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SAI VIDYA INSTITUTE OF TECHNOLOGY

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PRINCIPLES OF COMMUNICATION SYSTEMS (15EC45)

IV SEMESTER ECE

MODULE 4: NOISE IN ANALOG MODULATION

SYLLABUS: Introduction, Receiver Model, Noise in DSB-SC receivers, Noise in AM receivers, Threshold effect, Noise in FM receivers, Capture effect, FM threshold effect, FM threshold reduction, Pre-emphasis and De-emphasis in FM.

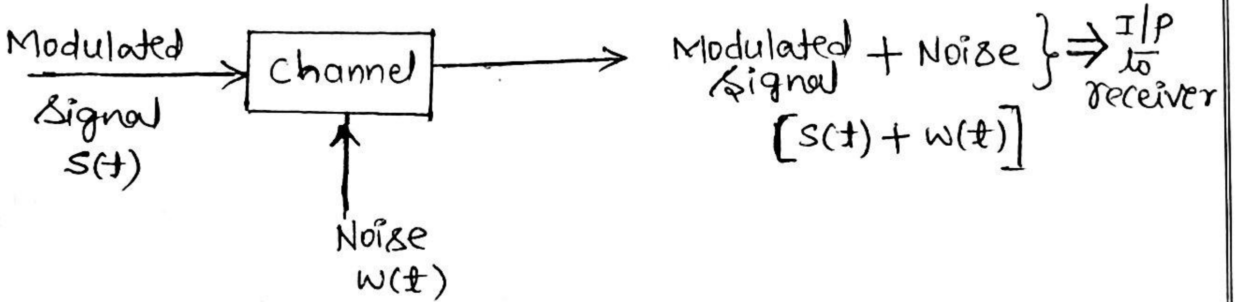
Module- 4

Noise in Analog Modulation

Introduction :-

Any unwanted frequency component which appears within the operating frequency is considered as "Noise".

↳ Noise gets added to the transmitted signal s(t), during the transmission over Communication channel.



↳ In this module the effect of Noise on receiver performance parameters such as

(i) pre-SNR (or) channel signal to Noise ratio $(SNR)_{channel}$

(ii) post-SNR (or) output signal to Noise ratio $(SNR)_{output}$

(iii) Figure-of-Merit (FOM), for analog modulation techniques such as DSBSC, AM and FM are discussed in detail.

↳ This module also gives the overview of pre-emphasis and De-emphasis methods used to improve signal to Noise ratio at transmitter and receiver respectively.

↳ Higher the value of Figure-of-Merit <FOM> better the performance of the receiver.

↳ FOM, for various analog Modulation techniques is found to be

- for DSBSC ⟶ FOM = 1
- for AM ⟶ $FOM = \frac{\mu^2}{2+\mu^2}$
- for FM ⟶ $FOM = 1.5\beta^2$

4.1 : Receiver Model :-

<Q> With relevant equations, Explain how noise is produced in a receiver model.

June/July - 2017 (8-Marks)

↳

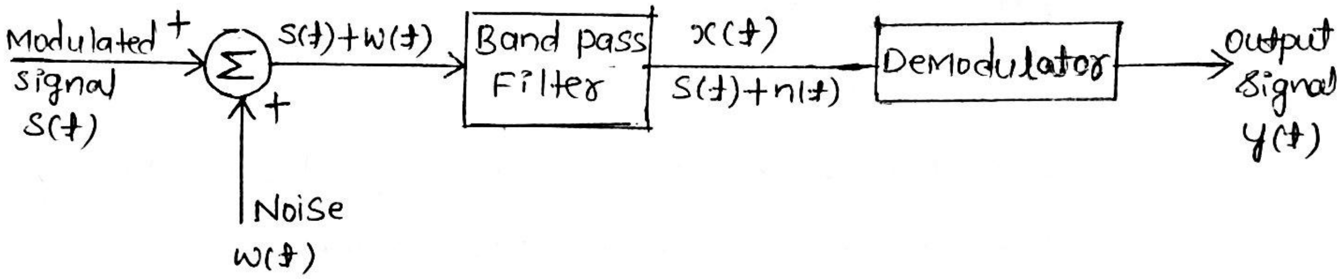


Fig.1 : Receiver Model

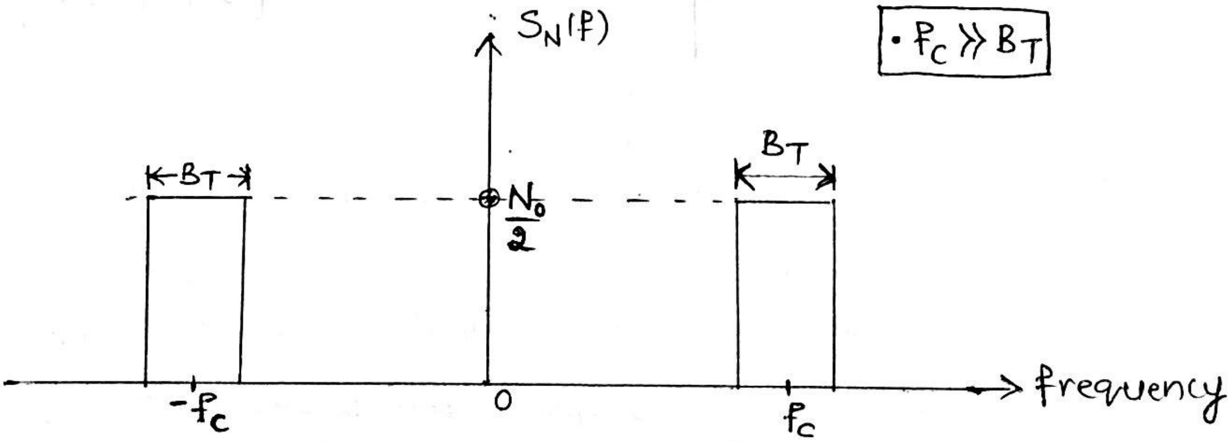


Fig.2 : Idealized characteristics of Band pass filter noise

Figure.1, shows the basic form of receiver model.

Let $s(t) \Rightarrow$ Modulated signal

$w(t) \Rightarrow$ Noise signal (wide-band noise)

↳ The receiver input signal is the sum of $s(t)$ and $w(t)$.

↳ The band pass filter in the receiver model represents the combined filtering action of the tuned amplifiers used in the actual receiver.

↳ The Bandwidth of a Band pass Filter <BPF> is kept just wide enough to pass the modulated signal $s(t)$ without distortion.

↳ The Demodulator block represented in Figure 1, depends on the type of modulation used to generate modulated signal, $s(t)$.

↳ The BPF, shown in receiver model is assumed to be ideal with characteristics of band pass filtered noise as shown in figure. 2.

↳ For the receiver model shown in figure. 1, we can define the following parameters

- We denote $\frac{N_0}{2}$ as the power spectral density of the noise $w(t)$ for both positive and Negative frequencies. where N_0 = Average noise power per unit bandwidth
- Mid-band frequency is equal to the carrier frequency and is denoted by "f_c".
- Typically the carrier frequency, $f_c \gg B_T$ as shown in figure. 2.

↳ We consider the filter noise, $n(t)$ as a narrowband noise and is defined in Canonical form by

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \longrightarrow (1)$$

where, $n_I(t)$ is the inphase noise component and $n_Q(t)$ is the Quadrature noise component, both components are measured with respect to the carrier wave $A_c\cos(2\pi f_c t)$

↳ The filtered signal $x(t)$ available for demodulation is defined by

$$x(t) = s(t) + n(t) \longrightarrow (2)$$

The Average Noise power is given by "$N_0 W$" $\left(\because \frac{N_0}{2} \times 2W = N_0 W\right)$

$x(t)$ is the output signal obtained from channel and is available for demodulation. Therefore pre-SNR (SNR-before demodulation) or Channel signal to Noise ratio $(SNR)_C$ is defined as

- $(SNR)_C = \dfrac{\text{Average power of the Modulated signal}}{\text{Average power of the noise in message Bandwidth}}$ ⟶ (3)

↳ The Demodulated signal <output signal> of the receiver model shown in figure 1, is given by

$$y(t) = m_d(t) + n_d(t) \longrightarrow (4)$$

where, $m_d(t)$ = demodulated signal Extracted from $s(t)$ and

$n_d(t)$ = demodulated output noise signal.

- Therefore, post-SNR (SNR, after demodulation) (or) output signal-to-noise-ratio $(SNR)_0$ is defined as

- $(SNR)_0 = \dfrac{\text{Average power of the demodulated output signal}}{\text{Average power of output Noise.}}$ ⟶ (5)

- Therefore, the Figure-of-Merit <FOM> for the receiver is given by,

$$\text{Figure of Merit} = \frac{(SNR)_0}{(SNR)_C} \longrightarrow (6)$$

Higher the value of Figure of merit, better the performance of the receiver. Its value depends on the type of modulation.

Note: (i) The average power of any Continuous time signal is given by its second-order-moment.

i.e., • average power of $s(t) = E[(s(t))^2]$ ⟶ depends on $s(t)$

• average power of $n(t) = E[(n(t))^2] = N_0 \times W$ ⇐ constant for all modulation types

• Average power of "$A_c \cos(2\pi f_c t)$" $= E[(A_c \cos 2\pi f_c t)^2] = \dfrac{A_c^2}{2}$ ⇐ constant for Sine & Cosine functions.

4.2 Noise in DSB-SC Receivers :-

⟨Q⟩ Show that Figure of merit for DSBSC System is unity.

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↳

Let • m(t) be the message signal and 'P' be the average power in m(t).

• c(t) be the carrier signal, then ~~time~~ domain expression for DSBSC-Signal is given by the product of m(t) and c(t).

∴ DSBSC-modulated signal, s(t) is

$$s(t) = m(t) \cdot c(t)$$

$$s(t) = m(t) \cdot [A_c \cos(2\pi f_c t)] \longrightarrow (1)$$

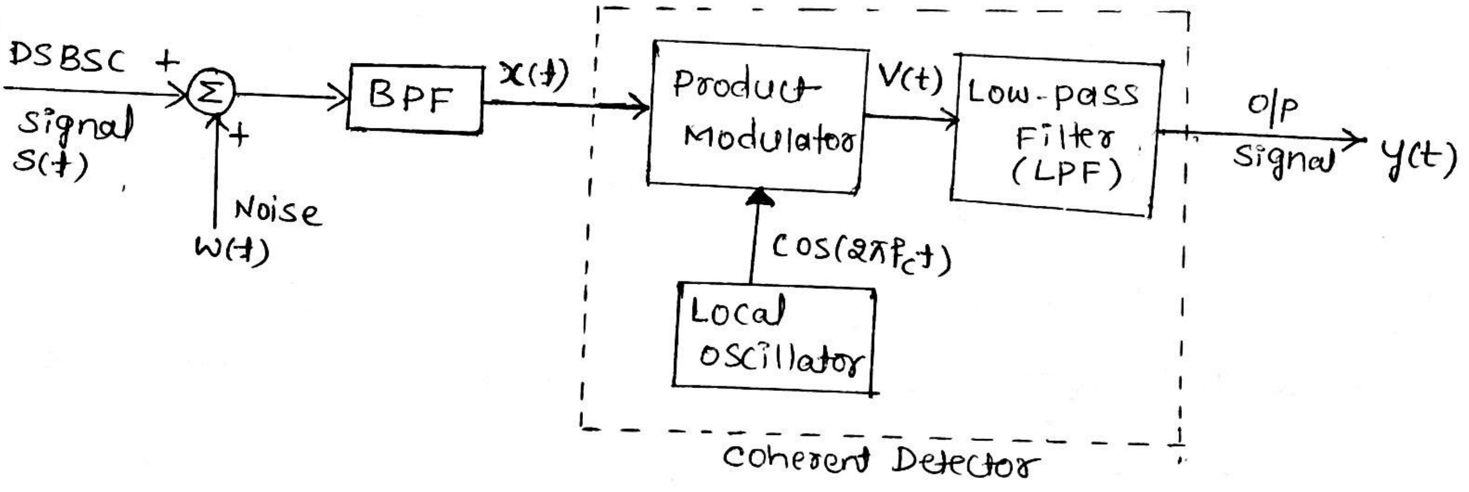


Figure 1 : Model of DSBSC receiver using Coherent Detector

• Figure 1, shows the model of a DSBSC receiver using a Coherent Detector.

• In figure 1, the filtered signal $[x(t) = s(t) + n(t)]$ is applied to product modulator.

• The product modulator multiplies the filtered signal x(t) with locally generated carrier "$\cos 2\pi f_c t$" & produces the product signal $v(t) = x(t) \cdot \cos(2\pi f_c t)$

• v(t) is applied to Low-pass-filter it eliminates all higher frequency components & produces output signal $y(t) = m_d(t) + n_d(t)$.

To find channel SNR $(SNR)_c$:-

↳ The DSBSC signal is given by

$$S(t) = m(t) \times A_c \cos(2\pi f_c t) \quad \text{——} (*)$$

• Therefore, the average power of the modulated signal $s(t)$ is

$$E\left[(s(t))^2\right] = E\left[(m(t))^2 \cdot (A_c \cos 2\pi f_c t)^2\right] = \frac{A_c^2}{2} \cdot P$$

where P_m = Average power of message signal $= E\left[(m(t))^2\right]$.

• Average power of the noise in message bandwidth is given by "$N_0 W$". where W = Bandwidth of message signal, $m(t)$.

∴ channel signal to noise ratio is

$$(SNR)_C = \frac{\text{Average power of the modulated signal, } s(t)}{\text{Average power of the noise in message bandwidth}}$$

$$(SNR)_C = \frac{A_c^2 P}{2 N_0 W} \quad \text{——} (A)$$

To find output SNR $(SNR)_0$:-

Total signal at the input of Coherent detector is

$$x(t) = s(t) + n(t) \longrightarrow (2)$$

We know that the narrow band noise signal $n(t)$ in its Canonical Form is represented by

$$n(t) = n_I(t) \cdot \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t) \longrightarrow (3)$$

where $n_I(t)$ = Inphase noise Component and $n_Q(t)$ = Quadrature phase noise Component, measured with respect to Carrier signal $\cos(2\pi f_c t)$.

Substitute equation (3) in equation (2) we get

$$x(t) = s(t) + n_I(t) \cdot \cos(2\pi f_c t) - n_Q(t) \cdot \sin(2\pi f_c t) \longrightarrow (4)$$

Therefore, the output of product Modulator is given by,

$$V(t) = x(t) \times \cos(2\pi f_c t) \quad \text{——} (5)$$

Substitute $x(t)$ equation (4) in (5) we get

$$V(t) = \left[s(t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)\right]\cos(2\pi f_c t)$$

$$V(t) = s(t).\cos(2\pi f_c t) + n_I(t)\cos^2(2\pi f_c t) - n_Q(t).\sin(2\pi f_c t).\cos(2\pi f_c t)$$

DSBSC signal, $s(t) = m(t).A_c\cos(2\pi f_c t)$, Therefore.

$$V(t) = A_c.m(t).\cos^2(2\pi f_c t) + n_I(t)\cos^2(2\pi f_c t) - n_Q(t).\sin(2\pi f_c t).\cos(2\pi f_c t) \longrightarrow (6)$$

Using Trignometric Identities

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \quad \text{and} \quad \sin\theta.\cos\theta = \frac{\sin 2\theta}{2}$$

$$V(t) = \frac{A_c.m(t)}{2}\left(1+\cos(4\pi f_c t)\right) + \frac{n_I(t)}{2}\left(1+\cos(4\pi f_c t)\right) - \frac{n_Q(t)}{2}\sin(4\pi f_c t)$$

$$V(t) = \frac{A_c m(t)}{2} + \frac{A_c.m(t)}{2}.\cos(4\pi f_c t) + \frac{n_I(t)}{2} + \frac{n_I(t)}{2}.\cos(4\pi f_c t) - \frac{n_Q(t)}{2}\sin(4\pi f_c t) \longrightarrow (7)$$

The output of product Modulator $V(t)$, is applied to low pass filter it allows only $\frac{m(t).A_c}{2}$ & $\frac{n_I(t)}{2}$ components & eliminates all other higher frequency terms.

∴ The output signal of coherent detector is

$$y(t) = \underbrace{\frac{A_c m(t)}{2}}_{\text{demodulated signal}} + \underbrace{\frac{n_I(t)}{2}}_{\text{output noise}}$$

Therefore Average power of demodulated output signal $E\left[\frac{A_c^2}{4}.m^2(t)\right]$ $\Big\} = \frac{A_c^2}{4}.P$

Average power of output Noise $E\left[\left(\frac{n_I(t)}{2}\right)^2\right]$ $\Big\} = \frac{N_0 W}{2}$ ⇐ Half of input noise power.

∴ Output signal to Noise ratio is

$$(SNR)_0 = \frac{\text{Average power of the demodulated signal}}{\text{Average power of output noise}}$$

$$(SNR)_0 = \frac{(A_c^2/4)P}{(N_0 W/2)} = \frac{A_c^2 P}{2N_0 W} \longrightarrow (B)$$

∴ Figure-of-Merit for DSBSC-receiver system is

$$\text{Figure of Merit} = \frac{(SNR)_0}{(SNR)_C} \longrightarrow (8)$$

Substitute equation (A) and equation (B) in equation (8) we get

$$FOM = \frac{(SNR)_0}{(SNR)_C} = \frac{\left(\frac{A_c^2 P}{2N_0 W}\right)}{\left(\frac{A_c^2 P}{2N_0 W}\right)} = \underline{1}$$

∴ Figure-of-Merit (FOM) for DSBSC receiver is Unity.

4.3 : Noise in AM receivers :-

(Q) obtain the expression for Figure of Merit of AM receivers using Envelope detector.

↳ Let m(t) be the message signal with average power 'P'

$$P = E[m^2(t)] = E\left[(A_m \sin(2\pi f_m t))^2\right] = \frac{A_m^2}{2}$$

c(t) be the carrier signal with $c(t) = A_c \cos 2\pi f_c t$. Then the Amplitude modulated (AM)-signal, s(t) is given by

$$s(t) = A_c[1 + K_a m(t)] \cos(2\pi f_c t) \longrightarrow (1)$$

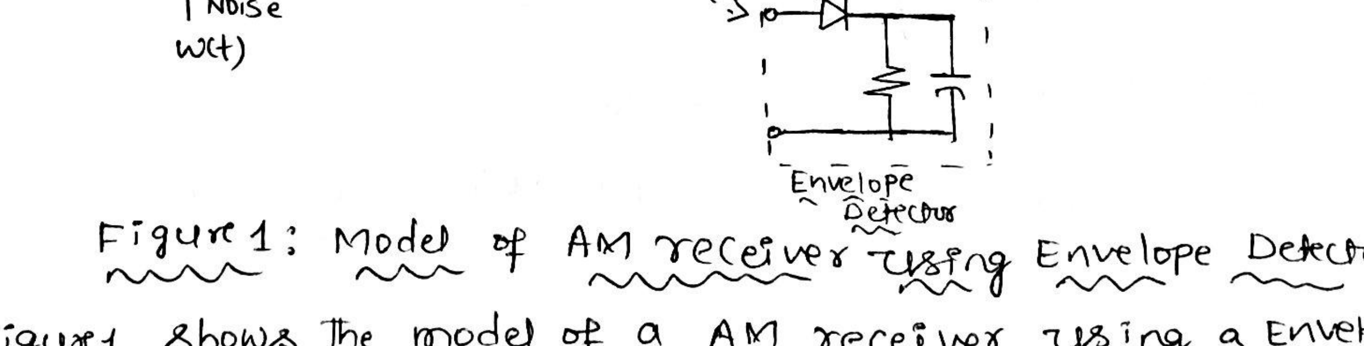


Figure 1: Model of AM receiver using Envelope Detector

Figure 1, shows the model of a AM receiver using a Envelope detector.

· Consider noise in AM receivers using Envelope detection.

• To determine channel SNR $(SNR)_C$:-

↳ The AM signal is given by

$$s(t) = A_c[1+K_a m(t)]\cos(2\pi f_c t) \longrightarrow (1)$$

↳ ∴ The Average power of modulated signal $s(t)$ $= E[\{s(t)\}^2]$

$$= E[\{A_c[1+K_a m(t)]\cos(2\pi f_c t)\}^2]$$

$$= E[\{A_c^2 \cdot (1+K_a m(t))^2 \cdot \cos^2(2\pi f_c t)]$$

$$= E[\{1+K_a m(t)\}^2] \cdot E[(A_c\cos(2\pi f_c t))^2]$$

$$= E[1+K_a^2 m^2(t)+2K_a m(t)] \cdot \left(\frac{A_c^2}{2}\right)$$

$$= \left(E[1] + E[K_a^2 m^2(t)] + E[2K_a m(t)]\right) \cdot \frac{A_c^2}{2}$$

($E[2K_a m(t)] \to 0$ since $m(t)$ is pure ac-cosine signal)

$$= (1 + K_a^2 P) \cdot \frac{A_c^2}{2}$$

∴ Average power of modulated signal $s(t)$ $= \dfrac{(1+K_a^2 P) A_c^2}{2} \longrightarrow (2)$

↳ Average power of the noise in message bandwidth is given by "$N_0 W$". where W = Bandwidth of message signal $m(t)$.

∴ channel signal to Noise ratio is

$$(SNR)_C = \frac{\text{Average power of modulated signal}}{\text{Average power of Noise in message Bandwidth}}$$

$$\therefore (SNR)_C = \frac{(1+K_a^2 P) A_c^2}{2 N_0 W} \longrightarrow (A)$$

- To Determine output SNR $(SNR)_0$:-

The total signal at the input of Envelope detector is

$$x(t) = s(t) + n(t) \longrightarrow (3)$$

where n(t) represents narrowband noise in terms of in-phase and Quadrature Components.

$$n(t) = n_I(t)\cdot \cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \quad (4)$$

Substitute s(t) & n(t) in equation (3) we get-

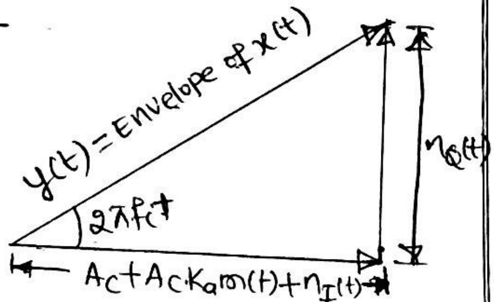
$$x(t) = A_c[1+K_a m(t)]\cos(2\pi f_c t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

$$= [A_c + A_c K_a m(t)]\cos(2\pi f_c t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

$$x(t) = \{(A_c + A_c K_a m(t) + n_I(t))\}\cos 2\pi f_c t - n_Q(t)\sin(2\pi f_c t) \quad (5)$$

∴ The output of the Envelope detector is

$$y(t) = \sqrt{[\underbrace{A_c + A_c K_a m(t) + n_I(t)}_{a}]^2 + \underbrace{n_Q^2(t)}_{b}} \longrightarrow (6)$$



Phasor-diagram of x(t)

Equation (6) gives the o/p of an ideal envelope detector.
Let us assume that the signal is much larger than Noise.
Then $\sqrt{a^2+b^2} \approx a$ when $a \gg b$.

$$\therefore y(t) = A_c + A_c K_a m(t) + n_I(t) \longrightarrow (7)$$

(A_c: DC Component; $A_c K_a m(t)$: Signal Component; $n_I(t)$: Noise Component)

The DC-component (first term) can be removed with a capacitor (—|(—) placed next to envelope detector as shown in Figure 1.

∴ The resultant demodulated signal is

$$y(t) = \underbrace{A_c K_a m(t)}_{\text{demodulated signal}} + \underbrace{n_I(t)}_{\text{output noise}} \longrightarrow (8)$$

∴ Average power of demodulated output signal $\{E[A_c^2 K_a^2 m^2(t)]\}$ $= A_c^2 K_a^2 P$

Average power of output Noise $E\{n_I^2(t)\}$ $= N_0 \times B_T = 2N_0 W$ ∵ $B_T = 2W$ for Amplitude Modulation

∴ The output signal to noise ratio is

$$(SNR)_0 = \frac{A_c^2 K_a^2 P}{2N_0 W} \longrightarrow (B)$$

∴ Figure of Merit for AM receiver is

$$\text{Figure-of-Merit} = \frac{(SNR)_0}{(SNR)_C} \longrightarrow (C)$$

Substituting equation (A) and (B) in equation (C) we get,

$$\text{Figure-of-Merit} = \frac{\left(\frac{A_c^2 K_a^2 P}{2N_0 W}\right)}{\left(\frac{A_c^2 (1+K_a^2 P)}{2N_0 W}\right)}$$

$$FOM = \frac{A_c^2 K_a^2 P}{A_c^2 (1+K_a^2 P)}$$

$$FOM = \frac{K_a^2 P}{(1+K_a^2 P)} \longrightarrow (D)$$

The Average power of the modulating wave $m(t)$ is

$$P = \frac{A_m^2}{2}$$

Substituting value of 'P' in equation (D) we get

$$FOM = \frac{K_a^2 \cdot A_m^2/2}{(1+K_a^2 A_m^2/2)}$$

∵ W.K.T Modulation Index of AM is $\mu = K_a \cdot A_m$

$$∴ \quad FOM = \frac{\mu^2/2}{1+\mu^2/2} = \frac{\mu^2/2}{2+\mu^2/2} = \frac{\mu^2}{2+\mu^2}$$

∴ Figure of Merit for AM receiver is $FOM = \frac{\mu^2}{2+\mu^2}$ //

4.4 : Noise in FM receivers :-

<Q> prove that Figure- of merit for single tone Frequency modu- -lated signal is $1.5\beta^2$.

↳ The single-tone Frequency modulated wave s(t) is given by,

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right) \longrightarrow (1)$$

Where m(t) = Message signal.

Let $\phi(t) = 2\pi k_f \int_0^t m(t)dt$, then

$$s(t) = A_c \cos\left(2\pi f_c t + \phi(t)\right) \longrightarrow (2)$$

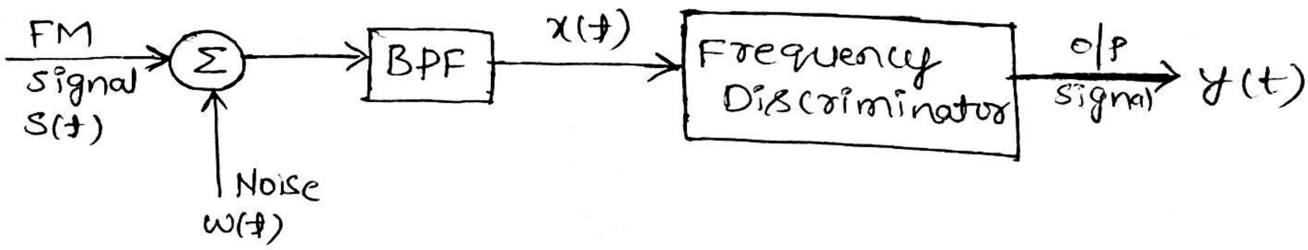


Fig1: Model of FM receiver using Frequency Discriminator

Fig1, shows the model of FM receiver using frequency discriminator.

To determine Channel SNR $(SNR)_c$:-

W.K.T the FM signal is

$$s(t) = A_c \cos\left(2\pi f_c t + \phi(t)\right) \text{ —— } (*)$$

∴ Average power of Modulated signal s(t) $= \dfrac{A_c^2}{2}$

Average power of noise in message bandwidth is $= N_0 \times W$

$$\therefore (SNR)_C = \frac{A_c^2}{2N_0W} \longrightarrow (A)$$

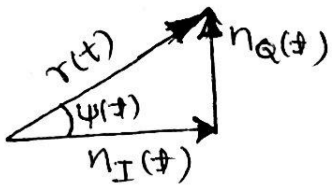
- To Determine output SNR $(SNR)_0$:-

The total signal at the input of frequency discriminator is,

$$x(t) = s(t) + n(t) \longrightarrow (3)$$

For output SNR, analysis Let us express $n(t)$ in terms of its magnitude $[r(t)]$ and phase $[\psi(t)]$ given by the equation

$$n(t) = r(t)\cos(2\pi f_c t + \psi(t)) \longrightarrow (4)$$



where $r(t) = \sqrt{n_I^2(t) + n_Q^2(t)} \longrightarrow (5)$

$\psi(t) = \tan^{-1}\left(\frac{n_Q(t)}{n_I(t)}\right) \longrightarrow (6)$

∴ Total signal at the input of demodulator is

$x(t) = s(t) + n(t)$ becomes

$$x(t) = A_c\cos(2\pi f_c t + \phi(t)) + r(t)\cos(2\pi f_c t + \psi(t)) \longrightarrow (7)$$

The relative phase $\theta(t)$ can be expressed as

$$\theta(t) \simeq \phi(t) + \frac{n_Q(t)}{A_c} \longrightarrow (8)$$

; where $n_Q(t) = r(t)\cdot\sin\psi(t)$

With an Ideal phase discriminator, the output $y(t)$ is proportional to the phase deviation $\frac{d\theta(t)}{dt}$.

i.e., the output signal,

$$y(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} \longrightarrow (9)$$

Substitute $\theta(t)$ from equation (8) in equation (9) we get

$$y(t) = \frac{1}{2\pi}\frac{d}{dt}\left[\phi(t) + \frac{n_Q(t)}{A_c}\right] \quad \because \phi(t) = 2\pi k_f \int_0^t m(t)\,dt$$

$$= \frac{1}{2\pi}\frac{d}{dt}\left[2\pi k_f \int_0^t m(t)\,dt + \frac{n_Q(t)}{A_c}\right]$$

$$y(t) = \underbrace{k_f m(t)}_{\text{demodulated signal}} + \underbrace{\frac{1}{2\pi A_c}\frac{d\,n_Q(t)}{dt}}_{\text{O/P Noise}}$$

∴ Average power of demodulated O/P signal $= k_f^2 P \longrightarrow (10)$

Where P = power in message signal $m(t) = \frac{A_m^2}{2}$

Average power of output Noise $= \frac{N_0}{A_c^2}\int_{-W}^{W} f^2\,df = \frac{N_0}{A_c^2}\left(\frac{f^3}{3}\right)_{-W}^{W}$

$$= \frac{2N_0W^3}{3A_c^2}$$

∴ output signal to noise ratio is

$$(SNR)_0 = \frac{k_f^2 P}{\left(\frac{2N_0W^3}{3A_c^2}\right)} = \frac{3A_c^2 k_f^2 P}{2N_0W^3} \longrightarrow (B)$$

∴ Figure-of-Merit for FM receiver is

$$FOM = \frac{(SNR)_0}{(SNR)_C} \longrightarrow (C)$$

Substitute equation (A) & equation (B) in equation (C) we get,

$$FOM = \frac{\left(\frac{3A_c^2 k_f^2 P}{2N_0W^3}\right)}{\left(\frac{A_c^2}{2N_0W}\right)} = \frac{3k_f^2 P}{W^2} \longrightarrow (D)$$

∴ Substitute $P = \frac{A_m^2}{2}$; Average power of message signal in equation (D) we get

$$FOM = \frac{3 K_f^2 A_m^2}{2W^2} = \frac{3}{2}\left(\frac{K_f A_m}{W}\right)^2 \longrightarrow (E)$$

We know that the modulation Index of FM-Signal

$$\beta = \frac{\Delta f}{f_m} = \frac{K_f A_m}{W}$$

∴ using the value of 'β' in FOM equation (E) we get Figure-of-Merit of FM receiver

$$\boxed{FOM = \frac{3}{2}\beta^2 = 1.5\,\beta^2} \quad \text{where } \beta = \frac{K_f A_m}{W}$$

—— * —— * ——

• Problems on Figure-of-Merit Calculation:-

List of Formulae:-

⟨1⟩ $FOM = \frac{\mu^2}{2+\mu^2}$ for Amplitude Modulation

⟨2⟩ $FOM = 1.5\,\beta^2$ for frequency Modulation

where β = Modulation Index of FM.

⟨3⟩ General definition of FOM is

$$FOM = \frac{(SNR)_0}{(SNR)_C}$$

⟨4⟩ $(SNR)_{0\,dB} = 10 \log (SNR)_0$

⟨5⟩ $(SNR)_{C\,dB} = 10 \log (SNR)_C$

<1> An AM receiver operating with a sinusoidal wave of 80% modulation has an output signal to noise ratio of 30dB. Calculate the corresponding channel s/l-to-noise ratio. prove the formula used.

↳

Given data:

$\mu = 0.8$

$(SNR)_{o\,dB} = 30\,dB$

$(SNR)_c = ?$

W.K.T. The FOM of AM receiver is

$$FOM = \frac{\mu^2}{2+\mu^2} = \frac{0.8^2}{2+0.8^2} = 0.2424$$

$$W.K.T.,\ FOM = \frac{(SNR)_0}{(SNR)_c}$$

$$\therefore (SNR)_c = \frac{(SNR)_0}{FOM}$$

$$(SNR)_0 = 10^{\wedge}\left(\frac{30}{10}\right) = \underline{1000}$$

$$\therefore (SNR)_c = \frac{(SNR)_0}{FOM} = \frac{1000}{0.2424} = 4125$$

$$(SNR)_{c\,dB} = 10\log(4125)$$

$$(SNR)_{c\,dB} = 36.15\,dB$$

· For derivation of FOM for AM receiver refer the AM-receiver FOM derivation.

<2> The average noise per/unit BW measured at the front end of AM receiver is 10^{-3} Watt/Hz. The Modulating wave is sinusoidal with a carrier power of 80KW and sideband power of 10KW per side band. The message BW is 4KHZ. Determine the $(SNR)_0$ of the system and FOM.

VTU Q.P

↳ Given: $N_0 = 10^{-3}$ Watt/Hz ; $P_c = \frac{A_c^2}{2} = 80KW \Rightarrow A_c = 400V$

Side band power $P_s = \frac{A_c^2\mu^2}{2} = 10\times10^3 \Rightarrow \boxed{\mu = 0.707}$

Message bandwidth, $W = 4KHZ$

$$\therefore (SNR)_0 = \frac{A_c^2\mu^2}{2N_0W} = \frac{(400)^2\times 0.707^2}{2\times10^{-3}\times4\times10^3} = \underline{5000}\ \ \&\ \ FOM = \frac{\mu^2}{2+\mu^2} = 0.2$$

** (3) Find FOM of AM receiver when depth of Modulation is (a) 100% (b) 50% (c) 30% VTU Q.P

↳ W.K.T. FOM of AM receiver is given by

$$FOM = \frac{\mu^2}{2+\mu^2} \quad \text{---(1)}$$

(a) when $\mu = 100\% = 1$.

$$FOM = \frac{\mu^2}{2+\mu^2} = \frac{1}{2+1} = \frac{1}{3} = 0.3333$$

(b) when $\mu = 50\% = 0.5$

$$FOM = \frac{\mu^2}{2+\mu^2} = \frac{0.5^2}{2+0.5^2} = 0.1111$$

(c) when $\mu = 30\% = 0.3$

$$FOM = \frac{\mu^2}{2+\mu^2} = \frac{0.3^2}{2+0.3^2} = 0.043$$

** (4) An FM signal with a maximum frequency deviation of 75KHZ is applied to an FM demodulator. When the input (channel) SNR is 15dB and the modulating frequency is 10KHZ. Estimate FOM and SNR at demodulator output. VTU.Q.P

↳ Given data :-

$\Delta f = 75KHZ$

$(SNR)_{cdB} = 15dB$

$f_m = W = 10KHZ$

$FOM = ?$

$(SNR)_0 = ?$

W.K.T FOM of FM receiver is

$$FOM = 1.5\beta^2$$

$$\beta = \frac{\Delta f}{f_m} = \frac{75K}{10K} = 7.5$$

$$\therefore FOM = 1.5\beta^2 = 1.5\,(7.5)^2$$

$$FOM = 84.375$$

To find output SNR:-

W.K.T. General definition of Figure-of-Merit

$$FOM = \frac{(SNR)_0}{(SNR)_C} \longrightarrow (1)$$

from given data $(SNR)_{C_{dB}} = 15\,dB = 10\log[(SNR)_c]$

$\therefore$ $(SNR)_C = 10^{\wedge}\left(\frac{15}{10}\right) = 10^{1.5}$

$(SNR)_C = 31.6227$

Also W.K.T FOM = 84.375

$\therefore$ substitute FOM = 84.375 & $(SNR)_C = 31.6227$ in equation (1) we get

$$(SNR)_0 = FOM \times (SNR)_C$$

$$= 84.375 \times 31.6227 = 2668.16 = 34.262\,dB$$

5) An FM receiver recieves an FM signal $s(t) = 10\cos[(2\pi\times10^8 t) + 6\sin(2\pi\times10^3 t)]$. Calculate the figure-of-Merit for this receiver. VTU Q.P

↳ Given

$s(t) = 10\cos[2\pi\times10^8 t + 6\sin(2\pi\times10^8) t]$ —(1)

General equation of FM is

$s(t) = A_C\cos[2\pi f_c t + \beta\sin(2\pi f_m t)]$ —(2)

Comparing (1) & (2) we get

Modulation index, $\boxed{\beta = 6}$

$\therefore$ Figure-of-Merit $= 1.5\beta^2$

$= 1.5\times6^2$

$= 54.$

4.5: Capture effect :-

<Q> Write a short note on Capture effect in FM. (4-Marks)

↳ In FM system, the signal can be affected by another frequency modulated signal whose frequency is close to the carrier frequency of the desired FM-signal. Then the receiver may lock such an interference signal and suppresses the desired FM-signal & interference signal becomes more stronger than the desired signal.

↳ When the strength of the desired signal and interference signal are nearly equal, the receiver locks interference signal for some time and desired signal for the some time and this goes on randomly and receiver captures the stronger signal. This effect is known as "Capture- effect".

4.6 : FM-Threshold Effect :-

<Q> Explain FM threshold effect in FM-System. V.T.U (6-Marks)

↳ The $(SNR)_0$ of an FM-signal is valid only if the (CNR) mesured at the frequency discriminator input is very much greater than unity.

i.e., $$(SNR)_0 = \frac{3A_c^2 k_f^2 P}{2N_0 W^3} \text{ is Valid iff } CNR \gg 1$$

if CNR < 1 then FM signal is corrupted by noise and FM receiver breaks down. & is called Threshold effect of FM.

i.e., Threshold effect is defined as the minimum carrier to noise ratio (CNR) that gives the $(SNR)_0$ not less than the value predicted by the usual SNR-formula assuming a small noise power.

↳ The Threshold effect can be avoided by keeping carrier to noise ratio of FM-system much greater than 1.

↳ when CNR ≤ 1, FM receiver breaks down due to FM-threshold effect. This point is called Threshold point.

4.7 : FM Threshold reduction :-

<Q> Explain about the FM-Threshold effect and its reduction Method (V.T.U. 8M)

↳ FM-Threshold can be reduced by using FM-demodulators with negative feedback (or) by using a phase-Locked loop demodulator. Such devices are referred as Extended threshold demodulators.

↳ FM-demodulator with negative feedback is also known as FMFB-demodulator.

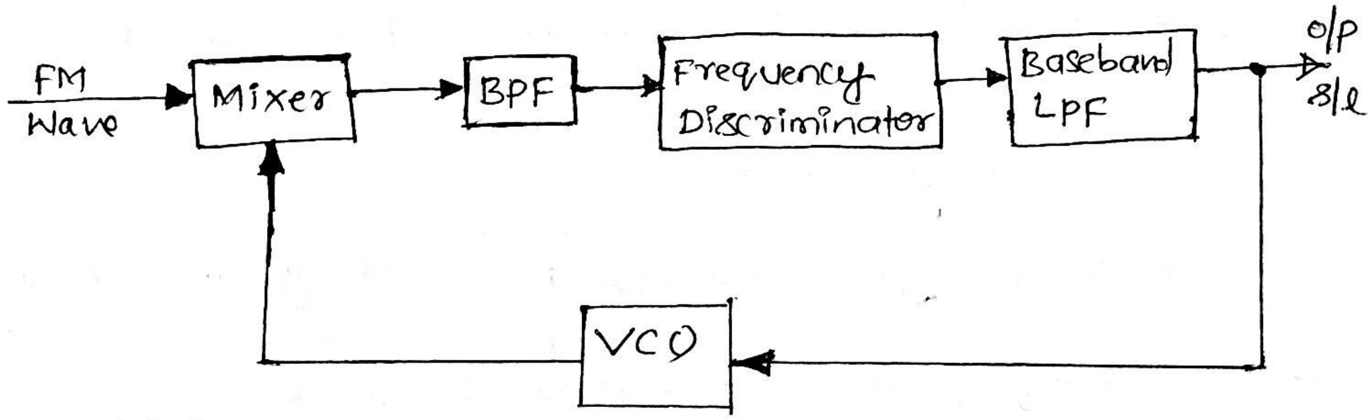


Fig1: Block diagram of FMFB demodulator for threshold reduction

↳ Figure 1, shows the block diagram of FMFB demodulator. In this system local oscillator is replaced by Voltage Controlled Oscillator <VCO>.

↳ The instantaneous output frequency of such VCO is controlled by demodulated output signal.

↳ The bandwidth of noise to which the FMFB receiver responds is precisely the band of noise that the VCO-tracks, thus the FMFB receiver acts as a tracking filter, that can track only the slowly varying frequency of a

Wide-band FM signal. 21

↳ Therefore it responds only to a narrow band of noise Centered about frequency "f_c", as a result FMFB-receivers allows a threshold Extension upto 5dB to 7dB as shown in Figure. 2.

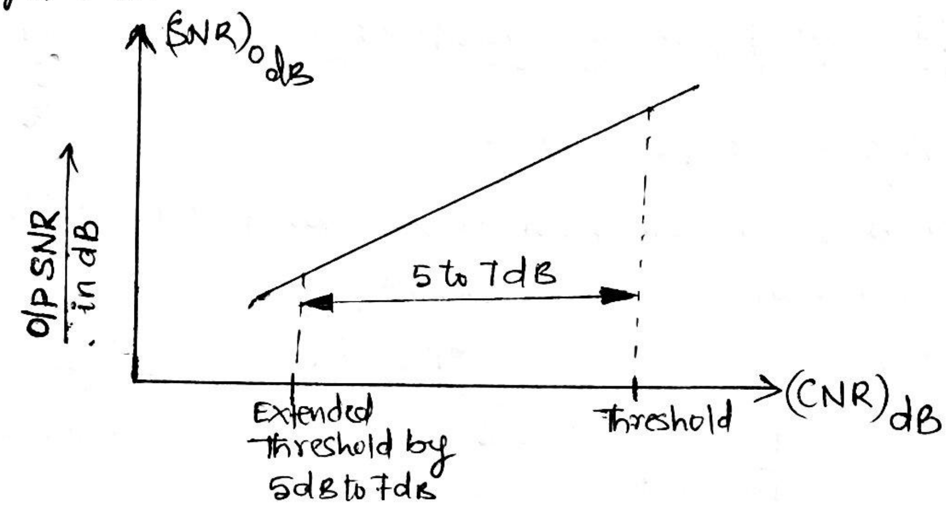


Fig. 2: Graph showing the Extended threshold effect

↳ ∴ FMFB demodulator with negative feedback provides 5dB to 7dB Enhancement in (CNR) & it always maintains CNR≫1 and it avoids FM-threshold effect.

** Pre-emphasis and De-emphasis in FM :-

<Q> With circuits and characteristics, explain the importance of pre-emphasis and De-emphasis in FM-systems.

VTU - 8M -

↳ pre-emphasis and De-emphasis methods are commonly used in FM-transmitter and FM-receiver respectively to improve the Threshold.

↳ pre-emphasis and De-emphasis are simple RC networks used to improve threshold upto 13dB to 16dB.

↳ Figure 1 shows the FM transmitter with pre-emphasis filter having transfer function $H_{pre}(f)$.

↳ Figure 1, shows the pre-emphasis filter used before FM-transmitter.

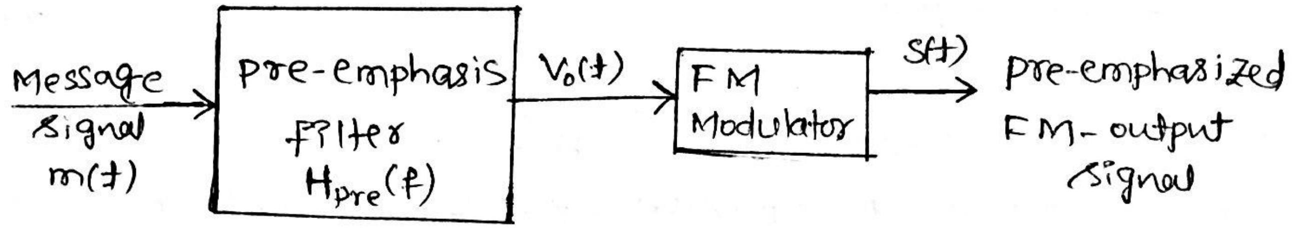
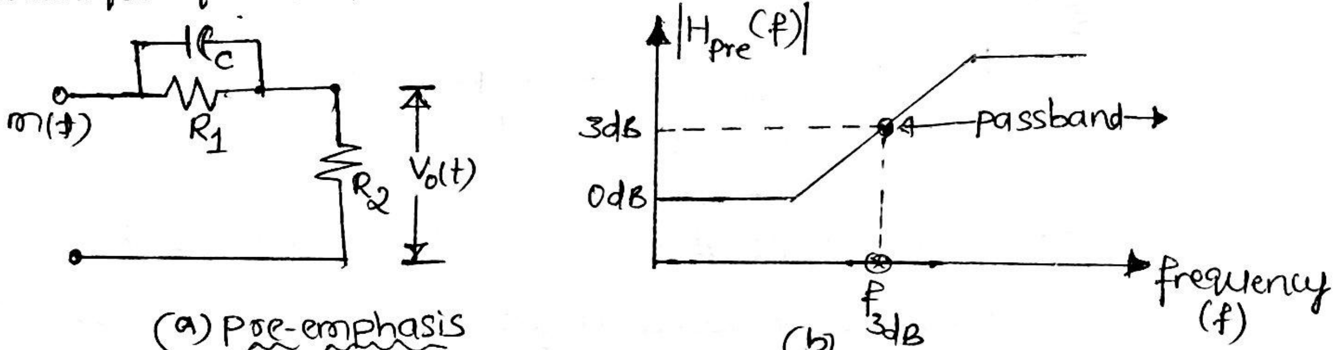


Fig1: Use of pre-emphasis filter in FM transmitter

* pre-emphasis filter (or) circuit :-

↳ pre-emphasis circuit is a High-pass-Filter (HPF) with transfer function shown in figure 2.



(a) Pre-emphasis circuit (b)

Fig.2: pre-emphasis filter circuit diagram with frequency response

↳ To improve $(SNR)_0$ at the FM-Modulator output, the high frequency components of the message signal m(t) are artificially emphasized at the transmitter, before the Modulation-process & is shown in fig.1.

↳ After pre-emphasis, m(t) occupies entire range of bandwidth assigned. Then at the frequency discriminator (FM-demodulator) of FM receiver, inverse operation of pre-emphasis called De-emphasis is performed.

- Figure 3, shows the FM receiver with de-emphasis filter having frequency response $|H_{de}(f)|$.
- De-emphasis filter/circuit is used after FM-demodulator as shown in Figure.3.

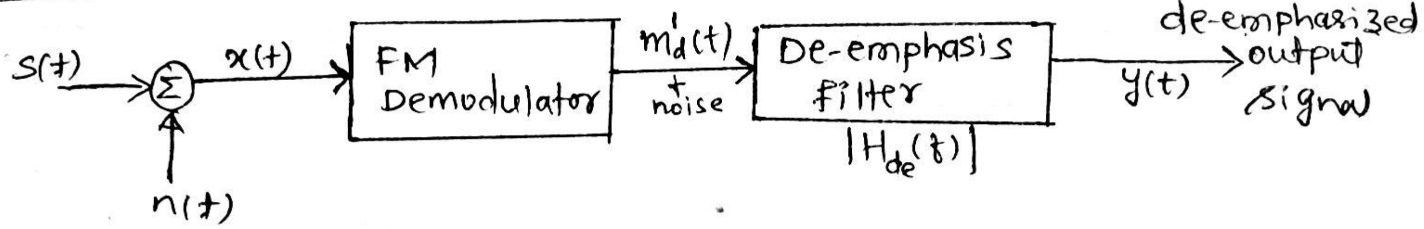
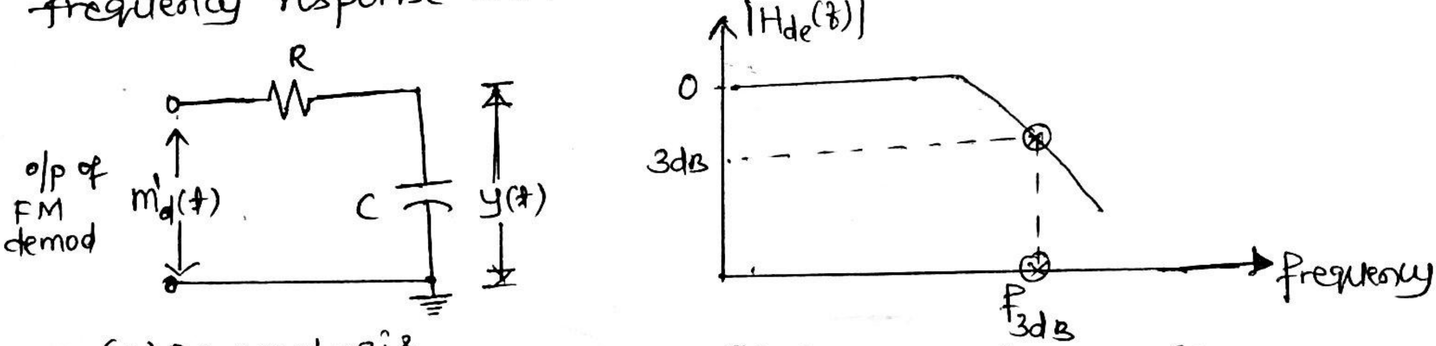


Figure 3: FM receiver with De-emphasis filter

De-emphasis filter (or) circuit :-

↳ De-emphasis circuit is a simple RC-Lowpass filter with frequency response as shown in Figure 4.



(a) De-emphasis circuit

(b) Frequency response

Fig. 4: De-emphasis filter circuit diagram & its frequency response

↳ High-frequency components present at the output of FM-demodulator o/p signal ($m'_d(t)$) are de-emphasized to restore the original message signal power distribution.

↳ During, de-emphasis process, High frequency components of the noise are also reduced and thereby efficiently increases output signal to noise ratio. $(SNR)_0$.

↳ The frequency response of de-emphasis filter & De-emphasis filter are related by,

$$|H_{pre}(f)| = \frac{1}{|H_{de}(f)|}$$

where $|H_{de}(f)| = \frac{1}{1 + j\frac{f}{f_{3dB}}}$ and $|H_{pre}(f)| = 1 + j\frac{f}{f_{3dB}}$

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↳ The effect of de-emphasis filter on output noise spectrum is illustrated in Figure 5.

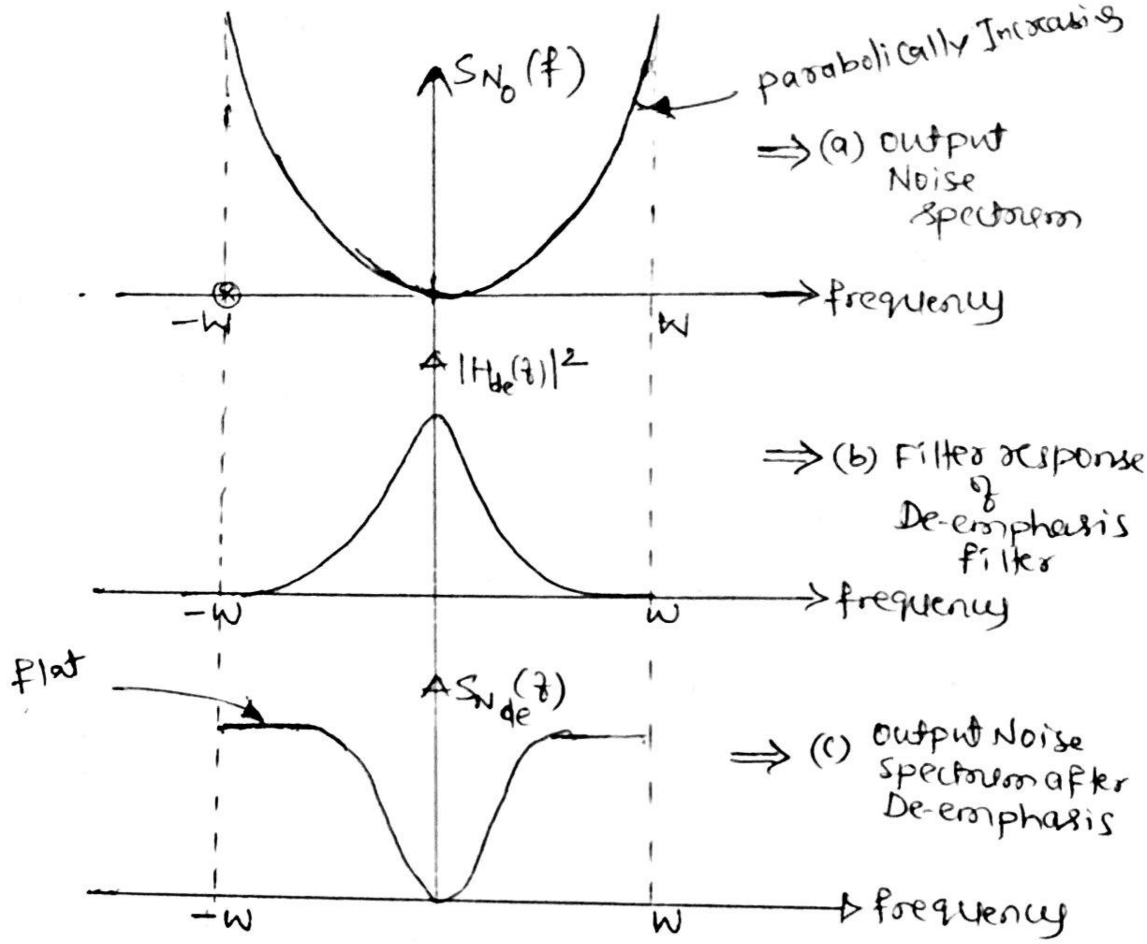

Figure 5: Effect of De-emphasis filter on output Noise

↳ Fig 5(a) shows output noise spectrum which is parabolically increasing towards the edge of ±W. Therefore $(SNR)_0$ reduces before deemphasis.

↳ When Noise is filtered by using de-emphasis filter having frequency response shown in fig 5(b), the output noise spectrum after de-emphasis becomes almost flat at ±W Hz as a result $(SNR)_0$ increases and it avoids FM-Threshold.

∴ pre-emphasis and De-emphasis circuits are Effectively used to increase (SNR) at the transmitter (before modulation) and receiver (after demodulation) of FM-system respectively.

~ end ~