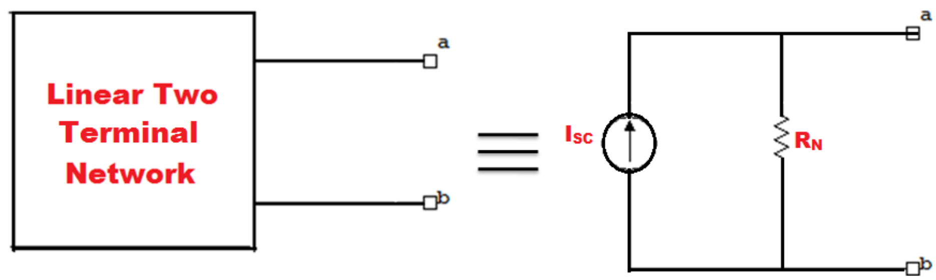
Theorem 1: Norton's Theorem

Statement :

Norton's Theorem states that a linear two terminal network can be replaced by an equivalent circuit consisting of a current I_N in parallel with a resistor R_N, where

- R_N is the equivalent resistance at the terminals when the independent sources are turned off
- I_N is short circuit current through the terminals.

If the circuit consists of the dependent sources the Norton's resistance has to be found out as R_N = Voc / Isc

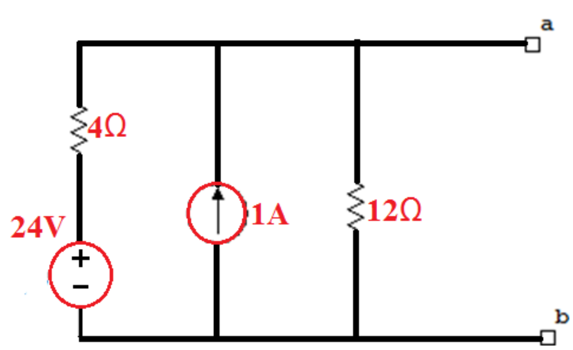


There can be two types of problems,

1. To find the Norton's equivalent circuit across the open circuit terminals
2. To find a voltage or a current in the circuit by Norton's Theorem.

Problems:

P1. Find the Norton's equivalent circuit across the terminals a-b

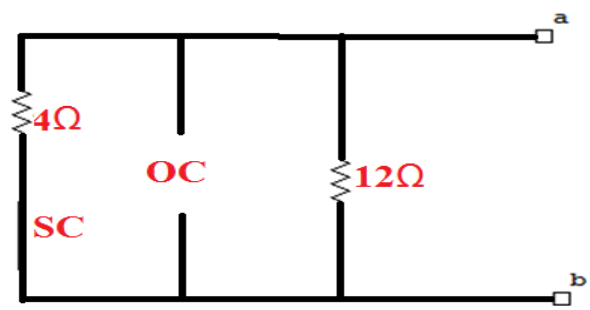


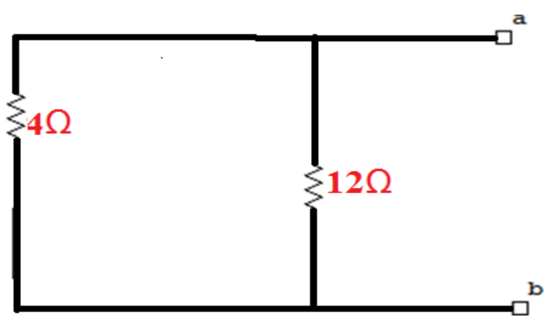
Solution:

Steps to find out the Norton's Resistance R_N :

Step 1: Turn off the independent sources

(open-circuit the current source and short-circuit the voltage source)





Step 2: Find the equivalent resistance looking into the open circuit terminals

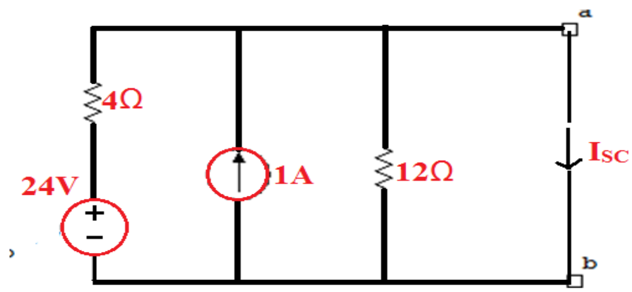
R_N= 12 x 4 / 12 + 4

R_N= 3 Ω

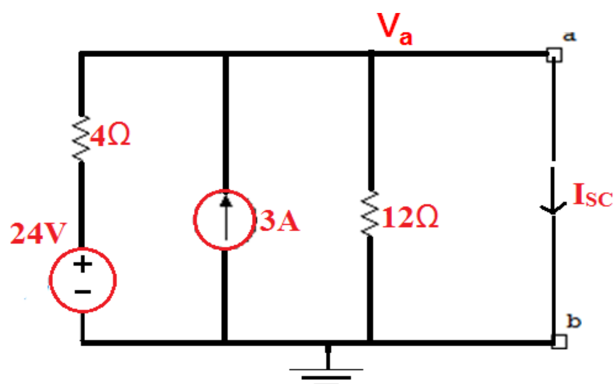
Steps to find out the Norton's Current I_N (Short circuit current):

Step 1: Short circuit the open circuit terminals and mark the I_{SC} as shown.

Step 2: Find the short circuit current by a suitable technique



By Node Analysis:



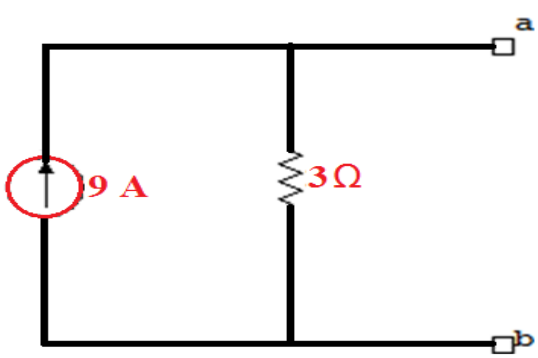
Applying KCL at node a :

$$\frac{Va - 24}{4} + \frac{Va}{12} + Isc = 3$$

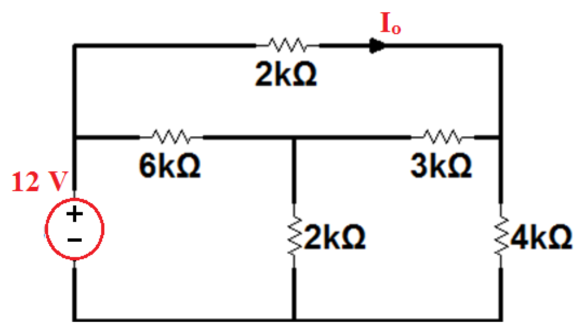
Substituting Va = 0 V in the above equation implies

Isc= 9 A

Therefore the Norton's equivalent circuit across terminals a-b is



P2. Find I_0 in the network shown, using Norton's Theorem



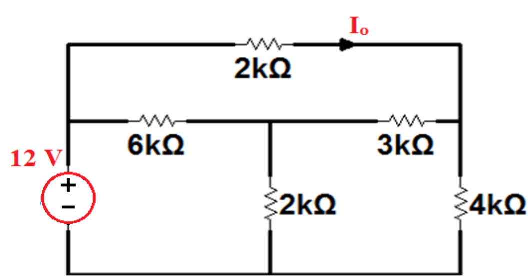
Solution:

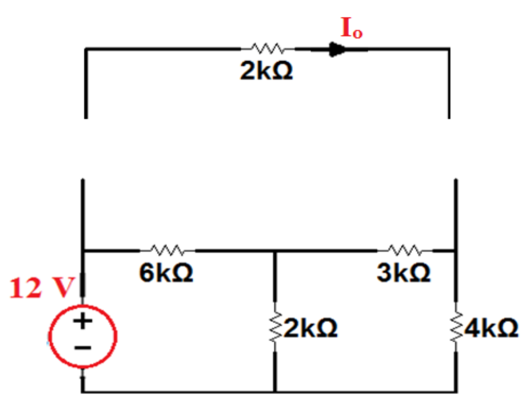
Step 1: Separate the branch through which I_0 is flowing

Step 2: Find the Norton's equivalent network across the open circuit terminals

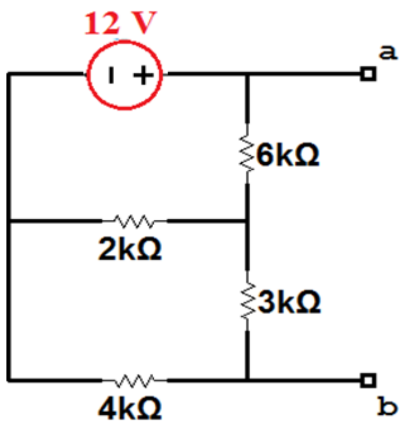
Step 3: Connect the branch separated, back to the Norton's equivalent circuit to find I_0

Step 1: Separate the branch through which I_0 is flowing

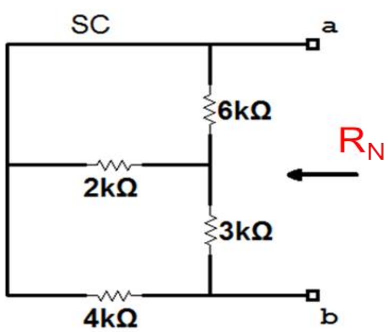




Step 2: Find the Norton's equivalent network across the open circuit terminals a-b



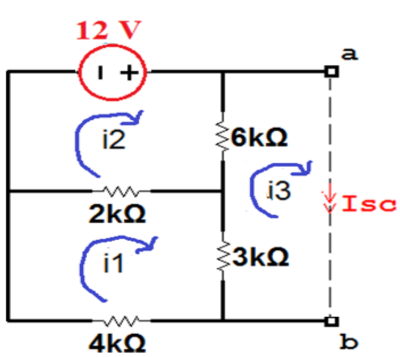
Find the R_N across the open circuit terminals a-b by short-circuiting12 V source



R_N = [(6 K || 2 K) + 3 K] || 4 K

R_N = 2.12 K Ω

Find the I_{SC} or I_N through terminals a-b by short-circuiting a-b as shown



By Mesh Analysis:

Mark i1, i2, i3 as shown

KVL to Mesh 1:

$4Ki1 + 2K(i1 - i2) + 3K(i1 - i3) = 0$

$9K\,i1 - 2K\,i2 - 3K\,i3 = 0$ Eq1

KVL to mesh 2:

$-12 + 6K(i2 - i3) + 2K(i2 - i1) = 0$

$-2K\,i1 + 8K\,i2 - 6K\,i3 = 12$........Eq2

KVL to mesh 3:

$3K(i3 - i1) + 6K(i3 - i2) = 0$

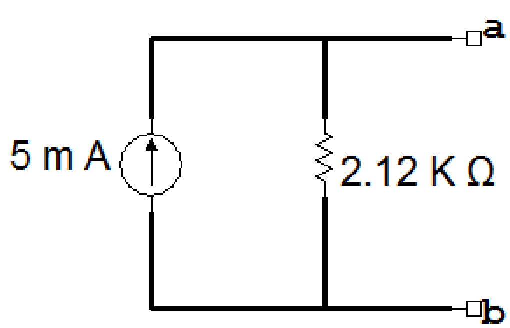
$-3K\,i1 - 6K\,i2 + 9K\,i3 = 0$.........Eq3

Solving Eq1, Eq2 and Eq3 we have,

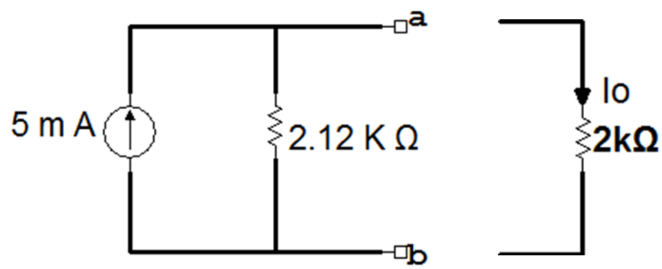
i1= 3mA,   i2=6mA,   i3=5mA

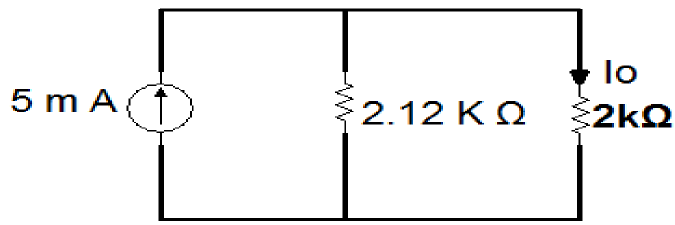
Isc = i3 = 5mA

Therefore the Norton's equivalent circuit across terminals a-b is



Step 3: Connect the branch separated, back to the Norton's equivalent circuit to find I_0

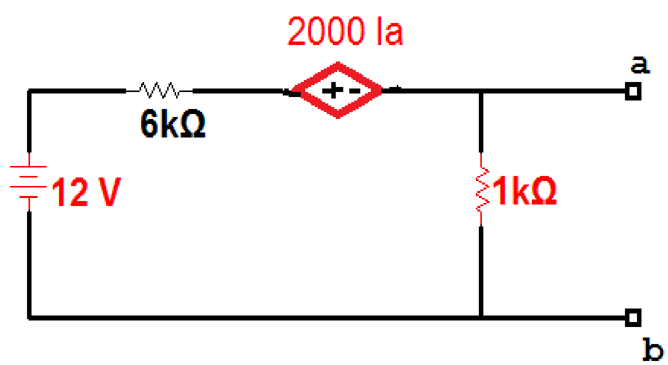




By Current Division Method

$$Io = \frac{5m \times 2.12\ K}{2\ K + 2.12\ K} = 2.57\ mA$$

P3. Find the Norton's Equivalent network across the terminals a-b



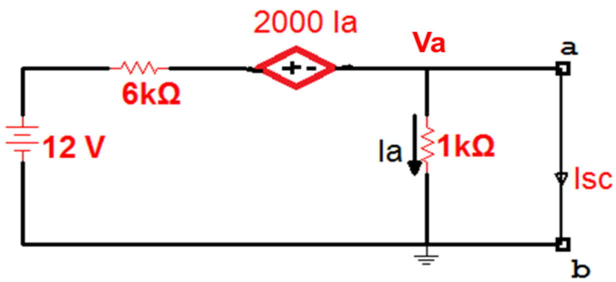
Solution:

Since the network consists of the dependent source (Dependant sources cannot be turned off) the Norton's resistance has to be found out as

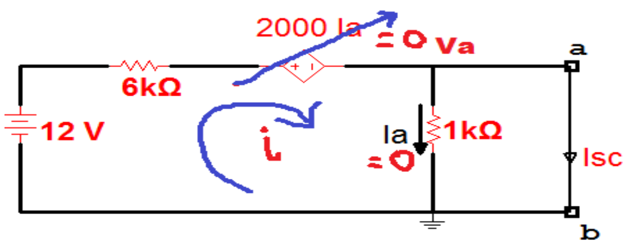
$$R_N = Voc / Isc$$

<u>Step 1: To find out I_{SC} (I_N)</u>

Short Circuit the terminals a-b and mark I_{SC} as shown

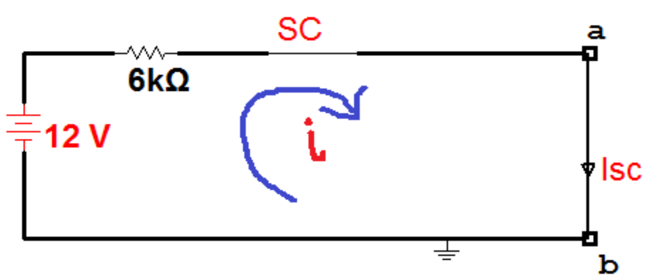


Va = Ia = 0



Since Va is connected to ground through short circuit terminals a-b Va=0.

Hence the circuit gets reduced to…

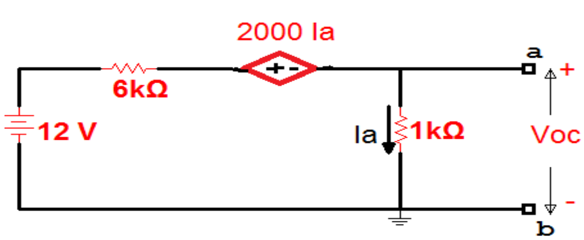


KVL: -12 + 6K i =0

i = 12/6K = 2 m A

<u>I_{SC} = i = 2 m A</u>

<u>Step 2: To find out V_{OC}</u>



KCL at node a:

$$\frac{Voc + 2000Ia - 12}{6K} + \frac{Voc}{1K} = 0$$

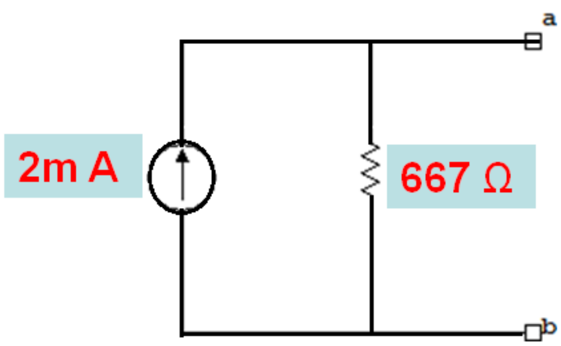
2000 Ia + 7 Va =12

Substituting Ia= $\frac{Voc}{1K}$

V_{OC} = 4/3 V

Therefore $R_N = V_{OC} / I_{SC} = 667\ \Omega$

Therefore Norton's equivalent circuit across the terminals a-b is given by



Theorem 2: Thevenin's Theorem

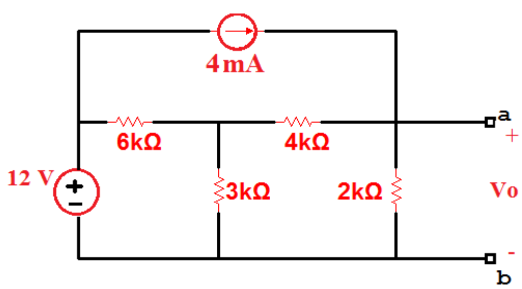
Definition :

Thevenin's Theorem states that a linear two terminal network can be replaced by an equivalent network consisting of an Voltage V_T in series with a resistor R_T , where

- R_T is the equivalent resistance at the terminals when the independent sources are turned off
- V_T is open circuit voltage across the terminals.

If the circuit consists of the dependent sources the Norton's resistance has to be found out as R_T = Voc / Isc

P1. Find V_o by Thevenin's Theorem



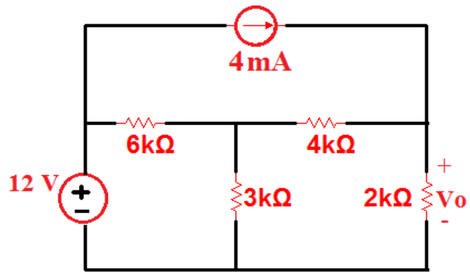
Solution:

Step 1: Remove resistor 2K Ω from the circuit across which V_O is dropping

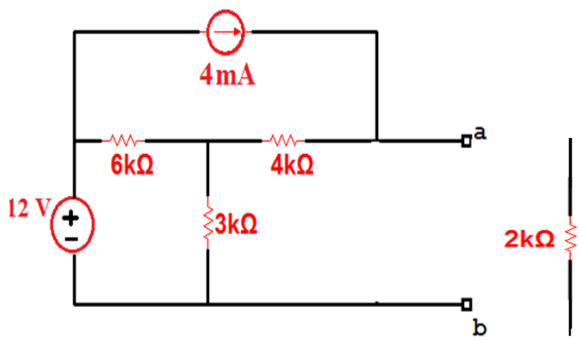
Step 2: Find the Thevenin's network across the open circuit terminals a-b

Step 3: Connect 2K Ω (Disconnected in Step 1) across the open circuit terminals a-b and find V_O.

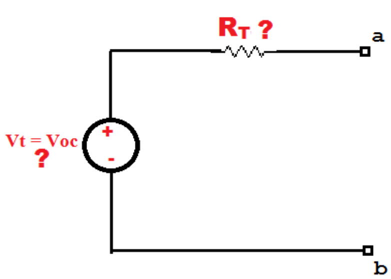
Circuit can be visualized as,



Step 1: Remove resistor 2K Ω from the circuit across which V_O is dropping and mark terminals a-b

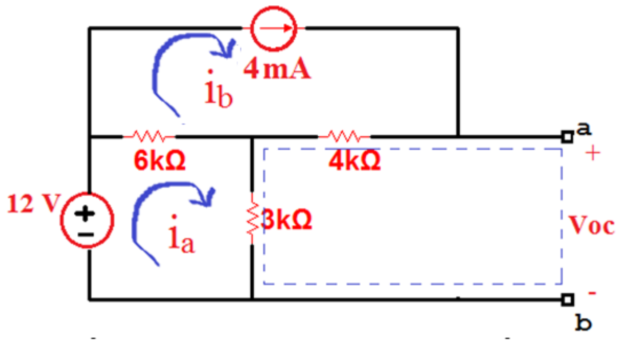


Step 2: Find the Thevenin's network across the open circuit terminals a-b



<u>To find V_{OC}:</u>

Mark V_{OC} across the open circuit terminals as shown:



Mark Mesh currents i_a and i_b:

By Observation:

$I_a = 4$ m A

Applying KVL to Mesh 1:

$- 12 + 6K (i_a - i_b) + 3K i_a = 0$

$9K i_a - 6K i_b = 12$

Sub. $I_a = 4$ mA,

$I_b = 4$ m A

To find V_{oc} apply KVL along the dotted path:

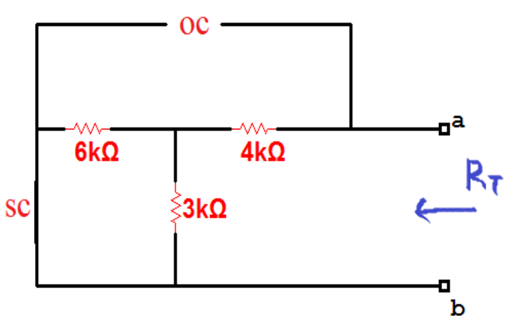
$- 3K I_a - 4K I_b + Voc = 0$

Sub. I_a and I_b,

$\boxed{Voc= 28\ V}$

To find R_T :

Deactivate the independent sources



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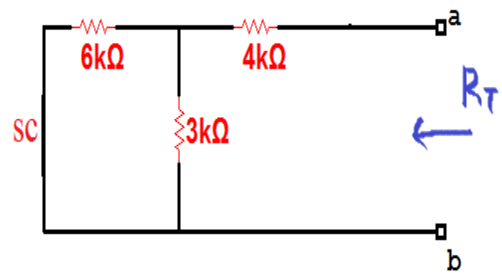
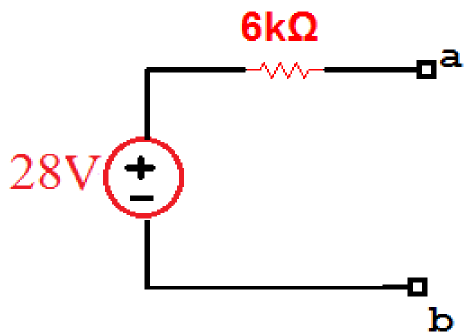


$R_T = (6K \,||\, 3K) + 4K$

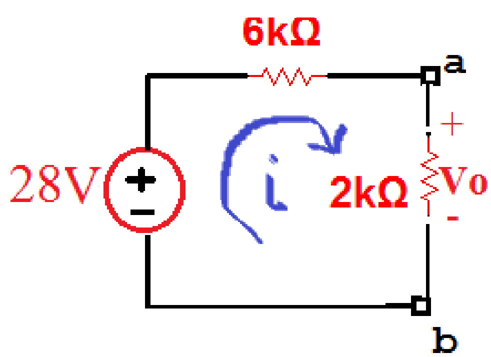
$\boxed{R_T = 6\text{ K}}$

Therefore the Thevenini's network is



<u>Step 3: To find V_O</u>

Now connect 2 K Ω across a-b to find V_O



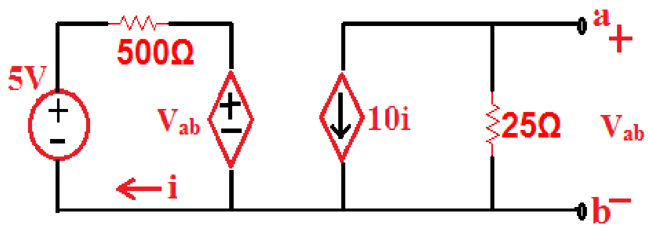
KVL gives,

$-28 + 6K\,i + 2K\,i = 0$

$i = 28/8K = 3.5\text{ mA}$

$\boxed{V_o = 2K\,i1 = 7\text{ V}}$

P2. Find the Thevenin's Equivalent circuit across terminals a-b

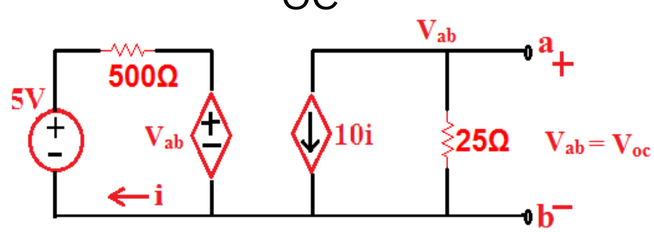


Solution:

Since the dependant sources are involved R_T is given by

$$R_T = V_{oc} / I_{SC}$$

Step 1: To find V_{OC}



Applying KVL to LHS part:

$$-5 + 500\,i + V_{ab} = 0$$
$$500\,i + V_{ab} = 5$$

Applying KCL to RHS part:

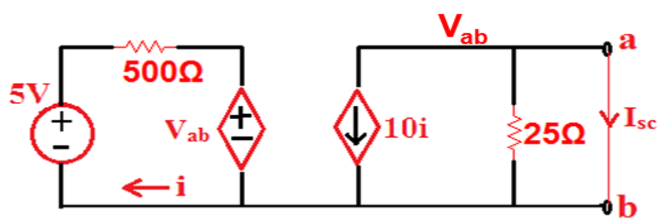
$$10\,i + V_{ab}/25 = 0$$
$$250\,i + V_{ab} = 0$$

Solving equations we have

$i = 0.02$ A $\quad V_{ab} = -5$ V

$V_{oc} = V_{ab} = -5$ V

Step 2:To find I_{SC}



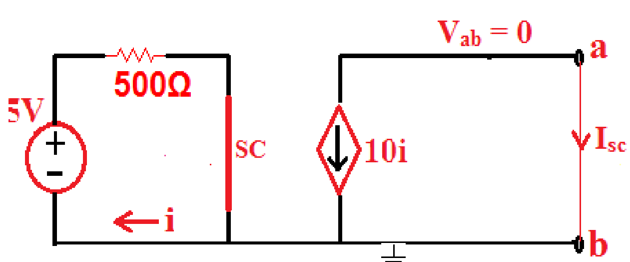
Short circuit terminals a-b and mark I_{SC} as shown

Mark V_{ab}

Since V_{ab} is connected to ground through a-b, $\mathbf{V_{ab} = 0}$

Since 25 Ω is in parallel with a short, 25 Ω is redundant

Therefore the circuit reduces to,



From LHS part, KVL gives

$-5 + 500\, i = 0$

From RHS part,

$I_{SC} = -10\, i$

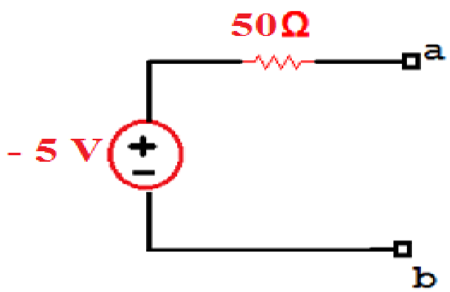
and sub. i = 0.01 A

$\boxed{I_{SC} = -0.1\ A}$

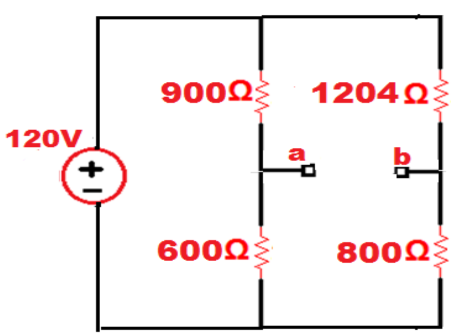
Therefore $R_T = V_{OC} / I_{SC} = -5 / -0.1$

$R_T = 50\ \Omega$

Therefore the Thevenin's network is,



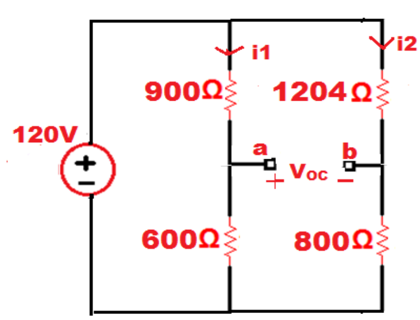
P3. Find the Thevenin's Equivalent network across terminals a-b



Solution:

Step1: To find Mark V_{OC} (V_T) across terminals a-b

Mark the branch currents i1 and i2 as shown



Applying KVL to mesh 1

$-120 + 900\,i1 + 600\,i1 = 0$

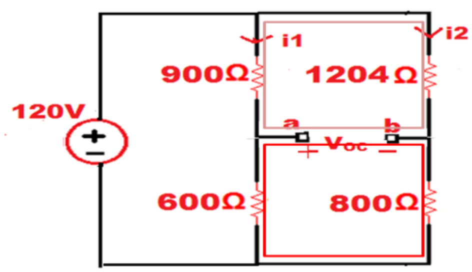
$i1 = 0.08$ A

Applying KVL to mesh 2

$-120 + 1204\,i2 + 800\,i2 = 0$

$i2 = 0.05988$ A

To find V_{OC}:



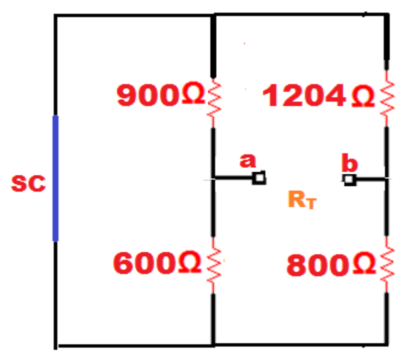
Applying KVL along the pink path

$-900\,i1 + 1204\,i2 - V_{OC} = 0$

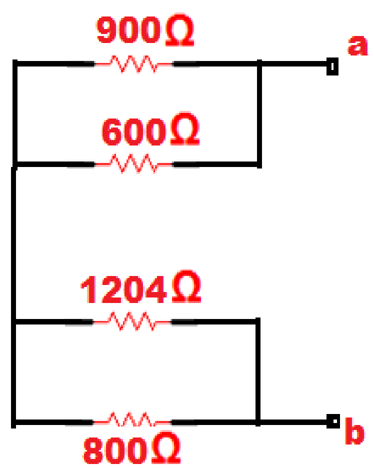
$V_{OC} = 0.095$ V

Step 2: To find R_T

Turning off 120 V source



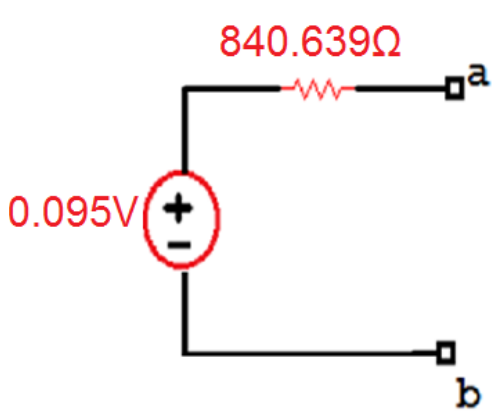
which can be visualized as



R_T = (900 || 600) + (1204 || 800)

R_T = 840.638 Ω

Therefore Thevenin's network is



Summary:

1. Thevenin’s network is a Voltage in series with a resistor

2. Thevenin’s voltage is V_{OC} across the terminals

3. Thevenin’s resitance and Norton’s resistance are the same.

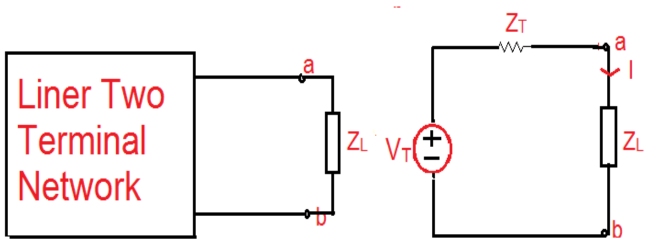
4. Thevenin’s and Norton’s equivalent networks can be obtained by source trensformatiom.

Theorem 3: Maximum Power Transfer Theorem

There are three cases to be considered in this

1. AC circuits with Impedance (Z_L) as load
2. AC circuits with purely resistive load (R_L)
3. DC circuits with resistive load (R_L)

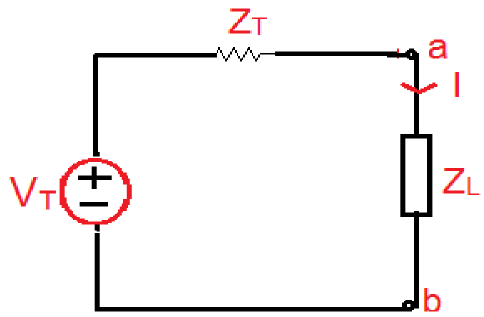
Conditions for Maximum Power Transfer :



where,

$Z_T = R_T + j X_T$

$Z_L = R_L + j X_L$



KVL to closed path:

$-V_T + Z_T\, I + Z_L I = 0$

$$I = \frac{V_T}{Z_T + Z_L} = \frac{V_T}{(R_T + j\, X_L) + (R_L + jX_L)}$$

The average power delivered to the load is

$$P = \frac{1}{2}|I^2|R \qquad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$I^2 = \frac{V_T^2}{[(R_T + j\, X_T) + (R_L + jX_L)]^2}$$

$$I^2 = \frac{V_T^2}{[(R_T + R_L) + j(X_T + X_L)]^2}$$

$$|I|^2 = \frac{|V_T|^2}{\left[\sqrt{(R_T + R_L)^2 + (X_T + X_L)^2}\right]^2}$$

Subtituting in equation in 1

$$P = \frac{R_L}{2}\, \frac{|V_T|^2}{(R_T + R_L)^2 + (X_T + X_L)^2}$$

For this P to be P_{Max} we can vary two parameters

– R_L and X_L in the load impedance.

Mathematically it can be done by differentiating P with respect to R_L and X_L partially and equating it to zero respectively.

i.e,

$$\frac{\partial P}{\partial R_L} = 0 \quad \text{and} \quad \frac{\partial P}{\partial X_L} = 0$$

Performing $\frac{\partial P}{\partial R_L} = 0$ results in

$$(R_T + R_L)^2 + (X_T + X_L)^2 - 2R_L(R_T + R_L) = 0$$

This implies

$$R_L = \sqrt{R_T^2 + (X_T + X_L)^2} \quad \ldots\ldots\ldots\ldots 2$$

Performing $\frac{\partial P}{\partial X_L} = 0$ results in

$$X_L = -X_T \quad \ldots\ldots\ldots\ldots 3$$

Substituting 3 in 2

$$R_L = R_T \quad \ldots\ldots\ldots\ldots\ldots\ldots 4$$

From equations 3 and 4

$$Z_L = R_L + j\,X_L = R_T - j\,X_T$$

$$Z_L = Z_T^*$$

If the Load Z_L is purely resistive then

$X_L = 0$ and $Z_L = R_L$

Substituting $X_L = 0$ in 2

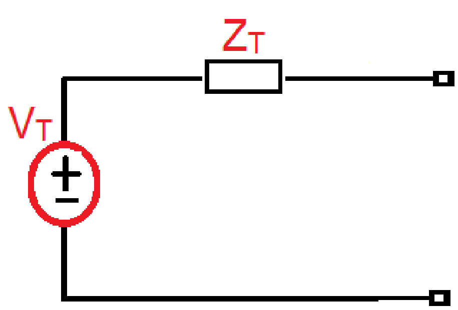
$$R_L = \sqrt{R_T^2 + X_T{}^2} \quad \text{..................}5$$

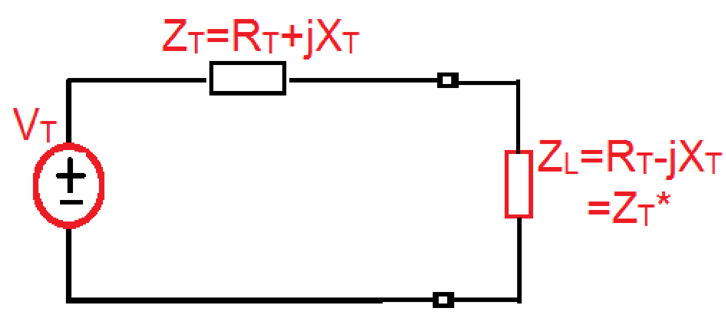
$$R_L = |Z_T| \quad \text{...............}6$$

Equations 4 , 5 and 6 are the conditions for which the maximum power would be transferred to the load.

Highlights:

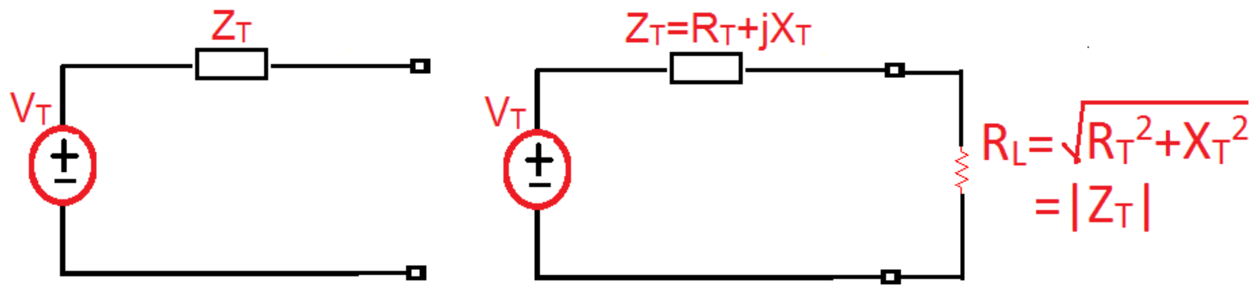
1. AC circuits with Impedance (Z_L) as load





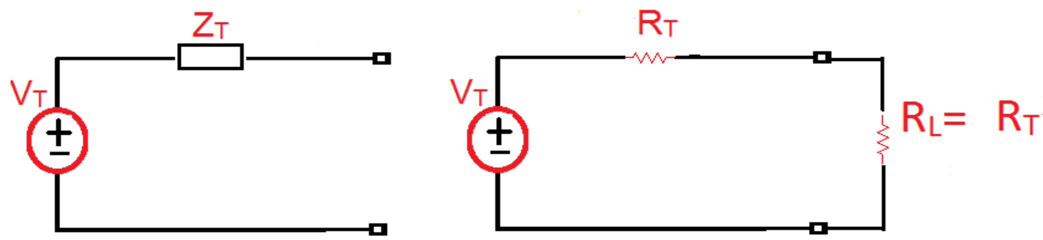
$P_{max} = |\ i\ |^2 R_L$

2. AC circuits with Pure Resistive (R_L) load



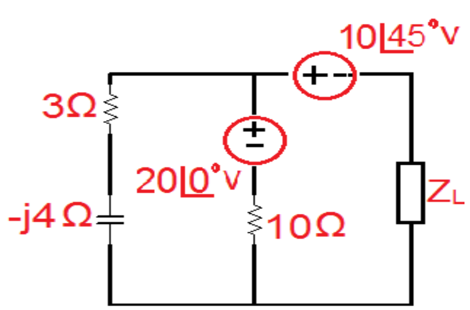
$P_{max} = |\ i\ |^2 R_L$

3. DC circuits with Resistor (R_L) as the load



$P_{max} = i^2 R_L$

P1. Calculate the value of Z_L for maximum power transfer and also calculate the maximum power.



Solution:

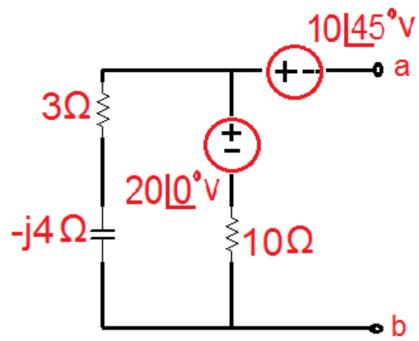
Step1. Remove the Impedance Z_L

Step2. Find the Thevenin's equivalent network across the terminals a-b

Step3. Connect $Z_L = Z_T^*$ across the terminals a-b for the maximum power transfer.

Step4. Find $P_{max} = |\ I\ |^2 R_L$

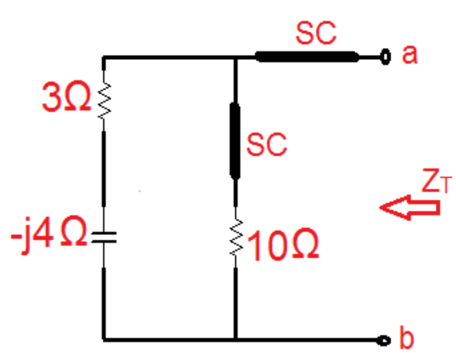
Step1. Remove the Impedance Z_L and mark terminals a-b



Step2. Find the Thevenin's equivalent network across the terminals a-b.

To find Thevenin's Impedance Z_L:

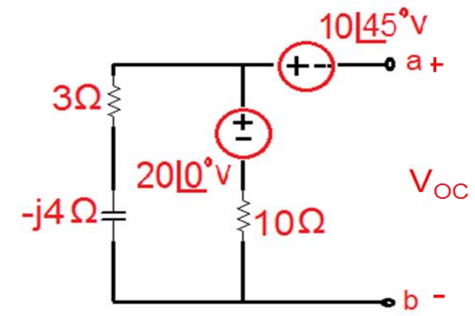
Deactivating the independent sources we have,

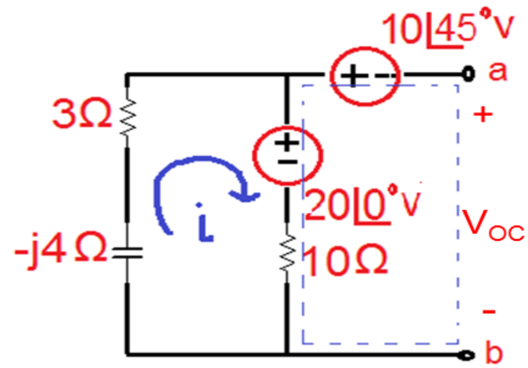


$Z_T = 10 \,||\, (3 - j\,4)$

$Z_T = 2.97 - j\,2.16\,\Omega$

To find Thevenin's Voltage V_T or V_{OC}:





<u>KVL implies:</u>

(3-j4) i + 20 +10 i = 0

i = -1.405 - j 0.432

KVL along the dotted path to find V_{OC}:

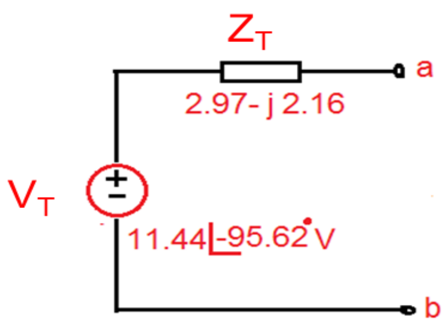
- 10 i – 20 + 10∟45 + V_{OC} = 0

Substituting i

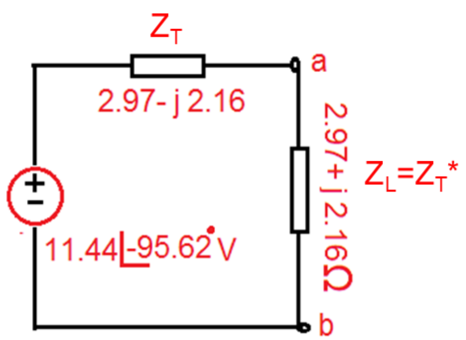
V_T = -1.121- j 1.391

= 11.44 ∟-95.62 V

Therefore Thevenin's equivalent network is



Step3. Connect Z_L=Z_T* across the terminals a-b to find the maximum power transfer.



KVL implies:

$-11.44 \angle -95.62 + (2.9729)i + (2.9729)i = 0$

$i = -0.185 - j\,1.916$ A

$i = 1.925 \angle -95.62$ A

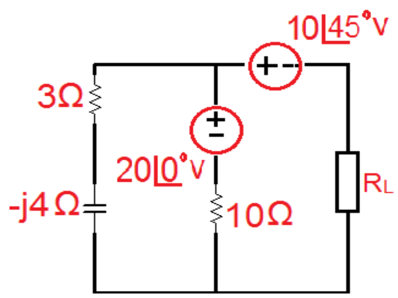
Step 4. To find P_{max}

$P_{max} = |\,i\,|^2 R_L$

$= (1.925)^2 \times 2.9729$

$P_{max} = 11$ Watts

P2. Calculate the value of R_L for maximum power transfer and also calculate the maximum power.



Solution:

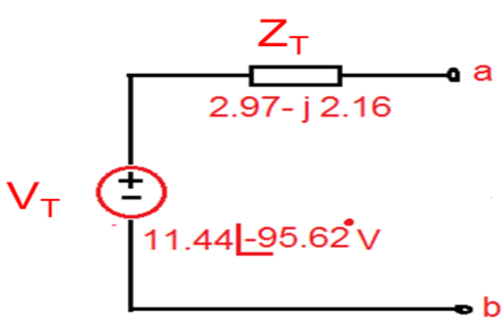
Step1. Remove the Impedance Z_L

Step2. Find the Thevenin's equivalent network across the terminals a-b

Step3. Connect Z_L=| Z | across the terminals a-b for the maximum power transfer.

Step4. Find $P_{max} = |I|^2 R_L$

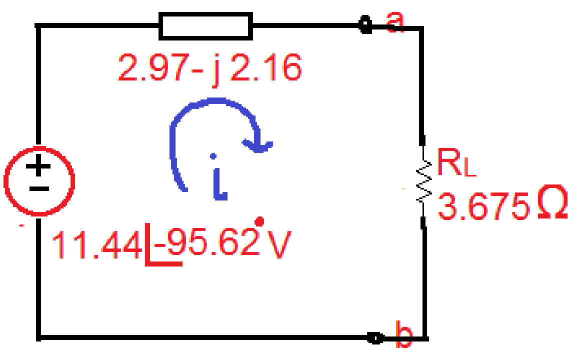
From Step1 and Step2 (Refer P1), the Thevenin's equivalent is



Step3. Connect R_L=| Z |across the terminals a-b to find the maximum power transfer.

$$R_L = |Z_T| = \sqrt{(2.97)^2 + (2.16)^2}$$

R_L = 3.675 Ω



KVL implies

$-11.44 \angle -95.62 + (2.97 - j\,2.16)\,i + 3.675\,i = 0$

$i = 1.6377 \angle -77.62$ A

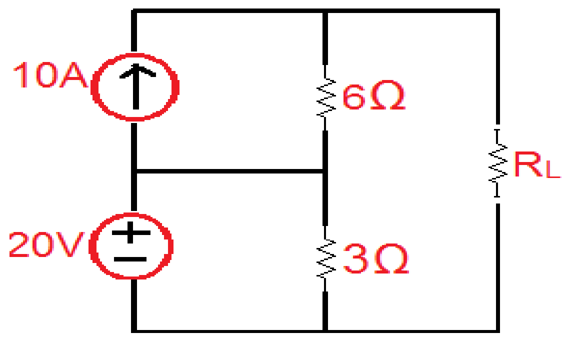
<u>Step 4. To find P_{max}</u>

$P_{max} = |\,i\,|^2 R_L$

$= (1.6377)^2 \times 3.675$

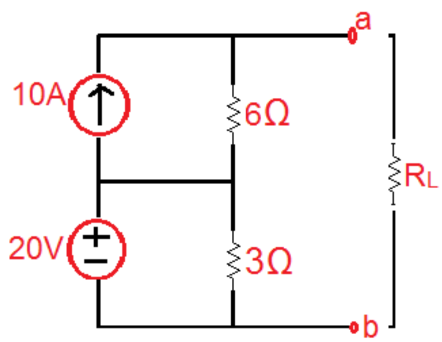
$P_{max} = 9.85$ W

P3. Find the R_L across the load for which maximum power will be transferred to the load and hence find the maximum power



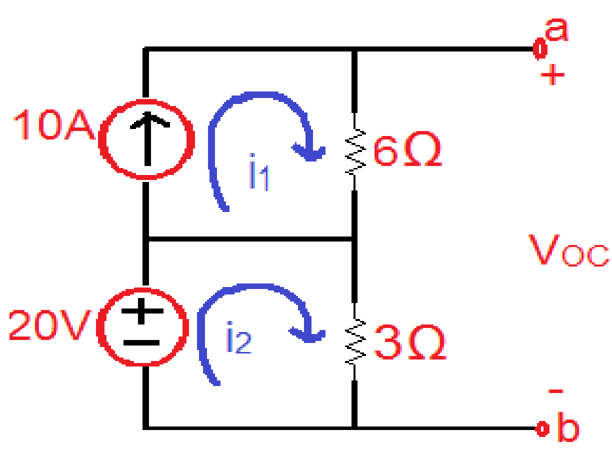
Solution:

Step 1: Remove the resistor R_L and mark terminals a-b as shown



Step 2: Find the Thevenin's network across the terminals a-b

To find V_{OC}:



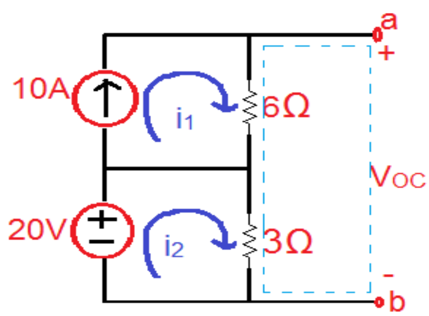
By observation:

$i_1 = 10$ A

KVL to mesh 2:

$-20 + 3\ i_2 = 0$

$i_2 = 20/3$ A



$- 3i_2 – 6i_1 + V_{OC} = 0$

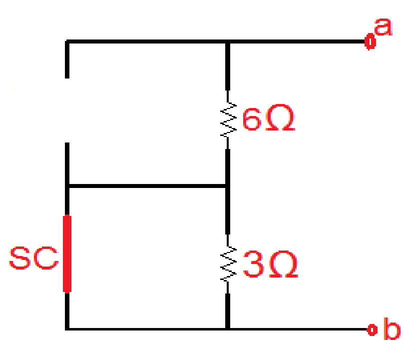
KVL along the dotted path

$V_{OC} = 6\ i_1 + 3\ i_2$

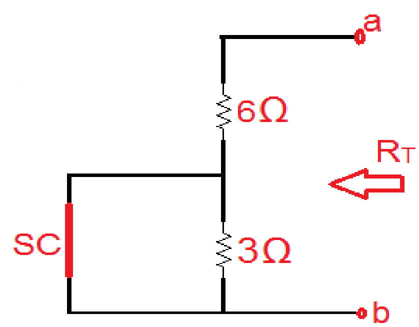
Substituting i_1 and i_2

$V_T = V_{OC} = 80$ V

To find R_T:



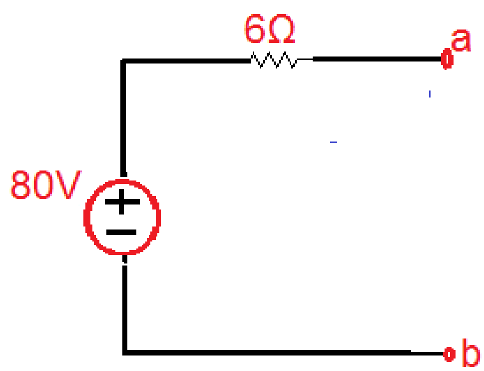
which can be visualized as



Since 3 Ω is in parallel with the short, it is redundant.

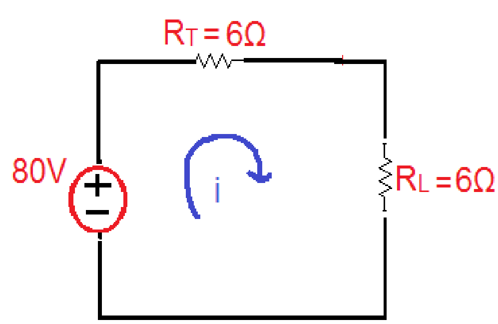
Therefore R_T= 6 Ω

Therefore Thevenin's network is



Step 3: To find P_{max}

Connect R_L = R_T across the terminals a-b



KVL implies:

$- 80 + 6i + 6i = 0$

$i = 20/3$ A

$P_{max} = i^2 R_L = (20/3)^2 \times 6 = 266.66$ W

Summary:

1. Maximum power transfer theorem is the extention of Thevenin's theorem.
2. The coditions for Maximum power to be transferred to the load are

   i) For AC circuits if load is impedance then $Z_L = Z_T^*$

   ii) For AC circuits if load is purely resistive then $R_L = |Z_T|$

   iii) For DC circuits $R_L = R_T$

3. Power is always a real entity and therefore for power calculations always real part of Z_L (i.e., R_L) is used.

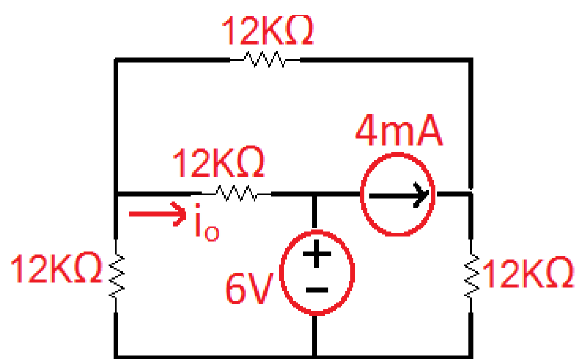
Theorem 4: Superposition Theorem

Statement:

In any Linear circuit containing multiple independent sources, a current or a voltage at any point in the circuit can be calculated as algebraic sum of Individual contributions of each source when acting alone.

Problems:

P1. Find i_o by Super position theorem.



Solution:

Let $i_0 = i_{01} + i_{02}$

where,

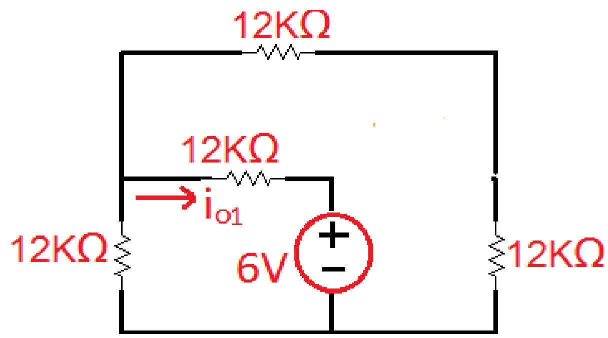
i_{01} is the contribution of 6 V source when acting alone and

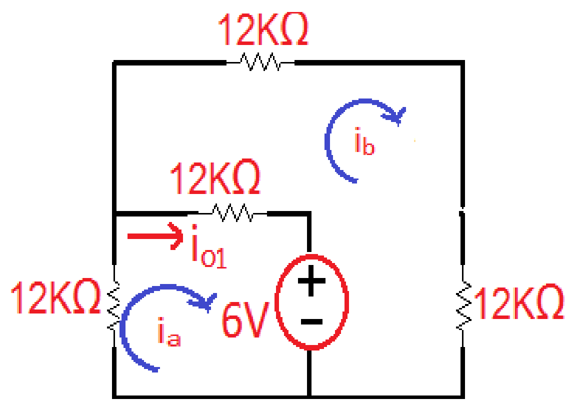
i_{02} is the contribution of 4mA source when acting alone

Steps:

Step 1 : To find i_{01} which is the contribution of 6 V acting alone

Deactivating the 4mA source the circuit becomes





<u>Applying KVL to mesh 1:</u>

$12K\,i_a + 12K\,(i_a - i_b) + 6 = 0$

$24K\,i_a - 12K\,i_b = -6$ Eq1

<u>Applying KVL to mesh 2:</u>

$12K\,(i_b - i_a) + 12K\,i_b + 12K\,i_b - 6 = 0$

$-12K\,i_a + 36K\,i_b = 6$................Eq2

Solving equations Eq1 and Eq2,

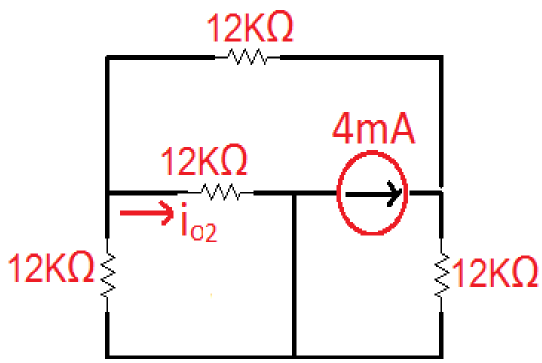
$i_a = -0.2$ mA

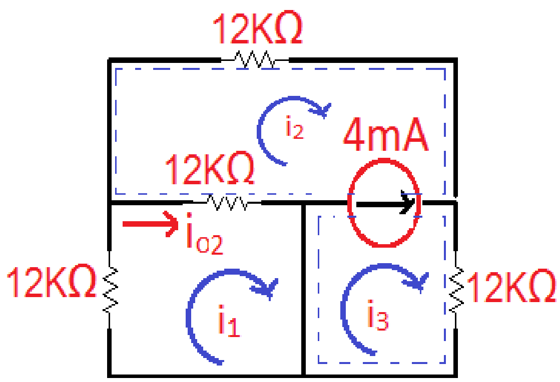
$i_b = 0.1$ mA

$i_{o1} = i_a - i_b = -0.3$ mA

Step 2 : To find i_{o2} which is the contribution of 4mA source acting alone

Deactivating the 6 V source the circuit becomes





Constraint equation:

$i_3 - i_2 = 4mA$

Applying KVL to mesh 1:

$12K\, i_1 + 12K\,(i_1 - i_2) = 0$

$24K\, i_1 - 12K\, i_2 = 0$

Applying KVL to Supermesh:

$12K(i_2 - i_1) + 12K\, i_2 + 12K\, i_3 = 0$

$-12K\, i_1 + 24K\, i_2 + 12K\, i_3 = 0$

Applying KVL to mesh 1:

$12K\, i_1 + 12K\,(i_1 - i_2) = 0$

$24K\, i_1 - 12K\; i_2 = 0$

Solving equations 1, 2 and 3

$i_1 = -0.8$ mA; $i_2 = -1.6$ mA; $i_3 = 2.4$mA

$\mathbf{i_{o2} = i_1 - i_2 = 0.8\ mA}$

Step 3 : To find i_o

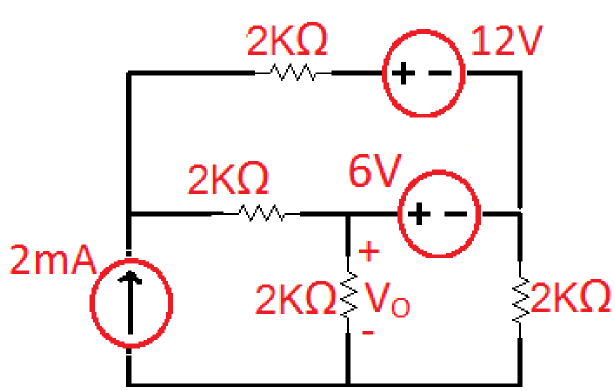
By Super Position Theorem,

$$i_0 = i_{01} + i_{02}$$

$$i_o = -0.3m + 0.8m$$

$$i_o = 0.5\ m\ A$$

P2. Find V_o by Super position theorem.



Solution:

Let $V_0 = V_{01} + V_{02} + V_{03}$

where,

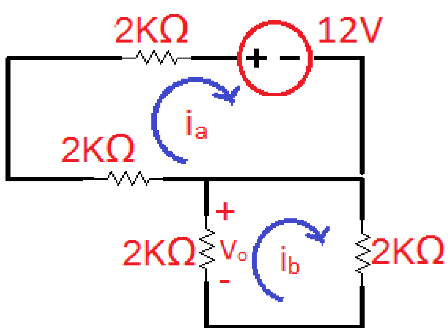
V_{01} is the contribution of 12V source when acting alone

V_{02} is the contribution of 6V source when acting alone

V_{03} is the contribution of 2mA source when acting alone

Step 1: To find V_{o1}

Deactivate 6V and 2mA sources



KVL to mesh2:

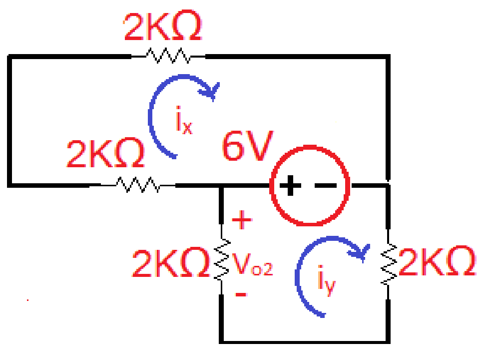
$2K\, i_b + 2K\, i_b = 0$

$i_b = 0$

$V_{o1} = -\,2K\, i_b = 0V$

Step 2: To find V_{o2}

Deactivate 12V and 2mA sources



KVL to mesh2:

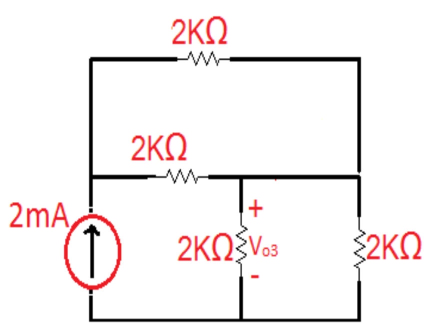
$2K\, i_y + 6 + 2K\, i_y = 0$

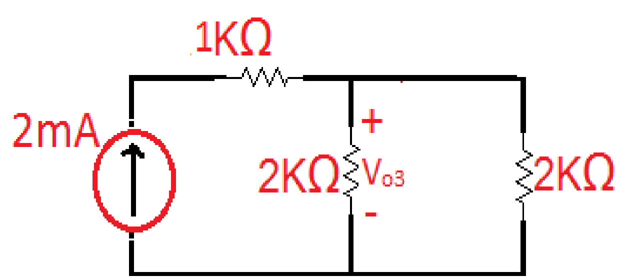
$i_Y = -1.5mA$

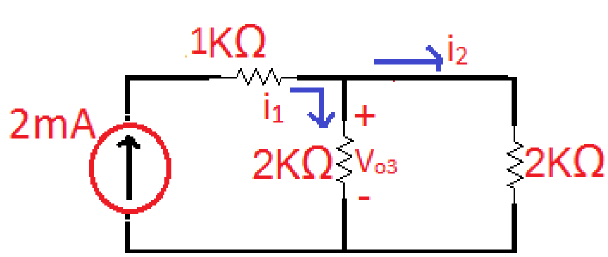
$V_{O2} = -\, 2K\, i_Y = 3\ V$

Step 3: To find V_{o3}

Deactivate 12V and 6V sources







$i_1 = i_2 = 1mA$

$V_{O3} = 2K\, i_1 = 2V$

Step 4:

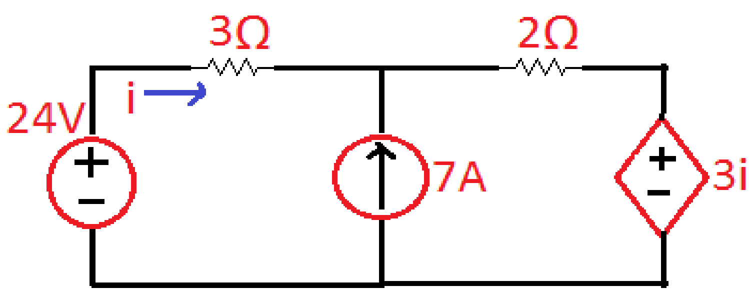
By Super position Theorem

$V_0 = V_{01} + V_{02} + V_{03}$

$V_0 = 0 + 3 + 2$

$V_0 = 5$ V

P3. Find i by Super position theorem.



Solution:

Let $i = i_1 + i_2$

where,

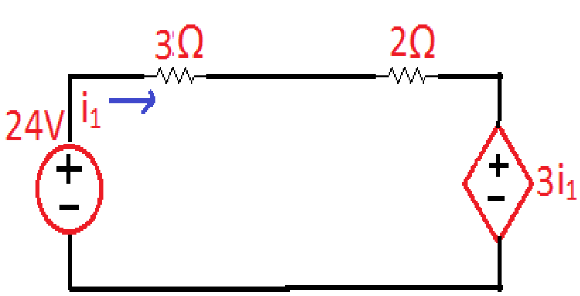
i_1 is the contribution of 24V source when acting alone

i_2 is the contribution of 7A source when acting alone

The dependant voltage source cannot be deactivated - keep it as it is.

Step 1: To find i_1

Deactivate 7A source



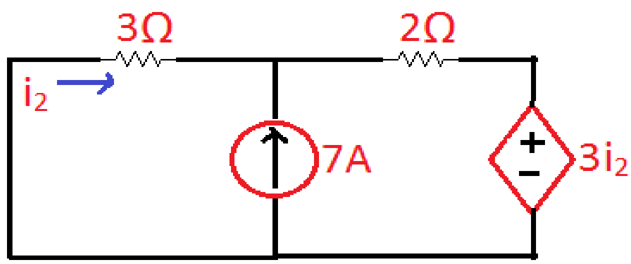
Applying KVL:

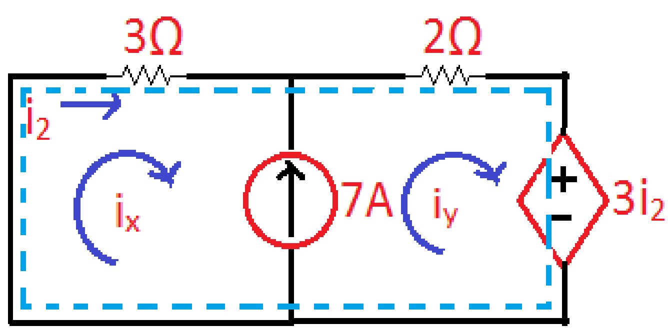
$-24 + 3\,i_1 + 2\,i_1 + 3\,i_1 = 0$

$i_1 = 3$ A

<u>Step 2: To find i_2</u>

Deactivate 24V source





<u>Constraint equation:</u>

$-i_X + i_Y = 7A$

<u>KVL to Supermesh:</u>

$3\,i_X + 2\,i_Y + 3\,i_2 = 0$

Sub. $i_2 = i_X$

$3\,i_X + 2\,i_Y + 3\,i_X = 0$

$6\,i_X + 2\,i_Y = 0$

Solving the equations

$-i_X + i_Y = 7A$

$6\ i_X + 2\ i_Y = 0$

Implies,

$i_X = -1.75$ A and $i_Y = 5.25$A

$i_2 = i_x = -1.75$A

Step 3:

By Super position Theorem

$i = i_1 + i_2$

$i = 3 - 1.75$

$i = 1.25$ A

Summary:

1. Superposition theorem is applicable to circuits with multiple independent sources only.
2. Dependant sources can be present.
3. At a time only one independent source should be acting, which gives its individual contribution.
4. Algebraic summation of the individual contributions gives the actual current/voltage in a circuit.
5. It is as good as cutting down complex problems into simpler ones.

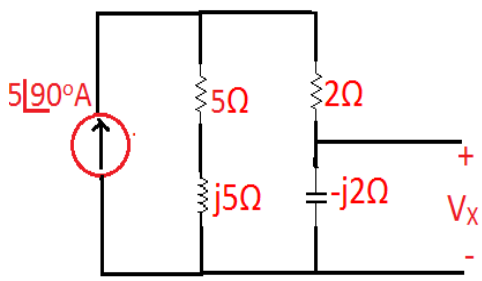
Theorem 5: Reciprocity Theorem

Statement:

In any Linear Bilateral single source circuit, the ratio of Excitation to Response is constant when the positions of Excitation and Response are interchanged.

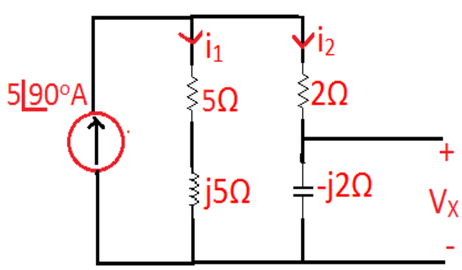
Problems:

P1. Find V_X and verify Reciprocity theorem.



Solution:

Step 1: To find the response V_X



Mark the branch currents i_1 and i_2

By current Division Rule:

$$i2 = \frac{(5+j5)5\angle 90}{(5+j5)+(2-j2)}$$

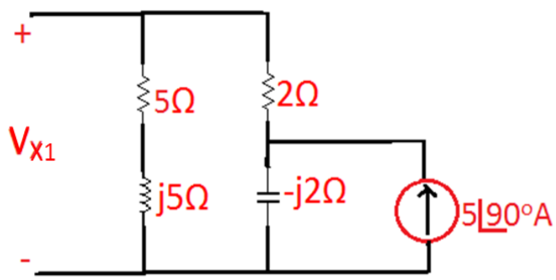
$$i_2 = -1.72 + j4.13 = 4.64\angle 111.8$$

Therefore V_X is given by,

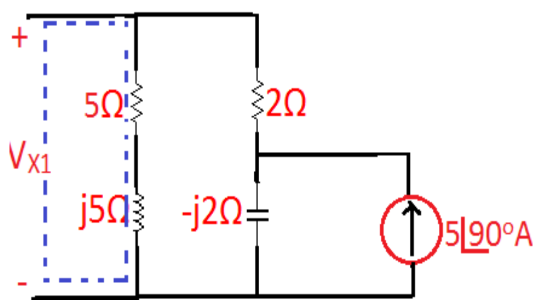
$$V_x = (-j2)i_2$$

$$\boxed{V_X = 8.62 + j3.44 = 9.28\angle 21.8^{o}\text{V}}$$

Step 2: Interchange the Excitation and Response



To find V_{X1} :



By Observation:

$$i_2 = -5\angle 90 \text{ A}$$

KVL to Mesh1:

$(5+j5)\ i_1 + 2\ i_1 - j2\ (i_1 - i_2) = 0$

Sub.

$$i_2 = -5\ \ 90^{o}\text{A}$$

$i_1 = -1.2 + j\,0.5 = 1.31\ \underline{|156.8^0}$A

KVL along the dotted path:

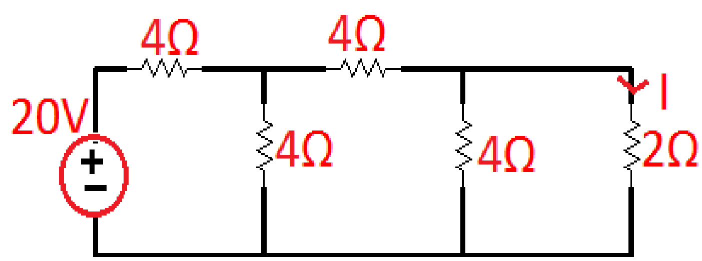
$-V_{X1} - (5+j5)\, i_1 = 0$

$Sub.\ i_1 = 1.31\ \underline{|156.8^0}$

$$\boxed{V_{X1} = 9.28\ \underline{|21.8^0}\mathrm{V}}$$

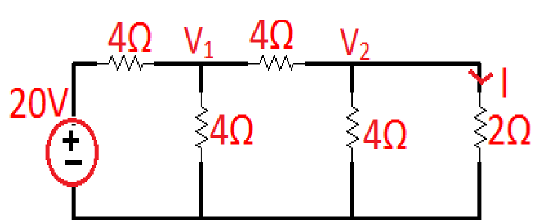
Since $V_X = V_{X1}$, Reciprocity Theorem is Verified .

P2. Find I and verify Reciprocity theorem.



<u>Solution:</u>

<u>Step 1: To find the response I</u>



<u>KCL at node 1:</u>

$$\frac{V_1 - 20}{4} + \frac{V_1}{4} + \frac{V_1 - V_2}{4} = 0$$

$$\frac{V_1 - 20 + V_1 + V_1 - V_2}{4} = 0$$

$3V_1 - V_2 = 20$ Eq1

<u>KCL at node 2:</u>

$$\frac{V_2}{4} + \frac{V_2 - V_1}{4} + \frac{V_2}{2} = 0$$

$$\frac{V_2 + V_2 - V_1 + 2V_2}{4} = 0$$

$-V_1 + 4V_2 = 0$ Eq2

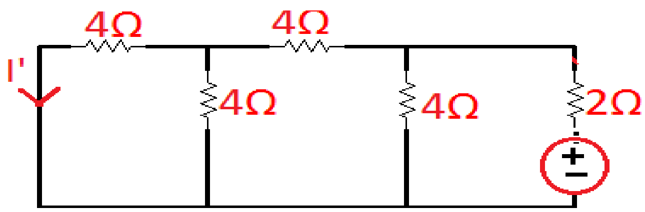
Solving equations 1 and 2

V1=7.27 V ; V2=1.8181V

$$\boxed{I = \frac{V_2}{2} = 0.909A}$$

Step 2: To find the response $I^{\prime}$

Interchange the positions of Excitation and Response



KCL at node 1:

$$\frac{V_1}{4}+\frac{V_1}{4}+\frac{V_1-V_2}{4}=0$$

$$\frac{V_1+V_1+V_1-V_2}{4}=0$$

$3V_1 - V_2=0$ -----------Eq A

KCL at node 2:

$$\frac{V_2-V_1}{4}+\frac{V_2}{4}+\frac{V_2-20}{2}=0$$

$$\frac{V_2-V_1+V_2+2V_2-40}{4}=0$$

$-V_1 + 4V_2=40$ ....Eq B

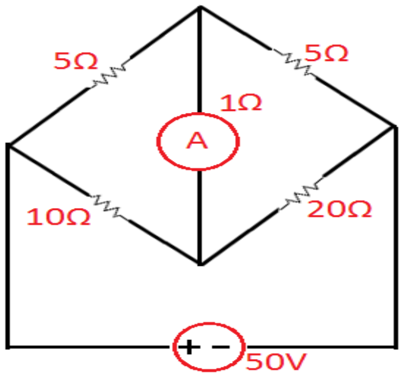
Solving equations 1 and 2

V1=3.6363 V ; V2=10.9V

$$\boxed{I' = \frac{V_1}{4} = 0.909A}$$

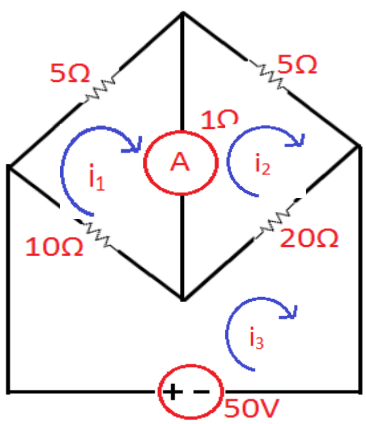
Since I = I^{I}, Reciprocity Theorem is Verified.

P3. Find The current through the ammeter and verify Reciprocity theorem.



Solution:

Step 1: To find the Ammeter current



KVL at mesh 1:

5 i1 + 1 (i1-i2) + 10 (i1-i3) = 0

KVL at mesh 2:

1 (i2-i1)+ 5 i2 + 20 (i2-i3) = 0

KVL at mesh 3:

10 (i3-i1)+ 20 (i3-i2)- 50 = 0

Which give,

16 i1 - i2 + 10 i3 = 0

- i1 + 26 i2 - 20 i3 = 0

- 10 i1 - 20 i2 + 30 i3 = 50

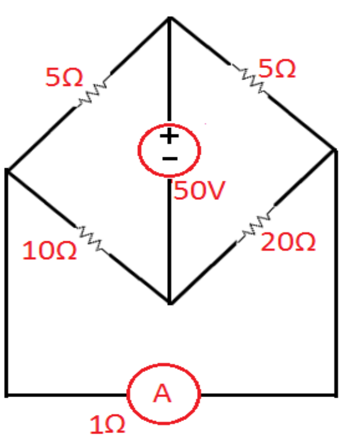
Solving the above for i1, i2 and i3

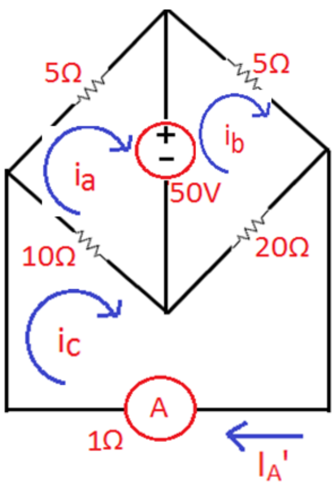
i1 = 4.59 A , i2 = 5.4098 A and i3 = 6.8 A

I A = i2 – i = 0.8 A Flowing upwards

Step 2: To find the response IA^{I}

Interchange the positions of Excitation and Response





KVL at mesh 1:

5 ia + 50 + 10 (ia-ic) = 0

KVL at mesh 2:

-50 + 5 ib + 20 (ib-ic) = 0

KVL at mesh 3:

10 (ic-ia)+ 20 (ic-ib)+ 1 ic = 0

Which give

15 ia - 10 ic = -50

25 ib - 20 ic = 50

- 10 ia - 20 ib + 31 ic = 0

Solving the above for ia, ib and ic

ia = -2.8 A , ib = 2.64 A and ic = 0.8 A

I'A = ic = 0.8 A

Since IA = I
I A, Reciprocity Theorem is Verified

Theorem 6: Millman's Theorem

Statement:

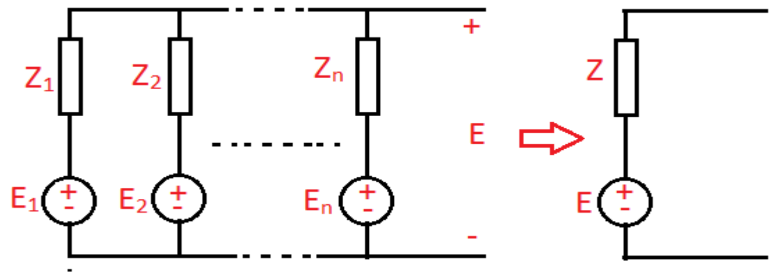
If 'n' generators of EMFs $E_1, E_2, \ldots E_n$ with internal impedances $Z_1, Z_2, \ldots Z_n$ are connected in parallel then the EMFs and the impedances can be combined to give a single EMF E with internal Impedance Z, where

$$E = \frac{E_1 Y_1 + E_2 Y_2 \ldots\ldots + E_n Y_n}{Y_1 + Y_2 \ldots\ldots + Y_n}$$

and

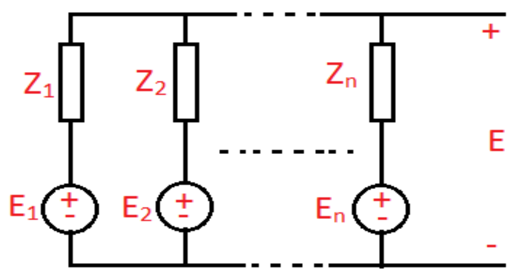
$$Z = \frac{1}{Y_1 + Y_2 \ldots\ldots + Y_n}$$

Proof:



Note that Z_1, Z_2......Z_n are the internal impedances

Consider,



KCL at node E:

$$\frac{E-E_1}{Z_1}+\frac{E-E_2}{Z_2}\ldots\ldots+\frac{E-E_n}{Z_n}=0$$

$$\frac{E-E_1}{Z_1}+\frac{E-E_2}{Z_2}\ldots\ldots+\frac{E-E_n}{Z_n}=0$$

$$E\left[\frac{1}{Z_1}+\frac{1}{Z_2}\ldots\ldots\frac{1}{Z_n}\right]=\frac{E_1}{Z_1}+\frac{E_2}{Z_2}\ldots.+\frac{E_n}{Z_n}$$

$$E[Y_1+Y_2\ldots\ldots+Y_n]=E_1Y_1+E_2Y_2\ldots+E_nY_n$$

$$\boxed{E=\frac{E_1Y_1+E_2Y_2\ldots\ldots+E_nY_n}{Y_1+Y_2\ldots\ldots+Y_n}}$$

Since all the internal impedances are in parallel,

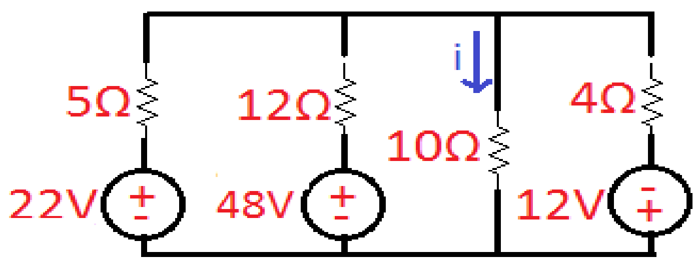
$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} \ldots\ldots \frac{1}{Z_n}$$

$$Y = Y_1 + Y_2 \ldots\ldots + Y_n$$

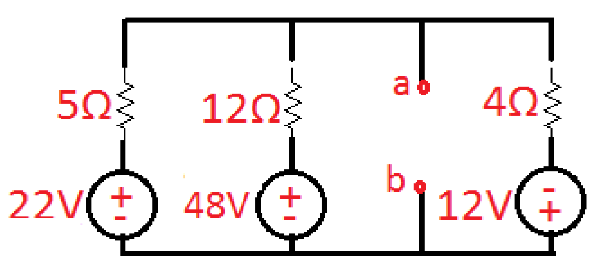
$$\boxed{Z = \frac{1}{Y_1 + Y_2 \ldots\ldots + Y_n}}$$

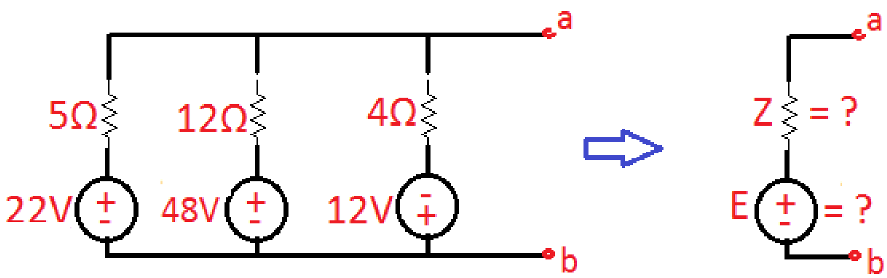
<u>Problems:</u>

P1. Find the current through 10 Ω by Millman'sTheorem.

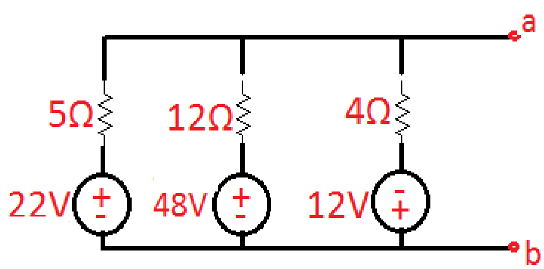


<u>Step 1: Remove 10Ω and mark terminals a-b</u>





Step 2: To find E and Z



$$E = \frac{E_1 Y_1 + E_2 Y_2 + E_3 Y_3}{Y_1 + Y_2 + Y_3}$$

$$E = \frac{22\left(\frac{1}{5}\right) + 48\left(\frac{1}{12}\right) - 12\left(\frac{1}{4}\right)}{\frac{1}{5} + \frac{1}{12} + \frac{1}{4}}$$

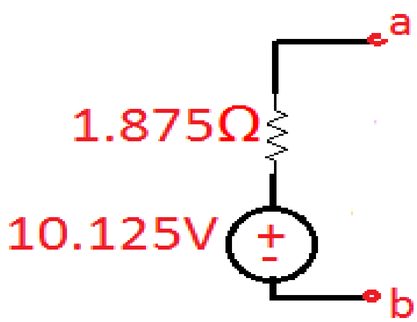
E = 10.125 Ω

$$Z = \frac{1}{Y_1 + Y_2 + Y_3}$$

$$Z = \frac{1}{\frac{1}{5} + \frac{1}{12} + \frac{1}{4}}$$

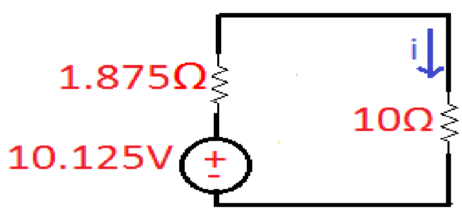
$$\boxed{Z = 1.875\ \Omega}$$

Therefore by Millman's Theorem



<u>Step 3: To find i through 10 Ω</u>

Connect 10 Ω across terminals a-b

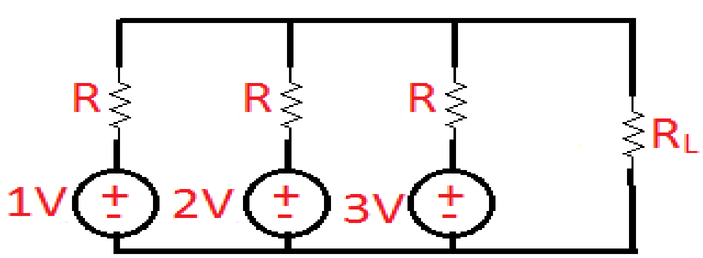


KVL:

$- 10.125 + 1.875\,i + 10\,i = 0$

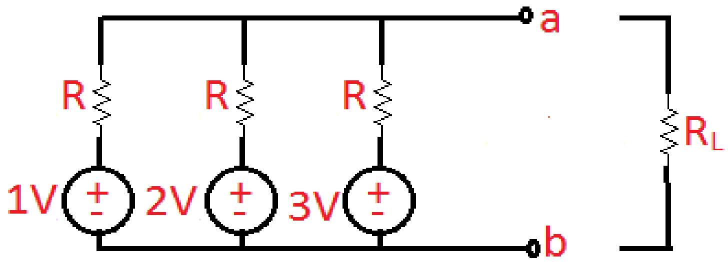
$$\boxed{i = 0.85\ A}$$

P2. Find R such that the maximum Power delivered to the load is 3mW

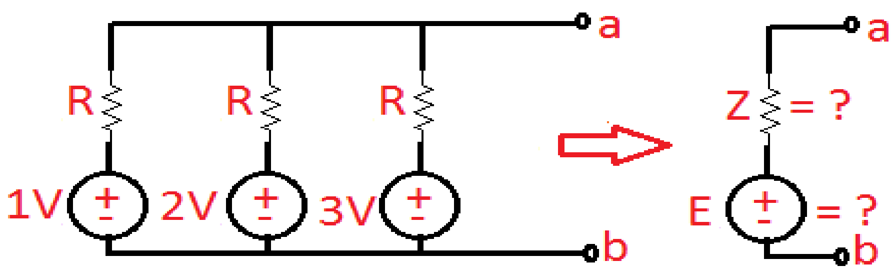


<u>Solution:</u>

Step 1: Remove R_L and mark the terminals a-b



Step 2: Using Millman's Theorem obtain one generator of emf E and internal impedance Z across a-b



To find E:

$$E = \frac{E_1Y_1 + E_2Y_2 + E_3Y_3}{Y_1 + Y_2 + Y_3}$$

$$E = \frac{1\left(\frac{1}{R}\right) + 2\left(\frac{1}{R}\right) + 3\left(\frac{1}{R}\right)}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R}}$$

$$E = \frac{6/R}{3/R} = 2V$$

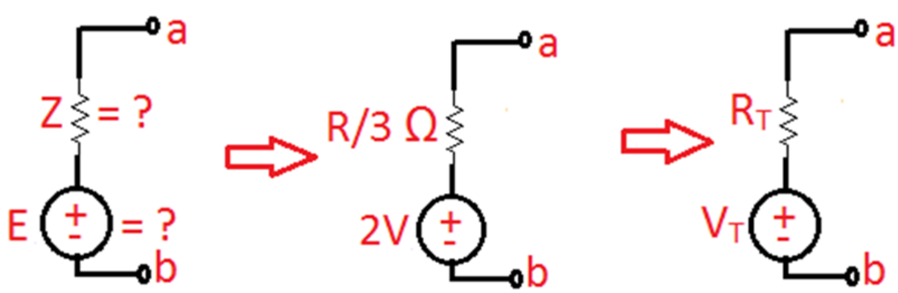
$$\boxed{E = 2V}$$

To find Z:

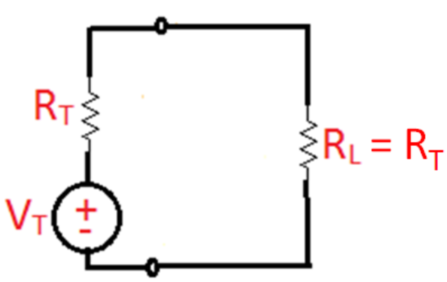
$$Z = \frac{1}{Y_1 + Y_2 + Y_3}$$

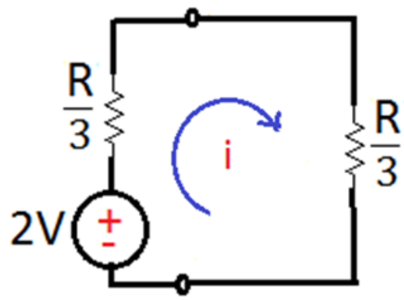
$$Z = \frac{1}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R}} = \frac{R}{3}\Omega$$

$$\boxed{Z = \frac{R}{3}\ \Omega}$$



Therefore by Maximum Power Transfer Theorem, for Maximum Power to be transferred to the load in DC circuits $R_L = R_T$





Applying KVL:

$$-2 + \frac{R}{3}i + \frac{R}{3}i = 0$$

$$i = \frac{3}{R}$$

<u>To find R for P_{max} = 3mW</u>

$$P_{max} = i^2 R_L$$

$$3m = \left(\frac{3}{R}\right)^2 \frac{R}{3}$$

$$3m = \frac{9}{R^2} \frac{R}{3}$$

$$\boxed{R = \frac{3}{3m} = 1K\Omega}$$

Therefore 1KΩ Resistor has to be connected as the load resistor for maximum power of 3mW to be delivered to the load.