Module 4 : HYDRAULIC TURBINES

4.1 Introduction:

The energy source which does not depend on thermal energy input to produce mechanical output is hydraulic energy. It may be either in the form of potential energy as we find in dams, reservoirs or in the form of kinetic energy in flowing water. Hydraulic turbines are the machines which convert the hydraulic energy in to mechanical energy.

4.2 Classification of Hydraulic Turbines:

Question No 4.1(a): Classify hydraulic turbines with examples. (VTU, Dec-06/Jan-04, May/Jun-10) **Answer:** Hydraulic turbines are classified based on the following important factors:

1. Based on the action of water on blades or the energy available at the turbine inlet, hydraulic turbines are classified as impulse and reaction turbines.

Impulse turbine: In this type of turbine the energy of the fluid entering the rotor is in the form of kinetic energy of jets. Example: Pelton turbine.

Reaction turbine: In this turbine the energy of the fluid entering the rotor is in the form of kinetic energy of jets and pressure energy of turbine. Example: Francis turbine and Kaplan turbine.

2. Based on the direction of fluid flow through the runner, turbine are classified as tangential flow turbine, radial flow turbine, axial flow turbine and mixed flow turbine.

Tangential flow turbine: In this type of turbine water strikes the runner along the tangential direction, these turbines are also known as peripheral flow turbines. Example: Pelton turbine.

Radial flow turbine: In this type of turbine water flow through the runner along the radial direction. Example: Francis turbine.

Axial flow turbine: In this type of turbine water flow through the runner along the axial direction. Example: Kaplan turbine.

Mixed flow turbine: In this type of turbine water enters the runner radially and leaves the runner axially. Example: Francis turbine.

3. Based on specific speed of runner, turbines are classified as low specific speed turbines, medium specific speed turbines and high specific speed turbines.

Low specific speed turbines: Such turbines have usually high head in the range of 200 m to 1400 m and these machines require low discharge. These turbines have specific speed in the range of 10 to 30 for single jet and 30 to 50 for double jet. Example: Pelton turbine

Medium specific speed turbines: Such turbines have usually medium head in the range of 50 m to 200 m and these machines require medium discharge. These turbines have specific speed in the range of 60 to 400. Example: Francis turbine.

High specific speed turbines: Such turbines have usually very low head in the range of 2.5 m to 50 m and these machines require high discharge. These turbines have specific speed in the range of 300 to 1000. Example: Kaplan turbine.

Question No 4.1(b): Mention the general characteristics features of Pelton, Francis and Kaplan turbines. (VTU, Dec-12)

Answer: Pelton wheel turbine is an impulse turbine. These turbines have usually high head in the range of 200 m to 1400 m and these machines require low discharge, hence the specific speed is low in the range of 10 to 30. In this type of turbine water strikes the runner along the tangential direction, these turbines are also known as peripheral (tangential) flow turbines.

Francis turbine is a reaction turbine. These turbines have usually medium head in the range of 50 m to 200 m and these machines require medium discharge, hence the specific speed is medium in the range of 60 to 400. In this type of turbine water enters radially and leaves axially or vice versa, these turbines are also known as mixed flow turbines.

Kaplan turbine is also a reaction turbine. These turbines have usually very low head in the range of 2.5 m to 50 m and these machines require high discharge, hence the specific speed is high in the range of 300 to 1000. In this type of turbine water flow through the runner along the axial direction, these turbines are also known as axial flow turbines.

4.3 Heads and Efficiencies of Hydraulic Turbines:

Question No 4.2: Define the following terms (i) Gross head (ii) Net head (iii) Volumetric efficiency (iv) Hydraulic efficiency (v) Mechanical efficiency (vi) Overall efficiency.

(VTU, Jul/Aug-04, Dec-04/Jan-08, Dec-12)

Answer: (i) Gross head (H_g) : It is the head of water available for doing useful work. It is the difference between the head race and tail race level when there is no flow. It is also known as static head.

(ii) Net head (H): It is the head available at the inlet of the turbine. It is obtained by considering all losses, like loss in kinetic energy of water due to friction, pipe bends and fittings. If h_f is the total loss, then net head is given by $H = H_g - h_f$.

(iii) Volumetric efficiency (η_v): It is the ratio of the quantity of water striking the runner of the turbine to the quantity of water supplied at the turbine inlet.

$$\eta_{v} = \frac{Q - \Delta Q}{Q}$$

Where ΔQ is the amount of water that slips directly to the tail race.

(iv) Hydraulic efficiency (η_H): It is the ratio of work done by the runner to the energy available at the inlet of the turbine.

Or,

$$\eta_{H} = \frac{U(V_{u1} \pm V_{u2})}{gH} = \frac{g(H - h_{L})}{gH} = \frac{H - h_{L}}{H}$$

Where *H* is net head and $h_L = (h_{Lr} + h_{Lc})$ is head loss in the runner and casing.

If leakage losses are considered then actual hydraulic efficiency is,

$$\eta_{H,act} = \frac{(Q - \Delta Q)U(V_{u1} \pm V_{u2})}{gQH} = \frac{g(Q - \Delta Q)(H - h_L)}{gQH} = \left(\frac{Q - \Delta Q}{Q}\right) \left(\frac{H - h_L}{H}\right)$$
$$\eta_{H,act} = \eta_v \eta_H$$

(v) Mechanical efficiency (η_m) : It is the ratio of shaft work output by the turbine to the work done by the runner.

$$\eta_m = \frac{w_{sft}}{U(V_{u1} \pm V_{u2})} = \frac{w_{sft}}{g(H - h_L)}$$

(vi) Overall efficiency (η_o): It is the ratio of shaft work output by the turbine to the energy available at the inlet of the turbine.

$$\eta_o = \frac{w_{sft}}{gH}$$
$$\eta_o = \eta_H \eta_v \eta_m = \eta_{H,act} \eta_m$$

4.4 Pelton Wheel Turbines:

Question No 4.3: With a neat sketch explain the working of a Pelton turbine. What is the reason for the provision of a splitter in a Pelton wheel bucket?

Answer: Pelton wheel turbine is an impulse turbine working under high head and low discharge. In this turbine water carried from the penstock enters the nozzle emerging out in the form of high velocity water jet. The potential energy of water in the penstock is converted in to kinetic energy by nozzle which is used to run the turbine runner.



Fig. 4.1 Pelton wheel

Figure 4.1 shows main components of Pelton wheel, water from a high head source or reservoir like dam enters the turbine runner through large diameter pipes known as penstocks. Each penstock pipe is branched in such a way that it can accommodate a nozzle at the end. Water flows through these nozzles as a high speed jet striking the vanes or buckets attached to the periphery of the runner. The runner rotates and supplies mechanical work to the shaft. Water is discharged at the tail race after doing work on the runner.

In a Pelton wheel the jet of water strikes the bucket and gets deflected by the splitter into two parts, this negates the axial thrust on the shaft.

4.5 Force, Power and Efficiency of a Pelton Wheel:

Question No 4.4: Derive an expression for force, power and efficiency of a Pelton wheel with the help of velocity triangles. (VTU, Jun/Jul-14) Or, Obtain an expression for the workdone per second by water on therunner a Pelton wheel and hydraulic efficiency. (VTU, Jun/Jul-14)

Answer: The inlet and outlet velocity triangles for Pelton wheel turbine is as shown in figure 4.2. As the runner diameter is same at inlet and outlet, tangential velocity of wheel remains the same. In practical case the relative at the outlet is slightly less than that at inlet due to frictional loss over the inner surface of the bucket. Some velocity is also lost due to the jet striking over the splitter. Hence $V_{r2} = C_b V_{r1}$, where C_b is bucket velocity coefficient.



Fig. 4.2 Velocity triangles for Pelton wheel turbine

From inlet velocity triangle,

$$V_1 = V_{u1}$$
$$V_{r1} = V_1 - U$$

From outlet velocity triangle,

$$V_{r2} = C_b V_{r1}$$

$$V_{u2} = x_2 - U = V_{r2} cos\beta_2 - U$$

$$V_{u2} = C_b V_{r1} cos\beta_2 - U = C_b (V_1 - U) cos\beta_2 - U$$
Then, $(V_{u1} + V_{u2}) = V_1 + C_b (V_1 - U) cos\beta_2 - U = (V_1 - U) + C_b (V_1 - U) cos\beta_2$
 $(V_{u1} + V_{u2}) = (V_1 - U) [\mathbf{1} + C_b cos\beta_2]$

Force exerted by the jet on the wheel is,

$$F = \dot{m}(V_{u1} + V_{u2})$$
$$F = \rho Q(V_1 - U)[1 + C_b \cos\beta_2]$$

Power output by the wheel is,

$$P = FU$$
$$P = \rho QU(V_1 - U)[1 + C_b cos\beta_2]$$

Hydraulic efficiency of the wheel is,

$$\eta_{H} = \frac{U(V_{u1} + V_{u2})}{gH} = \frac{U(V_{u1} + V_{u2})}{\frac{V_{1}^{2}}{2}}$$
$$\eta_{H} = \frac{2U(V_{1} - U)[1 + C_{b}cos\beta_{2}]}{V_{1}^{2}}$$

Or,

$$\eta_H = \frac{2(UV_1 - U^2)}{V_1^2} [1 + C_b \cos\beta_2]$$
$$\eta_H = 2(\varphi - \varphi^2) [1 + C_b \cos\beta_2]$$

Where $\varphi = \frac{U}{V_1}$ speed ratio.

Question No 4.5: Derive an expression for force, power and efficiency of a Pelton wheel assuming no frictional losses with the help of velocity triangles. (VTU, Dec-04/Jan-08)

Answer: By assuming no frictional losses over the blades the relative velocity of jet is remains constant, i.e., $V_{r2} = V_{r1}$

From inlet velocity triangle,

$$V_1 = V_{u1}$$
$$V_{r1} = V_1 - U$$

From outlet velocity triangle,

$$V_{r2} = V_{r1}$$

$$V_{u2} = x_2 - U = V_{r2}\cos\beta_2 - U$$

$$V_{u2} = V_{r1}\cos\beta_2 - U = (V_1 - U)\cos\beta_2 - U$$
Then, $(V_{u1} + V_{u2}) = V_1 + (V_1 - U)\cos\beta_2 - U = (V_1 - U) + (V_1 - U)\cos\beta_2$

$$(V_{u1} + V_{u2}) = (V_1 - U)[1 + \cos\beta_2]$$

Force exerted by the jet on the wheel is,

$$F = \dot{m}(V_{u1} + V_{u2})$$
$$F = \rho Q(V_1 - U)[1 + \cos\beta_2]$$

Power output by the wheel is,

$$P = FU$$
$$P = \rho Q U (V_1 - U) [1 + \cos\beta_2]$$

Hydraulic efficiency of the wheel is,

$$\eta_{H} = \frac{U(V_{u1} + V_{u2})}{gH} = \frac{U(V_{u1} + V_{u2})}{\frac{V_{1}^{2}}{2}}$$
$$\eta_{H} = \frac{2U(V_{1} - U)[1 + \cos\beta_{2}]}{V_{1}^{2}}$$

Or,

$$\eta_H = \frac{2(UV_1 - U^2)}{V_1^2} [1 + \cos\beta_2]$$
$$\eta_H = 2(\varphi - \varphi^2) [1 + \cos\beta_2]$$

Where $\varphi = \frac{U}{V_1}$ speed ratio

Question No 4.6: Show that for maximum utilization (maximum efficiency), the speed of the wheel is equal to half the speed of jet. (VTU, Dec-11)

Answer: The hydraulic efficiency of the Pelton wheel is given by,

$$\eta_H = 2(\varphi - \varphi^2)[1 + C_b cos\beta_2]$$

For given machine C_b and β_2 are constant, so η_H varies only with φ . The slope for the maximum hydraulic efficiency is,

$$\frac{d\eta_H}{d\varphi} = 0$$

$$\frac{d}{d\varphi} \{ 2(\varphi - \varphi^2) [1 + C_b \cos\beta_2] \} = 0$$

$$2(1 - 2\varphi) [1 + C_b \cos\beta_2] = 0$$

$$\varphi_{opt} = \frac{1}{2}$$

But $\varphi = \frac{U}{V_1}$ speed ratio

$$\frac{U}{V_1} = \frac{1}{2}$$
$$U = \frac{V_1}{2}$$

For maximum utilization (maximum hydraulic efficiency), the speed of the wheel is equal to half the speed of jet.

Question No 4.4: Draw the inlet and outlet velocity triangles for a Pelton wheel. Derive an expression for the maximum hydraulic efficiency of a Pelton wheel in terms of bucket velocity co-efficient and discharge blade angle. (VTU, Jun/Jul-11, Dec-14/Jan-15) Or,

Draw the inlet and outlet velocity triangles for a Pelton wheel. Show that the maximum hydraulic efficiency of a Pelton wheel turbine is given by, $\eta_{H,max} = \frac{1+C_b \cos \beta_2}{2}$ where C_b is bucket velocity coefficient and β_2 is exit blade angle. (VTU, Jun-12)Or,

Draw the inlet and outlet velocity triangles for a Pelton wheel. Show that the maximum hydraulic efficiency of a Pelton wheel turbine is given by, $\eta_{H,max} = \frac{1+\cos\beta_2}{2}$. Assume that relative velocity remains constant. (VTU, Dec-13)

Answer: The hydraulic efficiency of the Pelton wheel is given by,

$$\eta_{H} = 2(\varphi-\varphi^{2})[1+C_{b}cos\beta_{2}]$$

When $\varphi_{opt} = \frac{1}{2}$, the hydraulic efficiency of Pelton wheel will be maximum.

$$\eta_{H,max} = 2\left[\frac{1}{2} - \left(\frac{1}{2}\right)^2\right] [1 + C_b \cos\beta_2] = 2\left[\frac{1}{2} - \frac{1}{4}\right] [1 + C_b \cos\beta_2]$$
$$\eta_{H,max} = 2\left[\frac{1}{4}\right] [1 + C_b \cos\beta_2]$$
$$\eta_{H,max} = \frac{[1 + C_b \cos\beta_2]}{2}$$

If relative velocity remains constant (i.e. no frictional losses over the bucket), $C_b=1$

$$\eta_{H,max} = \frac{[1 + \cos\beta_2]}{2}$$

4.6 Design Parameters of Pelton Wheel:



1. Velocity of jet from the nozzle $V_1 = C_v \sqrt{2gH}$

Where C_{ν} is coefficient of velocity for nozzle ranges from 0.94 to 0.99

2. Tangential velocity of buckets $U = \varphi \sqrt{2gH}$

Where φ is speed ratio varies ranges from 0.43 to 0.48

3. Least diameter of the jet (d):

Fotal discharge,
$$Q = n \frac{\pi}{4} d^2 V_1$$

Where 'n' is number of jets or nozzles

4. Mean diameter or pitch diameter of buckets (D):

Tangential velocity, $U = \frac{\pi DN}{60}$

5. Jet ratio: It is the ratio of mean diameter of the runner to the minimum diameter of the jet.

$$J_r = \frac{D}{d}$$
 ranges between 6 to 35

6. Minimum number of buckets:

$$Z = \frac{J_r}{2} + 15$$

4. Angle of deflection usually ranges from 165° to 140°, hence vane angle at outlet

$$\beta_2 = (180^o - Angle of deflection)$$

8. Head loss due to friction in penstock:

$$h_f = \frac{4fLV_p^2}{2gD_p}$$

Where D_p is diameter of penstock, L is length of the penstock, V_p is fluid velocity through penstock and f is friction coefficient for penstock.

9. Bucket dimensions:

Width of the bucket B = 2.8d to 4d

Length of the bucket L = 2.4d to 2.8d

Depth of bucket T = 0.6d to 0.95d

10. Number of jets: Theoretically six jets or nozzles can be used with one Pelton wheel. However, practically not more than two jets per runner are used for a vertical runner and not more than four jets per runner are used for a horizontal runner.

4.4 Francis Turbine:

Question No 4.8: With a neat sketch explain the working of Francis turbine. Draw the velocity triangles of Francis turbine. (VTU, Jul/Aug-05, Dec-06/Jan-04, Dec-09/Jan-10)

Answer: Francis turbine is a reaction type turbine. Earlier Francis turbines were purely radial flow type but modern Francis turbines are mixed flow type in which water enters the runner radially and leaves axially at the centre. Figure 4.3 shows the main components of Francis turbines.

(i) Scroll (spiral) casing: It is also known as spiral casing. The water from penstock enters the scroll casing which completely surrounds the runner. The main function of spiral casing is to provide a uniform distribution of water around the runner and hence to provide constant velocity. In order to

provide constant velocity, the cross sectional area of the casing gradually decreases as the water reaching the runner.



Fig. 4.3 Francis turbine

(ii) Guide vanes (blades): After the scroll ring water passes over to the series of guide vanes or fixed vanes, which surrounds completely around the turbine runner. Guide vanes regulate the quantity of water entering the runner and direct the water on to the runner.

(iii) **Runner** (**Rotor**): The runner of turbine is consists of series of curved blades evenly arranged around the circumference. The vanes or blades are so shaped that water enters the runner radially at outer periphery and leaves it axially at its centre. The change in direction of flow from radial to axial when passes over the runner causes the appreciable change in circumferential force which is responsible to develop power.

(iv) **Draft tube:** The water from the runner flows to the tail race through the draft tube. A draft tube is a pipe or passage of gradually increasing area which connect the exit of the runner to the tail race. The exit end of the draft tube is always submerged below the level of water in the tail race and must be airtight.

Velocity triangles for Francis turbine: In the slow, medium and fast runners of a Francis turbine the inlet blade angle (β_1) is less than, equal to and greater than 90° respectively. The whirl component of velocity at the outlet is zero (i.e., V_{u2}=0).



Fig. 4.4 Velocity triangle for Francis turbine

4.8 Design Parameters of Francis Turbine:

1. Flow velocity or radial velocity at the turbine inlet is given by, $V_{f1} = \psi \sqrt{2gH}$

Where ψ is flow ratio ranging from 0.15 to 0.30

2. Tangential velocity of the runner or wheel at the inlet is given by, $U_1 = \varphi \sqrt{2gH}$

Where φ is speed ratio ranging from 0.6 to 0.9

3. Diameter of runner:

Inlet diameter (D₁) of the runner, $U_1 = \pi D_1 N$

Outlet diameter (D₂) of the runner, $U_2 = \pi D_2 N$

Where U_1 and U_2 are inlet and outlet runner velocity respectively

4. Discharge at the outlet is radial then the guide blade angle at the outlet is 90° .

i.e., $\alpha_2 = 90^o and V_{u2} = 0$

5. Head at the turbine inlet assuming no energy loss is given by, $gH = (U_1V_{u1} \pm U_2V_{u2}) + \frac{V_2^2}{2}$

$$H = \frac{1}{g} \left[(U_1 V_{u1} \pm U_2 V_{u2}) + \frac{V_2^2}{2} \right]$$

6. Discharge through the turbine is given by, $Q = A_f V_f = \pi D_1 B_1 V_{f1} = \pi D_2 B_2 V_{f2}$

Where A_f is area of flow through the runner, D is diameter of the runner, B is width of the runner and V_f is flow velocity.

If 'n' is the number of vanes in the runner and 't' is the thickness of the vane, then

$$Q = (\pi D_1 - nt_1)B_1V_{f1} = (\pi D_2 - nt_2)B_2V_{f2}$$

Normally it is assumed that, $D_1 = 2D_2$, $V_{f1} = V_{f2}$, and $B_2 = 2B_1$

4. Ratio of width to diameter is given by, $r = \frac{B_1}{D_1}$ ranging from 0.10 to 0.38.

4.9 Kaplan Turbine:

Question No 4.9: Explain the functioning of a Kaplan turbine with the help of a sectional arrangement diagram. Draw the velocity triangles of Kaplan turbine. (VTU, Jul-04, Jun/Jul-08)

Answer: The Kaplan turbine is an axial flow reaction turbine in which the flow is parallel to the axis of the shaft as shown in figure 4.5. In which water enters from penstock into the spiral casing. The guide vanes direct water towards the runner vanes without shock or formation of eddies. Between the guide vanes and the runner, the fluid gets deflected by 90° so that flow is parallel to the axis of rotation of the runner which is known as axial flow. The guide vanes impart whirl component to flow and runner vanes nullify this effect making flow purely axial. As compared to Francis turbine runner blades (16 to 24 numbers) Kaplan turbine uses only 3 to 8 blades. Due to this, the contact surface with water is less which reduces frictional resistance and losses.



Fig. 4.5 Kaplan turbine

Velocity triangles for Kaplan turbine: At the outlet, the discharge is always axial with no whirl velocity component (i.e., $V_{u2}=0$). The inlet and outlet velocity triangles for Kaplan turbine are as shown in figure 4.6.



Fig. 4.6 Velocity triangles for Kaplan turbine

4.10 Design Parameters of Kaplan Turbine:

1. Flow velocity or radial velocity at the turbine inlet is given by, $V_{f1} = \psi \sqrt{2gH}$

Where ψ is flow ratio ranging from 0.35 to 0.45

- 2. Flow velocity is remains constant throughout the runner, $V_{f1} = V_{f2} = V_f$
- 3. Discharge through the runner is given by,

$$Q = \frac{\pi}{4} (D^2 - d^2) V_f$$

Where D is tip diameter or outer diameter of the runner and d is hub diameter or boss diameter of the runner.

4. Discharge at the outlet is axial then the guide blade angle at the outlet is 90°.

i.e.,
$$\alpha_2 = 90^o and V_{u2} = 0$$

5. Head at the turbine inlet assuming no energy loss is given by,

$$H = \frac{1}{g} \left[U(V_{u1} \pm V_{u2}) + \frac{V_2^2}{2} \right]$$

4.11 Draft Tubes:

Question No 4.10 (a): Write a short note on draft tubes in a reaction hydraulic turbines. (VTU, Dec-14/Jan-15)

Water, after passing through the runner is discharged through a gradually expanding tube called draft tube. The free end of the draft tube is submerged deep into the water. Thus the entire water passage from the head race to tail race is completely closed and hence doesn't come in contact with atmospheric air. It is a welded steel plate pipe or a concrete tunnel with gradually increasing cross sectional area at the outlet.

4.11.1 Functions of Draft Tube:

Question No 4.10 (b): Explain the functions of a draft tube in a reaction hydraulic turbine.

(VTU, Jun/Jul-11, Dec-11, Dec-12)

Answer: The functions of a draft tube are as follows,

- 1. A reaction turbine is required to be installed above the tail race level for easy of maintenance work, hence some head is lost. The draft tube recovers this head by reducing the pressure head at the outlet to below the atmospheric level. It increases the working head of the turbine by an amount equal to the height of the runner outlet above the tail race. This creates a negative head or suction head.
- 2. Exit kinetic energy of water is a necessary loss in the case of turbine. A draft tube recovers part of this exit kinetic energy.
- 3. The turbine can be installed at the tail race level, above the tail race level or below the tail race level.

4.11.2 Types of Draft Tube:

Question No 4.11: Briefly explain with neat sketches different types of draft tube used in reaction hydraulic turbines. (VTU, Jun/Jul-09)

Answer: The important types of draft tubes are (i) Conical (straight divergent tube) type, (ii) Moddy's bell mouthed tube type, (iii) Simple elbow type, and (iv) Elbow tube type having square outlet and circular inlet.



Fig. 4.4 Types of draft tube

The first form is the straight conical type stretching from the turbine to the tail-race. The second type is also a straight draft tube except that is bell-shaped. This type of draft tube has an advantage that it can allow flow with whirl component to occur with very small losses at the turbine exit.

The third form is the simple elbow tube, is used when the turbine must be located very close to or below the tail-race level. However, the efficiency of simple elbow tube is usually not as great as that of the first two types. The fourth form of draft tube is similar to the third one except that the exit shape is square or rectangular instead of cylindrical as in the simple elbow tube.

In these types, the conical type is most efficient and commonly used. For the straight divergent type conical draft tube, the centre cone angle should not exceed 8°. If this angle exceeds 8°, the water flowing through the draft tube will not remain in contact with its inner surface and hence eddies are formed and the efficiency will be reduced. Draft tube efficiencies range generally from 0.4 to 0.9 for the first two types while they are 0.6 to 0.85 for the elbow tube types.

4.11.3 Efficiency of Draft Tube:

Question No 4.12: Show that the efficiency of draft tube is given by $\eta_d = \frac{V_2^2 - V_3^2 - 2gh_f}{V_2^2}$ where V_2 is absolute velocity of water at rotor exit, V_3 is absolute velocity of water at draft tube exit, h_f is loss of head due to friction. (VTU, Dec-08/Jan-09)

Answer: Efficiency of the draft tube is defined as the ratio of actual conversion of kinetic head into pressure head to the kinetic head available at the inlet of the draft tube.

Consider V_2 is absolute velocity of water at rotor exit, V_3 is absolute velocity of water at draft tube exit and h_f is loss of head due to friction.



Mathematically,

$$\eta_{d} = \frac{PH_{obt}}{KH_{avail}}$$
$$\eta_{d} = \frac{\frac{(V_{2}^{2} - V_{3}^{2})}{2g} - h_{f}}{\frac{V_{2}^{2}}{2g}}$$
$$\eta_{d} = \frac{V_{2}^{2} - V_{3}^{2} - 2gh_{f}}{V_{2}^{2}}$$