Reciprocating Compressor

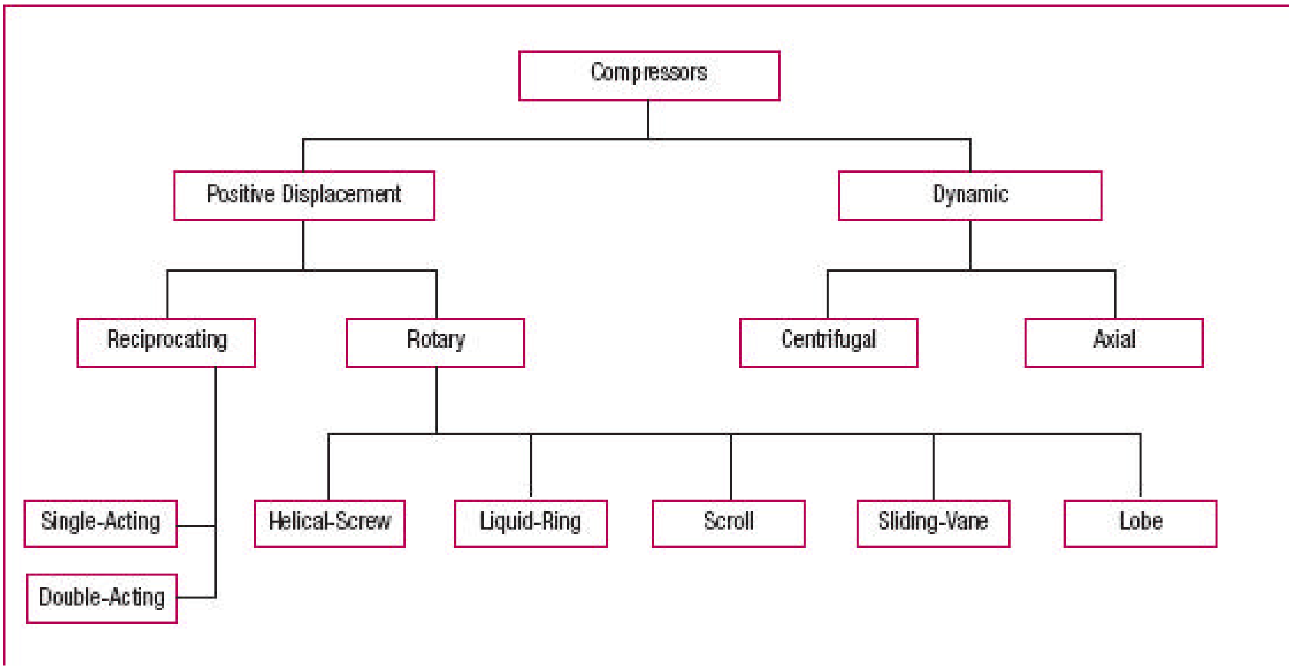
5.1 INTRODUCTION

Compressors are work absorbing devices which are used for increasing pressure of fluid at the expense or work done on fluid.

The compressors used for compressing air are called air compressors. Compressors are invariably used for all applications requiring high pressure air. Some of popular applications of compressor are, for driving pneumatic tools and air operated equipments, spray painting, compressed air engine, supercharging surface cleaning, refrigeration and air conditioning, chemical industry etc. compressors are supplied with low pressure air (or any fluid) at inlet which comes out as high pressure air (or any fluid) at outlet. Work required for increasing pressure of air is available from the prime mover driving the compressor. Generally, electric motor, internal combustion engine or steam engine, turbine etc. are used as prime movers. Compressors are similar to fans and blowers but differ in terms of pressure ratios. Fan is said to have pressure ratio up to 1.1 and blowers have pressure ratio between 1.1 to 4 while compressors have pressure ratios more than 4.

5.2 CLASSIFICATION OF COMPRESSORS

Table-5.1 Types of Compressors



Compressors can be classified in the following different ways.

(a) **Based on principle of operation:** Based on the principle of operation compressors can be classified as.

   (i) Positive displacement compressor.

   (ii) Non-positive displacement compressors.

In positive displacement compressors the compression is realized by displacement of solid boundary and preventing fluid by solid boundary from flowing back in the direction of pressure gradient. Due to solid wall displacement these are capable of providing quite large pressure ratios. Positive displacement compressors can be further classified based on the type of mechanism used for compression. These can be

(i) Reciprocating type positive displacement compressors

(ii) Rotary type positive displacement compressors.

Reciprocating compressors generally, employ piston-cylinder arrangement where displacement of piston in cylinder causes rise in pressure. Reciprocating compressors are capable of giving large pressure ratios but the mass handling capacity is limited or small. Reciprocating compressors may also be single acting compressor or double acting compressor. Single acting compressor has one delivery stroke per revolution while in double acting there are two delivery strokes per revolution of crank shaft. Rotary compressors employing positive displacement have a rotary part whose boundary causes positive displacement of fluid and thereby compression. Rotary compressors of this type are available in the names as given below;

(i) Roots blower

(ii) Vane type compressors

Rotary compressors of above type are capable of running at higher speed and can handle large mass flow rate than reciprocating compressors of positive displacement type.

Non-positive displacement compressors, also called as steady flow compressors use dynamic action of solid boundary for realizing pressure rise. Here fluid is not contained in definite volume and subsequent volume reduction does not occur as in case of positive displacement compressors. Non-positive displacement compressor may be of ‘axial flow type’ or ‘centrifugal type’ depending upon type of flow in compressor.

(b) **Based on number of stages:** Compressors may also be classfied on the basis of number of stages. Generally, the number of stages depend upon the maximum delivery pressure. Compressors can be single stage or multistage. Normally maximum compression ratio of 5 is realized in single stage compressors. For compression ratio more than 5 the multistage compressors are used.

Type values of maximum delivery pressures generally available from different type of compressor are,

(i) Single stage Compressor, for delivery pressure upto 5 bar.

(ii) Two stage Compressor, for delivery pressure between 5 to 35 bar

(iii) Three stage Compressor, for delivery pressure between 35 to 85 bar.

(iv) Four stage compressor, for delivery pressure more than 85 bar

(c) **Based on Capacity of compressors** : Compressors can also be classified depending upon the capacity of Compressor or air delivered per unit time. Typical values of capacity for different compressors are given as;

(i) Low capacity compressors, having air delivery capacity of 0.15 m^3/s or less

(ii) Medium capacity compressors, having air delivery capacity between 0.15 to 5 m^3/s.

(iii) High capacity compressors, having air delivery capacity more than 5 m^3/s

(d) Based on highest pressure developed: Depending upon the maximum pressure available from compressor they can be classified as low pressure, medium pressure, high pressure and super high pressure compressors. Typical values of maximum pressure developed for different compressors are as under:

(i) Low pressure compressor, having maximum pressure upto 1 bar

(ii) Medium pressure compressor, having maximum pressure from 1 bar to 8 bar

(iii) High pressure compressor, having maximum pressure from 8 to 10 bar

(iv) Super high pressure compressor, having maximum pressure more than 10 bar.

5.3 Reciprocating Compressors

Reciprocating Compressor has piston cylinder arrangement as shown Fig.5.1

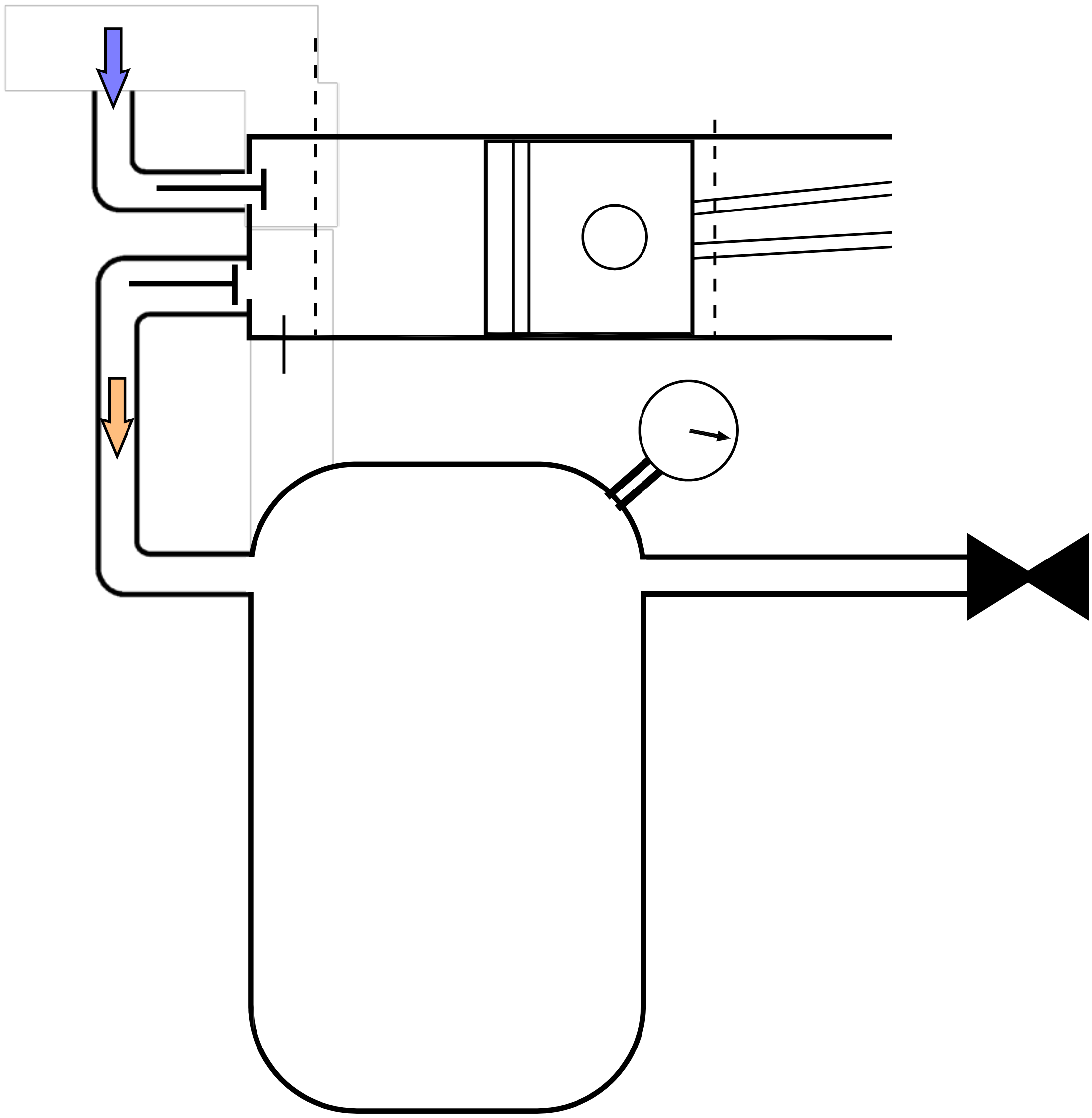


Fig.5.1 Reciprocating Compressor

Reciprocating Compressor has piston, cylinder, inlet valve, exit valve, connecting rod, crank, piston pin, crank pin and crank shaft. Inlet valve and exit valves may be of spring loaded type which get opened and closed due to pressure differential across them. Let us consider piston to be at top dead centre (TDC) and move towards bottom dead centre (BDC). Due to this piston movement from TDC to BDC suction pressure is created causing opening of inlet valve. With this opening of inlet valve and suction pressure the atmospheric air enters the cylinder.

Air gets into cylinder during this stroke and is subsequently compressed in next stroke with both inlet valve and exit valve closed. Both inlet valve and exit valves are of plate type and spring loaded so as to operate automatically as and when sufficient pressure difference is available to cause deflection in spring of valve plates to open them. After piston reaching BDC it reverses its motion and compresses the air inducted in previous stroke. Compression is continued till the pressure of air inside becomes sufficient to cause deflection in exit valve. At the moment when exit valve plate gets lifted the exhaust of compressed air takes place. This piston again reaches TDC from where downward piston movement is again accompanied by suction. This is how reciprocating compressor. Keeps on working as flow device. In order to counter for the heating of piston-cylinder arrangement during compression the provision of cooling the cylinder is there in the form of cooling jackets in the body. Reciproting compressor described above has suction, compression and discharge as three prominent processes getting compelted in two strokes of piston or one revolution of crank shaft.

5.4 Thermodynamic Analysis

Compression of air in compressor may be carried out following number of thermodynamic processes such as isothermal compression, polytropic compressor or adiabatic compressor. Fig.16.3 shows the thermodynamic cycle involved in compressor. Theoretical cycle is shown neglecting clearance volume but in actual cycle clearance volume can not be negligible. Clearance volume is necessary in order to prevent collision of piston with cylinder head, accommodating valve mechanism etc., Compression process is shown by process1-2, $1\text{-}2^{1}$, $1\text{-}2^{11}$ and $1\text{-}2^{111}$ following isothermal, polytropic and adiabatic processes.

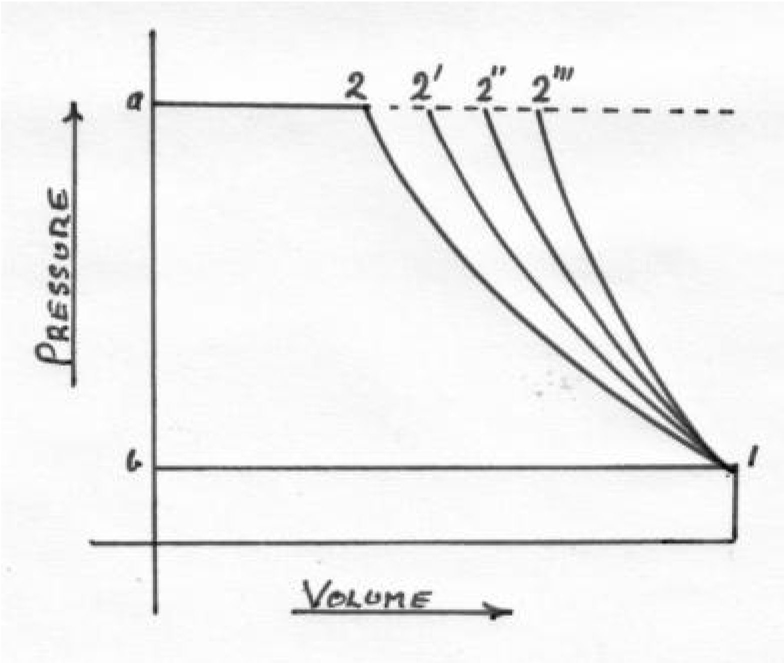


Fig.5.2 P-V diagram for Reciprocating Compressor without Clearance

On P-V diagram process 4-1 shows the suction process followed by compression during 1-2 and discharge through compressor is shown by process 2-3.

Air enters compressor at pressure p_1 and is compressed upto p_2. Compression work requirement can be estimated from the area below the each compression process. Area on p-V diagram shows that work requirement shall be minimum with isothermal process 1-2”. Work requirement is maximum with process 1-2 ie., adiabatic process. As a designer one shall be interested in a compressor having minimum compression work requirement. Therefore, ideally compression should occur isothermally for minimum work input. In practice it is not possible to have isothermal compression because constancy of temperature during compression can not be realized. Generally, compressors run at substantially high speed while isothermal compression requires compressor to run at very slow speed so that heat evolved during compression is dissipated out and temperature remains constant. Actually due to high speed running of compressor the compression process may be assumed to be near adiabatic or polytropic process following law of compression as $pV^n=C$ with of ‘n’ varying between 1.25 to 1.35 for air. Compression process following three processes is also shown on T-s diagram in Fig.16.4. it is thus obvious that actual compression process should be compared with isothermal compression process. A mathematical parameter called isothermal efficiency is defined for quantifying the degree of deviation of actual compression process from ideal compression process. Isothermal efficiency is defined by the ratio is isothermal work and actual indicated work in reciprocating compressor.

$$\text{Isothermal efficiency} = \frac{\text{Isothermal work}}{\text{Actual indicated Work}}$$

Practically, compression process is attempted to be closed to isothermal process by air/water cooling, spraying cold water during compression process. In case of multistage compression process the compression in different stages is accompanied by intercooling in between the stages. P_2 V_2

Mathematically, for the compression work following polytropic process, PV^n=C. Assuming negligible clearance volume the cycle work done.

Wc = Area on p-V diagram

$$\text{Wc} = \left[p_2V_2 + \left(\frac{p_2V_2 - p_1V_1}{n-1}\right)\right] - p_1V_1 \qquad 1$$

$$= \left[\left(\frac{n}{n-1}\right)[p_2V_2 - p_1V_1]\right]$$

$$= \left(\frac{n}{n-1}\right)(p_1V_1)\left[\frac{p_2V_2}{p_1V_1} - 1\right]$$

$$= \left(\frac{n}{n-1}\right)(p_1V_1)\left[\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)} - 1\right]$$

$$= \left(\frac{n}{n-1}\right)(mRT_1)\left[\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)} - 1\right]$$

$$= \left(\frac{n}{n-1}\right)(mR)(T_2 - T_1)$$

In case of compressor having isothermal compression process, n = 1, ie., $p_1V_1 = p_2V_2$

$$W_{iso} = p_2V_2 + p_1V_1 \ln r - p_1V_1$$

$$W_{iso} = p_1V_1 \ln r, \qquad where, r = \frac{V_1}{V_2}$$

In case of compressor having adiabatic compression process,

$$W_{adiabatic} = \left(\frac{\gamma}{\gamma - 1}\right)(mR)(T_2 - T_1)$$

Or

$$W_{adiabatic} = (mC_p)(T_2 - T_1)$$

$$W_{adiabatic} = (m)(h_2 - h_1)$$

$$\eta_{iso} = \frac{p_1 V_1 \ln r}{\left(\frac{n}{n-1}\right)(p_1 V_1)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]}$$

The isothermal efficiency of a compressor should be close to 100% which means that actual compression should occur following a process close to isothermal process. For this the mechanism be derived to maintain constant temperature during compression process. Different arrangements which can be used are:

(i) Faster heat dissipation from inside of compressor to outside by use of fins over cylinder. Fins facilitate quick heat transfer from air being compressed to atmosphere so that temperature rise during compression can be minimized.

(ii) Water jacket may be provided around compressor cylinder so that heat can be picked by cooling water circulating through water jacket. Cooling water circulation around compressor regulates rise in temperature to great extent.

(iii) The water may also be injected at the end of compression process in order to cool the air being compressed. This water injection near the end of compression process requires special arrangement in compressor and also the air gets mixed with water and needs to be separated out before being used. Water injection also contaminates the lubricant film inner surface of cylinder and may initiate corrosion etc, The water injection is not popularly used.

(iv) In case of multistage compression in different compressors operating serially, the air leaving one compressor may be cooled upto ambient state or somewhat high temperature before being injected into subsequent compressor. This cooling of fluid being compressed between two consecutive compressors is called intercooling and is frequently used in case of multistage compressors.

Considering clearance volume: With clearance volume the cycle is represented on Fig.5.3 The work done for compression of air polytropically can be given by the are a enclosed in cycle 1-2-3-4. Clearance volume in compressors varies from 1.5% to 35% depending upon type of compressor.

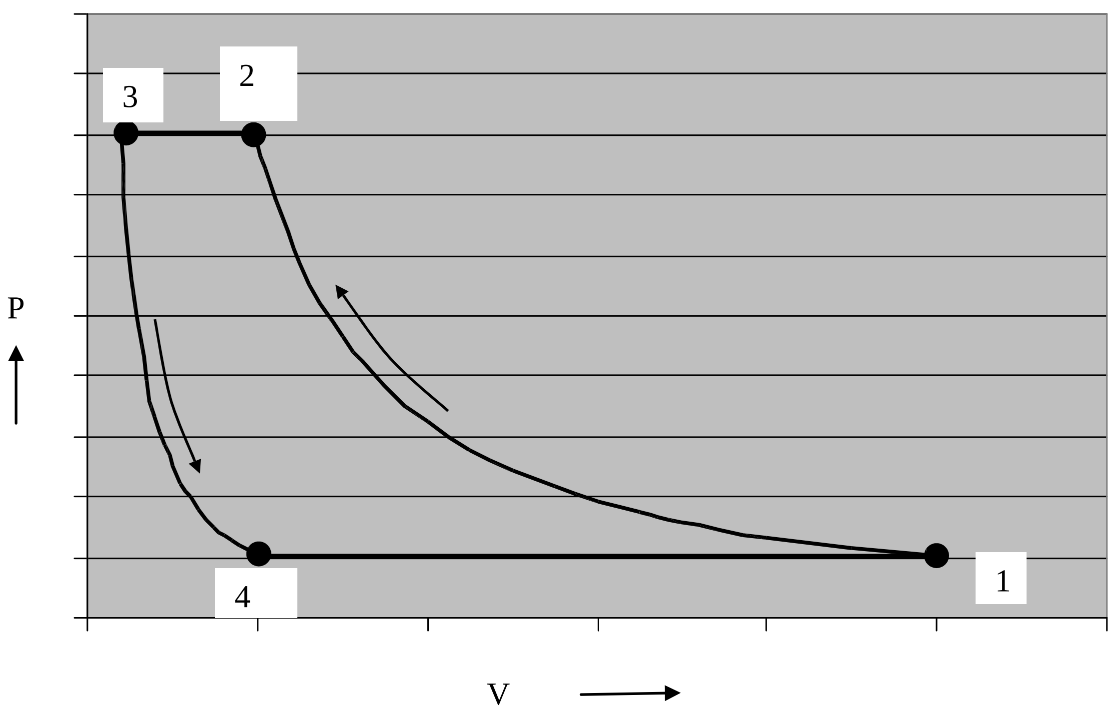


Fig.5.3 P-V diagram for Reciprocating Compressor with Clearance

$W_{c,with\ CV}$ = Area 1234

$$= \left(\frac{n}{n-1}\right)(p_1V_1)\left[\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)} - 1\right] - \left(\frac{n}{n-1}\right)(p_4V_4)\left[\left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} - 1\right]$$

Here $P_1 = P_4$, $P_2 = P_3$

$$= \left(\frac{n}{n-1}\right)(p_1V_1)\left[\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)} - 1\right] - \left(\frac{n}{n-1}\right)(p_1V_4)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \left(\frac{n}{n-1}\right)(p_1)\left[\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)} - 1\right](V_1 - V_4)$$

In the cylinder of reciprocating compressor (V_1-V_4) shall be the actual volume of air delivered per cycle. $V_d = V_1 - V_4$. This ($V_1 - V_4$) is actually the volume of air in hated in the cycle and delivered subsequently.

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)(p_1 V_d)\left[\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)} - 1\right]$$

If air is considered to behave as perfect gas then pressure, temperature, volume and mass can be inter related using perfect gas equation. The mass at state 1 may be given as m_1 mass at state 2 shall be m1, but at state 3 after delivery mass reduces to m_2 and at state 4 it shall be m_2.

So, at state 1, $p_1 V_1 = m_1 R T_1$

at state 2, $p_2 V_2 = m_1 R T_2$

at state 3, $p_3 V_3 = m_2 R T_3$ or $p_2 V_3 = m_2 R T_3$

at state 4, $p_4 V_4 = m_2 R T_4$ or $p_1 V_4 = m_2 R T_4$

Ideally there shall be no change in temperature during suction and delivery
i.e., $T_4 = T_1$ and $T_2 = T_3$ from earlier equation

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)(p_1)\left[\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)} - 1\right](V_1 - V_4)$$

Temperature and pressure can be related as,

$$\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} = \frac{T_2}{T_1} \quad \text{and} \quad \left(\frac{p_4}{p_3}\right)^{\frac{(n-1)}{n}} = \frac{T_4}{T_3} \quad \Longrightarrow \quad \left(\frac{p_1}{p_2}\right)^{\frac{(n-1)}{n}} = \frac{T_4}{T_3}$$

Substiuting

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)(m_1 R T_1 - m_2 R T_4)\left[\frac{T_2}{T_1} - 1\right]$$

Substituting for constancy of temperature during suction and delivery.

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)(m_1 R T_1 - m_2 R T_1)\left[\frac{T_2 - T_1}{T_1}\right]$$

Or

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)(m_1 - m_2) R (T_2 - T_1)$$

Thus (m_1-m_2) denotes the mass of air sucked or delivered. For unit mass of air delivered the work done per kg of air can be given as,

$$W_{c,withCV} = \left(\frac{n}{n-1}\right) R\left(T_2 - T_1\right) \quad \text{per kg of air}$$

Thus from above expressions it is obvius that the clearance volume reduces the effective swept volume i.e., the masso f air handled but the work done per kg of air delivered remains unaffected.

From the cycle work estimated as above the theoretical power required for running compressor shall be,

For single acting compressor running with N rpm, power input required, assuming clearance volume.

$$Powerrequired = \left[\left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)} - 1\right] p_1\left(V_1 - V_4\right)\right]\left(N\right)$$

For double acting compressor, Power

$$Powerrequired = \left[\left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)} - 1\right] p_1\left(V_1 - V_4\right)\right]\left(2N\right)$$

Volumetric efficiency: Volumetric efficiency of compressor is the measure of the deviation from volume handling capacity of compressor. Mathematically, the volumetric efficiency is given by the ratio of actual volume of air sucked and swept volume of cylinder. Ideally the volume of air sucked should be equal to the swept volume of cylinder, but it is not so in actual case. Practically the volumetric efficiency lies between 60 to 90%.

Volumetric efficiency can be overall volumetric efficiency and absolute volumetric efficiency as given below.

$$\text{Overall volumetric efficiency} = \frac{\text{Volume of free air sucked in cylinder}}{\text{Swept volume of LP cylinder}}$$

$$\left(\text{Volumetric efficiency}\right)_{\text{freeaircondition}} = \frac{\text{Volume of free air sucked in cylinder}}{\left(\text{Swept volume of LP cylinder}\right)_{freeaircondition}}$$

Here free air condition refers to the standard conditions. Free air condition may be taken as 1 atm or 1.01325 bar and 15^{o}C or 288K. consideration for free air is necessary as otherwise the different compressors can not be compared using volumetric efficiency because specific volume or density of air varies with altitude. It may be seen that a compressor at datum level (sea level) shall deliver large mass than the same compressor at high altitude.

This concept is used for giving the capacity of compressor in terms of 'free air delivery' (FAD). "Free air delivery is the volume of air delivered being reduced to free air conditions". In case of air the free air delivery can be obtained using perfect gas equation as,

$$\frac{p_a V_a}{T_a} = \frac{p_1 (V_1 - V_4)}{T_1} = \frac{p_2 (V_2 - V_3)}{T_2}$$

Where subscript a or p_a, V_a, T_a denote properties at free air conditions

$$V_a = \frac{p_1 T_a}{p_a} \frac{p_1 (V_1 - V_4)}{T_1} = \text{ FAD per cycle}$$

This volume V_a gives 'free air delivered' per cycle by the compressor.

Absolute volumetric efficiency can be defined, using NTP conditions in place of free air conditions.

$$\eta_{vol} = \frac{FAD}{Swept volume} = \frac{V_a}{(V_1 - V_2)} = \frac{p_1 T_a (V_1 - V_4)}{p_a T_1 (V_1 - V_3)}$$

$$\eta_{vol} = \left(\frac{p_1 T_a}{p_a T_1} \right) \left\{ \frac{(V_s + V_c) - V_4}{V_s} \right\}$$

Here V_s is the swept volume = $V_1 - V_3$

V_c is the clearance volume = V_3

$$\eta_{vol} = \left(\frac{p_1 T_a}{p_a T_1} \right) \left\{ 1 + \left(\frac{V_c}{V_s} \right) - \left(\frac{V_4}{V_s} \right) \right\}$$

Here $\frac{V_4}{V_s} = \frac{V_4}{V_c} \cdot \frac{V_c}{V_s} = \left(\frac{V_4}{V_3} \cdot \frac{V_c}{V_s} \right)$

Let the ratio of clearance volume to swept volume be given by C. = $\frac{V_c}{V_s}$

$$\eta_{vol} = \left(\frac{p_1 T_a}{p_a T_1} \right) \left\{ 1 + C - C \left(\frac{V_4}{V_3} \right) \right\}$$

$$\eta_{vol} = \left(\frac{p_1 T_a}{p_a T_1}\right)\left\{1 + C - C\left(\frac{p_2}{p_1}\right)^{1/n}\right\}$$

Volumetric efficiency depends on ambient pressure and temperature, suction pressure and temperature, ratio of clearance to swept volume, and pressure limits. Volumetric efficiency increases with decrease in pressure ratio in compressor.

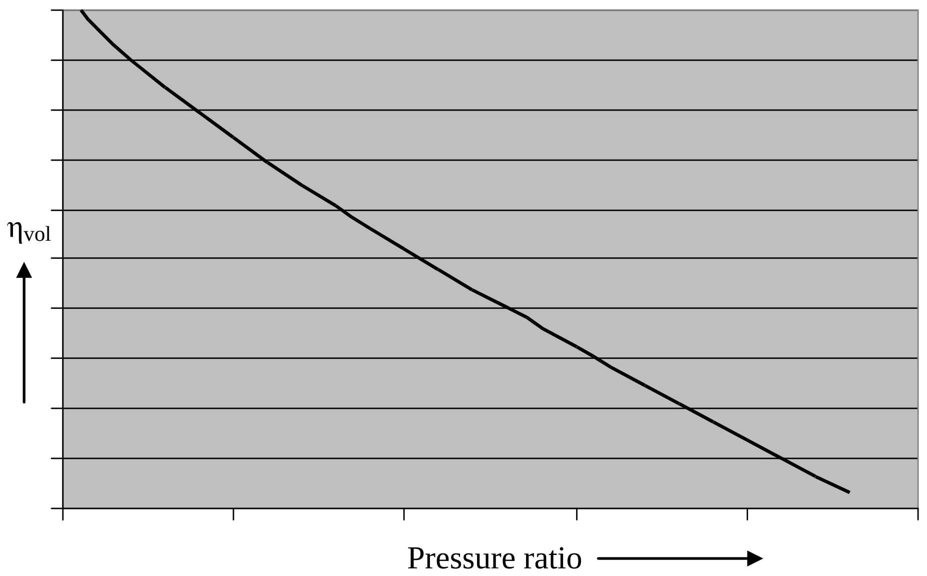


Fig.5.4 Volumetric efficiency v/s Pressure ratio

Multistage Compression

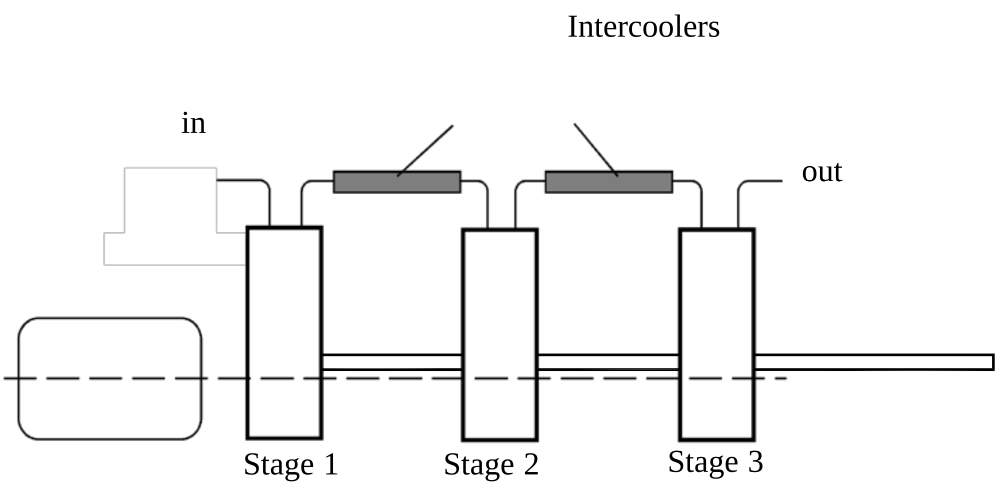


Fig.5.5 Multistage Compressor with inter coolers

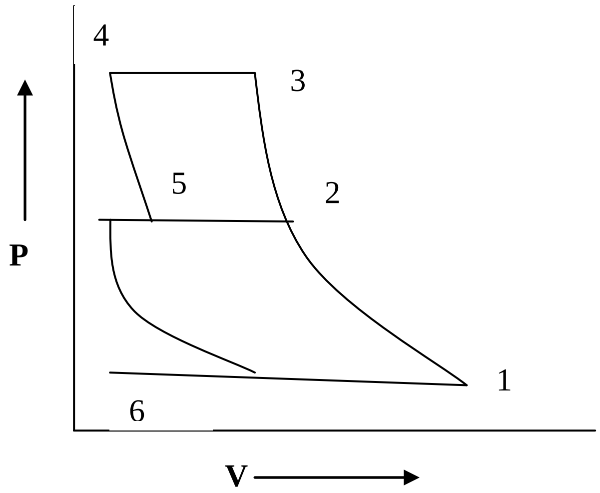


Fig.5.6 P-V diagram for Multistage Compressor

Multistage compression refers to the compression process completed in more than one stage i.e., a part of compression occurs in one cylinder and subsequently compressed air is sent to subsequent cylinders for further compression. In case it is desired to increase the compression ratio of compressor then multi-stage compression becomes inevitable. If we look at the expression for volumetric efficiency then it shows that the volumetric efficiency decreases with increase in pressure ratio. This aspect can also be explained using p-V representation shown in Fig.5.6.

Therefore, the volumetric efficiency reduces with increasing pressure ratio in compressor with single stage compression. Also for getting the same amount of free air delivery the size of cylinder is to be increased with increasing pressure ratio. The increase in pressure ratio also requires sturdy structure from mechanical strength point of view fro withstanding large pressure difference.

The solution to number of difficulties discussed above lies in using the multistage compression where compression occurs in parts in different cylinders one after the other. Fig.16.6b, shows the multistage compression occurring in two stages. Here first stage of compression occurs in cycle 12671 and after first stage compression partly compressed enters second stage of compression and occurs in cycle 2345. In case of multistage compression the compression in first stage occurs at low temperature and subsequent compression in following stages occurs at higher temperature. The compression work requirement depends largely upon the average temperature during compression. Higher average temperature during compression has larger work requirement compared to low temperature so it is always desired to keep the low average temperature during compression.

Apart from the cooling during compression the temperature of air at inlet to compressor can be reduced so as to reduce compression work. In multistage compression the partly compressed air leaving first stage is cooled upto ambient air temperature in intercooler and then sent to subsequent cylinder (stage) for compression. Thus, intercoolers when put between the stages reduce the compression work and compression is called intercooled compression. Intercooling is called perfect when temperature at inlet to subsequent stages of compression is reduced to ambient temperature. Fig.16.6c, shows multi-stage (two stage) intercooled compression. Intercooling between two stages causes temperature drop from 2 to 2' i.e discharge from first stage (at 2) is cooled upto the ambient temperature stage (at2') which lies on isothermal compression process 1-2'-3". In the absence of intercooling the discharge from first stage shall enter at 2. Final discharge from second stage occurs at 3' in case of intercooled compression compared to discharge at 3 in case of non-intercooled compression. Thus, intercooling offers reduced work requirement by the amount shown by area 22'3'3 on p-V diagram. If the intercooling is not perfect then the inlet state to second/subsequent stage shall not lie on the isothermal compression process line and this stage shall lie between actual discharge state from first stage and isothermal compression line.

Fig.16.7 shows the schematic of multi stage compressor (double stage) with inter cooler between stage T-s representation is shown in Fig.16.8. The total work requirement for running this shall be algebraic summation of work required for low pressure (LP) and high pressure (HP) stages. The size of HP cylinder is smaller than LP cylinder as HP cylinder handles high pressure air having smaller specific volume.

Mathematical analysis of multistage compressor is done with following assumptions:

(i) Compression in all the stages is done following same index of compression and there is no pressure drop in suction and delivery pressures in each stage. Suction and delivery pressure remains constant in the stages.

(ii) There is perfect intercooling between compression stages.

(iii) Mass handled in different stages is same i.e, mass of air in LP and HP stages are same.

(iv) Air behaves as perfect gas during compression.

From combined p-V diagram the compressor work requirement can be given as,

Work requirement in LP cylinder, $W_{LP} = \left(\frac{n}{n-1}\right) P_1 V_1 \left\{ \left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1 \right\}$

Work requirement in HP cylinder, $W_{HP} = \left(\frac{n}{n-1}\right) P_2 V_2 \left\{ \left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1 \right\}$

For perfect intercooling, $p_1V_1 = p_2V_2'$ and

$$W_{HP} = \left(\frac{n}{n-1}\right) P_2 V_{2'} \left\{ \left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1 \right\}$$

Therefore, total work requirement, $W_c = W_{LP} + W_{HP}$, for perfect inter cooling

$$W_C = \left(\frac{n}{n-1}\right) \left[P_1 V_1 \left\{ \left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1 \right\} + P_2 V_{2'} \left\{ \left(\frac{P_{2'}}{P_2}\right)^{\frac{n-1}{n}} - 1 \right\} \right]$$

$$= \left(\frac{n}{n-1}\right) \left[P_1 V_1 \left\{ \left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1 \right\} + P_1 V_1 \left\{ \left(\frac{P_{2'}}{P_2}\right)^{\frac{n-1}{n}} - 1 \right\} \right]$$

$$W_C = \left(\frac{n}{n-1}\right) P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} + \left(\frac{P_{2'}}{P_1}\right)^{\frac{n-1}{n}} - 2 \right]$$

Power = W_c X N - Watts

If we look at compressor work then it shows that with the initial and final pressures p_1 and $P_{2'}$ remaining same the intermediate pressure p_2 may have value floating between p_1 and $P_{2'}$ and change the work requirement Wc. Thus, the compressor work can be optimized with respect to intermediate pressure P

$_2$. Mathematically, it can be differentiated with respect to P_2.

$$\frac{dW_C}{dP_2} = \frac{d}{dP_2}\left[\left(\frac{n}{n-1}\right)P_1V_1\left\{\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} + \left(\frac{P_{2'}}{P_2}\right)^{\frac{n-1}{n}} - 2\right\}\right]$$

$$\frac{dW_C}{dP_2} = \left[\left(\frac{n}{n-1}\right)P_1V_1\frac{d}{dP_2}\left\{\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} + \left(\frac{P_{2'}}{P_2}\right)^{\frac{n-1}{n}} - 2\right\}\right]$$

$$\frac{dW_C}{dP_2} = \left(\frac{n}{n-1}\right)P_1V\left\{\left(\frac{n-1}{n}\right)P_1^{\frac{1-n}{n}} \cdot P_2^{\frac{-1}{n}} - \left(\frac{n-1}{n}\right) \cdot P_{2'}^{\frac{1-n}{n}} \cdot P_2^{\frac{1-2n}{n}}\right\}$$

Equating to zero yields

$$P_1^{\frac{1-n}{n}} \cdot P_2^{\frac{-1}{n}} = \cdot P_{2'}^{\frac{1-n}{n}} \cdot P_2^{\frac{1-2n}{n}}$$

$$P_2^{\frac{-2+2n}{n}} = \cdot P_{2'}^{\frac{1-n}{n}} \cdot P_1^{\frac{n-1}{n}}$$

$$P_2^{2\left(\frac{n-1}{n}\right)} = \left(P_1 \cdot P_{2'}\right)^{\left(\frac{n-1}{n}\right)}$$

$$P_2^{\;2} = \left(P_1 \cdot P_{2'}\right), P_2 = \sqrt{P_1 \cdot P_{2'}}$$

Pressure ratio in Ist stage = Pressure ratio in IInd stage

Thus, it is established that the compressor work requirement shall be minimum when the pressure ratio in each stage is equal.

In case of multiple stages, say *i* number of stages, for the delivery and suction pressures of P_{i+1} and P_1 the optimum stage pressure ratio shall be,

Optimum stage pressure ratio = $\left(\frac{P_{i+1}}{P_1}\right)^{\frac{1}{i}}$ for pressure at stages being P_1, P_2, P_3,……P_{i-1}, P_i, P_{i+1}

Minimum work required in two stage compressor can be given by

$$W_{C,\min} = \left(\frac{n}{n-1}\right)P_1V_1 \cdot 2\left\{\left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1\right\}$$

For *i* number of stages, minimum work,

$$W_{C,\min} = i \cdot \left(\frac{n}{n-1}\right)P_1V_1\left\{\left(\frac{P_{i+1}}{P_i}\right)^{\frac{(n-1)}{n \cdot i}} - 1\right\}$$

It also shows that for optimum pressure ratio the work required in different stages remains same for the assumptions made for present analysis. Due to pressure ration being equal in all stages the temperature ratios and maximum temperature in each stage remains same for perfect intercooling.

Cylinder dimensions: In case of multistage compressor the dimension of cylinders can be estimated basing upon the fact that the mass flow rate of air across the stages remains same. For perfect intercooling the temperature of air at suction of each stage shall be same.

If the actual volume sucked during suction stroke is V_1, V_2, V_3. . . . for different stages they by perfect gas law, $P_1 V_1 = RT_1$, $P_2 V_2 = RT_2$, Pc, $V_3 = RT_3$

For perfect intercooling

$P_1 V_1 = RT_1$, $p_2 V_2 = RT_1$, p_3, $V_3 = RT_1$

$P_1 V_1 = P_2 V_2 = RT_2$, P_3, $V_3 =$ . . . . .

If the volumetric efficiency of respective stages in $\eta_{V_1}, \eta_{V_2}, \eta_{V_3}, ...$

Then theoretical volume of cylinder1, $V_{1,th} = \dfrac{V_1}{\eta_{V_1}}; V_1 = \eta_{V_1} \cdot V_{1,th}$

Cylinder 2, $V_{2,th} = \dfrac{V_1}{\eta_{V_1}}; V_2 = \eta_{V_2} \cdot V_{2,th}$

Cylinder 3, $V_{3,th} = \dfrac{V_3}{\eta_{V_3}}; V_3 = \eta_{V3} \cdot V_{3,th}$

Substituting,

$$P_1 \cdot \eta_{V_1} \cdot V_{1,th} = P_2 \cdot \eta_{V_2} \cdot V_{2,th} = P_3 \cdot \eta_{V_3} \cdot V_{3,th} = ...$$

Theoretical volumes of cylinder can be given using geometrical dimensions of cylinder as diameters D_1, D_2, D_3 . . . and stroke lengths L_1, L_2, L_3 . . . .

Or $$V_{1,th} = \frac{\pi}{4} \cdot D_1^{\,2} \cdot L_1$$

$$V_{2,th} = \frac{\pi}{4} \cdot D_2^{\,2} \cdot L_2$$

$$V_{3,th} = \frac{\pi}{4} \cdot D_3^{\,2} \cdot L_3$$

Or $$P_1 \cdot \eta_{Vi} \cdot \frac{\pi}{4} \cdot D_1^{\,2} \cdot L_1 = P_2 \cdot \eta_{V2} \cdot \frac{\pi}{4} \cdot D_2^{\,2} \cdot L_2$$

$$= P_3 \cdot \eta_{V3} \cdot \frac{\pi}{4} \cdot D_3^{\ 2} \cdot L_3 = ...$$

$$P_1 \cdot \eta_{V1} \cdot \frac{\pi}{4} \cdot D_1^{\ 2} \cdot L_1 = P_2 \cdot \eta_{V2} \cdot \frac{\pi}{4} \cdot D_2^{\ 2} \cdot L_2$$

$$= P_3 \cdot \eta_{V3} \cdot D_3^{\ 2} \cdot L_3 = ...$$

If the volumetric efficiency is same for all cylinders, i.e $\eta_{V1} = \eta_{V2} = \eta_{V3} = ...$ and stroke for all cylinder is same i.e. $L_1 = L_2 = L_3 = ...$

Then, $\quad D_1^{\ 2} P_1 = D_2^{\ 2} P_2 = D_3^{\ 2} P_3 = ...$

These generic relations may be used for getting the ratio of diameters of cylinders of multistage compression.

Energy balance: Energy balance may be applied on the different components constituting multistage compression.

For LP stage the steady flow energy equation can be written as below:

$$m \cdot h_1 + W_{LP} = m \cdot h_2 + Q_{LP}$$

$$Q_{LP} = W_{LP} - m(h_2 - h_1)$$
$$Q_{LP} = W_{LP} - m \cdot C_p (T_2 - T_1)$$

For intercooling (Fig. 5.5) between LP and HP stage steady flow energy equation shall be;

$$m \cdot h_2 = m \cdot h_{2'} + Q_{Int}$$
$$Q_{Int} = m(h_2 - h_{2'})$$
$$Q_{Int} = m \cdot C_p (T_2 - T_{2'})$$

For HP stage (Fig.5.5) the steady flow energy equation yields.

$$m \cdot h_{2'} + W_{HP} = m \cdot h_{3'} + Q_{HP}$$
$$Q_{HP} = W_{HP} + m(h_{2'} - h_{3'})$$
$$Q_{HP} = W_{HP} + m \cdot C_p (T_{2'} - T_{3'}) = W_{HP} - m \cdot C_p (T_{3'} - T_{2'})$$

In case of perfect intercooling and optimum pressure ratio, $T_{2'} = T_1$ and $T_2 = T_{3'}$

Hence for these conditions,

$$Q_{LP} = W_{LP} - m \cdot C_p (T_2 - T_1)$$
$$Q_{Int} = m \cdot C_p (T_2 - T_1)$$
$$Q_{HP} = W_{HP} - m \cdot C_p (T_2 - T_1)$$

Total heat rejected during compression shall be the sum of heat rejected during compression and heat extracted in intercooler for perfect intercooling.

Heat rejected during compression for polytropic process $= \left(\frac{\gamma - n}{\gamma - 1} \right) \times Work$

UNIT-5
Air Compressors

Review of equations

Work done in a single stage compressor

$$= \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] = \frac{n}{n-1} \; mR(T_2 - T_1)$$

Work done in a two stage compressor for perfect inter cooling

$$= \frac{2n}{n-1} P_1 V_1 \left[\left(\frac{P_3}{P_1} \right)^{\frac{n-1}{2n}} - 1 \right]$$

Work done in a two stage compressor

$$\eta_\upsilon = \frac{P_1 T_a}{P_a T_1} \left[1 + c - c \left(\frac{P_2}{P_1} \right)^{\frac{1}{n}} \right]$$

Volumetric Efficiency

$$= \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{n-1} P_2 V_2 \left[\left(\frac{P_2}{P_2} \right)^{\frac{n-1}{n}} - 1 \right]$$

Problem 1

A single stage acting air compressor 30 cm bore and 40 cm stroke is running at a speed of 100 RPM. It takes in air at 1 bar and 20ºC and drive it when the compresses it to a pressure of 5 bar. Find the power required to drive it when compression is (i) isothermal (ii) PV1.2 = C and (iii) adiabatic. Also find the isothermal efficiencies for the cases (ii) and (iii) Neglect clearance.

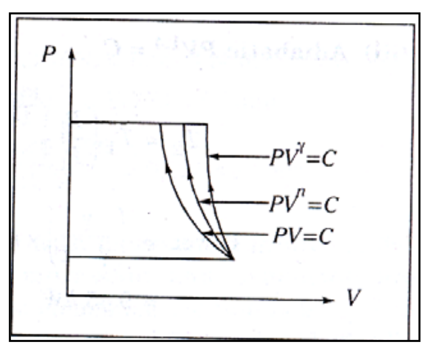
$N_1 = 100 RPM, d = 30 cm$

$L = 40 cm \qquad P_1 = 1\,\text{bar}$

$T_1 = 20°C \qquad P_2 = 5\,\text{bar}$

$$V_s = \frac{\Pi}{4} d^2 L = \frac{\Pi}{4} 0.3^2 \chi \, 0.4$$

$$= 0.028\,\text{m}^3 / cycle$$



$$V_1 = V_s = 0.028 \chi \frac{100}{60} = 0.047\,\text{m}^3 / s$$

$$m = \frac{P_1 V_1}{RT_1} = \frac{1x10^2 x0.047}{0.287x293} = 0.055\,\text{kg/s}$$

(i) Isothermal $\mathrm{PV} = \mathrm{C}$

$$\text{Power} = P_1 V_1 Ln \frac{P_2}{P_1} = mRT_1 Ln \frac{P_2}{P_1}$$

$$= 0.055x0.287x293Ln\frac{5}{1} = 7.56kW$$

(i) $\mathrm{PV}^{1.2} = \mathrm{C}$

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 293\left(\frac{5}{1}\right)^{\frac{1.2-1}{1.2}} = 383.14\,\mathrm{K}$$

$$\text{Power} = \frac{n}{n-1}\left(P_2V_2 - P_1V_1\right) = \frac{n}{n-1}\; x\; mR\left(T_2 - T_1\right)$$

$$= \frac{1.2}{1.2-1}\; x\; 0.055 \text{ x } 0.287\left(383.14 - 293\right) = 8.53\,\mathrm{kW}$$

$$Isothermaleffiency = \frac{\text{Isothermal Power}}{Actualpower}$$

$$= \frac{7.56}{8.53} = 0.8854$$

$$\eta_{Isothermal} = 88.54\,\%$$

(iii) Adiabatic $\mathrm{PV}^{1.4} = \mathrm{C}$

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 293\left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}} = 464\,\mathrm{K}$$

$$Isothermal effiency = \frac{\text{Isothermal Power}}{Actual power}$$

$$= \frac{7.56}{8.53} = 0.8854$$

$$\eta_{Isothermal} = 88.54\,\%$$

(iii) Adiabatic $\mathrm{PV}^{1.4} = \mathrm{C}$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 293\left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}} = 464\ \mathrm{K}$$

$$\text{Power} = \frac{\gamma}{\gamma-1}\, x\, mR(T_2 - T_1)$$

$$= \frac{1.4}{1.4-1}\, x\, 0.055 \text{ x } 0.287(464-293) = 9.45\ \mathrm{kW}$$

$$\eta_{Isothermal} = \frac{7.56}{9.45} = 0.80$$

$$= 80\,\%$$

Problem 2

A single acting stage acting air compressor with clearance running at 360 rpm has a bore of 10 cm. The compression and expansion are polytrophic with n = 1.25 for each. The clearance volume is 80cm3 . *If the suction and delivery pressures are 98.1 kPa and 706.32 kPa absolute, find the free air at 101 kPa and 15*ºCdelivered per minute. What is the work done per cycle? The temperature at the beginning of compression may be taken as 30ºC.Find also the power required to drive the compressor.

$T_a = 15°C,\quad d = 10cm$

$L = 8.5cm\quad \mathrm{P}_1 = 98.1kPa$

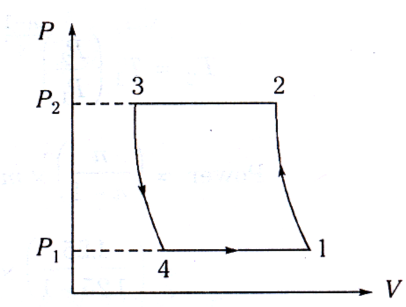
$\mathrm{T}_1 = 30°C\quad \mathrm{P}_2 = 706.32kPa$

$\mathrm{P}_a = 101kPa$

$$V_s = \frac{\Pi}{4}d^2L = \frac{\Pi}{4}0.1^2 \chi\ 0.85$$

$$= 6.675 \text{ x } 10^{-4}\ \mathrm{m}^3$$

$$V_c = 80cm = 0.8\, x\, 10^{-4}\ \mathrm{m}^3$$



$$Clearance \text{ ratio } \mathrm{C} = \frac{\mathrm{V_c}}{\mathrm{V_s}} = \frac{0.8 \text{ x } 10^{-4}}{6.675 \text{ x } 10^{-4}} = 0.12$$

Volumetric efficiency

$$\eta_\upsilon = \frac{P_1 T_a}{P_a T_1}\left[1 + c - c\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right]$$

$$= \frac{98.1 \text{ x } 288}{101 \text{ x } 303}\left[1 + 0.12 - 0.12\left(\frac{706.32}{98.1}\right)^{\frac{1}{1.25}}\right] = 0.496$$

$$m = \frac{P_a T_a}{R T_a} = \frac{101 \text{ x } 0.1193}{0.287 \text{ x } 288}$$

$$= 0.1457 \ \text{ kg/min}$$

$$\eta_\upsilon = \frac{V_a}{V_s}$$

$$\therefore \text{Volume of free air } \mathrm{V_a} = 0.496 \text{ x } 6.675 \text{ x } 10^{-4}$$

$$= 0.0003314 \text{ m}^3 / \text{cycle}$$

$$= 0.0003314 \text{ x } 360 = 0.1193 \text{ m}^3 / \text{min}$$

$$\mathrm{m} = \frac{0.1457}{60} = 0.00242 kg / \sec$$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 303\left(\frac{706.32}{98.1}\right)^{\frac{1.25-1}{1.25}} = 449.67 \text{ K}$$

$$\text{Power} = \left(\frac{\mathrm{n}}{n-1}\right) \text{ x mR}(T_2 - T_1)$$

$$= \left(\frac{1.25}{1.25-1}\right) \text{ x } 0.00242 \text{ x } 0.287(449.67 - 303)$$

$$= 0.5 \text{ kJ/s or kW}$$

$$Workdone / cycle = \frac{0.5}{360/60} = 0.0084 kJ / cycle$$

Problem 3

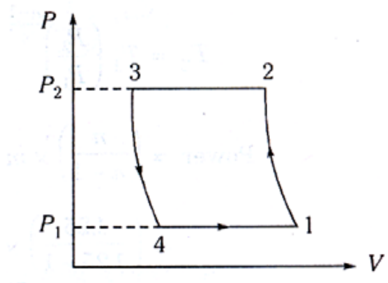
A single stage double acting air compressor is required to deal with 17 m3/min of air measured at 1 bar and 15°C. the pressure and temperature at the end of suction is 0.98 bar and 30°C. The delivery pressure is 6.3bar. The rpm of the compressor is 500.assuming a clearance volume of 5% of the stroke volume, laws of the compression and expansion as PV1.32 =C, calculate the necessary stroke of volume, temperature of the air delivered and power of the compressor.

$T_a = 15°C,\qquad V_a = 17 m^3/\min$

$P_a = 1 bar \qquad P_1 = .98 bar$

$T_1 = 30°C \qquad P_2 = 6.3 bar$

$N = 500 rpm$



$$C = \frac{5}{100} = 0.05$$

$$\eta_\upsilon = \frac{P_1 T_a}{P_a T_1}\left[1 + C - C\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right]$$

$$\eta_\upsilon = \frac{98.1 \text{ x } 288}{1 \text{ x } 303}\left[1 + 0.05 - 0.05\left(\frac{6.3}{0.98}\right)^{\frac{1}{1.32}}\right] = 0.8452$$

$$\eta_\upsilon = \frac{V_a}{V_s}$$

$$V_s = Stroke Volume = \frac{17}{0.8452} = 20.11\, m^3/\min$$

$$V_s = \frac{20.11}{500} = 0.0402 m^3/\min$$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 303\left(\frac{6.3}{0.98}\right)^{\frac{1.32-1}{1.32}}$$

$$Temperature \text{ of air delivered} = 475.71 \text{ K}$$

$$m = \frac{P_a T_a}{R T_a} = \frac{1 \text{ x } 100 \text{ x } 17}{0.287 \text{ x } 288} = 20.56 \text{kg/min}$$

$$= \frac{20.56}{60} = 0.3427 \text{ kg/sec}$$

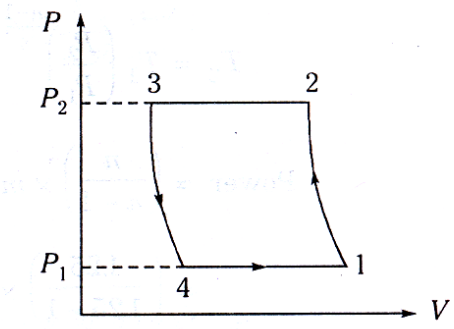
$$\text{Power} = \left(\frac{\text{n}}{n-1}\right) \text{x mR}\left(T_2 - T_1\right)$$

$$= \left(\frac{1.32}{1.32-1}\right) \text{x } 0.3427 \text{ x } 0.287\left(495.71 - 303\right)$$

$$= 70 \text{ kJ/s or kW}$$

Problem 4

A single stage double acting air compressor delivers 15 m3 of air per min of air measured at 1.013 bar and 27°C. delivers at 7bar.The condition at the end of the suction stroke are pressure 0.98 bar and temperature 4°C. The a clearance volume is 4% of the swept volume, and stroke to bore ration is 1.3:1 and compressor runs at 300rpm. calculate the Volumetric efficiency of the compressor. Assume the index of compression and expansion to be 1.3.



$$T_a = 27°C, \qquad \text{V}_a = 15m^3/\text{min}$$

$$\text{P}_\text{a} = 1.013bar \qquad \text{P}_1 = .98bar$$

$$\text{T}_1 = 40°C \qquad \text{P}_2 = 7bar$$

$$N = 300rpm, Clearanceratio = 0.04$$

$$L/d = 1.3$$

$$\eta_\upsilon = \frac{P_1 T_a}{P_a T_1}\left[1 + C - C\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right]$$

$$\eta_\upsilon = \frac{98.1 \text{ x } 300}{1.013 \text{ x } 313}\left[1 + 0.04 - 0.04\left(\frac{7}{0.98}\right)^{\frac{1}{1.3}}\right] = 0.796$$

$$= 79.6\%$$

$$\eta_\upsilon = \frac{V_a}{V_s}$$

$$V_s = Swept\,Volume = \frac{15}{0.796} = 18.84\, m^3/\text{min}$$

But for double acting compressor

$$\text{V}_s = \frac{\Pi}{4} d^2 \text{ x L 2NL} = 1.3\text{d}$$

$$18.84 = \frac{\Pi}{4} d^2 \text{ x } 1.3\text{d x 2 x 300}$$

$$\text{d} = \sqrt[3]{\frac{18.8 \text{ x } 44}{\Pi \text{ x } 1.3 \text{ x 2 x 300}}} = 0.313m = 31.3cm$$

$$L = 1.3x31.3 = 40.72\text{cm}$$

$$m = \frac{P_a T_a}{RT_a} = \frac{1.013 \text{ x } 100 \text{ x } 15}{0.287 \text{ x } 300} = 17.6 \text{kg/min}$$

$$= \frac{17.6}{60} = 0.294 \text{ kg/s}$$

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = 313 \left(\frac{7}{0.98} \right)^{\frac{1.3-1}{1.3}} = 492.7 \text{ K}$$

$$\text{Power} = \left(\frac{\text{n}}{n-1} \right) \text{x mR} \left(T_2 - T_1 \right)$$

$$\text{Power} = \left(\frac{1.3}{1.3-1} \right) \text{x } 0.294 \text{ x } 0.287 \left(492.71 - 313 \right)$$

$$= 65.7 \text{ kJ/s or kW}$$

$$\text{Isothermal Power} = P_1 V_1 Ln \frac{P_2}{P_1} = mRT_1 Ln \frac{P_2}{P_1}$$

$$= 0.294 x 0.287 x 313 Ln \frac{7}{0.98} = 51.92 kW$$

$$Isothermal \text{ efficiency} = \frac{51.92}{65.7} x 100 = 79\%$$

Problem 5

A two-stage compressor compresses 1kg/min of air from 1bar to 42.18 bar. Initial temperature is 15ºC. At the intermediate pressure the intercooling is perfect. The compression takes place according to PV1.35 =C. Neglecting the effect of clearance, determine the minimum power required to run the compressor. Also find the mass of cooling water required in the intercooler , if the temperature rise of water is limited to 5°C.

$$M = 1kg / \text{min} = \frac{1}{60} = 0.0166 kg / s$$

$$P_1 = 1bar \quad \text{T}_1 = 15°C \quad \text{P}_3 = 42.18 bar$$

$$For \text{ perfect intercooling}$$

$$\text{P}_2 = \sqrt{\text{P}_1 \text{P}_3} = \sqrt{1 \text{x} 42.18} = 6.48 bar$$

$$\text{T}_2 = \text{T}_1 \left(\frac{\text{P}_2}{\text{P}_1} \right)^{\frac{n-1}{n}} = 288 \left(\frac{6.49}{1} \right)^{\frac{1.35-1}{1.35}} = 467.7 K$$

$$\text{Maximum Power} = \left(\frac{\text{n}}{n-1}\right) \text{x mR}\left(T_2 - T_1\right)$$

$$= \left(\frac{2\text{x}1.35}{1.35-1}\right) \text{x } 0.0166 \text{ x } 0.287\left(467.7 - 288\right)$$

$$= 6.6 \text{ kJ/s or kW}$$

$$\text{Heat rejected in the intercooler} \quad \text{Q} = \text{mC}_\text{p}\left(T_2 - T_1\right)$$

$$= 0.0166x1.005\left(467.7 - 288\right)$$

$$= 2.99\text{kJ/Sec}$$

$$But \text{ Q is also} = \text{m}_\omega x C_{p\omega} x \text{ temperature rise}$$

$$299 = \text{m}_\omega x 4.187 \text{ x } 5$$

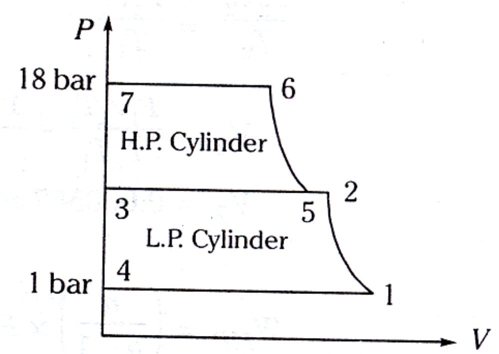
$$\text{m}_\omega = 0.1432 kg/\text{sec}$$

Problem 6

A two stage reciprocating compressor delivers 150 m3/hr of free air measured at 1.03 bar and15ºC. The final pressure 18bar. The pressure and temperature of the air in LP cylinder before compression is 1 bar and 30°C. the diameter of the LP cylinder is twice that of HP cylinder and air enters the HP cylinder at 40ºC. If compression follows the law PV1.22=C, determine

(i) Intermediate pressure and power required if the intercooler is imperfect.

(ii) Ration of cylinder diameter and minimum power required for perfect intercooling.



$$T_a = 15°C, \qquad \text{V}_a = 150 m^3/\text{min}$$

$$\text{P}_\text{a} = 1.03 bar \qquad \text{P}_1 = 1 bar$$

$$\text{T}_1 = 30°C \qquad \text{P}_3 = 18 bar$$

$$\text{T}_2 = 40°C$$

Neglet the effect of clearance

(i) Imperfect intercooling

$$\text{d}_{\text{LP}} = 2d_{HP}$$

$$P_1V_1^{1.22} = P_2V_2^{1.22}$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{1.22} = \left(\frac{\Pi/4xd_{LP}^2 xL}{\Pi/4xd_{HP}^2 xL}\right)^{1.22}$$

$$\frac{P_2}{P_1} = \left(\frac{d_{LP}}{d_{HP}}\right)^{2x1.22}$$

$$P_2 = 1x(2)^{2.44}$$

$$Intercooling\ pressure\ \text{P}_2 = 5.426 bar$$

$$\frac{P_1V_1}{T_1} = \frac{P_aV_a}{T_a}$$

$$V_1 = \frac{P_aV_aT_1}{T_aP_1} = \frac{1.03x10^2\,x0.02916x303}{1x10^2\,x288}$$

$$= 0.045\text{m}^3/\text{sec}$$

$$W_{LP} = \frac{n}{n-1}P_1V_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$W_{LP} = \frac{1.22}{1.22-1}1x10^2\,x0.045\left[\left(\frac{5.426}{1}\right)^{\frac{1.22-1}{1.22}} - 1\right]$$

$$= 8.89\text{ kW}$$

Air enters HP cylinder at $T_2 = 40°C$

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$$

$$V_2 = \frac{P_1V_1T_2}{T_1P_2} = \frac{1x10^2\,x0.045x313}{5.426x10^2\,x303}$$

$$= 0.008567\text{m}^3/\text{sec}$$

$$W_{HP} = \frac{n}{n-1}P_2V_2\left[\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} - 1\right]$$

$$W_{HP} = \frac{1.22}{1.22-1}5.426x10^2\,x0.008567\left[\left(\frac{18}{5.426}\right)^{\frac{1.22-1}{1.22}} - 1\right]$$

$$= 6.12\text{ kW}$$

$$\text{Total Power} = 8.89 + 6.12$$

$$= 15.01\text{ kW}$$

Problem 7

A multi stage compressor compressing air is to be designed to elevate the pressure from 1 bar to 120bar such that the stage pressure ratio should not exceed 4. Determine

(i) The number of stages
(ii) Exact stage pressure ratio
(iii) Intermediate pressure

Solution:

$P_1 = 1bar \quad P_{N+1} = 120bar$

$$Stage \text{ pressure ratio} = \frac{P_2}{P_1} = \frac{P_2}{P_1} = \frac{P_3}{P_2} = \frac{P_4}{P_3} = \frac{P_{N+1}}{P_N} = 4$$

Assuming the intercooling to be perfect we have

$$\frac{P_{N+1}}{P_N} = \left(\frac{P_{N+1}}{P_1}\right)^{\frac{1}{N}}$$

$$4 = \left(\frac{120}{1}\right)^{\frac{1}{N}}$$

$$N = \frac{Ln120}{Ln4} = 3.453$$

$\therefore$ Number of stages = 4

$$\text{Exact stage pressure ratio} = \frac{P_{N+1}}{P_N} = (120)^{\frac{1}{4}} = 3.31$$

$$\frac{P_5}{P_4} = 3.31, \; P_4 = \frac{120}{3.310} = 36.25\,bar$$

$$\frac{P_4}{P_3} = 3.31, \; P_3 = \frac{36.25}{3.31} = 10.95\,bar$$

$$\frac{P_3}{P_2} = 3.31, \; P_2 = \frac{10.95}{3.31} = 3.308\,bar$$

$\therefore$ Intermediat pressure are 36.25 bar, 10.95 bar and 3.308 bar

Problem 8

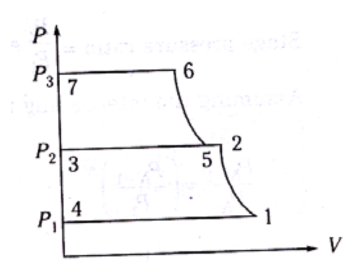
A two-stage compressor delivers air at pressure of 19 bar. The free air conditions are 1.03 bar and 25ºC. The pressure of the air before compression is 0.98bar. The intermediate pressure is 4.5bar. The temperature of the air entering each cylinder is 35°C. The law of compression and expansion being PV1.25 =C. The clearance volume is 5% of the swept volume. Determine the volumetric efficiency and the work done per kg of air.

$T_a = 25°C, \quad P_2 = 4.5bar$

$P_a = 1bar \quad P_1 = 0.98bar$

$T_1 = 35°C \quad P_3 = 19bar$

$Clearance\ ratio\ C = 0.05$



$$\eta_\upsilon = \frac{P_1 T_a}{P_a T_1}\left[1 + C - C\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right]$$

$$\eta_\upsilon = \frac{98.1 \text{ x } 298}{1 \text{ x } 308}\left[1 + 0.05 - 0.05\left(\frac{4.5}{0.98}\right)^{\frac{1}{1.25}}\right] = 0.835$$

$$= 83.5\%$$

Even through the temperature of air entering each cylinder is same, work is not saem in both cylinder since $P_2 \neq \sqrt{P_1 P_3}$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 308\left(\frac{4.5}{0.98}\right)^{\frac{1.25-1}{1.25}} = 416.1K$$

$$W_{LP} = \frac{n}{n-1}R(T_2 - T_1) = \frac{1.25}{1.25-1}x0.287(416.1 - 308)$$

$$= 155.12 \text{ kJ/kg}$$

Temperature of air entering HP cylinder $T_5 = 35°C$

$$T_6 = T_5\left(\frac{P_6}{P_5}\right)^{\frac{n-1}{n}} = 308\left(\frac{19}{4.5}\right)^{\frac{1.25-1}{1.25}} = 410.82K$$

$$W_{HP} = \frac{n}{n-1}R(T_6 - T_5) = \frac{1.25}{1.25-1}0.287(410.82 - 308)$$

$$= 147.55 \text{kJ/kg}$$

Total work $= 155.12 + 147.55$

$= 302.75$kJ/kg of air

Problem 9

A two-stage double acting air compressor takes in air at 1 bar and 25°C. It runs at 200 rpm. The diameter of LP cylinder is 35cm. The stroke of both LP and HP cylinders are 40cm. The clearance volume of both the cylinders is 4%. The index of compression is 1.3. The LP cylinder discharges air at a pressure of 4 bar. The air passes through the intercooler so that it enters the HP cylinder at 27°C and 3.6 bar, finally it is discharged from the compressor at 14.4 bar.

Calculate

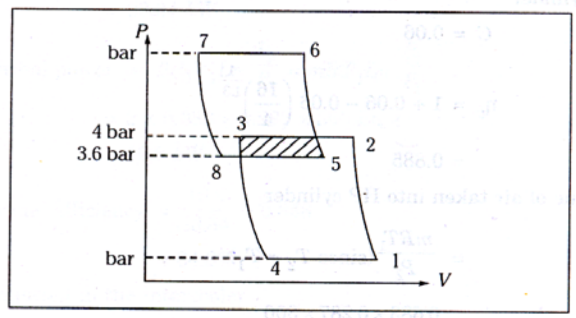
(i) Diameter of HP cylinder
(ii) The heat rejected in the intercooler
(iii) The power required to drive the HP cylinder

$P_1 = 1$ bar $\quad T_1 = 25°C \quad N = 200$ For a double acting compressor, swept volume of LP cylinder

$$= \frac{\pi}{4} D_{LP}^2 \times L_{LP} \times \frac{2N}{60}$$

$$= \frac{\pi}{4} \times 0.35^2 \times 0.4 \times \frac{2 \times 220}{60}$$

$$V_s = 0.2822 \ m^3/\text{sec}$$



Volumatric effiency referred to the suction condition 1

$$\eta_\upsilon = 1 + C - C\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$$

Volume of air referred to condition 1

$$V_1 = 0.9238x0.2822 = 0.26071 m^3 / S$$

$$Mass\ of\ air \quad m = \frac{P_1 V_1}{RT_1} = \frac{1x10^2 x0.26071}{0.287x298}$$

$$= 0.30483 \text{ kg/s}$$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 298\left(\frac{4}{1}\right)^{\frac{1.3-1}{1.3}} = 410.34K$$

Heat rejected in the intercooler

$$= mC_p(T_2 - T_1) = 0.30483x1.005(410.34 - 298)$$

$$= 34.14 \text{kJ/s}$$

Volume of air draw to HP cylinder

$$V_5 = \frac{mRT_5}{P_5} = \frac{0.30483x0.287x298}{3.6x10^2} = 0.0724 m^3 / s$$

Pressure ratio in LP cylinder = 4
Pressure ratio in HP cylinder =14.4/3.6=4
Since the pressure ratio and clearance percentage in both HP and LP cylinders are the same, the volumetric efficiency of the cylinders referred to the condition at the start of compression is same.

$$\therefore \text{Swept volume of HP cylinder} = \frac{V_5}{\eta_\upsilon} = \frac{0.0724}{0.9238}$$

$$= 0.07837 \text{ m}^3 / s$$

$$\frac{\Pi}{4} D^2{}_{HP} x L_{HP} x \frac{2x200}{60} = 0.7837$$

$$\frac{\Pi}{4} D^2{}_{HP} x 0.4x \frac{2x200}{60} = 0.1934 m$$

$$= 19.34 \text{ cm}$$

Since pressure ratio is same $T_6 = T_2$

Power required for HP cylinder

$$= \left(\frac{n}{n-1}\right) \text{x mR}(T_6 - T_5)$$

$$= \left(\frac{n}{n-1}\right) \text{x mR}(T_2 - T_1)$$

$$= \left(\frac{1.3}{1.3-1}\right) \text{x } 0.30483x0.287(410.34 - 298)$$

$$= 42.58 \text{ kW}$$

Problem 10

A two stage air compressor compresses air from 17^0C and 1 bar to 63 bar. The air is cooled in the intercooler to 30^0C and intermediate pressure is steady at 7.7 bar. The low pressure cylinder is 10 cm diameter and the stroke for both cylinders is 11.25 cm. Assuming a compression law of $PV^{1.35}$ = constant, and that the volume of air at atmospheric conditions drawn in per stroke is equal to the low pressure cylinder swept volume, find the power of the compressor while running at 250 rpm. Find also the diameter of HP cylinder.

Solution:

d_{LP} = 10cm L = 11.25cm

P_1 = 1 bar P_2 = 7.7 bar $T_1 = 17^0C$

P_3= 63 bar

Volume of LP cylinder

$V_1 = \pi/4 \text{ X } 0.1^2 \text{ X } 0.1125 = 0.00088 \text{ m}^3$

$m = P_1V_1/R\,T_1 = (1 \text{ X } 100 \text{ X } 0.00088)/(0.287 \text{ X } 290) = 0.00106$ kg

$T_2 = 30^0C$

Volume of air entering the HP cylinder

$V_2 = mR\,T_2/\,P_2 = (0.00106 \text{ X } 0.283 \text{ X } 303)/\,(7.7 \text{ X } 10^2) = 0.0001198 \text{ m}^3$

$V_2 = \pi/4 \text{ X } d_2^2 \text{ X } L$

$0.0001198 = \pi/4 \text{ X } d_2^2 \text{ X } 0.1125$

d_2 = diameter of HP cylinder

= 0.0368 m

Diameter of HP cylinder = 3.68 cm

Work required/cycle = $(n/n\text{-}1)[\, P_1V_1\{(\,P_2\,/\,P_1)^{n-1/n} - 1\} + P_2V_2(\,P_3\,/\,P_2)^{n-1/n} - 1\,]$

$W = (1.35/1.35\text{-}1)\,[\,1 \text{ X } 10^2 \text{ X } 0.00088\,\{(7.7/1)^{1.35-1/1.35} - 1\} + 7.7 \text{ X } 10^2 \text{ X } 0.0001198\,\{(63/7.7)^{1.35-1/1.35} - 1\}]$

= 0.49456 kJ/cycle

Power = 0.49456 X Number of cycles/sec

= 0.49456 X 250/60

Power = 2.06 kW

Problem 11

A three stage air compressor draws 8 m^3/min of air at 1 bar and 18^0C and delivers the same at 55 bar and 20^0C. The index of compression is 1.32. The air while passing through the intercoolers and aftercoolers suffers a pressure loss of 4% and is cooled to the intial temperature. Determine the shaft power required to drive the compressor if mechanical efficiency is 85%.

Solution:

$V_1 = 8 \text{ m}^3/\text{min} = 8/60 = 0.133 \text{ m}^3/\text{sec}$

P_1 = 1 bar $T_1 = 18^0C$ P_4 = 55 bar

The pressure drop of 4% in the intercooler is accounted for by the factor C = 0.96

For 3 stage compressor,

Power = 3(n/n-1) X P_1V_1 [{P_4/C^3P_1}$^{n-1/3n}$ - 1]

= 3 (1.32/1.32-1) X 1 X 10^2 X 0.133 [{55/0.96^3 X 1}$^{1.32-1/1.32}$ - 1]

= 283.kW

Actual shaft power required = Power/η_{mech} = 283.32/0.85 = **333.32kW**

STEAM NOZZLES

8.1 Introduction

In the impulse steam turbine, the overall transformation of heat into mechanical work is accomplished in two distinct steps. The available energy of steam is first changed into kinetic energy, and this kinetic energy is then transformed into mechanical work. The first of these steps, viz., the transformation of available energy into kinetic energy is dealt with in this chapter.

A nozzle is a passage of varying cross-sectional area in which the potential energy of the steam is converted into kinetic energy. The increase of velocity of the steam jet at the exit of the nozzle is obtained due to decrease in enthalpy (total heat content) of the steam. The nozzle is so shaped that it will perform this conversion of energy with minimum loss.

8.2 General Forms of Nozzle Passages

A nozzle is an element whose primary function is to convert enthalpy (total heat) energy into kinetic energy. When the steam flows through a suitably shaped nozzle from zone of high pressure to one at low pressure, its velocity and specific volume both will increase.

The equation of the continuity of mass may be written thus :

...(8.1)

where *m* *m* mass flow in kg/sec.,

V = velocity of steam in m/sec.,

A = area of cross-section in m , and

v = specific volume of steam in m3/kg.

In order to allow the expansion to take place properly, the area at any section of the nozzle must be such that it will accomodate the steam whatever volume and velocity may prevail at that point.

As the mass flow (*m*) is same at all sections of the nozzle, area of cross-section (*A*) varies as —. The manner in which both *V* and *v* vary depends upon the properties of the substance flowing. Hence, the contour of the passage of nozzle depends upon the nature of the substance flowing.

For example, consider a *liquid*- a substance whose specific volume *v* remains almost constant with change of pressure. The value of will go on increasing with change of pressure. Thus, from eqn. (8.1), the area of cross-section should decrease with the decrease of pressure. Fig. 8-1 (a) illustrates the proper contour of longitudinal section of a nozzle suitable for liquid. This also can represent convergent nozzle for a fluid whose peculiarity is that while both velocity and specific volume increase, the rate of specific volume increase is less than that of the velocity, thus resulting in increasing value of $\frac{V}{v}$.

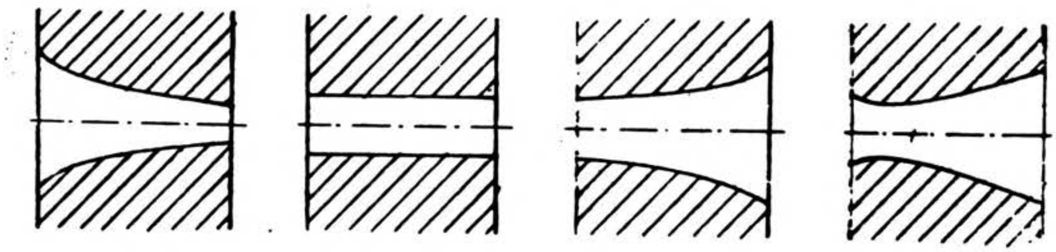


Fig. 8-1. General forms of Nozzles.

Fig. 8-1(b) represents the correct contour for some hypothetical substance for which both velocity and specific volume increase at the same rate, so that their ratio $\frac{V}{v}$ is a constant at all points. The area of cross-section should therefore, be constant at all points, and the nozzle becomes a plain tube.

Fig. 8-1(c) represents a divergent nozzle for a fluid whose peculiarity is that $\frac{V}{v}$ decreases with the drop of pressure, i.e., specific volume increases at a faster rate than velocity with the drop of pressure. The area of cross-section should increase as the pressure decreases.

Table 8-1

Properties of steam at various pressures when expanding dry saturated steam from 14 bar to 0.15 bar through a nozzle, assuming frictionless adiabatic flow.

Pressure p bar	Dryness fraction x	Enthalpy drop $H_1 - H_2$ kJ	Velocity V m/sec.	Specific Volume v_s m^3/kg	Discharge per unit area kg/m^2	Area A m^2	Diameter D metre
14	1·000	-	-	-	-	-	-
12	0·988	38·6	278	0·1633	1,723	0·00058	0·0272
10	0.974	84·1	410	0·1944	2,165	0·00046	0·0242
7	0·950	164·7	574	0·2729	*2,214	0·00045	x0·0239
3·5	0·908	309	786	0·5243	1,651	0·00061	0·0279
1·5	0·872	441·2	939	1·1593	929	0·0011	0·0374
0·70	0·840	555·6	1,054	2·365	531	0·00188	0·049
0·15	0·790	736·7	1,214	10·022	153	0·0065	0·091

* Maximum discharge per unit area x Smallest diameter

Fig. 8-1(d) shows the general shape of convergent-divergent nozzle suitable for gases and vapours. It can be shown that in practice, while velocity and specific volume both increase from the start, velocity first increases faster than the specific volume, but after a certain critical point, specific volume increases more rapidly than velocity. Hence the value of $\frac{V}{v}$ first increases to maximum and then decreases, necessitating a nozzle of *convergent-divergent* form. The above statement may be verified by referring to table 8-1, which shows the properties of steam at various pressures when expanding dry saturated steam from 14 bar to 0·15 bar through a nozzle, assuming frictionless adiabatic flow.

8.3 Steam Nozzles

The mass flow per second for wet steam, at a given pressure during expansion is given by

$$m = \frac{AV}{v} = \frac{AV}{xv_s} \text{ kg/sec.} \qquad \text{...(8.2)}$$

where A = Area of cross-section in m^2,

V = Velocity of steam in m/sec,

v_s = Specific volume of dry saturated steam, m^3/kg,

x = Dryness fraction of steam, and

$v = x\ v_s$ = Specific volume of wet steam, m^3/kg.

As the mass of steam per second (m) passing through any section of the nozzle must be constant, the area of cross-section (A) of nozzle will vary according to the variation of $\frac{V}{xv_s}$ i.e., product of A and $\frac{V}{xv_s}$ is constant. If the factor $\frac{V}{xv_s}$ increases with the drop in pressure, the cross-sectional area should decrease and hence a *convergent* shaped nozzle. The decrease of the factor $\frac{V}{xv_s}$ with pressure drop will require increasing cross-sectional area to maintain mass flow constant and hence the *divergent* shaped nozzle.

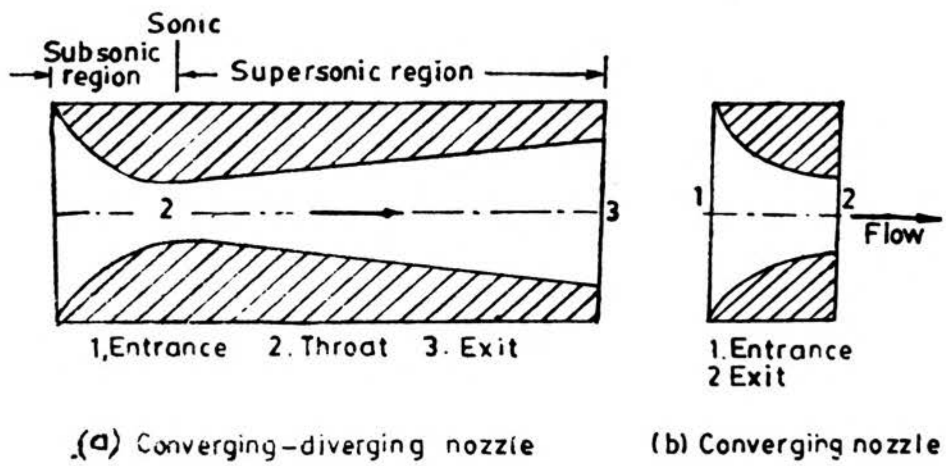


Fig. 8-2. Longitudinal sectional view of steam nozzles.

In practice at first the nozzle cross-section tapers to a smaller section in order to allow for increasing value of $\frac{V}{xV_s}$; after this smallest diameter is reached, it will diverge to a larger cross-section. The smallest section of the nozzle is known as the *throat.*

A nozzle which first converges to throat and then diverges, as in fig. 8-2(a), is termed as *converging-diverging nozzle.* It is used for higher pressure ratio $\left(\frac{p_2}{p_1}\right)$.

Some form of nozzles finish at the throat and no diverging portion is fitted; this type shown in fig. 8-2(b), is known as *converging nozzle.* In this the greatest area is at the entrance and minimum area is at the exit which is also the throat of the nozzle. This nozzle is used when the pressure ratio, $\frac{P_2}{p_1}$ is less than 0.58 (critical).

8.4 Flow Through Steam Nozzles

From the point of view of thermodynamics, the steam flow through nozzles may be spoken as adiabatic expansion. During the flow of steam through the nozzle, heat is neither supplied nor rejected. Moreover, as the steam expands from high pressure to low pressure, the heat energy is converted into kinetic energy, i.e., work is done in expanding to increase the kinetic energy. Thus the expansion of steam through a nozzle is an adiabatic, and the flow of steam through nozzle is regarded as an adiabatic flow.

It should be noted that the expansion of steam through a nozzle is not a free expansion, and the steam is not throttled, because it has a large velocity at the end of the expansion. Work is done by the expanding steam in producing this kinetic energy.

In practice, some kinetic energy is lost in overcoming the friction between the steam and the side of the nozzle and also internal friction, which will tend to regenerate heat. The heat thus formed tends to dry the steam. About 10% to 15% of the enthalpy drop from inlet to exit is lost in friction. The effect of this friction, in resisting the flow and in drying the steam, must be taken into account in the design of steam nozzles, as it makes an appreciable difference in the results.

Another complication in the design of steam through a nozzle is due to a phenomenon known as *supersaturation;* this is due to a time lag in the condensation of the steam during the expansion. The expansion takes place very rapidly and if the steam is initially dry or superheated, it should become wet as the pressure falls, because the expansion is adiabatic. During expansion the steam does not have time to condense, but remains in an unnatural dry or superheated state, then at a certain instant, it suddenly condenses to its natural state. See illustrative problem no. 14.

Thus, the flow of steam through a nozzle may be regarded as either an ideal adiabatic (isentropic) flow, or adiabatic flow modified by friction and supersaturation.

If friction is negligible, three steps are essential in the process of expansion from pressure p_1 to p_2 :

(i) Driving of steam upto the nozzle inlet from the boiler. The 'flow-work' done on the steam is p_1v_1 and results in similar volume of steam being forced through the exit to make room for fresh charge (steam).

(ii) Expansion of steam through the nozzle while pressure changes from p_1 to p_2, the work done being $\frac{1}{n-1}(p_1v_1 - p_2v_2)$

where n is the index of the isentropic expansion,

v_1 = volume occupied by 1 kg of steam at entrance to nozzle, and

v_2 = volume occupied by 1 kg of steam as it leaves the nozzle.

Alternatively, this work done is equal to the change of internal energy, $\mu_1 - \mu_2$ as during isentropic expansion work is done at the cost of internal energy.

(iii) Displacement of the steam from the low pressure zone by an equal volume discharged from the nozzle. This work amounts to p_2v_2 which is equal to the final flow work spent in forcing the steam out to make room for fresh charge (steam).

Thus, the new work done in increasing kinetic energy of the steam,

$$W = p_1v_1 + \left[\frac{1}{n-1}(p_1v_1 - p_2v_2)\right] - p_2v_2$$

$$W = \frac{n}{n-1}(p_1v_1 - p_2v_2) \qquad \text{... (8.3)}$$

This is same as the work done during Rankine cycle.

Alternatively, $W = p_1v_1 + (\mu_1 - \mu_2) - p_2v_2$

$$= (p_1v_1 + \mu_1) - (\mu_2 + p_2v_2) = H_1 - H_2 \qquad \text{...(8.4)}$$

where, H_1 and H_2 are the values of initial and final enthalpies allowing for the states of superheating or wetness as the case may be. This is exactly equivalent to the enthalpy drop equivalent to the work done during the Rankine cycle. The value of $H_1 - H_2$ may be found very rapidly from the Mollier chart ($H - \Phi$ chart) or more slowly but with greater accuracy from the steam tables.

In the design of steam nozzles the calculations to be made are :

(i) the actual velocity attained by the steam at the exit,

(ii) the minimum cross-sectional area (throat area) required for a given mass flow per second,

(iii) the exit area, if the nozzle is converging-diverging, and

(iv) the general shape of the nozzle – axial length.

8.4.1 Velocity of steam leaving nozzle : The gain of kinetic energy is equal to the enthalpy drop of the steam. The initial velocity of the steam entering the nozzle (or velocity of approach) may be neglected as being relatively very small compared with exit velocity.

For isentropic (frictionless adiabatic) flow and considering one kilogram of steam

$$\frac{V^2}{2 \times 1{,}000} = H_1 - H_2 = H$$

where H is enthalpy drop in kJ/kg and V = velocity of steam leaving the nozzle in m/sec.

$$\therefore\ V = \sqrt{2 \times 1{,}000H} = 44{\cdot}72\sqrt{H}\ \text{m/sec.} \qquad \text{... (8.5)}$$

Let the available enthalpy drop after deducting frictional loss be kH,

i.e. $(1 - k)H$ is the friction loss,

Then, $V = 44{\cdot}72\sqrt{kH}$ m/sec. ... (8.6)

If the frictional loss in the nozzle is 15 per cent of the enthalpy drop, then $k = 0.85$.

8.4.2 Mass of steam discharged : The mass flow of steam in kg per second through a cross-sectional area A and at a pressure p_2 can be written as

$$m = \frac{AV_2}{v_2}$$ where v_2 = specific volume of steam at pressure p_2.

But $v_2 = v_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = v_1\left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}$ ...(8.7)

where, v_1 = specific volume of steam at pressure p_1.

Using the value of velocity V from eqns. (8.3.) and (8.5),

$$m = \frac{A}{v_2}\sqrt{\left[2{,}000\frac{n}{n-1}(p_1v_1 - p_2v_2)\right]} = \frac{A}{v_2}\sqrt{2{,}000\frac{n}{n-1}p_1v_1\left[1 - \frac{p_2v_2}{p_1v_1}\right]}$$

Putting the value of v_2 from eqn. (8.7), we get,

$$m = \frac{A}{v_1\left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}}\sqrt{2{,}000\frac{n}{n-1}p_1v_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]}$$

$$m = A\sqrt{2{,}000\frac{n}{n-1}\times\frac{p_1}{v_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]} \quad \text{... (8.8)}$$

8.4.3 Critical pressure ratio : Using eqn. (8.8), the rate of mass flow per unit area is given by

$$\frac{m}{A} = \sqrt{2{,}000\frac{n}{n-1}\times\frac{p_1}{v_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]}$$

The mass flow per unit area has the maximum value at the throat which has minimum area, the value of pressure ratio $\left(\frac{p_2}{p_1}\right)$ at the throat can be evaluated from the above expression corresponding to the maximum value of $\frac{m}{A}$.

All the items of this equation are constant with the exception of the ratio $\left(\frac{p_2}{p_1}\right)$.

Hence, $\frac{m}{A}$ is maximum when $\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]$ is the maximum.

Differentiating the above expression with respect to $\left(\frac{p_2}{p_1}\right)$ and equating to zero for a maximum discharge per unit area

$$\frac{d}{d\left(\frac{p_2}{p_1}\right)}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right] = 0$$

$$\therefore \frac{2}{n}\left[\frac{p_2}{p_1}\right]^{\frac{2}{n}-1} - \frac{n+1}{n}\left[\frac{p_2}{p_1}\right]^{\frac{n+1}{n}-1} = 0$$

Hence, $\left[\frac{p_2}{p_1}\right]^{\frac{2-n}{n}} = \frac{n+1}{n}\left[\frac{p_2}{p_1}\right]^{\frac{1}{n}}$ or $\left[\frac{p_2}{p_1}\right]^{2-n} = \left[\frac{n+1}{2}\right]^{n}\left(\frac{p_2}{p_1}\right)$

from which $\left[\frac{p_2}{p_1}\right]^{1-n} = \left[\frac{n+1}{2}\right]^{n}$ or $\frac{p_2}{p_1} = \left[\frac{2}{n+1}\right]^{\frac{n}{n-1}}$ ... (8.9)

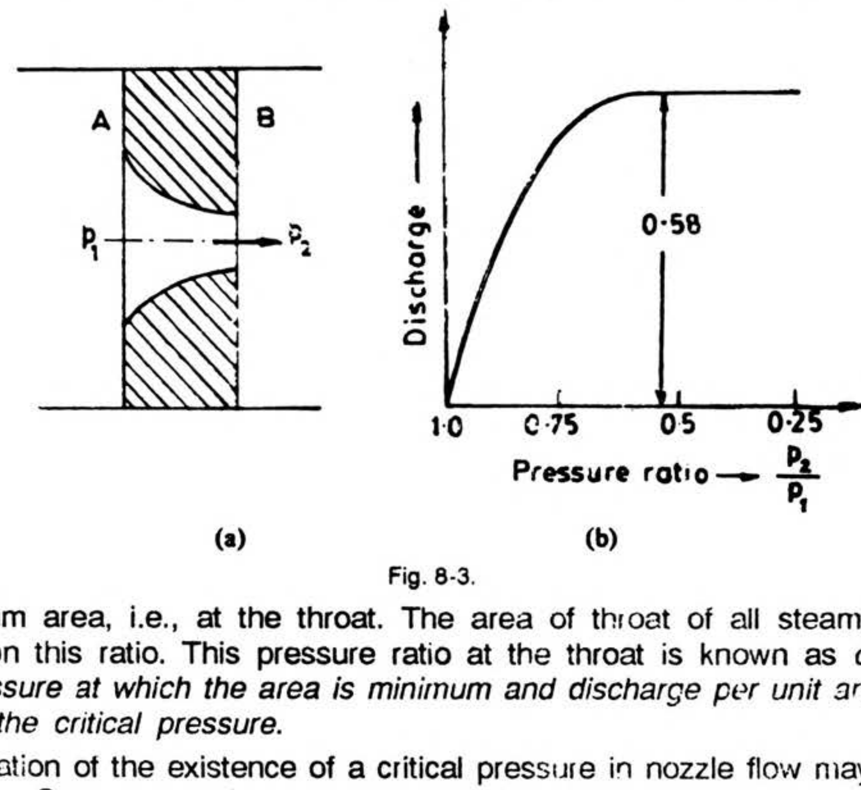
$\frac{p_2}{p_1}$ is known as *critical pressure ratio* and depends upon the value of index n.

The following approximate values of index n and corresponding values of critical pressure ratios may be noted :

Initial condition of steam	Value of index n for isentropic expansion	Nozzle critical pressure ratio $\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$
Superheated or supersaturated	1·300	0·546
Dry saturated	1·135	0·578
Wet	1·113	0·582

Dr. Zeuner has suggested a well known equation for value of n in the adiabatic expansion of steam viz. $n = 1{\cdot}035 + 0{\cdot}1x_1$, where x_1 is the initial dryness fraction of steam.

The eqn. (8.9) gives the ratio between the throat pressure (p_2) and the inlet pressure (p_1) for a maximum discharge per unit area through the nozzle. The mass flow being constant for all sections of nozzle, maximum discharge per unit area occurs at the section



(a) (b)

Fig. 8-3.

having minimum area, i.e., at the throat. The area of throat of all steam nozzle should be designed on this ratio. This pressure ratio at the throat is known as *critical pressure ratio. The pressure at which the area is minimum and discharge per unit area is maximum is termed as the critical pressure.*

The implication of the existence of a critical pressure in nozzle flow may be expressed in another way. Suppose we have two vessels A and B. A containing steam at a high and steady pressure p_1. Suppose that the pressure in B may be varied at will. A and B are connected by a diaphragm containing a convergent nozzle, as shown in fig. 8-3(a).

Assume at first that p_2 is equal to p_1, then there is no flow of steam through the nozzle. Now let p_2 be gradually reduced. The discharge m through the nozzle will increase as shown by the curve of fig. 8-3(b). As the pressure p_2 approaches the critical value, the discharge rate gradually approaches its maximum value, and when p_2 is reduced below the critical value, the discharge rate does not increase but remains at the same value as that at the critical pressure. The extraordinary result that p_2 can be reduced well below the critical pressure without influencing the mass flow was first discovered by R.D. Napier.

Another explanation can be visualised as follows : the critical pressure will give velocity of steam at the throat equal to the velocity of the sound *(sonic velocity)*. The flow of steam in the convergent portion of the nozzle is *sub-sonic.* Thus, to increase the velocity of steam above sonic velocity *(super sonic)* by expanding steam below critical pressure, divergent portion is necessary [fig· 8-2(a)]·