

FORMAT-1B

MODULE-2: Field Effect Transistors (FET)

Definition:

FET is a three terminal electronic device used for variety of applications that match with BJT. In FET, an electric field is established by the charges present, which controls the conduction path of the output circuit without the need for direct contact between controlling and controlled quantities. In a Field effect device current is controlled by the action of an electron field, rather than carrier injection.

The main difference between BJT and FET is BJT is a current controlled device while FET is a voltage controlled device. This is shown in fig 1.

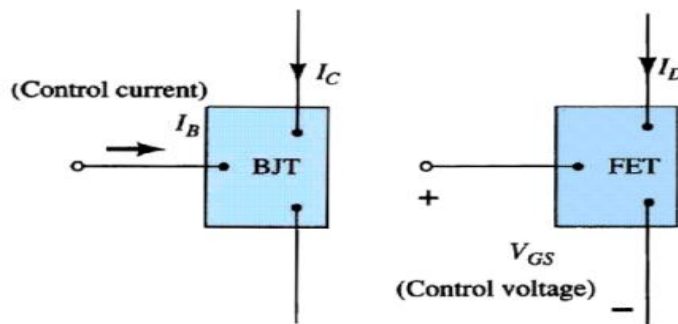


Fig 1: Comparison between BJT and FET

TYPES OF FETS:

1. Junction Field Effect Transistors(JFETs)
2. Metal Oxide Semiconductor Field Effect Transistors (MOSFETs)

JUNCTION FIELD EFFECT TRANSISTORS(JFETS):

JFET is a unipolar device as conduction in the device is dependent on either electrons or holes. Accordingly there are two types of JFET; namely: n-Channel JFET and p-Channel JFET.

Features of FET:

- FET is a voltage controlled device.
- FET is a unipolar device.
- FET has high input impedance
- AC voltage gain of JFET is low
- FET has higher temperature stability.
- FET are small in size and hence are useful in ICs.

CONSTRUCTION AND CHARACTERISTICS OF N-CHANNEL JFET:

The basic construction of the n-channel JFET is as shown in fig 2. The major part of the structure is the n-type material which forms the channel between embedded layers of p-

type material. The top of the n-type channel is connected through an ohmic contact to a terminal referred to as the drain(D), whereas the lower end of the material is connected through an ohmic contact referred to as source(S). The 2 p-type materials are connected together to the gate (G) terminal. In the absence of any applied potentials, JFET has 2 p-n junctions under no bias condition. As a result, depletion region is formed at each junction.

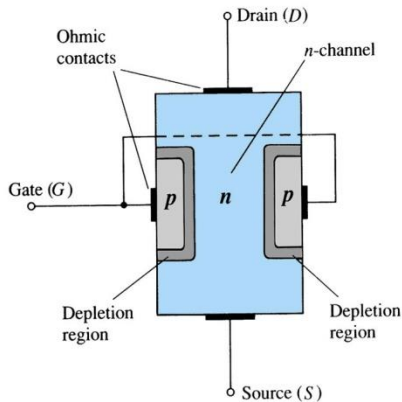


Fig 2 Construction of n-channel JFET

OPERATION:

Fig 3 shows the working of n-channel JFET for different gate-source voltage (V_{GS}) and drain to source voltage (V_{DS}) = 0V.

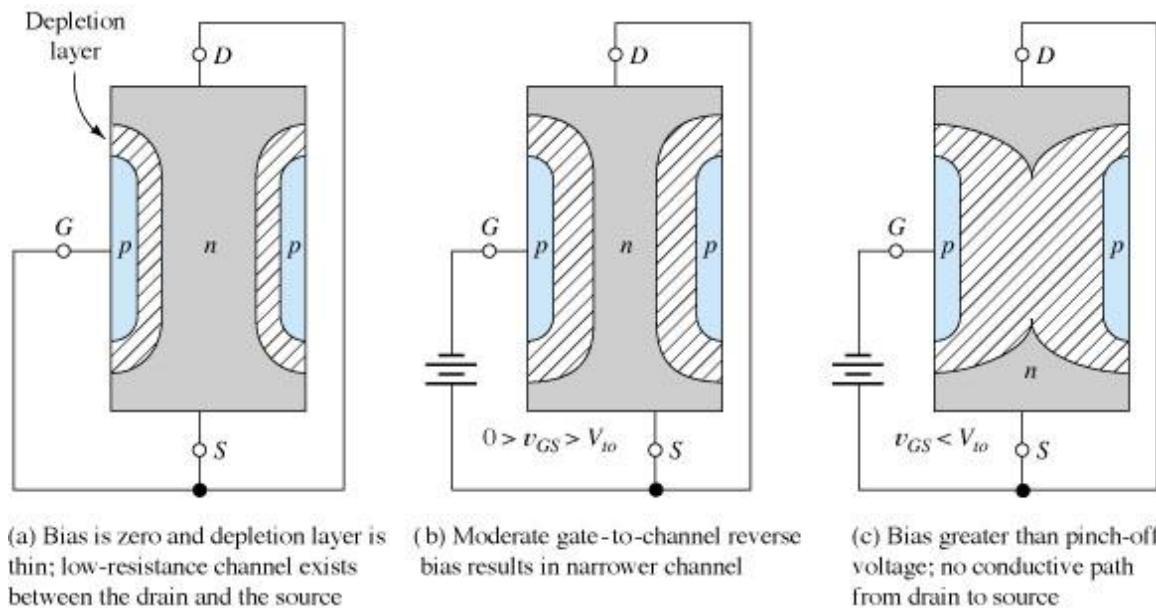


Fig 3: Operation of n-channel JFET

Case i: $V_{GS} = 0$ and $V_{DS} = 0$

- Under zero bias condition depletion region around the p-n junction is thin and thus exhibits low channel resistance.

Case ii: $V_{GS} = 0$ and $V_{DS} = +$ small voltage.

The gate and source are at the same potential and the instant the voltage V_{DS} is applied the electrons in the n-channel are drawn towards the drain terminal establishing drain current (I_D).

Due to reverse biasing of the p-n junction for the length of the channel results in gate current = 0.

As V_{DS} is increased further, the drain current increases. When $V_{DS} = V_P$, the depletion region widens causing reduction in the channel width. The reduced path of conduction causes the resistance to increase and the current saturates. When V_{DS} is further increased, the two depletion regions touch resulting in pinch-off condition. The drain characteristics (plot of I_D vs V_{DS} for $V_{GS} = \text{constant}$ is as shown in fig 4.

Case iii: $V_{GS} = -ve$ voltage and $V_{DS} = +$ small voltage.

The effect of applied reverse bias on gate and source widens the depletion regions around the p-n junctions but at the lower levels of V_{DS} . The resulting saturation level for I_D is reduced and will continue to decrease as V_{GS} is made more and more negative. The drain characteristics is as shown in fig 5 for different values of V_{GS} . When $V_{GS} = -V_P$, pinch-off condition occurs resulting in $I_D = 0$. V_P is called pinch-off voltage.

The region to the left of pinch off locus is called ohmic region and the region to the right of pinch-off locus is saturation region. This region of JFET is employed for linear amplifiers. In ohmic region JFET can be employed as a variable resistor. The resistance is controlled by V_{GS} . As V_{GS} becomes more and more negative, the slope of the characteristics becomes more and more horizontal indicating increasing resistance level.

The resistance is given by the equation 1.

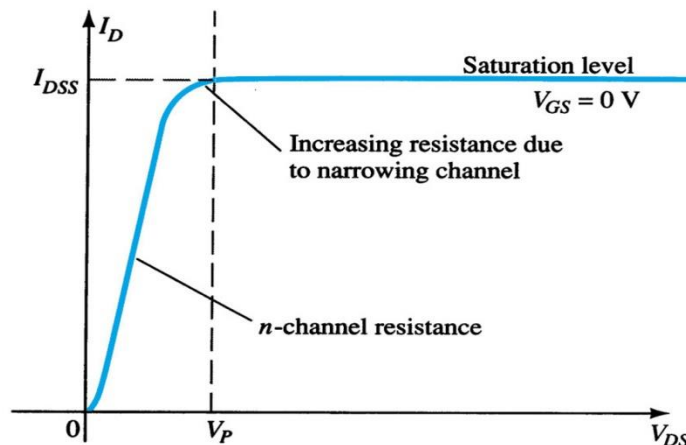


Fig 4 : Drain characteristics of n-channel JFET for $V_{GS} = 0V$.

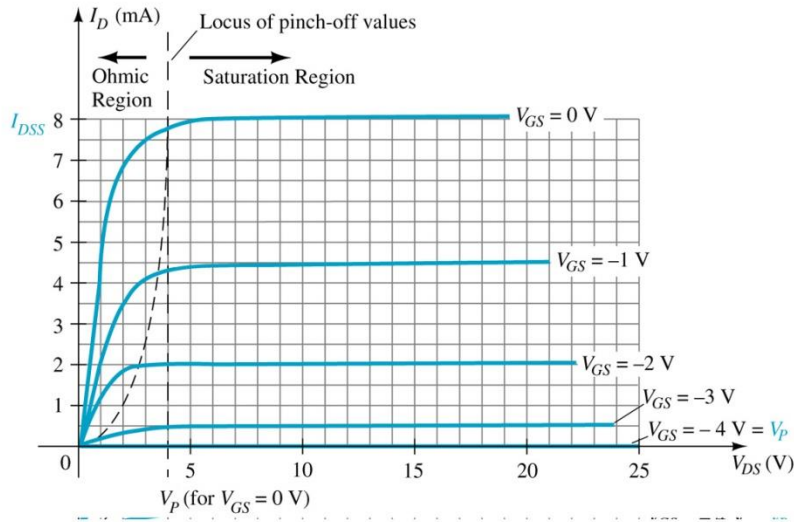


Fig 5 :Drain characteristics of JFET for different V_{GS} values.

TRANSFER CHARACTERISTICS OF N-CHANNEL JFET:

Transfer characteristics are a plot of I_D as a function of V_{GS} with V_{DS} as constant. Shockley Equation as in equation 1 is used to plot transfer characteristics.

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2 \quad \text{----- Eqn (1)}$$

I_D depends on V_{GS} in a non-linear manner. As a result, FET's are often referred to square law devices. Using the drain characteristics on the right of Y-axis, we can draw a horizontal line from the saturation region of the curve denoted as $V_{GS} = 0V$ to the I_D axis. The resulting current level for both the graphs is I_{DSS} .

When $V_{GS} = V_P$, the drain current is 0mA, defining another point on transfer curve. Transfer curve is a direct transfer from input to output variables. Transfer characteristics are a parabolic curve as shown in fig 6.

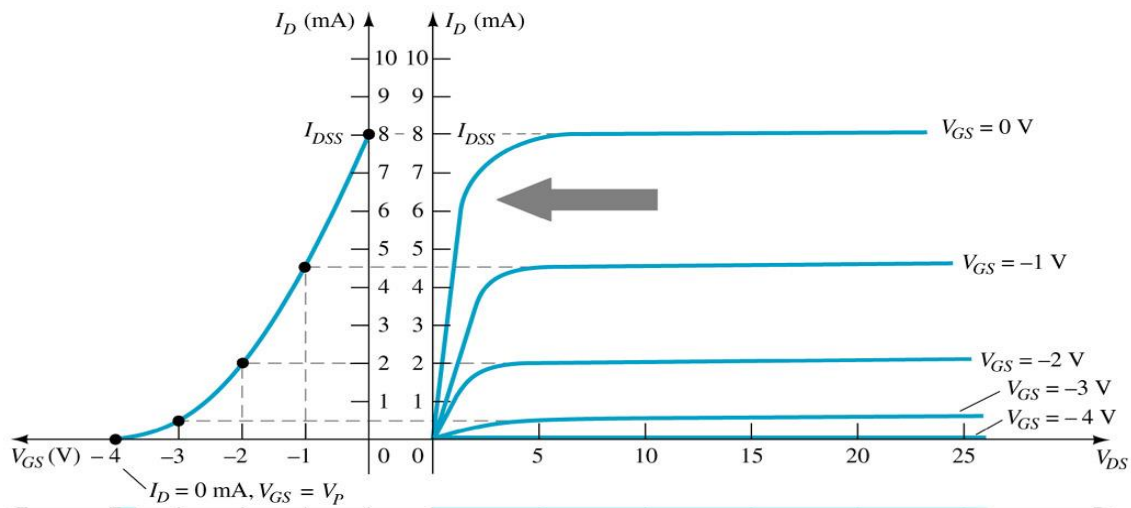


Fig 6: Transfer characteristics from drain characteristics

TRANSFER CHARACTERISTICS: SHORT-HAND METHOD

Transfer characteristics can also be obtained by applying following conditions to Schokley's equation (1).

Condition 1: $V_{GS} = 0$

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2$$

Therefore $I_D = I_{DSS}$.

Condition 2: $V_{GS} = V_P$

Therefore from equation 1 $I_D = 0 \text{ mA}$.

Condition 3: $V_{GS} = V_P/2$

Therefore from equation 1, $I_D = I_{DSS} \left(1 - \frac{1}{2} \right)^2$

$I_D = I_{DSS}/4$

Condition 4: $I_D = I_{DSS}/2$

From eq(1) , $V_{GS} = V_P(1 - \sqrt{I_D/I_{DSS}})$

$$V_{GS} = V_P(1 - \sqrt{0.5})$$

$$V_{GS} = 0.3V_P$$

Points are marked for these conditions of V_{GS} and I_D and the co-ordinates are joined using smooth curve.

CONSTRUCTION AND CHARACTERISTICS OF P-CHANNEL JFET:

The basic construction of the p-channel JFET is as shown in fig 2. The major part of the structure is the p-type material which forms the channel between embedded layers of p-type material. The top of the p-type channel is connected through an ohmic contact to a terminal referred to as the drain(D), whereas the lower end of the material is connected through an ohmic contact referred to as source(S). The 2 n-type materials are connected together to the gate (G) terminal. In the absence of any applied potentials , JFET has 2 p-n junctions under no bias condition. As a result, depletion region is formed at each junction.

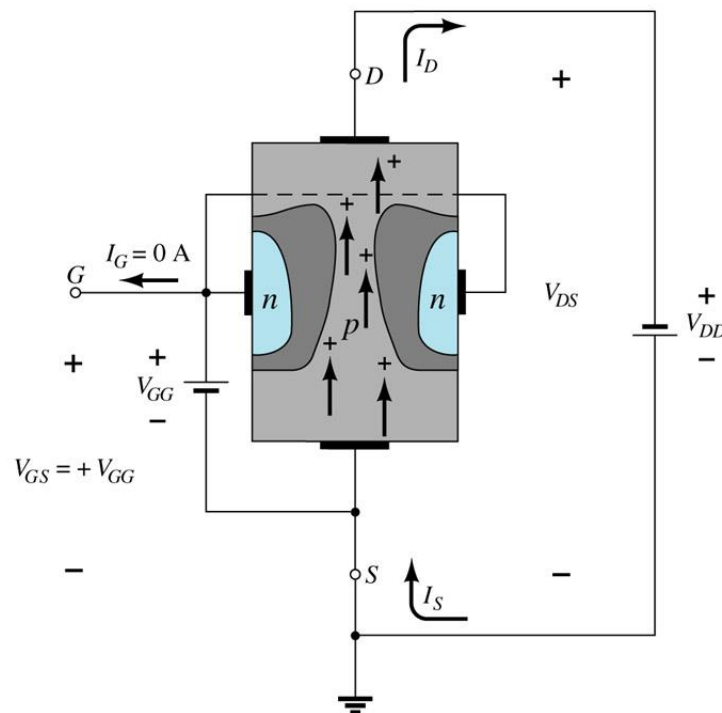


Fig 7 Construction of n- channel JFET

OPERATION OF P-CHANNEL JFET:

Case i: $V_{GS} = 0$ and $V_{DS} = 0$

- Under zero bias condition depletion region around the p-n junction is thin and thus exhibits low channel resistance.

Case ii: $V_{GS} = 0$ and $V_{DS} = -ve$ small voltage.

The gate and source are at the same potential and the instant the voltage V_{DS} is applied the holes in the p-channel are drawn towards the drain terminal establishing drain current (I_D).

Due to reverse biasing of the p-n junction for the length of the channel results in gate current = 0.

As V_{DS} is increased further, the drain current increases. When $V_{DS} = -V_P$, the depletion region widens causing reduction in the channel width. The reduced path of conduction causes the resistance to increase and the current saturates. When V_{DS} is further increased, the two depletion regions touch resulting in pinch-off condition. The drain characteristics (plot of I_D vs V_{DS} for $V_{GS} = \text{constant}$ is as shown in fig 8).

Case iii: $V_{GS} = +ve$ voltage and $V_{DS} = -ve$ small voltage.

The effect of applied reverse bias on gate and source widens the depletion regions around the p-n junctions but at the lower levels of V_{DS} . The resulting saturation level for I_D is reduced and will continue to decrease as V_{GS} is made more and more positive. The drain characteristics are as shown in fig 8 for different values of V_{GS} . When $V_{GS} = -V_P$, pinch-off condition occurs resulting in $I_D = 0$. V_P is called pinch-off voltage.

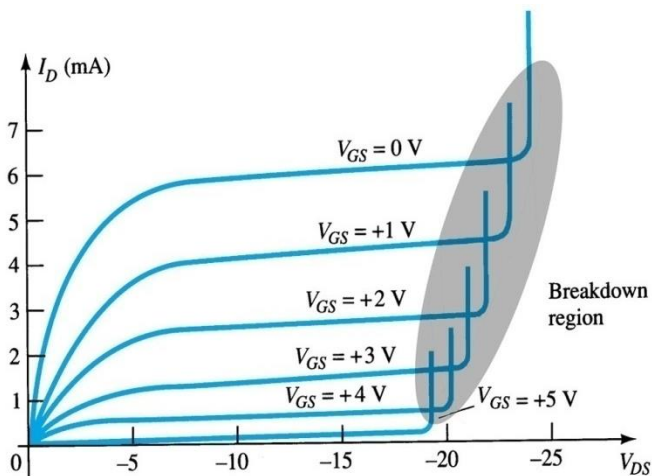


Fig 8: Drain Characteristics of p-channel JFET

Symbols of JFET:

Fig 9(a) and 9 (b) shows the symbols of n-channel and p- channel FET respectively.

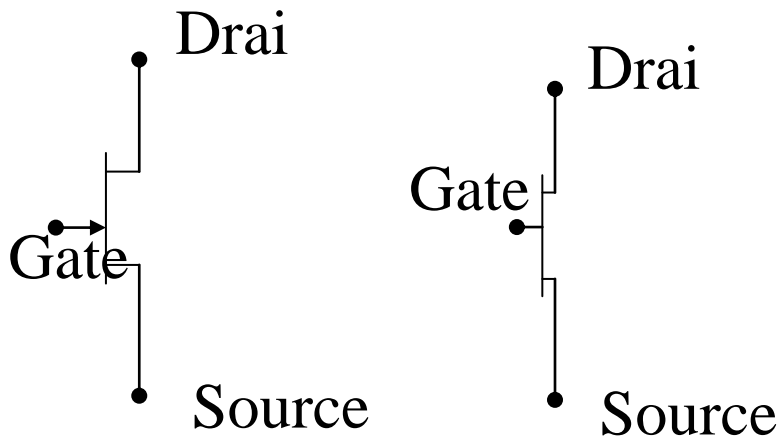


Fig 9: JFET Symbols.
9(a) n-Channel JFET

9(b)p-channel JFET

Metal Oxide Semiconductor Field Effect Transistors (MOSFETs):

MOSFET is a type of Field Effect Transistor in which majority charge carriers flow in the channel. The width of the channel is controlled by an electrode called gate. Channel width determines how well the device conducts.

MOSFETS are useful in high-speed switching circuits and in Integrated Circuits.

There are two types of MOSFET's:

- (i) Depletion type MOSFET
- (ii) Enhancement type MOSFET

DEPLETION TYPE MOSFET:

Depletion-type MOSFETs are further classified as

- (i) N-channel D-type MOSFET
- (ii) P-Channel D-type MOSFET

N-CHANNEL DEPLETION TYPE MOSFET:

The basic construction of the n-channel depletion type MOSFET is as shown in fig (10). A slab of p-type material is formed from a Si base and is referred to as the substrate. The source and drain terminals are connected through metallic contacts to n-doped regions linked by a n-channel. The gate is also connected to a metal contact surface but remains insulated from the n-channel by a very thin SiO₂ layer. The presence of SiO₂ layer accounts for very high input impedance of the device. The input impedance of MOSFET is higher than JFET.

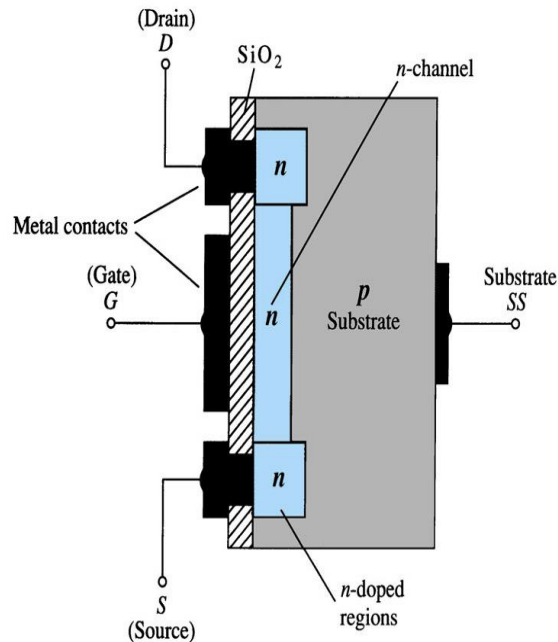


Fig 10: Construction n-Channel Depletion type MOSFET.

A small n layer is implanted in the region below SiO_2 to create n-channel. The insulating layer between gate and the channel has resulted in another name for the device : Insulated-gate FET or IGFET.

OPERATION OF N-CHANNEL DEPLETION MODE MOSFET:

Case i: $V_{GS} = 0$ and $V_{DS} = +ve$ voltage

Since drain is positive with respect to source, the free electrons are attracted from source to drain to constitute drain current I_D . The drain characteristics and transfer characteristics of depletion mode MOSFET is as shown in fig 11.

Case ii: $V_{GS} = -ve$ Voltage and $V_{DS} = +ve$ small voltage

The negative potential at the gate will cause the electrons to move towards p-type substrate as charges repel while holes from p-type substrate are attracted toward gate. Depending on the magnitude of negative bias established by V_{GS} , a level of recombination between electrons and holes will occur that will reduce the number of free electrons in the n-channel available for conduction. The more negative the bias, higher is the rate of recombination. The resulting level of I_D is reduced with the increasing levels of negative bias for V_{GS} as in fig11.

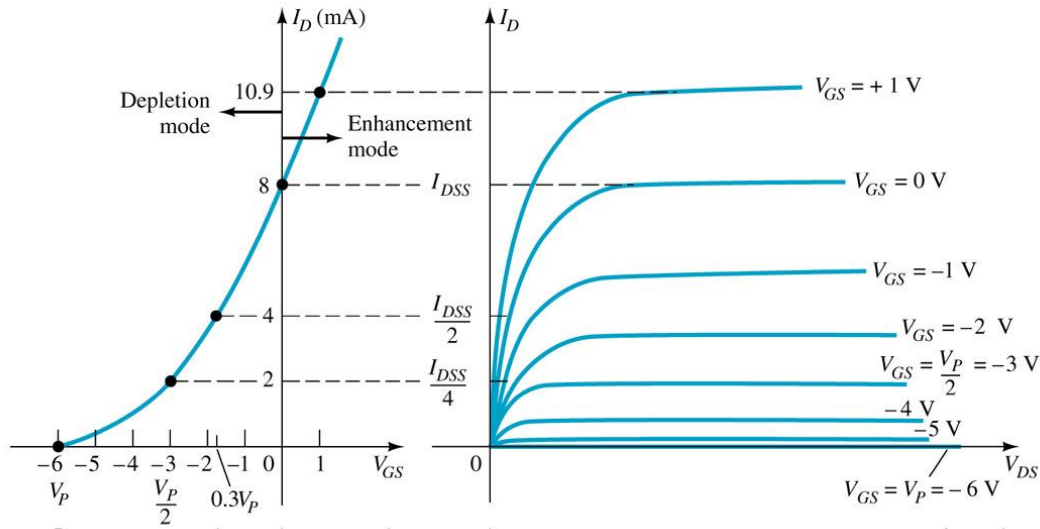


Fig 11(a) Transfer Characteristics

Fig 11(b) Drain Characteristics.

Case iii: $V_{GS} = +ve$ Voltage and $V_{DS} = +ve$ small voltage

For positive values of V_{GS} , the +ve gate will draw additional electrons from p-type substrate as minority charge carriers are attracted towards gate. New carriers are generated due to collisions and I_D will increase at a rapid rate. Thus, application of $+V_{GS}$ has enhanced the level of free carriers in the channel compared to $V_{GS} = 0V$. The region of +ve gate voltage on the drain or transfer characteristics is referred as enhancement region. The region between the cut-off and the saturation level of I_{DSS} is referred as the depletion region.

Transfer characteristics are a plot of I_D as a function of V_{GS} with V_{DS} as constant. Shockley Equation as in equation 2 is used to plot transfer characteristics.

----- Eqn (2) $I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2$

Also, Short hand method can be used to plot transfer characteristics curve.

Condition1: $V_{GS} = 0$, Hence from eq(2),

$$I_D = I_{DSS}$$

Condition 2: $V_{GS} = V_P$

Therefore from equation 2 $I_D = 0\text{mA}$.

Condition 3: $V_{GS} = V_P/2$

Therefore from equation 2, $I_D = I_{DSS}(1 - \frac{1}{2})^2$

$$I_D = I_{DSS}/4$$

Condition 4: $I_D = I_{DSS}/2$

From eq(1), $V_{GS} = V_P(1 - \sqrt{I_D/I_{DSS}})$

$$V_{GS} = V_P(1 - \sqrt{0.5})$$

$$V_{GS} = 0.3V_P$$

$$V_{GS} = 0.3V_P$$

Points are marked for these conditions of V_{GS} and I_D and the co-ordinates are joined using smooth curve.

P-CHANNEL DEPLETION TYPE MOSFET:

The basic construction of the p-channel depletion type MOSFET is as shown in fig (12a). A slab of n-type material is formed from a Si base and is referred to as the substrate. The source and drain terminals are connected through metallic contacts to p-doped regions linked by a p-channel. The gate is also connected to a metal contact surface but remains insulated from the p-channel by a very thin SiO_2 layer. The presence of SiO_2 layer accounts for very high input impedance of the device. The input impedance of MOSFET is higher than JFET.

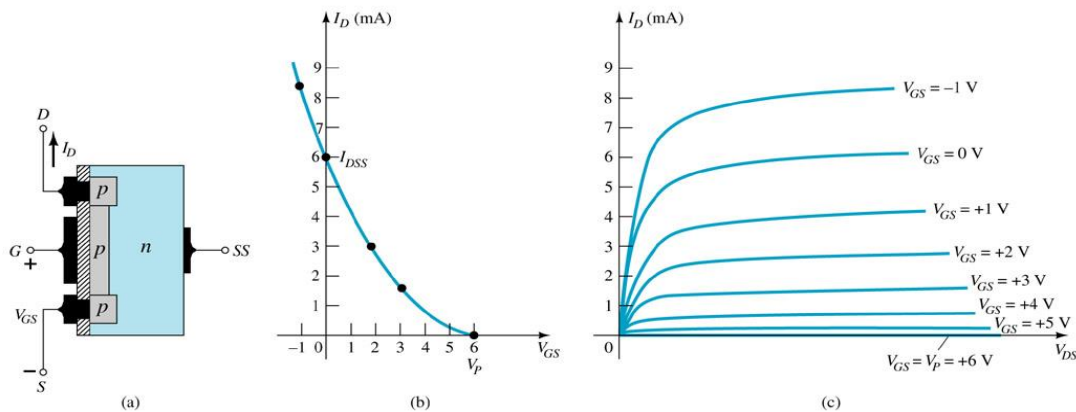


Fig 12: (a) Construction (b) Transfer Characteristics (c) Drain characteristics

OPERATION OF P-CHANNEL DEPLETION MODE MOSFET:

Case i: $V_{GS} = 0$ and $V_{DS} = -ve$ voltage

Since drain is negative with respect to source, the holes are attracted from source to drain to constitute drain current I_D . The drain characteristics and transfer characteristics of depletion mode MOSFET is as shown in fig 12(c) and (b) respectively.

Case ii: $V_{GS} = +ve$ Voltage and $V_{DS} = -ve$ small voltage

The positive potential at the gate will cause the holes to move towards n-type substrate as charges repel while electrons from n-type substrate are attracted toward gate. Depending on the magnitude of positive bias established by V_{GS} , a level of recombination between electrons and holes will occur that will reduce the number of holes in the p-channel available for conduction. The more positive the bias, higher is the rate of recombination. The resulting level of I_D is reduced with the increasing levels or positive bias for V_{GS} as in fig 12(c).

Case iii: $V_{GS} = -ve$ Voltage and $V_{DS} = -ve$ small voltage

For positive values of V_{GS} , the -ve gate will draw additional holes from n-type substrate as minority charge carriers are attracted towards gate. New carriers are generated due to

collisions and I_D will increase at a rapid rate. Thus, application of $-V_{GS}$ has enhanced the level of free carriers in the channel compared to $V_{GS} = 0V$.

Transfer characteristics are a plot of I_D as a function of V_{GS} with V_{DS} as constant. Shockley Equation as in equation 2 is used to plot transfer characteristics.

$$\text{----- Eqn (3)} \quad I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2$$

Also, Short hand method can be used to plot transfer characteristics curve.

Condition1: $V_{GS} = 0$, Hence from eq(3),

$$I_D = I_{DSS}$$

Condition 2: $V_{GS} = V_P$

Therefore from equation 2 $I_D = 0mA$.

Condition 3: $V_{GS} = V_P/2$

Therefore from equation 2, $I_D = I_{DSS} \left(1 - \frac{1}{2} \right)^2$

$$I_D = I_{DSS}/4$$

Condition 4: $I_D = I_{DSS}/2$

$$\text{From eq(1), } V_{GS} = V_P \left(1 - \sqrt{I_D/I_{DSS}} \right)$$

$$V_{GS} = V_P \left(1 - \sqrt{0.5} \right)$$

$$V_{GS} = 0.3V_P$$

$$V_{GS} = 0.3V_P$$

Points are marked for these conditions of V_{GS} and I_D and the co-ordinates are joined using smooth curve.

SYMBOLS OF DEPLETION TYPE MOSFET

Fig 13 shows the symbols of n-channel and p-channel Depletion mode MOSFET.

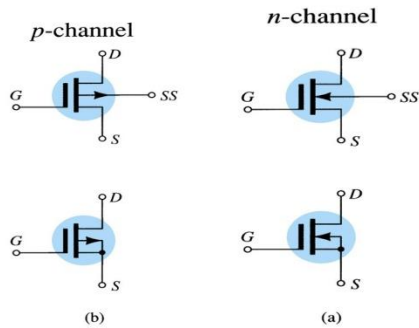


Fig 13 (a) n-channel depletion type MOSFET (b) p-channel depletion type MOSFET

N-CHANNEL ENHANCEMENT-MODE MOSFET (E-MOSFET):

The construction of n-channel enhancement mode MOSFET is as shown in fig 14. The starting material is a p-type substrate into which highly doped n-regions are diffused to form source and drain regions. A layer of SiO₂ is grown all over the p-type substrate and is etched to create window for n-diffusion. The source and drain terminals are taken out through metallic contacts to n-doped regions as shown in fig 14. Metal is deposited on SiO₂ to create Gate. The presence of SiO₂ between gate and p-substrate provides electrical isolation between the two regions. No channel exists between source and drain in E-MOSFET.

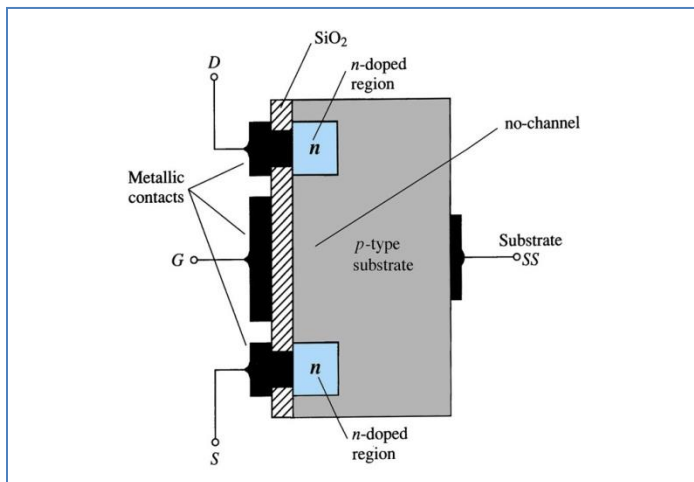


Fig 14: Construction of n-channel E-MOSFET

OPERATION OF N-CHANNEL E-MOSFET:

Case i: $V_{GS} = 0$ and $V_{DS} = +ve$ voltage

The application of drain to source voltage while gate and source are shorted will cause no I_D to flow as no channel exists for this condition.

Case ii: $V_{GS} = +ve$ Voltage and $V_{DS} = +ve$ small voltage

When gate is made positive with respect to source, electrons are attracted towards the gate but holes are repelled back into p-type substrate. Since the region under the gate is p-type substrate, the positive voltage on gate causes holes which are majority charge carriers in p-type substrate to repel and move towards substrate. A positive V_{GS} and positive V_{DS} causes the two pn junctions to be reverse biased and depletion region is formed. Now the device is said to be in depletion mode. The positive V_{GS} also causes electrons to be attracted towards the gate. Now, device is said to be in accumulation mode. Since the region below the gate was p-substrate and accumulation of electrons has caused the type to change to n-type. Thus the device is said to be in inversion mode as shown in fig 15. A positive V_{GS} has caused a thin layer of negative charge to be formed in the substrate under the gate. Thus, channel is said to be created. The value of V_{GS} which causes channel to be formed under the gate is called threshold voltage (V_T). A small I_D flows. When V_{GS} is increased above V_T , conductivity of the channel is enhanced and thus pulling more electrons into the channel. When $V_{GS} < V_T$, there is no channel. Since channel is formed by the application of $+V_{GS}$, the type of MOSFET is Enhancement type. As V_{GS} is increased further, higher level of I_D flows as shown in fig 16. A positive V_{GS} cause potential drop across the channel. For large V_{DS} this voltage may not be sufficient to invert the channel near the drain end there by causing drain current to saturate. The channel is said to be pinched off. I_D flows due to diffusion.

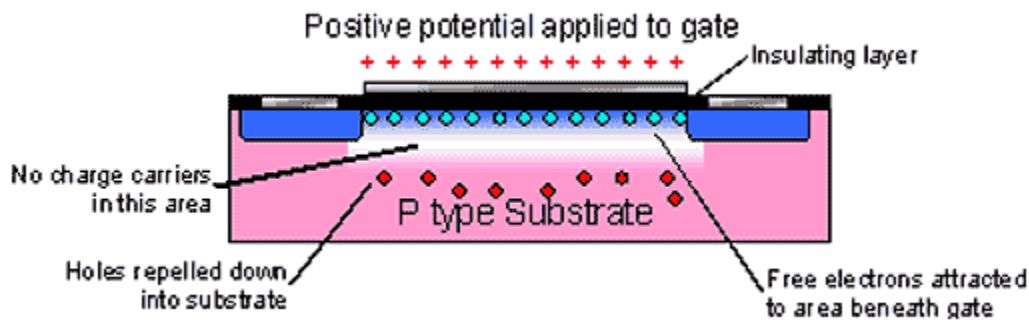


Fig 15: Formation of Inversion layer

TRANSFER CHARACTERISTICS OF N-CHANNELE-MOSFET:

The transfer characteristics of n-channel E-MOSFET is as shown in fig 16. For $V_{GS} > V_T$, the relationship between drain current and V_{GS} is nonlinear and is given by eqn 4.

$$I_D = K(V_{GS} - V_T)^2 \quad \text{-----Eq 4}$$

Where K is a constant and is a function of the construction of the device as given by Eqn 5.

$$K = \frac{I_{D(ON)}}{(V_{GS(ON)} - V_T)^2} \quad \text{-----Eq 5}$$

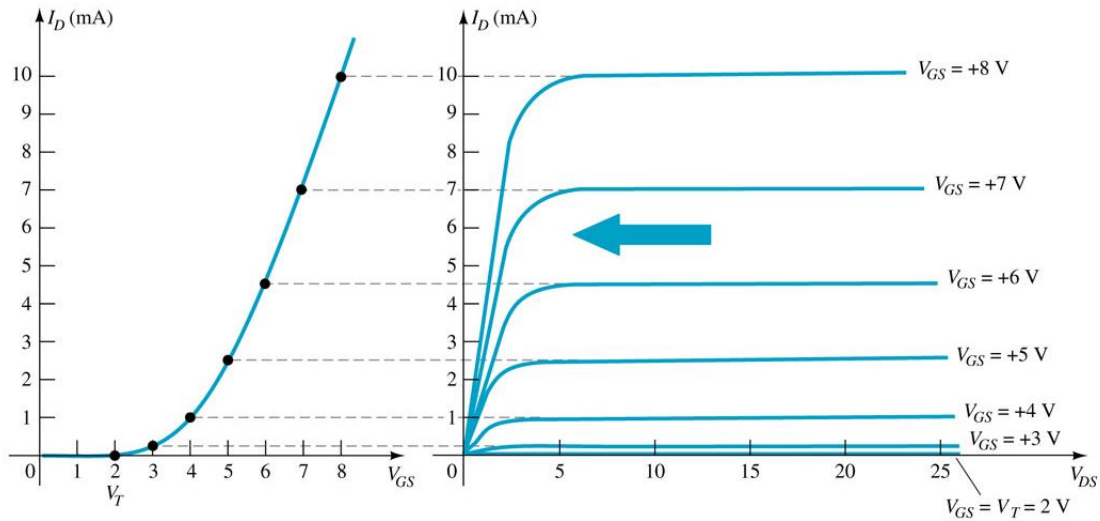


Fig16: Transfer Characteristics

Drain Characteristics

Thus I_D increases steadily when $V_{GS} > V_T$ and I_D is zero when $V_{GS} < V_T$.

p-CHANNEL ENHANCEMENT-MODE MOSFET (E-MOSFET):

The construction of p-channel E-MOSFET is opposite to that of n-channel E-MOSFET. Substrate is of n-type and source, drain are of p-type as in fig17(a). The voltage polarities and current directions are reversed in p-channel E-MOSFET. The drain and transfer characteristics of p-channel E-MOSFET are as shown in Fig 17 (c) and (b) respectively.

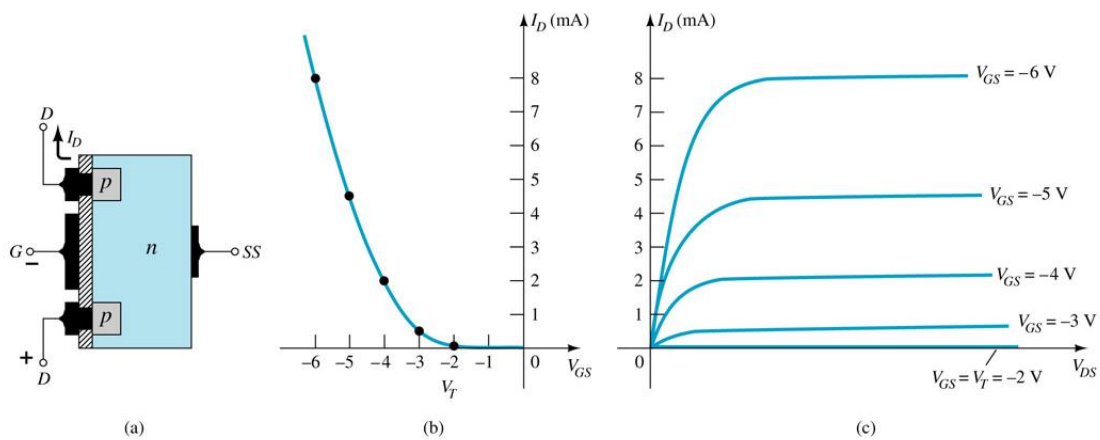


Fig 17: (a) Construction

(b) Transfer Characteristics

(c) Drain Characteristics

OPERATION OF P-CHANNEL E-MOSFET:

Case i: $V_{GS} = 0$ and $V_{DS} = -ve$ voltage

The application of drain to source voltage while gate and source are shorted will cause no I_D to flow as no channel exists for this condition.

Case ii: $V_{GS} = -ve$ Voltage and $V_{DS} = -ve$ small voltage

When gate is made negative with respect to source, holes are attracted towards the gate but electrons are repelled back into n-type substrate. Since the region under the gate is n-type substrate, the negative voltage on gate causes electrons which are majority charge carriers in n-type substrate to repel and move towards substrate. A negative V_{GS} and negative V_{DS} causes the two pn junctions to be reverse biased and depletion region is formed. Now the device is said to be in depletion mode. The negative V_{GS} also causes holes to be attracted towards the gate. Now, device is said to be in accumulation mode. Since the region below the gate was n-substrate and accumulation of holes has caused the type to change to p-type. Thus the device is said to be in inversion mode. A negative V_{GS} has caused a thin layer of positive charges to be formed in the substrate under the gate. Thus, channel is said to be created. The value of V_{GS} which causes channel to be formed under the gate is called threshold voltage (V_T). A small I_D flows. When V_{GS} is decreased below V_T , conductivity of the channel is enhanced and thus pulling more electrons into the channel. When $V_{GS} > V_T$, there is no channel. Since channel is formed by the application of $-V_{GS}$, the type of MOSFET is Enhancement type. The drain characteristics are as shown in fig 17 (c).

TRANSFER CHARACTERISTICS OF P-CHANNEL E-MOSFET:

The V_{GS} is negative and I_D flows in opposite direction. The transfer characteristics of p-channel E-MOSFET is as shown in fig 17(b). I_D increases steadily with V_{GS} .

E-MOSFET SYMBOLS:

Fig 18 (a) and 18(b) shows the symbols of n-channel and p-channel E-MOSFET.

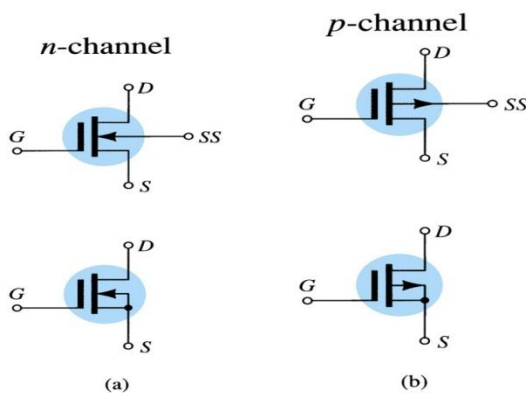


Fig 18: E-MOSFET Symbols

FET CONFIGURATION:

The three types of FET configuration are:

- (i) Common Source (CS) Configuration
- (ii) Common Drain (CD) Configuration
- (iii) Common Gate (CG) Configuration

FET BIASING:

Biasing is done to establish proper levels of DC voltages and currents for desired region of operation. It establishes Q-point.

TYPES OF BIASING:

- (i) Fixed Bias
- (ii) Self Bias
- (iii) Voltage divider Bias

Voltage divider bias most widely used biasing technique in amplifiers.

FET AS AN AMPLIFIER:

Fig.19 Shows Common Source Circuit. The Voltage V_{GG} provides the necessary reverse-bias between gate and source of JFET. The signal to be amplified is V_S . The transfer Characteristics of JFET is as shown in Fig. 20. A DC load line is drawn on the characteristics. The point of intersection of DC load line on Transfer characteristics for specific V_{GS} is called Q point. Let Q point be situated at the middle of DC load line.

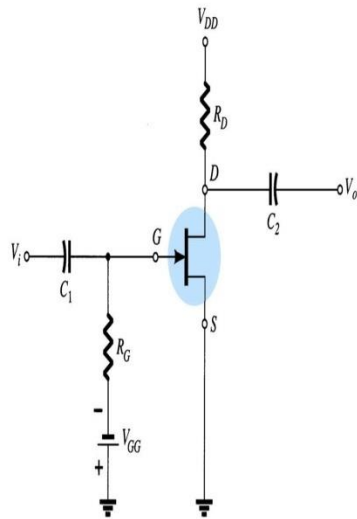


Fig 19: CS Amplifier with Fixed Bias

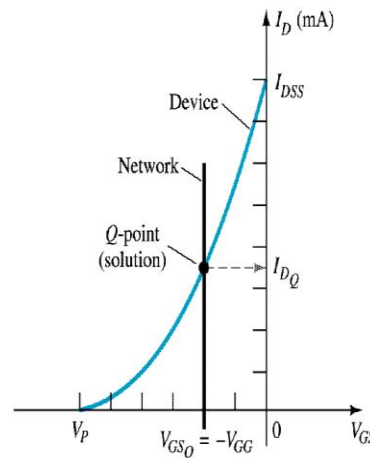


Fig 20: Locate Q-Point

The instantaneous V_{gs} is

$$V_{gs} = V_S - V_{GG} \text{ ----- (6)}$$

Both I_D and V_{DS} can be considered as sinusoid superimposed on the DC values.

Then $V_{GS} = -V_{GG} + V_{gs}$ ----- (7)

$$I_D = i_d + I_{DQ}$$

$$V_{OUT} = V_{DS} = V_{DSQ} + V_{ds}$$
 ----- (8)

Since output signal is greater than input signal, amplification has occurred. The magnitude of Voltage gain is the ratio of output voltage to input voltage.

$$|A_v| = \frac{V_o}{V_s}$$

The selection of Q point at the middle gives undistorted output. If the operating point is located either closer to ohmic region or near pinch-off voltage, the output waveform will be clipped during +Ve or -Ve half cycles. In Common Source circuit, output is 180° out of phase with input.

To locate Q point, the following procedure is used

- Plot transfer characteristics
- Draw a vertical line (load line) from $V_{GS} = -V_{GG}$.
- Intersection of load line with transfer characteristics will give Q point.

Fig 20 shows the position of Q point using the above procedure.

JFET parameters

Transconductance: The change in the Drain Current due to change in Gate to Source voltage is defined as Transconductance 'g_m'.

$$g_m = \frac{\Delta I_d}{\Delta V_{GS}}$$

We know that, $I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$ ----- (9)

And $g_m = \frac{\Delta I_D}{\Delta V_{GS}}$

Differentiate Eq. (9) w.r.t V_{GS}

$$\frac{\Delta I_D}{\Delta V_{GS}} = I_{DSS} \times 2 \left(1 - \frac{V_{GS}}{V_P}\right) \left(-\frac{1}{V_P}\right)$$

$$g_m = -\frac{2I_{DSS}}{V_P} \left(1 - \frac{V_{GS}}{V_P}\right)$$
 ----- (10)

Also from Eq. (9) $1 - \frac{V_{GS}}{V_P} = \sqrt{\frac{I_D}{I_{DSS}}}$ ----- (11)

Therefore substitute Eq. (11) in Eq. (10),

we get, $g_m = -\frac{2I_{DSS}}{V_P} \sqrt{\frac{I_D}{I_{DSS}}} = -\frac{2}{V_P} \sqrt{I_D I_{DSS}^2}$

therefore $g_m = -\frac{2}{V_P} \sqrt{I_D I_{DSS}}$ ----- (12)

when $V_{GS} = 0$, $g_m = g_{m0}$

therefore from Eq. (10), $g_{m0} = -\frac{2I_{DSS}}{V_P}$ ----- (13)

Substitute Eq. (13) in Eq. (10)

we get, $g_m = g_{m0} \left(1 - \frac{V_{GS}}{V_P}\right)$

Drain Resistance, r_d : The ratio of change in Drain to Source voltage to change in Drain current is called Drain resistance, r_d with constant V_{GS} .

$$r_d = \left. \frac{\Delta V_{DS}}{\Delta I_D} \right|_{V_{GS}=\text{constant}}$$

r_d determines the output impedance Z_O of the JFET amplifier.

JFET small signal model:

Fig. (21) shows low frequency small signal model for n-channel JFET. The relationship between I_D and V_{GS} is

$$\Delta I_D = g_m \Delta V_{gs}$$

and hence a current Source is connected from Drain to Source. The input impedance of JFET is high and hence $I_G=0$. Thus in the small signal model input impedance is represented by open circuit. The output impedance is represented by r_d from Drain to Source.

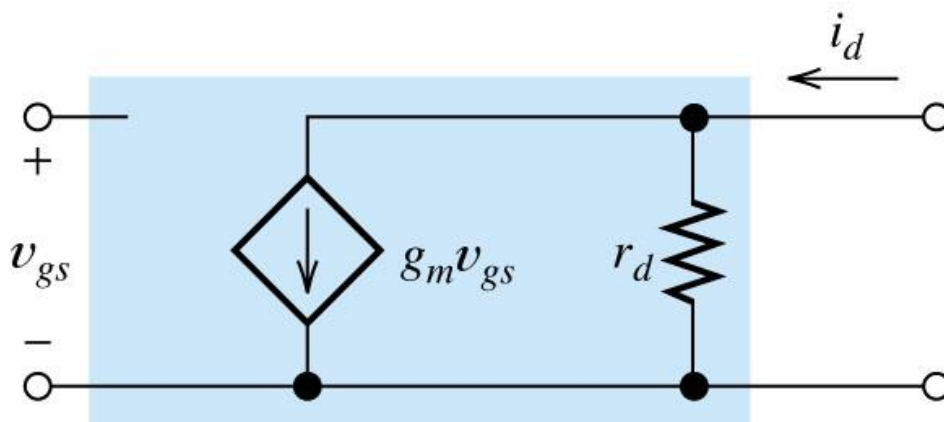


Fig. (21) n-JFET small signal model

Approximate model

Since $r_d \gg$ external Drain resistance R_D , r_d can be replaced by open circuit as shown in Fig. (22).

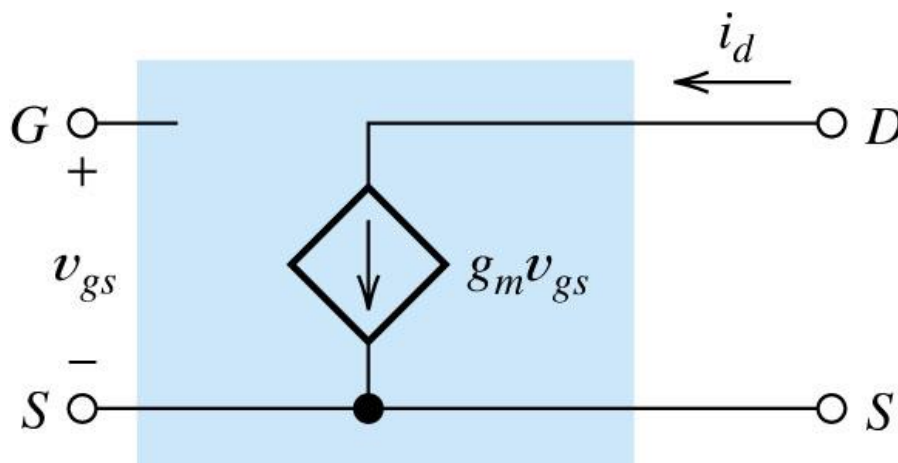


Fig. (22) Approximate small signal model of n-JFET.

COMMON SOURCE (CS) AMPLIFIER WITH FIXED BIASING:

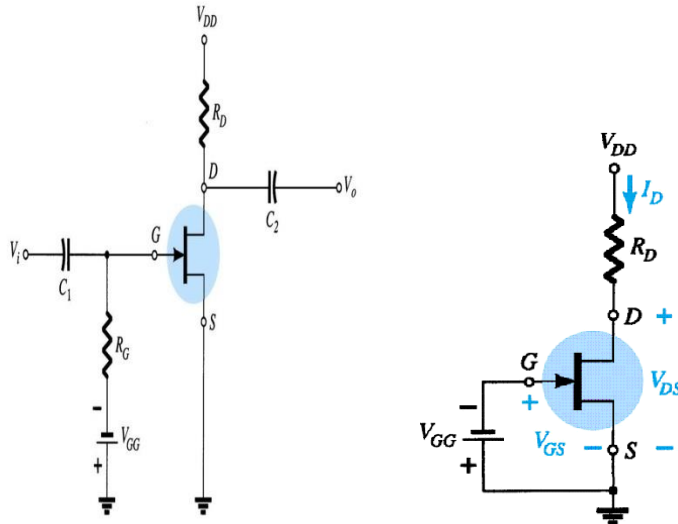


Fig 23: CS Amplifier with Fixed Bias Fig 24: DC equivalent Circuit

Fig 23 shows CS amplifier with fixed bias. R_G is used to limit current in case V_{GG} is connected with wrong polarity

This would forward bias the gate-source junction causing high currents, which would destroy the transistor

DC Analysis:

Open circuit C_1 & C_2 and current through R_G i.e. $I_G=0$. Therefore R_G is represented by short circuits as shown in Fig. (24)

Apply KVL to the input circuit of Fig. (24)

$$V_{GG} - V_{GS} = 0$$

$$V_{GS} = -V_{GG} \text{ ----- (14)}$$

Since V_{GG} is constant, V_{GS} is fixed and hence the name fixed bias.

Apply KVL to output circuit

$$V_{DD} - I_D R_D - V_{DS} = 0$$

$$V_{DSQ} = V_{DD} - I_D R_D \text{ ----- (15)}$$

And I_D for fixed bias is $I_{DQ} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$

Therefore Q point is $[V_{DS}, I_{DQ}]$

Small signal analysis:

- (i) To obtain AC equivalent circuit, short circuit C_1 , C_2 and reduce DC voltages to zero.

- (ii) Replace JFET by its small signal model to obtain AC equivalent circuit Fig. (25).

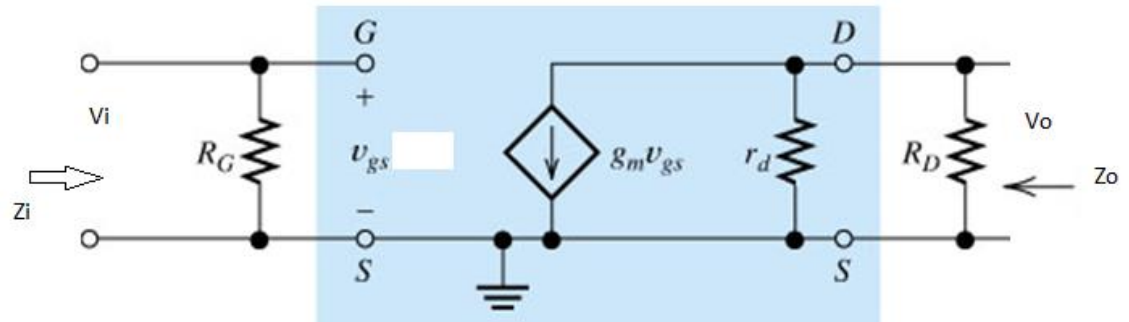


Fig 25: ac equivalent circuit

Z_i : From the circuit, Fig. (25)

$$Z_i = R_G$$

Z_o :

Reduce $V_i=0$, $V_{gs}=0$ therefore $g_m V_{gs}=0$.

$$Z_o = R_D \parallel r_d$$

If $r_d \gg R_D$, Then $Z_o = R_D$

Voltage gain, A_v :

$$A_v = \frac{V_{ds}}{V_{gs}} = \frac{V_o}{V_i}$$

From Fig. (25) $V_o = -g_m V_{gs} (r_d \parallel R_D)$ and $V_i = V_{gs}$

$$A_v = -g_m V_{gs} (r_d \parallel R_D)$$

If $r_d \gg R_D$, $A_v = -g_m R_D$

The negative sign indicates there is a phase shift of 180° between input and output voltages.

COMMON SOURCE (CS) AMPLIFIER WITH SELF BIAS:

Fig 26 shows Common Source amplifier with self bias. Voltage across R_S determines gate to source voltage. Dc equivalent circuit is obtained by open circuiting all capacitors as shown in fig 27.

Apply KVL to Fig27,

$$-V_{GS} - V_S = 0$$

$$V_S = -V_{GS}$$

$$\text{Also, } V_S = I_D R_S$$

$$\text{Therefore, } V_{GS} = -I_D R_S$$

Apply KVL to output circuit of fig27,

$$V_{DS} = V_{DD} - I_D R_S - I_D R_D$$

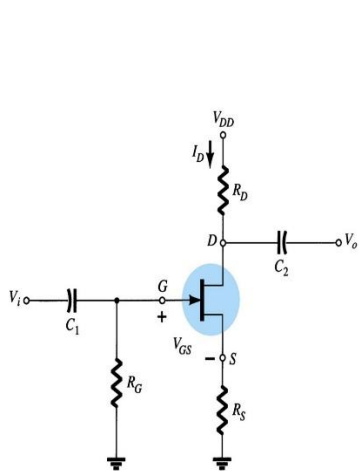


Fig 26. CS amplifier with Self Bias

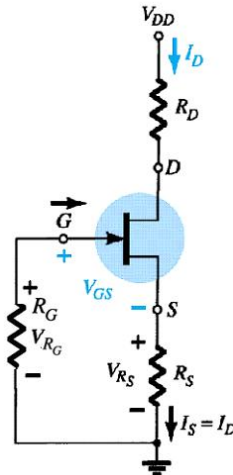


Fig 27 DC equivalent Circuit

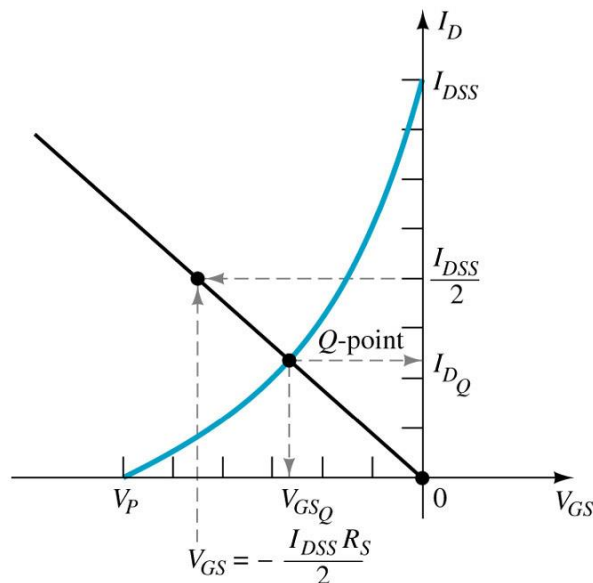


Fig 28 : Q point

Fig 8 shows the location of Q point obtained by following procedure.

- Plot transfer characteristics
- Plot one point of load line at $V_{GS} = 0, I_D = 0$.
- Second point can be obtained by choosing I_D and finding V_{GS} .
- $I_D = I_{DSS}/2$, then

$$V_{GS} = -I_D R_S = -I_{DSS} R_S / 2$$

Join two points to draw DC load line

- Intersection of load line with transfer characteristics will give Q point.

CS amplifier with Self Bias(Bypassed R_s) –ac analysis

Fig 29 shows CS amplifier with Self bias and R_S is bypassed by C_S . Fig 30 shows ac equivalent Circuit obtained by short circuiting C_1, C_2, C_S .

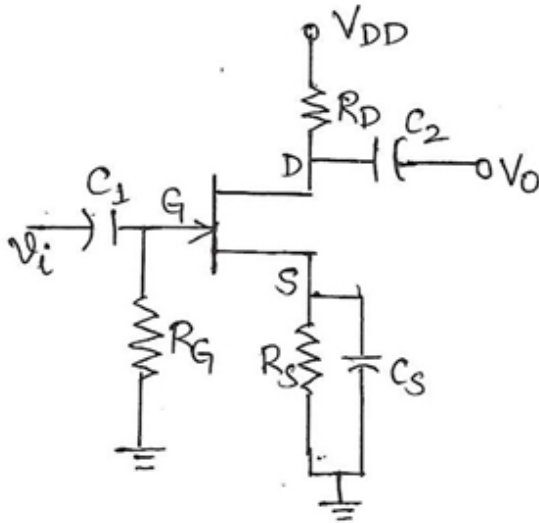


Fig 29: CS amplifier with Self Bias

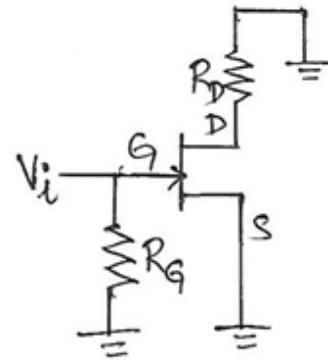


Fig 30: ac Equivalent Circuit.

Replacing JFET by its equivalent small signal model results in circuit shown in Fig 31.

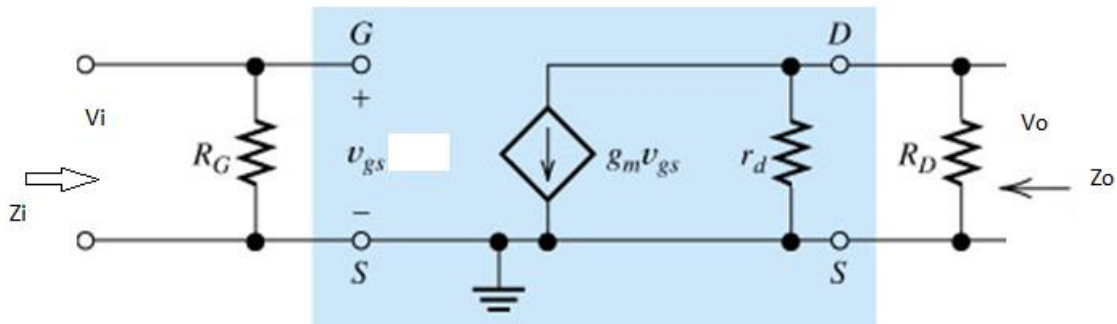


Fig 31: AC equivalent model of CS amplifier with Self bias

Z_i : From the circuit, Fig. (31)

$$Z_i = R_G$$

Z_o :

Reduce $V_i = 0$, $V_{gs} = 0$ therefore $g_m V_{gs} = 0$.

$$Z_o = R_D \parallel r_d$$

If $r_d \gg R_D$, Then $Z_o = R_D$

Voltage gain, A_v :

$$A_v = \frac{V_{ds}}{V_{gs}} = \frac{V_o}{V_i}$$

From Fig. (31) $V_o = -g_m V_{gs} (r_d \parallel R_D)$ and $V_i = V_{gs}$

$$A_v = -g_m V_{gs} (r_d \parallel R_D)$$

If $r_d \gg R_D$, $A_v = -g_m R_D$

The negative sign indicates there is a phase shift of 180° between input and output voltages.

CS amplifier with Self Bias(UnBypassedRs) –ac analysis

Fig 32 shows CS amplifier with self bias but R_S is unbypassed. To obtain ac equivalent circuit, C_1 and C_2 are short circuited as shown in Fig33. Replacing JFET by its equivalent small signal model, we get the circuit shown in fig 34.

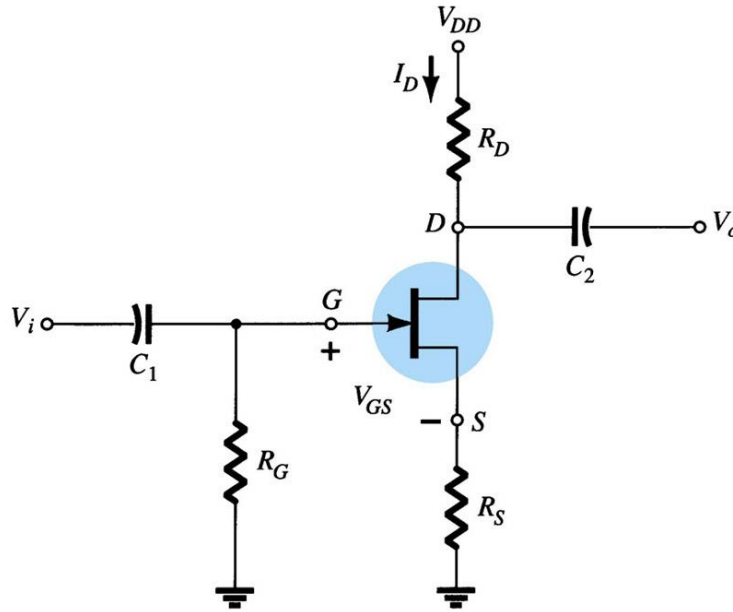


Fig 32: CS amplifier with self bias

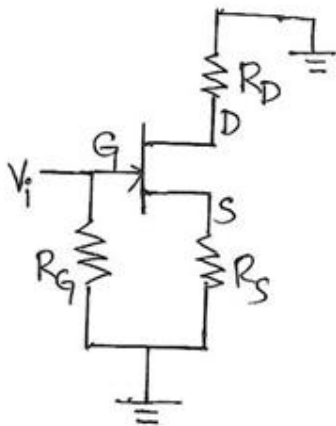


Fig 33: ac Equivalent Circuit

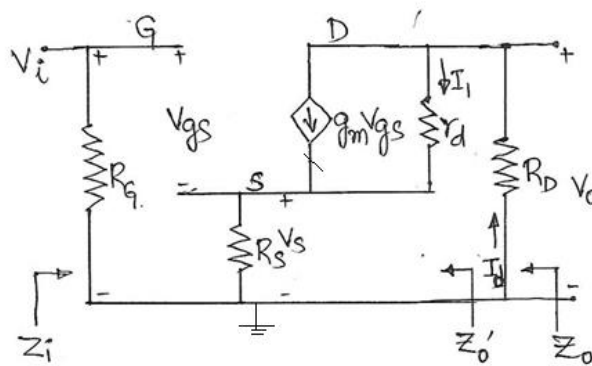


Fig 34: ac Equivalent model

Z_i : From Fig 34,
 $Z_i = R_G$.

Z_o' : Output impedance excluding R_D .

$$Z_o' = V_o/I_d$$

Apply KVL to the output circuit of Fig 34,

$$V_o = I_1 r_d + I_d R_S$$

But, $I_1 = I_d - g_m V_{gs}$.

Therefore, $V_o = (I_d - g_m V_{gs}) + I_d R_S$ -----(16)

Apply KVL to the input circuit of Fig 34,

$$V_i - V_{gs} - I_d R_S = 0$$

$$V_{gs} = -I_d R_S + V_i$$

For output impedance, $V_i = 0$.

Therefore, $V_{gs} = -I_d R_S$ -----(17)

Substituting Eq (17) in (16)

$$V_o = I_d (r_d + g_m R_S r_d + R_S)$$

Therefore, $Z_o' = V_o/I_d = r_d + g_m R_S r_d + R_S$

But, $\mu = g_m r_d$

Therefore, $Z_o' = r_d + R_S (\mu + 1)$ -----(18)

Thus, output impedance with unbypassed R_S is increased.

Z_o : Output impedance considering R_D

$$Z_o = Z_o' \parallel R_D$$

Voltage Gain , A_V :

From Fig 34, $V_o = -I_d R_D$ -----(19)

Apply KVL to the outer part of Fig 34,

$$(I_d - g_m V_{gs}) r_d + I_d R_D + I_d R_S = 0$$
-----(20)

Also $V_{gs} = V_i - I_d R_S$ -----(21)

Eq (21) in Eq (20)

$$I_d = (g_m V_i r_d) / (r_d + g_m R_S r_d + R_S + R_D)$$
-----(22)

Eq (22) in Eq (19)

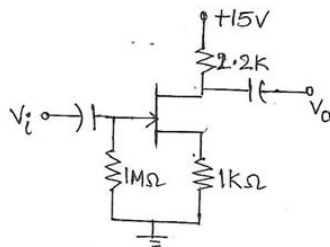
$$V_o = (-g_m V_i r_d R_D) / (r_d + g_m R_S r_d + R_S + R_D)$$

$$A_V = V_o/V_i = -g_m r_d R_D / (r_d + g_m R_S r_d + R_S + R_D)$$

If $r_d \gg R_S + R_D$,

$$A_V = -g_m R_D / (1 + g_m R_S)$$

Example: For the CS amplifier shown, operating point is defined by $V_{GSQ} = -2.5V$, $V_P = -6V$ and $I_{DQ} = 2.5mA$ with $I_{DSS} = 8mA$. Calculate g_m , r_d , Z_i , Z_o and A_V . Take $Y_{os} = 20\mu S$



(i) $g_m = g_{mo} \left(1 - \frac{V_{GS}}{V_P} \right)$

$$g_{m0} = -\frac{2I_{DSS}}{V_P} = \frac{2 \times 8 \times 10^{-3}}{6} = 2.67 \text{ mS}$$

$$g_m = g_{m0} \left(1 - \frac{V_{GS}}{V_P} \right) = \left(1 - \frac{(-2.5V)}{(-6V)} \right) = 1.58 \text{ mS}$$

$$(ii) \quad r_d = \frac{1}{Y_{OS}} = \frac{1}{20 \times 10^{-6}} = 50 \text{ k}\Omega$$

$$(iii) \quad Z_i = R_G = 1 \text{ M}\Omega$$

$$(iv) \quad Z_o = Z_o' \parallel R_D = r_d + R_S(\mu+) \parallel R_D = 2163.41 \Omega$$

(v)

$$A_V = \frac{V_o}{V_i} = -g_m r_d R_D / r_d + g_m R_S r_d + R_S + R_D = -1.315.$$

CS amplifier with Voltage Divider Bias(Bypassed Rs):

Fig 35 Shows voltage divider bias circuit.

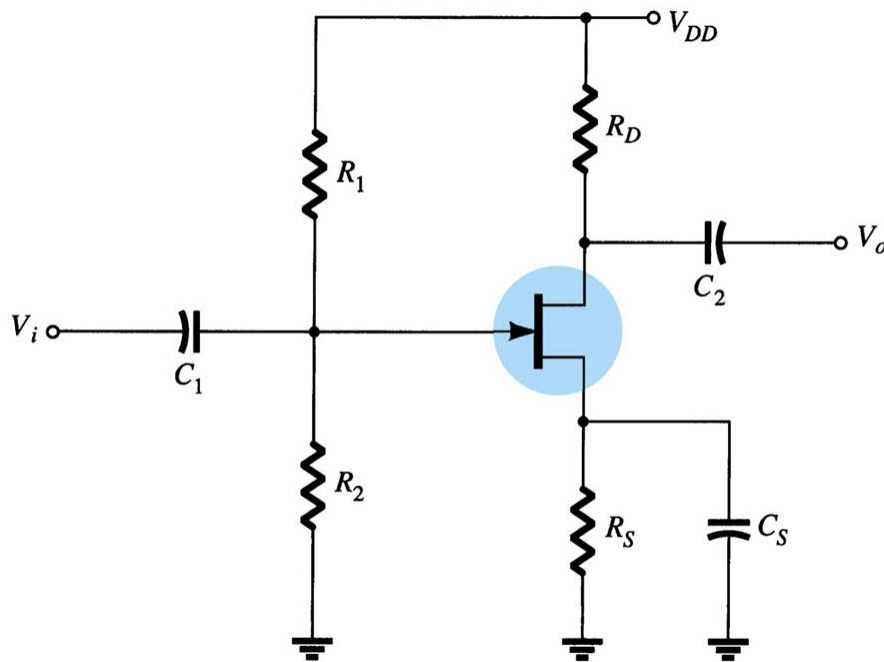


Fig 35: Voltage Divider Bias

DC analysis: Open circuit C1, C2, CS and the resultant circuit is as shown in Fig 36.

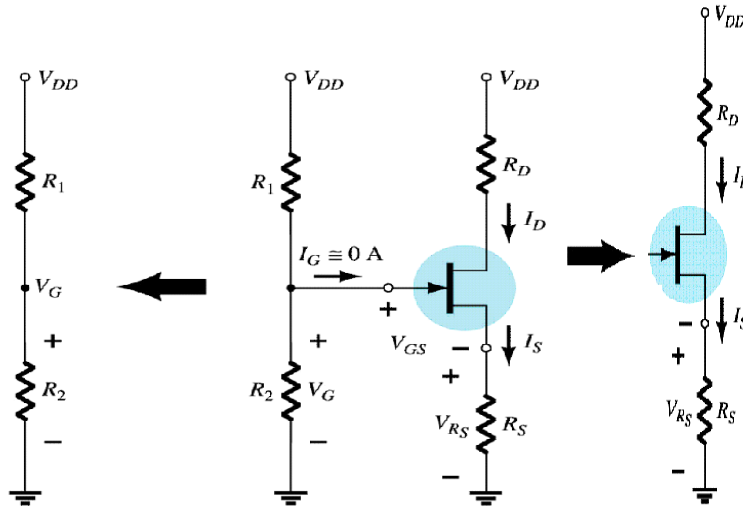


Fig 36: DC Equivalent Circuit.

From Fig 36,

$$V_G = V_{DD} \left(\frac{R_2}{R_1 + R_2} \right)$$

$$-V_G + V_{GS} + I_D R_S = 0$$

$$V_{GS} = V_G - I_D R_S$$

$$V_{DS} = V_{DD} - I_D (R_D + R_S)$$

Fig 37 shows the procedure to fix Q point in voltage divider bias.

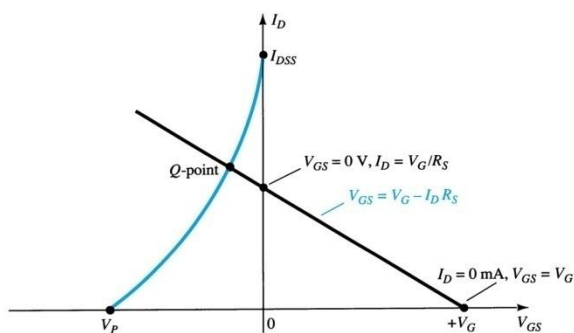


Fig 37: Fixing Q Point.

AC Analysis: AC Equivalent Circuit is obtained by Shorting C1, C2 and CS as shown in Fig 38. Replacing JFET by its small signal model, we get the circuit shown in Fig 39.

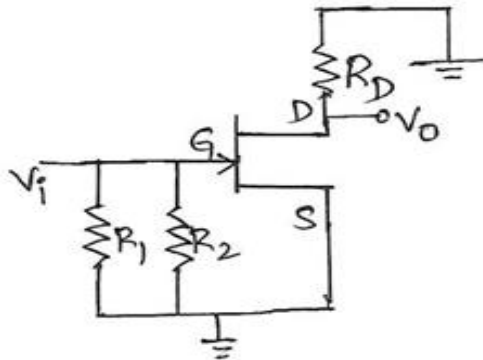


Fig 38: AC Equivalent Circuit

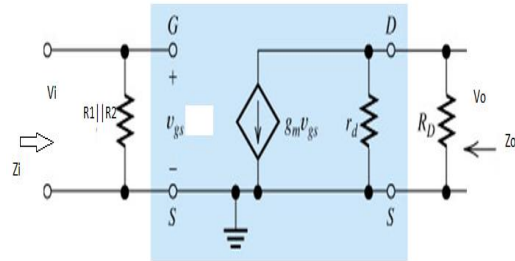


Fig 39: AC Equivalent model

Z_i : From the circuit, Fig. (39)

$$Z_i = R_1 \parallel R_2$$

Z_o :

Reduce $V_i=0$, $V_{gs}=0$ therefore $g_m V_{gs}=0$.

$$Z_o = R_D \parallel r_d$$

If $r_d \gg R_D$, Then $Z_o = R_D$

Voltage gain, A_v :

$$A_v = \frac{V_{ds}}{V_{gs}} = \frac{V_o}{V_i}$$

From Fig. (39) $V_o = -g_m V_{gs} (r_d \parallel R_D)$ and $V_i = V_{gs}$

$$A_v = -g_m V_{gs} (r_d \parallel R_D)$$

If $r_d \gg R_D$, $A_v = -g_m R_D$

The negative sign indicates there is a phase shift of 180° between input and output voltages.

CS amplifier with Voltage Divider Bias (UnBypassed R_s): ac Analysis

Fig 40 Shows CS voltage divider bias with un-bypassed R_s . AC Analysis is obtained by short circuiting C1, C2 and the resultant circuit is shown in Fig 41. Replacing JFET by its equivalent small signal model, we get circuit shown in Fig 42.

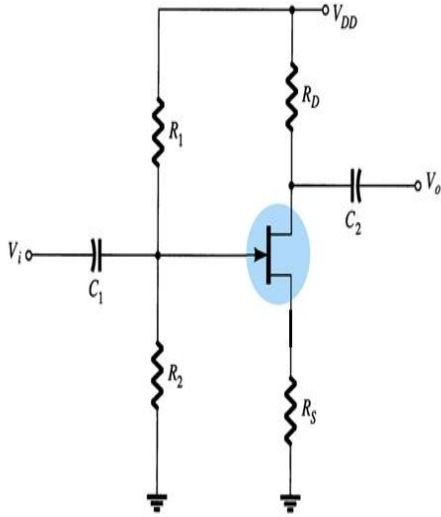


Fig 40: CS amplifier with Voltage Diver Bias

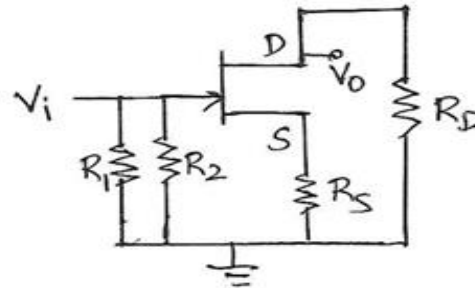


Fig 41: Ac Equivalent Circuit

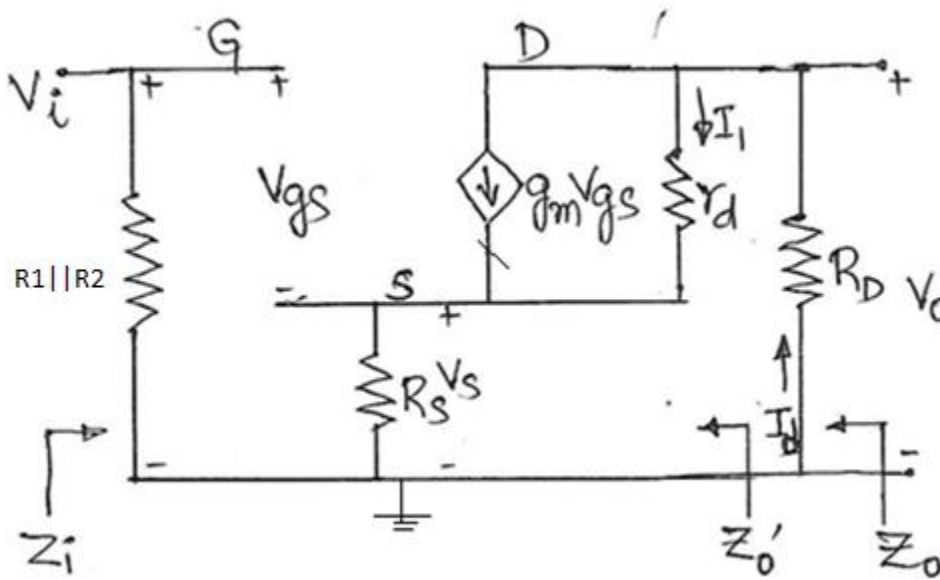


Fig 42: AC equivalent Model

Zi: From Fig 42,

$$Z_i = R_1 || R_2.$$

Zo': Output impedance excluding RD.

$$Z_o' = V_o / I_d$$

Apply KVL to the output circuit of Fig 42,

$$V_o = I_d r_d + I_d R_S$$

$$\text{But, } I_d = I_d - g_m V_{gs}.$$

$$\text{Therefore, } V_o = (I_d - g_m V_{gs}) + I_d R_S \text{-----(23)}$$

Apply KVL to the input circuit of Fig 42,

$$V_i - V_{gs} - I_d R_S = 0$$

$$V_{gs} = -I_d R_S + V_i$$

For output impedance, $V_i = 0$.

$$\text{Therefore, } V_{gs} = -I_d R_S \text{-----(24)}$$

Substituting Eq (24) in (23)

$$V_o = I_d(r_d + g_m R_S r_d + R_S)$$

$$\text{Therefore, } Z_o' = V_o/I_d = r_d + g_m R_S r_d + R_S$$

$$\text{But, } \mu = g_m r_d$$

$$\text{Therefore, } Z_o' = r_d + R_S(\mu + 1) \text{-----(25)}$$

Thus, output impedance with unbypassed R_S is increased.

Z_o : Output impedance considering R_D

$$Z_o = Z_o' || R_D$$

Voltage Gain , A_V :

$$\text{From Fig 42, } V_o = -I_d R_D \text{-----(26)}$$

Apply KVL to the outer part of Fig 42,

$$(I_d - g_m V_{gs})r_d + I_d R_D + I_d R_S = 0; \text{-----(27)}$$

$$\text{Also } V_{gs} = V_i - I_d R_S \text{-----(28)}$$

Eq (27) in Eq (26)

$$I_d = (g_m V_i r_d) / (r_d + g_m R_S r_d + R_S + R_D) \text{-----(29)}$$

Eq (22) in Eq (19)

$$V_o = (-g_m V_i r_d R_D) / (r_d + g_m R_S r_d + R_S + R_D)$$

$$A_V = V_o/V_i = -g_m r_d R_D / (r_d + g_m R_S r_d + R_S + R_D)$$

If $r_d \gg R_S + R_D$,

$$A_V = -g_m R_D / (1 + g_m R_S)$$

Common Drain (CD)/ Source Follower Configuration:

Fig. (43) shows Common Drain configuration. The input is applied between Gate and Source and output between Source and Ground (i.e. Drain is grounded during AC analysis)

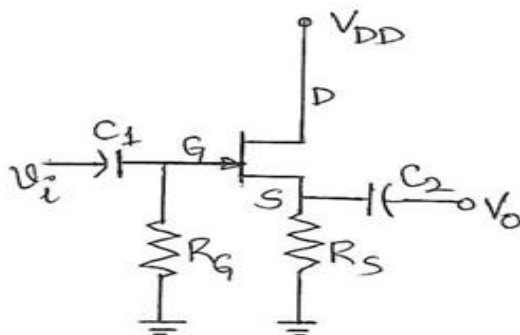


Fig 43: Source Follower Circuit

From the circuit in Fig 43,

$$V_G + V_{GS} - V_S = 0$$

$$\text{Therefore } V_G + V_{GS} = V_S$$

When a signal is applied to JFET gate via C_1 , V_G varies with the signal. As V_{GS} is constant and $V_S = V_G + V_{GS}$ varies with V_i . As the output voltage at the Source (V_S)

follows changes in the signal voltage applied to the gate, this circuit is also called Source follower.

The AC equivalent circuit and low frequency equivalent model for Source follower is as shown in Fig. (44) and Fig. (45) respectively.

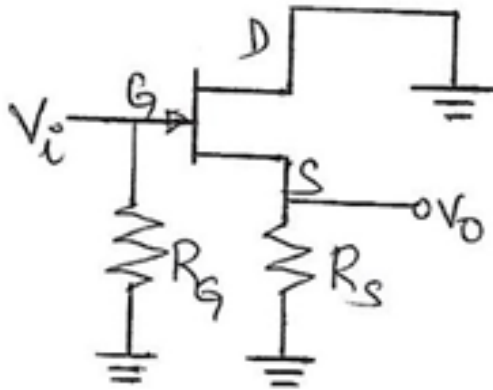


Fig. (44) AC Equivalent Circuit

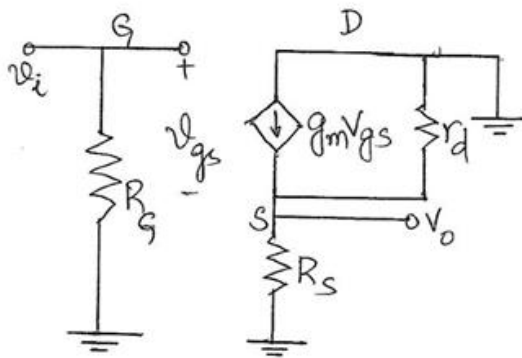


Fig. (45) AC Equivalent Model

$$V_i - V_{gs} - V_o = 0$$

$$V_o = V_i - V_{gs}$$

$$V_o + V_{gs} - V_i = 0$$

$$V_{gs} = V_i - V_o$$

$$V_i = 0, V_{gs} = -V_o$$

Z_i:

From the input circuit,

$$Z_i = R_G$$

Output Z:Z_o

Fig. (45) can also be written as Fig. (46)

$$Z_o = \frac{V_o}{I_d}$$

Apply KVL to the output loop of Fig 46,

$$V_i - V_{gs} - V_o = 0$$

$$\text{For } Z_o, V_i = 0, V_o = V_{gs}$$

$$\text{But from Fig. (46), } I_d = g_m V_{gs}$$

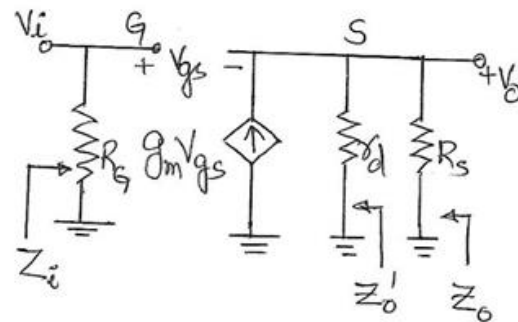


Fig (46) AC Equivalent Model

Therefore $g_m V_o = I_d$

$$Z_o = \frac{V_o}{I_d} = \frac{1}{g_m}$$

Therefore $Z_o = Z_o \parallel R_S$

$$Z_o = \frac{1}{g_m} \parallel R_S$$

Voltage Gain, A_v

$$A_v = \frac{V_o}{V_i}$$

From Fig. (46),

$$V_o = I_d (r_d \parallel R_S)$$

$$\text{And } I_d = g_m V_{gs}$$

$$\text{Therefore } V_o = g_m V_{gs} (r_d \parallel R_S)$$

From input circuit

$$V_i = -V_{gs} + V_o$$

$$\text{Therefore } V_i = -g_m V_{gs} (r_d \parallel R_S) - V_{gs}$$

$$\text{Therefore } A_v = \frac{V_o}{V_i} = \frac{-g_m V_{gs} (r_d \parallel R_S)}{-V_{gs} [g_m (r_d \parallel R_S) + 1]}$$

$$A_v = \frac{g_m (r_d \parallel R_S)}{1 + [g_m (r_d \parallel R_S)]}$$

If $r_d \gg R_S$; $r_d \parallel R_S \approx R_S$

$$A_v = \frac{g_m R_S}{1 + g_m R_S}$$

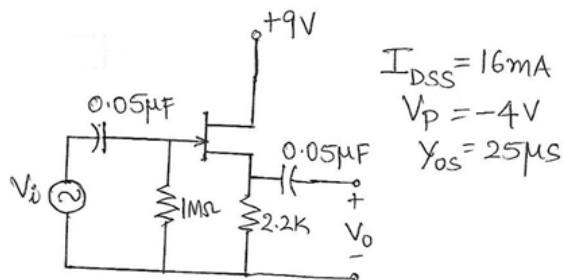
If $g_m R_S \gg 1$, $A_v \approx 1$ but it is always less than one.

There is no phase shift between input and output voltages.

Source Follower exhibits following Characteristics;

- High input Impedance.
- Low Output Impedance
- Voltage gain is less than 1.
- No phase shift between input and output.

Example: A DC analysis of Source Follower network shown in Fig. below results in $V_{GSQ} = -2.86V$ and $I_{DQ} = 4.56mA$. Determine (i) g_m (ii) r_d (iii) Z_i (iv) Z_o with and without r_d (v) A_v with and without r_d . Take $I_{DSS} = 16mA$, $V_p = -4V$, $Y_{OS} = 25\mu S$.



(i) $g_{m0} = \frac{2I_{DSS}}{V_P} = \frac{2 \times 16 \times 10^{-3}}{4} = 8 \text{ mS}$
 $g_m = g_{m0} \left(1 - \frac{V_{GSQ}}{V_P}\right) = 8 \times 10^{-3} \left(1 - \frac{(-2.86)}{(-4)}\right)$
 $g_m = 2.28 \times 10^{-3} \text{ S}$

(ii) $r_d = \frac{1}{Y_{OS}} = \frac{1}{25 \times 10^{-6}} = 40 \text{ K}\Omega$

(iii) $Z_i = R_G = 1 \text{ M}\Omega$

(iv) With r_d
 $Z_o = r_d \parallel R_S \parallel \frac{1}{g_m} = 40 \text{ K} \parallel 2.2 \text{ K} \parallel \frac{1}{2.28 \times 10^{-3}}$
 $= 362.52 \Omega$
 Without r_d
 $Z_o = R_S \parallel \frac{1}{g_m} = 2.2 \times 10^3 \parallel \frac{1}{2.28 \times 10^{-3}}$
 $= 365.69 \Omega$

(v) A_V With r_d
 $A_V = \frac{g_m(r_d \parallel R_S)}{1 + [g_m(r_d \parallel R_S)]} = 0.826$
 A_V Without r_d
 $A_V = \frac{g_m R_S}{1 + g_m R_S} = \frac{2.28 \times 10^{-3} \times 2.2 \times 10^3}{1 + 2.28 \times 10^{-3} \times 2.2 \times 10^3} = 0.833$

Common Gate (CG) Configuration:

Fig. (47) shows CG configuration, the input is applied between Source and Gate and output is taken between Drain and Gate.

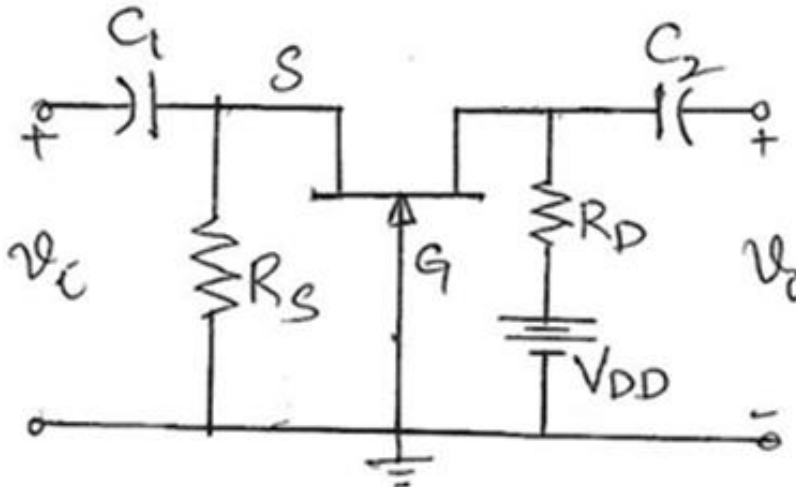


Fig. (47) Common Gate Configuration

In CG configuration, Gate voltage is constant. An increase in V_i in positive direction increases the $-V_e$ Gate to Source bias voltages. Due to this Drain current reduces, reducing the drop $I_D R_D$. Since $V_D = V_{DD} - I_D R_D$, the reduction in I_D results in an

increase in output V_g V_D . Similarly when input V_g reduces, opposite action takes place which reduces the output voltage. Thus there is no phase shift between input and output in a Common Gate amplifier. AC equivalent model of CG amplifier is as shown in Fig. (48).

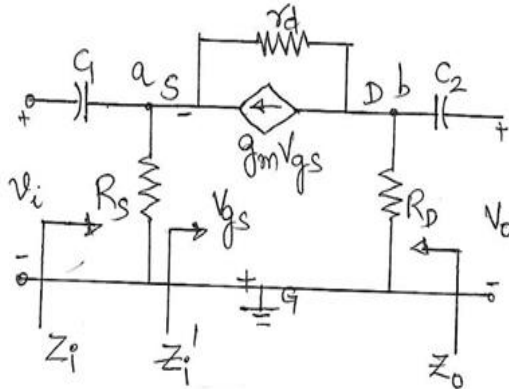


Fig. (48) AC Equivalent Model

$$\hat{Z}_i = \frac{V_i}{I}$$

Current through r_d (Fig 49)

$$I_{rd} = I + g_m V_{gs}$$

$$\text{Therefore } I = I_{rd} - g_m V_{gs} \text{ ----- (30)}$$

From the circuit,

$$I_{rd} = \frac{V_i - IR_D}{r_d} \text{ ----- (31)}$$

Substitute Eq. (31) in Eq. (30)

$$I = \frac{V_i - IR_D}{r_d} - g_m V_{gs} \text{ ----- (32)}$$

But $V_i = -V_{gs}$ (from Fig. (49))

$$\text{Therefore } I = \frac{V_i - IR_D}{r_d} + g_m V_i$$

$$I = \frac{V_i}{r_d} - \frac{IR_D}{r_d} + g_m V_i$$

$$I \left[1 + \frac{R_D}{r_d} \right] = V_i \left[\frac{1}{r_d} + g_m \right]$$

$$\frac{V_i}{I} = \frac{1 + \frac{R_D}{r_d}}{\frac{1}{r_d} + g_m} = \frac{r_d + R_D}{g_m r_d + 1} = \hat{Z}_i$$

$$Z_i = \hat{Z}_i \parallel R_S = R_S \parallel \frac{r_d + R_D}{1 + g_m r_d}$$

If $r_d \gg R_D$ and $g_m r_d \gg 1$,

$$Z_i = R_S \parallel \frac{r_d}{g_m r_d} = R_S \parallel \frac{1}{g_m}$$

Input impedance of CG amplifier is less than CS and CD amplifier.

Z_o : when $V_i=0$ i.e. when input is short circuited, the equivalent circuit is

$$Z_o = r_d \parallel R_D$$

If $r_d \gg R_D$, $Z_o \approx R_D$

$$A_v = \frac{V_o}{V_i}$$

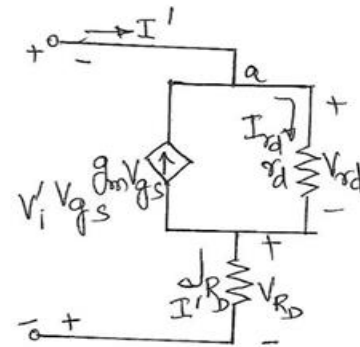


Fig (49) AC Equivalent Model Redrawn

$$V_o = -I_D R_D \text{ and } V_i = -V_{gs}$$

Apply KVL to Fig. (b) outer loop

$$V_i + (I_d - g_m V_{gs}) r_d + I_D R_D = 0$$

$$\text{But } V_{gs} = -V_i$$

$$\text{Therefore } V_i + I_d R_d + g_m V_i r_d + I_D R_D = 0$$

$$V_i [1 + g_m r_d] + I_d [r_d + R_D] = 0$$

$$-I_d [r_d + R_D] = V_i [1 + g_m r_d]$$

$$V_i = \frac{-I_d (r_d + R_D)}{1 + g_m r_d}$$

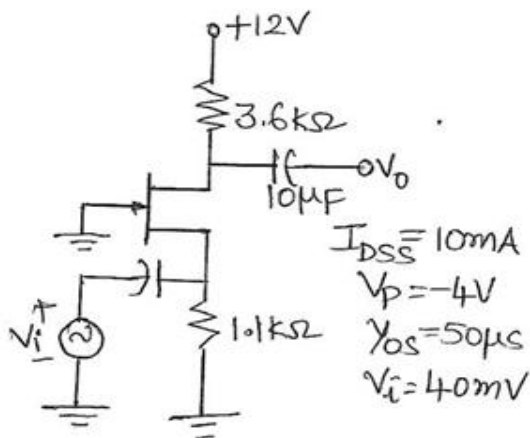
$$A_v = \frac{V_o}{V_i} = \frac{R_D (1 + g_m r_d)}{r_d + R_D}$$

If $r_d \gg R_D$, $g_m r_d \gg 1$

$$A_v = \frac{R_D (g_m r_d)}{r_d} = R_D g_m$$

Thus there is no phase shift between input and output in CG amplifier.

Example: For the network shown, if $V_{GSQ} = -2.2V$ and $I_{DQ} = 2.03mA$. Determine g_m, r_d . Calculate Z_i with and without r_d , Z_o with and without r_d . Determine v_o with and without r_d .



$$(i) \quad g_{m0} = \frac{2I_{DSS}}{V_p} = \frac{2 \times 10 \times 10^{-3}}{4} = 5 \text{ mS}$$

$$g_m = g_{m0} \left(1 - \frac{V_{GSQ}}{V_p} \right) = 5 \times 10^{-3} \left(1 - \frac{(-2.2)}{(-4)} \right)$$

$$g_m = 2.25 \times 10^{-3} \text{ S.}$$

$$(ii) \quad r_d = \frac{1}{Y_{OS}} = \frac{1}{50 \times 10^{-6}} = 20 \text{ k}\Omega$$

$$(iii) \quad \frac{r_d + R_D}{g_m r_d + 1} = Z_i = 0.31 \text{ k}\Omega$$

$$(iv) \quad Z_i = Z_i \parallel R_S = R_S \parallel \frac{r_d + R_D}{1 + g_m r_d} = 0.35 \text{ k}\Omega$$

$$(v) \quad Z_o \approx R_D = 3.6 \text{ k}\Omega$$

$$(vi) \quad Z_o = r_d \parallel R_D = 3.05 \text{ k}\Omega$$

(vii) $A_V = R_D g_m = 8.1$

(viii) $A_V = \frac{V_o}{V_i} = \frac{R_D(1 + g_m r_d)}{r_d + R_D} = 7.02$

(ix) $v_o \text{ without } r_d = 324\text{mV}$

$v_o = 280.8\text{mV}$