MODULE 4

FREQUENCY DOMAIN ANALYSIS

LESSON STRUCTURE

OBJECTIVES:

- To demonstrate Stability Determine Gain & Phase Margins Medium effort.
- To demonstrate applications of the frequency response to analysis of system stability (the Nyquist criterion), relating the frequency response to transient performance specifications.
- To demonstrate frequency response and to determine stability of control system applying using Bode plot.
- To demonstrate to plot graph of amplitude plot, usually in the log-log scale and a phase plot, which is usually a linear-log plot.

4.1. Nyquist Stability criterion

This graphical method, which was originally developed for the stability analysis of feedback amplifiers, is especially suitable for different control applications. With this method the closed-loop stability analysis is based on the locus of the open-loop frequency response $G_0(j\omega)$. Since only knowledge of the frequency response $G_0(j\omega)$ is necessary, it is a versatile practical approach for the following cases:

a) For many cases $G_0(j\omega)$ can be determined by series connection of elements whose parameters are known.

b) Frequency responses of the loop elements determined by experiments or $G_0(j\omega)$ can be considered directly.

c) Systems with dead time can be investigated.

d) Using the frequency response characteristic of $G_0(j\omega)$ not only the stability analysis, but also the design of stable control systems can be easily performed.

4.2. Nyquist criterion using Nyquist plots

To derive this criterion one starts with the rational transfer function of the *open loop*

$$G_0(s) = \frac{N_0(s)}{D_0(s)}$$

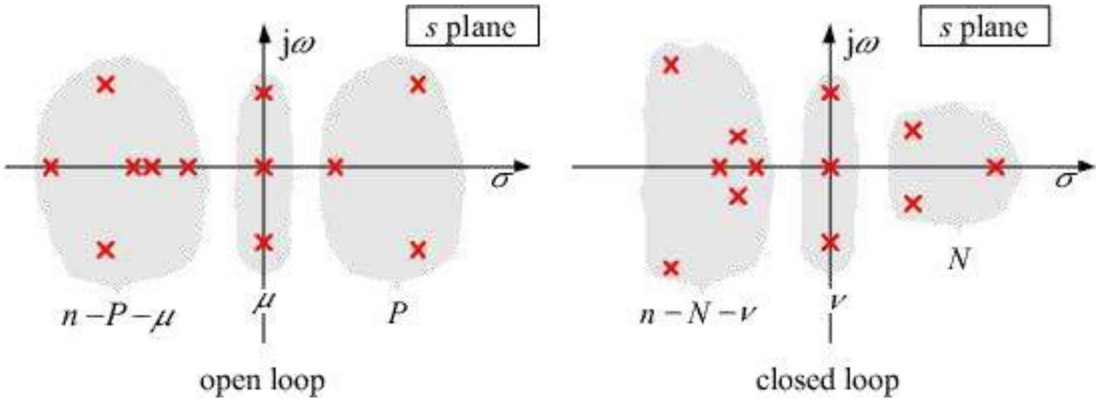


Figure: Poles of the open and closed loop in the s plane (multiple poles are counted according to their multiplicity)

To determine $\Delta\varphi_S$, the locus $G'(j\omega) = 1 + G_0(j\omega)$ can be drawn on the Nyquist diagram and the phase angle checked. Expediently one moves this curve by 1 to the left in the $G_0(j\omega)$ plane. Thus for stability analysis of the closed loop the locus $G_0(j\omega)$ of the open loop according to Figure 5.5 has to be drawn.

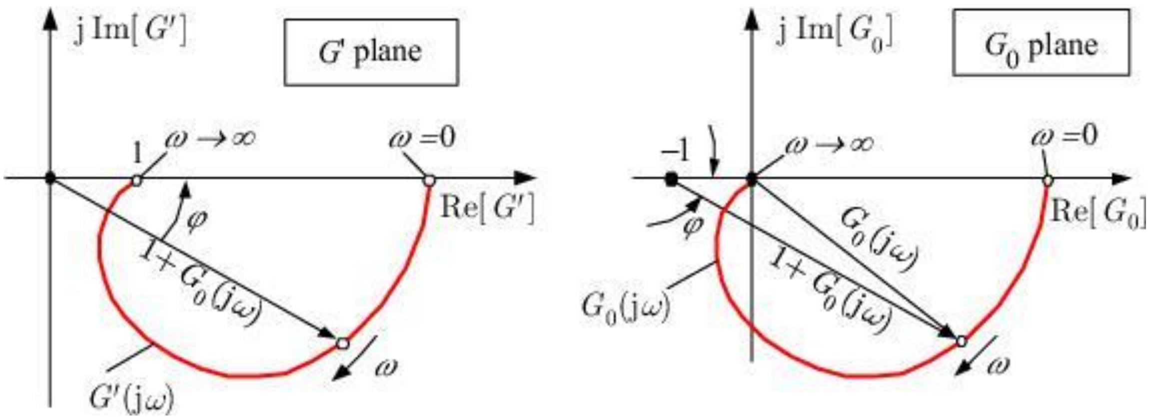


Figure : Nyquist diagrams of $G'(j\omega)$ and $G_0(j\omega)$

Here $\Delta\varphi_S$ is the continuous change in the angle of the vector from the so called *critical point* (-1,j0) to the moving point on the locus of $G_0(j\omega)$ for $0 \le \omega \le \infty$. Points where the locus passes through the point (-1,j0) or where it has points at infinity correspond to the zeros and poles of $G'(s)$ on the imaginary axis, respectively. These discontinuities are not taken into account for the derivation of .

Figure shows an example of a $G_0(j\omega)$

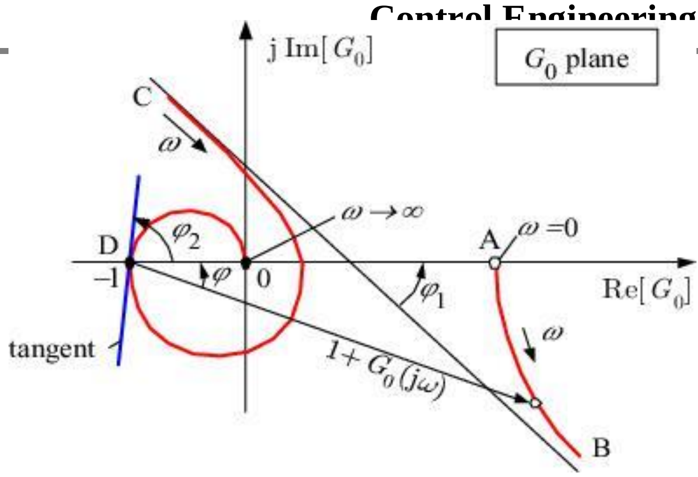


Figure: Determination of continuous changes in the angle $\Delta\varphi_S$

where two discontinuous changes of the angle occur. Thereby the continuous change of the angle consists of three parts

$$\Delta\varphi_S = \Delta\varphi_{AB} + \Delta\varphi_{CD} + \Delta\varphi_{DO}$$

$$= -\varphi_1 - (2\pi - \varphi_1 - \varphi_2) - \varphi_2 = -2\pi \ .$$

The rotation is counter clockwise positive.

As the closed loop is only asymptotically stable for $N = \nu = 0$, then from the *general case of the Nyquist criterion* follows:

The closed loop is asymptotically stable, if and only if the continuous change in the angle of the vector from the critical point (-1,j0) to the moving point of the locus $G_0(j\omega)$ of the open loop is

$$\Delta\varphi_S = (P + \mu/2)\pi \ .$$

For the case with a *negative* gain K_0 of the open loop the locus is rotated by 180° relative to the case with a positive K_0. The Nyquist criterion remains valid also in the case of a *dead time* in the open loop.

4.3. Simplified forms of the Nyquist criterion

It follows from that for an open-loop stable system, that is $P = 0$ and $\mu = 0$, then $\Delta\varphi_S = 0$. Therefore the Nyquist criterion can be reformulated as follows:

If the open loop is asymptotically stable, then the closed loop is only asymptotically stable, if the frequency response locus of the open loop does neither revolve around or pass through the critical point (-1,j0).

Another form of the simplified Nyquist criterion for $G_0(s)$ with poles at $s = 0$ is the so called 'left-hand rule':

The open loop has only poles in the left-half s plane with the exception of a single or double pole at $s = 0$ (P, I or I_2 behaviour). In this case the closed loop is only stable, if the critical point (-1,j0) is on the *left* hand-side of the locus $G_0(j\omega)$ in the direction of increasing values of ω.

This form of the Nyquist criterion is sufficient for most cases. The part of the locus that is significant is that closest to the critical point. For very complicated curves one should go back to the general case. The left-hand rule can be graphically derived from the generalised locus

The orthogonal (σ, ω)-net is observed and asymptotic stability of the closed loop is given, if a curve with $\sigma < 0$ passes through the critical point (-1,j0). Such a curve is always on the left-hand side of $G_0(j\omega)$.

4.4. The Nyquist criterion using Bode plots

Because of the simplicity of the graphical construction of the frequency response characteristics of a given transfer function the application of the Nyquist criterion is often more simple using Bode plots. The continuous change of the angle $\Delta\varphi_S$ of the vector from the critical point (-1,j0) to the locus of $G_0(j\omega)$ must be expressed by the amplitude and phase response of $G_0(j\omega)$. From figure

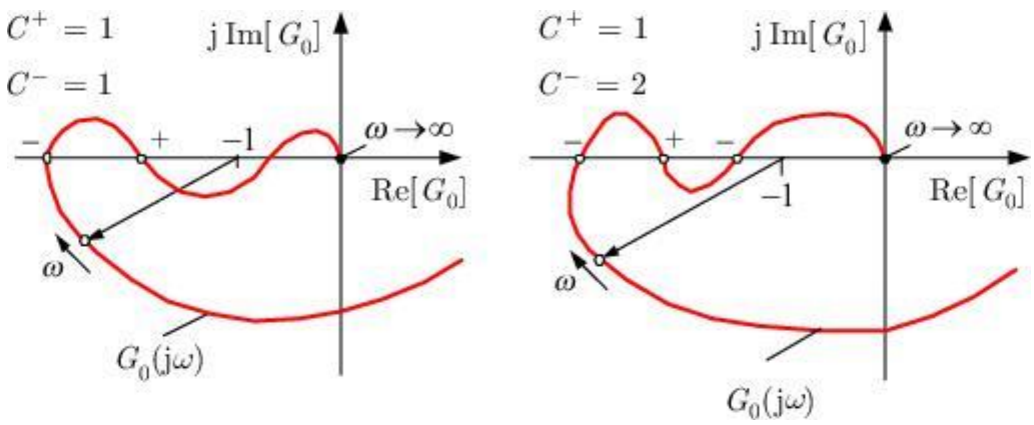


Figure : Positive (+) and negative (-) intersections of the locus $G_0(j\omega)$ with the real axis on the left-hand side of the critical point

it can be seen that this change of the angle is directly related to the count of intersections of the locus with the real axis on the left-hand side of the critical point between $(-\infty, -1)$. The Nyquist criterion can therefore also represented by the count of these intersections if the gain of the open loop is positive.

Regarding the intersections of the locus of $G_0(j\omega)$ with the real axis in the range $(-\infty, -1)$, the transfer from the upper to the lower half plane in the direction of increasing ω values are treated as *positive intersections* while the reverse transfer are *negative intersections*

(Figure 5.7). The change of the angle is zero if the count of positive intersections S^+ is equal to the count of negative intersections S^-. The change of the angle $\Delta\varphi_S$ depends also on the number of positive and negative intersections and if the open loop does not have poles on the imaginary axis, the change of the angle is

$$\Delta\varphi_S = 2\pi(C^+ - C^-)\,.$$

In the case of an open loop containing an integrator, i.e. a single pole in the origin of the complex plane ($\mu = 1$), the locus starts for $\omega = 0$ at $\delta - j\infty$, where an additional $+\pi/2$ is added to the change of the angle. For proportional and integral behaviour of the open loop

$$\Delta\varphi_S = 2\pi(C^+ - C^-) + \mu\pi/2 \qquad \mu = 0,1$$

is valid. In principle this relation is also valid for $\mu = 2$, but the locus starts forat $\omega = 0$ $-\infty + j\delta$ (Figure 5.8), and this intersection would be counted

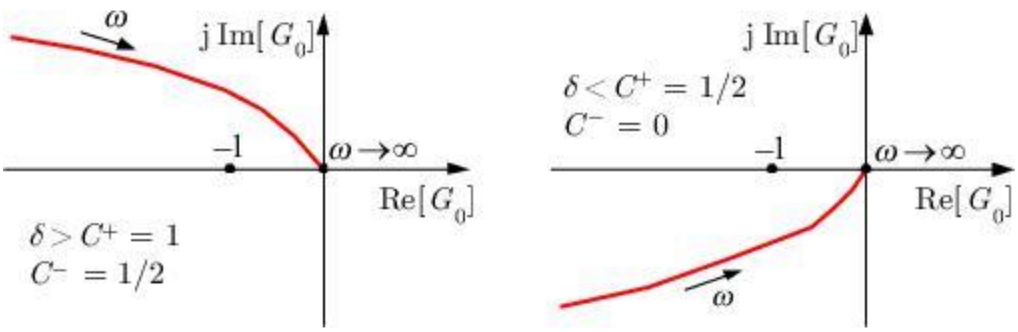


Figure : Count of the intersections on the left-hand side of the critical point for I_2 behaviour of the open loop

as a negative one if $\delta > 0$, i.e. if the locus for small ω is in the upper half plane of the real axis. But de facto there is for $\delta > 0$ (and accordingly $\delta < 0$) no intersection. This follows from the detailed investigation of the discontinuous change of the angle, which occurs at $\omega = 0$. As only a continuous change of the angle is taken into account and because of reason of symmetry the start of the locus at $\omega = 0$ is counted as a half intersection, positive for $\delta < 0$ and negative for $\delta > 0$, which is analogous to the definition given above For continuous changes of the angle

$$\Delta\varphi_S = 2\pi(C^+ - C^-) \qquad (\mu = 2)$$

The open loop with the transfer function $G_0(s)$ has P poles in the left-half s plane and possibly a single ($\mu = 1$) or double pole ($\mu = 2$) at $s = 0$. If the locus of $G_0(j\omega)$ has C^+ positive and C^- negative intersections with the real axis to the left of the critical point, then the closed loop is only asymptotically stable, if

$$D^* = C^+ - C^- = \begin{cases} \dfrac{P}{2} & \text{for } \mu = 0,1 \\ \dfrac{P+1}{2} & \text{for } \mu = 2 \end{cases}$$

is valid. For the special case, that the open loop is stable ($P = 0$, $\mu = 0$), the number of positive and negative intersections must be equal.

From this it follows that the difference of the number of positive and negative intersections in the case of $\mu = 0,1$ is an integer and for $\mu = 2$ not an integer. From this follows immediately, that for $\mu = 0,1$ the number P is even, for $\mu = 2$ the number $P+1$ is uneven and therefore in all cases P is an even number, such that the closed loop is asymptotically stable. This is only valid if $D^* \geq 1$.

The Nyquist criterion can now be transferred directly into the representation using frequency response characteristics. The magnitude response $A_0(\omega)_{dB}$, which corresponds to the locus $G_0(j\omega)$, is always positive at the intersections of the locus with the real axis in the range of $(-\infty, -1)$. These points of intersection correspond to the crossings of the phase response $\varphi_0(\omega)$ with lines $\pm 180°$, $\pm 540°$ etc., i.e. a uneven multiple of 180°. In the case of a positive intersection of the locus, the phase response at the $\pm(2k+1)\,180°$ lines crosses from below to top and reverse from top to below on a negative intersection as shown in Figure 5.9. In the following these crossings

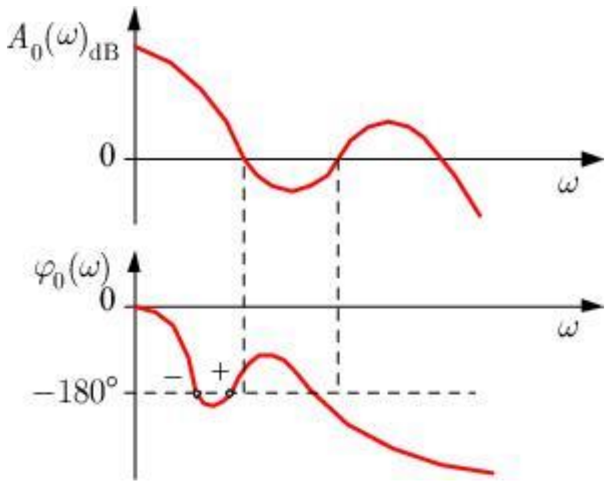


Figure : Frequency response characteristics of $G_0(j\omega) = A_0(\omega)\,e^{j\varphi_0(\omega)}$ and definition of positive (+) and negative (-) crossings of the phase response $\varphi_0(\omega)$ with the -180° line

will be defined as positive (+) and negative (-) crossings of the phase response $\varphi_0(\omega)$ over the particular $\pm(2k+1)\,180°$ lines, where $k = 0, 1, 2, \ldots$ may be valid. If the phase response starts at -180° this point is counted as a half crossing with the corresponding sign. Based on the discussions above the Nyquist criterion can be formulated in a form suitable for frequency response characteristics:

The open loop with the transfer function $G_0(s)$ has P poles in the right-half s plane, and possibly a single or double pole at $s = 0$. C^+ are the number of positive and C^- of negative crossings of the phase response $\varphi_0(\omega)$ over the $\pm(2k+1)\,180°$ lines in the frequency range where $A_0(\omega)_{dB} > 0$ is valid. The closed loop is only asymptotically stable, if

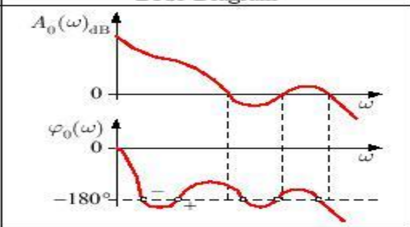
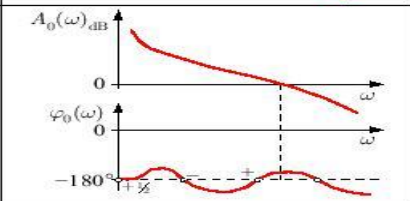
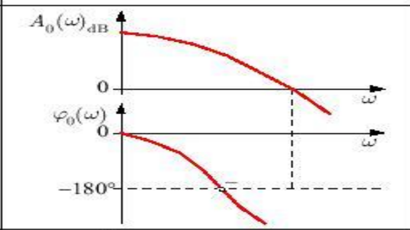
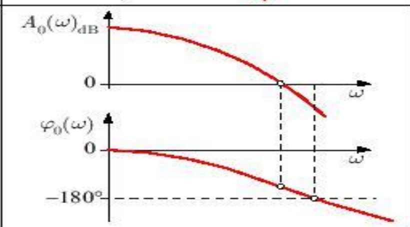
$$D^* = C^+ - C^- = \begin{cases} \frac{P}{2} & \text{for } \mu = 0,1 \\ \frac{P+1}{2} & \text{for } \mu = 2 \end{cases}$$

is valid. For the special case of an open-loop stable system ($P = 0$, $\mu = 0$)

$$D^* = C^+ - C^- = 0$$

must be valid.

Table 7.1: Examples of stability analysis using the Nyquist criterion with frequency response characteristics

No.	Bode Diagram	Stability Analysis
1		$\left.\begin{matrix} S^+ = 1 \\ S^- = 2 \\ D^* = -1 \\ P = 2 \end{matrix}\right\} \Rightarrow D^* \neq P/2$: unstable
2		$\left.\begin{matrix} S^+ = 3/2 \\ S^- = 1 \\ D^* = 1/2 \\ P = 0 \end{matrix}\right\} \Rightarrow D^* = \frac{P+1}{2}$: stable if 2 poles in the origin
3		$\left.\begin{matrix} S^+ = 0 \\ S^- = 1 \\ D^* = -1 \\ P = 0 \end{matrix}\right\} \Rightarrow D^* \neq P/2$: unstable
4		$\left.\begin{matrix} S^+ = 0 \\ S^- = 0 \\ D^* = 0 \\ P = 0 \end{matrix}\right\} \Rightarrow D^* = P/2$: stable

Finally the 'left-hand rule' will be given using Bode diagrams, because this version is for the most cases sufficient and simple to apply.

The open loop has only poles in the left-half s plane with the exception of possibly one single or one multiple pole at $s = 0$ (P, I or I_2 behaviour). In this case the closed loop is only asymptotically stable, if $G_0(j\omega)$ has a phase of $\varphi_0 > -180°$ for the *crossover frequency* ω_C at $A_0(\omega_C)_{dB} = 0$.

This stability criterion offers the possibility of a practical assessment of the 'quality of stability' of a control loop. The larger the distance of the locus from the critical point the farther is the closed loop from the stability margin. As a measure of this distance the terms gain margin and phase margin are introduced according to Figure below

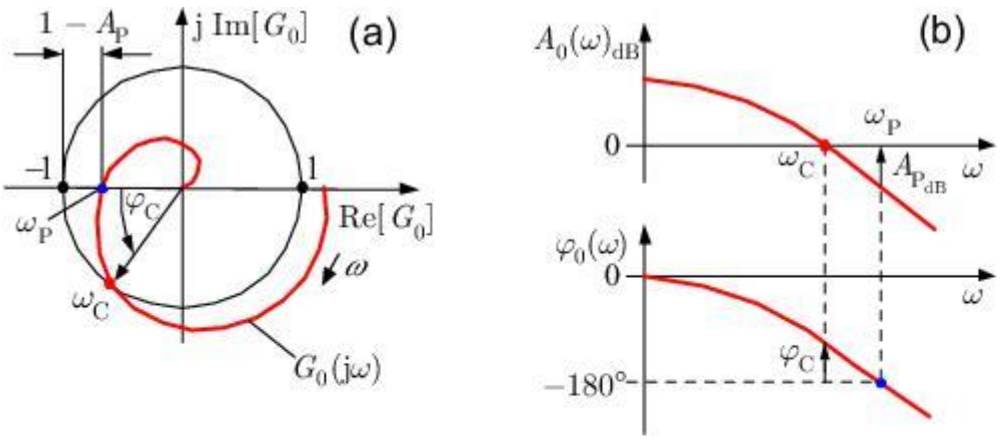


Figure : Phase and gain margin φ_C and A_P or $A_{P_{dB}}$, respectively, in the (a) Nyquist diagram and (b) Bode diagram

Example Problems:

Q1 The polar plot of the open-loop transter of feedback control system intersects the real axis at—2 Calculate gain margin (in dB) of the system.

Ans. Given a = −2

$$\text{Gain margin} = 20 \log_{10} \frac{1}{|a|}$$

$$= 20 \log_{10} |0.5|$$

gain margin = −6.02 dB.

Q2. What is the gain margin of a system in decibels if its Nyquist plot cuts the negative real axis at — 0.7?

Ans.

a = —0.7

$$\text{gain margin} = -20 \log_{10} \frac{1}{|a|}$$

$$= -20 \log_{10} \frac{1}{|0.7|}$$

gain margin = −3 dB.

Q4. Consider a feed lock system with the open-loop transfer function. Given by

$$G(s) = \frac{K}{s\,(2s+1)}$$

Examine the stability of the closed-loop system. Using Nyquist stability theory

Ans. $G(s)H(s) = \frac{K}{s(2s+1)}$

Here poles are $s = 0, -\frac{1}{2}$. One pole is at origin and one is at $-\frac{1}{2}$. But no pole is at right half of s-plane.

$\therefore \quad P = 0$

For stability,

$N = Z - P$

$Z = P + N$

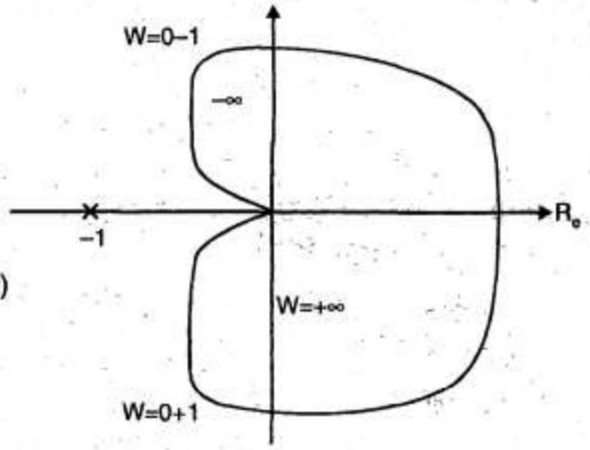
$N \Rightarrow$ Number of clockwise enrichment about $(-1 + j0)$

As there is no enrichment, so $N = 0$

$Z = 0 + 0$

$= 0$

Thus system is stable.



Q 5. Draw the Nyquest plot for the open loop transfer function given below:

$$G(s)H(s) = \frac{1}{s(1+2s)(1+s)}$$

and obtain the gain margin and phase margin

Ans. Given $G(s)H(s) = \frac{1}{s(1+2s)(1+s)}$

Put $s = j\omega$

$$G(j\omega)H(j\omega) = \frac{1}{j\omega(1+2j\omega)(1+j\omega)}$$

Rationalizing

$$G(j\omega)H(j\omega) = \frac{-3}{(1+4\omega^2)(1+\omega^2)} - j\frac{1-2\omega^2}{\omega(1+4\omega^2)(1+\omega^2)}$$

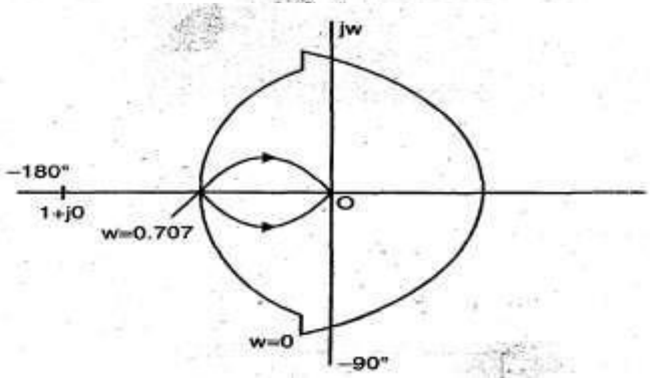
Equating imaginary parts to zero, real axis intersection is at

$$1 - 2\omega^2 = 0$$

$$\omega = 0.707$$

$|G(j\omega)H(j\omega)|_{j\omega=0.707} = 0.66$

Nyquist plot is as shown :



Q6. Consider a feed lock system with the open-loop transfer function. Given by

$$G(s) = \frac{K}{s(2s+1)}$$

Examine the stability of the closed-loop system. Using Nyquist stability theory.

Ans. $G(s)H(s) = \frac{K}{s(2s+1)}$

Here poles are $s = 0, -\frac{1}{2}$. One pole is at origin and one is at $-\frac{1}{2}$. But no pole is at right half of s-plane.

$\therefore \quad P = 0$

For stability,

$N = Z - P$

$Z = P + N$

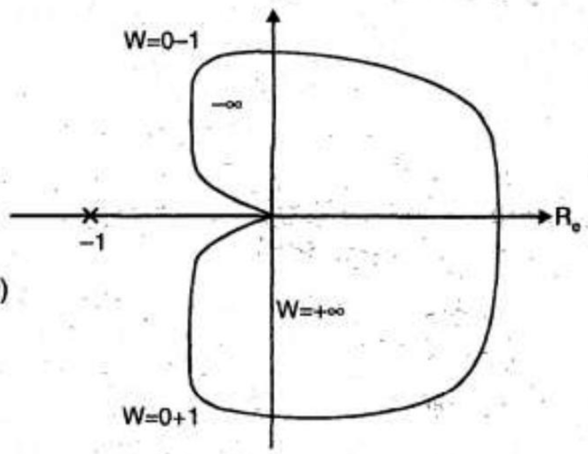
$N \Rightarrow$ Number of clockwise enrichment about $(-1 + j0)$

As there is no enrichment, so $N = 0$

$Z = 0 + 0$

$= 0$

Thus system is stable.



Q7. Sketch the Nyquist plot for the system with the open loop transfer function

$$\frac{K}{(j\omega+1)(j\omega+1.5)(j\omega+2)}$$

and determine the range of K for which the system is

Ans. Given $G(s)H(s) = \frac{K}{(s+1)(s+1.5)(s+2)}$

Put $\quad s = j\omega$

$$G(j\omega)H(j\omega) = \frac{K}{(s+1)(s+1.5)(s+2)}$$

Rationalizing and separating real and imaginary parts

$$= \frac{\left(3-4.5\,\omega^2\right)K}{\left(1+\omega^2\right)\left(2.25+\omega^2\right)\left(4+\omega^2\right)} - \frac{jK\left(6.5\omega-\omega^3\right)}{\left(1+\omega^2\right)\left(2.25+\omega^2\right)\left(4+\omega^2\right)}$$

To get point of intersection on real axis, equate imaglnary part to zero.

$$\Rightarrow \frac{K\left(6.5\omega-\omega^3\right)}{\left(1+\omega^2\right)\left(2.25+\omega^2\right)\left(4+\omega^2\right)} = 0$$

$$\omega = 2.55 \text{ rad/sec}$$

$$|G(j\omega)|_{\omega=2.25} = -0.24\,K$$

Intersection with imaginary axis :

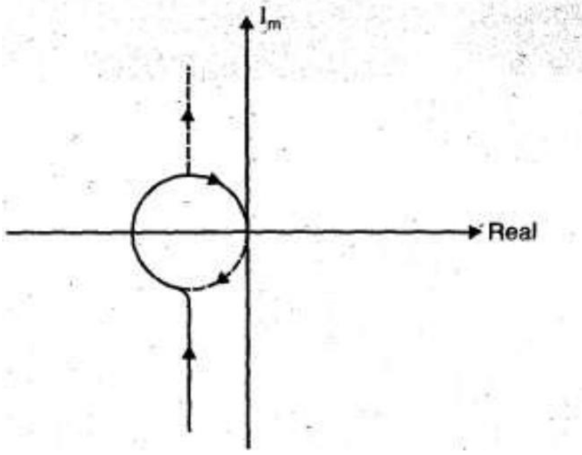
$$\omega = \sqrt{\frac{3}{4.5}} = 0.81$$

$|G(j\omega)\,H(j\omega)|_{\omega=2.25} = -0.028\,K$

For stability $-0.028\,K < -1$

$$K < 35.03.$$

The plot is as shown below :



Q.8. Sketch the Nyquist plot for system with

$$G(s)\,H(s) = \frac{(1+0.5s)}{s^2(1+0.1s)(1+0.02s)}$$

Comment on the stability.

Ans. $G(s)H(s) = \dfrac{(1+0.5s)}{s^2(1+0.1s)(1+0.02s)}$

Put $s = j\omega$

$$G(j\omega)H(j\omega) = \frac{(1+0.5j\omega)}{(j\omega)^2(1+0.1j\omega)(1+0.02j\omega)}$$

The mapping for Nyquist contour is as follow.
Along jω axis for various values of ω, G (jω) H (jω) is plotted.

ω	0	∞	0.1	1.0	2.0	4.0	10.0	20.00

where $|G(j\omega)H(j\omega)| = \dfrac{\sqrt{1+0.25\,\omega^2}}{\omega^2\sqrt{1+0.01\omega^2}\,\sqrt{1+0.0004\,\omega^2}}$

$$\angle G(j\omega)H(j\omega) = \tan^{-1}0.5\,\omega - 180^\circ - \tan^{-1}0.1\,\omega - \tan^{-1}0.02\,\omega$$

Point of intersection of G (jω) H (jω)

$$\angle G(j\omega)H(j\omega) = -180^\circ + \tan^{-1}0.5\,\omega - 180^\circ - \tan^{-1}0.1\,\omega - \tan^{-1}0.02\,\omega$$

$$= 180^\circ$$

$$\tan^{-1}0.5\,\omega = \tan^{-1}0.1\,\omega + \tan^{-1}0.02\,\omega$$

$$0.5\,\omega = \frac{(0.1)(\omega+0.02\,\omega)}{1-0.002\,\omega^2}$$

$$(1-0.002\,\omega^2)(0.5\,\omega) = 0.1\,\omega + 0.02\,\omega$$

$$0.5\,\omega - 0.001\,\omega^3 = 0.12\,\omega$$

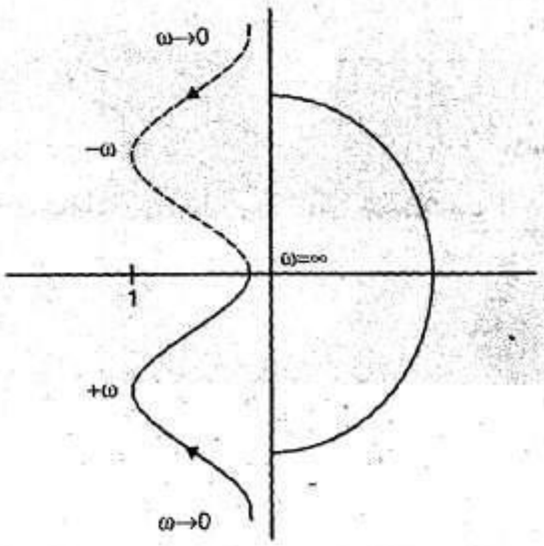
$$0.38 = 0.001\,\omega^2$$

$$\omega = 19.49 \text{ rad/sec}$$

$$|G(j\omega)H(j\omega)|_{\omega=19.49} = \frac{1+j0.25\times19.49}{(j19.49)^2(1+j0.1\times19.49)(1+j0.002\times19.49)}$$

$$= \frac{1+4.8725j}{(19.49j)^2(1+1.949j)(1+0.0389j)}$$

The plot is as shown below :



There is no encirclement of 1 + j0, hence system is stable.

Q 9. How is it possible to make assessment of relative stability using Nyquist criterion? Construct Nyquist plot for the system whose open loop transfer function is

$$G(s)\,H(s) = \frac{K(1+s)^2}{s^3}.$$

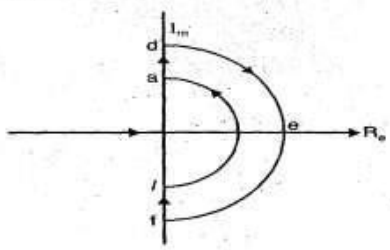
Find the range of K for stability.

Ans.

- Nyquist critierion can be used to make assessment of relative stability.
- Using the characteristic equation the Nyquist plot is drawn. A feedback system is sable if and only if, the i.e. contour in the G (s) plane does not encircle the (—1, 0) point when the number of poles of G(s) in the right hand s plane is zero.
- If G (a) has P poles in the right hand plane, then the number of anticlockwise encirciements of the (—1, 0) point must be equal to P for a stable system, N=—P0
  where N = No of clockwise encirclements about (—1, 0) point in C (s) plane P0 = No of poles G (s) in RHP 0

Now given $G(s)\,H(s) = \frac{K(1+s)^2}{s^3}$

No. of poles at RHS of s-plane P = 0
For stability N = 0
Nyquist path is shown :



For path a – d, put $s = j\omega$, $0 < \omega < \infty$

$\omega = 0$, $G(j\omega)\,H(j\omega) = \infty \angle -270°$

$\omega = \infty$, $G(j\omega)\,H(j\omega) = 0 \angle -90°$

Rotational angle = −90° − (270°) = 180° anticlockwise

∴ Polar plot is shown by dark circle in following figure.

Draw mirror image for path f – i (in previous figure) path d – e – f will be origin

As term $\frac{1}{s^3}$ is present, there will be three semicircles of ∞ radius.

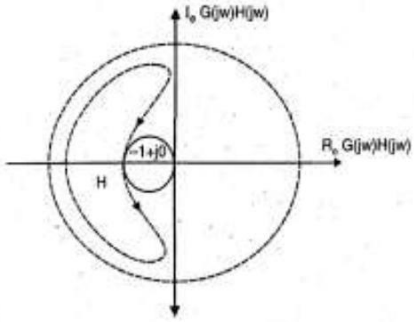
Or

Start point

At ω = −0 (i.e. pt 'i') G (jω) H (jω) = ∞ ∠270°

End point ω = −0 (i.e. pt 'a') G (jω) H (jω) = ∞ ∠−270°

Hence plot is as shown below :



4.5. Bode attenuation diagrams

If the absolute value $A(\omega)$ and the phase $\varphi(\omega)$ of the frequency response $G(j\omega) = A(\omega)\, e^{j\varphi(\omega)}$

are separately plotted over the frequency ω, one obtains the

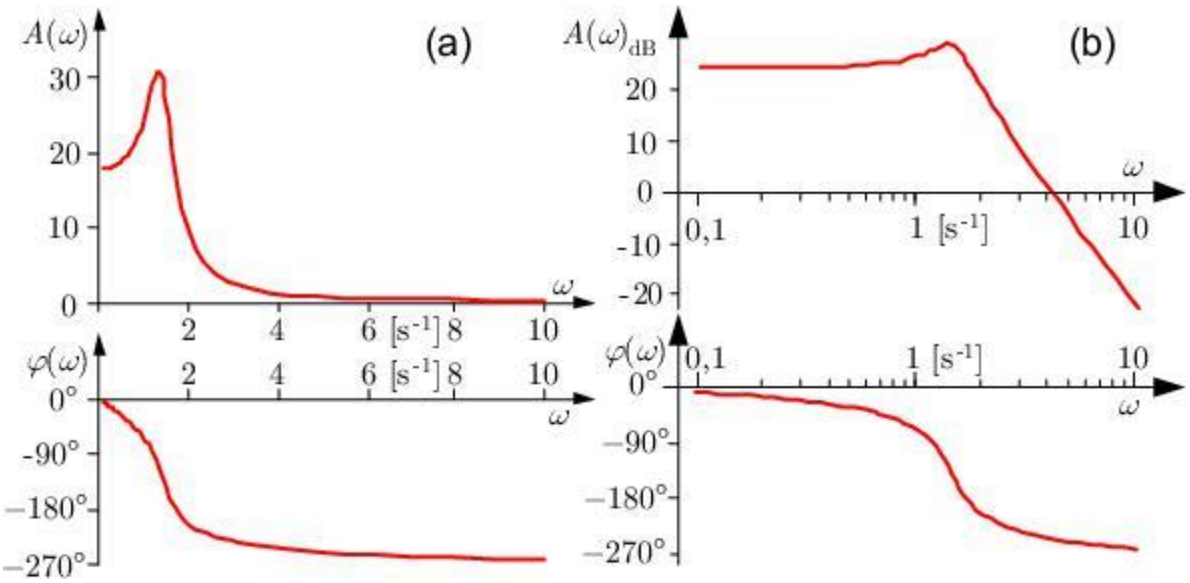


Figure 6.1: Plot of a frequency response: (a) linear, (b) logarithmic presentation (ω on a logarithmic scale) (Bode plot)

amplitude response and the *phase response*. Both together are the *frequency response characteristics*. $A(\omega)$ and ω are normally drawn with a logarithm and $\varphi(\omega)$ with a linear scale. This representation is called a *Bode diagram* or *Bode plot*. Usually $A(\omega)$ will be specified in decibels [dB] By definition this is

$$A(\omega)_{dB} = 20 \log_{10} A(\omega) \quad [\text{dB}]$$

The logarithmic representation of the amplitude response $A(\omega)_{dB}$ has consequently a linear scale in this diagram and is called the *magnitude*.

4.6. Stability analysis using Bode plots:

- The magnitude and phase relationship between sinusoidal input and steady state output of a system is known as frequency response.
- The polar plot of a sinusoidal transfer function G (jw) is plot of the magnitude of G (jw) versus the phase angle of G (jw) on polar coordinates as _co‘ varied from zero to infinity.
- The phase margin is that amount, of additional phase lag at the gain crossover frequency required to bring the system to the verge of instability.
- The gain margin is the reciprocal of the magnitude l G(jw) l at the frequency at which the phase angle as _1800.
- The inverse polar plot at G (jw) is a graph of 1/G (jw) as a function of w.
- Bode plot is a graphical representation of the transfer function for determining the stability of control system.
- Bode plot is a combination of two plot - magnitude plot and phase plot

- The transfer function having no poles and zeros in the right -half s-plane are called minimum phase transfer function.
- System with minimum phase transfer function are called minimum phase systems.
- The transfer function having poles and zeros in the right half s-plane are called non-minimum phase transfer functions systems with non-minimum phase transfer function. are called non-minimum phase system.
- In bode plot the relative stability of the system is determined from the gain margin and phase margin. .
- If gain cross frequency is less than phase cross over frequency then gain margin and phase margin both are positive and system is stable.
- If gain cross over frequency is greater than the phase crossover frequency than both gain margin and‘phase margin are negative.
- It gain cross over frequency is equal to me phase cross over trequency me gain marg and phase margin are zero and system is marginally stable.
- The maximum value of magnitude is known as resonant peak.
- The magnitude of resonant peak gives the information about the relative stability of the system.
- The frequency at which magnitude has maximum value is known as resonant frequency.
- Bandwidth is defined a the range of frequencies in which the magnitude of closed loop does not drop —3 db.

Example Problems:

Q1. Sketch the Bode Plot for the transfer function given by,

G(s) H(s) = 2 (s+0.25)/s² (s+1) (s+0.5)

and from Plot find (a) Phase and Gain cross rer frequencies (b) Gain Margin and Phase Margin. Is this System Stable?

Ans. Given $G(s)\,H(s) = \dfrac{2(s+0.25)}{s^2(s+1)(s+0.5)}$

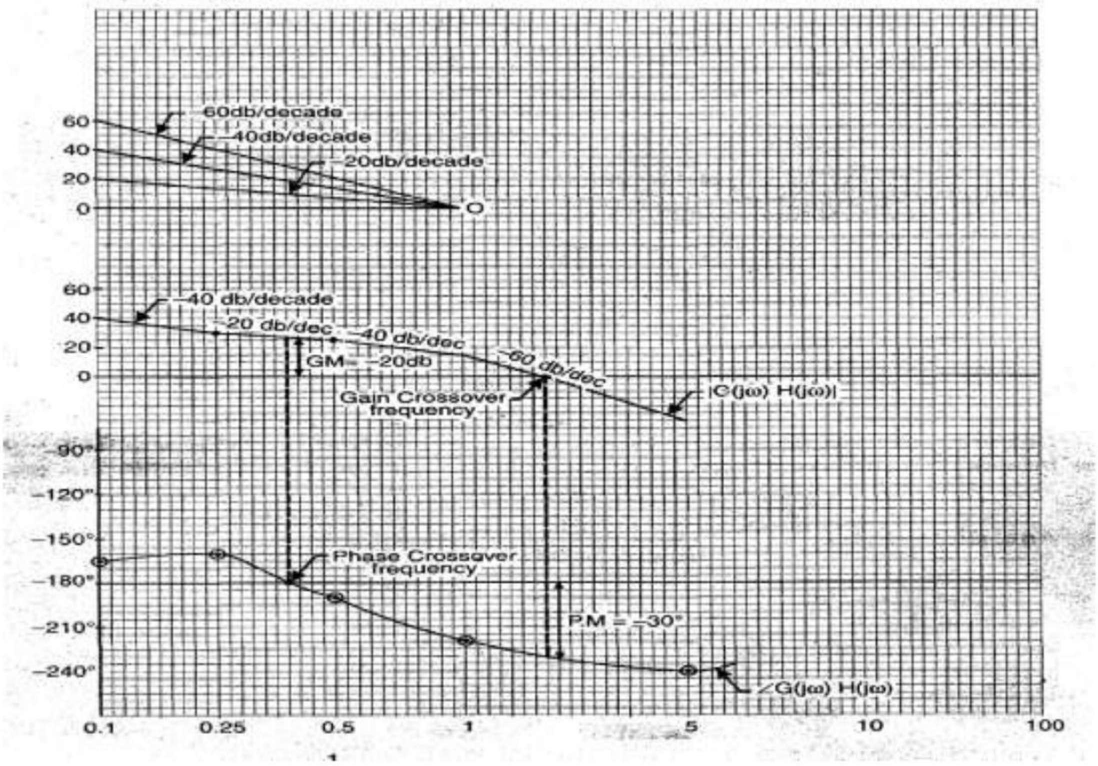
$$= \frac{\frac{2\times 0.25}{0.5}\left[\frac{s}{0.25}+1\right]}{s^2(s+1)\left[\frac{s}{0.5}+1\right]}$$

$$= \frac{1(4s+1)}{s^2(s+1)(2s+1)}$$

Put $s = j\omega$

$$G(j\omega)\,H(j\omega) = \frac{(j4\omega+1)}{(j\omega^2)(j\omega+1)(2j\omega+1)}$$

This is type 2 system, hence initial slope of bode plot = −40 dB/decade and the plot intersects 0 dB axis at $\omega = \sqrt{K} = \sqrt{1} = 1$ rad/sec. The comer frequencies are :



$\omega = \frac{1}{4} = 0.25$ rad/sec

$\omega = \frac{1}{2} = 0.5$ rad/sec

$\omega = 1$ rad/sec.

Frequency range is considered from $\omega = 0.1$ rad/sec to $\omega = 10$ rad/sec.

The plot is as shown.

As initial slope of plot is −40 dB/dec and comer frequency is 0.25 rad/sec. The plot after $\omega = 0.25$ has slope = −20 dB/dec.

After $\omega = 0.5$, slope is −40 dB/decade

After $\omega = 1$, slope is −60 dB/dec.

Phase Angle :

$\angle G(j\omega) H(j\omega) = \tan^{-1}(4\omega) - 180° - \tan^{-1}\omega - \tan^{-1} 2\omega.$

The phase angle for frequency range considered are calculated as :

ω	0.1	0.25	0.5	1	5
∠G (jω) H (jω)	−175.2	−175.2	−188	−212.4	−225

The gain crosses 0db axis at co = 1.24 rad/sec, the gain crossover frequency is co = 1.24 rad/sec.
The phase crosses —180° line at co = 0.4 rad/sec, therefore phase crossover frequency is co = 0.4 rad/sec.
At phase cross over the gain is 20 dB, therefore gain margin is —20 dB.
At gain crossover the phase angle is 2150, the phase margin is 180° + (—215°) = —35°.
As both gain and phase margins are negative, the system is unstable.

Q3. Sketch the bode plot for the transfer function given by

$$G(s) = \frac{23.7\,(1+j\omega)\,(1+0.2j\omega)}{(j\omega)\,(1+3j\omega)\,(1+0.5j\omega)\,(1+0.1j\omega)}$$

and from plot find gain margin and phase margin.

Ans.

On 0)-axis mark the point at 23.7 rad/sec. since in denominator (jw) term is having power one, from 23.7 draw a line of slope —20 db/decade to meet y-axis. This will be the starting point.

Step 1.

From the starting point to I corner frequency (0.33) the slope of the line is —20 db/decade.

From I corner frequency (0.33) to second corner frequency (1) the slope of the line will be —20 ÷ (—20) = —40 db/decade.

From II corner frequency to IV corner frequency (2) the slope of the line be —40 + (÷20) = —20 db/decade.

From III corner frequency to IV corner frequency, the slope of line will be —20 + (—20) = —40 db/decade.

From IV corner frequency (5) to V corner frequency the slope will be —40 ÷ (+20) = —20 db/decade.

After V corner frequency, the slope will be (—20) ÷ (—20) = —40 db/decade.

Step 2.
Draw the phase plot.

Step 3.
From graph
Phase margin = +34°
Gain margin =infinity

$$G(j\omega)\,H(j\omega) = \frac{k}{j\omega\,(j0.1\omega+1)(j0.05\omega+1)}.$$

Ans. Advantages of Bode Plot :
Please refer to Q. No. 1 (i) of May 2009.

As $G(j\omega)\,H(j\omega) = \dfrac{k}{j\omega\,(j0.1\omega+1)(j0.5\omega+1)}$

Comer frequencies are

$$\omega_1 = \frac{1}{0.1}$$
$$= 10 \text{ rad/sec}$$

$$\omega_2 = \frac{1}{0.05}$$
$$= 20 \text{ rad/sec}$$

Draw magnitude plot without K.
For phase plot

ω	Arg jω ϕ_1	Arg (1 + j0.1ω) ϕ_2	Arg (1 + j0.05ω) ϕ_3	Resultant $\phi_1 + \phi_2 + \phi_3$
4	−90°	−21.8°	−11.3°	−123.1°
6	−90°	−30.96°	−16.69°	−137.65°
8	−90°	−38.56°	−21.8°	−150.36°
10	−90°	−45°	−26.56°	−161.56°
12	−90°	−50.19°	−30.96°	−171.46°
14	−90°	−54.46°	−35°	−179.48°
18	−90°	−60.9°	−42°	−192.9°
20	−90°	−63.43°	−45°	−198.43°

ω	$-\tan^{-1} j\omega$	$-\tan^{-1} 3\omega$	$-\tan^{-1} 0.5\omega$	$-\tan^{-1} 0.1\omega$	$\tan^{-1} \omega$	$\tan^{-1} 2\omega$	Resultant
0.1	−90°	−16.7°	−2.86°	−0.57°	+5.71°	1.14°	−103°
0.2	−90°	−31°	−5.71°	−1.14°	+11.3°	2.3°	−114.25°
0.5	−90°	−56.3°	−14.03°	−2.86°	+26.56°	5.71°	−130.92°
0.8	−90°	−67.4°	−21.8°	−4.57°	+38.65°	9.09°	−136.03°
1.0	−90°	−71.56°	−26.56°	−5.71°	+45°	11.3°	−137.5°
2.0	−90°	−80.54°	−45°	−11.3°	+63.43°	21.8°	−141.61°
5.0	−90°	−86.18°	−68.19°	−26.56°	+78.7°	45°	−147.23°
8.0	−90°	−87.61°	−76°	−38.65°	+82.87°	58°	−151.39°
10.0	−90°	−88°	−78.7°	−45°	+84.3°	63.4°	−154.0°
20.0	−90°	−89°	−84.3°	−63.43°	+87.13°	76°	−163.6°

OUTCOMES:

At the end of the module, the students are able to:

- To Determine Gain & Phase Margins effect.
- Applications of the frequency response to analysis of system stability (the Nyquist criterion), relating the frequency response to transient performance specifications.
- Determine stability of control system applying Nyquist stability criterion and using Bode plot.
- Plot a graph of amplitude plot, usually in the log-log scale and a phase plot, which is usually a linear-log plot.

SELF-TEST QUESTIONS:

1. Apply Nyquist stability criterion for the system with transfer function **G(S)H(S)=** $\frac{K}{S(S+2)(S+4)}$ find the stability.

2. The open loop transfer function of a system is given by **G(S)H(S) =** $\frac{10(S+10)}{S(S+2)(S+5)}$.
   Draw Bode diagram, Find Gain cross over frequency (GCF), Phase cross over frequency (PCF), Gain margin (GM), Phase margin (PM). Find stability of the system.
3. The open loop transfer function of a system is given by **G(S)H(S) =** $\frac{50K}{S(S+10)(S+6)(S+1)}$
   Draw Bode diagram, Find Gain cross over frequency (GCF), Phase cross over frequency (PCF), Gain margin (GM), Phase margin (PM). Find the value of **K** to have GM=10 decibels.

FURTHER READING:

1. **Control engineering,** Swarnakiran S, Sunstar publisher, 2018.
2. **Feedback Control System**, Schaum's series. 2001.