

MODULE 4

BRIDGES

Introduction:

- Bridges is a circuit which is used for measuring various components like R, C and L
- Bridge as a simple circuit consists of having 4 resistance arms in a closed loop, with dc current source applied to 2 opposite junction and current detector connected to other 2 junction as shown in Fig. 4.1.
- In this the unknown component is measured in comparison with known component called as standard.
- This method of measurement is very accurate and the accuracy of measurement is directly proportional to the bridge component.

There are 2 types of bridges

- ac bridge – impedances consisting of C and L
- dc bridges – measure resistance

The dc bridge used for measuring resistance is called **Wheatstone's bridge**.

Wheatstone's Bridge:

- It is the most accurate method for measuring resistance and a common method used in laboratory.
- The circuit is shown in Fig 4.1.

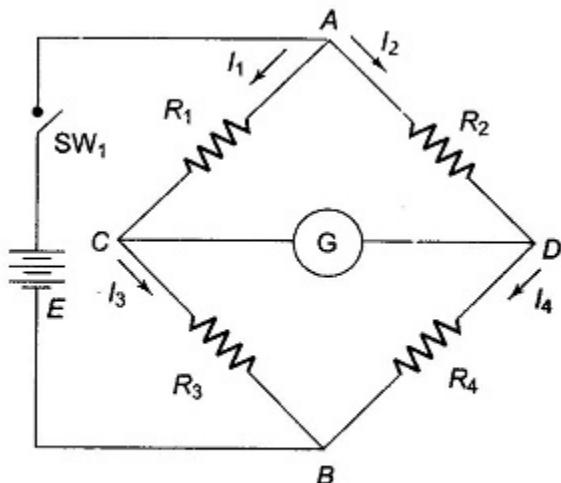


Fig. 4.1 Wheatstone's Bridge

- It has an emf source E and switch Sw connected between points A and B.
- A sensitive current indicating meter is connected to point C and D. Meter used is a zero center scale, when at rest it is mid scale at 0 current. Current in one direction causes pointer to deflect in one direction and for current in the opposite direction causes the pointer to deflect in opposite direction. When no current flowing in the circuit, the pointer rests at '0'.
- When Sw is closed current flows and divides into 2 arms at point A as I1 and I2.

- Bridge is balanced when current through G is ‘0’ i.e potential difference at C and D should be equal.

i.e

$$I_1 R_1 = I_2 R_2 \quad \dots \quad (1)$$

For galvanometer current to be zero, $I_1 = I_3$ and $I_2 = I_4$

Thus under balanced condition,

$$I_1 = I_3 = \frac{E}{R_1 + R_3} \quad (a)$$

And

$$I_2 = I_4 = \frac{E}{R_2 + R_4} \quad (b)$$

Using (a) and (b) in equation (1), we get

$$\frac{E * R_1}{R_1 + R_3} = \frac{E * R_2}{R_2 + R_4}$$

Simplifying the above equation we get,

$$R_1(R_2 + R_4) = R_2(R_1 + R_3)$$

$$R_1 R_2 + R_1 R_4 = R_1 R_2 + R_2 R_3$$

$$R_1 R_4 = R_2 R_3$$

Now $R_4 = \frac{R_2 R_3}{R_1}$

This is the equation for bridge to be balance.

For balancing one of the resistance will be made adjustable and if R_4 is the unknown resistance then

$$R_x = \frac{R_2 R_3}{R_1}$$

Sensitivity of Wheatstone's bridge:

- When there is unbalance in the bridge, there is deflection in the pointer of galvanometer (G) which depends on the sensitivity of the galvanometer.
- If the G is more sensitive then, the deflection is more for the same amount of current. Thus sensitivity is considered as **deflection/unit current**. i.e $S = D/I$, D = deflection and I = current in μA
- Sensitivity can be expressed in linear or angular with the units as $S = \text{mm}/\mu\text{A}$ (Linear) and $S = \text{degree}/\mu\text{A}$ or $S = \text{radian}/\mu\text{A}$ (Angular)
- Thus total deflection $D = S * I$

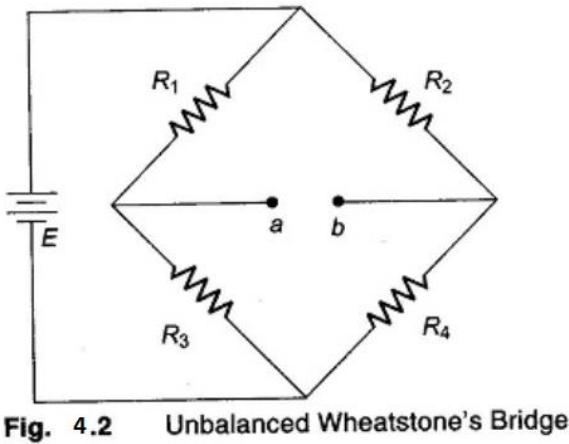
Unbalanced Wheatstone's bridge:

This is the analysis of Wheatstone's bridge under unbalanced condition and this determines the amount of current flowing in the G.

The circuit analysis can be done using any general circuit analysis, considering “Thevenin's Theorem” will determine the current through G.

Since the interest is to find the current through G under unbalanced condition we need to find the Thevenin's equivalent circuit as seen by G

The first step is to remove G and find open circuit voltage between terminals a and b as shown in fig 4.2



Applying voltage divider at point 'a' and 'b', we get

$$E_a = \frac{R3 E}{R1 + R3}$$

$$E_b = \frac{R4 E}{R2 + R4}$$

Voltage between a and b is the difference between E_a and E_b and this represents the Thevenin's equivalent voltage,

$$Eth = E_a - E_b = \frac{R3 E}{R1 + R3} - \frac{R4 E}{R2 + R4}$$

Thus

$$Eth = E \left(\frac{R3 E}{R1+R3} - \frac{R4 E}{R2+R4} \right)$$

Thevenin's equivalent resistance can be determined by replacing voltage source by its internal impedance or with a short and looking into a and b as shown in fig 4.3.,

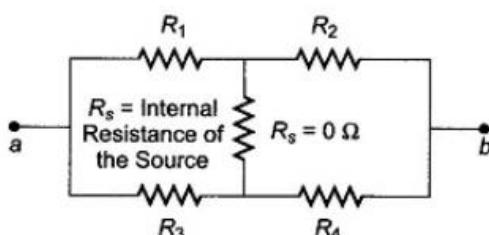


Fig. 4.3 Thévenin's Resistance

$$Rth = (R1 \parallel R3) + (R2 \parallel R4)$$

Thus

$$Rth = \frac{R1 R3}{R1+R3} + \frac{R2 R4}{R2+R4}$$

Thevenin's equivalent circuit is shown in fig 4.4

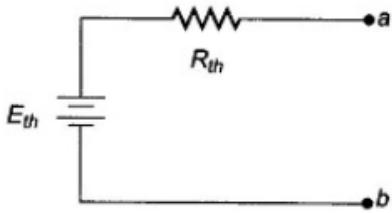


Fig. 4 .4 Thévenin's Equivalent

If G is connected between a and b in the above circuit and its original circuit then both experiences same deflection.

The magnitude of current is limited by R_{th} and the resistance seen with G i.e R_g (internal resistance of G)

Thus the deflection of current in galvanometer is given by

$$I_g = \frac{E_{th}}{R_{th} + R_g}$$

Slightly unbalanced Wheatstone's bridge:

If three of the four resistor in a bridge are equal to R and the fourth differs by 5% or less, we can develop an approximate but accurate expression for Thevenin's equivalent voltage and resistance as follows. The circuit is shown in Fig 4.5

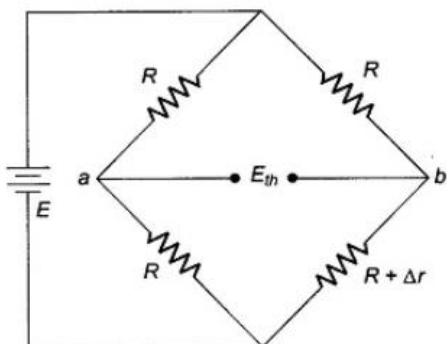


Fig. 4.5 Slightly Unbalanced Wheatstone's Bridge

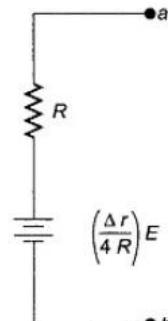


Fig. 4.6 Thévenin's Equivalent of a Slightly Unbalanced Wheatstone's Bridge

Voltage at point 'a' is given by

$$E_a = \frac{ER}{R+R} = \frac{ER}{2R} = \frac{E}{2}$$

Voltage at point 'b' is given by

$$E_b = \frac{E(R+\Delta r)}{R+(R+\Delta r)} = \frac{E(R+\Delta r)}{2R+\Delta r}$$

Thevenin's equivalent voltage is given by,

$$Eth = Ea - Eb = Eb - Ea$$

$$Eth = \frac{E(R + \Delta r)}{2R + \Delta r} - \frac{E}{2}$$

Simplifying this,

$$Eth = E \left(\frac{2R + 2\Delta r - 2R - \Delta r}{2(2R + \Delta r)} \right)$$

$$Eth = E \left(\frac{\Delta r}{4R + 2\Delta r} \right)$$

Now if Δr is 5% of R or less, then Δr can be neglected at the denominator without appreciable error. Thus Eth now is

$$Eth = \frac{\Delta r}{4R}$$

The equivalent resistance can be calculated by replacing the voltage source with its internal impedance,

$$R_{th} = \frac{R \cdot R}{R + R} + \frac{R(R + \Delta r)}{R + (R + \Delta r)}$$

Simplifying the above equation,

$$R_{th} = \frac{R}{2} + \frac{R(R + \Delta r)}{2R + \Delta r}$$

If Δr is small compared to R, then it can be neglected

$$R_{th} = \frac{R}{2} + \frac{R \cdot R}{2R}$$

$$R_{th} = \frac{R}{2} + \frac{R}{2} = \frac{2R}{2} = R$$

Thus the Thevenin's equivalent circuit is shown in Fig 4.6

The current through the G is given by,

$$I_g = \frac{Eth}{R_{th} + R_g}$$

Applications of Wheatstone's bridge:

- Wheatstone bridge is used to measure resistance in the range of 1Ω to low $M\Omega$.
- Used to measure the dc resistance of various types of wire, either for the purpose of quality control of the wire itself, or of some assembly in which it is used.
- To find the resistance of motor windings, transformers, solenoids, and relay coils.
- Wheatstone Bridge Circuit is also used extensively by telephone companies and others to locate cable faults.

Advantages of Wheatstone's bridge:

- It operates on null deflection i.e., indication is independent on indicating instrument's characteristics and this is reason it has high degrees of accuracy.
- The variation in the source does not alter the balance of bridge, hence the corresponding errors are completely avoided.
- In Wheatstone bridge potential errors are canceled out including the bridge excitation, and temperature errors.

Limitations of Wheatstone's bridge:

- For low resistance measurement, the resistance of the leads and contacts becomes significant and this may introduce error.
- While measuring high resistance, the resistance presented by the bridge becomes so large that the galvanometer will be insensitive to imbalance. Thus for high resistance measurements in mega ohms, the Wheatstones bridge cannot be used.
- Another problem in Wheatstone Bridge Circuit is the change in resistance of the bridge arms due to the heating effect of current through the resistance. The rise in temperature causes a change in the value of the resistance, and sometimes high current may cause a permanent change in value.

Kelvin's bridge

- When the resistance to be measured is of the order of magnitude of bridge contact and lead resistance, a modified form of Wheatstone's bridge, the Kelvin's bridge is used.
- Kelvin's bridge is used to measure values of resistance below $1\ \Omega$. In low resistance measurement, the resistance of the leads connecting the unknown resistance to the terminal of the bridge circuit may affect the measurement.
- Thus in Kelvin's bridge, the effect of contact and lead resistance is important.
- Consider the circuit in Fig.4.7, where R_y represents the resistance of the connecting leads from R_3 to R_x (unknown resistance). The galvanometer can be connected either to point c or to point a.
- When it is connected to point a, the resistance R_y , of the connecting lead is added to the unknown resistance R_x , resulting in too high indication for R_x .
- When the connection is made to point c, R_3 , is added to the bridge arm R_3 and resulting measurement of R_x is lower than the actual value.
- If the galvanometer is connected to point b, in between points c and a, in such a way that the ratio of the resistance from c to b and that from a to b equals the ratio of resistances R_1 and R_2 , then

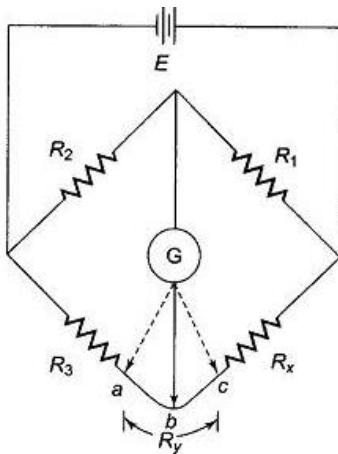
$$\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}$$

The bridge balance equation is given by,

$$R_1 * R_3 = R_2 * R_x$$

But R_3 is now $R_3 + R_{ab}$ and R_x is now $R_x + R_{cb}$

$$\text{Therefore } R_1 * (R_3 + R_{ab}) = R_2 * (R_x + R_{cb}) \text{ ----- (1)}$$

**Fig. 4.7** Kelvin's Bridge

From the Fig 4.7,

$$R_{ab} + R_{cb} = R_y \quad \text{--- (A)}$$

and

$$\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2} \quad \text{--- (B)}$$

Adding 1 to both sides of equation (B), we get

$$\frac{R_{cb}}{R_{ab}} + 1 = \frac{R_1 + R_2}{R_2} + 1$$

$$\frac{R_{cb} + R_{ab}}{R_{ab}} = \frac{R_1 + R_2}{R_2}$$

using equation (A) in the above equation, we get

$$\frac{R_{cb} + R_{ab}}{R_{ab}} = \frac{R_y}{R_{ab}} = \frac{R_1 + R_2}{R_2}$$

and R_{ab} is now

$$R_{ab} = \frac{R_2 * R_y}{R_1 + R_2}$$

Rearranging equation (A) and using equation for R_{ab} ,

$$R_{cb} = R_y - R_{ab}$$

$$R_{cb} = R_y - \frac{R_2 * R_y}{R_1 + R_2}$$

$$R_{cb} = \frac{R_1 R_y + R_2 R_y - R_2 R_y}{R_1 + R_2}$$

$$R_{cb} = \frac{R_1 R_y}{R_1 + R_2}$$

Substituting R_{ab} and R_{cb} in equation (1)

$$R_1 * (R_3 + \frac{R_2 R_y}{R_1 + R_2}) = R_2 * (R_x + \frac{R_1 R_y}{R_1 + R_2})$$

$$Rx + \frac{R1 Ry}{R1 + R2} = \frac{R1}{R2} \left(R3 + \frac{R2 Ry}{R1 + R2} \right)$$

$$Rx + \frac{R1 Ry}{R1 + R2} = \frac{R1 R3}{R2} + \frac{R2 Ry}{R1 + R2} * \frac{R1}{R2}$$

$$Rx + \frac{R1 Ry}{R1 + R2} = \frac{R1 R3}{R2} + \frac{R1 Ry}{R1 + R2}$$

Upon simplification, we get

$$Rx = \frac{R1 R3}{R2}$$

- The above equation is the normal Wheatstone's bridge under balanced condition.
- Also the effect of lead resistance connecting from a to c is eliminated by connecting galvanometer to intermediate position 'b'.
- This is the principle of constructing Kelvin's double bridge also known as Kelvin's bridge. It is called double bridge as it incorporates 2nd set of resistance ratio arms.
- The schematic of Kelvin's double bridge is shown in Fig 4.8
- In this 2nd set of arms a and b connect galvanometer to point c
- The galvanometer gives null indication when potential at k and c are equal i.e $Elk = Elm_c$

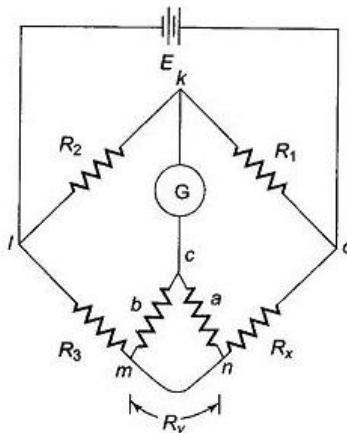


Fig. 4.8 Kelvin's Double Bridge

Now Elk is given by

$$Elk = \frac{R2 E}{R1 + R2}$$

and E is given by

$$E = I * R$$

$$Elk = \frac{R2}{R1 + R2} * I * R$$

R is to be determined considering path l-m-n-o-l, in this path at point m - n, the resistance is $Ry \parallel (a+b)$. Thus total resistance now becomes as

$$R = R3 + Rx + \frac{Ry (a + b)}{a + b + Ry}$$

Now Elk is given by

$$Elk = \frac{R2}{R1 + R2} * I * \left[R3 + Rx + \frac{Ry (a + b)}{a + b + Ry} \right]$$

Similarly Elmc is given by, $Elmc = Elm + Emc$

$$Elm = I R_3$$

$$Emc = \frac{b Emn}{a+b} \quad \text{using voltage divider rule}$$

$$Emn = (a+b) \parallel Ry * I = \frac{Ry(a+b)}{a+b+Ry} * I$$

Thus now

$$Emc = \frac{b}{a+b} * I * \frac{Ry(a+b)}{a+b+Ry}$$

Therefor

$$Elmc = I R_3 + \frac{b}{a+b} * I * \frac{Ry(a+b)}{a+b+Ry}$$

$$Elmc = I \left[R_3 + \frac{b}{a+b} * \frac{Ry(a+b)}{a+b+Ry} \right]$$

But under balanced condition,

$$Elk = Elmc$$

$$\text{i.e. } \frac{IR_2}{R_1 + R_2} \left(R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} \right) = I \left[R_3 + \frac{b}{a+b} \left\{ \frac{(a+b)R_y}{a+b+R_y} \right\} \right]$$

$$\therefore R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \frac{R_1 + R_2}{R_2} \left(R_3 + \frac{bR_y}{a+b+R_y} \right)$$

$$\therefore R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \left(\frac{R_1}{R_2} + 1 \right) \left(R_3 + \frac{bR_y}{a+b+R_y} \right)$$

$$R_x + \frac{(a+b)R_y}{a+b+R_y} + R_3 = \frac{R_1 R_3}{R_2} + R_3 + \frac{b R_1 R_y}{R_2 (a+b+R_y)} + \frac{b R_y}{a+b+R_y}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_1 R_y}{R_2 (a+b+R_y)} + \frac{b R_y}{a+b+R_y} - \frac{(a+b)R_y}{a+b+R_y}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_1 R_y}{R_2 (a+b+R_y)} + \frac{b R_y - a R_y - b R_y}{a+b+R_y}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_1 R_y}{R_2 (a+b+R_y)} - \frac{a R_y}{a+b+R_y}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_y}{(a+b+R_y)} \left(\frac{R_1}{R_2} - \frac{a}{b} \right)$$

But $\frac{R_1}{R_2} = \frac{a}{b}$

Therefore, $R_x = \frac{R_1 R_3}{R_2}$

- The above is the equation for Kelvin's bridge.
- From the above equation, R_y i.e., resistance of the connecting lead has no effect on the measurement provided the resistance ratio of arms are equal.
- This bridge can measure resistance in the range of $1\Omega - 10\mu\Omega$ with accuracy of $\pm 0.05\%$ to $\pm 0.02\%$.

AC Bridges:

- The ac bridges are similar to dc bridge except that the bridge arms have impedances and the bridge is excited by ac source rather than dc source.
- Impedances at audio frequency and radio frequency can be determined by means of ac bridges.

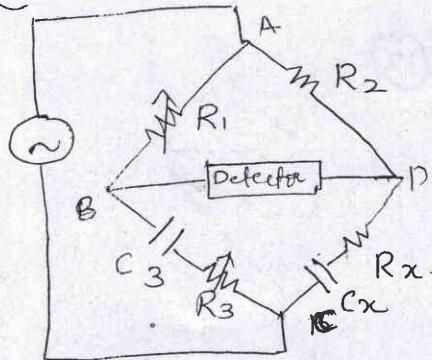
Capacitance comparison bridge:-

E.I

(10)

(11)

- * The resistance ratio arms R_1 & R_2 are resistive.
- * The std. known capacitance C_3 is in series with R_3 which is variable in order to balance the bridge.



- * C_x is the unknown capacitor and R_x is leakage resistance of the capacitor.
- * The unknown capacitor is determined by comparing it with std capacitance along with R_x and also R_x value is found.

$$Z_1 = R_1$$

$$Z_2 = R_2$$

$$Z_3 = R_3 - jX_3 = R_3 - j(1/\omega C_3)$$

$$Z_4 = R_x - jX_4 = R_x - j(1/\omega C_x)$$

- * The condition for bridge balance is

$$Z_1 Z_4 = Z_2 Z_3$$

$$R_1 (R_x - j/\omega C_x) = R_2 (R_3 - j/\omega C_3)$$

$$R_1 R_x - \frac{R_1 j}{\omega C_x} = R_2 R_3 - \frac{R_2 j}{\omega C_3}$$

- * Two complex quantities are said to be equal when their real & imaginary items are equal.

$$R_1 R_x = R_2 R_3$$

$$\Rightarrow R_x = \frac{R_2 R_3}{R_1}$$

$$\frac{R_1 j}{\omega C_x} = \frac{R_2 j}{\omega C_3}$$

$$\Rightarrow C_x = \frac{R_1 C_3}{R_2}$$

$$C_x = \frac{R_1 C_3}{R_2} = C_x - \frac{R_1 C_3}{R_2}$$

(12)

Using both equations the unknown capacitance & its leakage resistance can be measured.

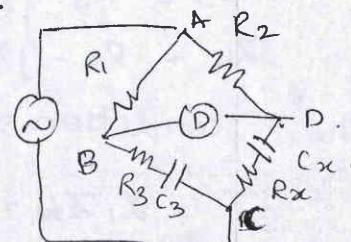
Ex:- A capacitance comparison bridge is used to measure capacitive impedance at freq. of 2kHz . The bridge constants at balance are $C_3 = 100\mu\text{F}$, $R_1 = 10\text{k}\Omega$, $R_2 = 50\text{k}\Omega$, $R_3 = 100\text{k}\Omega$. Find the equivalent series dt of unknown impedance.

$$\rightarrow R_1 = 10\text{k}\Omega, R_2 = 50\text{k}\Omega, R_3 = 100\text{k}\Omega, C_3 = 100\mu\text{F}.$$

$$R_x = \frac{R_2 R_3}{R_1} = \frac{50\text{k} \times 100\text{k}}{10\text{k}} = 500\text{k}\Omega$$

$$C_x = \frac{C_3 R_1}{R_2} = \frac{100\mu\text{F} \times 10\text{k}}{50\text{k}} = 20\mu\text{F}$$

$$C_x = 20\mu\text{F}$$



Ex:- An ac bridge is balanced at 2kHz with the following components in each arm; AB: $10\text{k}\Omega$, Arm BC = $100\mu\text{F}$ with series $100\text{k}\Omega$, Arm AD = $50\text{k}\Omega$. Find the unknown impedance in arm CD, if the detector is b/w BD.

$$\rightarrow Z_1 = 10\text{k}\Omega$$

$$Z_2 = 50\text{k}\Omega$$

$$Z_3 = 100\text{k} - j \left(\frac{1}{2\pi 2\text{k} \times 100\mu\text{F}} \right)$$

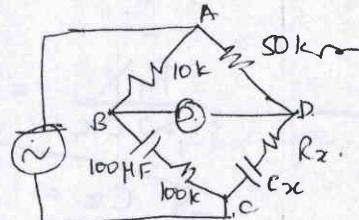
$$= 100\text{k} - j \left(\frac{1}{2\pi 2\text{k} \times 100\mu\text{F}} \right)$$

$$Z_3 = 100\text{k} - j 0.795$$

$$Z_4 = Z_x$$

$$Z_1 Z_4 = Z_2 Z_3$$

$$Z_4 = Z_x = \frac{Z_2 Z_3}{Z_1}$$



$$Z_x = \frac{500 \times [100k - j0.795]}{10k}$$

(11)

$$Z_x = 500k - j0.795 \times 5$$

$$Z_x = R_x + jX_c$$

$$X_c = 3.975$$

$$\therefore R_x = 500k$$

$$X_c = \frac{1}{2\pi f C_x}$$

$$\Rightarrow C_x = \frac{1}{2\pi \times 5000 \times 3.975}$$

$$C_x = 200 \text{ nF}$$

Inductance Comparison bridge:-

* In this the unknown inductance & its internal resistance R_x is obtained by comparing with std inductor & resistance ie L_3 & R_3 .

* The circuit is shown.

* The bridge balance is obtained as

$$Z_1 Z_4 = Z_2 Z_3$$

$$Z_1 = R_1$$

$$\Rightarrow Z_1 Z_x = Z_2 Z_3$$

$$Z_2 = R_2$$

$$Z_3 = R_3 + jX_3$$

$$Z_x = \frac{Z_2 Z_3}{Z_1}$$

$$Z_3 = R_3 + j\omega L_3$$

$$Z_x = \frac{R_2 (R_3 + j\omega L_3)}{R_1}$$

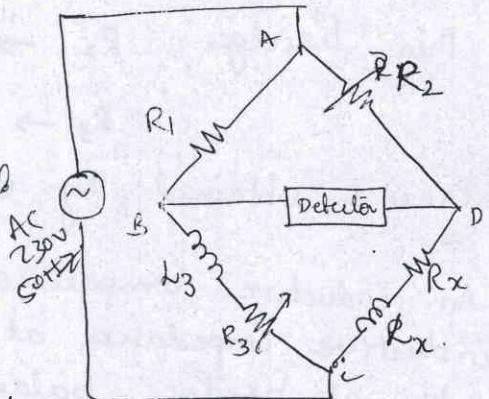
$$Z_4 = R_x + jX_x$$

$$Z_x = \frac{R_2 R_3 + R_2 j\omega L_3}{R_1}$$

$$Z_4 = R_x + j\omega L_x$$

$$Z_x = R_x + j\omega L_x$$

$$R_x = \frac{R_2 R_3}{R_1}$$



$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_x + j\omega L_x) (R_3 + j\omega L_3)$$

$$\boxed{Z_1 Z_x = Z_2 Z_3} \rightarrow \text{Balanced Bridge, } \\ R_1 (R_x + j\omega L_x) = R_2 (R_3 + j\omega L_3)$$

$$R_1 R_x + j\omega L_x R_1 = R_2 R_3 + j\omega L_3 R_2$$

Equating real & imaginary parts

(14)

$$R_1 R_x = R_2 R_3$$

$$\therefore R_x = \frac{R_2 R_3}{R_1}$$

$$j\omega L_x R_1 = j\omega L_3 R_2$$

$$L_x = R_2 R_3$$

$$L_x = \frac{R_2 R_3}{R_1}$$

~~$$R_1 R_x + j\omega L_x R_1 = R_2 R_3 + j\omega L_3 R_2$$~~
~~$$R_1 R_x + j\omega \frac{R_2 R_3}{R_1} R_1 = R_2 R_3 + j\omega L_3 R_2$$~~
~~$$R_1 R_x + j\omega R_2 R_3 = R_2 R_3 + j\omega L_3 R_2$$~~

~~$$R_1 R_x = R_2 R_3$$~~
~~$$R_x = \frac{R_2 R_3}{R_1}$$~~

IIBalanc

* In this bridge $R_2 \rightarrow$ is used for inductive balance control.

$R_3 \rightarrow$ is used for resistance balance

Balance is obtained by varying L_3 or R_3 alternately.

Ex:- An inductive comparison bridge is used to measure the inductive impedance at a freq of 1.5 KHz. The bridge constants at bridge balance are $L_3 = 8mH$, $R_1 = 1k\Omega$, $R_2 = 25k\Omega$, $R_3 = 50k\Omega$. Find the equivalent series circuit of unknown impedance:

$$\rightarrow R_1 = 1k\Omega, R_2 = 25k\Omega, R_3 = 50k\Omega, L_3 = 8mH$$

$$R_x = \frac{R_2 R_3}{R_1} = \frac{25k \times 50k}{1k} = 1250k = \underline{\underline{1.25M\Omega}}$$

$$L_x = \frac{R_2 L_3}{R_1} = \frac{25k \times 8mH}{1k} = 200mH = \underline{\underline{=}}$$

Maxwell's bridge:-

* Maxwell's bridge is used to measure unknown inductance in comparison with known capacitor (std) or using variable std self inductance.

→ Maxwell's inductance bridge.

→ Maxwell's inductance Capacitance bridge.

→ The circuit shown Maxwell's inductance capacitance bridge, where L is found in comparison with C (known std C).

→ One of the ratio arm consists of R in \parallel with C . & thus it is easier to write the bridge balance equation using in admittance form.

→ The bridge balance eqn is

$$Z_1 Z_4 = Z_2 Z_3$$

$$Z_1 Z_x = Z_2 Z_3$$

$$Z_x = \frac{Z_2 Z_3}{Z_1} = Z_2 Z_3 \cdot \frac{1}{Z_1}$$

$$\Rightarrow Z_x = Z_2 Z_3 Y_1$$

$$Z_2 = R_2, Z_3 = R_3, Z_1 = \frac{R_1 + j\omega C_1}{R_1}$$

$$Z_x = R_x + j\omega L_x$$

$$\text{Admittance} = \frac{1}{\text{Impedance}}$$

$$Y = \frac{1}{Z}$$

$$\begin{aligned} Z_1 &= R_1 \parallel C_1 & \therefore Y_1 &= \frac{1}{Z_1} \\ Z_1 &= \frac{R_1}{1 + j\omega C_1 R_1} & C \text{ is } \parallel \text{ with } R \\ Y_1 &= \frac{1}{R_1} + j\omega C_1 \end{aligned}$$

$$Z_x = Z_2 Z_3 Y_1$$

$$R_x + j\omega L_x = R_2 \cdot R_3 \cdot \left(\frac{1}{R_1} + j\omega C_1 \right)$$

$$R_x + j\omega L_x = \frac{R_2 R_3}{R_1} + R_2 R_3 j\omega C_1$$

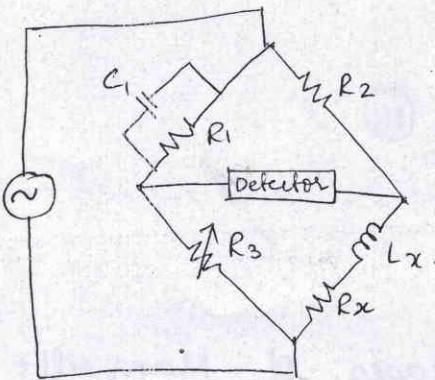
Equating real & imaginary parts

Q or Quality factor of L
is given by

$$Q = \frac{\omega L_x}{R_x}$$

$$Q = \frac{\omega R_2 R_3 C_1 \cdot R_1}{R_2 R_3}$$

$$Q = \omega C_1 R_1$$



(12)

(15)

(16)

Advantages of Maxwell's bridge:-

- 1) Balance eqn is independent of losses associated with inductance.
- 2) Balance eqn is independent of freq of measurement.
- 3) Scale of resistance can be calibrated to read the inductance directly.
- 4) R_1 can be calibrated to read Ω value of coil directly.

Disadvantages of Maxwell's bridge:-

- * It is limited to measure values of Ω b/w $1 - 10$. Not suitable for Ω value < 1 & > 10 .
- * Balance adjustment is little difficult due to interaction b/w resistance & reactance balances.
- * Bridge balance eqn are independent of freq, but practically the coil under test vary with freq which may cause error.

Watson's bridge:-

The arms of ac maxwell's bridge are adjusted as

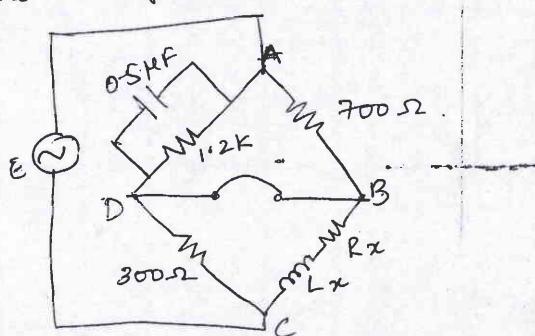
Ex - The arms of ac maxwell's bridge are adjusted as
 Arm AB: Nonreactive resistance of 700Ω
 Arm CD: Nonreactive resistance of 300Ω
 Arm AD: Nonreactive resistance of 1200Ω is 11^{th} with C of $0.5\mu\text{F}$. If the bridge is balanced under this condition., find the components of the branch BC?

$$\rightarrow \text{Given: } C_1 = 0.5\mu\text{F}$$

$$R_1 = 1.2\text{k}\Omega$$

$$R_2 = 700\Omega$$

$$R_3 = 300\Omega$$



$$\text{From eqns } R_x = \frac{R_2 R_3}{R_1} = \frac{175}{\frac{700 \times 300}{4}} = 175 \Omega \quad (13)$$

$$L_x = R_2 R_3 C_1$$

$$= 700 \times 300 \times 0.5 \mu F$$

$$L_x =$$

$$\boxed{R_x = 175 \Omega}$$

$$\boxed{L_x = 0.105 H}$$

(A)

Ex: The arms of an a.c. Maxwell's bridge are arranged as follows. AB & BC are non-reactive resistor of 100Ω each. DA is standard variable inductor L_1 of resistance 32.7Ω & CD comprises a std variable resistance R 32.7Ω & CD comprises a std variable resistance R of 1.36Ω . Balance was obtained with $L_1 = 47.8 \text{ mH}$ & $R = 1.36 \Omega$. Find the resistance & inductance of coil.

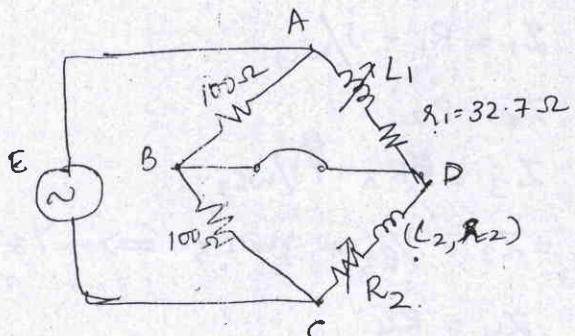
$$\rightarrow \text{Given: } L_1 = 47.8 \text{ mH}$$

$$R = 1.36 \Omega$$

Balance will be obtained

$$\text{when } L_2 = 47.8 \text{ mH}$$

$$R_2 = 1.36 \Omega$$



At balance

$$100 [(\Re_1 + j\omega L_1)] = 100 [(R_2 + r_2) + j\omega L_2]$$

$$100 \Re_1 + 100 j\omega L_1 = 100 (R_2 + r_2) + 100 j\omega L_2$$

Equating real & imaginary parts

$$\Re_1 = R_2 + r_2$$

$$\boxed{L_2 = L_1 = 47.8 \text{ mH}}$$

$$\Re_2 = \Re_1 - R_2$$

$$= 32.7 - 1.36$$

$$\boxed{\Re_2 = 31.34 \Omega}$$

(18)

Wein bridge:-

- * Wein bridge in its basic form is used to measure frequency.
 - * But it can be used to measure unknown capacitor with great accuracy.
 - * The Wein bridge is shown in fig.
- It has series RC & parallel across RC combination in the adjoining arm.
- * The impedances of arms are

$$Z_1 = R_1 - j/\omega C_1$$

$$Z_2 = R_2$$

$$Z_3 = R_3 + j/\omega C_3$$

$$Y_3 = Z_3 = 1/R_3 + j\omega C_3 \Rightarrow Y_3 = 1/R_3 + j\omega C_3 \Rightarrow \text{Admittance}$$

$$Z_4 = R_4$$

- * The balance eqn is

$$R_1 Z_1 Z_4 = R_2 Z_3$$

$$\text{i.e. } \cancel{Z_1 Z_4} = \cancel{Z_2 Z_3}$$

$$Z_1 Z_4 = Z_2 Y_3$$

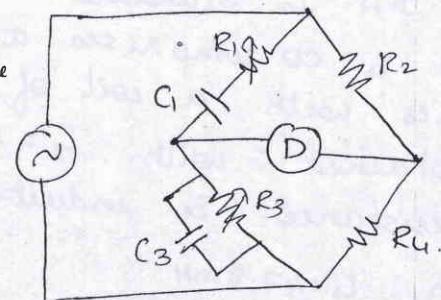
$$Z_2 = Z_1 Z_4 Y_3$$

$$R_2 = (R_1 - j/\omega C_1) R_4 (1/R_3 + j\omega C_3)$$

$$= R_4 (R_1 - j/\omega C_1) (1/R_3 + j\omega C_3)$$

$$= R_4 \left[\frac{R_1}{R_3} + j\omega C_3 R_1 - j/\omega C_1 R_3 + \frac{C_3}{C_1} \right]$$

$$R_2 = \frac{R_1 R_4}{R_3} + j\omega C_3 R_1 R_4 - \frac{j R_4}{\omega C_1 R_3} + \frac{R_4 C_3}{C_1}$$



$$R_2 = \left[\frac{R_1 R_4}{R_3} + \frac{R_4 C_3}{C_1} \right] - j \left[\frac{R_4}{\omega C_1 R_3} - \omega C_3 R_1 R_4 \right]. \quad (14)$$

Equating real & imaginary parts. (19)

$$\frac{R_1 R_4}{R_3} + \frac{R_4 C_3}{C_1} = R_2 \Rightarrow \text{Real part}$$

$$\frac{R_4}{\omega C_1 R_3} = \omega C_3 R_1 R_4 \Rightarrow \text{imaginary part.}$$

Considering real parts:

$$R_4 \left[\frac{R_1}{R_3} + \frac{C_3}{C_1} \right] = R_2$$

$$\boxed{\frac{R_2}{R_4} = \left[\frac{R_1}{R_3} + \frac{C_3}{C_1} \right]} \quad \text{--- (A)}$$

Considering imaginary parts

$$R_4 = \cancel{\omega^2 C_1 C_3 R_1^2 R_3}$$

$$\frac{R_4}{\omega C_1 R_3} = \omega C_3 R_1 R_4$$

$$1 = \omega^2 C_1 C_3 R_1 R_3.$$

$$\omega^2 = \frac{1}{C_1 C_3 R_1 R_3}$$

$$\omega = \frac{1}{\sqrt{C_1 C_3 R_1 R_3}}$$

$$2\pi f = \omega.$$

$$\boxed{f = \frac{1}{2\pi \sqrt{C_1 C_3 R_1 R_3}}} \quad \text{--- (B)}$$

(20)

~~Eqⁿ A & B result~~

Eqⁿ A & B helps in determining resistance ratio.

R_2/R_4 & freq.

* Thus if we satisfy $\frac{R_2}{R_4}$ ratio i.e. eqn A & excite the bridge with freq + (eqn B) then the bridge will be balanced.

* In most of the Wein bridge ckt, the components are chosen such that $R_1 = R_3 = R$
 $C_1 = C_3 = C$.

$$\frac{R_2}{R_1} = 1+1 = 2$$

$$f = \frac{1}{2\pi RC} \rightarrow \text{General form of equation for the freq of the bridge ckt.}$$

Applications:-

- Bridge is used to measure freq in audio range 20 Hz - 20 kHz. Resistance → used for range changing capacitors → used for freq control.
- Bridge can be used to measure capacitor if operating freq is known.
- Bridge can be used in harmonic distortion analyzer, as a notch filter & in AF & RF oscillators as freq determining element.
- Accuracy of 0.5% - 1% can be obtained using this bridge.

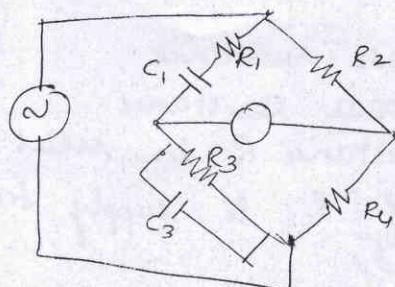
Ex:- Find the equivalent parallel resistance or capacitance (15) that causes a Wein bridge to null with the following component values. $R_1 = 3.1 \text{ k}\Omega$, $C_1 = 5.2 \mu\text{F}$, $R_2 = 25 \text{ k}\Omega$, $f = 2.5 \text{ kHz}$ (21)
 $\& R_4 = 100 \text{ k}\Omega$.

→ From given data Wein bridge can be drawn as.

From the bridge balance eqn. we have

$$\omega^2 = \frac{1}{C_1 C_3 R_1 R_3} \quad \text{---(1)}$$

$$\frac{R_2}{R_4} = \left[\frac{R_1}{R_3} + \frac{C_3}{C_1} \right] \quad \text{---(2)}$$



(i) Rearranging (1)

$$C_3 = \frac{1}{C_1 \omega^2 R_1 R_3}$$

Using C_3 in (2)

$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{1}{C_1^2 \omega^2 R_1 R_3}$$

$$\frac{25 \text{ k}}{100 \text{ k}} = \frac{3.1 \text{ k}}{R_3} + \frac{1}{(5.2 \mu\text{F})^2 \times (2\pi \times 2.5 \text{ kHz})^2 \times 3.1 \text{ k} \times R_3}$$

$$0.25 = \frac{3.1 \text{ k}}{R_3} + \frac{0.0483}{R_3} = \frac{3.1 \text{ k} + 0.0483}{R_3}$$

$$\boxed{R_3 = 12.4 \text{ k}\Omega}$$

Using R_3

$$C_3 = \frac{1}{C_1 \omega^2 R_1 R_3} = \frac{1}{5.2 \mu\text{F} \times (2\pi \times 2.5 \text{ kHz})^2 \times 3.1 \text{ k} \times 12.4 \text{ k}}$$

$$\boxed{C_3 = 20.28 \mu\text{F}}$$

Ex:- The Wien bridge ABCD, supplied with a sinusoidal voltage, have the following values:
 $AB = 330 \text{ }\Omega$ in parallel with $0.2 \mu\text{F}$ capacitor.

(22)

$$BC = 400 \Omega \text{ resistance}$$

$$CD = 800 \Omega \text{ resistance}$$

DA resistance R is series with a $1.5 \mu F$ capacitor. Determine value of R & supply freq at which bridge will be balanced.

$$\rightarrow Z_1 = R_1 - j/\omega C_1 = R_1 - j/2\pi f C_1$$

$$Z_2 = R_2 = 800 \Omega$$

$$Z_3 = \cancel{R_3}$$

$$Z_4 = R_4 = 400 \Omega$$

$$Y_3 = \frac{1}{Z_3} = \frac{1}{R_3} + j\omega C_3$$

The balance eqn is

$$Z_1 Z_4 = Z_2 Z_3$$

$$Z_2 = \frac{Z_1 Z_4}{Z_3}$$

$$Z_2 = Z_1 Z_4 Y_3$$

$$800 = \left(R_1 - \frac{j}{\omega C_1} \right) (400) \left(\frac{1}{R_3} + j\omega C_3 \right)$$

$$2800 = \left(R_1 - j \frac{666.667 \times 10^3}{\omega} \right) (400) \left(3.030 \times 10^{-3} + j\omega 0.2 \mu F \right)$$

~~$$2800 = R_1 \times 3.030 \times 10^{-3}$$~~

$$2 = \left(R_1 - j \frac{666.667 \times 10^3}{\omega} \right) \left(3.030 \times 10^{-3} + j\omega 0.2 \mu H \right)$$

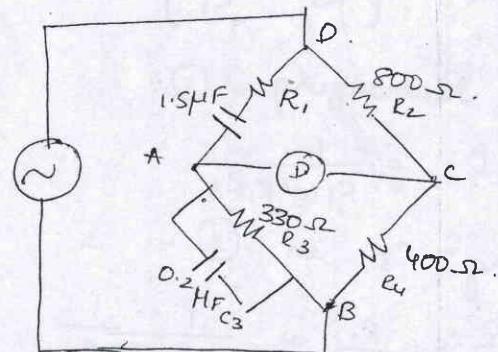
$$= 3.030 \times 10^{-3} R_1 + j\omega 0.2 \mu H R_1 - j \frac{666.667 \times 10^3}{\omega} \cdot 3.030 \times 10^{-3}$$

$$+ \frac{666.667 \times 10^3 \times \omega 0.2 \mu H}{\omega}$$

$$2 = [3.030 \times 10^{-3} R_1 + 0.1333] + j \left[\omega 0.2 \mu H R_1 - \frac{2020.20}{\omega} \right]$$

Equating real & imaginary part

$$2 = 3.030 \times 10^{-3} R_1 + 0.13334$$



$$R_1 = \frac{2 - 0.1333}{3.0303 \times 10^3}$$

(16)

$$R_1 = 615.9 \Omega$$

(23)

$$\boxed{R_1 = 616 \Omega = R} \text{ unknown resistance}$$

Imaginary part

$$\omega 0.2 \mu R_1 - \frac{2020 \cdot 20}{\omega} = 0.$$

$$\omega^2 0.2 \mu R_1 - 2020 \cdot 20 = 0.$$

$$(2\pi f)^2 0.2 \times 10^{-6} \times 616 - 2020 \cdot 20 = 0$$

$$(2\pi f)^2 = \frac{2020 \cdot 20}{0.2 \times 10^{-6} \times 616} = 16.397 \times 10^6$$

$$2\pi f = \sqrt{16.397 \times 10^6} = 4.049 \times 10^3$$

$$f = \frac{4.049 \times 10^3}{2\pi}$$

$$\boxed{f = 644.48 \text{ Hz}}$$

Ex:- Find the parallel 'R' & 'C' that causes a Wein bridge to null with the following component values.
R₁ = 2.7 kΩ, R₂ = 22 kΩ, C₁ = 5 μF, R₄ = 100 kΩ & the operating freq is 2.2 kHz

→ we have eqns R₃ & C₃ = ?

$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1}$$

$$\omega = \frac{1}{\sqrt{R_1 C_1 R_3 C_3}} \Rightarrow \omega^2 = \frac{1}{R_1 C_1 R_3 C_3}$$

$$C_3 = \frac{1}{\omega^2 R_1 R_3 C_1 C_3}$$

(24)

Using C_3 in R_2/R_4 Eqⁿ.

$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{1}{\omega^2 R_1 C_1^2 R_3}$$

$$R_3 = 12.273 \text{ k}\Omega$$

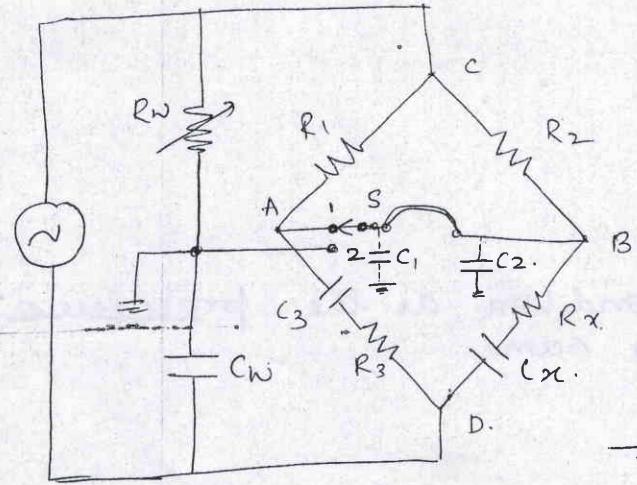
$$C_3 = \frac{1}{\omega^2 R_1 C_1 R_3}$$

$$C_3 = 31.58 \text{ pF}$$

Wagner's Earth Connection :-

~~Noise~~ In electrical ckt, parasite or stray capacitances is an unavoidable & unwanted capacitance that exists b/w parts of electronic component or ckt due to proximity to each other.

- * All actual elements such as diode, transistor & conductors have internal capacitance which can cause their behaviour to vary from that of ideal ckt element.
- While performing high freq measurement, stray capacitance b/w bridge elements & ground & b/w the bridge arms exists & they become significant.
- This introduces error in the measurement when we are measuring small capacitance & large inductance values.
- This can be avoided or controlled by providing shielding & grounding the shield. However this does not eliminate the capacitance but makes its constant value.
- Another effective & popular method for eliminating stray capacitance is by using "Wagner's ground connection."
- ckt is shown. The ckt is a capacitance bridge measuring

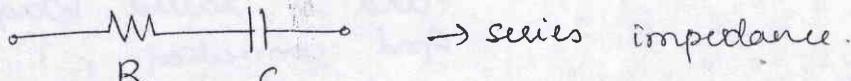


- (25)
- In the ckt C_1 & C_2 represents the stray capacitance
 - In Wagner's Earth/gnd connection ~~one~~ one more arm having R_w & C_w which acts /forms a potential divider is used.
 - The R_w & C_w ~~in~~ is grounded and is called Wagner's gnd connection..

- The adjustment procedure is as follows.
- The switch S is connected to point 1 & adjusted for null or min sound in headphone, by varying R_1
- Then S is connected to point 2 ~~or~~ i.e. Wagner's gnd point & R_w is adjusted to get min or null sound in headphone.
- When S is connected again to point 1 there will be some imbalance so R_1 & R_3 are varied to get min sound
- S is again connected to point 2 & vary R_w to get min sound. This procedure is repeated until null is obtained at both point 1 & 2. This gives the "ground potential".
- At ~~this~~ this condition, C_1 & C_2 i.e stray capacitance are effectively short ckted & have no effect on the normal bridge. ($\frac{1}{I}$)
- From point C to D the capacitance that exists are also eliminated by the Wagner's ground connection. As the current thru' capacitance will enter the Wagner's ground connection & their effect will also be nullified.
- The addition of Wagner's ground connection will

not affect the balance condition as the procedure for measurement remains same.

Note

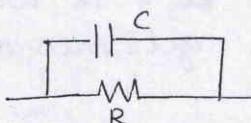


→ series impedance.

$$R + jX_c \quad X_c = \frac{1}{j\omega c}$$

$$R + \frac{1}{j\omega c}$$

$$\boxed{R - j\omega c = Z}$$



→ Parallel impedance.

$$\frac{R \times X_c}{R + X_c} = \frac{R * \frac{1}{j\omega c}}{R + \frac{1}{j\omega c}}$$

$$= \frac{R/j\omega c}{jR\omega c + 1}$$

$$Z = \frac{R}{jR\omega c + 1}$$

$$= \frac{\cancel{R}}{\cancel{jR\omega c} + R}$$

$$= \frac{R}{j\omega c + R}$$

$$Y = \frac{1}{Z}$$

$$Y = \frac{1}{R}$$

$$- \frac{1}{jR\omega c + 1}$$

$$= \frac{1 + jR\omega c}{R}$$

$$= \frac{1}{R} + j \frac{\omega c}{R}$$

$$\boxed{Y = \frac{1}{R} + j\omega c}$$

MEASURING INSTRUMENTS

①

- Are the devices for measuring physical quantities.
- They play an important role in all phases of electronics & helps in determining how an electronic ckt is performing.
- The fundamental electrical measurements are voltage, current & resistance / impedance.
- The instruments which are used for measuring these quantities form the building blocks for more complex equipments/devices used for measuring other parameters like power, freq & other special measurement.
- A measuring device converts primary indication into some other form of energy that can be easily displayed on a scale.

Q meter :-

- Q factor also called Quality factor or storage factor.
- The overall efficiency of coils & capacitors which are used in RF applications can be evaluated using Q value.
- It is an instrument designed to measure electrical properties of coils & capacitors.
- The principle is based on series resonance
- It is also defined as ratio of reactance to resistance of a reactive element.

Working principle

- It works on series resonance — A freq point at which inductive reactance of inductor becomes equal to capacitive reactance of capacitor. ie $X_L = X_C$
- In other words : The voltage drop across the coil or capacitor is Q times applied voltage.

→ Series resonant ckt is shown in which
 E = Applied Voltage

E_C = Capacitor voltage

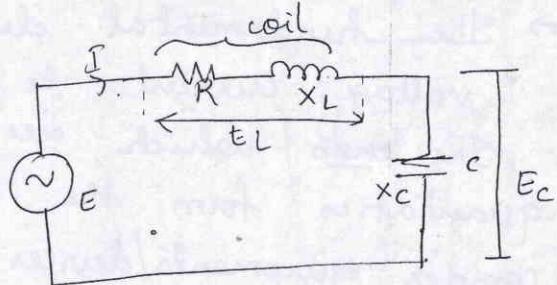
E_L = inductive voltage

X_L = inductive reactance

X_C = capacitive reactance.

R = coil resistance

I = current



At resonance $X_L = X_C$

$$E_C = I X_C$$

$$E_L = I X_L$$

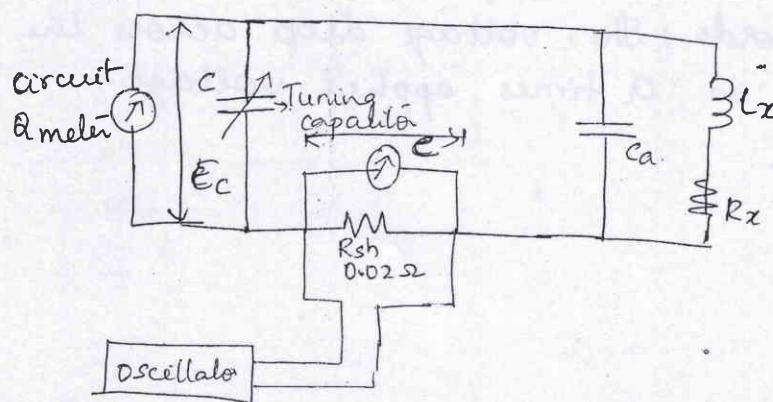
$$E = I R.$$

$\therefore Q = \frac{\text{Reactance}}{\text{Resistance}}$

$$\therefore Q = \frac{X_C}{R} = \frac{X_L}{R} = \frac{E_C}{E}$$

From the above eqn if E is kept constant, then voltage across capacitor is Q times E .
 Thus a voltmeter when calibrated across capacitor gives the Q value.

Practical Q meter:



- Wide range oscillator with freq range 50 kHz to 50 MHz is used as source to provide current to resistance R_{sh} whose value is 0.02 Ω.
- Current through R_{sh} represents voltage source of magnitude 'e' with small internal resistance.
- The voltage across shunt is measured using a thermocouple meter and voltage across capacitor is measured using electronic voltmeter corresponding to E_c . This is adjusted directly to read Q.

$$Q = \frac{E_c}{e}$$

or $E_c = Q e$.

- If inductance of coil has to be determined then it has to be connected to the test terminals of the instrument.
- The circuit is tuned to resonance by varying either capacitance or oscillator freq.
- If C is varied, then oscillator freq is adjusted to given freq to obtain resonance.
- If oscillator freq is varied, then C is pre-set to desired value to get resonance.
- To get the actual Q value, the output obtained should be multiplied by index setting or "Multiply Q by" switch.
- The inductance of the coil can be found from the known values of oscillator freq e & resonating capacitor C.

$$X_L = X_C \Rightarrow 2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{4\pi^2 LC} \quad f = \frac{1}{2\pi\sqrt{LC}}$$

$$\text{or } L = \frac{1}{4\pi^2 f^2 C}$$

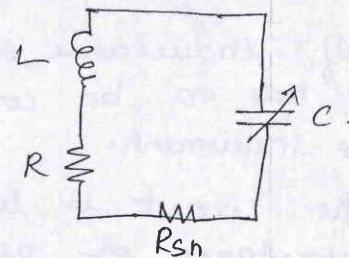
Factors that may cause error:-

1. Due to R_{sh} :

- * At high freq., the electronic voltmeter will suffer from losses (due to transit time effect) & this effect results in introducing R_{sh} into the tank ckt as shown below.

$$Q_{act} = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{2\pi f L}{R}$$

$$Q_{obs} = \frac{\omega L}{R + R_{sh}}$$



$$\therefore \frac{Q_{act}}{Q_{obs}} = \frac{\frac{\omega L}{R}}{\frac{\omega L}{R + R_{sh}}} = \frac{\omega L}{R} \times \frac{R + R_{sh}}{\omega L} = 1 + \frac{R_{sh}}{R}$$

$$\therefore \frac{Q_{act}}{Q_{obs}} = \therefore Q_{act} = Q_{obs} \left[1 + \frac{R_{sh}}{R} \right].$$

Thus in order to make Q_{act} close to Q_{obs} , R_{sh} should be made as small as possible.

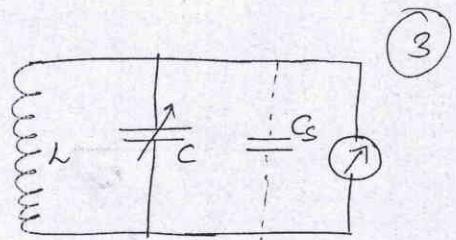
2. Due to stray capacitance:-

- * The presence of stray capacitance modifies the actual ω inductance of coil.
- * At resonant freq., $X_L = X_C$ & the ckt impedance purely resistive & this can be used to measure stray / distributed / self capacitance.
- * One method of measuring stray or distributed capacitance C_s of coil is by making 2 measurements at different freq.
- * The capacitor C of Q meter is calibrated to indicated capacitance value.
- * The coil under test is connected to Q meter terminal as shown in fig.

* The ~~L~~ C is kept to high value
ie max & ckt is resonated
by varying oscillator freq.

* At resonance, the oscillatory
freq & capacitor be denoted by

$$f_1 \text{ Hz} \& C_1$$



(3)

* The oscillator freq is now ↑ to twice the original freq i.e. $f_2 = 2f_1$ & the C is varied to obtain resonance at f_2 . Thus at resonance the freq & capacitor values are.

$$f_2 = 2f_1 \text{ Hz}$$

$$C_2$$

* The resonant freq of LC ckt is given

$$X_C = X_L \quad 2\pi f L = \frac{1}{2\pi f C}$$

$$\therefore f = \frac{1}{2\pi \sqrt{LC}}$$

* At the initial or 1st resonance condition, the total capacitance in the ckt is

$$C_1 + C_S \quad \& \text{ the resonant freq is now}$$

$$f_1 = \frac{1}{2\pi \sqrt{(C_1 + C_S)L}}$$

* At the 2nd resonance condition, the total capacitor in the ckt is

$$f_2 = \frac{1}{2\pi \sqrt{L(C_2 + C_S)}}$$

* But we have $f_2 = 2f_1$

$$\frac{1}{2\pi \sqrt{L(C_2 + C_S)}} = \frac{2}{2\pi \sqrt{L(C_1 + C_S)}}$$

$$\sqrt{\frac{1}{L(C_2 + C_s)}} = \sqrt{\frac{2}{(C_1 + C_s)L}} \quad \text{squaring both sides.}$$

~~Eqn 1 & Eqn 2~~

$$\frac{1}{K(C_2 + C_s)} = \frac{4}{K(C_1 + C_s)}$$

$$C_1 + C_s = 4(C_2 + C_s)$$

$$C_1 + C_s = 4C_2 + 4C_s$$

$$C_1 = 4C_2 + 4C_s - C_s$$

$$C_1 = 4C_2 + 3C_s.$$

$$C_s = \frac{C_1 - 4C_2}{3}$$

Thus the shunt capacitance can be calculated using above eqn.

Ex:- The self capacitance of coil is measured by making 2 measurements at diff frequencies. The first measurement is at $f_1 = 1\text{ MHz}$ & $C_1 = 500\text{ pF}$. The 2nd measurement is at $f_2 = 2\text{ MHz}$ & $C_2 = 110\text{ pF}$. Determine distributed capacitor. Also calculate value of L .

$$\rightarrow C_s = \frac{C_1 - 4C_2}{3}$$

$$C_s = \frac{\frac{500 \times 10^{-12}}{3} - 4 \times 110 \times 10^{-12}}{3} = \frac{(500 - 440)\text{ pF}}{3} = \frac{20}{3}\text{ pF}$$

$$C_s = 20\text{ pF}$$

$$f_1 = \frac{1}{2\pi\sqrt{L(C_1 + C_s)}}$$

$$1\text{ MHz} = \frac{1}{2\pi\sqrt{L(500\text{ pF} + 20\text{ pF})}}$$

Squaring both sides.

$$(1\text{ MHz})^2 = \frac{1}{(2\pi L)(500\text{ pF} + 20\text{ pF})}$$

$$L = \frac{1}{4\pi (1 \text{ MHz})^2 \times 520 \text{ pF}}$$

(4)

$$L = \underline{48.712 \text{ HF}}$$

Ex:- Determine the value of self capacitance when the following measurements are performed.

$$f_1 = 2 \text{ MHz} \quad C_1 = 500 \text{ pF}$$

$$f_2 = 6 \text{ MHz} \quad C_2 = 50 \text{ pF}$$

→ Given $f_2 = 3f_1$

$$\text{At resonance} \quad \frac{1}{2\pi\sqrt{L(C_2 + C_s)}} = \frac{3}{2\pi\sqrt{L(C_1 + C_s)}}$$

Squaring both sides.

$$\frac{1}{C_2 + C_s} \propto \frac{9}{C_1 + C_s}$$

$$C_1 + C_s = 9C_2 + 9C_s$$

$$C_1 - 9C_2 = 8C_s$$

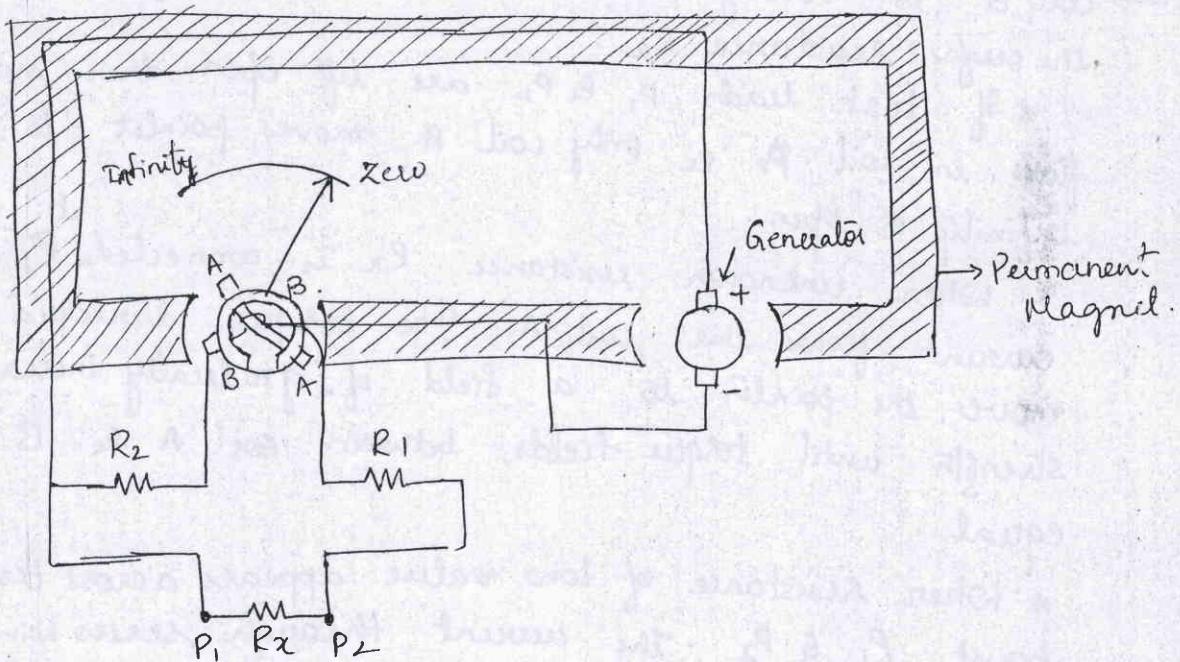
$$\therefore C_s = \frac{C_1 - 9C_2}{8}$$

$$C_s = \frac{500 \text{ p} - 9(150 \text{ p})}{8} = \frac{500 \text{ p} - 1350 \text{ p}}{8} = \underline{\underline{62.5 \text{ pF}}}$$

$$\boxed{C_s = 6.25 \text{ pF}}$$

Megger

- Resistances of order $0.1\text{M}\Omega$ and upwards are classified as high resistances and these can be measured using portable instrument called as "Megger".
- It is used to measure high resistances seen in cable insulation, transformer windings, between motor windings etc.
- It works on the principle of electromagnetic induction.
- Megger is an portable ohmmeter which has an inbuilt high voltage source.
- The Megger is shown in fig below.



- It has 2 main elements.
 1. Magnet type dc generator → To supply current for measurement.
 2. Ohmmeter → Measure resistance value.
- The dc generator is the high voltage source, which produces high voltages such as 500V, 1000V, 2500V or 5000V depending on model.
- The meter has 2 windings.* One winding B is in series with R_2 and is connected to output of generator (ie +ve). This winding moves pointer towards high resistance end on the scale

when generator is operating.

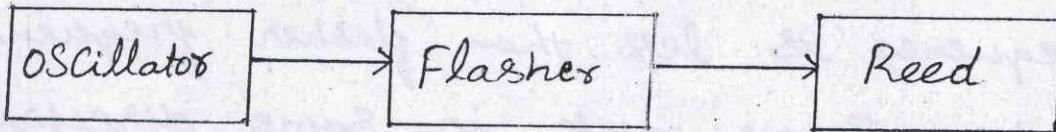
- * Other winding A which is in series with R_1 is connected between -ve terminal of generator & the test line. This winding moves the pointer towards zero end when current flows.
- * Both windings are mounted on same shaft and they are at right angle to each other.
- The test leads are P_1 & P_2 to which unknown resistance to be measured is connected.
- Coil A is 'current coil' connected to -ve o/p of generator & is in series with R_1 to the test lead P_2 .
- Test lead P_1 is connected to the +ve o/p of generator.
- Coil B is 'voltage coil' connected across generator o/p ~~at~~ the shunt resistance R_2 .
 - * If test leads P_1 & P_2 are left open then no current flows in coil A & only coil B moves pointer to indicate infinity or open.
 - * When unknown resistance R_x is connected ^{at} P_1 & P_2 current flows in coil A. The torque developed moves the pointer to a field of gradually increasing strength until torque fields between coil A & B are equal.
 - * When resistance of low value appears across test point P_1 & P_2 , the current through series winding causes pointer to move towards zero.

Applications:-

- Megger is used to determine high resistance between conducting part of cbt and ground (called as insulation).
- Used to test continuity between any 2 points.
 - * When pointer points to full scale deflection there is electrical continuity between them.

Stroboscope

- * Stroboscope consists of an oscillator, a reed and a flasher.

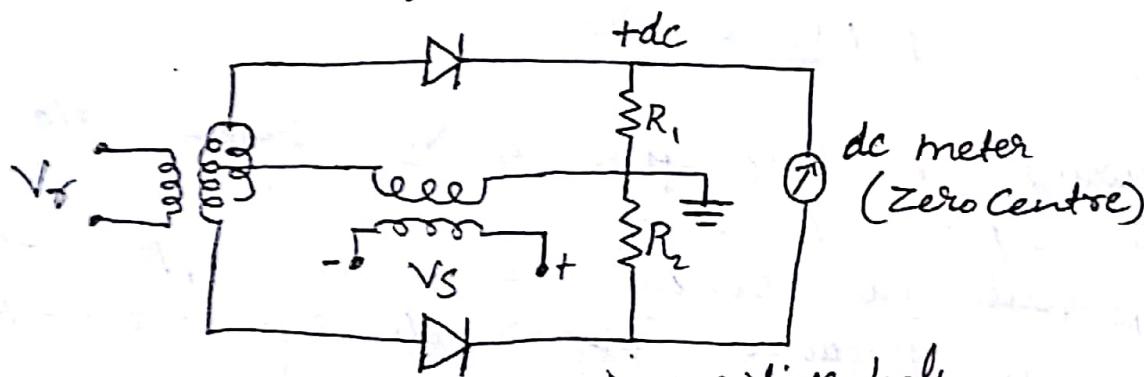


- * Working principle of Stroboscope
A high intensity light is flashed at precise intervals directed upon rotating or vibrating object.
- * Oscillator : It is externally triggered multivibrator which provide trigger pulses to flasher mechanism to control flash rate.
- * Flasher : It is tube fired by capacitor discharge, which is controlled by trigger pulses. Tube is filled with inert gas which produces light when it is ionised.
- * Reed : is driven from ac line & vibrates at 7200 times per minute.
- * When frequency of moving object exactly matches Stroboscope frequency, moving object appear as single stationary image.
- * When Image appears to rotate in opposite direction to that of actual rotation. Rotation

- frequency is less than flasher frequency.
- * when Image rotate in same direction as actual rotation, rotation frequency is higher than the flasher frequency
 - * Stroboscope is used to check motor ^(or) generator speeds ranging from 60 - 1,000,000 rpm.
 - * It has an accuracy ^{of measurement} as close as 0.1 %

Phase meter.

- * Also Called as phase sensitive detector used for comparing AC Signal with Reference signal.
- * Detector produces a rectified O/P which is given to DC meter to clearly read O/P of phase detector. Swings zero Center pointer in one direction for in phase error & in opposite direction for out-phase Condition.
- * Detector distinguishes only b/w in phase & 180° out of phase, without regard for other phase angles.



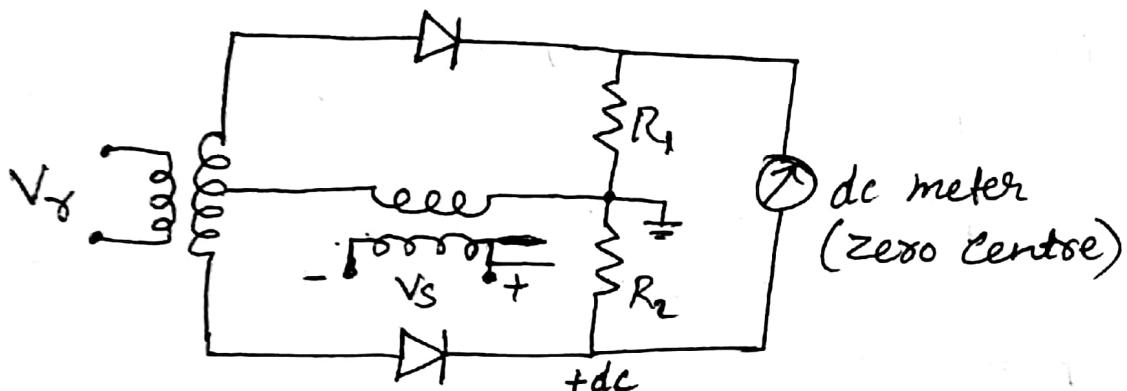
a) positive half.

* When $V_S = 0$, During +ve half cycle of reference V_x V_x causes rectified current to flow through D_1 , producing +ve V_{dg} to ground across R_1 , which deflects meter to right.

* During -ve half cycle V_x Causes an equal rectified current to flow through diode D_2 , producing equal tendency for meter to deflect to the left.

* So equal & opposite tendencies, the galvanometer reads zero over full cycle.

- * When I/P V_s is Applied, and V_s is inphase with V_o , produces larger Current through D_1 & larger dc o/p V_{lg} on first half.
- * D_2 does not conduct, so V_{lg} across R_2 is $V_s - V_o$.
- * And V_{lg} across R_1 is $V_s + V_o$

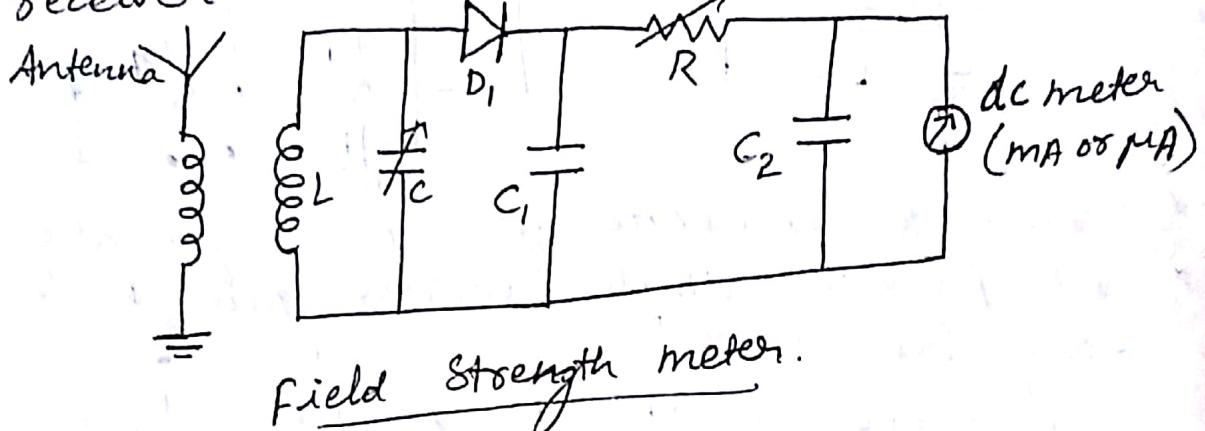


b) Negative half

- * During -ve half Cycle of V_s , signal V_{lg} is in opposite direction.
- * D_1 will not Conduct & Signal oppose instantaneous ac V_{lg} produces smaller dc V_{lg} across R_2 .
- * Galvanometer deflects to right proportional to magnitude of inphase I/P Signal V_s .
- * Now, if V_s is 180° out of phase with V_o , V_{lg} add on lower half on transformer secondary, so galvanometer deflects to left proportional to magnitude of I/P Signal.

Field Strength meter.

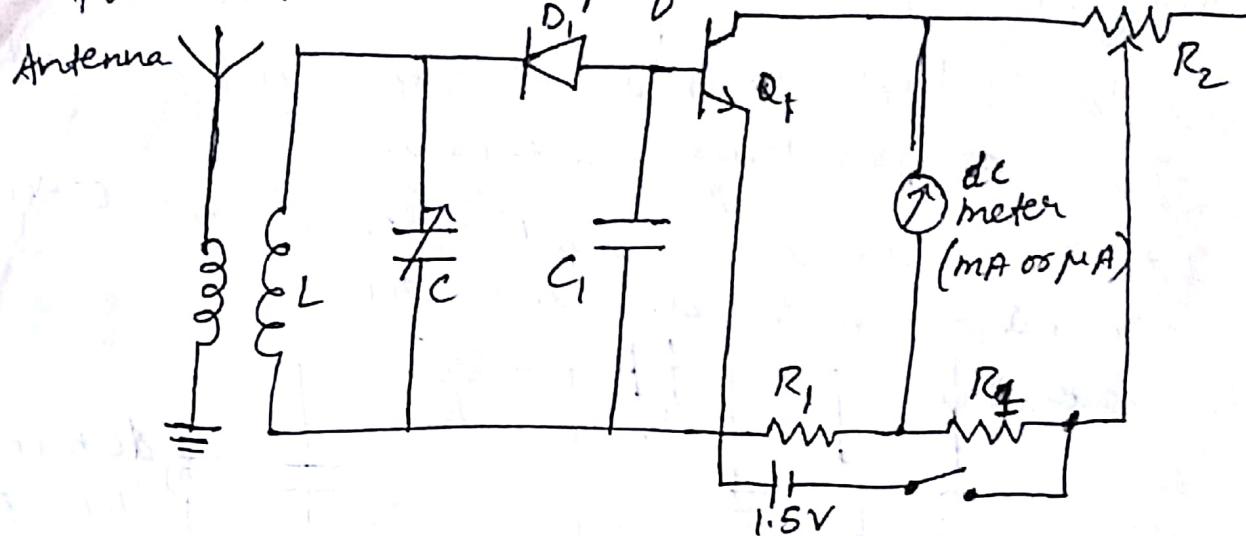
- * It is used to measure radiation intensity of a transmitting antenna.
- * It has its own small whip antenna for essential receiver with an indicator.



Field Strength meter.

- * wave meter Ckt with rectified -meter indicator is often equipped with small whip Antenna, & it is called as field strength meter.
- * Although we can get indications by this setup fairly positioning close to transmitting antenna but sensitivity is not high enough for use with ordinary low powered transmitters.
- * Also field strength measurement to be made at distance of several wavelengths from transmitting antenna to avoid misleading reading obtained due to combination of radiation field with induction field close to transmitter.
- * To enable wavemeter to measure field strength greater sensitivity is obtained with addition

of transistor DC amplifier. as shown below.



- * Transistor Connected in CE Configuration.
- * It provides ample Current gain, with Satisfactory Sensitivity.
- * Quiescent Current is balanced by back up current through Variable Resistor R_2 .
- * Quiescent current is checked at intervals. since it vary with temp changes.
- * Collector Current through meter provides indication of strength of RF wave obtained.
- * Current is not strictly proportional to field strength because of non-linearities of Semiconductor Diode & transistors.
- * the response is satisfactory for relative comparison of field strength.