Module 1 : INTRODUCTION TO TURBOMACHINES

1.1 Introduction:

The turbomachine is used in several applications, the primary ones being electrical power generation, aircraft propulsion and vehicular propulsion for civilian and military use. The units used in power generation are steam, gas and hydraulic turbines, ranging in capacity from a few kilowatts to several hundred and even thousands of megawatts, depending on the application. Here, the turbomachines drives the alternator at the appropriate speed to produce power of the right frequency. In aircraft and heavy vehicular propulsion for military use, the primary driving element has been the gas turbine.

1.2 Turbomachines and its Principal Components:

Question No 1.1: Define a turbomachine. With a neat sketch explain the parts of a turbomachine.
(VTU, Jan-07, Dec-12, Jan-14, Jul-15)

Answer: A turbomachine is a device in which energy transfer takes place between a flowing fluid and a rotating element due to the dynamic action, and results in the change of pressure and momentum of the fluid.

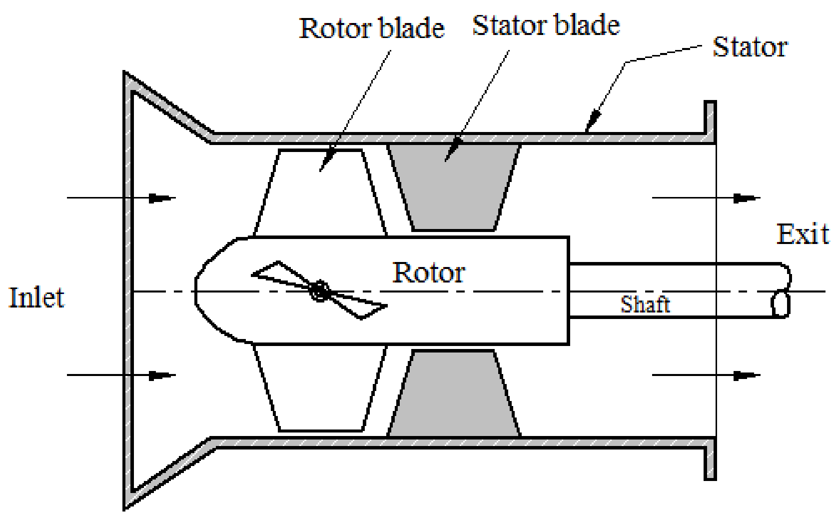

Fig. 1.1 Principal components of turbomachine

The following are the principal components of turbomachine: (i) Rotor, (ii) Stator and (iii) Shaft.

Rotor is a rotating element carrying the rotor blades or vanes. Rotor is also known by the names runner, impellers etc. depending upon the particular machine. Here energy transfer occurs between the flowing fluid and the rotating element due to the momentum exchange between the two.

Stator is a stationary element carrying the guide vanes or stator blades. Stator blades are also known by guide blades or nozzle depending upon the particular machine. These blades usually control the direction of fluid flow during the energy conversion process.

Shaft is transmitting power into or out of the machine depending upon the particular machine. For power generating machines, it may call as output shaft and for power absorbing machines; it may called as input shaft.

1.3 Classification of Turbomachines:

Question No 1.2: Explain how turbomachines are classified. Give at least one example of each. (VTU, Feb-06, Jul-13, Jun/Jul 14)

Answer: Turbomachines are broadly classified into power generating, power absorbing and power transmitting turbomachines.

In power-generating turbomachines, fluid energy (decrease in enthalpy) is converted into mechanical energy which is obtained at the shaft output, whereas in power-absorbing turbomachines, mechanical energy which is supplied at the shaft input is converted to fluid energy (increase in enthalpy). The power-transmitting turbomachines are simply transmitting power from input shaft to an output shaft. That means, these devices act merely as an energy transmitter to change the speed and torque on the driven member as compared with the driver.

Again power-generating and power-absorbing turbomachines are classified by the direction of the fluid flow as: (i) axial flow, (ii) radial flow and (iii) mixed flow. In the axial flow and radial flow turbomachines, the major flow directions are approximately axial and radial respectively, while in the mixed flow machine, the flow enters axially and leaves radially or vice versa. A radial flow machine may also be classified into radial inward flow (centripetal) or radial outward flow (centrifugal) types depending on whether the flow is directed towards or away from the shaft axis.

Question No 1.3: Explain with examples the power generating, power absorbing and power transmitting turbomachines. (VTU, Aug-02, Jul-13, Jul-14)

Answer: Power generating turbomachine is one which converts fluid energy in the form of kinetic energy or pressure energy into mechanical energy in terms of rotating shaft. Turbines are the best example for this type.

Power absorbing turbomachine is one which converts mechanical energy into fluid energy. Compressors, fans, pumps and blowers are the best example for this type.

Power transmitting is one which is used to transmit power from driving shaft to driven shaft with the help of fluid. There is no mechanical connection between the two shafts. The best examples for this type are hydraulic coupling and hydraulic torque converter.

Question No 1.4: What is an axial flow turbomachine? How is it different from a radial flow turbomachine? Give one example each.

Answer: In axial flow turbomachine, the major flow direction is approximately axial, example: Kaplan turbine. Whereas in radial flow turbomachine, the major flow direction is radial, example: Francis turbine.

1.4 Positive-Displacement Devices and Turbomachines:

Question No 1.5: Compare the turbomachines with positive displacement machines. ***(VTU, Feb-02, Feb-03, Feb-04, Jun-12, Dec-12, Jul-13, Jan-16, Jul-16, Jan-17, Jul-17)***

Answer: The differences between positive-displacement machines and turbomachines are given by comparing their modes of action, operation, energy transfer, mechanical features etc. in the following table.

Modes	Positive-displacement Machine	Turbomachine
Action	(a) It creates thermodynamic and mechanical action between a nearly static fluid and a relatively slowly moving surface.	(a) It creates thermodynamic and dynamic interaction between a flowing fluid and rotating element.
	(b) It involves a change in volume or a displacement of fluid.	(b) It involves change in pressure and momentum of the fluid.
	(c) There is a positive confinement of the fluid in the system.	(c) There is no positive confinement of the fluid at any point in the system.
Operation	(a) It involves a reciprocating motion of the mechanical element and unsteady flow of the fluid. But some rotary positive displacement machines are also built. Examples: Gear pump, vane pump	(a) It involves a purely rotary motion of mechanical element and steady flow of the fluid. It may also involve unsteady flow for short periods of time, especially while starting, stopping or during changes of load.
	(b) Entrapped fluid state is different from the surroundings when the machine is stopped, if heat transfer and leakages are avoided.	(b) The fluid state will be the same as that of the surroundings when the machine is stopped.
Mechanical Features	(a) Because of the reciprocating masses, vibrations are more. Hence low speeds are adopted.	(a) Rotating masses can be completely balanced and vibrations eliminated. Hence high speeds can be adopted.
	(b) Heavy foundations are required.	(b) Light foundations sufficient.

	(c) Mechanical design is complex because of valves.	(c) Design is simple.
	(d) Weight per unit output is more.	(d) Weight per unit output is less.
Efficiency of conversion process	(a) High efficiency because of static energy transfer.	(a) Efficiency is low because of dynamic energy transfer.
	(b) The efficiencies of the compression and expansion processes are almost the same.	(b) The efficiency of the compression process is low.
Volumetric efficiency	(a) Much below that of a turbomachine because of valves.	(a) It is almost 100%.
	(b) Low fluid handling capacity per unit weight of machine.	(b) High fluid handling capacity per unit weight of machine.
Fluid phase change and surging	No such serious problems are encountered.	(a) Causes cavitation in pumps and turbines. Therefore leads to erosion of blades.
		(b) Surging or pulsation leads to unstable flow. And also causes vibrations and may destroy the machine.
		(c) These factors deteriorate the performance of the machine.

Question No 1.6: Are vane compressors and gear pumps turbomachines? Why? (VTU, Dec-10)

Answer: No, vane compressors and gear pumps are positive displacement machines and work by moving a fluid trapped in a specified volume (i.e., fluid confinement is positive).

1.5 First and Second Laws of Thermodynamics Applied to Turbomachines:

Question No 1.7: Explain the applications of first and second laws of thermodynamics to turbomachines. (VTU, Jul/Aug-02) Or,

Starting from the first law, derive an expression for the work output of a turbomachine in terms of properties at inlet and outlet. Or,

Deducing an expression, explain the significance of first and second law of thermodynamics applied to a turbomachine. (VTU, Dec-12, Dec 14/Jan 15)

Answer: Consider single inlet and single output steady state turbomachine, across the sections of which the velocities, pressures, temperatures and other relevant properties are uniform.

Application of first law of thermodynamics: The steady flow equation of the first law of thermodynamics in the unit mass basis is:

$$q + h_1 + \frac{V_1^2}{2} + gz_1 = w + h_2 + \frac{V_2^2}{2} + gz_2 \tag{1.1}$$

Here, q and w are heat transfer and work transfer per unit mass flow across the boundary of the control volume respectively.

Since, the stagnation enthalpy: $h_o = h + \frac{V^2}{2} + gz$.

Then, equation (1.1) becomes: $q - w = h_{o2} - h_{o1} = \Delta h_o$ (1.2)

Generally, all turbomachines are well-insulated devices, therefore q=0. Then equation (1.2) can be written as: $\Delta h_o = -w$ (1.3)

The equation (1.3) represents that, *the energy transfer as work is numerically equal to the change in stagnation enthalpy of the fluid between the inlet and outlet of the turbomachine.*

In a power-generating turbomachine, w is positive as defined so that Δh_o is negative, i.e., the stagnation enthalpy at the exit of the machine is less than that at the inlet. The machine produces out work at the shaft. In a power-absorbing turbomachine, w is negative as defined so that Δh_o is positive. The stagnation enthalpy at the outlet will be greater than that at the inlet and work is done on the flowing fluid due to the rotation of the shaft.

Application of second law of thermodynamics: The second law equation of states, applied to stagnation properties is:

$$T_o ds_o = dh_o - v_o dp_o \tag{1.4}$$

But equation (1.3) in differential form is, $dh = -dw$.

Then equation (1.4) can be written as:

$$-dw = v_o dp_o + T_o ds_o \tag{1.5}$$

In a power-generating machine, dp_o is negative since the flowing fluid undergoes a pressure drop when mechanical energy output is obtained. However, the Clausius inequality for a turbomachine is given that $T_o ds_o \geq 0$. The sign of equality applies only to a reversible process which has a work output $dw_{rev} = v_o dp_o$. In a real machine (irreversible machine), $T_o ds_o > 0$, which has a work output $dw_{irr} = v_o dp_o - T_o ds_o$. So that $dw_{rev} - dw_{irr} = T_o ds_o$ and represents the decrease in work output due to the irreversibilities in the machine. Therefore the reversible power-generating machine exhibits the highest mechanical output of all the machines undergoing a given stagnation pressure change. A similar argument may be used to prove that the reversible power-absorbing machine needs the minimum work input of all the machines for a given stagnation pressure rise (i.e., $dw_{irr} - dw_{rev} = T_o ds_o$).

1.6 Efficiency of Turbomachines:

Question No 1.8: Define: (i) adiabatic efficiency and (ii) mechanical efficiency for power generating and power absorbing turbomachines. ***(VTU, Dec-12)***

Answer: The performance of a real machine is always inferior to that of a frictionless and loss-free ideal machine. A measure of its performance is the efficiency, defined differently for power-generating and power-absorbing machines.

For power-generating machine, the efficiency is defined as:

$$\eta_{pg} = \frac{Actual\text{ Shaft Work Output}}{Ideal\ Work\ Output} = \frac{w_{sft}}{w_i}$$

$$Or, \qquad \eta_{pg} = \frac{Actual\text{ Shaft Work Output}}{Hydrodynamic\ Energy\ Available\ from\ the\ Fluid} = \frac{w_{sft}}{w_i}$$

For power-absorbing machine, the efficiency is defined as:

$$\eta_{pa} = \frac{Ideal\ Work\ Input}{Actual\text{ Shaft Work Input}} = \frac{w_i}{w_{sft}}$$

$$Or, \qquad \eta_{pa} = \frac{Hydrodynamic\ Energy\ Supplied\ to\ the\ Fluid}{Actual\text{ Shaft Work Input}} = \frac{w_i}{w_{sft}}$$

Generally, losses occur in turbomachines are due to: (a) mechanical losses like bearing friction, windage, etc., (b) fluid-rotor losses like unsteady flow, friction between the blade and the fluid, leakage across blades etc. If the mechanical and fluid-rotor losses are separated, the efficiencies may be rewritten in the following forms:

For power-generating turbomachine,

$$\eta_{pg} = \frac{\text{Mechanical Energy Supplied by the Rotor}}{Hydrodynamic\ Energy\ Available\ from\ the\ Fluid} \times \frac{Actual\text{ Shaft Work Output}}{\text{Mechanical Energy Supplied by the Rotor}}$$

Or, $$\eta_{pg} = \eta_a \times \eta_m$$

For power-absorbing turbomachines,

$$\eta_{pa} = \frac{Hydrodynamic\ Energy\ Supplied\ to\ the\ Fluid}{\text{Mechanical Energy Supplied to the Rotor}} \times \frac{\text{Mechanical Energy Supplied to the Rotor}}{Actual\text{ Shaft Work Input}}$$

Or, $$\eta_{pg} = \eta_a \times \eta_m$$

where η_a and η_m are adiabatic and mechanical efficiencies respectively.

For power-generating turbomachine, adiabatic or isentropic or hydraulic efficiency may be written as,

$$\eta_a = \frac{\text{Mechanical Energy Supplied by the Rotor}}{Hydrodynamic\ Energy\ Available\ from\ the\ Fluid} = \frac{w_r}{w_i}$$

For power-absorbing turbomachine, adiabatic or isentropic or hydraulic efficiency may be written as,

$$\eta_a = \frac{Hydrodynamic\ Energy\ Supplied\ to\ the\ Fluid}{\text{Mechanical Energy Supplied to the Rotor}} = \frac{w_i}{w_r}$$

Note: (i) Hydrodynamic energy is defined as the energy possessed by the fluid in motion.

(ii) Windage loss is caused by fluid friction as the turbine wheel and blades rotate through the surrounding fluid.

(iii) Leakage loss is caused by the fluid when it passes over the blades tip without doing any useful work.

1.7 Dimensional Analysis:

The dimensional analysis is a mathematical technique deals with the dimensions of the quantities involved in the process. Basically, dimensional analysis is a method for reducing the number and complexity of experimental variable that affect a given physical phenomenon, by using a sort of compacting technique.

The three primary purposes of dimensional analysis are:

1. To generate non-dimensional parameters that help in the design of experiments and in the reporting of experimental results.
2. To obtain scaling laws so that prototype performance can be predicted from model performance.
3. To predict the relationship between the parameters.

1.7.1 Fundamental Quantities: Mass (M), length (L), time (T) and temperature (ɵ) are called fundamental quantities since there is no direct relation between these quantities. There are seven basic quantities in physics namely, mass, length, time, electric current, temperature, luminous intensity and amount of a substance.

1.7.2 Secondary Quantities or Derived Quantities: The quantities derived from fundamental quantitie are called derived quantities or secondary quantities. Examples: area, volume, velocity, force, acceleration, etc.

1.7.3 Dimensional Homogeneity: An equation is said to be dimensionally homogeneous if the fundamental dimensions have identical powers of M, L, T on both sides.

For example: $Q = AV$

In dimensional form:

$$\frac{L^3}{T} = L^2 \times \frac{L}{T} = \frac{L^3}{T}$$

1.8 Buckingham's π-Theorem:

The Buckingham's π-theorem states that "if there are 'n' variables in a dimensionally homogeneous equation and if these variables contain 'm' fundamental dimensions such as M, L, T then they may be grouped into (n-m), non-dimensional independent π-terms".

Let a variable X_1 depends upon independent variables X_2, X_3,.....X_n. The functional equation may be written as:

$$X_1 = f(X_2, X_3, \dots \dots X_n) \tag{1.6}$$

The above equation can also be written as:

$$f(X_1, X_2, X_3, \dots \dots X_n) = C \tag{1.7}$$

Where, C is constant and f is some function.

In the above equation (1.7), there are 'n' variables. If these variables contain 'm' fundamental dimensions, then according to Buckingham's π-theorem,

$$f_1(\pi_1, \pi_2, \pi_3, \dots \dots \pi_{n-m}) = C \quad (1.8)$$

1.9 Procedure for Applying Buckingham's π-Theorem:

1) With a given data, write the functional relationship.
2) Write the equation in its general form.
3) Choose repeating variables and write separate expressions for each π-term, every π-term must contain the repeating variables and one of the remaining variables. In selecting the repeating variable the following points must be considered:

   (a) Never pick the dependent variable.

   (b) The chosen repeating variables must not by themselves be able to form a dimensionless group. Example: V, L and t are not considered as a repeating variable, because $\frac{Vt}{L}$ will be a non-dimensional.

   (c) The chosen repeating variables must represent all the primary dimensions in the problem.

   (d) Never pick the variables that are already dimensionless. These are π's already, all by themselves.

   (e) Never pick two variables with the same dimensions or with dimensions that differ by only an exponent. That is one variable contains geometric property, second variable contains flow property and third containing fluid property.

   (f) Pick simple variables over complex variables whenever possible.

   (g) Pick popular parameters since they may appear in each of the π's.
4) The repeating variables are written in exponential form.
5) With the help of dimensional homogeneity, find the values of exponents by obtaining simultaneous equations.
6) Now, substitute the values of these exponents in the π terms.
7) Write the functional relation in the required form.

1.8.1 Geometric Variables: The variables with geometric property in turbomachines are *length, diameter, thickness, height* etc.

1.8.2 Kinematic Variables: The variables with flow property in turbomachines are *velocity, speed, volume flow rate, acceleration, angular velocity* etc.

1.8.3 Dynamic Variables: The variables with fluid property in turbomachines are *mass flow rate, gas density, dynamic viscosity, bulk modulus, pressure difference, force, power, elasticity, surface tension, specific weight, stress, resistance* etc.

Note: (1) For power generating turbomachines, the performance of a machine is referred to the power developed (P), workdone (W), pressure ratio (P_1/P_2) or efficiency (η) which depend on independent variables.

(2) For power absorbing turbomachines, the performance is referred to the discharge (Q), enthalpy rise (Δh), pressure ratio (P_2/P_1) or efficiency (η) which depend on independent variables.

Question No 1.9: Performance of a turbomachine depends on the variables discharge (Q), speed (N), rotar diameter (D), energy per unit mass flow (gH), power (P), density of fluid (ρ), dynamic viscosity of fluid (μ). Using the dimensional analysis obtain the π-terms. **(VTU, Jul/Aug-02)**

Answer: General relationship is:

$$f(Q, N, D, gH, P, \rho, \mu) = constant$$

Dimensions: $Q = L^3T^{-1}, N = T^{-1}, D = L, gH = L^2T^{-2}, P = ML^2T^{-3}, \rho = ML^{-3}, \mu = ML^{-1}T^{-1}$

Number of variables, n = 7

Number of fundamental variables, m = 3

Number of π-terms required, (n-m) = 4

Repeating variables are: *D,N,ρ*

π_1-term: $\boldsymbol{\pi_1 = D^aN^b\rho^cQ}$

In dimensional form: $M^0L^0T^0 = L^a(T^{-1})^b(ML^{-3})^cL^3T^{-1}$

Equating the powers of M L T on both sides:

For M, $0 = c$

For L, $0 = a - 3c + 3 \Longrightarrow a = -3$

For T, $0 = -b - 1 \Longrightarrow b = -1$

Then, $\pi_1 = D^{-3}N^{-1}\rho^0Q$

$$\boldsymbol{\pi_1 = \frac{Q}{ND^3}}$$

π_2-term: $\boldsymbol{\pi_2 = D^aN^b\rho^c gH}$

In dimensional form: $M^0L^0T^0 = L^a(T^{-1})^b(ML^{-3})^cL^2T^{-2}$

Equating the powers of M L T on both sides:

For M, $0 = c$

For L, $0 = a - 3c + 2 \Longrightarrow a = -2$

For T, $0 = -b - 2 \Longrightarrow b = -2$

Then, $\pi_1 = D^{-2}N^{-2}\rho^0 gH$

$$\boldsymbol{\pi_2 = \frac{gH}{N^2D^2}}$$

π_3-term: $\boldsymbol{\pi_3 = D^aN^b\rho^cP}$

In dimensional form: $M^0L^0T^0 = L^a(T^{-1})^b(ML^{-3})^cML^2T^{-3}$

Equating the powers of M L T on both sides:

For M, $0 = c + 1 \Rightarrow c = -1$

For L, $0 = a - 3c + 2 \Rightarrow a = -5$

For T, $0 = -b - 3 \Rightarrow b = -3$

Then, $\pi_3 = D^{-5}N^{-3}\rho^{-1}P$

$$\boldsymbol{\pi_3 = \frac{P}{\rho N^3 D^5}}$$

π3-term: $\boldsymbol{\pi_4 = D^a N^b \rho^c \mu}$

In dimensional form: $M^0L^0T^0 = L^a(T^{-1})^b(ML^{-3})^c ML^{-1}T^{-1}$

Equating the powers of M L T on both sides:

For M, $0 = c + 1 \Rightarrow c = -1$

For L, $0 = a - 3c - 1 \Rightarrow a = -2$

For T, $0 = -b - 1 \Rightarrow b = -1$

Then, $\pi_4 = D^{-2}N^{-1}\rho^{-1}\mu$

$$\boldsymbol{\pi_4 = \frac{\mu}{\rho N D^2}}$$

Question No 1.10: Give the significance of the dimensionless terms (i) Flow coefficient (ii) Head coefficient (iii) Power coefficient with respect to turbomachines. ***(VTU, Jan-07)*** ***Or,***

Explain capacity coefficient, head coefficient and power coefficient referring to a turbomachines. ***(VTU, Feb-02, Feb-03, Feb-04, Jan-16, Jul-17)***

Answer: The various π-terms have the very significant role in a turbomachine as explained below.

(i) Flow Coefficient: It is also called as capacity coefficient or specific capacity. The term $\frac{Q}{ND^3}$ is the capacity coefficient, which signifies the volume flow rate of fluid through a turbomachine of unit diameter of runner operating at unit speed. The specific capacity is constant for dynamically similar conditions. Hence for a fan or pump of certain diameter running at various speeds, the discharge is proportional to the speed. This is the *First fan law.*

Speed ratio: The specific capacity is related to another quantity called speed ratio and is obtained as follows: $\frac{Q}{ND^3} \propto \frac{D^2V}{ND^3} \propto \frac{V}{ND} \propto \frac{V}{U} = \frac{1}{\varphi}$ $\quad (Because\ Q = AV = \frac{\pi D^2 V}{4} \propto D^2V\ and\ also\ U \propto ND)$

Where $\varphi = \frac{U}{V}$ is called the speed ratio, which is defined as the ratio of tangential velocity of runner to the theoretical jet velocity of fluid. For the given machine, the speed ratio is fixed.

(ii) Head Coefficient: The term $\frac{gH}{N^2D^2}$ is called the head coefficient or specific head. It is a measure of the ratio of the fluid potential energy (column height H) and the fluid kinetic energy while moving at the rotational speed of the wheel U. The term can be interpreted by noting that: $\frac{gH}{N^2D^2} \propto \frac{gH}{U^2}$

The head coefficient is constant for dynamically similar machines. For a machine of specified diameter, the head varies directly as the square of the tangential speed of wheel. This is the Second fan law.

(iii) Power Coefficient: The term $\frac{P}{\rho N^3 D^5}$ is called the power coefficient or specific power. It represents the relation between the power, fluid density, speed and wheel diameter. For a given machine, the power is directly proportional to the cube of the tangential speed of wheel. This is the Third fan law.

Question No 1.11: Discuss the effect of Reynolds number on turbomachine. ***(VTU, Jun/Jul-08)***

Answer: The Reynolds number defined as the ratio of the inertial force to the viscous force. It is an important parameter, which represents the nature of flow. If the Reynolds number is greater than 4000, the flow is termed as turbulent, in which the inertia effect is more than the viscous effects. And, if Reynolds number is less than 2000, then flow is laminar in which viscous effects are more than the inertia effect.

The values of Reynolds number in turbines are much higher than the critical values. Most of the turbines use relatively low viscosity fluids like air, water and light oil. Therefore, the Reynolds number has very little effect on the power output of the machine. But, Reynolds number is an important parameter for small pumps, compressors, fans and blowers. Their performance improves with an increase in Reynolds number.

The Reynolds number for the pipe flow is expressed as $R_e = \frac{\rho VD}{\mu}$

1.10 Specific Speed:

The specific speed is the dimensionless term and is the parameter of greatest importance in incompressible flow machines. The specific speed is only the parameter that doesn't contain the linear dimension of the runner. Hence, while operating under the same conditions of flow and head, all geometrically similar machines have the same specific speed, irrespective of their sizes.

The specific speed can be expressed in terms of discharge (Q) for power absorbing machine or the power (P) for power generating machine.

Specific power is referred as the ratio of Power in or out of turbomachine to its weight/Unit Mass/ Unit Volume.

1.10.1 Specific Speed of a Pump:

Question No 1.12: Define specific speed of a pump. Derive an expression for specific speed of a pump from fundamentals. ***(VTU, Aug-05, Jun-12, Jan 15, Jul-15)***

Answer: Specific speed can be defined as *"a speed of geometrically similar machines discharging one cubic meter per second of water under head of one meter"*.

Head coefficient is given by $\frac{gH}{N^2D^2}$

$$N^2D^2 \propto gH$$

or $$D \propto \frac{(gH)^{1/2}}{N} \tag{1.9}$$

Flow coefficient is given by $\frac{Q}{ND^3}$

or $$Q \propto ND^3$$

From equation (1.9) $$Q \propto \frac{(gH)^{3/2}}{N^2}$$

or $$Q = C\frac{(gH)^{3/2}}{N^2} \tag{1.10}$$

Where C is proportionality constant, from the definition of specific speed of pump:

$$If\ Q = 1\,m^3/s, \qquad and\ \ H = 1m\,, then\ N = N_s$$

Then equation (1.10) can be written as, $C = \frac{N_s^2}{g^{3/2}}$ (1.11)

Substitute equation (1.11) in equation (1.10), then $\boldsymbol{N_s = \frac{NQ^{1/2}}{H^{3/4}}}$ (1.12)

The equation (1.12) gives the specific speed of a pump.

1.10.2 Specific Speed of a Turbine:

Question No 1.13: Define specific speed of a turbine. Obtain an expression for the same in terms of shaft power, speed and head. ***(VTU, Jul-08, Jul-13, Dec 14/ Jan 1, Jan-175)***

Answer: Specific speed of a turbine is defined as *"a speed of a geometrically similar machine which produces one kilowatt power under a head of one meter"*.

Power coefficient is given by $\frac{P}{\rho N^3D^5}$ (1.13)

From equation (1.9) $D \propto \frac{(gH)^{1/2}}{N}$, then equation (1.13) can be written as, $P \propto \frac{\rho(gH)^{5/2}}{N^2}$

or $$P = C\frac{\rho(gH)^{5/2}}{N^2} \tag{1.14}$$

Where C is proportionality constant, from the definition of specific speed of turbine:

$$If\ P = 1kW\ and\ H = 1m, then\ N = N_s$$

Then, equation (1.14) becomes $$C = \frac{N_s^2}{\rho g^{5/2}} \tag{1.15}$$

Substitute equation (1.15) in equation (1.14), then $\boldsymbol{N_s = \frac{NP^{1/2}}{H^{5/4}}}$ (1.16)

The equation (1.16) gives the specific speed of a turbine.

1.10.3 Significance of Specific Speed:

Question No 1.14: Briefly explain the significance of specific speed related to turbomachines.

(VTU, Jul-06, Jan-14)

Answer: In incompressible flow pumps, it possible to guess the approximate rotor shape from the specific speed. Small specific speed impellers have narrow and small openings whereas large specific speed impellers have wide openings and are expected to have large flow rates. Thus, a centrifugal pump has a nearly pure radial outward flow has the small inlet area. The flow rate is small because of the small inlet area but the head against which it works is high. So for the centrifugal pumps specific speed is small. Thus, to accommodate the large flow a relatively large impeller is needed for centrifugal pumps ($H \propto D^2$). A volute or mixed-flow pump has a bigger opening because of its mixed-flow characteristic though the head developed is not as large as that of the centrifugal pump. Its specific speed is higher than that of the centrifugal pump. At the extreme end is the axial-flow pump, which has a relatively large flow area and therefore a considerable volume flow rate. The head it develops is therefore small compared with that of radial-flow pumps. Its specific speed is very large.

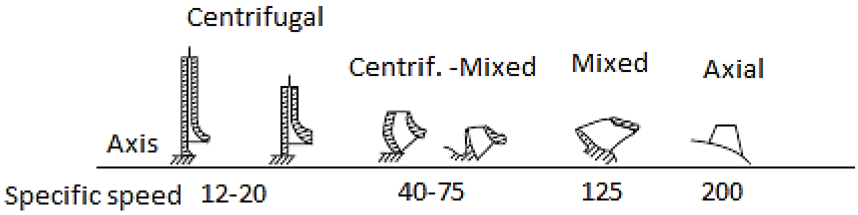

Fig. 1.2 Impeller shape variation with specific speed in pumps.

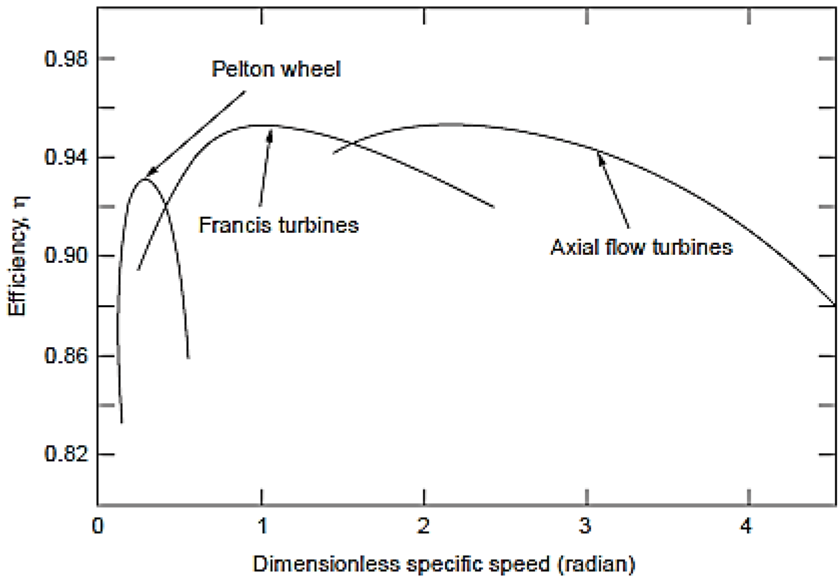

Fig. 1.3 Efficiency variation with specific speed in turbines.

Similarly, the specific speed determines the approximate shapes of the rotors as well. Consider for example the Pelton wheel which is a low specific speed, high head turbine. The volumetric flow rate is small since the turbine utilizes one or more nozzles from which the fluid emerges as jets. The Francis turbine covers a wide range of specific speeds and is suitable for intermediate heads. The Kaplan turbine operates at low heads and need large fluid flow rates to produce reasonable amounts of power. Their specific speeds are therefore high. Generally, specific speed is used as a guide to select a type of turbine under given condition of head and flow (i.e. site conditions). Therefore, such a thumb rule gives rise to a maximum efficiency. Thus, when specific speed is very high, Kaplan turbine is best selection to give rise to very high efficiency. When specific speed is very low, higher efficiencies are possible only if Pelton wheel is selected.

1.10.4 Range of Specific Speed of Various Turbomachines:

Specific speed in SI units		
1	Pelton wheel	
	Single jet	3 to 30
	Double jet	31 to 43
	Four jet	44 to 60
2	Francis turbine	
	Radial	61 to 102
	Mixed (Medium speed)	103 to 188
	Mixed (Fast speed)	189 to 368
3	Kaplan (Propeller) turbine	369 to 856
4	Centrifugal pumps	
	Turbine pump	12 to 25
	Volute pump	26 to 95
5	Mixed flow pump	96 to 210
6	Axial flow pump	211 to 320
7	Centrifugal compressor	32 to 74
8	Axial compressor	75 to 120
9	Blowers	121 to 1050

1.11 Unit Quantities:

Question No 1.15: Define unit quantities. Derive expressions to each of them. ***(VTU, Jan-08, Jul-16)***

Answer: In hydraulic turbines, it is usual to define quantities as unit flow, unit speed and unit power, which are the values of the quantities under consideration per unit head.

Unit flow (Q_u): Unit flow is the flow that occurs through the turbine while working under unit head.

Flow of fluid is given by, $Q = AC_v\sqrt{2gH}$ (1.17)

Where A is area of nozzle and C_v is coefficient of velocity.

or $Q = K\sqrt{H}$ (1.18)

Where $K = AC_v\sqrt{2g}$ proportionality constant.

But, from definition, $H = 1m, Q = Q_u$

Substitute in equation (1.18), $Q_u = K$

Then, equation (1.18) can be written as, $Q = Q_u\sqrt{H}$

or $\boldsymbol{Q_u = \frac{Q}{\sqrt{H}}}$

Unit speed (N_u): Unit speed is the speed at which the machine runs under unit head.

Head coefficient is given by $\pi_2 = \frac{gH}{N^2D^2}$

or $N^2 = KH$ (1.19)

Where $K = \frac{gH}{D^2\pi_2}$ proportionality constant.

From definition, $N = N_u, H = 1m$

Substitute in equation (1.19), $N_u^2 = K$

Then, equation (1.19) can be written as, $N^2 = N_u^2H$

or $\boldsymbol{N_u = \frac{N}{\sqrt{H}}}$

Unit power (P_u): Unit power is the power developed by the hydraulic machine while working under a unit head.

Power developed by hydraulic machine is given by $P = \rho gQH$

But, from equation (1.18), $Q = K\sqrt{H}$

Then, $P = K\rho gH^{3/2}$

or $P = CH^{3/2}$ (1.20)

Where $C = K\rho g$ proportionality constant.

From definition, $P = P_u, H = 1m$

Substitute in equation (1.20), $P_u = C$

Then, equation (1.20) can be written as, $P = P_uH^{3/2}$

or $\boldsymbol{P_u = \frac{P}{H^{3/2}}}$

1.12 Model Studies:

The principal of all model designs is to prepare a model, from its behaviour can produce a trustworthy, consistent and accurate prediction of the prototype performance. For this prediction the model and prototype should be geometrically, kinematically and dynamically similar. Model is a small scale replica of the actual machine and the actual machine is called prototype.

1.12.1 Geometric Similarity: It is the similarity of form or shape. Two systems, the model and prototype are said to be geometrically similar if the ratios of all corresponding linear dimensions of the systems are equal or homologous at all points.

For geometric similarity: $\frac{l_m}{l_p} = \frac{b_m}{b_p} = \frac{d_m}{d_p}$

Where l, b and d are the length, width and depth respectively and m and p are the suffixes that indicate model and prototype.

1.12.2 Kinematic Similarity: It is the similarity of motion. Two systems are considered to be kinematically similar if they are geometrically similar and ratios of components of velocity at all homologous points are equal.

For kinematic similarity:$\frac{(V_1)_m}{(V_1)_p} = \frac{(V_2)_m}{(V_2)_p} = \frac{(V_3)_m}{(V_3)_p} = \cdots$

Where $(V_1)_m, (V_2)_m, (V_3)_m$ are resultant velocities at points 1, 2, and 3 in the model and $(V_1)_p, (V_2)_p, (V_3)_p$ are resultant velocities at the corresponding points in the prototype.

1.12.3 Dynamic Similarity: Two systems are considered to be dynamically similar if they are geometrically and kinematically similar and the ratios of the corresponding forces acting at the corresponding points are equal.

For dynamic similarity:$\frac{(F_1)_m}{(F_1)_p} = \frac{(F_2)_m}{(F_2)_p} = \frac{(F_3)_m}{(F_3)_p} = \cdots$

Where $(F_1)_m, (F_2)_m, (F_3)_m$ are forces acting at points 1, 2, and 3 in the model and $(F_1)_p, (F_2)_p, (F_3)_p$ are forces acting at the corresponding points in the prototype.

1.13 Moody's Formula:

Machines of different sizes handling oils and other viscous fluids undergo efficiency changes under varying load conditions. For this reason, Moody has suggested an equation to determine turbine efficiencies from experiments on a geometrically similar model.

For heads smaller than 150 m, the efficiencies of model and prototype are related by the equation:

$$\eta_p = 1 - (1 - \eta_m)\left(\frac{D_m}{D_p}\right)^{0.2}$$

For heads larger than 150 m, the efficiencies of model and prototype are related by the equation:

$$\eta_p = 1 - (1 - \eta_m)\left(\frac{D_m}{D_p}\right)^{0.25}\left(\frac{H_m}{H_p}\right)^{0.1}$$

Since the power outputs for the prototype and model hydraulic turbines are $P_p = \eta_p \rho Q_p g H_p$ and $P_m = \eta_m \rho Q_m g H_m$, the power-ratio may be written as:

$$\frac{P_p}{P_m} = \left(\frac{\eta_p}{\eta_m}\right)\left(\frac{Q_p}{Q_m}\right)\left(\frac{H_p}{H_m}\right)$$

It has been assumed here that similarity equations may be applied and the power incremented in proportion to the machine efficiency.

From the flow coefficient,

$$\frac{Q_p}{Q_m} = \left(\frac{N_p}{N_m}\right)\left(\frac{D_p}{D_m}\right)^3$$

But, from the head coefficient,

$$\frac{N_p}{N_m} = \left(\frac{D_m}{D_p}\right)\left(\frac{H_p}{H_m}\right)^{\frac{1}{2}}$$

Then flow–ratio may be written as,

$$\frac{Q_p}{Q_m} = \left(\frac{H_p}{H_m}\right)^{\frac{1}{2}}\left(\frac{D_p}{D_m}\right)^2$$

Finally the power-ratio may be written as,

$$\frac{P_p}{P_m} = \left(\frac{\eta_p}{\eta_m}\right)\left(\frac{D_p}{D_m}\right)^2\left(\frac{H_p}{H_m}\right)^{\frac{3}{2}}$$

From the above relation the power output-ratio can be calculated using geometric ratio, head-ratio and efficiency-ratio.

1.14 Important Dimensionless Numbers:

Question No 1.16: Explain the following dimensionless numbers: (i) Froude's number, (ii) Weber's number, (iii) Mach's number and (iv) Euler's number. ***(VTU, Dec-07/Jan-08)***

Answer:

(i) Froude's number: It is defined as the ratio of inertia force to gravity force. Froude's number has considerable practical significance in free surface flow problems, like flow in orifices, flow over notches, flow over the spillways etc. The flow in these problems has predominant gravitational forces.

The Froude's number is given by $\frac{V^2}{gL}$.

(ii) Weber's number: It is defined as the ratio of inertia force to the surface tension force. Weber's number has considerable practical significance in problems influenced by surface tension, like gas-liquid and liquid-liquid interfaces and contact of such interfaces with a solid boundary. These problems have predominant surface tension force.

The Weber's number is given by $\frac{\rho L V^2}{\sigma}$.

(iii) Mach's number: It is defined as the ratio of inertia force to elastic force. Mach's number has considerable practical significance in compressible flow problems, like shells, bullets, missiles and rockets fired into air. These problems have predominant elastic force.

The Mach's number is given by $\frac{V}{\sqrt{K/\rho}}$

(iv) Euler's number: It is defined as the ratio of pressure force to inertia force. Euler's number has considerable practical significance in modelling of hydraulic turbines and pumps. The flow in these machines has predominant pressure forces.

The Euler's number is given by $\frac{P}{\rho V^2}$.

Module 1 : THERMODYNAMICS OF FLUID FLOW

This chapter deals with the some basic definitions of thermodynamics applies to the turbomacines and the discussions on the thermodynamics of the fluid flow through turbomachines.

3.1 Sonic Velocity and Mach Number:

Question No 3.1: Define Mach number and hence explain subsonic flow, sonic flow and supersonic flow. Or, write a note on Mach number. ***(VTU, Dec-09/Jan-10) Or,***

Give classification of fluid flow based on Mach number and explain in brief. ***(VTU, Dec-12)***

Answer: Sonic velocity (velocity of the sound) is referred to the speed of propagation of pressure wave in the medium. The velocity of the sound in a fluid at a local temperature T for an isentropic flow is given by

$$c = \sqrt{\gamma RT}.$$

Where γ, R and T are the ratio of specific heats, characteristic gas constant and the local temperature of the fluid respectively. At sea level the velocity of sound in air is given as 340 m/s.

Mach number is defined as the ratio of local velocity of fluid (V) to the sonic velocity (c) in that fluid. Thus

$$M = \frac{V}{c} = \frac{V}{\sqrt{\gamma RT}}$$

The fluid flow can be generally classified into subsonic flow, sonic flow and supersonic flow based on the value of Mach number.

Subsonic flow: If the Mach number is less than 1, then that type of flow is called subsonic flow, in which the velocity of the fluid is less than the velocity of the sound in that medium.

Sonic flow: If the Mach number is equal to 1, then that type of flow is called sonic flow, in which the velocity of the fluid is same as the velocity of the sound in that medium.

Supersonic flow: If the Mach number is greater than 1, then that type of flow is called supersonic flow, in which the velocity of the fluid is greater than the velocity of the sound in that medium.

3.2 Isentropic Flow for a Varying Flow Area:

Question No 3.2: For the isentropic flow through varying flow area, show that $\frac{dA}{A} = \frac{dp}{p}\left(\frac{1-M^2}{\gamma M^2}\right)$ ***and discuss the physical significance. Or, derive an expression for area ratio for isentropic flow through a passage of varying cross sectional area and discuss the significance of the expression.*** ***(VTU, Jun/Jul-13)***

Answer: The Continuity equation is given by,

$$\frac{dA}{A} + \frac{dV}{V} + \frac{d\rho}{\rho} = 0$$

Or,

$$\boldsymbol{\frac{dA}{A} = -\left(\frac{dV}{V} + \frac{d\rho}{\rho}\right)}$$

But isentropic equation is,

$$\frac{dp}{p} = \gamma\frac{d\rho}{\rho} \Rightarrow \boldsymbol{\frac{dp}{\gamma p} = \frac{d\rho}{\rho}}$$

But Euler's equation is,

$$\frac{dp}{\rho} + VdV = 0 \Rightarrow \frac{dp}{\rho V^2} + \frac{dV}{V}$$

$$\frac{dV}{V} = -\frac{dp}{\rho V^2}$$

From Mach number,

$$V^2 = M^2\gamma RT = M^2\gamma\left(\frac{p}{\rho}\right)$$

$$\rho V^2 = M^2\gamma p$$

Then,

$$\boldsymbol{\frac{dV}{V} = -\frac{dp}{M^2\gamma p}} \tag{3.1}$$

Therefore,

$$\frac{dA}{A} = -\left(-\frac{dp}{M^2\gamma p} + \frac{dp}{\gamma p}\right)$$

$$\frac{dA}{A} = \frac{dp}{M^2\gamma p} - \frac{dp}{\gamma p} = \frac{dp}{p}\left(\frac{1}{M^2\gamma} - \frac{1}{\gamma}\right)$$

$$\boldsymbol{\frac{dA}{A} = \frac{dp}{p}\left(\frac{1 - M^2}{\gamma M^2}\right)} \tag{3.2}$$

The significance of the equations (3.1) and (3.2) is discussed below:

The equation (3.1) shows that for nozzle pressure decreases as velocity increases and for diffuser velocity decreases as pressure increases.

For subsonic flow (M<1) the quantity $\left(\frac{1-M^2}{\gamma M^2}\right)$ is positive. In the nozzle pressure decreases, so the quantity $\frac{dp}{p}$ is negative; therefore from equation (3.2) the quantity $\frac{dA}{A}$ is also negative and hence area must decrease for subsonic nozzle in the direction of fluid flow. The shape of the subsonic nozzle (convergent nozzle) is as shown in figure 3.1.

. In the diffuser pressure increases, so the quantity $\frac{dp}{p}$ is positive; therefore from equation (3.2) the quantity $\frac{dA}{A}$ is also positive and hence area must increase for subsonic diffuser in the direction of fluid flow. The shape of the subsonic diffuser (divergent diffuser) is as shown in figure 3.1.

For supersonic flow (M>1) the quantity $\left(\frac{1-M^2}{\gamma M^2}\right)$ is negative. In the nozzle pressure decreases, so the quantity $\frac{dp}{p}$ is negative; therefore from equation (3.2) the quantity $\frac{dA}{A}$ is positive and hence area must increase for supersonic nozzle in the direction of fluid flow. The shape of the supersonic nozzle (divergent nozzle) is as shown in figure 3.2.

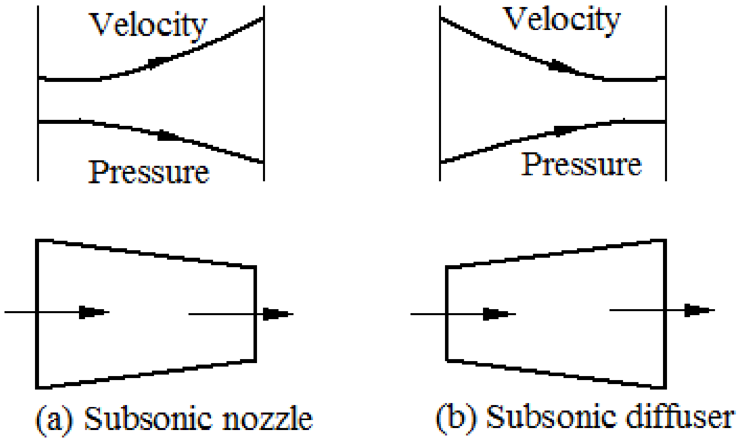

Fig. 3.1 Subsonic nozzle and diffuser

. In the diffuser pressure increases, so the quantity $\frac{dp}{p}$ is positive; therefore from equation (3.2) the quantity $\frac{dA}{A}$ is negative and hence area must decrease for supersonic diffuser in the direction of fluid flow. The shape of the supersonic diffuser (convergent diffuser) is as shown in figure 3.2.

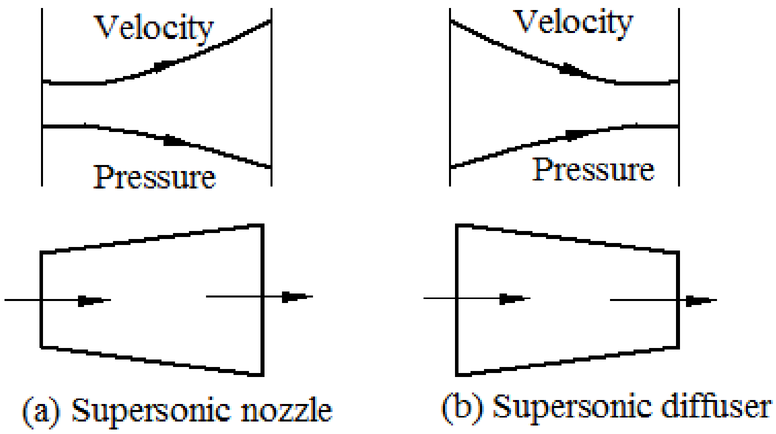

Fig. 3.2 Supersonic nozzle and diffuser

For sonic flow (M=1) the quantity $\left(\frac{1-M^2}{\gamma M^2}\right)$ is zero, from equation (3.2) the quantity $\frac{dA}{A}$ is also zero, i.e., area must be constant. This is the situation occurs at the throat portion of the convergent-divergent nozzle.

Note: The subsonic diffuser, subsonic nozzle and the supersonic nozzle are all of practical importance as for as the turbomachines are concerned, while the supersonic diffuser is of interest for wind tunnel and ram jet.

3.3 Static and Stagnation States:

Question No 3.3: Define static state and stagnation state for a fluid.

(VTU, Dec-11, Dec-12, Dec-14/Jan-15)

Answer: There are two kinds of state for the flowing fluid, namely static state and stagnation state.

(i) Static state: It is the state refers to those properties like pressure, temperature, density etc. which are measured when the measuring instruments are at rest relative to the flow of fluid.

(ii) Stagnation state: It is the final state of a fictitious, isentropic and work free process during which the final kinetic and potential energies of the fluid reduces to zero in a steady flow.

Question No 3.4: Write expressions for (i) stagnation enthalpy, (ii) stagnation temperature, (iii) stagnation pressure and (iv) stagnation density.

Answer: For a fictitious, isentropic and work free process the initial state is always the static state and final state is stagnation state. A steady flow energy equation (SFEE) for this fictitious process can be written as:

$$h_o + \frac{1}{2}V_o^2 + gZ_o + w = h + \frac{1}{2}V^2 + gZ + q$$

For isentropic and work free process, q=0 and w=0 and at the final state (stagnation state) of this process, ke=0 and pe=0. Thus steady flow energy equation is:

$$h_o = h + \frac{1}{2}V^2 + gZ$$

(i) Stagnation Enthalpy: It is defined as the enthalpy of a fluid when it is adiabatically decelerated to zero velocity. The stagnation enthalpy can be written as:

$$\boldsymbol{h_o = h + \frac{1}{2}V^2 + gZ}$$

Or,

$$\boldsymbol{h_o = h + \frac{1}{2}V^2}$$

(ii) Stagnation Temperature: It is defined as the temperature of a fluid when it is adiabatically decelerated to zero velocity. The stagnation temperature defined through stagnation enthalpy as:

$$c_pT_o = c_pT + \frac{1}{2}V^2$$

$$\boldsymbol{T_o = T + \frac{V^2}{2c_p}}$$

Or,

$$\frac{T_o}{T} = 1 + \frac{V^2}{2c_pT} = 1 + \frac{V^2(\gamma-1)}{2\gamma RT} = 1 + \left(\frac{\gamma-1}{2}\right)\frac{V^2}{c^2}$$

$$\boldsymbol{\frac{T_o}{T} = 1 + \left(\frac{\gamma-1}{2}\right)M^2}$$

(iii) Stagnation Pressure: It is defined as the pressure of a fluid when it is adiabatically decelerated to zero velocity. The relation between the stagnation and static pressures can be written as:

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\boldsymbol{\frac{p_o}{p} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]^{\frac{\gamma}{\gamma-1}}}$$

For incompressible flows, $h = \frac{p}{\rho}$

$$\frac{p_o}{\rho} = \frac{p}{\rho} + \frac{V^2}{2}$$

$$\boldsymbol{p_o = p + \frac{\rho V^2}{2}}$$

(**iv) Stagnation Density:** The stagnation density can be defined by using stagnation pressure and temperature. For an isentropic process,

$$\frac{\rho_o}{\rho} = \left(\frac{T_o}{T}\right)^{\frac{1}{\gamma-1}}$$

$$\boldsymbol{\frac{\rho_o}{\rho} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]^{\frac{1}{\gamma-1}}}$$

3.4 Compression Process in Compressor:

3.4.1 Efficiency of Compression Process:

Question No 3.5: Define the following, with the help of a h-s diagram, for the power absorbing turbomachines: (i) Total-to-total efficiency, (ii) Total-to-static efficiency, (iii) Static-to-total efficiency, (iv) Static-to static efficiency. (VTU, Dec-06/Jan-07)

Answer: The h-s diagram for the compression process is shown in figure 3.3. The fluid has initially the static pressure and temperature determines by state 1, the state 01 is the corresponding stagnation state. After passing through the turbomachine, the final static properties of the fluid are determined by state 2 and state 02 is corresponding stagnation state. If the process is reversible, the final fluid static state would be 2' while stagnation state would be 02'. Line 1-2 in static coordinates and line 01-02 in stagnation coordinates represent the real process.

The actual work input for compression process is,

$$w = h_{02} - h_{01}$$

The ideal work input can be calculated by any one of the following four equations:

(i) Totol-to-total work input is the ideal work input for the stagnation ends,

$$w_{t-t} = h_{02'} - h_{01}$$

(ii) Total-to-static work input is the ideal work input for the stagnation inlet to the static exit,

$$w_{t-s} = h_{2\prime} - h_{01}$$

(iii) Static-to-total work input is the ideal work input for the static inlet to the stagnation exit,

$$w_{s-t} = h_{02\prime} - h_1$$

(iv) Static-to-static work input is the ideal work input for the static inlet to the static exit,

$$w_{s-s} = h_{2\prime} - h_1$$

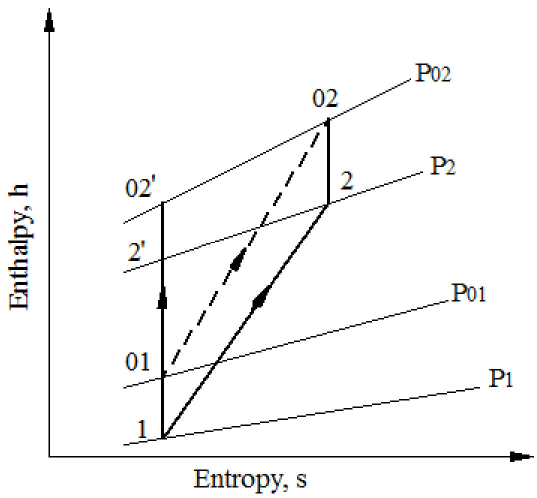

Fig. 3.3 h-s diagram for compression process

The efficiency of the compression process can be expressed by any one of the following equations:

(i) Total-to-total efficiency is defined as the ratio of total-to-total work input to the actual work input.

$$\eta_{t-t} = \frac{w_{t-t}}{w} = \frac{h_{02\prime} - h_{01}}{h_{02} - h_{01}}$$

(ii) Total-to-static efficiency is defined as the ratio of total-to-static work input to the actual work input.

$$\eta_{t-s} = \frac{w_{t-s}}{w} = \frac{h_{2\prime} - h_{01}}{h_{02} - h_{01}}$$

(iii) Static-to-total efficiency is defined as the ratio of static-to-total work input to the actual work input.

$$\eta_{s-t} = \frac{w_{s-t}}{w} = \frac{h_{02\prime} - h_1}{h_{02} - h_{01}}$$

(iv) Static-to-static efficiency is defined as the ratio of static-to-static work input to the actual work input.

$$\eta_{s-s} = \frac{w_{s-s}}{w} = \frac{h_{2\prime} - h_1}{h_{02} - h_{01}}$$

3.4.2 Effect of Pre-heat:

Question No 3.6: With the help of T-s diagram, show that the preheat factor in a multistage compressor is less than unity. ***(VTU, May/Jun-10, Jun/Jul-11)***

Answer: The preheat factor for a compressor may be defined as the ratio of direct or Rankine isentropic work to the cumulative isentropic work.

The thermodynamic effect of multistage compression can be studied by considering three stage compressor working between inlet pressure p_1 and the delivery pressure p_2 as shown in the figure 3.4. The intermediate pressures are being p_A and p_B. The stage pressure ratio, p_r and the stage efficiency, η_{st} are assumed to be same for all stages. The process 1-2' and 1-2 are the isentropic and actual compression process respectively.

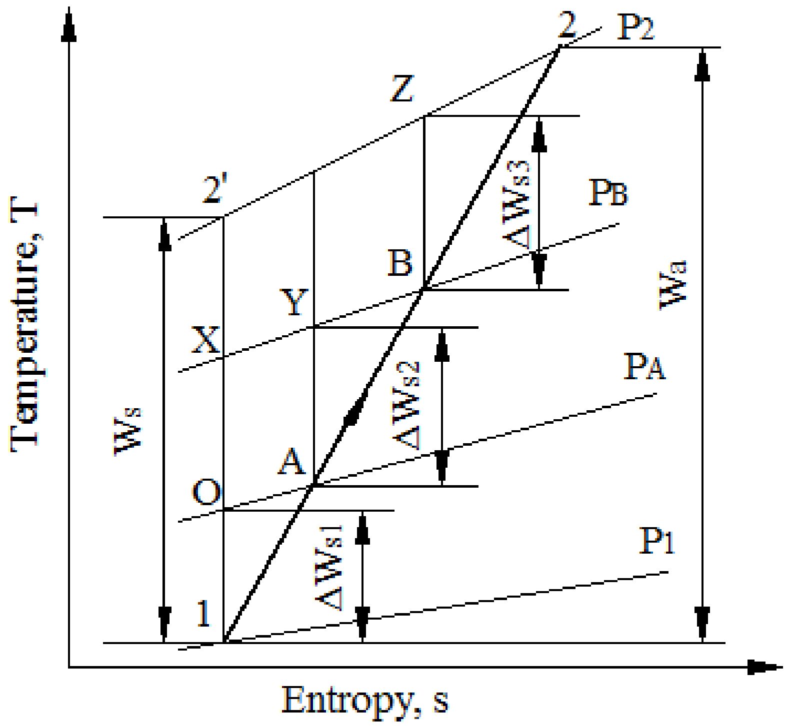

Fig. 3.4 Effect of preheat on compression process

As the constant pressure lines are diverging towards the right hand side of the temperature-entropy diagram, the isentropic work per stage increases as the temperature difference increases for the same pressure ratio and the stage efficiency. For example, in the second stage between pressures p_A and p_B, the isentropic temperature difference represented by the line A-Y is greater than that represented by the line X-O. It is therefore the isentropic work for the stage is greater by virtue of the inefficiency of the previous stage. Similarly for the next stage also.

Therefore, $$w_s < (\Delta w_{s1} + \Delta w_{s2} + \Delta w_{s3})$$

Or, $$w_s < \Sigma \Delta w_s$$

$$\frac{\boldsymbol{w_s}}{\boldsymbol{\Sigma \Delta w_s}} < 1$$

Therefore, the Preheat factor $\frac{w_s}{\Sigma\Delta w_s}$ is always less than unity for multistage compressor. This is due to the preheating of the fluid at the end of each compression stage and this appears as the losses in the subsequent stages.

Question No 3.7: For a multistage compressor, show that the overall efficiency is less than the stage efficiency using T-s diagram. ***(VTU, Jun/Jul-08)***

Answer: Consider three stage compressor working between inlet pressure p_1 and the delivery pressure p_2 as shown in the figure 3.4. The intermediate pressures are being p_A and p_B. The stage pressure ratio, p_r and the stage efficiency, η_{st} are assumed to be same for all stages. The process 1-2' and 1-2 are the isentropic and actual compression process respectively.

If the overall efficiency of the multistage compressor is η_o, then the total actual work is given by,

$$w_a = \frac{w_s}{\eta_o}$$

Or,
$$\boldsymbol{w_s = \eta_o w_a}$$

The total actual work can also be written as the sum of the actual work done in each stage,

$$w_a = w_{a1} + w_{a2} + w_{a3} = \frac{\Delta w_{s1}}{\eta_{st}} + \frac{\Delta w_{s2}}{\eta_{st}} + \frac{\Delta w_{s3}}{\eta_{st}}$$

$$w_a = \frac{1}{\eta_{st}}(\Delta w_{s1} + \Delta w_{s2} + \Delta w_{s3})$$

$$w_a = \frac{1}{\eta_{st}}\Sigma\Delta w_s$$

Or,
$$\boldsymbol{\Sigma\Delta w_s = \eta_{st} w_a}$$

As the constant pressure lines are diverging towards the right hand side of the temperature-entropy diagram, the isentropic work per stage increases as the temperature difference increases for the same pressure ratio and the stage efficiency.

Therefore,
$$w_s < (\Delta w_{s1} + \Delta w_{s2} + \Delta w_{s3})$$

Or,
$$w_s < \Sigma\Delta w_s$$

$$\eta_o w_a < \eta_{st} w_a$$

$$\boldsymbol{\eta_o < \eta_{st}}$$

For multistage compressor, the overall isentropic efficiency is less than the stage efficiency.

3.4.3 Infinitesimal Stage Efficiency or Polytropic Efficiency:

Question No 3.8: Obtain an expression for polytropic efficiency for a compressor in terms of pressure ratio and temperature ratio. Further express stage efficiency in terms of polytropic efficiency and pressure ratio. Also draw the relevant T-s diagram. ***(VTU, Jun/Jul-13) Or, Define the***

term infinitesimal stage efficiency of a compressor. Show that the polytropic efficiency during the compression process is given by $\boldsymbol{\eta_p = \dfrac{\frac{\gamma-1}{\gamma} ln\left(\frac{p_2}{p_1}\right)}{ln\left(\frac{T_2}{T_1}\right)}}$ ***.(VTU, Dec-14/Jan-15)***

Answer: A finite compressor stage is made up of number of infinitesimal stages; the efficiency of these small stages is called polytropic efficiency or infinitesimal stage efficiency.

Consider a single stage compressor having its stage efficiency η_{st}, operates between the pressures p_1 and p_2, and an infinitesimal stage of efficiency η_p, working between the pressures p and p+dp as shown in figure 3.5.

The infinitesimal stage efficiency is given by,

$$\eta_p = \frac{Isentropic\ Temperature\ Rise}{Actual\ Temperature\ Rise} = \frac{dT'}{dT}$$

The actual temperature rise for infinitesimal stage is given by,

$$dT = \frac{dT'}{\eta_p} = \frac{T' - T}{\eta_p} = \frac{T\left(\frac{T'}{T} - 1\right)}{\eta_p}$$

$$\frac{dT}{T} = \frac{\left[\left(\frac{p + dp}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{\eta_p}$$

$$\frac{dT}{T} = \frac{1}{\eta_p}\left[\left(1 + \frac{dp}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

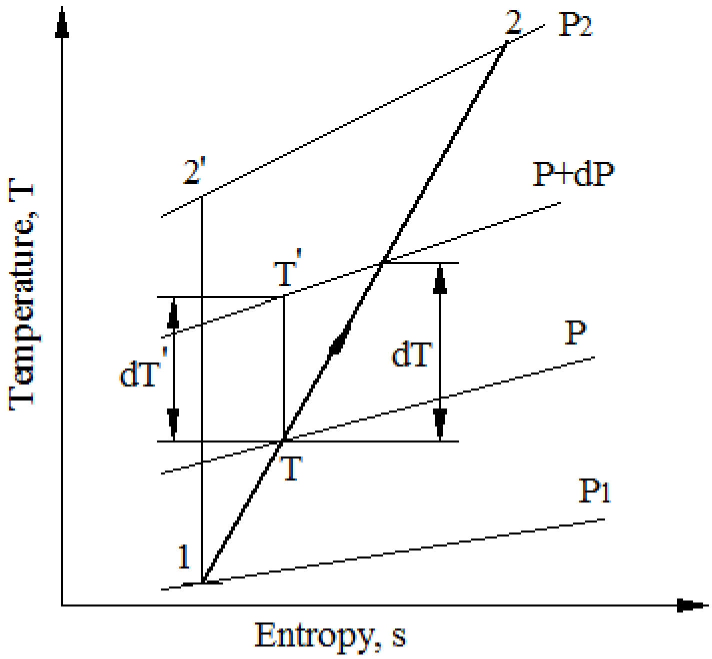

Fig. 3.5 Infinitesimal stage of a compressor

By series of expansion, $(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots$ and neglecting second order differentials,

$$\frac{dT}{T} = \frac{1}{\eta_p}\left[1 + \frac{\gamma - 1}{\gamma}\frac{dp}{p} - 1\right]$$

$$\frac{dT}{T} = \frac{1}{\eta_p}\frac{\gamma - 1}{\gamma}\frac{dp}{p} \tag{3.3}$$

By integration with limits 1 to 2,

$$ln\left(\frac{T_2}{T_1}\right) = \frac{1}{\eta_p}\frac{\gamma - 1}{\gamma} ln\left(\frac{p_2}{p_1}\right)$$

$$\boldsymbol{\eta_p = \frac{\frac{\gamma - 1}{\gamma} ln\left(\frac{p_2}{p_1}\right)}{ln\left(\frac{T_2}{T_1}\right)}}$$

Question No 3.9: With the help of T-s diagram, show that polytropic efficiency during the compression process is given by $\boldsymbol{\eta_p = \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{n}{n-1}\right)}$***(VTU, Jun/Jul-13)***

Answer: From equation (3.3),

$$\frac{dT}{T} = \frac{1}{\eta_p}\frac{\gamma - 1}{\gamma}\frac{dp}{p}$$

By integration,

$$ln(T) = \frac{1}{\eta_p}\frac{\gamma - 1}{\gamma} ln(p) + Const$$

$$\frac{p^{\frac{\gamma-1}{\eta_p \gamma}}}{T} = Const$$

For actual compression process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\eta_p \gamma}}$$

Assume actual compression process having polytropic index '*n*',

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

Therefore,

$$\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\eta_p \gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

Equating the indices,

$$\frac{\gamma - 1}{\eta_p \gamma} = \frac{n - 1}{n}$$

Or,

$$\boldsymbol{\eta_p = \left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{n}{n-1}\right)}$$

Question No 3.10: Derive an expression for stage efficiency of a compressor in terms of stage pressure ratio, polytropic efficiency and ratio of specific heats. Indicate the process on T-s diagram. ***(VTU, Dec-12) Or,***

With the help of T-s diagram, show that stage efficiency of compressor is given by

$$\boldsymbol{\eta_{st} = \frac{P_r^{\frac{\gamma-1}{\gamma}} - 1}{P_r^{\frac{\gamma-1}{\eta_p \gamma}} - 1}}$$

Answer: From the T-s diagram shown in figure 3.5, the compressor stage efficiency is given by,

$$\eta_{st} = \frac{Isentropic\ Temperature\ Rise}{Actual\ Temperature\ Rise}$$

$$\eta_{st} = \frac{T_{2\prime} - T_1}{T_2 - T_1} = \frac{T_1\left(\frac{T_{2\prime}}{T_1} - 1\right)}{T_1\left(\frac{T_2}{T_1} - 1\right)} = \frac{\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\eta_p \gamma}} - 1\right]}$$

Let, $p_r = \frac{p_2}{p_1}$

$$\boldsymbol{\eta_{st} = \frac{P_r^{\frac{\gamma-1}{\gamma}} - 1}{P_r^{\frac{\gamma-1}{\eta_p \gamma}} - 1}}$$

3.4.4 Multistage Compressors:

Question No 3.11: Derive an expression for an overall isentropic efficiency for multistage compression in terms of pressure ratio, polytropic efficiency, number of stages and ratio of specific heats for a compressor. Or,

Show that for a multistage compression the overall isentropic efficiency is given by

$$\boldsymbol{\eta_o = \frac{P_r^{K\frac{\gamma-1}{\gamma}} - 1}{P_r^{K\frac{\gamma-1}{\eta_p \gamma}} - 1}}$$

Where K= number of stages, P_r= pressure ratio per stage, η_p= polytropic efficiency, γ= ratio of specific heats.

Answer: The figure 3.6 shows the T-s diagram for compression process in multistage compressor operating between the pressures p_1 and p_{K+1}. If there are K stages with the overall pressure ratio $\frac{p_{K+1}}{p_1}$ and having equal stage efficiency and stage pressure ratio.

The overall efficiency of the multistage compressor is,

$$\eta_o = \frac{Total\ Isentropic\ Temperature\ Rise}{Total\ Actual\ Temperature\ Rise}$$

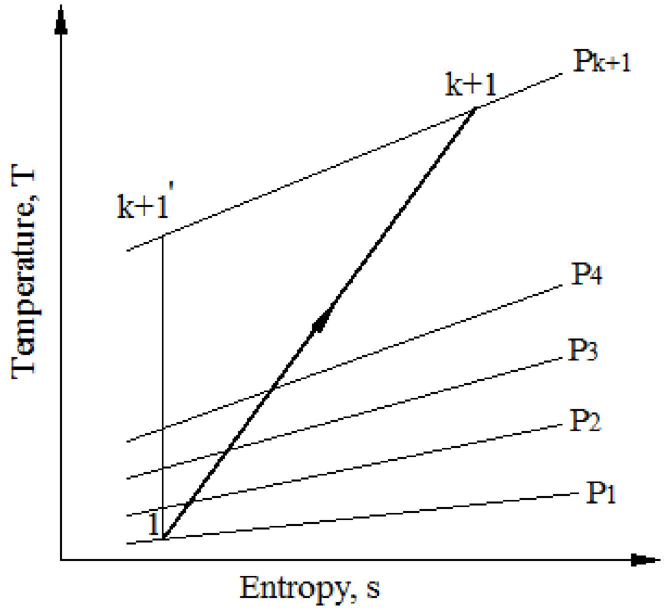

Fig. 3.6 Compression process in multistage compressor

$$\eta_o = \frac{T_{(K+1)\prime} - T_1}{T_{K+1} - T_1} = \frac{T_1\left(\frac{T_{(K+1)\prime}}{T_1} - 1\right)}{T_1\left(\frac{T_{K+1}}{T_1} - 1\right)}$$

$$\boldsymbol{\eta_o = \frac{P_{ro}^{\frac{\gamma-1}{\gamma}} - 1}{P_{ro}^{\frac{\gamma-1}{\eta_p \gamma}} - 1}}$$

The overall pressure ratio can be written as, $p_{ro} = p_r^K$

Then overall efficiency of multistage compressor is,

$$\boldsymbol{\eta_o = \frac{P_r^{K\frac{\gamma-1}{\gamma}} - 1}{P_r^{K\frac{\gamma-1}{\eta_p \gamma}} - 1}}$$

Question No 3.12: Derive an expression for an overall isentropic efficiency for finite number of stages of compression in terms of pressure ratio, stage efficiency, number of stages and ratio of specific heats for a compressor. ***(VTU, May/Jun-10)*** ***Or, Show that for a finite number of stages for compression the overall isentropic efficiency is given by***

$$\boldsymbol{\eta_o = \frac{P_r^{K\frac{\gamma-1}{\gamma}} - 1}{\left[1 + \frac{P_r^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{st}}\right]^K - 1}}$$

Where K= number of stages, P_r= pressure ratio per stage, η_{st}= stage efficiency, γ= ratio of specific heats. ***(VTU, Jan/Feb-06)***

Answer: If T_1 is the initial temperature at which the fluid enters the multistage compressor, K is the number of stages having equal pressure ratio p_r in each stage, then the actual temperature rise in each stage can be given as follows:

For first stage:

$$\Delta T_1 = (T_2 - T_1) = \frac{(T_{2'} - T_1)}{\eta_{st}} = \frac{T_1\left(\frac{T_{2'}}{T_1} - 1\right)}{\eta_{st}} = T_1 \frac{P_r^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{st}}$$

Let $A = \frac{P_r^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{st}}$

$$\boldsymbol{\Delta T_1 = AT_1}$$

For second stage:

$$\Delta T_2 = (T_3 - T_2) = AT_2 = A(T_1 + AT_1)$$

$$\boldsymbol{\Delta T_2 = AT_1(1 + A)}$$

For third stage:

$$\Delta T_3 = (T_4 - T_3) = AT_3 = A[T_2 + AT_1(1 + A)]$$

$$\Delta T_3 = A[(T_1 + AT_1) + AT_1(1 + A)] = A[T_1(1 + A) + AT_1(1 + A)]$$

$$\Delta T_3 = A[(1 + A)(T_1 + AT_1)] = AT_1[(1 + A)(1 + A)]$$

$$\boldsymbol{\Delta T_3 = AT_1(1 + A)^2}$$

Similarly for fourth stage:

$$\boldsymbol{\Delta T_4 = AT_1(1 + A)^3}$$

And for Kth stage:

$$\boldsymbol{\Delta T_K = AT_1(1 + A)^{K-1}}$$

Total temperature rise across the multistage compressor is:

$$\Delta T_o = \Delta T_1 + \Delta T_2 + \Delta T_3 + \Delta T_4 + \cdots + \Delta T_K$$

$$\Delta T_o = AT_1 + AT_1(1 + A) + AT_1(1 + A)^2 + AT_1(1 + A)^3 + \cdots + AT_1(1 + A)^{K-1}$$

$$\Delta T_o = AT_1[1 + (1 + A) + (1 + A)^2 + (1 + A)^3 + \cdots + (1 + A)^{K-1}]$$

$$\Delta T_o = AT_1 S$$

Where $$\boldsymbol{S = 1 + (1 + A) + (1 + A)^2 + (1 + A)^3 + \cdots + (1 + A)^{K-1}}$$

$$S = 1 + (1 + A)[1 + (1 + A) + (1 + A)^2 \ldots + (1 + A)^{K-2}]$$

Or, $$S = 1 + (1 + A)[1 + (1 + A) + (1 + A)^2 \ldots + (1 + A)^{K-2} + (1 + A)^{K-1} - (1 + A)^{K-1}]$$

$$S = 1 + (1 + A)[S - (1 + A)^{K-1}]$$

$$S = 1 + S(1 + A) - (1 + A)^K = 1 + S + SA - (1 + A)^K$$

$$\boldsymbol{SA = (1 + A)^K - 1}$$

But, $$\Delta T_o = SAT_1$$

$$\Delta T_o = T_1[(1+A)^K - 1]$$

$$\boldsymbol{\Delta T_o = T_1\left[\left(1 + \frac{P_r^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{st}}\right)^K - 1\right]}$$

The overall efficiency of the multistage compressor is,

$$\eta_o = \frac{Total\ Isentropic\ Temperature\ Rise}{Total\ Actual\ Temperature\ Rise} = \frac{\Delta T_o{}'}{\Delta T_o}$$

$$\eta_o = \frac{T_{(K+1)\prime} - T_1}{\Delta T_o} = \frac{T_1\left(\frac{T_{(K+1)\prime}}{T_1} - 1\right)}{\Delta T_o} = \frac{T_1\left(P_{ro}^{\frac{\gamma-1}{\gamma}} - 1\right)}{\Delta T_o}$$

$$\eta_o = \frac{T_1\left(P_r^{K\frac{\gamma-1}{\gamma}} - 1\right)}{T_1\left[\left(1 + \frac{P_r^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{st}}\right)^K - 1\right]}$$

$$\boldsymbol{\eta_o = \frac{P_r^{K\frac{\gamma-1}{\gamma}} - 1}{\left[1 + \frac{P_r^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{st}}\right]^K - 1}}$$

3.5 Expansion Process in Turbine:

3.5.1 Efficiency of Expansion Process:

Question No 3.13: Define the following, with the help of a h-s diagram, for the power generating turbomachines: (i) Total-to-total efficiency, (ii) Total-to-static efficiency, (iii) Static-to-total efficiency, (iv) Static-to static efficiency. ***(VTU, Dec-07/Jan-08, May/Jun-10, Dec-10, Jun/Jul-11)***

Answer: The h-s diagram for the expansion process is shown in figure 3.7. The fluid has initially the static pressure and temperature determines by state 1, the state 01 is the corresponding stagnation state. After passing through the turbomachine, the final static properties of the fluid are determined by state 2 and state 02 is corresponding stagnation state. If the process is reversible, the final fluid static state would be 2' while stagnation state would be 02'. Line 1-2 in static coordinates and line 01-02 in stagnation coordinates represent the real process.

The actual work output for expansion process is,

$$w = h_{01} - h_{02}$$

The ideal work output can be calculated by any one of the following four equations:

(i) Totol-to-total work output is the ideal work output for the stagnation ends,

$$w_{t-t} = h_{01} - h_{02'}$$

(ii) Total-to-static work output is the ideal work output for the stagnation inlet to the static exit,

$$w_{t-s} = h_{01} - h_{2'}$$

(iii) Static-to-total work output is the ideal work output for the static inlet to the stagnation exit,

$$w_{s-t} = h_1 - h_{02'}$$

(iv) Static-to-static work output is the ideal work output for the static inlet to the static exit,

$$w_{s-s} = h_1 - h_{2'}$$

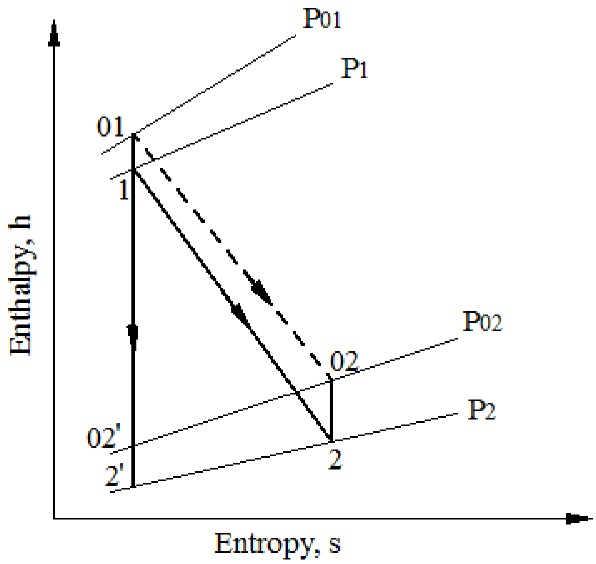

Fig. 3.7 h-s diagram for expansion process

The efficiency of the compression process can be expressed by any one of the following equations:

(i) Total-to-total efficiency is defined as the ratio of actual work output to the total-to-total work output.

$$\eta_{t-t} = \frac{w}{w_{t-t}} = \frac{h_{01} - h_{02}}{h_{01} - h_{02'}}$$

(ii) Total-to-static efficiency is defined as the ratio of actual work output to the total-to-static work output.

$$\eta_{t-s} = \frac{w}{w_{t-s}} = \frac{h_{01} - h_{02}}{h_{01} - h_{2'}}$$

(iii) Static-to-total efficiency is defined as the ratio of actual work output to the static-to-total work output.

$$\eta_{s-t} = \frac{w}{w_{s-t}} = \frac{h_{01} - h_{02}}{h_1 - h_{02'}}$$

(iv) Static-to-static efficiency is defined as the ratio of actual work output to the static-to-static work output.

$$\eta_{s-s} = \frac{w}{w_{s-s}} = \frac{h_{01} - h_{02}}{h_1 - h_{2'}}$$

3.5.2 Effect of Reheat:

Question No 3.14: What is Reheat factor? Show that the reheat factor is greater than unity in a multistage turbine. ***(VTU, Dec-06/Jan-07, Jun/Jul-09, Dec-11, Jun/Jul-14, Dec-14/Jan-15)***

Answer: The reheat factor for a turbine may be defined as the ratio of cumulative isentropic work to the direct or Rankine isentropic work.

The thermodynamic effect of multistage expansion can be studied by considering three stage turbine working between inlet pressure p_1 and the delivery pressure p_2 as shown in the figure 3.8. The intermediate pressures are being p_A and p_B. The stage pressure ratio, p_r and the stage efficiency, η_{st} are assumed to be same for all stages. The process 1-2' and 1-2 are the isentropic and actual expression process respectively.

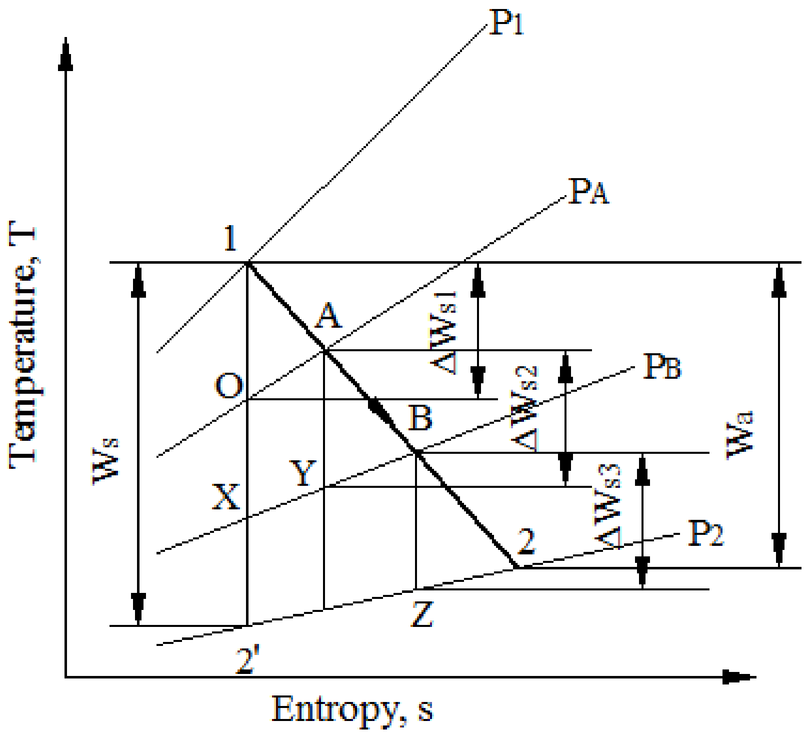

Fig. 3.8 Effect of reheat on expansion process

As the constant pressure lines are diverging towards the right hand side of the temperature-entropy diagram, the isentropic work per stage increases as the temperature difference increases for the same pressure ratio and the stage efficiency. For example, in the second stage between pressures p_A and p_B, the isentropic temperature difference represented by the line A-Y is greater than that represented by the line O-X. It is therefore the isentropic work for the stage is greater by virtue of the inefficiency of the previous stage. Similarly for the next stage also.

Therefore, $(\Delta w_{s1} + \Delta w_{s2} + \Delta w_{s3}) > w_s$

Or, $\Sigma \Delta w_s > w_s$

$$\frac{\boldsymbol{\Sigma\Delta w_s}}{\boldsymbol{w_s}} > 1$$

Therefore, the Reheat factor $\frac{\Sigma\Delta w_s}{w_s}$ is always greater than unity for multistage turbine. This is due to the reheating of the fluid at the end of each expansion stage and this appears as the losses in the subsequent stages.

Question No 3.15: For a multistage turbine, show that the overall efficiency is greater than the stage efficiency using T-s diagram. ***(VTU, Jun/Jul-08)***

Answer: Consider three stage turbine working between inlet pressure p_1 and the delivery pressure p_2 as shown in the figure 3.8. The intermediate pressures are being p_A and p_B. The stage pressure ratio, p_r and the stage efficiency, η_{st} are assumed to be same for all stages. The process 1-2' and 1-2 are the isentropic and actual expansion process respectively.

If the overall efficiency of the multistage expansion is η_o, then the total actual work is given by,

$$w_a = \eta_o w_s$$

Or,

$$\boldsymbol{w_s = \frac{w_a}{\eta_o}}$$

The total actual work can also be written as the sum of the actual work done in each stage,

$$w_a = w_{a1} + w_{a2} + w_{a3} = \eta_{st}\Delta w_{s1} + \eta_{st}\Delta w_{s2} + \eta_{st}\Delta w_{s3}$$

$$w_a = \eta_{st}(\Delta w_{s1} + \Delta w_{s2} + \Delta w_{s3})$$

$$w_a = \eta_{st}\Sigma\Delta w_s$$

Or,

$$\boldsymbol{\Sigma\Delta w_s = \frac{w_a}{\eta_{st}}}$$

As the constant pressure lines are diverging towards the right hand side of the temperature-entropy diagram, the isentropic work per stage increases as the temperature difference increases for the same pressure ratio and the stage efficiency.

Therefore, $(\Delta w_{s1} + \Delta w_{s2} + \Delta w_{s3}) > w_s$

Or, $\Sigma\Delta w_s > w_s$

$$\frac{w_a}{\eta_{st}} > \frac{w_a}{\eta_o}$$

$$\boldsymbol{\eta_o > \eta_{st}}$$

For multistage turbine, the overall isentropic efficiency is greater than the stage efficiency.

3.5.3 Infinitesimal Stage Efficiency or Polytropic Efficiency:

Question No 3.16: Define the term infinitesimal stage efficiency of a turbine. Show that the polytropic efficiency during the expansion process is given by $\boldsymbol{\eta_p = \dfrac{ln\left(\frac{T_2}{T_1}\right)}{\frac{\gamma-1}{\gamma} ln\left(\frac{p_2}{p_1}\right)}}$

(VTU, Dec-09/Jan-10, Jun-12, Dec-13/Jan-14)

Answer: A finite turbine stage is made up of number of infinitesimal stages; the efficiency of these small stages is called polytropic efficiency or infinitesimal stage efficiency.

Consider a single stage turbine having its stage efficiency η_{st}, operates between the pressures p_1 and p_2, and an infinitesimal stage of efficiency η_p, working between the pressures p and p-dp as shown in figure 3.9.

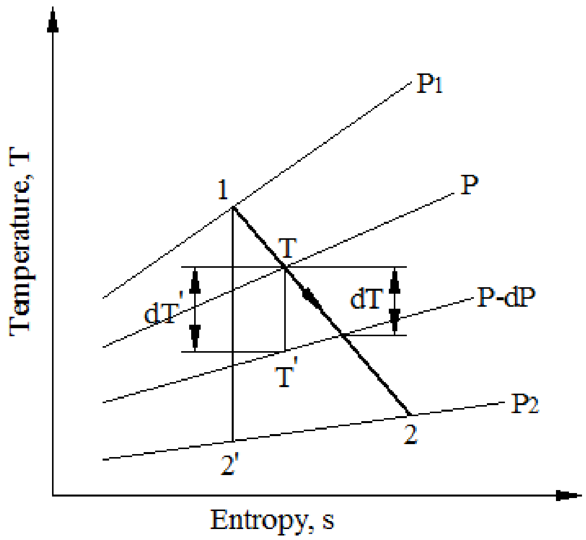

Fig. 3.9 Infinitesimal stage of a turbine

The infinitesimal stage efficiency is given by,

$$\eta_p = \frac{Actual\ Temperature\ Drop}{Isentropic\ Temperature\ Drop} = \frac{dT}{dT'}$$

The actual temperature rise for infinitesimal stage is given by,

$$dT = \eta_p dT' = \eta_p (T - T') = \eta_p T\left(1 - \frac{T'}{T}\right)$$

$$\frac{dT}{T} = \eta_p \left[1 - \left(\frac{p - dp}{p}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$\frac{dT}{T} = \eta_p \left[1 - \left(1 - \frac{dp}{p}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

By series of expansion, $(1-x)^n = 1 - nx + \frac{n(n-1)}{2}x^2 - \cdots$ and neglecting second order differentials,

$$\frac{dT}{T} = \eta_p \left[1 - \left(1 - \frac{\gamma - 1}{\gamma}\frac{dp}{p}\right)\right]$$

$$\frac{dT}{T} = \eta_p \frac{\gamma - 1}{\gamma}\frac{dp}{p} \tag{3.4}$$

By integration with limits 1 to 2,

$$ln\left(\frac{T_2}{T_1}\right) = \eta_p \frac{\gamma - 1}{\gamma} ln\left(\frac{p_2}{p_1}\right)$$

$$\boldsymbol{\eta_p = \frac{ln\left(\frac{T_2}{T_1}\right)}{\frac{\gamma - 1}{\gamma} ln\left(\frac{p_2}{p_1}\right)}}$$

Or,

$$\boldsymbol{\eta_p = \frac{\frac{\gamma}{\gamma - 1} ln\left(\frac{T_2}{T_1}\right)}{ln\left(\frac{p_2}{p_1}\right)}}$$

Question No 3.17: With the help of T-s diagram, show that polytropic efficiency during expansion process is given by $\boldsymbol{\eta_p = \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{n-1}{n}\right)}$ ***(VTU, Dec-08/Jan-09, Jun/Jul-14)***

Answer: From equation (3.4),

$$\frac{dT}{T} = \eta_p \frac{\gamma - 1}{\gamma}\frac{dp}{p}$$

By integration,

$$ln(T) = \eta_p \frac{\gamma - 1}{\gamma} ln(p) + Const$$

$$\frac{p^{\eta_p \frac{\gamma - 1}{\gamma}}}{T} = Const$$

For actual compression process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\eta_p \frac{\gamma - 1}{\gamma}}$$

Assume actual compression process having polytropic index ‘n’,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

Therefore,

$$\left(\frac{p_2}{p_1}\right)^{\eta_p \frac{\gamma - 1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

Equating the indices,

$$\eta_p \frac{\gamma - 1}{\gamma} = \frac{n - 1}{n}$$

Or,

$$\boldsymbol{\eta_p} = \left(\frac{\boldsymbol{\gamma}}{\boldsymbol{\gamma - 1}}\right)\left(\frac{\boldsymbol{n - 1}}{\boldsymbol{n}}\right) \tag{3.5}$$

Question No 3.18: Show that the index 'n' of polytropic expansion in a turbine of infinitesimal stage efficiency η_p is given by $n = \frac{\gamma}{\gamma-(\gamma-1)\eta_p}$, where γ is a ratio of specific heats. (VTU, Dec-10)

Answer: From equation (3.5),

$$\eta_p = \left(\frac{\gamma}{\gamma - 1}\right)\left(\frac{n - 1}{n}\right)$$

$$\eta_p(\gamma - 1)n = \gamma(n - 1)$$

$$\eta_p\gamma n - \eta_p n = \gamma n - \gamma$$

$$\eta_p\gamma n - \eta_p n - \gamma n = -\gamma$$

$$n\left(\eta_p\gamma - \eta_p - \gamma\right) = -\gamma$$

$$n = \frac{-\gamma}{\eta_p\gamma - \eta_p - \gamma} = \frac{\gamma}{-\eta_p\gamma + \eta_p + \gamma}$$

$$\boldsymbol{n = \frac{\gamma}{\gamma - (\gamma - 1)\eta_p}}$$

Question No 3.19: With the help of T-s diagram, show that stage efficiency of turbine is given by

$$\boldsymbol{\eta_{st} = \frac{1 - P_r^{-\left(\eta_p\frac{\gamma-1}{\gamma}\right)}}{1 - P_r^{-\left(\frac{\gamma-1}{\gamma}\right)}}} \textit{ (VTU, Jun-12)}$$

Answer: From the T-s diagram shown in figure 3.9, the turbine stage efficiency is given by,

$$\eta_{st} = \frac{Actual\ Temperature\ Drop}{Isentropic\ Temperature\ Drop}$$

$$\eta_{st} = \frac{T_1 - T_2}{T_1 - T_{2\prime}} = \frac{T_1\left(1 - \frac{T_2}{T_1}\right)}{T_1\left(1 - \frac{T_{2\prime}}{T_1}\right)} = \frac{\left[1 - \left(\frac{p_2}{p_1}\right)^{\eta_p\frac{\gamma-1}{\gamma}}\right]}{\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

Let, $p_r = \frac{p_1}{p_2}$

$$\boldsymbol{\eta_{st} = \frac{1 - P_r^{-\left(\eta_p\frac{\gamma-1}{\gamma}\right)}}{1 - P_r^{-\left(\frac{\gamma-1}{\gamma}\right)}}}$$

3.5.4 Multistage Turbines:

Question No 3.20: Derive an expression for an overall isentropic efficiency for multistage expansion in terms of pressure ratio, polytropic efficiency, number of stages and ratio of specific heats for a turbine. Or,

Show that for a multistage expansion the overall isentropic efficiency is given by

$$\boldsymbol{\eta_o} = \frac{\mathbf{1 - P_r}^{-\left(\eta_p \frac{\gamma-1}{\gamma} K\right)}}{\mathbf{1 - P_r}^{-\left(\frac{\gamma-1}{\gamma} K\right)}}$$

Where K= number of stages, P_r= pressure ratio per stage, η_p= polytropic efficiency, γ= ratio of specific heats.

Answer: The figure 3.10 shows the T-s diagram expansion process in multistage turbine operating between the pressures p_1 and p_{K+1}. If there are K stages with the overall pressure ratio $\frac{p_1}{p_{K+1}}$ and having equal stage efficiency and stage pressure ratio.

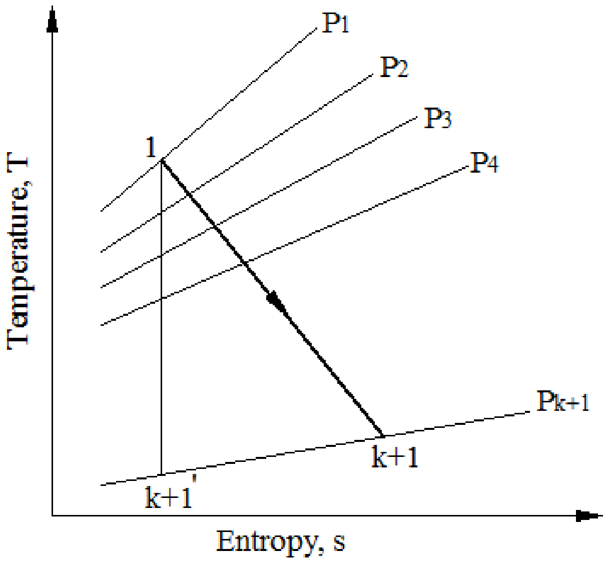

Fig. 3.10 Expansion process in multistage turbine

The overall efficiency of the multistage turbine is,

$$\eta_o = \frac{Total\ Actual\ Temperature\ Drop}{Total\ Isentropic\ Temperature\ Drop}$$

$$\eta_o = \frac{T_1 - T_{K+1}}{T_1 - T_{(K+1)'}} = \frac{T_1\left(1 - \frac{T_{K+1}}{T_1}\right)}{T_1\left(1 - \frac{T_{(K+1)'}}{T_1}\right)} = \frac{\left[1 - \left(\frac{p_{K+1}}{p_1}\right)^{\eta_p \frac{\gamma-1}{\gamma}}\right]}{\left[1 - \left(\frac{p_{(K+1)'}}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

Let, $p_{ro} = \frac{p_1}{p_{K+1}}$

$$\boldsymbol{\eta_o} = \frac{\mathbf{1} - \boldsymbol{P_{ro}}^{-\left(\eta_p \frac{\boldsymbol{\gamma}-\mathbf{1}}{\boldsymbol{\gamma}}\right)}}{\mathbf{1} - \boldsymbol{P_{ro}}^{-\left(\frac{\boldsymbol{\gamma}-\mathbf{1}}{\boldsymbol{\gamma}}\right)}}$$

The overall pressure ratio can be written as, $p_{ro} = p_r^K$

Then overall efficiency of multistage turbine is,

$$\boldsymbol{\eta_o} = \frac{\mathbf{1} - \boldsymbol{P_r}^{-\left(\eta_p \frac{\boldsymbol{\gamma}-\mathbf{1}}{\boldsymbol{\gamma}}\boldsymbol{K}\right)}}{\mathbf{1} - \boldsymbol{P_r}^{-\left(\frac{\boldsymbol{\gamma}-\mathbf{1}}{\boldsymbol{\gamma}}\boldsymbol{K}\right)}}$$

Question No 3.21: Derive an expression for an overall isentropic efficiency for finite number of stages of expansion in terms of pressure ratio, stage efficiency, number of stages and ratio of specific heats for a turbine. ***(VTU, Jul-07) Or,***

Show that for a finite number of stages for expansion the overall isentropic efficiency is given by

$$\boldsymbol{\eta_o} = \frac{\mathbf{1} - \left\{\mathbf{1} - \boldsymbol{\eta_{st}}\left[\mathbf{1} - \left(\frac{\mathbf{1}}{\boldsymbol{p_r}}\right)^{\frac{\boldsymbol{\gamma}-\mathbf{1}}{\boldsymbol{\gamma}}}\right]\right\}^{\boldsymbol{K}}}{\left[\mathbf{1} - \left(\frac{\mathbf{1}}{\boldsymbol{p_r}}\right)^{\boldsymbol{K}\frac{\boldsymbol{\gamma}-\mathbf{1}}{\boldsymbol{\gamma}}}\right]}$$

Where K= number of stages, P_r= pressure ratio per stage, η_{st}= stage efficiency, γ= ratio of specific heats. ***(VTU, Jun/Jul-09)***

Answer: If T_1 is the initial temperature at which the fluid enters the multistage turbine, K is the number of stages having equal pressure ratio p_r in each stage, then the actual temperature drop in each stage can be given as follows:

For first stage:

$$\Delta T_1 = (T_1 - T_2) = \eta_{st}(T_1 - T_{2'}) = T_1\eta_{st}\left(1 - \frac{T_{2'}}{T_1}\right) = T_1\eta_{st}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$\Delta T_1 = T_1\eta_{st}\left[1 - \left(\frac{1}{p_r}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

Let, $B = \eta_{st}\left[1 - \left(\frac{1}{p_r}\right)^{\frac{\gamma-1}{\gamma}}\right]$

$$\boldsymbol{\Delta T_1 = BT_1}$$

For second stage:

$$\Delta T_2 = (T_2 - T_3) = BT_2 = B(T_1 - BT_1)$$

$$\boldsymbol{\Delta T_2 = BT_1(1 - B)}$$

For third stage:

$$\Delta T_3 = (T_3 - T_4) = BT_3 = B[T_2 - BT_1(1 - B)]$$

$$\Delta T_3 = B[T_1 - BT_1 - BT_1(1-B)] = BT_1[(1-B) - B(1-B)]$$

$$\boldsymbol{\Delta T_3 = BT_1(1-B)^2}$$

Similarly for fourth stage:

$$\boldsymbol{\Delta T_4 = BT_1(1-B)^3}$$

And for Kth stage:

$$\boldsymbol{\Delta T_K = BT_1(1-B)^{K-1}}$$

Total temperature drop across the multistage turbine is:

$$\Delta T_o = \Delta T_1 + \Delta T_2 + \Delta T_3 + \Delta T_4 + \cdots + \Delta T_K$$

$$\Delta T_o = BT_1 + BT_1(1-B) + BT_1(1-B)^2 + BT_1(1-B)^3 + \cdots + BT_1(1-B)^{K-1}$$

$$\Delta T_o = BT_1[1 + (1-B) + (1-B)^2 + (1-B)^3 + \cdots + (1-B)^{K-1}]$$

Let, $$\boldsymbol{S = 1 + (1-B) + (1-B)^2 + (1-B)^3 + \cdots + (1-B)^{K-1}}$$

$$S = 1 + (1-B)[1 + (1-B) + (1-B)^2 + \cdots + (1-B)^{K-2}]$$

$$S = 1 + (1-B)[1 + (1-B) + (1-B)^2 + \cdots + (1-B)^{K-2} + (1-B)^{K-1} - (1-B)^{K-1}]$$

$$S = 1 + (1-B)[S - (1-B)^{K-1}] = 1 + S(1-B) - (1-B)^K$$

$$S = 1 + S - SB - (1-B)^K$$

$$\boldsymbol{SB = 1 - (1-B)^K}$$

But, $$\Delta T_o = SBT_1$$

$$\Delta T_o = T_1[1 - (1-B)^K]$$

$$\boldsymbol{\Delta T_o = T_1\left\{1 - \left\{1 - \eta_{st}\left[1 - \left(\frac{1}{p_r}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^K\right\}}$$

The overall efficiency of the multistage turbine is,

$$\eta_o = \frac{Total\ Actual\ Temperature\ Drop}{Total\ Isentropic\ Temperature\ Drop}$$

$$\eta_o = \frac{\Delta T_o}{T_1 - T_{(K+1)'}} = \frac{T_1\left\{1 - \left\{1 - \eta_{st}\left[1 - \left(\frac{1}{p_r}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^K\right\}}{T_1\left(1 - \frac{T_{(K+1)'}}{T_1}\right)} = \frac{1 - \left\{1 - \eta_{st}\left[1 - \left(\frac{1}{p_r}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^K}{\left[1 - \left(\frac{p_{(K+1)'}}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

But, $\frac{p_1}{p_{(K+1)'}} = p_{ro} = p_r^K$

$$\boldsymbol{\eta_o = \frac{1 - \left\{1 - \eta_{st}\left[1 - \left(\frac{1}{p_r}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^K}{\left[1 - \left(\frac{1}{p_r}\right)^{K\frac{\gamma-1}{\gamma}}\right]}}$$