Module 1: Gas Power Cycles

Introduction

An important application of thermodynamics is the analysis of power cycles through which the energy absorbed as heat can be continuously converted into mechanical work. A thermodynamic analysis of the heat engine cycles provides valuable information regarding the design of new cycles or for improving the existing cycles.

The purpose of a thermodynamic cycle is either to produce power, or to produce refrigeration/pumping of heat. Therefore, the cycles are broadly classified as follows:

1. Heat engine or power cycles.

2. Refrigeration/heat pump cycles.

Any thermodynamic cycle is essentially a closed cycle in which, the working substance undergoes a series of processes and is always brought back to the initial state. However, some of the power cycles operate on open cycle. It means that the working substance is taken into the unit from the atmosphere at one end and is discharged into the atmosphere after undergoing a series of processes at the other end.

Analysis of Cycles

In air standard analysis, air is considered as the working medium. The analysis is carried out with the following assumptions.

Assumptions

1. The working substance consists of a fixed mass of air and behaves as a perfect gas. The closed system is considered which under goes a cycle process. Therefore, there are no intake or exhaust process.

2. The combustion process is replaced by an equivalent heat addition process form an external source. Thus there is no change in the chemical equilibrium of the working fluid and also composition.

3. There is no exhaust process; this is replaced by an equivalent heat rejection process.

4. Compression and expansion processes in the cycle are considered as reversible adiabatic process.

5. The specific heats C_p and C_v of air remains constant and does not vary with temperature.

Carnot Cycle

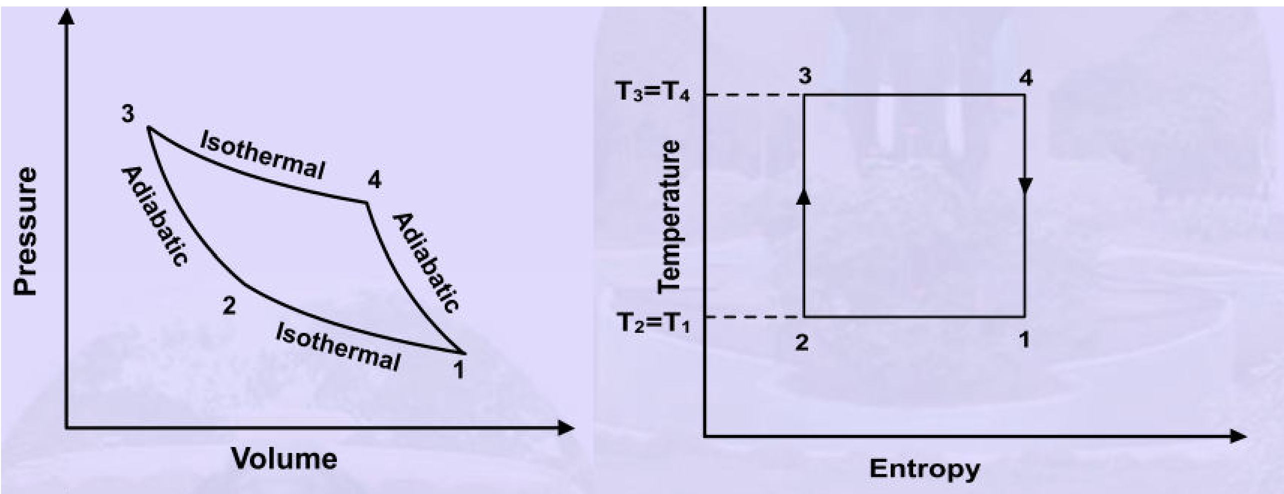


Figure 1: P-V and T-S diagram of Carnot Cycle

The T-s and p-v diagrams for a Carnot power cycle are shown in Fig.1. The cycle consists of two reversible adiabatic and two reversible isothermal processes, The working of the cycle is as follows:

Process 1-2: Reversible isothermal heat rejection of the working substance from state 1 to state 2.

Process 2-3: Isentropic compression of the working substance from state 2 to state 3. During this process work is done on the working substance by the surroundings.

Process 3-4: Reversible isothermal heat addition of the working substance from state 3 to state 4.

Process 4-1: Isentropic expansion of the working substance so that it comes back to its initial state. During this process work is done by the working substance on the surroundings.

Expression for Thermal Efficiency

From the thermodynamics, for unit mass of air
Heat supplied from point 3 to 4

$$p_3v_3 \ln \frac{v_4}{v_3} \tag{1}$$

Heat rejected from point 1 to 2

$$p_1v_1 \ln \frac{v_2}{v_1} \tag{2}$$

From characteristic gas equation we can write $p_3v_3 = RT_{max}$ and $p_1v_1 = RT_{min}$
Substituting in the above equations, we get

$$RT_{min} \ln \frac{v_2}{v_1}$$
$$RT_{max} \ln \frac{v_4}{v_3}$$

So the Work-done during the cycle is given by W = Heat supplied - Heat rejected

$$W = RT_{max} \ln \frac{v_4}{v_3} - RT_{min} \ln \frac{v_1}{v_2} \tag{3}$$

As the compression ratio

$$r_c = \frac{v_4}{v_3} = \frac{v_1}{v_2}$$

Substituting the above in the Work-done equation, we get

$$W = RT_{max} \ln r - RT_{min} \ln r$$

Carnot Thermal Efficiency is given by $= \dfrac{\text{Workdone}}{\text{Heat Supplied}}$
So, Mathematically it can written as,

$$\eta_{th} = \frac{RT_{max} \ln r - RT_{min} \ln r}{RT_{max} \ln r} \rightarrow \frac{T_{max} - T_{min}}{T_{max}} \tag{4}$$

Carnot cycle can be executed in a closed system (a piston and cylinder device or in a steady flow device. It can be seen that the thermal efficiency depends only on two temperatures T_{max} and T_{min} and is independent of working substance. The Carnot cycle is the most efficient cycle that can be executed between a heat source at temperature T_{max} and a heat sink at temperature T_{min}. But reversible isothermal heat transfer process is difficult to achieve in practice, because, it would require very large heat exchangers and it would take a very long time (a power cycle in a typical engine has to be completed in a fraction of a second). Therefore it is not practical to build an engine that would operate on a cycle that closely approximates a Carnot cycle.

The real value of the Carnot cycle comes from the fact that it is used as a standard against which the actual or other ideal power cycles are compared. It can be seen from the equation that the thermal efficiency the

Carnot power cycle increases with increase in T_{max} and with decrease in T_{min}. Hence in actual or other ideal cycles attempts are made in increasing the average temperature at which heat is supplied or by decreasing the average temperature at which heat is rejected. It should also be noted that the source and sink temperatures that can be used in practice have their limitations. The highest temperature in the cycle is limited by the maximum temperature the components of the engine can withstand and the lowest temperature is limited by the temperature of the cooling medium used in the cycle such as the atmospheric air, ocean, lake or a river.

Air Standard Otto Cycle

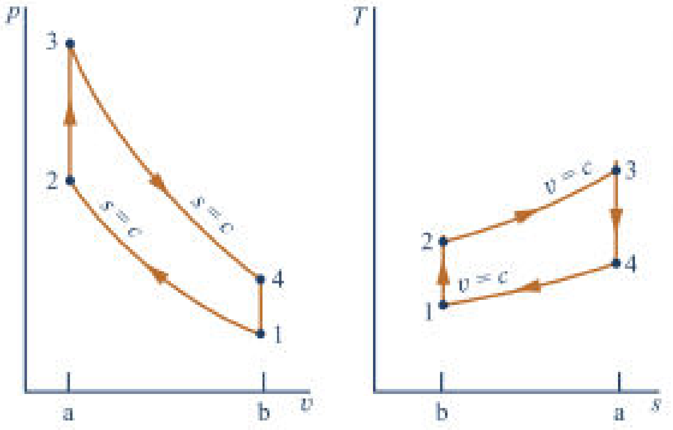


Figure 2: P-V and T-S diagram of Otto Cycle

Otto cycle is the ideal cycle for spark ignition engines. The cycle is named after Nikolaus A Otto, a German who built a four – stroke engine in 1876 in Germany using the cycle proposed by Frenchman Beau de Rochas in 1862.The p–V and T–s diagrams for an Otto cycle are shown in Fig.2. The cycle consists of the following processes:

Process 1-2: Isentropic compression of air from state 1 to state 2. During this process. Work is done on air by the surroundings.

Process 2-3: Constant volume of heating of air from state 2 till the maximum permissible temperature is reached.

Process 3–4: Isentropic expansion of air from state 3 to state 4. During this process work is done by air on the surroundings.

Process 4–1: Constant volume cooling of air till the air comes back to its original state.

Expression for Thermal Efficiency and Work output

No heat is added during the isentropic compression processes 1-2 and 3-4.

Heat added during the constant volume heating process 2-3 is given by,

$$Q_H = c_v * (T_3 - T_2) \tag{5}$$

Heat rejected during the constant volume heating process 4-1 is given by,

$$Q_L = c_v * (T_1 - T_4) \tag{6}$$

Work done during the cycle is given by,

$$W = Q_H - Q_L = c_v(T_3 - T_2) - c_v(T_4 - T_1)$$

Thermal Efficiency is given by, $\eta_{th} = \frac{W}{Q_H}$

$$\eta_{th} = \frac{c_v(T_3 - T_2) - c_v(T_4 - T_1)}{c_v(T_3 - T_2)} \tag{7}$$

Further simplified as,

$$\eta_{th} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$\eta_{th} = 1 - \frac{1}{\frac{(T_3 - T_2)}{(T_4 - T_1)}} \tag{8}$$

The equation 8, gives the expression for thermal efficiency of the Otto cycle in terms of the temperatures at the salient points of the cycles. It is possible to express the net work output and thermal efficiency of the Otto cycle in terms of compression ratio. So, for the isentropic processes 1-2 and 3-4, we can write,

$$\frac{T_3}{T_4} = (\frac{v_4}{v_3})^{(\gamma-1)} \quad \text{and} \quad \frac{T_2}{T_1} = (\frac{v_1}{v_2})^{(\gamma-1)}$$

But $v_2 = v_3$ and $v_1 = v_4$ Therefore we can write,

$$\frac{(T_3 - T_2)}{(T_4 - T_1)} = \frac{(T_4(\frac{v_1}{v_2})^{(\gamma-1)} - T_1(\frac{v_1}{v_2})^{(\gamma-1)})}{(T_4 - T_1)} = \frac{(T_4 - T_1)}{(T_4 - T_1)}(\frac{v_1}{v_2})^{(\gamma-1)}$$

$$\frac{(T_3 - T_2)}{(T_4 - T_1)} = (\frac{v_1}{v_2})^{(\gamma-1)}$$

So the equation 8 for thermal efficiency can be written as

$$\eta_{th} = 1 - \frac{1}{(\frac{v_1}{v_2})^{(\gamma-1)}} = 1 - \frac{1}{r_c^{(\gamma-1)}} \tag{9}$$

From the above equation, it can be observed that the efficiency of the Otto cycle is mainly the function of compression ratio for the given ratio of C_p and C_v . If we plot the variations of the thermal efficiency with increase in compression ratio for different gases, the curves are obtained as shown in Fig.3, beyond certain values of compression ratios, the increase in the thermal efficiency is very small, because the curve tends to be asymptotic. However, practically the compression ratio of petrol engines is restricted to maximum of 9 or 10 due to the phenomenon of knocking at high compression ratios.

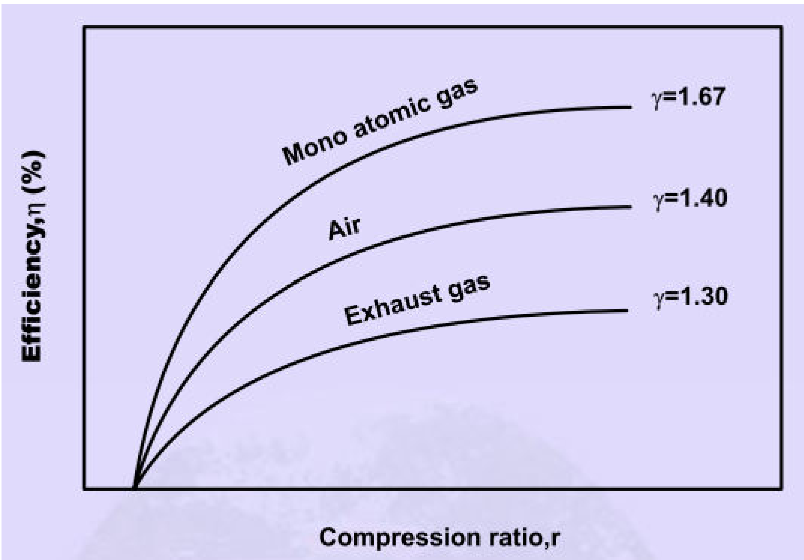


Figure 3: Variation of thermal efficiency with compression ratio

Expression for Mean effective pressure

Generally it is defined as the ratio of new workdone to the displacement volume of the pistion. Let us consider unit kg of working substance, Work done is given by,

$$W = c_v(T_3 - T_2) - c_v(T_4 - T_1) \tag{10}$$

Displacement volume is given by,

$$v_s = (v_1 - v_2)$$
$$v_s = v_1(1 - \frac{1}{r}), \quad \text{But}, v_1 = \frac{RT_1}{p_1}$$
$$v_s = \frac{RT_1}{p_1}(1 - \frac{1}{r})$$
$$v_s = \frac{c_v(\gamma - 1)T_1}{p_1}(\frac{r-1}{r}) \quad \text{Since}, \quad R = c_v(\gamma - 1) \tag{11}$$

Mean effective pressure is given by;

$$p_m = \frac{(T_3 - T_2) - (T_4 - T_1)}{\frac{c_v(\gamma-1)T_1}{p_1}(\frac{r-1}{r})}$$
$$p_m = \frac{1}{c_v(\gamma - 1)} \frac{p_1}{T_1} \frac{r}{r-1}[(T_3 - T_2) - (T_4 - T_1)] \tag{12}$$

As $T_2 = T_1(r)^{\gamma-1)}$. Let Pressure ratio is $r_p = \frac{p_3}{p_2} = \frac{T_3}{T_2}$ and $T_3 = \frac{p_3}{p_2}T_2 = r_pT_2 = r_pT_1(r)^{(\gamma-1)}$

$$\frac{T_4}{T_3} = (\frac{v_3}{v_4})^{(\gamma-1)} = (\frac{1}{r})^{(\gamma-1)}$$
$$T_4 = r_pT_1(r)^{(\gamma-1)} * (\frac{1}{r})^{(\gamma-1)}$$
$$T_4 = r_pT_1 \tag{13}$$

So the mean effective pressure is given by

$$p_m = \frac{p_1 r}{c_v(r-1)(\gamma-1)}\left[(r_p r^{\gamma-1} - r^{\gamma-1}) - (r_p - 1)\right]$$
$$p_m = p_1 r\left[\frac{(r^{\gamma-1}(r_p - 1) - (r_p - 1)}{c_v(r-1)(\gamma-1)}\right]$$
$$p_m = p_1 r\left[\frac{(r^{\gamma-1} - 1)(r_p - 1)}{c_v(r-1)(\gamma-1)}\right] \tag{14}$$

The above expression is the equation for Mean effective pressure in terms of compression ration r, pressure ratio r_p and specific heat ratio γ

Air Standard Diesel Cycle

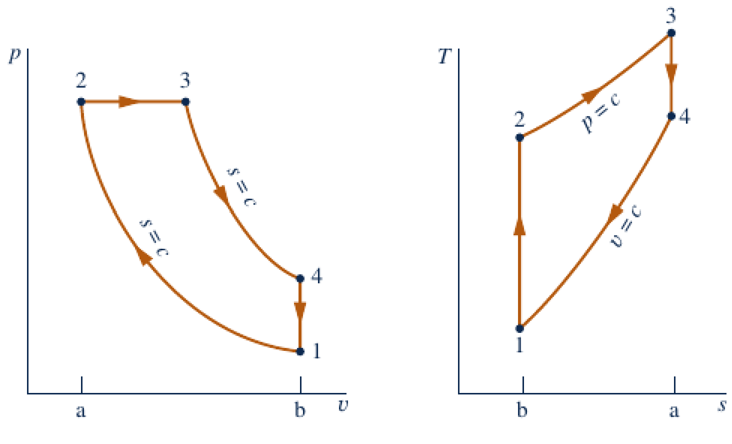


Figure 4: P-V and T-S diagram of Diesel Cycle

The diesel cycle is the ideal cycle for compression ignition engines (CI engines). CI engine was first proposed by Rudolph Diesel in 1890. The diesel engine works on the principle of compression ignition. In such an engine, only air is compressed and at the end of the compression process, the fuel is sprayed into the engine cylinder containing high pressure air, so that the fuel ignites spontaneously and combustion occurs. Since only air is compressed during the compression stroke, the possibility of auto ignition is completely eliminated in diesel engines. Hence diesel engines can be designed to operate at much higher compression ratios (between 12 and 24). Also another benefit of not having to deal with auto ignition is that fuels used in this engine can be less refined (thus less expensive).

Air standard diesel cycle is a idealized cycle for diesel engines. It is as shown on P-v and T-s diagrams. The processes in the cycle are as follows:

Process 1-2: Reversible adiabatic Compression.During this process the work is done on the air by the surroundings.

Process 2-3: Constant pressure heat addition till the maximum permissible temperature is reached.

Process 3-5: Reversible adiabatic Compression. During this process work is done by the air on the surroundings.

Process 4-1: Constant volume heat rejection so that air comes back to its original state to complete the cycle.

Expression for Work output and Thermal Efficiency

Consider unit kg of working fluid. Since the compression and expansion processes are reversible adiabatic processes, we can write,

$$\begin{aligned} \text{Heat supplied} \quad Q_H &= c_p(T_3 - T_2) \\ \text{Heat Rejected} \quad Q_L &= c_v(T_1 - T_4) \end{aligned}$$

Work done during the cycle is given by,

$$\begin{aligned} W &= Q_H - Q_L \\ W &= c_p(T_3 - T_2) - c_v(T_4 - T_1) \end{aligned}$$

Now, we can write Thermal efficiency as,

$$\begin{aligned} \eta_{th} &= \frac{W}{Q_H} \\ \eta_{th} &= \frac{c_p(T_3 - T_2) - c_v(T_4 - T_1)}{c_p(T_3 - T_2)} \\ \eta_{th} &= 1 - \frac{1}{\gamma}(\frac{T_4 - T_1}{T_3 - T_2}) \end{aligned} \tag{15}$$

The above expression gives the equation for thermal efficiency in terms of temperatures and ratio of specific heat γ. We can get the expression for thermal efficiency in terms of compression ratio r_c, cut-off ratio ρ. Now expressing all the Temperatures in terms of T_1, γ and r_c, we get.,

$$\begin{aligned} & T_2 = T_1 r_c^{\gamma-1} \\ & \text{such that,} r = \frac{v_1}{v_2} \text{also cut-off ratio} \quad \rho = \frac{v_3}{v_2} = \frac{T_3}{T_2} \\ & T_3 = \rho T_2 = \rho T_1 r_c^{\gamma-1} \\ & \frac{T_4}{T_3} = (\frac{v_3}{v_4})^{\gamma-1} \\ & T_4 = \rho T_1 r_c^{\gamma-1}(\frac{v_3 v_2}{v_2 v_4})^{\gamma-1} \\ & T_4 = \rho T_1 r_c^{\gamma-1}(\frac{\rho}{r_c})^{\gamma-1} = \rho^{\gamma} T_1 \end{aligned}$$

Substituting the above temperature expressions in equation 23, we get

$$\eta_{th} = 1 - \frac{1}{\gamma}\frac{\rho^{\gamma}T_1 - T_1}{\rho r_c^{\gamma-1}T_1 - r_c^{\gamma-1}T_1}$$
$$\eta_{th} = 1 - \frac{\rho^{\gamma} - 1}{\gamma r_c^{\gamma-1}(\rho - 1)} \tag{16}$$

From the above equation, it is observed that the thermal efficiency of the diesel engine can be increased by increasing the compression ratio r_c, by decreasing the cut-off ratio ρ and by using a large value of γ. Since the quantity, $\frac{\rho^{\gamma}-1}{\gamma(\rho-1)}$ is always greater than unity, the efficiency of the diesel cycle is always lower than that of an Otto cycle having the same compression ratio. However, practical Diesel engines uses higher compression ratios compared to petrol engines.

Expression for Mean effective pressure

As already discussed during Otto cycle, it is defined as the ratio of new workdone to the displacement volume of the pistion. Let us consider unit kg of working substance, Heat supplied during the diesel cycle is given,

$$Q_H = c_p(T_3 - T_2)$$

Heat rejected during the diesel cycle is given,

$$Q_L = c_v(T_1 - T_4)$$

Work done is given by,

$$W = Q_H - Q_L = c_p(T_3 - T_2) - c_v(T_4 - T_1) \tag{17}$$

Displacement volume is given by,

$$v_s = (v_1 - v_2)$$
$$v_s = v_1\big(1 - \frac{1}{r_c}\big), \quad \text{But,}\ v_1 = \frac{RT_1}{p_1}$$
$$v_s = \frac{RT_1}{p_1}(1 - \frac{1}{r_c})$$
$$v_s = \frac{c_v(\gamma-1)T_1}{p_1}(\frac{r_c - 1}{r_c}) \quad \text{Since,} \quad R = c_v(\gamma - 1) \tag{18}$$

Mean effective pressure is given by;

$$p_m = \frac{\gamma(T_3 - T_2) - (T_4 - T_1)}{\frac{c_v(\gamma-1)T_1}{p_1}(\frac{r_c - 1}{r_c})}$$
$$p_m = \frac{1}{c_v(\gamma-1)}\frac{p_1}{T_1}\frac{r_c}{r_c - 1}[\gamma(T_3 - T_2) - (T_4 - T_1)] \tag{19}$$

As $T_2 = T_1(r_c)^{\gamma-1)}$, if Cut-off ratio $\rho = \frac{v_3}{v_2}$,r_e is the expansion ratio $= \frac{v_4}{v_3} = \frac{v_4 v_2}{v_2 v_3}$ so expansion ratio $r_e = \frac{r_c}{\rho}$
Then, $T_3 = \frac{v_3}{v_2}T_2 = \rho T_2 = \rho T_1(r_c)^{(\gamma-1)}$

$$\frac{T_4}{T_3} = (\frac{v_3}{v_4})^{(\gamma-1)} = \frac{1}{r_e^{\gamma-1}} = (\frac{\rho}{r_c})^{(\gamma-1)}$$
$$T_4 = \rho T_1(r_c)^{(\gamma-1)} * (\frac{\rho}{r_c})^{(\gamma-1)}$$
$$T_4 = \rho^{\gamma}T_1 \tag{20}$$

So the mean effective pressure is given by

$$p_m = \frac{p_1 r_c}{c_v(r_c - 1)(\gamma - 1)}\left[\gamma(\rho r_c^{\gamma-1} - r_c^{\gamma-1}) - (\rho^{\gamma} - 1)\right]$$
$$p_m = p_1 r_c\Big[\frac{\gamma r_c^{\gamma-1}(\rho - 1) - (\rho^{\gamma} - 1)}{c_v(r_c - 1)(\gamma - 1)}\Big] \tag{21}$$

The above expression is the equation for Mean effective pressure in terms of compression ration r, cut-off ratio ρ and specific heat ratio γ

Difference between Actual Diesel and Otto cycles

Otto Cycle

1. Homogeneous mixture of fuel and air formed in the Carburettor is supplied to the engine cylinder.
2. Ignition is initiated by means of electric spark plug.
3. Power output is varied by means of throttle-valve near Carburettor, which controls the fuel-air mixutre supply to the engine

Diesel Cycle

1. No carburetor is used. Air alone is supplied to the engine cylinder. Fuel is injected directly into the engine cylinder at the end of compression stroke by means of a fuel injector. Fuel-air mixture is heterogeneous.
2. No spark plug is used. Compression ratio is high and the high temperature of air ignites fuel.
3. No throttle value is used. Power output is controlled only by means of the mass of fuel injected by the fuel injector.

Air Standard Dual-Combustion or Limited Pressure or Semi-Diesel cycle

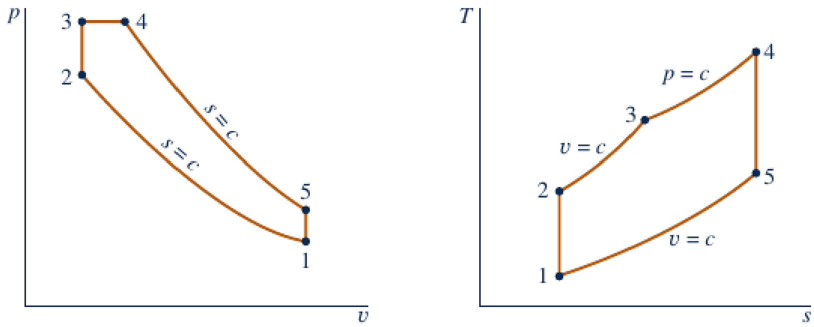


Figure 5: P-V and T-S diagram of Dual Cycle

The cycle is also called as the dual cycle, which is show in Figure. 5. Here the heat addition occurs partly at constant volume and partly at constant pressure. This cycle is a closer approximation to the behaviour of the actual Otto and Diesel engines because in the actual engines, the combustion process doesn't exactly occur at constant ovolume or at constant pressure but rather as in the dual cycle. Hence for most oil engines the ideal cycle is taken as the dual cycle. The P-V and T-S diagram for the dual cycle is shown above and it consists of follow processes,
Process 1-2: Reversible adiabatic compression.
Process 2-3: Constant volume heat addition.
Process 3-4: Constant pressure heat addition.
Process 4-5: Reversible adiabatic expansion.
Process 5-1: Constant volume heat rejection.
Consider unit kg of mass during the cyclic process,
Heat supplied during the process 2-3 and 3-4 is given by,

$$Q_H = c_v(T_3 - T_2) + c_p(T_4 - T_3)$$

Heat rejected during the process 5-1 is given by,

$$Q_L = c_v(T_1 - T_5)$$

Thermal efficiency of the cycle can be obtained by the equation
$\eta_{th} = \dfrac{\text{Work done}}{\text{Heat Supplied}}$

$$\eta_{th} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$
$$\eta_{th} = 1 - \frac{c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)}$$
$$\eta_{th} = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)} \tag{22}$$

The efficiency of the cycle can be expressed in terms of the following ratios, compression ratio $r_c = \frac{v_1}{v_2}$, Expansion ratio $r_c = \frac{v_5}{v_4}$, Cut-off ratio $\rho = \frac{v_4}{v_3}$ and pressure ratio $r_p = \frac{p_3}{p_2}$
We can express, $r_c = \rho * r_e$ such that $r_e = \frac{r_c}{\rho}$
For the process 3-4

$$\rho = \frac{v_4}{v_3} = \frac{T_4 p_3}{p_4 T_3} = \frac{T_4}{T_3}$$
$$T_3 = \frac{T_4}{\rho} \tag{23}$$

For the process 2-3

$$\frac{p_2 v_2}{T_2} = \frac{p_3 v_3}{T_3}$$
$$T_2 = T_3 \frac{p_2}{p_3} = \frac{T_4}{r_p \rho} \tag{24}$$

For the process 1-2

$$\frac{T_2}{T_1} = (\frac{v_1}{v_2})^{\gamma-1}$$
$$T_2 = T_1 (r_c)^{\gamma-1}$$
$$T_1 = \frac{T_4}{r_p.\rho.r_c^{\gamma-1}} \tag{25}$$

For the process 4-5

$$\frac{T_5}{T_4} = (\frac{v_4}{v_5})^{\gamma-1} = \frac{1}{r_e^{\gamma-1}}$$
$$T_5 = T_4 \frac{\rho^{\gamma-1}}{r_c^{\gamma-1}} \tag{26}$$

Substituting the values of $T_1, T_2, T_3 and T_5$ in the expression for the efficiency equation, we get

$$\eta_{th} = 1 - \frac{T_4 \frac{\rho^{\gamma-1}}{r_c^{\gamma-1}} - \frac{T_4}{r_p.\rho.r_c^{\gamma-1}}}{(\frac{T_4}{\rho} - \frac{T_4}{r_p.\rho} + \gamma(T_4 - \frac{T_4}{\rho})}$$
$$\eta_{th} = 1 - \frac{1}{r_c^{\gamma-1}} . \frac{r_p.\rho - 1}{(r_p - 1) + \gamma.r_p(\rho - 1)} \tag{27}$$

Comparison on Otto, Diesel and Dual cycles

The important variable factors which are used as the basis for comparison of the cycles are compression ratio, peak pressure, heat rejection and the net work. In order to compare the performance of the Otto, Diesel and Dual combustion cycles, some of the variable factors must be fixed. In this section, a comparison of these three cycles is made for the same compression ratio, constant maximum pressure and temperature, same heat rejection and net work output. This analysis will show which cycle is more efficient for a given set of operating conditions.

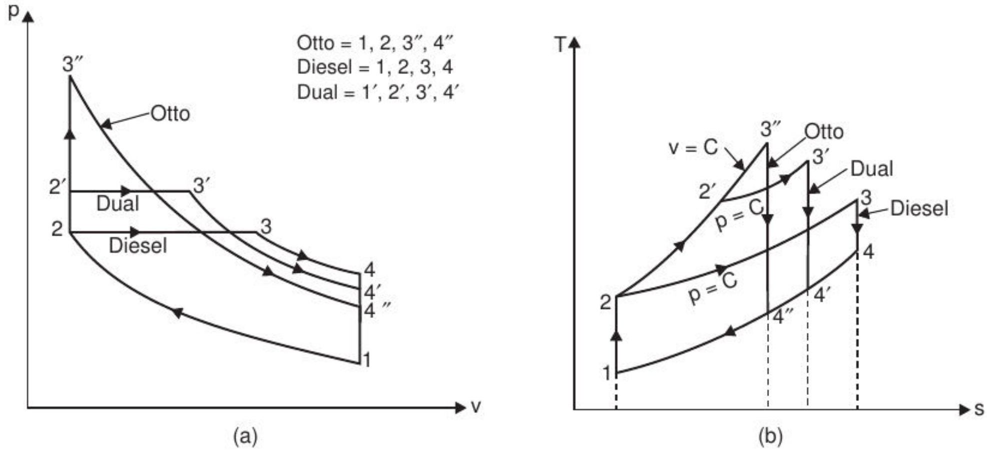


Figure 6: P-V and T-S diagram on same compression ratio and heat input

Case:1 Same compression ratio and heat input

A comparison of the cycles (Otto, Diesel and Dual) on the p-v and T-s diagrams for the same compression ratio and heat supplied is shown in the Fig.6.
We know that, $\eta = 1 - \frac{HeatSupplied}{HeatRejected}$
Since all the cycles reject their heat at the same specific volume, process line from state 4 to 1, the quantity of heat rejected from each cycle is represented by the appropriate area under the line 4 to 1 on the T-s diagram. As is evident from the efficiency equation, the cycle which has the least heat rejected will have the highest efficiency. Thus, Otto cycle is the most efficient and Diesel cycle is the least efficient of the three cycles.
i.e., $\eta_{otto} > \eta_{dual} > \eta_{diesel}$.

Case-2: Same Maximum Temperature, pressure and Heat rejection

The air-standard Otto, Dual and Diesel cycles are drawn on common p-v and T-s diagrams for the same maximum pressure and maximum temperature, for the purpose of comparison.

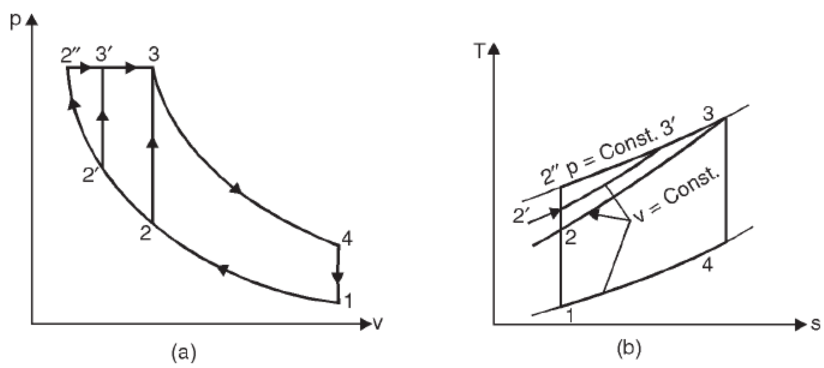


Figure 7: P-V and T-S diagram for comparison on same Maximum Temp, pressure and Heat rejection

$Otto\ 1-2-3-4-1, Dual\ 1-2'-3'-3-4-1, Diesel\ 1-2"-3-4-1$ as shown in Fig.7. Slope of constant volume lines on T-s diagram is higher than that of constant pressure lines.
Here the otto cycle must be limited to a low compression ratio(r_c) to fulfill the condition that point 3 (same maximum pressure and temperature) is to be a common state for all the three cycles. The construction of cycles on T-s diagram proves that for the given conditions the heat rejected is same for all the three cycles (area under process line 4-1).
Since, by definition, $\eta = 1 - \frac{HeatSupplied}{HeatRejected} = 1 - \frac{Constant}{Q_H}$
the cycle with greater heat addition will be more efficient. From the T-S diagram
$Q_{H-Diesel}$= Area under $2" - 3$, Q_{H-Dual}= Area under $2' - 3' - 3$, Q_{H-Otto}= Area under $2 - 3$
It can be seen that $Q_{H-Diesel} > Q_{H-Dual} > Q_{H-Otto}$
and thus efficiencies are given by, $\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}$.

Brayton Cycle

Gas turbine engines operate on either open or closed basis. The Fig. 8a is the open mode of operation which is commonly used. In this type the atmospheric air is continuously drawn into the compressor and compressed to a high

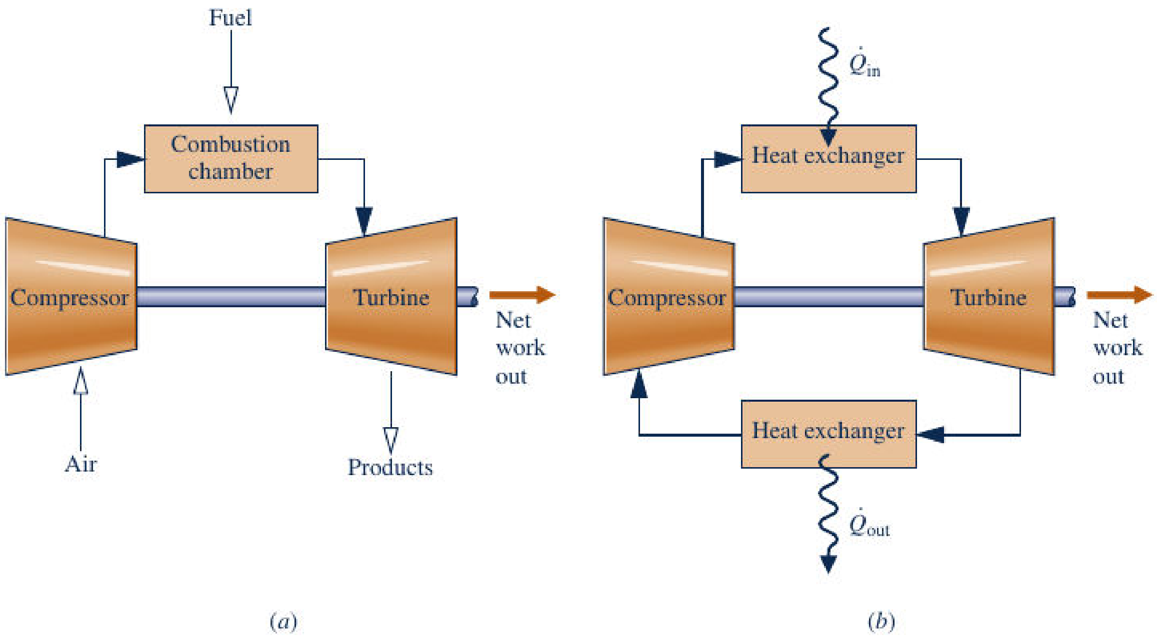


Figure 8: Simple Gasturbine (a) Open cycle (b) Close cycle

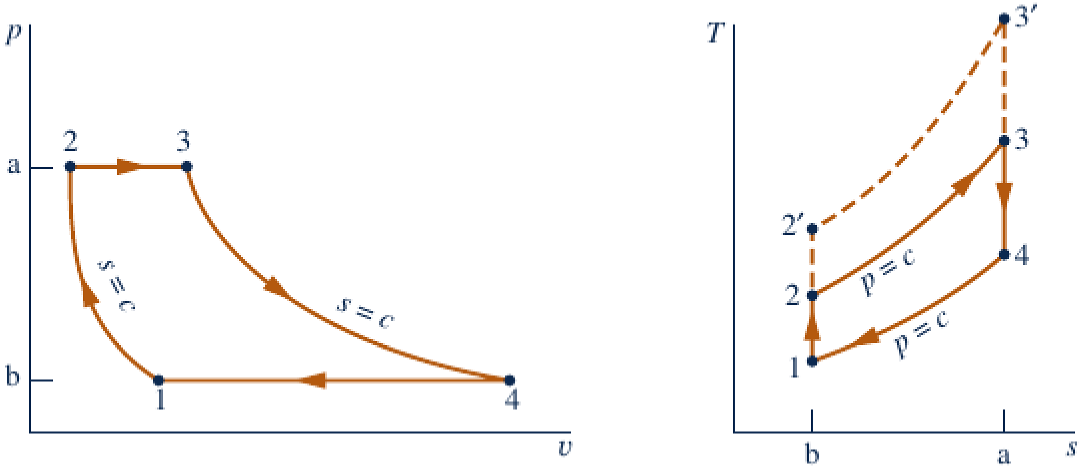
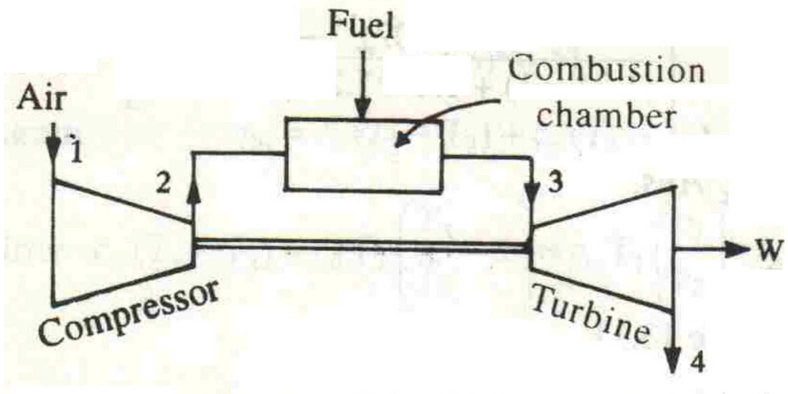


Figure 9: P-V and T-S diagram for Brayton cycle

GAS TURBINES

Gas Turbines and Jet Propulsion: Classification of Gas Turbines, Analysis of open cycle gas turbine cycle. Advantages and Disadvantages of closed cycle. Methods to improve thermal efficiency. Jet propulsion and Rocket propulsion.

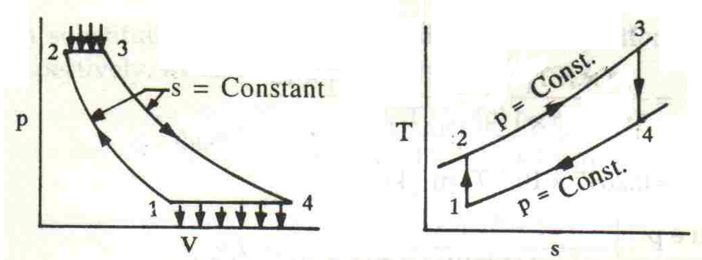
Simple Gas Turbine Cycle



Open type gas-turbine cycle

A schematic diagram of a simple gas turbine power plant is shown in figure. Air is drawn from the atmosphere into the compressor, where it is compressed reversibly and adiabatically. The relatively high pressure is then used in burning the fuel in the combustion chamber. The air fuel ratio is quite high (about 60:1) to limit the temperature of the burnt gases entering the turbine. The gases then expand isentropically in the turbine. A portion of the work obtained from the turbine is utilized to drive the compressor and the auxiliary drive, and rest of the power output is the net power of the gas turbine plant.

A gas turbine plant works using a Brayton or joule cycle. This cycle was originated by joule, a British engineer for use in a hot air reciprocating engine and later in about 1870 an American engineer George Brayton tried this cycle in a gas turbine. This cycle consists of two constant pressures and two adiabatic processes. The P-V and T-S diagrams of the cycle are as shown in figure.



p-v and T-s diagrams of air stander gas-turbine cycle

Process 1 – 2: isentropic compression in the compressor

Process 2 – 3: constant pressure heat addition in the combustion chamber

Process 3 – 4: isentropic expansion in the turbine

Process 4 -1: constant pressure heat rejection in the atmosphere or cooling of air in the intercooler (closed cycle).

Expression of net work output:

We have net work output, $W_N = W_T - W_C$
Turbine work, $W_T = h_3 - h_4$
$= C_P (T_3 - T_4)$ since the working fluid is a perfect gas

Compressor work, $W_C = h_2 - h_1$
$= C_P (T_2 - T_1)$

$\therefore W_N = C_P (T_3 - T_4) - C_P (T_2 - T_1)$

Let $R = \frac{P_2}{P_1}$ = pressure ratio for compression
$t = T_3/T_1$ = Temperature ratio

$$W_N = C_P T_1 \left[\frac{T_3}{T_1} - \frac{T_4}{T_1} - \frac{T_2}{T_1} + 1 \right]$$

We have $\frac{T_1}{P_1^{\frac{r-1}{r}}} = \frac{T_2}{P_2^{\frac{r-1}{r}}}$ $\quad \therefore \frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{r-1}{r}} = R^{\frac{r-1}{r}}$

$$\frac{T_4}{T_1} = \frac{T_4}{T_3} \frac{T_3}{T_1}$$

$$= \left(\frac{P_4}{P_3} \right)^{\frac{r-1}{r}} t = \left(\frac{1}{R} \right)^{\frac{r-1}{r}} t \qquad \because P_1 = P_4 \quad \& \quad P_2 = P_3$$

$$\therefore W_N = C_P T_1 \left[t - \frac{t}{R^{\frac{r-1}{r}}} - R^{\frac{r-1}{r}} + 1 \right]$$

Expression for Thermal Efficiency:

We have thermal efficiency, $\eta_{th} = \frac{W_N}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$

Heat added, $Q_H = h_3 - h_2 = C_P (T_3 - T_2)$
Heat rejected, $Q_L = h_4 - h_1 = C_P (T_4 - T_1)$

$$\therefore \eta_{th} = 1 - \frac{C_P (T_4 - T_1)}{C_P (T_3 - T_2)} = 1 - \frac{T_1 \left[\frac{T_4}{T_1} - 1 \right]}{T_2 \left[\frac{T_3}{T_2} - 1 \right]}$$

Now, $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{r-1}{r}} = R^{\frac{r-1}{r}}$ & $T_3 / T_4 = \left(P_3 / P_4 \right)^{\frac{r-1}{r}} = \left(\frac{1}{R} \right)^{\frac{r-1}{r}}$

But as $P_2 = P_3$ & $P_1 = P_4$, it follows that $\frac{T_2}{T_1} = \frac{T_3}{T_4}$ *or* $\frac{T_4}{T_1} = \frac{T_3}{T_1}$

$$\therefore \eta_{th} = 1 - \frac{T_1}{T_2} \quad i.e., \quad \eta_{th} = 1 - \frac{1}{\left(\frac{T_2}{T_1} \right)} \quad \text{or} \quad \eta_{th} = 1 - \frac{1}{R^{\frac{r-1}{r}}}$$

From the above equation, it is seen that the efficiency of the air standard gas turbine cycle increases with increase in pressure ratio (R) and the type of working fluid.

Optimum Pressure Ratio for Specific Power Output

In a gas turbine cycle, T_1 is the temperature of the atmosphere and T_3 is the temperature of the burnt gases entering the turbine. Temperature T_3 is fixed by the metallurgical consideration of the turbine and temperature T_1 is fixed by the atmospheric condition. Between these two extreme values of temperature, there exists an optimum pressure ratio for which the work output of the turbine is maximum.

We have, the net work output of the turbine is,

$$W_N = C_P T_1 \left[t - \frac{t}{R^{\frac{r-1}{r}}} - R^{\frac{r-1}{r}} - 1 \right] \qquad \text{--- (1)}$$

The optimum pressure ratio is obtained by differentiating the net work output w.r.t. the pressure ratio and putting the derivative equal to zero i.e., $\frac{dW_N}{dR} = 0$

Or $\frac{d}{dR} \left[C_P T_1 \left\{ t - \frac{t}{R^{\frac{\gamma-1}{\gamma}}} - R^{\frac{\gamma-1}{\gamma}} - 1 \right\} \right] = 0$

Differentiating with respect to R we get,

$$-t\frac{1-\gamma}{\gamma}R^{\frac{1-\gamma-\gamma}{\gamma}}-\frac{\gamma-1}{\gamma}R^{\frac{\gamma-1-\gamma}{\gamma}}=0$$

i.e., $$-t\left(\frac{1-\gamma}{\gamma}\right)R^{\frac{1-2\gamma}{\gamma}}=\frac{\gamma-1}{\gamma}R^{-1/\gamma}$$

$$t\left(\frac{\gamma-1}{\gamma}\right)R^{\frac{1-2\gamma}{\gamma}}=\frac{\gamma-1}{\gamma}R^{-1/\gamma}$$

or $$\frac{R^{-1/\gamma}}{R^{\frac{1-2\gamma}{\gamma}}}=t \quad or \quad R^{-\frac{1}{\gamma}-\frac{1-2\gamma}{\gamma}}=t$$

or $$R^{\frac{-1-1+2\gamma}{\gamma}}=t \quad or \quad R^{\frac{2(\gamma-1)}{\gamma}}=t$$

or $$(R)_{opt}=t^{\frac{\gamma}{2(\gamma-1)}} \qquad \text{i.e., } R_{opt}=\left[\frac{T_3}{T_1}\right]^{\frac{\gamma}{2(\gamma-1)}}$$

Substituting this value of R in the expression for W_N, we get

$$(W_N)_{opt}=C_PT_1\left[t-\frac{t}{\left[t^{\frac{r}{2(r-1)}}\right]^{\frac{r-1}{r}}}-\left[t^{\frac{r}{2(r-1)}}\right]^{\frac{r-1}{r}}+1\right]$$

$$=C_PT_1\left[t-\frac{t}{t^{\frac{1}{2}}}-t^{\frac{1}{2}}+1\right]$$

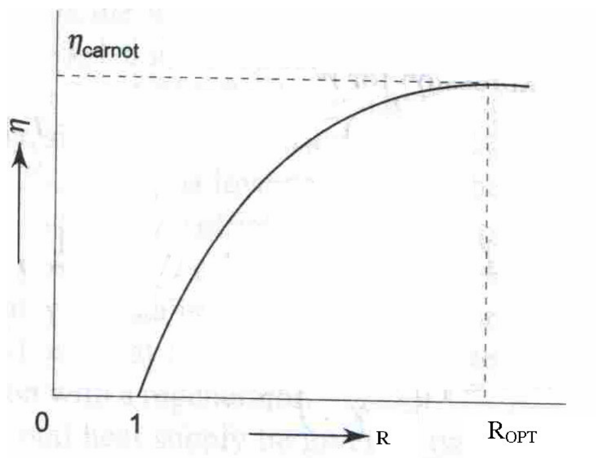
$$=C_PT_1\left[t-t^{\frac{1}{2}}-t^{\frac{1}{2}}+1\right]$$

$$=C_PT_1\left[t-2t^{\frac{1}{2}}+1\right]$$

$$(W_N)_{opt}=C_PT_1\left[t^{\frac{1}{2}}-1\right]$$

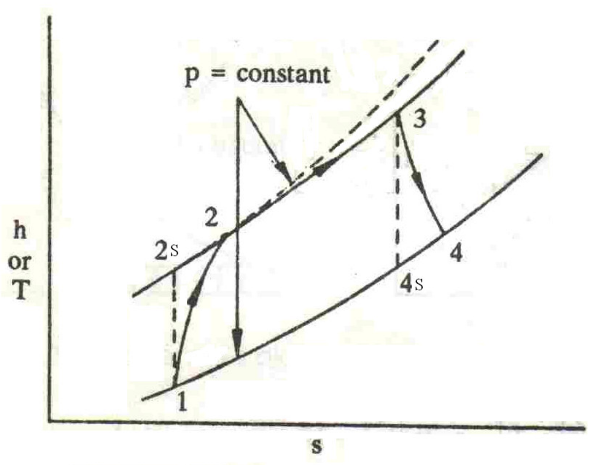
$$\eta_{th} = 1 - \frac{1}{R^{\frac{r-1}{r}}} \quad = \quad 1 - \frac{1}{\left[t^{\frac{r}{2(r-1)}} \right]^{\frac{r-1}{r}}}$$

$$\therefore (\eta_{th})_{opt} = 1 - \frac{1}{t^{\frac{1}{2}}}$$



Effect of pressure ratio on Brayton cycle efficiency

In an ideal gas turbine plant, the compression and expansion processes are isentropic and there is no pressure-drop in the combustion chamber. But because of irreversibilities associated in the compressor and the turbine, and the pressure-drop in the actual flow passages and combustion chamber, an actual gas turbine plant differs from ideal one. The T-S diagram of actual plant is shown in figure.



T-s diagram of the actual cycle

$$\therefore Compressor \quad efficeincy, \quad \eta_C = \frac{h_{2S} - h_1}{h_2 - h_1}$$

and the $turbine \quad efficeincy, \quad \eta_t = \dfrac{h_3 - h_4}{h_3 - h_{4S}}$

Classification: Gas turbine are mainly divided into two group

I Constant pressure combustion gas turbine
i) Open cycle, ii) Closed cycle

II Constant volume combustion gas turbine
In almost all the field open cycle gas turbine plants are used. Closed cycle plants were introduced at one stage because of their ability to burn cheap fuel.

Advantages and disadvantages of closed cycle over open cycle

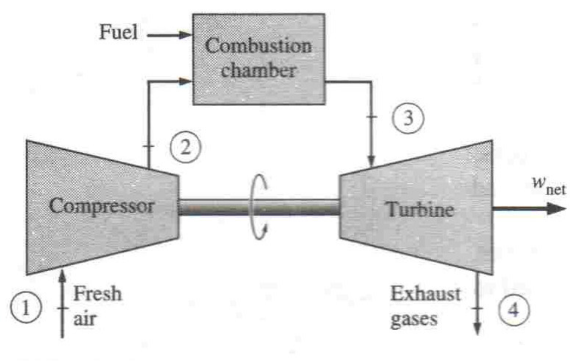
Advantages of closed cycle:

i) Higher thermal efficiency
ii) Reduced size
iii) No contamination
iv) Improved heat transmission
v) Improved part load η
vi) Lesser fluid friction
vii) No loss of working medium
viii) Greater output and
ix) Inexpensive fuel.

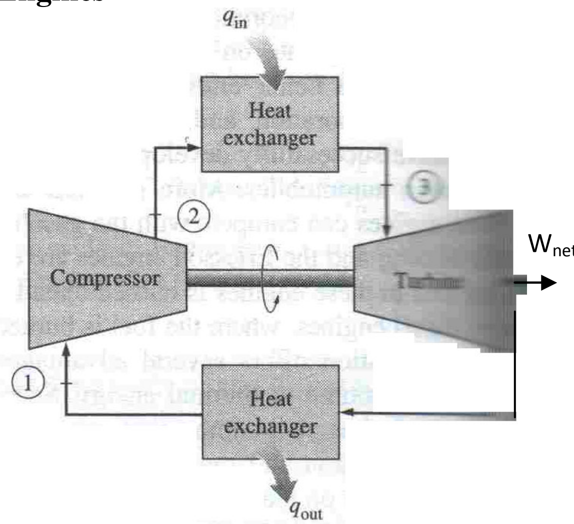
Disadvantages of closed cycle:

i) Complexity
ii) Large amount of cooling water is required. This limits its use of stationary installation or marine use
iii) Dependent system
iv) The wt of the system pre kW developed is high comparatively, $\therefore$ not economical for moving vehicles
v) Requires the use of a very large air heater.

An Open cycle and Closed cycle Gas Turbine Engines



An open-cycle gas-turbine engine.

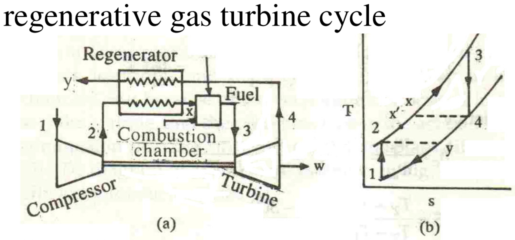


A closed-cycle gas-turbine engine

Methods to improve the performance of simple gas turbine plants

I Regenerative Gas Turbine Cycle: The temperature of the exhaust gases in a simple gas turbine is higher than the temperature of the air after compression process. The η of the Brayton cycle can be increased by utilizing part of the energy of the exhaust gas from the turbine in heating up the air leaving the compression in a heat exchanger called a regenerator, thereby reducing the amount of heat supplied from an external source and also the amount of heat rejected.

Figure shows a single stage regenerative gas turbine cycle



An ideal air standard regenerative gas turbine cycle

Air is drawn from the atmosphere into the compressor and is compressed isentropically to state 2. It is then heated at constant pressure in the regenerator to state x by the hot burnt gases from the gas turbine. Since the temperature of the air increases before it reaches the combustion chamber, less amount of fuel will be required to attain the designed turbine inlet temperature of the products of combustion. After combustion at constant pressure in the combustion chamber, the gas enters the turbine at state 3 and expands isentropically to state 4 in the turbine. It then enters the counter-flow regenerator as stated earlier, where it gives up a portion of its heat energy to the compressed air from the compressor and leaves the regenerator at state y.

In an ideal cycle, the temperature of the air leaving the regenerator is equal to the temperature of the burnt gases leaving the turbine, i.e., $T_x = T_4$. But in practice, the temperature of the air leaving the regenerator is less than T_x. In T-S diagram, T_x^1 is the temperature of the air leaving the regenerator in an actual plant.

$\therefore$ Effectiveness of a regenerator is $\varepsilon = \eta_r = \dfrac{T_{x^1} - T_2}{T_x - T_2}$ when C_P is constant.

In an ideal regenerator, heat loss by the burnt gases is equal to the heat gained by the air in the regenerator, i.e., $T_4 - T_y = T_x - T_2$,
Where $T_x = T_4$ and $T_y = T_2$ and hence $\eta_r = 1$

$$\eta_{th} = 1 - \frac{Q_L}{Q_H}$$

For an ideal regenerative gas turbine cycle,

$Q_L = C_P\,(T_y - T_1) = C_P\,(T_2 - T_1)$ and

$Q_H = C_P\,(T_3 - T_x) = C_P\,(T_3 - T_4)$

$$\therefore \eta_{th} = 1 - \frac{(T_2 - T_1)}{(T_3 - T_4)} = 1 - \frac{T_1\left[\dfrac{T_2}{T_1} - 1\right]}{T_3\left[1 - \dfrac{T_4}{T_3}\right]}$$

Since $\dfrac{T_2}{T_1} = \left(\dfrac{P_2}{P_1}\right)^{\frac{r-1}{r}} = R^{\frac{r-1}{r}}$ *and* $\dfrac{T_4}{T_3} = \left(\dfrac{P_4}{P_3}\right)^{\frac{r-1}{r}} = \left(\dfrac{P_1}{P_2}\right)^{\frac{r-1}{r}} = \left(\dfrac{1}{R}\right)^{\frac{r-1}{r}}$

$$\therefore \eta_{th} = 1 - \frac{1}{t}\frac{\left(R^{\frac{r-1}{r}} - 1\right)}{\left(1 - \dfrac{1}{R^{\frac{r-1}{r}}}\right)} = 1 - \frac{1}{t}.\quad R^{\frac{r-1}{r}}$$

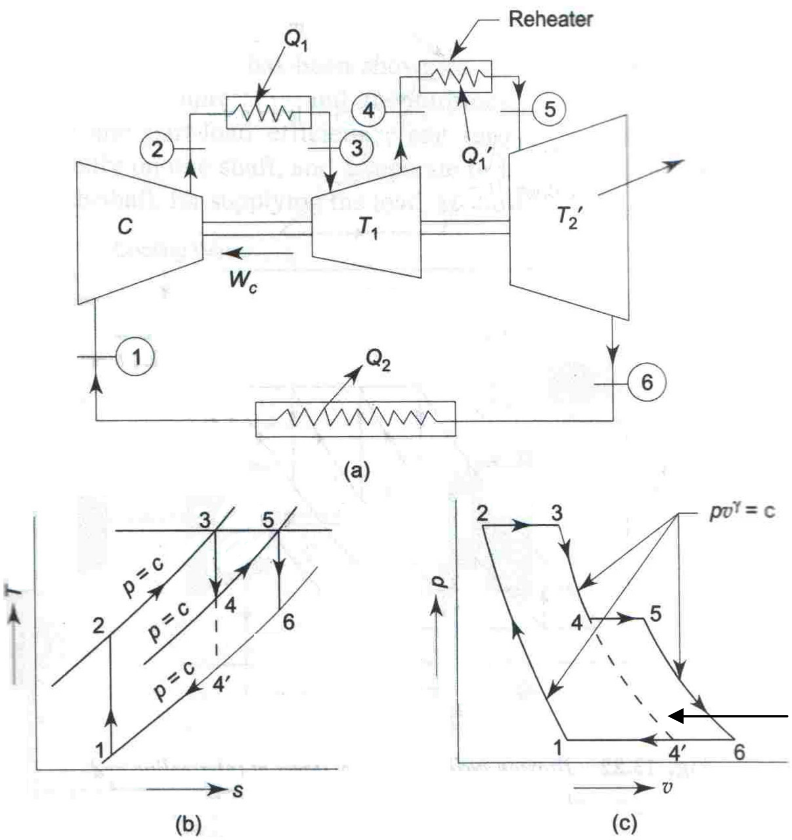
i.e., $\eta_{th} = 1 - \dfrac{1}{t} R^{\frac{r-1}{r}}$

It is evident that the η_{th} of an ideal regenerative gas turbine cycle depends not only on the pressure ratio but also on the ratio of the two extreme temperatures. For a fixed ratio of T_3/T_1, the cycle η drops with increasing pressure ratio.

In practice the regenerator is costly, heavy and bulky, and causes pressure losses which brings about a decrease in cycle η.

II Ideal regenerative cycle with inter cooling and reheat:

a) Gas turbine cycle with reheat and multistage expansion:

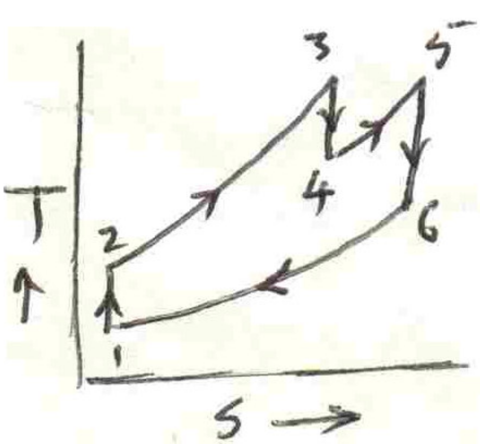


Increased work output due to reheating

Effect of reheat on Brayton cycle

Work output of the turbine is increased by multistage expansion with reheating. In the above illustration, two-stage expansion is done in low pressure (T_1) and high pressure (T_2^1) turbines with reheating the air in between stages.

Optimum work output for a two-stage reheat cycle:



Assumptions: The air after one stage of expansion is reheated back to its original temperature i.e., $T_3 = T_5$

Let $R = \frac{P_2}{P_1}, \quad R_1 = \frac{P_3}{P_4} \quad and \quad R_2 = \frac{P_5}{P_6}, t = \quad \frac{T_3}{T_1} = \max imum\ cycle\ temperature\ ratio$

$$\therefore R_1 R_2 = \frac{P_3}{P_4}\frac{P_5}{P_6} \quad \because P_5 = P_4$$

$$= \frac{P_2}{P_1} = R$$

Net work output is $W_N = C_P (T_3 - T_4) + C_P (T_5 - T_6) - C_P (T_2 - T_1)$

$$= C_P T_1 \left[\frac{T_3}{T_1} - \frac{T_4}{T_1} + \frac{T_3}{T_1} - \frac{T_6}{T_1} - \frac{T_2}{T_1} + 1\right]$$

we have $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}} = R^{\frac{r-1}{r}}$

$$\frac{T_4}{T_1} = \frac{T_4}{T_3}\frac{T_3}{T_1} = \left(\frac{P_4}{P_3}\right)^{\frac{r-1}{r}}\frac{T_3}{T_1} = t / R_1^{\frac{r-1}{r}}$$

$$\frac{T_6}{T_1} = \frac{T_6}{T_5}\frac{T_5}{T_1} = \left(\frac{P_6}{P_5}\right)^{\frac{r-1}{r}}\frac{T_3}{T_1} = t / R_2^{\frac{r-1}{r}}$$

$$\therefore W_N = C_P T_1 \left[t - \frac{t}{R_1^{\frac{r-1}{r}}} + t - \frac{t}{R_2^{\frac{r-1}{r}}} - R^{\frac{r-1}{r}} + 1 \right]$$

but $R_2 = R/R_1$

$$\therefore W_N = C_P T_1 \left[2t - \frac{t}{R_1^{\frac{r-1}{r}}} - t\left(\frac{R_1}{R}\right)^{\frac{r-1}{r}} - R^{\frac{r-1}{r}} + 1 \right]$$

For given values of R, t and T_1, W_N is maximum if $\frac{dW_N}{dR_1} = 0$

$$\therefore \frac{dW_R}{dR_1} = C_P T_1 \left[0 - t.\frac{1-\gamma}{\gamma}.R_1^{\frac{\gamma-1}{\gamma}-1} - \frac{t}{R^{\frac{\gamma-1}{\gamma}}}.\frac{\gamma-1}{\gamma}.R_1^{\frac{\gamma-1}{\gamma}-1} - 0 + 0 \right] = 0$$

i.e., $\frac{\gamma-1}{\gamma}.t\ R_1^{\frac{1-2\gamma}{\gamma}} - \frac{\gamma-1}{\gamma}\frac{t}{R^{\frac{\gamma-1}{\gamma}}} \quad R_1^{-1/\gamma} = 0$

$$R_1^{\frac{1-2r}{r}} = \frac{R_1^{-\frac{1}{\gamma}}}{R^{\frac{r-1}{r}}} \qquad or \qquad R^{\frac{r-1}{r}} = \frac{R_1^{-1/\gamma}}{R_1^{\frac{1-2r}{r}}}$$

$$R^{\frac{r-1}{r}} = R_1^{2\frac{(r-1)}{r}}$$

$$\text{Or } R = R_1^2 \qquad \text{i.e., } R_1 = \sqrt{R}$$

$$\therefore R_2 = \frac{R}{R_1} = \frac{R}{\sqrt{R}} = \sqrt{R}$$

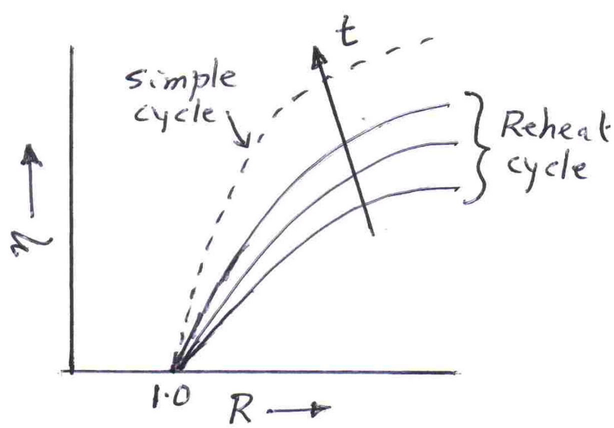
$\therefore$ For maximum work output, $R_1 = R_2 = \sqrt{R}$

Similarly if the cycle has 'N' stages of expansion, with reheating, then for maximum work output, pressure ratio for each = (compression pressure ratio)$^{1/N}$

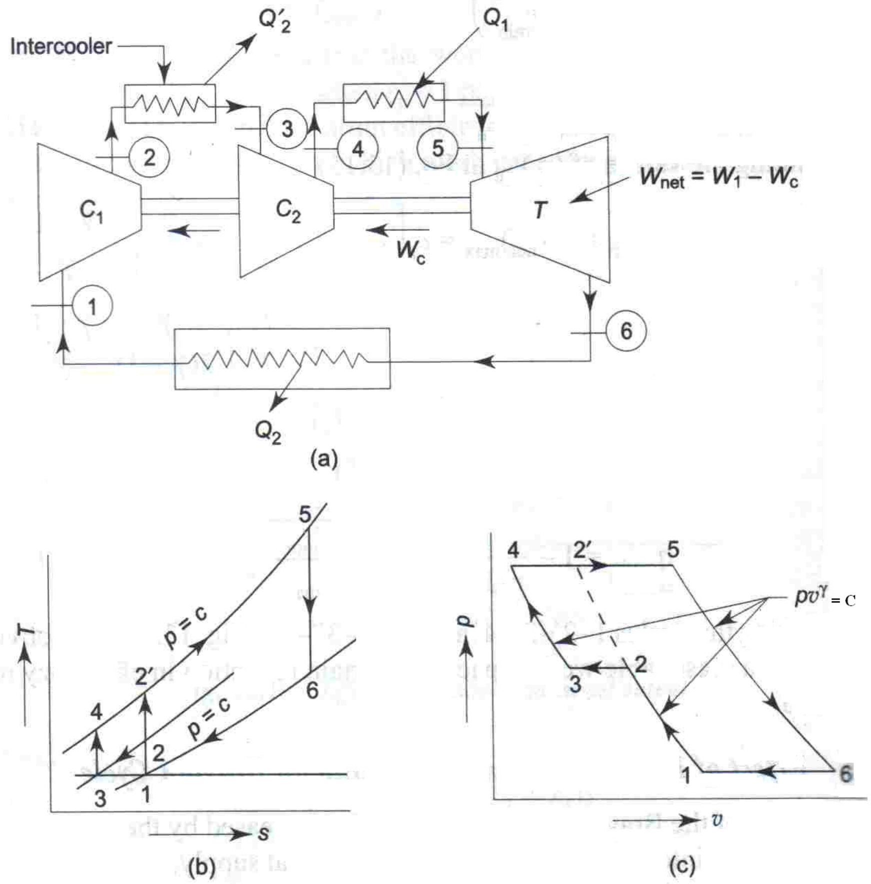
Advantages: By employing multistage expansion with reheating in between the stages, the net work output from the gas turbine cycle can be increased. This is illustrated on the T-S diagram shown for a 2-stage expansion with reheating in between the stages.

Disadvantages: But disadvantage of reheating is due to the fact that additional heat has to be supplied in order to reheat the air after each stage of expansion. This may result in a decrease in the thermal efficiency of the cycle. This is shown in figure below.

It can be seen that for a given value of t, the thermal η of the reheat cycle increases with increase in R and for a given value of R, the thermal η increases with increase in t. However, the thermal η of a reheat cycle will be less than that of a simple cycle for a given value of R.



b) Gas turbine cycle with multistage compression with inter cooling in between the stages:



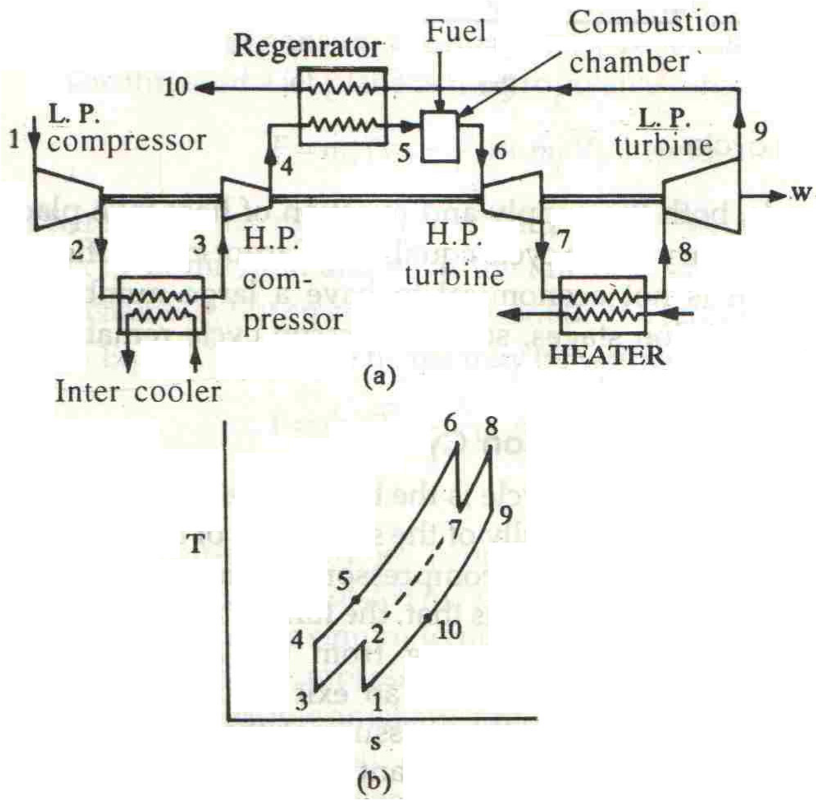
Effect of intercooling on Brayton cycle

Let $\frac{P_2}{P_1} = R_1 \quad P_4/P_3 = R_2 \quad P_5/P_6 = R$

For maximum work output, $R_1 = R_2 = \sqrt{R}$

The work output from a simple gas turbine cycle can be increased also by having multistage compression with inter cooling in between the stages. The effect of having two stage compression with inter cooling in between the stages is illustrated on the T-S diagram. It can be seen that, a higher work output has been achieved than that of simple cycle by an amount shown by the shaded area. The disadvantage of it is that more heat has to be supplied to heat the air than that is required for simple cycle. This may reduce the thermal $\eta$of the cycle.

c) Gas turbine cycle with two stage compression two-stage expansion and regenerator.



The thermal η of a gas turbine cycle may be improved by incorporating multistage compression with intercooling between the stages and multistage expansion with reheating between the turbines and also providing a regenerator. There is a definite saving of work due to multistage compression with intercooling arrangement between the stages. Similarly, the work output of the turbine is increased by multistage expansion with reheating. As a result, the net work of the plant increases.

The thermal efficiency of the cycle is given by $\eta_{th} = 1 - \frac{Q_L}{Q_H}$

$$= 1 - \frac{(h_{10} - h_1) + (h_2 - h_3)}{(h_6 - h_5) + (h_8 - h_7)}$$

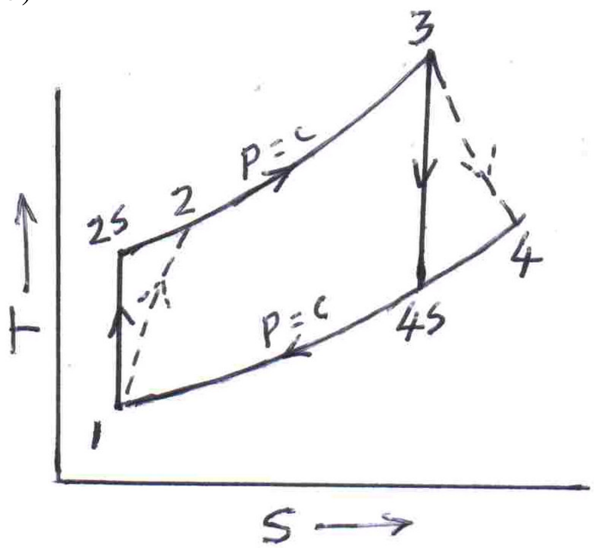
Deviation of Practical gas turbine cycle from ideal cycle:

1) The working substance will not be air through out the cycle. Air is compressed in compressor where as the products of combustion coming out of the combustion chamber is expanded in the turbine. The value of C_P and γ will be different for expansion and heating as compared to compression process.
   For compression, C_P = 1.005 kJ/kg-^{0}K, γ = 1.4
   For expansion, C_P = 1.135 kJ/kg-^{0}K, γ = 1.33

2) There will be pressure loss in the piping connecting the various components of the plant. ∴the pressure with which the products of combustion enters the turbine will be less than the

pressure with which air is coming out of the compressor i.e., the pressure ratio for expansion will be less than pressure ratio for compression.

3)



1-2s ⇒ isentropic work of compression
1-2 ⇒ actual work of compression
3-4s ⇒ isentropic turbine work
3-4 ⇒ Actual turbine work

In a practical gas turbine cycle the compression and expansion processes are not isentropic but adiabatic with certain amount of frictional losses. The friction losses are accounted for by defining a parameter called isentropic efficiency.

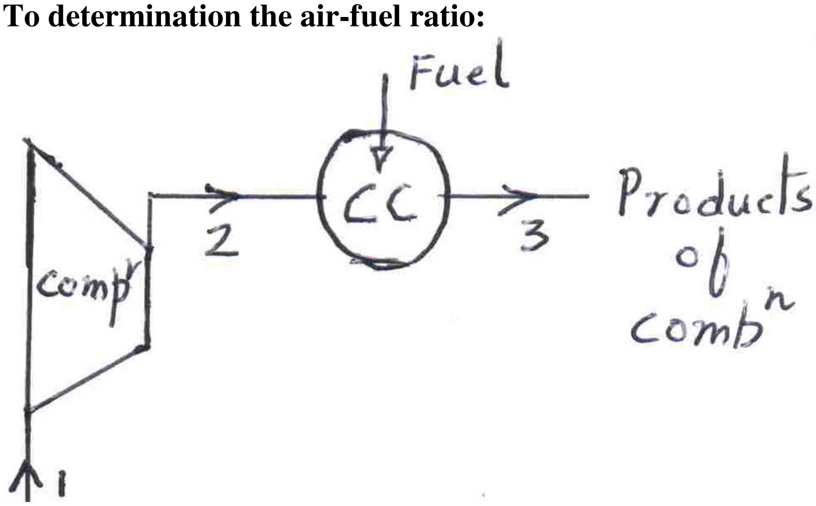
a) Isentropic η of compression (η_C)

$$\eta_C = \frac{Isentropic\ work\ of\ compression}{Actual\ work\ of\ compression} = \frac{T_{2S} - T_1}{T_2 - T_1}$$

b) Isentropic η of turbine (η_T)

$$\eta_T = \frac{Actual\ turbine\ work}{Isentropic\ turbine\ work} = \frac{T_3 - T_4}{T_3 - T_{4S}}$$

To determination the air-fuel ratio:



Let m_a = mass of air entering the combustion chamber
m_f = mass of fuel entering the combustion chamber
CV = Calorific value of fuel

Applying SFEE to combustion chamber, we get,

$$m_a h_2 + m_f CV = (m_a + m_f) h_3$$

$\div$by m_f

$$\frac{m_a}{m_f} h_2 + CV = \left(\frac{m_a}{m_f} + 1\right) h_3$$

$$\cong \frac{m_a}{m_f} h_3 \quad as \quad \frac{m_a}{m_f} >>>>> 1$$

$$\therefore \frac{m_a}{m_f}(h_3 - h_2) = CV$$

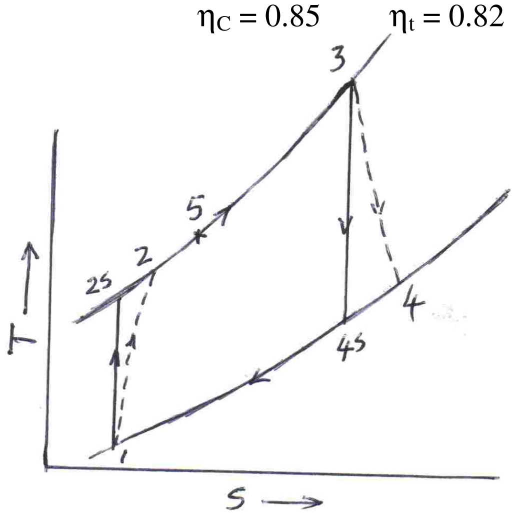
$$or \quad \frac{m_a}{m_f} = \frac{CV}{(h_3 - h_2)} \qquad i.e., \quad \frac{m_a}{m_f} = \frac{CV}{C_P(T_3 - T_2)}$$

Problems:

1. **In a G.T. installation, the air is taken in at 1 bar and 15^{0}C and compressed to 4 bar. The isentropic η of turbine and the compressor are 82% and 85% respectively. Determine (i) compression work, (ii) Turbine work, (iii) work ratio, (iv) Th. η.**

   What would be the improvement in the th. η if a regenerator with 75% effectiveness is incorporated in the cycle. Assume the maximum cycle temperature to be 825^{0}K.

Solution: $P_1 = 1$ bar $\quad T_1 = 288^0K \quad P_2 = 4$ bar $\quad T_3 = 825^0K$
$\eta_C = 0.85 \quad \eta_t = 0.82$



Process 1-2 is isentropic i.e., $\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}}$

$$\therefore T_{2s} = 288\,(4)^{\frac{0.4}{1.4}} = 428.14^0\,K$$

But $\eta_C = \frac{T_{2s} - T_1}{T_2 - T_1}$ $\quad$ *i.e.*, $0.85 = \frac{428.14 - 288}{T_2 - 288}$ $\quad \therefore T_2 = 452.87^0\,K$

Process 3-4 is isentropic $\quad$ *i.e.*, $\frac{T_{4s}}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{r-1}{r}}$ $\quad \therefore T_{4s} = 825\left(\frac{1}{4}\right)^{\frac{0.4}{1.4}} = 554.96$

But $\eta_t = \frac{T_3 - T_4}{T_3 - T_{4s}}$ $\quad$ *i.e.*, $0.82 = \frac{825 - T_4}{825 - 554.96}$ $\quad \therefore T_4 = 603.57^0\,K$

(i) Compressor work, W_C $= C_P\,(T_2 - T_1)$
$= 1.005\,(452.87 - 288) = 165.69$ kJ/kg

(ii) Turbine work, W_t $= C_P\,(T_3 - T_4)$
$= 1.005\,(825 - 603.57) = 222.54$ kJ/kg

(iii) Work ratio = $W_R = \frac{Net\ work\ output}{Turbine\ work} = \frac{W_T - W_C}{W_T} = 0.255$

(iv) Th. η, $\eta_{th} = \frac{W_{net}}{Q_H} = \frac{222.54 - 165.69}{C_P(825 - 452.87)} = \frac{56.85}{373.99}$
$= 15.2\%$

we have effectiveness $= \frac{T_5 - T_2}{T_4 - T_2} = 0.75 = \frac{T_5 - 452.87}{603.57 - 452.87}$

$$\therefore T_5 = 565.89^0 K$$

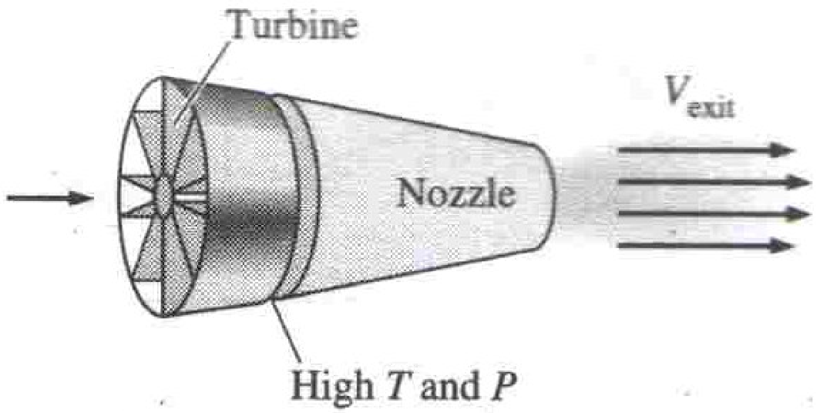
$\therefore$ Heat supplied, $Q_H^{1} = Q_{5\text{-}3} = C_P(T_3 - T_5)$
$= 1.005\,(825 - 565.89)$
$= 260.4$ kJ/kg

$$\therefore \eta_{th} = \frac{W_T - W_C}{Q_H^{1}} = \frac{56.85}{260.4} = 0.218$$

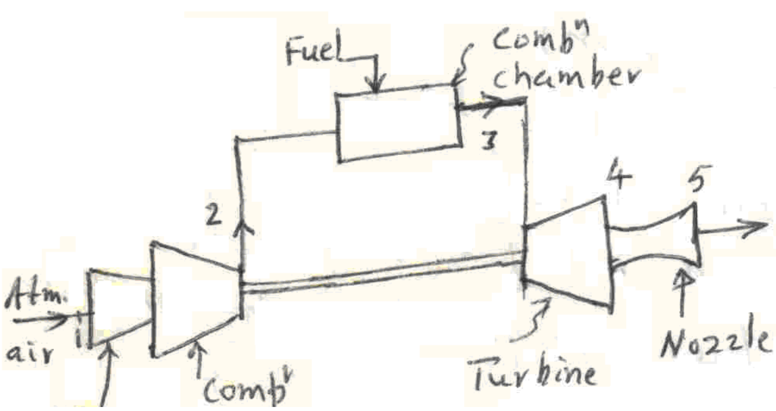
$\therefore$ Improvement in η_{th} due to regenerator $= \frac{0.218 - 0.152}{0.152}$
$= 0.436$ $\quad$ i.e., 43.6%

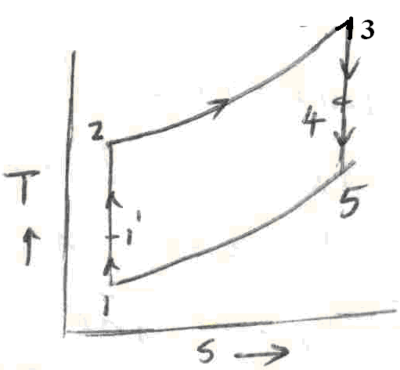
Ideal Jet-Propulsion Cycles:

Gas turbine engines are widely used to power air-crafts because they are light and compact and have a high power-to-weight ratio. Air craft gas turbine operates on an open cycle called a jet-propulsion cycle. The ideal jet propulsion cycle differs from the simple ideal Brayton cycle in that the gases are not expanded to the ambient pressure in the turbine. Instead, they are expanded to a pressure such that the power produced by the turbine is just sufficient to drive the compressor and the auxiliary equipment, such as a small generator and hydraulic pumps. That is, the net work output of a jet propulsion cycle is zero. The gases that exit the turbine at relatively high pressure are subsequently accelerated in a nozzle to provide the thrust to propel the air craft. Also, air craft gas turbine operate at higher pressure ratios (typically between 10-25), and the fluid passes through a diffuser first, where it is decelerated and its pressure is increased before it enters the compressor.



Air standard Jet Propulsion cycle:





An air standard Brayton or Joule cycle is the basic cycle for jet propulsion. The jet engine consists practically of the same type of components as in gas turbine plants, namely a compressor, a combustion chamber and a turbine. The only difference is that, the former also consists of an inlet diffuser, where the air entering from the atmosphere is decelerated and slightly compressed, and an exit nozzle, where the products of combustion expand to the pressure of the surroundings with increase in relative velocity. Such a plant and the corresponding T-S diagram are shown in Fig. In this plant, the work output of the turbine is just sufficient to drive the compressor.

Air enters the diffuser from the atmosphere, is slightly compressed from state 1 to state 1^1. It then enters the compressor, where it is further compressed to state 2. The compressed air then flows into the combustion chamber where it burns the fuel at constant pressure from state 2 to state 3 and the products of combustion then expand in the turbine from state 3 to state 4 developing power which is just sufficient to drive the compressor. Further expansion of the burnt gases takes place in the exit nozzle at state 5, after which the gases make an exit into the atmosphere with a very high velocity. The momentum of the exhaust gases flowing at high velocity from the nozzle result in a thrust upon the aircraft on which the engine is installed. In an actual jet propulsion plant, there is a slight pressure drop in the combustion chamber and the processes of combustion and expansion are not strictly reversible adiabatic. The thrust of a jet plane is the propulsive force i.e.,

$F = \dot{m}_a(\bar{v}_5 - \bar{v}_1) + \dot{m}_f \bar{v}_5$ where $\dot{m}_a$ = mass flow rate of air,

$\bar{v}_1$ & $\bar{v}_5$ = inlet and exit velocity of the fluid

$\dot{m}_f$ = mass flow rate of fuel in the combustion chamber

but since $\dot{m}_f$ is very small, $\dot{m}_f \bar{v}_5$ term is neglected.

$\therefore F = \dot{m}_a(\bar{v}_5 - \bar{v}_1)$

Turbo jet:

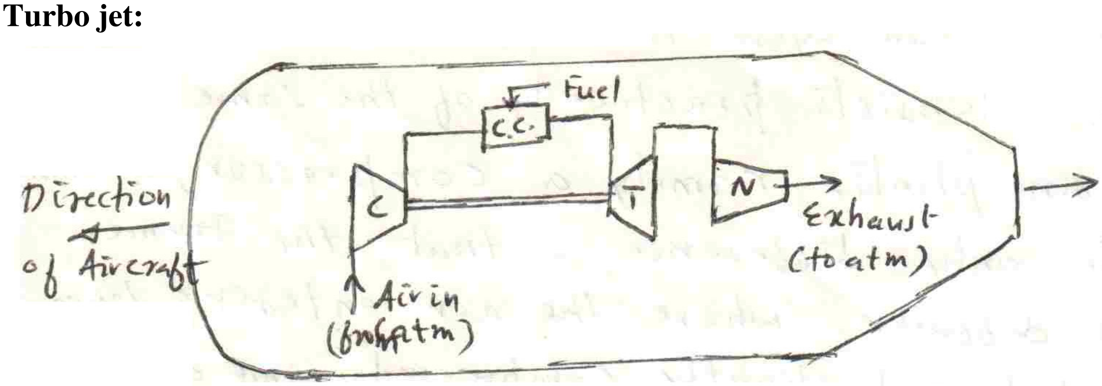


Figure shows a turbo jet unit. Power produced by the turbine is just sufficient to drive the compressor. The exhaust gases from the turbine which are at a higher pressure than atmosphere are expanded in the nozzle and a very high velocity jet is produced which provides a forward motion to the air-craft by the jet reaction (Newton's third law of motion).
At higher speeds the turbo jet gives higher propulsion efficiency. The turbo-jets are most suited to the air-crafts traveling above 800 km/hr. The overall η of a turbo jet is the product of the thermal η of the gas turbine plant and the propulsive η of the jet (nozzle).

Turbo-Propellers:

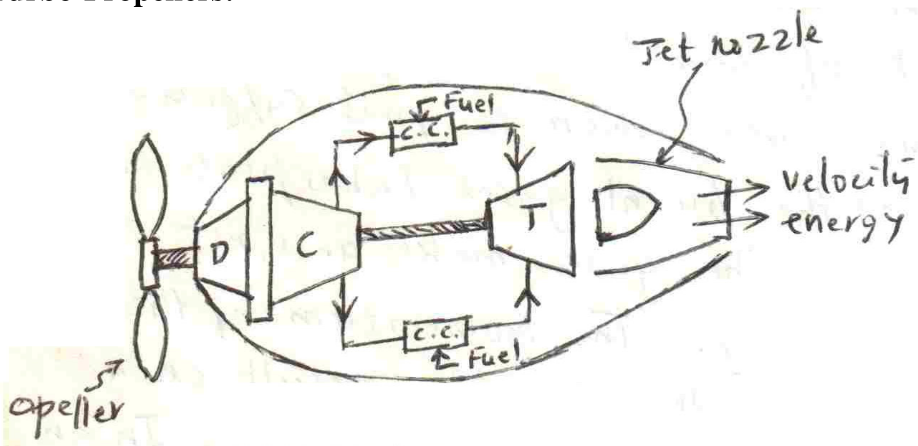


Figure shows a turbo-propeller system used in air-crafts. Here expansion takes place partly in turbine (80%) and partly (20%) in the nozzle. The power developed by the turbine is consumed in running the compressor and the propeller. The propeller and jet produced by the nozzle give forward motion to the aircraft.

It is having an added advantage over turbojet i.e., low specific weight and simplicity in design and propeller i.e., high power for take-off and high propulsion η at speeds below 600 km/hr.

Its overall η is improved by providing the diffuser before compressor, which increases the pressure rise. This pressure rise takes place due to conversion of kinetic energy of the incoming air (equal to air-craft velocity) into pressure energy by the diffuser. This type of compression is called 'ram effect'.

Ram jet: Ram jet engines have the capacity to fly at supersonic speeds. A diffuser increases the pressure of incoming air which is adequate to self ignite the fuel. In a ram jet engine the temperature of the rammed air is always above the self ignition temperature of the fuel employed. The ram-jet engine consists of a diffuser, combustion chamber and nozzle. The air enters the ram-jet plant with supersonic speed and is slowed down to sonic velocity in the supersonic diffuser, consequently the pressure suddenly increases to the formation of shock wave. The pressure of the air is further increased in the subsonic diffuser, increasing the temperature of air above the ignition temperature. The burning of the fuel takes place in the combustion chamber with the help of flame stabilization. The high pressure and high temperature gases pass through the nozzle where the pressure energy is converted to kinetic energy. The high velocity gases leaving the nozzle is a source of forward thrust to the ram-jet.

Advantages:

i) No moving parts
ii) Wide variety of fuels may be used
iii) Light in weight

The major short coming of ram-jet engine is that it cannot be started of its own. It has to be accelerated to certain flight velocity by some launching device. It is always equipped with a small turbo-jet which starts the ram-jet.

Rocket Propulsion: Similar to jet propulsion, the thrust required for rocket propulsion is produced by the high velocity jet of gases passing through the nozzle. But the main difference is that in case of jet propulsion the oxygen required for combustion is taken from the atmosphere and fuel is stored whereas for rocket engine, the fuel and oxidizer both are contained in a propelling body and as such it can function in vacuum also.

Rockets are classified as follows:

I According to the type of propellants

i) Solid propellant rocket
ii) Liquid propellant rocket

II According to the number of motors

i) Single-stage rocket (consists of one rocket motor)
ii) Multi-stage rocket (consists more than one rocket motor)

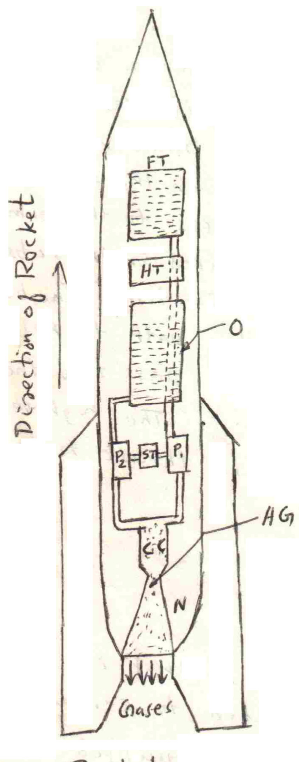


Fig :- Rocket

FT = Fuel tank
HT = Hydrogen peroxide tank
O = Oxidizer tank
ST = Steam turbine
P_1, P_2 = Pumps
C.C = Combustion chamber
HG = Hot gases
N = Nozzle

Figure shows a simple type single stage liquid propellant (the fuel and the oxidizer are commonly known as propellants) rocket. It consists of a fuel tank FT, an oxidizer tank O, two pumps P_1, P_2, a steam turbine ST and a combustion chamber C.C. The fuel tank contains alcohol and oxidizer tank contains liquid oxygen. The fuel and the oxidizer are supplied by the pumps to the combustion chamber where the fuel is ignited by electrical means. The pumps are driven with the help of steam turbine. Here the steam is produced by mixing a very concentrated hydrogen-peroxide with potassium permanganate. The products of combustion are discharged from the combustion chamber through the nozzle N. So the rocket moves in the opposite direction.

In some modified forms, this type of rockets may be used in missiles.

Uses:

1. Long range artillery
2. Signaling and fire work display
3. Jet assisted take-off
4. For satellites
5. For space ships
6. Research