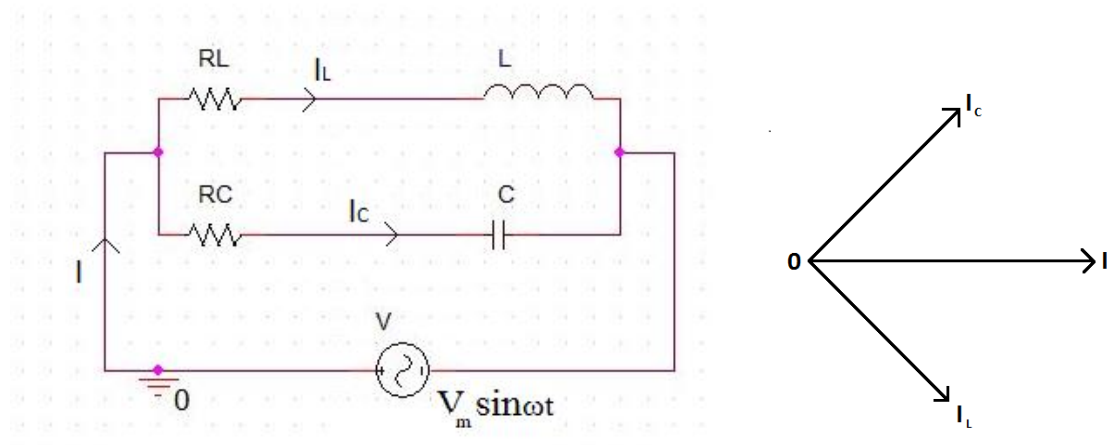
At anti resonance current through capacitor is $I_C = Q_o * I$

$I = \frac{V}{R_{ar}}$ at resonance. Similarly, the current through inductor is $I_L = Q_o * I$

∴ Since $Q_o > 1$ the current through inductor and capacitor is Q times the total current at resonance.

GENERAL CASE – RESISTANCE PRESENT IN BOTH BRANCHES:

Q1) Derive the expression for the resonance frequency in parallel resonant circuit containing resistance in both branches.



Admittance of the Inductive branch is

$$Y_L = \frac{1}{R_L + j\omega L} = \frac{1}{R_L + jX_L}$$

By rationalizing

$$Y_L = \frac{R_L - jX_L}{R_L^2 + X_L^2} \dots \dots \dots (1)$$

The admittance of capacitance branch is

$$Y_C = \frac{1}{R_C - \frac{j}{\omega C}} = \frac{1}{R_C - jX_C} = \frac{R_C + jX_C}{R_C^2 + X_C^2} \dots \dots (2)$$

Total admittance $Y = Y_L + Y_C$

$$Y = \frac{R_L - jX_L}{R_L^2 + X_L^2} + \frac{R_C + jX_C}{R_C^2 + X_C^2}$$

$$Y = \left(\frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2} \right) + j \left(\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2} \right) \dots \dots (3)$$

At resonance the Susceptance (imaginary part of Y) becomes zero

$$\therefore \left(\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2} \right) = 0$$

$$\frac{X_C}{R_C^2 + X_C^2} = \frac{X_L}{R_L^2 + X_L^2}$$

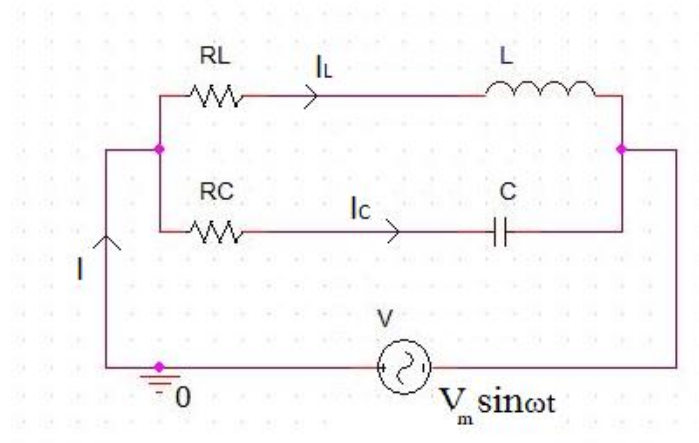
Substituting for X_C and X_L ; $X_C = \frac{1}{\omega_{ar}C}$ and $X_L = j\omega_{ar}L$ we get:

$$\omega_{ar}^2 = \frac{1}{LC} \left[\frac{L/C - R_L^2}{L/C - R_C^2} \right]$$

Anti-resonant frequency is

$$f_{ar} = \frac{1}{2\pi} \sqrt{\frac{1}{LC} \left[\frac{L/C - R_L^2}{L/C - R_C^2} \right]} \dots \dots \dots (4)$$

Q2) Prove that the circuit will resonate at all frequencies if $R_L = R_C = \sqrt{\frac{L}{C}}$



When net Susceptance of the circuit is zero with

$$R_L = R_C = \sqrt{\frac{L}{C}}$$

In this case, the circuit acts as a pure resistive circuit irrespective of frequency i.e. the circuit resonates at all frequencies. The admittance of the circuit is

$$Y = \frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2} \dots \dots \dots (1)$$

$$\text{If } R_L = R_C = \sqrt{\frac{L}{C}}, \text{ then } \Rightarrow (R_L)^2 = (R_C)^2 = (R)^2 = \frac{L}{C} = X_L X_C$$

$$\therefore Y = \left[\frac{R_L(R_C^2 + X_C^2) + R_C(R_L^2 + X_L^2)}{(R_C^2 + X_C^2)(R_L^2 + X_L^2)} \right]$$

With $R_L = R_C = R$,

$$Y = R \left[\frac{R^2 + X_C^2 + R^2 + X_L^2}{R^4 + R^2(X_L^2 + X_C^2) + X_L^2 X_C^2} \right]$$

Substitute R^4 for $X_L^2 X_C^2$ in denominator

$$= R \left[\frac{2R^2 + X_C^2 + X_L^2}{R^4 + R^2(X_L^2 + X_C^2) + R^4} \right]$$

$$= R \left[\frac{2R^2 + X_C^2 + X_L^2}{2R^4 + R^2(X_L^2 + X_C^2)} \right]$$

Take R^2 common from denominator and simplifying, we get:

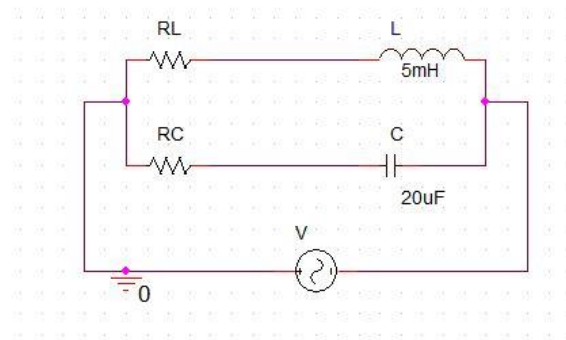
$$Y = \frac{1}{R}; Z = \frac{1}{Y} = R$$

$$W. k. t R = \sqrt{\frac{L}{C}}$$

Therefore, Impedance of the circuit at resonance is $Z = R = \sqrt{\frac{L}{C}}$. So, circuit is purely resistive, hence circuit resonates at all frequencies.

PROBLEMS:

1. Determine R_L and R_C for which the circuit shown resonates at all frequencies.



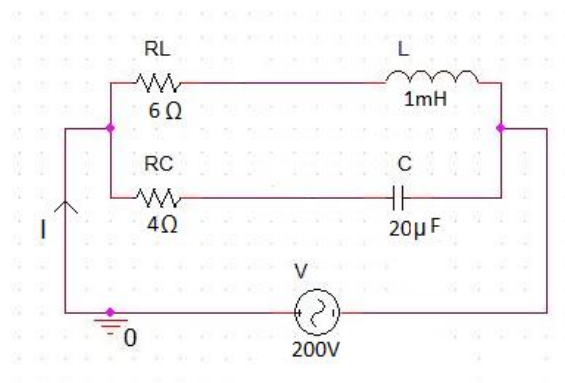
Solution:

$$Z = R = \sqrt{\frac{L}{C}}$$

On substituting we get,

$$R_L = R_C = 31.26$$

2. For the network shown find the resonant frequency and the current 'I' as indicated in the figure.



SOLUTION:

$$f_{ar} = \frac{1}{2\pi} \sqrt{\frac{1}{LC} \left[\frac{L/C - R_L^2}{L/C - R_C^2} \right]}$$

On substituting the values, we get

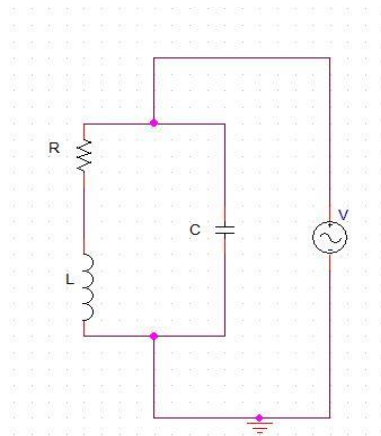
$$f_{ar} = 722.93 \text{ Hz}$$

$$I = VY_{ar} = 200 \left(\frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2} \right)$$

$$= 200 \left(\frac{R_L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_C^2 + \omega^2 C^2} \right)$$

$$\boxed{I = 27.03 \text{ A}}$$

3. In the circuit shown, an inductance of 0.1 H having a Q of 5 is in parallel with a capacitor. Determine the value of capacitance & coil resistance at resonant frequency of 500 rad/sec.



Solution:

$$f_{ar} = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{1}{Q_o^2}}$$

Or

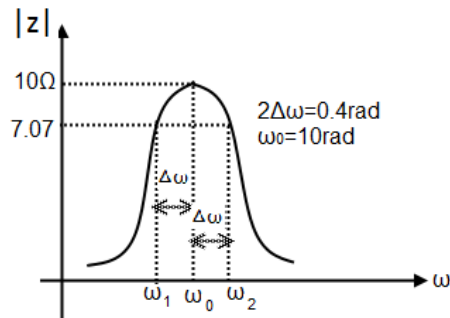
$$\omega_{ar} = \frac{1}{\sqrt{LC}} \sqrt{1 - \frac{1}{Q_o^2}}$$

$$\boxed{\therefore C = 3.84 \times 10^{-5} = 38.4 \mu\text{F}}$$

We have $= \frac{\omega_{ar} L}{R}$, therefore $R = \frac{\omega_{ar} L}{Q} = \frac{500 \times 0.1}{5}$

$$\boxed{\therefore R = 10\Omega}$$

4. Determine the RLC parallel circuit parameters whose response curve is shown. What are the new values of ω_{ar} & bw if 'C' is increased 4 times.



Solution:

From the resonance curve, we have

$$Z_{ar} = \frac{L}{CR} = 10 \Omega$$

$$BW = 0.4 \text{ rad/sec}$$

$$W_{ar} = 10 \text{ rad/sec}$$

$$Q \text{ factor} = \frac{\omega_{ar}}{BW} = \frac{10}{0.4} = 25 \dots\dots\dots (1)$$

We have,

$$Z_{ar} = R(1 + Q_o^2), \therefore R = \frac{Z_{ar}}{1 + Q_o^2}$$

$$R = \frac{10}{1 + (25)^2} = 0.01597 \Omega \dots\dots\dots (2)$$

We have

$$Z_{ar} = \frac{L}{CR}$$

$$\frac{L}{C} = Z_{ar} \times R = 10 \times 0.01597 = 0.1597$$

Or

$$L = (0.1597) \times C \dots\dots\dots (3)$$

By definition,

$$\omega_{ar} = \frac{1}{\sqrt{LC}} * \sqrt{1 - \frac{1}{90^2}}$$

Squaring on both sides and rearranging the eqn

$$LC = \frac{1}{\omega_{ar}^2} * \left(1 - \frac{1}{90^2}\right) = 9.984 \times 10^{-3} \dots\dots\dots (4)$$

Substituting eq (3) in (4)

$$(0.1597C)C = 9.984 \times 10^{-3}$$

$$\therefore C = 0.25 \text{ F}$$

$$\text{From eq (3) } L = (0.25) (0.1597) = 0.0399 \text{ H}$$

Therefore, to achieve resonance at 10 rad/sec & to have BW of 0.4 rad/sec the RLC parameters are

$$R = 0.01597 \ \Omega$$

$$L = 0.0399 \text{ H}$$

$$C = 0.25 \text{ F}$$

If 'C' is increased 4 times i.e. $C' = 4C = 4 \times 0.25 = 1 \text{ F}$

Then the new Anti-resonant frequency is

$$\omega'_{ar} = \frac{1}{\sqrt{LC'}} \sqrt{1 - \frac{1}{Q_o'^2}} = 5 \text{ rad/s}$$

$$Q_o' = \frac{1}{R} \sqrt{\frac{L}{C'}} = \frac{1}{0.01597} \sqrt{\frac{0.0399}{1}}$$

$$Q_o' = 12.5$$

$$BW = \frac{\omega_{ar'}}{Q_o'} = \frac{5}{12.5}$$

$$BW = 0.4 \text{ rad/sec}$$

5. A two branch anti-resonant circuit contains $L = 0.4 \text{ H}$, $C = 40 \ \mu\text{F}$, resonance is to achieved by variation of R_L & R_C . Calculate the resonant frequency f_{ar} for the following cases :

i) $R_L = 120 \ \Omega$, $R_C = 80 \ \Omega$ ii) $R_L = R_C = 100 \ \Omega$

Solution:

Case 1:

$$\frac{L}{C} = \frac{0.4}{40 \times 10^{-6}} = 10^4; \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.4 \times 40 \times 10^{-6}}} = 250$$

$$\omega_{ar} = \sqrt{\frac{1}{LC} \left[\frac{L/C - R_L^2}{L/C - R_C^2} \right]} = 250 \sqrt{\left[\frac{120^2 - 10^4}{80^2 - 10^4} \right]} \Rightarrow \text{(Imaginary)}$$

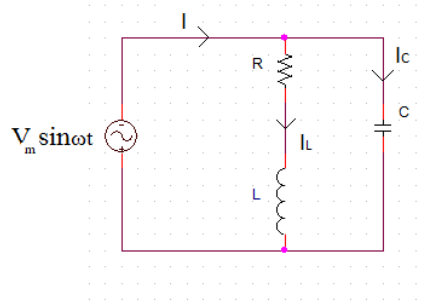
Hence resonance is not possible.

Case 2:

$$\omega_{ar} = 250 \sqrt{\left[\frac{100^2 - 10^4}{100^2 - 10^4} \right]} = \text{(Indeterminate)}$$

Hence resonance is possible at all frequencies.

6. For the circuit shown. Find the resonant frequency ω_{ar} , Q and band-width, if $R=25\Omega$; $L=0.5H$, $C=5\mu F$.



Solution:

$$\omega_{ar} = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \sqrt{\frac{1}{0.5 \times 5 \times 10^{-6}} - \frac{(25)^2}{(0.5)^2}}$$

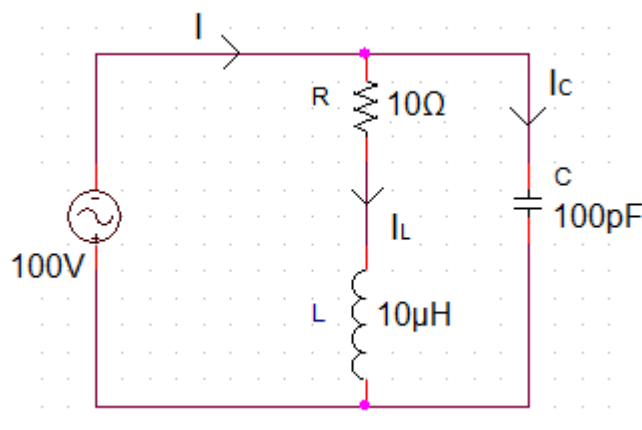
$$\omega_{ar} = 630.49 \text{ rad/s}$$

$$Q = \frac{\omega_{ar} L}{R} = \frac{630.47 \times 0.5}{25}$$

$$Q = 12.61$$

$$BW = \frac{\omega_{ar}}{Q} = \frac{630.47}{12.61}$$

7. For the parallel resonant circuit, Find I_C, I_L, I_O, f_0 & dynamic resistance.



SOLUTION:

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}; \text{ On substituting we get}$$

$$f_0 = 5.03 \times 10^6 \text{ Hz.}$$

At resonance dynamic resistance

$$Z_0 = \frac{L}{RC} = 10000 \Omega$$

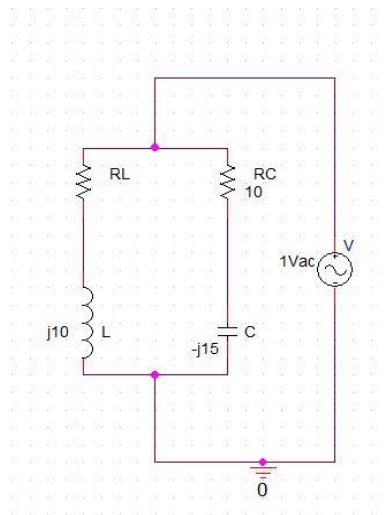
$$I_0 = \frac{V}{Z_0} = 0.01 \text{ A}$$

$$Q = \frac{\omega_{ar} L}{R} = \frac{2\pi f_{ar} L}{R} = 31.6$$

$$I_L = I_0 * Q = 31.6 \times 0.01 = 0.316 \text{ A}$$

$$I_C = I_0 * Q = 31.6 \times 0.01 = 0.316 \text{ A}$$

8. Find the value of R_L for which the circuit is resonant.



SOLUTION:

The admittance of the ckt at resonance is the real part of Y.

$$\begin{aligned}
 Y &= \frac{1}{R_L + j10} + \frac{1}{10 - j15} \\
 &= \frac{R_L - j10}{R_L^2 + 10^2} + \frac{10 + j15}{100 + 225} \text{ (Rationalizing)} \\
 &= \left(\frac{R_L}{R_L^2 + 10^2} + \frac{10}{325} \right) + j \left(\frac{15}{325} - \frac{10}{R_L^2 + 100} \right) \text{ (Separating real and imaginary terms)}
 \end{aligned}$$

At resonance, the imaginary part of 'Y' is zero.

$$\left(\frac{15}{325} - \frac{10}{R_L^2 + 100} \right) = 0$$

$$R_L = 10.8 \Omega$$