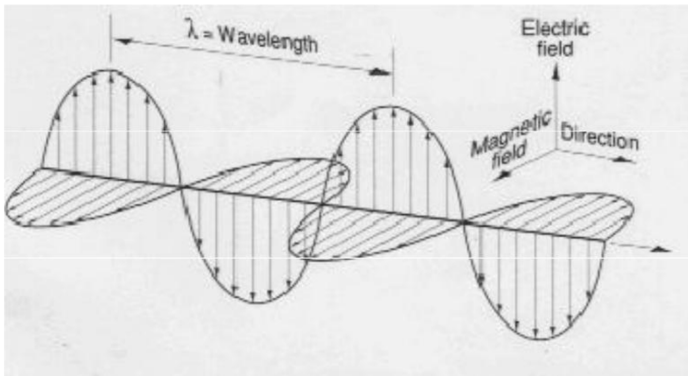
MODULE-IV

Plane Wave:

A uniform plane wave is the wave that the electric field, E or magnetic field, H in same direction, same magnitude and same phase in infinite planes perpendicular to the direction of propagation. A plane wave has no electric field, and magnetic field, components along its direction of propagation.



Wave Equations:

If the wave is in simple (linear, isotropic and homogeneous) nonconducting medium (=0), Maxwell's equation reduce to,

$$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \qquad \vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} \qquad \vec{\nabla} \cdot \vec{H} = 0$$

The first-order differential equations in the two variables E and H . They can combine to give E or H alone using second-order equation.

Using Maxwell's equation,

$$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (1) \qquad \vec{\nabla} \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} \quad (2) \qquad \vec{\nabla} \cdot \vec{E} = 0 \quad (3)$$

The curl of equation of (1)

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu \frac{\partial}{\partial t}\left(\vec{\nabla} \times \vec{H}\right)$$

Replacing in equation (2)

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

We know that $\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \vec{\nabla}\left(\vec{\nabla} \cdot \vec{E}\right) - \vec{\nabla}^2 \vec{E}$ because of equation (3), thus the wave equation is

$$\vec{\nabla}^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

The wave equation also can written as

$$\vec{\nabla}^2 \vec{E} - k^2 \vec{E} = 0$$ --------------------------(a)

Assuming an implicit time dependence $e^{j\omega t}$ in the field vector. Equation (a) also called Helmholtz equation. The k is called the wave number or propagation constant.

$$k - k_o\sqrt{\varepsilon_r} \qquad = \frac{2\pi f}{c}\sqrt{\varepsilon_r}$$

and

$$c - \frac{1}{\sqrt{\varepsilon\mu}}$$

where c is the velocity of light in free space.

For magnetic intensity domain, H, we have,

$$\vec{\nabla}^2 \vec{H} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

or

$$\vec{\nabla}^2 \vec{H} - \mu_r \varepsilon_r k_o^2 \vec{H} = 0$$

For a uniform plane wave with an electric field $E\ \hat{x}\ E_x$ traveling in the z-direction, the wave equation can be reduced as

$$\frac{\partial^2 \vec{E}_x(z)}{\partial z^2} - k^2 \vec{E}_x(z) = 0$$

The solution of this wave equation,

$$\vec{E}(z) = \hat{x}E_x = \hat{x}E_o e^{-kz} = \hat{x}E_o e^{-\alpha z} e^{-j\beta z}$$

Where is the attenuation constant of the medium and is its phase constant.

The associated magnetic field, H,

$$\vec{H}(z) = \hat{y}H_y = \hat{y}\frac{\vec{E}_x}{\eta} = \hat{y}\frac{E_o}{\eta} e^{-\alpha z} e^{-j\beta z}$$

where is the intrinsic impedance of the medium.

The k is called the wave number or propagation constant.

$$k^2 = k_o^2 \varepsilon_r . \mu_r$$

$$k^2 = k_o^2 \mu_r (\varepsilon_r' - j\varepsilon_r'')$$

The wave number can also be written in terms of and .

$$k^2 = (\alpha + j\beta)^2 = (\alpha^2 - \beta^2) + j2\alpha\beta$$

Thus,

$$\alpha^2 - \beta^2 = k_o^2 \mu_r \varepsilon_r' \qquad \textbf{(1)}$$

$$2\alpha\beta = -k_o^2 \mu_r \varepsilon_r'' \qquad \textbf{(2)}$$

By solving (1) & (2),

$$\alpha = \sqrt{\frac{k_o^2 \mu_r \varepsilon_r'}{2}\left(\sqrt{1+\left(\frac{\varepsilon_r''}{\varepsilon_r'}\right)^2} - 1\right)}$$

$$\beta = \sqrt{\frac{k_o^2 \mu_r \varepsilon_r'}{2}\left(\sqrt{1+\left(\frac{\varepsilon_r''}{\varepsilon_r'}\right)^2} + 1\right)}$$

So for different medium,

Lossless Medium $(\sigma = 0)$	Low-loss Medium $(\varepsilon''/\varepsilon' \neq 0)$	Conductor $(\varepsilon''/\varepsilon' \to \infty)$
$\alpha = 0$	$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$	$\alpha = \sqrt{\pi f \mu \sigma}$
$\beta = \omega\sqrt{\mu\varepsilon}$	$\beta = \omega\sqrt{\mu\varepsilon}$	$\beta = \sqrt{\pi f \mu \sigma}$

Electromagnetic Phenomena are described by using four Maxwell's equations:

	Maxwell's equation		
	Integral form:	**Description**	**Information**
Gauss's Law (Electric fields)	$\underbrace{\varepsilon_o \oint \vec{E} \cdot d\vec{S}}_{Left} = \underbrace{q}_{Right}$	**Left side:** The number of electric field lines – perpendicularly passing through to a closed surface, $\vec{S}$ **Right side:** Total amount of charge, q contained within that surface, .	Electric charge produces an electric field, $\vec{E}$ and the flux of that field passing through any closed surface is proportional to the total charge, q contained within that surface. Charge on an insulated conductor moves outward surface.
	Differential form: $\underbrace{\varepsilon_o \vec{\nabla} \cdot \vec{E}}_{Left} = \underbrace{\rho}_{Right}$	**Left side:** Divergence of the electric field, $\vec{E}$ – the tendency of the field to "flow" away from a specified location. **Right side:** Electric charge density, ρ	The electric field, $\vec{E}$ produced by electric charge diverges from positive charge and converges upon negative charge. The electric field, $\vec{E}$ is tendency to propagate perpendicularly away from a surface charge.

Gauss's Law **(Magnetic fields)**	**Integral form:** $\underbrace{\mu_o \oint \vec{H} \cdot d\vec{S}}_{Left} = \underbrace{0}_{Right}$	**Left side:** The number of magnetic field lines – perpendicularly passing through a closed surface. **Right side:** Identically zero.	The total magnetic flux passing through any closed surface is zero. Flux enter the closed surface is same with the flux come out from the surface. The divergence of the magnetic field at any point is zero.
	Differential form: $\underbrace{\mu_o \vec{\nabla} \cdot \vec{H}}_{Left} = \underbrace{0}_{Right}$	**Left side:** Divergence of the magnetic field – the tendency of the field to "flow" away from a point than toward it. **Right side:** Identically zero.	

Faraday's Law	**Integral form:** $\underbrace{\oint_C \vec{E} \cdot d\vec{l}}_{Left} = \underbrace{-\mu_o \int_S \frac{\partial \vec{H}}{\partial t} \cdot d\vec{S}}_{Right}$	**Left side:** The circulation of the vector electric field, $\vec{E}$ around a closed path, C. **Right side:** The rate of change with time (d/dt) of magnetic field, through any surface, $\vec{S}$.	Changing magnetic flux through a surface induces an emf in any boundary path, C of that surface, and a changing magnetic field, $\vec{H}$ induces a circulating electric field.
	Differential form: $\underbrace{\vec{\nabla} \times \vec{E}}_{Left} = \underbrace{-\mu_o \frac{\partial \vec{H}}{\partial t}}_{Right}$	**Left side:** Curl of the electric field, – the tendency of the field lines to circulate around a point. **Right side:** The rate of change of the magnetic field, $\vec{H}$ over time (d/dt)	A circulating electric field, is produced by a magnetic field, $\vec{H}$ that changes with time.

Ampere's Law	**Integral form:** $\underbrace{\oint_C \vec{H}\cdot d\vec{l}}_{Left} = \underbrace{\int_S \left(\vec{J}_c + \varepsilon_o \frac{\partial \vec{E}}{\partial t}\right)\cdot d\vec{S}}_{Right}$	**Left side:** The circulation of the magnetic field, $\vec{H}$ around a closed path, C. **Right side:** Two sources for the magnetic field, $\vec{H}$: a steady conduction current, $\vec{J}_c$ and a changing electric field, $\vec{E}$ through any surface, bounded by closed path, C.	An electric current or a changing electric flux through a surface produces a circulating magnetic field around any path, C that bounds that surface.
	Differential form: $\underbrace{\bar{\nabla}\times\bar{H}}_{Left} = \underbrace{\bar{J}_c + \varepsilon_o \frac{\partial \bar{E}}{\partial t}}_{Right}$	**Left side:** Curl of the magnetic field, – the tendency of the field lines to circulate around a point. **Right side:** Two terms represent the electric current density, $\vec{J}_c$ and the time rate of change of the electric field, $\vec{E}$.	A circulating electric field, is produced by a magnetic field, $\vec{H}$ that changes with time. An electric current, or a changing electric field, through a surface produces a circulating magnetic field, $\vec{H}$ around any path that bounds that surface.

Poynting Vector and Power Flow in Electromagnetic Fields:

Electromagnetic waves can transport energy from one point to another point. The electric and magnetic field intensities associated with a travelling electromagnetic wave can be related to the rate of such energy transfer.

Let us consider Maxwell's Curl Equations:

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$$

Using vector identity

$$\nabla.\left(\vec{E}\times\vec{H}\right) = \vec{H}.\nabla\times\vec{E} - \vec{E}.\nabla\times\vec{H}$$

The above curl equations we can write

$$\vec{H}.\frac{\partial\vec{B}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2\right) \quad , \quad \vec{E}.\frac{\partial\vec{D}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}\mu E^2\right)$$

And $\vec{E}.\vec{J} = \sigma E^2$.

In simple medium where $\in, \mu$ and σ are constant, we can write

$$\therefore \nabla.\left(\vec{E}\times\vec{H}\right) = -\frac{\partial}{\partial t}\left(\frac{1}{2}\in E^2 + \frac{1}{2}\mu H^2\right) - \sigma E^2$$

Applying Divergence theorem we can write,

$$\oint_S \left(\vec{E}\times\vec{H}\right).d\vec{S} = -\frac{\partial}{\partial t}\int_V\left(\frac{1}{2}\in E^2+\frac{1}{2}\mu H^2\right)dV - \int_V \sigma E^2 dV \quad \text{…………….(a)}$$

The term $\frac{\partial}{\partial t}\int_V\left(\frac{1}{2}\in E^2+\frac{1}{2}\mu H^2\right)dV$ represents the rate of change of energy stored in the electric and magnetic fields and the term $\int_V \sigma E^2 dV$ represents the power dissipation within the volume. Hence right hand side of the equation (a) represents the total decrease in power within the volume under consideration.

The left hand side of equation (6.36) can be written as $\oint_S \left(\vec{E}\times\vec{H}\right).d\vec{S} = \oint_S \vec{P}.d\vec{S}$ where $\vec{P}=\vec{E}\times\vec{H}$ (W/mt^2) is called the Poynting vector and it represents the power density vector associated with the electromagnetic field. The integration of the Poynting vector over any closed surface gives the net power flowing out of the surface. Equation (6.36) is referred to as Poynting theorem and it states that the net power flowing out of a given volume is equal to the time rate of decrease in the energy stored within the volume minus the conduction losses.

Poynting vector for the time harmonic case:

For time harmonic case, the time variation is of the form $e^{j\omega t}$, and we have seen that instantaneous value of a quantity is the real part of the product of a phasor quantity and $e^{j\omega t}$ when $\cos\omega t$ is used as reference. For example, if we consider the phasor

$$\vec{E}(z)=\hat{a}_x E_x(z)=\hat{a}_x E_0 e^{-j\beta z}$$

then we can write the instanteneous field as

$$\vec{E}(z,t)=\mathrm{Re}\left[\vec{E}(z)e^{j\omega t}\right]=E_0\cos(\omega t-\beta z)\hat{a}_x$$

when E_0 is real.

Let us consider two instanteneous quantities A and B such that

$$A=\mathrm{Re}\left(Ae^{j\omega t}\right)=|A|\cos(\omega t+\alpha), \quad B=\mathrm{Re}\left(Be^{j\omega t}\right)=|B|\cos(\omega t+\beta)$$

where A and B are the phasor quantities. i.e, $A=|A|e^{j\alpha}$

$$B=|B|e^{j\beta}$$

Therefore,

$$AB=|A|\cos(\omega t+\alpha)|B|\cos(\omega t+\beta)$$
$$=\frac{1}{2}|A||B|\left[\cos(\alpha-\beta)+\cos(2\omega t+\alpha+\beta)\right]$$

Since A and B are periodic with period $T=\frac{2\pi}{\omega}$, the time average value of the product form AB, denoted by $\overline{AB}$ can be written as

$$\overline{AB} = \frac{1}{T}\int_0^T AB\,dt$$

$$\overline{AB} = \frac{1}{2}|A||B|\cos(\alpha - \beta)$$

Further, considering the phasor quantities *A* and *B*, we find that

$$AB^* = |A|e^{j\alpha}|B|e^{-j\beta} = |A||B|e^{j(\alpha-\beta)}$$

and $\text{Re}(AB^*) = |A||B|\cos(\alpha - \beta)$, where * denotes complex conjugate.

$$\therefore \overline{AB} = \frac{1}{2}\text{Re}(AB^*)$$

The poynting vector $\vec{P} = \vec{E}\times\vec{H}$ can be expressed as

$$\vec{P} = \hat{a}_x(E_yH_z - E_zH_y) + \hat{a}_y(E_zH_x - E_xH_z) + \hat{a}_z(E_xH_y - E_yH_x)$$ ....(b)

If we consider a plane electromagnetic wave propagating in +z direction and has only E_x component, from (b) we can write:

$$\vec{P}_z = E_x(z,t)H_y(z,t)\hat{a}_3$$

Using (6.41)

$$\vec{P}_{zav} = \frac{1}{2}\text{Re}\left(E_x(z)H_y^*(z)\hat{a}_z\right)$$

$$\vec{P}_{zav} = \frac{1}{2}\text{Re}\left(E_x(z)\times H_y(z)\right)$$

where $\vec{E}(z) = E_x(z)\hat{a}_x$ and $\vec{H}(z) = H_y(z)\hat{a}_y$, for the plane wave under consideration.

For a general case, we can write

$$\vec{P}_{av} = \frac{1}{2}\text{Re}\left(\vec{E}\times\vec{H}^*\right)$$

We can define a complex Poynting vector

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^*$$

and time average of the instantaneous Poynting vector is given by $\vec{P}_{av} = \text{Re}\left(\vec{S}\right)$.

Polarisation of plane wave:

The polarization of a plane wave can be defined as the orientation of the electric field vector as a function of time at a fixed point in space. For an electromagnetic wave, the specification of the orientation of the electric field is sufficient as the magnetic field components are related to electric field vector by the Maxwell's equations.

Let us consider a plane wave travelling in the +z direction. The wave has both E_x and E_y components.

$$\vec{E} = \left(\hat{a}_x E_{ox} + \hat{a}_y E_{oy}\right) e^{-j\beta z}$$

The corresponding magnetic fields are given by,

$$\vec{H} = \frac{1}{\eta}\hat{a}_z \times \vec{E}$$
$$= \frac{1}{\eta}\hat{a}_z \times \left(\hat{a}_x E_{ox} + \hat{a}_y E_{oy}\right) e^{-j\beta z}$$
$$= \frac{1}{\eta}\left(-E_{oy}\hat{a}_x + E_{ox}\hat{a}_x\right) e^{-j\beta z}$$

Depending upon the values of E_{ox} and E_{oy} we can have several possibilities:

1. If $E_{oy} = 0$, then the wave is linearly polarised in the *x*-direction.
2. If $E_{oy} = 0$, then the wave is linearly polarised in the *y*-direction.
3. If E_{ox} and E_{oy} are both real (or complex with equal phase), once again we get a linearly polarised wave with the axis of polarisation inclined at an $\tan^{-1}\frac{E_{oy}}{E_{ox}}$, with respect to the x-axis. This is shown in angle fig 6.4.

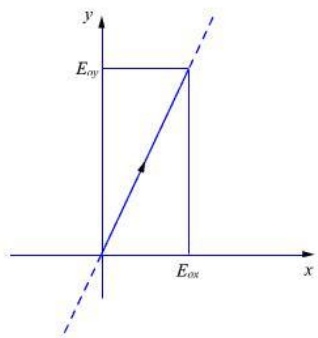


Fig 6.4 : Linear Polarisation

If *Eox* and *Eoy* are complex with different phase angles, $\vec{E}$ will not point to a single spatial direction. This is explained as follows:

Let $E_{ox} = |E_{ox}| e^{ja}$ $E_{oy} = |E_{oy}| e^{jb}$

Then,

and ……………………………….(c)

To keep the things simple, let us consider a =0 and $b = \frac{\pi}{2}$. Further, let us study the nature of the electric field on the z =0 plain.

From equation (c) we find that,

$$E_x(o,t) = |E_{ox}| \cos \omega t$$
$$E_y(o,t) = |E_{oy}| \cos\left(\omega t + \frac{\pi}{2}\right) = |E_{oy}|(-\sin \omega t)$$

$$\therefore \left(\frac{E_x(o,t)}{|E_{ox}|}\right)^2 + \left(\frac{E_y(o,t)}{|E_{oy}|}\right)^2 = \cos^2 \omega t + \sin^2 \omega t = 1$$

and the electric field vector at z = 0 can be written as

$$\vec{E}(o,t) = |E_{ox}|\cos(\omega t)\hat{a}_x - |E_{oy}|\sin(\omega t)\hat{a}_y \quad \text{.........(d)}$$

Assuming $|E_{ox}| > |E_{oy}|$, the plot of $\vec{E}(o,t)$ for various values of t is hown in figure 6.5.

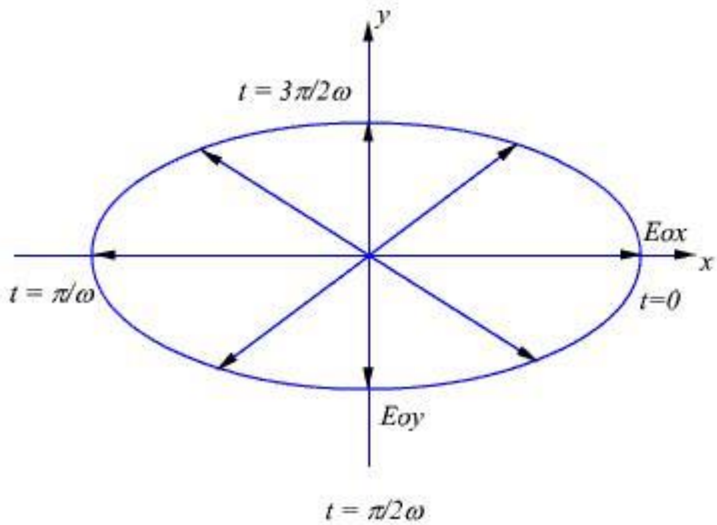


Figure 6.5 : Plot of *E*(*o*,*t*)

From equation (d) and figure (6.5) we observe that the tip of the arrow representing electric field vector traces an ellipse and the field is said to be elliptically polarized.

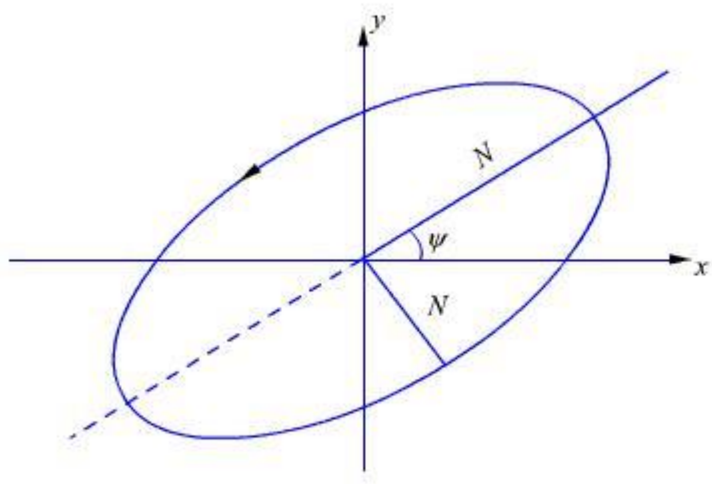


Figure 6.6: Polarisation ellipse

The polarisation ellipse shown in figure 6.6 is defined by its axial ratio(M/N, the ratio of semimajor to semiminor axis), tilt angle ψ (orientation with respect to xaxis) and sense of rotation(i.e., CW or CCW). Linear polarisation can be treated as a special case of elliptical polarisation, for which the axial ratio is infinite.

In our example, if $|E_{ox}| = |E_{oy}|$, from equation (6.47), the tip of the arrow representing electric field vector traces out a circle. Such a case is referred to as Circular Polarisation. For circular polarisation the axial ratio is unity.

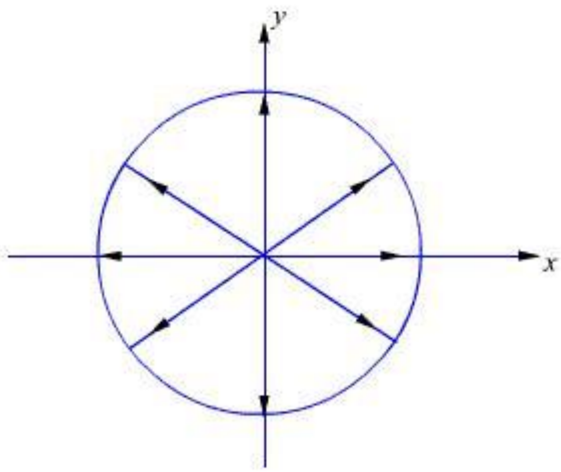


Figure 6.7: Circular Polarisation (RHCP)

Further, the circular polarisation is aside to be right handed circular polarisation (RHCP) if the electric field vector rotates in the direction of the fingers of the right hand when the thumb points in the direction of propagation-(same and CCW). If the electric field vector rotates in the opposite direction, the polarisation is asid to be left hand circular polarisation (LHCP) (same as CW).

In AM radio broadcast, the radiated electromagnetic wave is linearly polarised with the $\vec{E}$ field vertical to the ground(vertical polarisation) where as TV signals are horizontally polarised waves. FM broadcast is usually carried out using circularly polarised waves.
In radio communication, different information signals can be transmitted at the same frequency at orthogonal polarisation (one signal as vertically polarised other horizontally polarised or one as RHCP while the other as LHCP) to increase capacity. Otherwise, same signal can be transmitted at orthogonal polarisation to obtain diversity gain to improve reliability of transmission.

Behaviour of Plane waves at the inteface of two media:
We have considered the propagation of uniform plane waves in an unbounded homogeneous medium. In practice, the wave will propagate in bounded regions where several values of ε, μ, σ will be present. When plane wave travelling in one medium meets a different medium, it is partly reflected and partly transmitted. In this section, we consider wave reflection and transmission at planar boundary between two media.

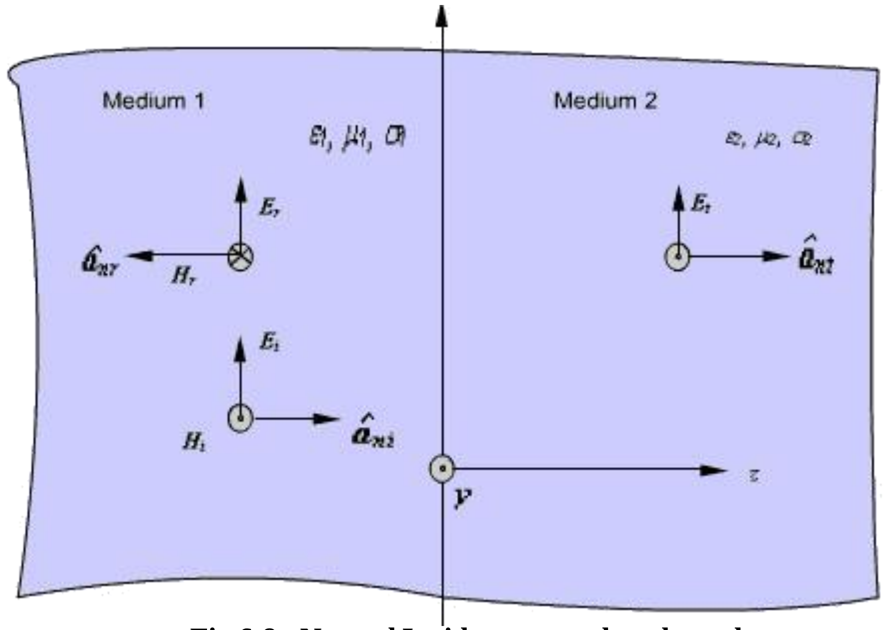


Fig 6.8 : Normal Incidence at a plane boundary

Case1: Let z = 0 plane represent the interface between two media. Medium 1 is characterised by $(\varepsilon_1, \mu_1, \sigma_1)$ and medium 2 is characterized by $(\varepsilon_2, \mu_2, \sigma_2)$.
Let the subscripts '*i*' denotes incident, '*r*' denotes reflected and '*t*' denotes transmitted field components respectively.

The incident wave is assumed to be a plane wave polarized along *x* and travelling in medium 1 along $\hat{a}_z$ direction. From equation (6.24) we can write

$$\vec{E}_i(z) = E_{io} e^{-\gamma_1 z} \hat{a}_x \quad \text{..................(e)}$$

$$\vec{H}_i(z) = \frac{1}{\eta_i} \hat{a}_z \times E_{io}(z) = \frac{E_o}{\eta_i} e^{-\gamma_1 z} \hat{a}_y \quad \text{.....................(f)}$$

where $\gamma_1 = \sqrt{j\omega\mu_1(\sigma_1 + j\omega\varepsilon_1)}$ and $\eta_1 = \sqrt{\frac{j\omega\mu_1}{\sigma_1 + j\omega\varepsilon_2}}$.

Because of the presence of the second medium at *z* =0, the incident wave will undergo partial reflection and partial transmission. The reflected wave will travel along $\hat{a}_z$ in medium 1.
The reflected field components are:

$$\vec{E}_r = E_{ro} e^{\gamma_1 z} \hat{a}_x \quad \text{...(g)}$$

$$\vec{H}_r = \frac{1}{\eta_1}\left(-\hat{a}_z\right) \times E_{ro} e^{\gamma_1 z} \hat{a}_x = -\frac{E_{ro}}{\eta_1} e^{\gamma_1 z} \hat{a}_y \quad \text{.........(h)}$$

The transmitted wave will travel in medium 2 along $\hat{a}_z$ for which the field components are

$$\vec{E}_t = E_{t0} e^{-\gamma_2 z} \hat{a}_x \quad \text{.......................................(i)}$$

$$\vec{H}_t = \frac{E_{t0}}{\eta_2} e^{-\gamma_2 z}\, \hat{a}_y \quad \text{...(j)}$$

where $\gamma_2 = \sqrt{j\omega\mu_2\left(\sigma_2 + j\omega\varepsilon_2\right)}$ and $\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\varepsilon_2}}$

In medium 1,

$$\vec{E}_1 = \vec{E}_i + \vec{E}_r \text{ and } \vec{H}_1 = \vec{H}_i + \vec{H}_r$$

and in medium 2,

$$\vec{E}_2 = \vec{E}_t \text{ and } \vec{H}_2 = \vec{H}_t$$

Applying boundary conditions at the interface z = 0, i.e., continuity of tangential field components and noting that incident, reflected and transmitted field components are tangential at the boundary, we can write

$$\vec{E}_i(0) + \vec{E}_r(0) = \vec{E}_i(0)$$

$$\& \quad \vec{H}_i(0) + \vec{H}_r(0) = \vec{H}_t(0)$$

From equation (e) to (j) we get,

$$E_{io} + E_{ro} = E_{to} \quad \text{...(k)}$$

$$\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2} \quad \text{...(l)}$$

Eliminating E_{to} ,

$$\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{1}{\eta_2}\left(E_{io} + E_{ro}\right)$$

$$\text{or, } E_{io}\left(\frac{1}{\eta_1} - \frac{1}{\eta_2}\right) = E_{ro}\left(\frac{1}{\eta_1} + \frac{1}{\eta_2}\right)$$

$$\text{or, } E_{ro} = \tau E_{io}$$

$$\tau = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad \text{...............(m)}$$

is called the reflection coefficient.

From equation (k) & (l), we can write

$$2E_{io} = E_{to}\left[1 + \frac{\eta_1}{\eta_2}\right]$$

$$\text{or, } E_{to} = \frac{2\eta_2}{\eta_1 + \eta_2} E_{io} = TE_{io}$$

$$T = \frac{2\eta_2}{\eta_1 + \eta_2}$$

is called the transmission coefficient.

We observe that,

$$T = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{\eta_2 - \eta_1 + \eta_1 + \eta_2}{\eta_1 + \eta_2} = 1 + \tau$$

The following may be noted

(i) both τ and T are dimensionless and may be complex

(ii) $0 \le |\tau| \le 1$

Let us now consider specific cases:

Case I: Normal incidence on a plane conducting boundary

The medium 1 is perfect dielectric $(\sigma_1 = 0)$ and medium 2 is perfectly conducting $(\sigma_2 = \infty)$.

$$\therefore \eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}$$

$$\eta_2 = 0$$

$$\gamma_1 = \sqrt{(j\omega\mu_1)(j\omega\epsilon_1)}$$

$$= j\omega\sqrt{\mu_1 \epsilon_1} = j\beta_1$$

From (k) and (l)

$$\tau = -1$$

and T =0

Hence the wave is not transmitted to medium 2, it gets reflected entirely from the interface to the medium 1.

$$\therefore \vec{E}_1(z) = E_{io}e^{-j\beta_1 z}\,\hat{a}_x - E_{io}e^{j\beta_1 z}\,\hat{a}_x = -2jE_{io}\sin\beta_1 z\,\hat{a}_x$$

& $$\therefore \vec{E}_1(z,t) = \mathrm{Re}\left[-2jE_{io}\sin\beta_1 z e^{j\omega t}\right]\hat{a}_x = 2E_{io}\sin\beta_1 z\sin\omega t\,\hat{a}_x$$

Proceeding in the same manner for the magnetic field in region 1, we can show that,

$$\vec{H}_1(z,t) = \hat{a}_y\frac{2E_{io}}{\eta_1}\cos\beta_1 z\cos\omega t$$

The wave in medium 1 thus becomes a **standing wave** due to the super position of a forward travelling wave and a backward travelling wave. For a given $'t'$, both $\vec{E}_1$ and $\vec{H}_1$ vary sinusoidally with distance measured from z = 0. This is shown in figure 6.9.

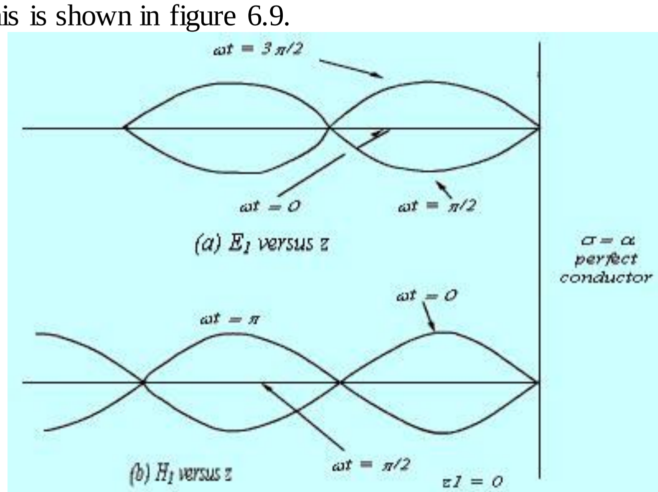


Figure 6.9: Generation of standing wave

Zeroes of $E_1(z,t)$ and
Maxima of $H_1(z,t)$.

$$\left.\right\} \text{occur at } \beta_1 z = -n\pi \quad \text{or } z = -n\frac{\lambda}{2}$$

Maxima of $E_1(z,t)$ and
zeroes of $H_1(z,t)$.

$$\left.\right\} \text{occur at } \beta_1 z = -(2n+1)\frac{\pi}{2} \quad \text{or } z = -(2n+1)\frac{\lambda}{4}, \ n = 0,1,2,....$$

Case2: Normal incidence on a plane dielectric boundary

If the medium 2 is not a perfect conductor (i.e. $\sigma_2 \neq \infty$) partial reflection will result. There will be a reflected wave in the medium 1 and a transmitted wave in the medium 2.Because of the reflected wave, standing wave is formed in medium 1.
From above equations we can write

$$\vec{E}_1 = E_{io}\left(e^{-\gamma_1 z} + \Gamma e^{\gamma_1 z}\right)\hat{a}_x$$

Let us consider the scenario when both the media are dissipation less i.e. perfect dielectrics ($\sigma_1 = 0, \sigma_2 = 0$)

$$\gamma_1 = j\omega\sqrt{\mu_1\varepsilon_1} = j\beta_1 \qquad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$$

$$\gamma_2 = j\omega\sqrt{\mu_2\varepsilon_2} = j\beta_2 \qquad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$

In this case both η_1 and η_2 become real numbers.

$$\vec{E}_1 = \hat{a}_x E_{io}\left(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}\right)$$
$$= \hat{a}_x E_{io}\left((1+T)e^{-j\beta_1 z} + \Gamma\left(e^{j\beta_1 z} - e^{-j\beta_1 z}\right)\right)$$
$$= \hat{a}_x E_{io}\left(Te^{-j\beta_1 z} + \Gamma(2j\sin\beta_1 z)\right) \qquad ....(n)$$

From (n), we can see that, in medium 1 we have a traveling wave component with amplitude TE_{io} and a standing wave component with amplitude $2JE_{io}$.
The location of the maximum and the minimum of the electric and magnetic field components in the medium 1from the interface can be found as follows. The electric field in medium 1 can be written as

$$\vec{E}_1 = \hat{a}_x E_{io} e^{-j\beta_1 z}\left(1 + \Gamma e^{j2\beta_1 z}\right)$$

If $\eta_2 > \eta_1$ i.e. $\Gamma > 0$
The maximum value of the electric field is

$$\left|\vec{E}_1\right|_{max} = E_{io}(1+T)$$

and this occurs when

$$2\beta_1 z_{max} = -2n\pi$$

or $z_{max} = -\frac{n\pi}{\beta_1} = -\frac{n\pi}{2\pi/\lambda_1} = -\frac{n}{2}\lambda_1$, n = 0, 1, 2, 3......................(o)

The minimum value of $|\vec{E}_1|$ is

$$|\vec{E}_1|_{min} = E_{io}(1-\Gamma) \quad\text{.................(p)}$$

And this occurs when

$$2\beta_1 z_{min} = -(2n+1)\pi$$

or $z_{min} = -(2n+1)\frac{\lambda_1}{4}$, n = 0, 1, 2, 3...........................(q)

For $\eta_2 < \eta_1$ i.e. $\Gamma < 0$

The maximum value of $|\vec{E}_1|$ is $E_{io}(1-\Gamma)$ which occurs at the z_{min} locations and the minimum value of $|\vec{E}_1|$ is $E_{io}(1+\Gamma)$ which occurs at z_{max} locations as given by the equations (o) and (q).

From our discussions so far we observe that $\frac{|E|_{max}}{|E|_{min}}$ can be written as

$$S = \frac{|E|_{max}}{|E|_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

The quantity S is called as the standing wave ratio.

As $0 \le |\Gamma| \le 1$ the range of S is given by $1 \le S \le \infty$

We can write the expression for the magnetic field in medium 1 as

$$\vec{H}_1 = \hat{a}_y \frac{E_{io}}{\eta_1} e^{-j\beta_1 z}\left(1 - \Gamma e^{j2\beta_1 z}\right)$$

From above equation we can see that $|\vec{H}_1|$ will be maximum at locations where $|\vec{E}_1|$ is minimum and vice versa.

In medium 2, the transmitted wave propagates in the + z direction.

Oblique Incidence of EM wave at an interface

So far we have discuss the case of normal incidence where electromagnetic wave traveling in a lossless medium impinges normally at the interface of a second medium. In this section we shall consider the case of oblique incidence. As before, we consider two cases

i. When the second medium is a perfect conductor.

ii. When the second medium is a perfect dielectric.

A plane incidence is defined as the plane containing the vector indicating the direction of propagation of the incident wave and normal to the interface. We study two specific cases when the incident electric field $\vec{E}_i$ is perpendicular to the plane of incidence (perpendicular polarization) and $\vec{E}_i$ is parallel to the plane of incidence (parallel polarization). For a general case, the incident wave may have arbitrary polarization but the same can be expressed as a linear combination of these two individual cases.

Oblique Incidence at a plane conducting boundary

i. Perpendicular Polarization

The situation is depicted in figure 6.10.

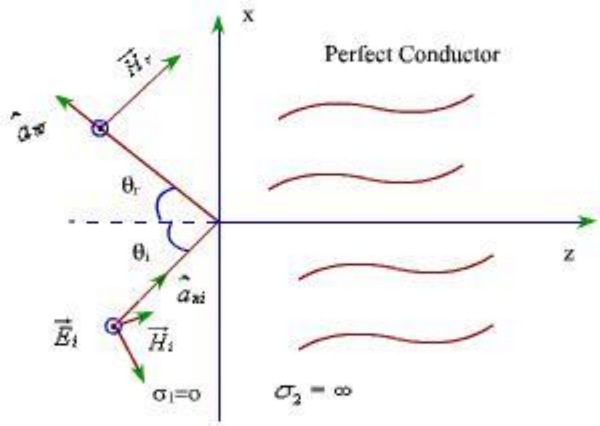


Figure 6.10: Perpendicular Polarization

As the EM field inside the perfect conductor is zero, the interface reflects the incident plane wave. $\hat{a}_{ni}$ and $\hat{a}_{nr}$ respectively represent the unit vector in the direction of propagation of the incident and reflected waves, θ_i is the angle of incidence and θ_r is the angle of reflection.

We find that

$$\hat{a}_{ni} = \hat{a}_z \cos\theta_i + \hat{a}_x \sin\theta_i$$
$$\hat{a}_{nr} = -\hat{a}_z \cos\theta_r + \hat{a}_x \sin\theta_r$$

Since the incident wave is considered to be perpendicular to the plane of incidence, which for the present case happens to be xz plane, the electric field has only y-component. Therefore,

$$\vec{E}_i(x,z) = \hat{a}_y E_{io} e^{-j\beta_1 \bar{a}_{ni}\cdot\bar{r}}$$
$$= \hat{a}_y E_{io} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

The corresponding magnetic field is given by

$$\vec{H}_i(x,z) = \frac{1}{\eta_1}\left[\hat{a}_n \times \vec{E}_i(x,z)\right]$$
$$= \frac{1}{\eta_1}\left[-\cos\theta_i \hat{a}_x + \sin\theta_i \hat{a}_z\right] E_{io} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

Similarly, we can write the reflected waves as

$$\vec{E}_r(x,z) = \hat{a}_y E_{ro} e^{-j\beta_1 \bar{a}_{nr} \cdot \bar{r}}$$
$$= \hat{a}_y E_{ro} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

Since at the interface z=o, the tangential electric field is zero.

$$E_{io} e^{-j\beta_1 x\sin\theta_i} + E_{ro} e^{-j\beta_1 x\sin\theta_r} = 0$$

The above equation is satisfied if we have

$$E_{ro} = -E_{io}$$
$$\text{and } \theta_i = \theta_r$$

The condition $\theta_i = \theta_r$ is Snell's law of reflection.

$$\therefore \vec{E}_r(x,z) = -\hat{a}_y E_{io} e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}$$

$$\text{and } \vec{H}_r(x,z) = \frac{1}{n_1}\left[\hat{a}_{nr} \times \vec{E}_r(x,z)\right]$$
$$= \frac{E_{io}}{n_1}\left[-\hat{a}_x \cos\theta_i - \hat{a}_z \sin\theta_i\right] e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}$$

The total electric field is given by

$$\vec{E}_1(x,z) = \vec{E}_i(x,z) + \vec{E}_r(x,z)$$
$$= -\hat{a}_y 2jE_{io} \sin(\beta_1 z\cos\theta_i) e^{-j\beta_1 x\sin\theta_i}$$

Similarly, total magnetic field is given by

$$\vec{H}_1(x,z) = -2\frac{E_{io}}{n_1}\left[\hat{a}_x \cos\theta_i \cos(\beta_1 z\cos\theta_i) e^{-j\beta_1 x\sin\theta_i} + \hat{a}_z j\sin\theta_i \sin(\beta_1 z\cos\theta_i) e^{-j\beta_1 x\sin\theta_i}\right]$$

From above two equations we observe that

1. Along z direction i.e. normal to the boundary

   y component of $\vec{E}$ and x component of $\vec{H}$ maintain standing wave patterns according to $\sin\beta_{1z} z$ and $\cos\beta_{1z} z$ where $\beta_{1z} = \beta_1 \cos\theta_i$. No average power propagates along z as y component of $\vec{E}$ and x component of $\vec{H}$ are out of phase.

2. Along x i.e. parallel to the interface

   y component of $\vec{E}$ and z component of $\vec{H}$ are in phase (both time and space) and propagate with phase velocity

$$v_{p1x} = \frac{\omega}{\beta_{1x}} = \frac{\omega}{\beta_1 \sin\theta_i}$$
$$\text{and } \lambda_{1x} = \frac{2\pi}{\beta_{1x}} = \frac{\lambda_1}{\sin\theta_i}$$

The wave propagating along the x direction has its amplitude varying with z and hence constitutes a **non uniform**plane wave. Further, only electric field is perpendicular to the direction of propagation (i.e. x), the magnetic field has component along the direction of propagation. Such waves are called transverse electric or TE waves.

ii. **Parallel Polarization**:

In this case also $\hat{a}_{ni}$ and $\hat{a}_{nr}$ are given by the derived equations. Here $\vec{H}_i$ and $\vec{H}_r$ have only y component.

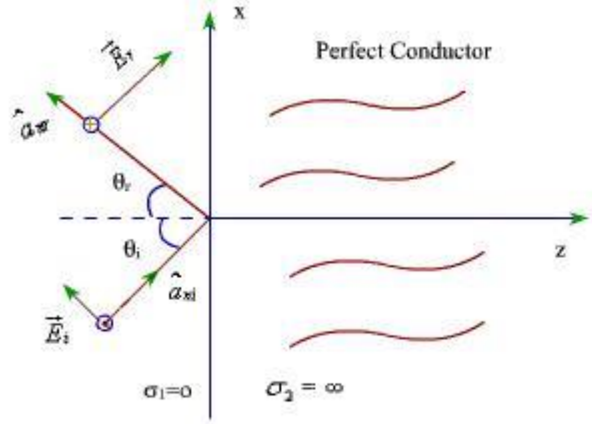


Figure 6.11: Parallel Polarization

With reference to fig (6.11), the field components can be written as:

Incident field components:

$$\vec{E}_i(x,z) = E_{io}\left[\cos\theta_i \hat{a}_x - \sin\theta_i \hat{a}_z\right] e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\vec{H}_i(x,z) = \hat{a}_y \frac{E_{io}}{\eta_1} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \qquad \text{..........................(r)}$$

Reflected field components:

$$\vec{E}_r(x,z) = E_{ro}\left[\hat{a}_x \cos\theta_r + \hat{a}_z \sin\theta_r\right] e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\vec{H}_r(x,z) = -\hat{a}_y \frac{E_{ro}}{\eta_1} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

Since the total tangential electric field component at the interface is zero.

$$E_i(x,0) + E(x,0) = 0$$

Which leads to $E_{io} = -E_{ro}$ and $\theta_i = \theta_r$ as before.

Substituting these quantities in (r) and adding the incident and reflected electric and magnetic field components the total electric and magnetic fields can be written as

$$\vec{E}_i(x,z) = -2E_{io}\left[\hat{a}_x j\cos\theta_i \sin(\beta_1 z\cos\theta_i) + \hat{a}_z \sin\theta_i \cos(\beta_1 z\cos\theta_i)\right] e^{-j\beta_1 x\sin\theta_i}$$

and $$\vec{H}_i(x,z) = \hat{a}_y \frac{2E_{io}}{\eta_1}\cos(\beta_1 z\cos\theta_i) e^{-j\beta_1 x\sin\theta_i}$$

Once again, we find a standing wave pattern along z for the x and y components of $\vec{E}$ and $\vec{H}$, while a non uniform plane wave propagates along x with a phase velocity given by $v_{p1x} = \frac{vp_1}{\sin\theta_i}$

$v_{p1} = \frac{\omega}{\beta_1}$

where . Since, for this propagating wave, magnetic field is in transverse direction, such waves are called transverse magnetic or TM waves.

Oblique incidence at a plane dielectric interface

We continue our discussion on the behavior of plane waves at an interface; this time we consider a plane dielectric interface. As earlier, we consider the two specific cases, namely parallel and perpendicular polarization.

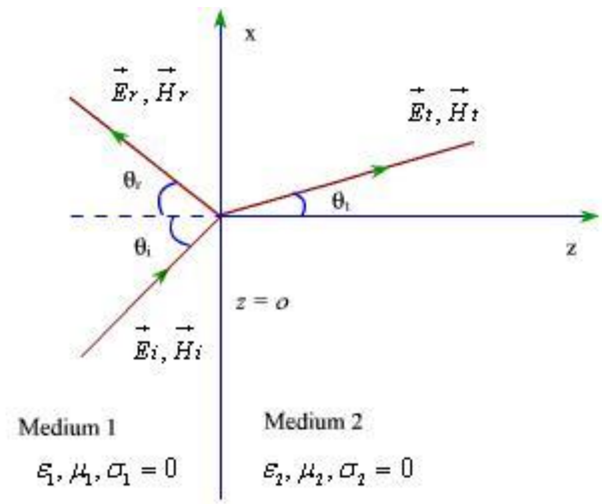


Fig 6.12: Oblique incidence at a plane dielectric interface

For the case of a plane dielectric interface, an incident wave will be reflected partially and transmitted partially.

In Fig(6.12), θ_i, θ_0 and θ_t corresponds respectively to the angle of incidence, reflection and transmission.

1. **Parallel Polarization**

As discussed previously, the incident and reflected field components can be written as

$$\vec{E}_i(x,z) = E_{io}\left[\cos\theta_i \hat{a}_x - \sin\theta_i \hat{a}_z\right] e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\vec{H}_i(x,z) = \hat{a}_y \frac{E_{io}}{\eta_1} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\vec{E}_r(x,z) = E_{ro}\left[\hat{a}_x \cos\theta_r + \hat{a}_z \sin\theta_r\right] e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\vec{H}_r(x,z) = -\hat{a}_y \frac{E_{ro}}{\eta_1} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

In terms of the reflection coefficient Γ

$$\vec{E}_r(x,z) = \Gamma E_{io}\left[\hat{a}_x \cos\theta_r + \hat{a}_z \sin\theta_r\right] e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\vec{H}_r(x,z) = -\hat{a}_y \frac{\Gamma E_{io}}{\eta_1} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

The transmitted filed can be written in terms of the transmission coefficient T

$$\vec{E}_t(x,z) = TE_{io}\left[\hat{a}_x \cos\theta_t - \hat{a}_z \sin\theta_t\right] e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\vec{H}_t(x,z) = \hat{a}_y \frac{TE_{io}}{\eta_2} e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

We can now enforce the continuity of tangential field components at the boundary i.e. $z=0$

$$\cos\theta_i e^{-j\beta_1 x\sin\theta_i} + \Gamma\cos\theta_r e^{-j\beta_1 x\sin\theta_r} = T\cos\theta_t e^{-j\beta_2 x\sin\theta_t}$$

$$\text{and } \frac{1}{\eta_1}e^{-j\beta_1 x\sin\theta_i} - \frac{\Gamma}{\eta_1}e^{-j\beta_1 x\sin\theta_r} = \frac{T}{\eta_2}e^{-j\beta_2 x\sin\theta_t} \quad \text{.........................(s)}$$

If both E_x and H_y are to be continuous at $z=0$ for all x, then form the phase matching we have

$$\beta_1 \sin\theta_i = \beta_1 \sin\theta_r = \beta_2 \sin\theta_t$$

$\therefore$ We find that

$$\theta_i = \theta_r$$

$$\text{and } \beta_1 \sin\theta_i = \beta_2 \sin\theta_t \quad \text{.........................(t)}$$

Further, from equations (s) and (t) we have

$$\cos\theta_i + \Gamma\cos\theta_i = T\cos\theta_t$$

$$\text{and } \frac{1}{\eta_1} - \frac{\Gamma}{\eta_1} = \frac{T}{\eta_2}$$

$$\therefore \quad \cos\theta_i(1+\Gamma) = T\cos\theta_t$$

$$\text{and } \frac{1}{\eta_1}(1-\Gamma) = \frac{T}{\eta_2}$$

$$\therefore T = \frac{\eta_2}{\eta_1}(1-\Gamma)$$

$$\cos\theta_i(1+\Gamma) = \frac{\eta_2}{\eta_1}(1-\Gamma)\cos\theta_t$$

$$\therefore (\eta_1\cos\theta_i + \eta_2\cos\theta_t)\Gamma = \eta_2\cos\theta_t - \eta_1\cos\theta_i$$

or

$$\Gamma = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$\text{and } T = \frac{\eta_2}{\eta_1}(1-\Gamma)$$

$$= \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \quad \text{.........................(u)}$$

From equation (u) we find that there exists specific angle $\theta_i = \theta_b$ for which $\Gamma = 0$ such that

$$\eta_2\cos\theta_t = \eta_1\cos\theta_b$$

or

$$\sqrt{1-\sin^2\theta_t} = \frac{\eta_1}{\eta_2}\sqrt{1-\sin^2\theta_b}$$

Further,

$$\sin\theta_t = \frac{\beta_1}{\beta_2}\sin\theta_b$$

For non magnetic material $\mu_1 = \mu_2 = \mu_0$
Using this condition

$$1-\sin^2\theta_t = \frac{\varepsilon_1}{\varepsilon_2}\left(1-\sin^2\theta_b\right)$$

$$\text{and } \sin^2\theta_t = \frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta_b \qquad \text{.......................(v)}$$

rom equation (v), solving for $\sin\theta_b$ we get

$$\sin\theta_b = \frac{1}{\sqrt{1+\frac{\varepsilon_1}{\varepsilon_2}}}$$

This angle of incidence for which Γ = *0* is called Brewster angle. Since we are dealing with parallel polarization we represent this angle by $\theta_{b\|}$ so that

$$\sin\theta_{b\|} = \frac{1}{\sqrt{1+\frac{\varepsilon_1}{\varepsilon_2}}}$$

2. **Perpendicular**

Polarization For this case

$$\vec{E}_i(x,z) = \hat{a}_y E_{io} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\vec{H}_i(x,z) = \frac{E_{io}}{\eta_1}\left[-\hat{a}_x\cos\theta_i + \hat{a}_z\sin\theta_i\right]e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\vec{E}_r(x,z) = \hat{a}_y \Gamma E_{io} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\vec{H}_r(x,z) = \frac{\Gamma E_{io}}{\eta_1}\left[\hat{a}_x\cos\theta_r + \hat{a}_z\sin\theta_r\right]e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\vec{E}_t(x,z) = \hat{a}_y T E_{io} e^{-j\beta_1(x\sin\theta_t + z\cos\theta_t)}$$

$$\vec{H}_t(x,z) = \frac{TE_{io}}{\eta_2}\left[-\hat{a}_x\cos\theta_t + \hat{a}_z\sin\theta_t\right]e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

Using continuity of field components at *z=0*

$$e^{-j\beta_1 x\sin\theta_i} + \Gamma e^{-j\beta_1 x\sin\theta_r} = TE_{io}e^{-j\beta_2 x\sin\theta_t}$$

$$\text{and } -\frac{1}{\eta_1}\cos\theta_i e^{-j\beta_1 x\sin\theta_i} + \frac{\Gamma}{\eta_1}\cos\theta_r e^{-j\beta_1 x\sin\theta_r} = -\frac{T}{\eta_2}\cos\theta_t e^{-j\beta_2 x\sin\theta_t}$$

As in the previous case

$$\beta_1\sin\theta_i = \beta_1\sin\theta_r = \beta_2\sin\theta_t$$

$$\therefore \quad \theta_i = \theta_r$$

$$\text{and } \sin\theta_t = \frac{\beta_1}{\beta_2}\sin\theta_i$$

Using these conditions we can write

$$1 + \Gamma = T$$

$$-\frac{\cos\theta_i}{n_1} + \frac{\Gamma\cos\theta_i}{n_1} = -\frac{T\cos\theta_t}{n_2} \qquad \text{.........................(w)}$$

From equation (w) the reflection and transmission coefficients for the perpendicular polarization can be computed as

$$\Gamma = \frac{n_2\cos\theta_i - n_1\cos\theta_t}{n_2\cos\theta_i + n_1\cos\theta_t}$$

$$\text{and } \quad T = \frac{2n_2\cos\theta_i}{n_2\cos\theta_i + n_1\cos\theta_t}$$

We observe that if Γ = *0* for an angle of incidence $\theta_i = \theta_b$

$$n_2\cos\theta_b = n_1\cos\theta_t$$

$$\therefore \cos^2\theta_t = \frac{n_2}{n_1}\cos^2\theta_b$$

$$= \frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}\cos^2\theta_b$$

$$\therefore 1 - \sin^2\theta_t = \frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}\left(1 - \sin^2\theta_b\right)$$

Again $\sin\theta_t = \dfrac{\beta_1}{\beta_2}\sin\theta_b$

$$\therefore \sin^2\theta_t = \frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}\sin^2\theta_b$$

$$\therefore \left(1 - \frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}\sin^2\theta_b\right) = \frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2} - \frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}\sin^2\theta_b$$

or $\sin^2\theta_b\left(\dfrac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2} - \dfrac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}\right) = \left(1 - \dfrac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}\right)$

or $\sin^2\theta_b\left(\dfrac{\mu_1^2 - \mu_2^2}{\mu_1\mu_2\varepsilon_2}\right)\varepsilon_1 = \left(\dfrac{\mu_1\varepsilon_2 - \mu_2\varepsilon_1}{\mu_1\varepsilon_2}\right)$

or $\sin^2\theta_b = \dfrac{\mu_2\left(\mu_1\varepsilon_2 - \mu_2\varepsilon_1\right)}{\varepsilon_1\left(\mu_1^2 - \mu_2^2\right)}$ ....(x)

We observe if $\mu_1 = \mu_2 = \mu_0$ i.e. in this case of non magnetic material Brewster angle does not exist as the denominator or equation (x) becomes zero. Thus for perpendicular polarization in dielectric media, there is Brewster angle so that Γ can be made equal to zero.

From our previous discussion we observe that for both polarizations

$$\sin\theta_t = \frac{\beta_1}{\beta_2}\sin\theta_i$$

If $\mu_1 = \mu_2 = \mu_0$

$$\sin\theta_t = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}\sin\theta_i$$

For $\varepsilon_1 > \varepsilon_2$; $\theta_t > \theta_i$

The incidence angle $\theta_i = \theta_c$ for which $\theta_t = \frac{\pi}{2}$ i.e. $\theta_c = \sin^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$ is called the critical angle of incidence. If the angle of incidence is larger than θ_c total internal reflection occurs. For such case an evanescent wave exists along the interface in the x direction (w.r.t. fig (6.12)) that attenuates exponentially in the normal i.e. z direction. Such waves are tightly bound to the interface and are called surface waves.

QUESTIONS:

1.Write down Maxwell's field equations in the differential and integral form for time harmonic fields
2.Derive the expressions for energy stored in electric and magnetic field. Which field is efficient.

3.In a uniform plane wave, E and H are at right angles to each other. Prove.
4.A lossy dielectric is characterized by $_R$=1.5, $_R$=1 and / =2.5x10^{-4}. At a frequency of 200MHz, how far can a uniform plane wave propagate in the material before

(i)it undergoes an attenuation 1Np

(ii)its amplitude is halved

5. Deduce the integral form of the theoram of Poynting and state the significance of the three terms appearing in the equation.

6.What are the properties of uniform plane wave?

7.Write Maxwell's equation in integral form and interpret

8.Show that characteristic impedance of free space is 377ohm

9.State and explain Poynting Vector(P) and Poynting theorem.

10.A brass(conductivity=10^7mho/m) pipe with inner and outer diameter of 3.4 and 4 cm carries a total current of 100A dc. Find Electric field (E), Magnetic field(H) and Poynting Vector(P) within the brass