MODULE 1

Measurement and Error

Introduction:

The measurement of any quantity plays very important role not only in science but in all branches of engineering, medicine and in almost all the human day to day activities. The technology of measurement is the base of advancement of science. The role of science and engineering is to discover the new phenomena, new relationships, the laws of nature and to apply these discoveries to human as well as other scientific needs. The science and engineering is also responsible for the design of new equipment. The operation, control and the maintenance of such equipment and the processes is also one of the important functions of the science and engineering branches. All these activities are based on the proper measurement and recording of physical, chemical, mechanical, optical and many other types of parameters.

Measurement: The measurement of a given parameter or quantity is the act or result of a quantitative comparison between a predefined standard and an unknown quantity to be measured.

An electronic instrument is the one which is based on electronic or electrical principles for its measurement function. The measurement of any electronic or electrical quantity or variable is termed as an electronic measurement.

Static characteristics: The static characteristics are defined for the instruments which measure the quantities which do not vary with time. The various static characteristics are accuracy, precision, resolution, error, sensitivity, threshold, zero drift, stability and linearity.

Accuracy: It is the degree of closeness with which the instrument reading approaches the true value of the quantity. It denotes the extent to which we approach the actual value of the quantity. It indicates the ability of instrument to indicate the true value of the quantity.

Ex: if voltmeter reads 100V with ±1% error, then true or actual value lies between 99V and 100V.

Precision: It is the measure of consistency or repeatability of measurements.

Resolution: It is the smallest increment of quantity being measured which can be detected with certainty by an instrument. OR The smallest change in a measured value to which device responds.

Ex: If a digital voltmeter indicates 8.135V and if the measured quantity increases or decreases by 0.001 or 1mV, then reading becomes either 8.136V or 8.134V respectively. Thus the resolution of the instrument is 1mV.

Significant Figures: The significant figures convey the actual information about the magnitude and also contributes to the resolution.

Ex: If 8.134V indicates a voltage measured, then it has significant figures of 4.

Error: The most important static characteristics of an instrument is its accuracy, which is generally expressed in terms of the error called static error. It is given by

$e = A_m - A_a$ where

e – Error
A_m – Measured value
A_a – Actual value

The static error is defined as the difference between the true value of the variable and the value indicated by the instrument. The static error may arise due to number of reasons. The static errors are classified as:

1) Gross errors: The gross errors mainly occur due to carelessness or lack of experience of a human being. These cover human mistakes in readings, recordings and calculating results. These errors also occur due to incorrect adjustments of instruments. These errors cannot be treated mathematically. These errors are also called personal errors.

2) Systematic errors: The systematic errors are mainly resulting due to the shortcomings of the instrument and the characteristics of the material used in the instrument, such as defective or worn parts, ageing effects, environmental effects, etc. A constant uniform deviation of the operation of an instrument is known as a systematic error.

There are three types of systematic errors as

1) Instrumental errors: these errors are inherent because of their mechanical structure and moving component. Ex: stretching of springs, irregular tension to spring, overloading and others. These errors can be avoided by
    - Selecting suitable instrument for measurement
    - Correction factors can be applied after determining instrumental errors
    - Calibrating the device against standard
2) Environmental errors: Due to external condition of a measuring device i.e the surrounding area of the instrument like temperature, humidity, magnetic or electrostatic fields. These errors can be avoided by
    - Air conditioning
    - Using magnetic shields
    - Hermetically scaling components
    - Using heat sinks
3) Observational errors: are the errors introduced by the observer. The most common error is the parallax error introduced in reading a meter scale

3) Random errors: these are the errors that remain after gross and systematic errors. These errors are due to unknown causes. These errors are small and can be treated mathematically.

When the error is specified in terms of an absolute quantity and not as a percentage, then it is called an **absolute error**. Thus the voltage of 10 ± 0.5 V indicated ± 0.5 V as an absolute error. When the error is expressed as a percentage or as a fraction of the total quantity to be measured, then it is called **relative error.**

Error may be expressed either as absolute or as percentage of error.

Absolute error may be defined as the difference between the expected value of the variable and the measured value of the variable, or

$$e = Y_n - X_n$$

where e = absolute error

Y_n = expected value

X_n = measured value

$$\text{Therefore } \% \text{ Error} = \frac{\text{Absolute value}}{\text{Expected value}} \times 100$$

$$= \frac{e}{Y_n} \times 100$$

$$\text{Therefore } \% \text{ Error} = \left(\frac{Y_n - X_n}{Y_n}\right) \times 100$$

It is more frequently expressed as a accuracy rather than error.

$$\text{Therefore} \qquad A = 1 - \left|\frac{Y_n - X_n}{Y_n}\right|$$

where A is the relative accuracy.

Accuracy is expressed as % accuracy

$$a = 100\% - \%\ \text{error}$$

$$a = A \times 100\ \%$$

where a is the % accuracy.

Statistical Analysis:

Statistical analysis of measurement helps in analytical determination of the uncertainty of the final test result. To make statistical analysis meaningful, large number of measurement is usually required. This method is used when deviation of measurement from its true value is to be determined and the reason for the error is unpredictable.

Arithmetic Mean: When quantity is measured many times and all the measurement are not same then this method is used. Using mean the best approximation to the actual value is found. The arithmetic mean of n measurements at a specific count of the variable x is given by the expression

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} = \frac{\sum_{n=1}^{n} x_n}{n}$$

where $\bar{x}$ = Arithmetic mean

x_n = nth reading taken

n = total number of readings

Deviation from Mean: This is the deviation of a given reading from the arithmetic mean of the group of readings. If the deviation of the first x_1, is called d_1 and for 2nd reading it is called d_2 and so on. The deviation may be positive or negative and the algebraic sum of all deviations must be zero. The deviations from the mean can be expressed as

$$d_1 = x_1 - \bar{x},\ d_2 = x_2 - \bar{x}\ \ldots\text{, similarly } d_n = x_n - \bar{x}$$

Average Deviation: is an indication of the precision of the instrument used in measurement. Average Deviation is defined as the sum of absolute values of the deviation divided by the number of readings. Average deviation may be expressed as

$$D_{av} = \frac{|d_1| + |d_2| + |d_3| + \cdots + |d_n|}{n}$$

or $$D_{av} = \frac{\sum |d_n|}{n}$$

where D_{av} = average deviation

$|d_1|, |d_2|, \ldots, |d_n|$ = Absolute value of deviations

and n = total number of readings

Standard Deviation: The standard deviation is the square root of the sum of all individual deviations squared, divided by the number of readings. It may be expressed as

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \cdots + d_n^2}{n}}$$

$$\sigma = \sqrt{\frac{d_n^2}{n}}$$

where σ = standard deviation

Standard deviation is also known as root mean square deviation and is an important factor in the statistical analysis of measurement. Reducing this quantity helps in improving the measurement.

The square of standard deviation is known as variance and it is expressed as

$$\sigma^2 = \frac{d_1^2 + d_2^2 + d_3^2 + \cdots + d_n^2}{n}$$

Probable Error: It is denoted by PE and is given by

PE = ±0.6745 σ

Dynamic Characteristics of Instruments:

The set of criteria defined for the instruments, which are changes rapidly with time, is called **'dynamic characteristics'.** The dynamic characteristics are

Speed of response: The speed of response of measuring instrument is defined as the quickness with which an instrument responds to a change in the output signal.

Lag: It is the retardation or delay in the response of a measurement system to changes in the measured quantity.

Fidelity: It is the ability of a measurement system to reproduce the output in the same form as the input.

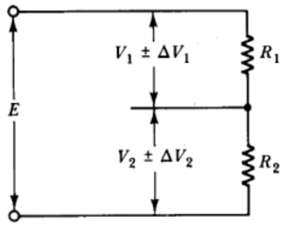
Dynamic error: It is the difference between the true value of the quantity changing with time and the value indicated by the measurement system if no static error is assumed. It is also called measurement error.

Measurement error combinations:

When a quantity is calculated from measurements made on two or more instruments, it must be assumed that errors due to instrument inaccuracy combine in worst possible way. The resulting error is then larger than the error in any one instrument.

Sum of quantities:
Where a quantity is determined as the sum of two measurements, the total error is the sum of the absolute errors in each measurement. As illustrated in Figure

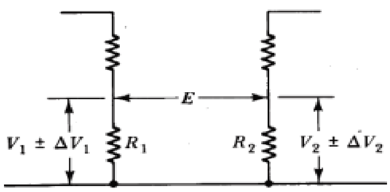
$$E = (V1 \pm \Delta V1) + (V2 \pm \Delta V2)$$



Error in sum of quantities equal sum of errors

Thus, % error in $E = (V1 + V2) \pm (\Delta V1 + \Delta V2)$

Difference of quantities:
Figure below illustrates a situation in which a potential difference is determined as the difference between two measured voltages. Here again, the errors are additive:

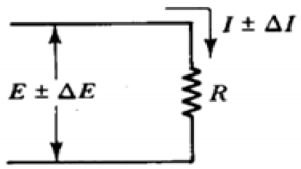


Error in difference of quantities equal sum of errors

$E = (V1 \pm \Delta V1) - (V2 \pm \Delta V2)$

Giving $E = (V1 - V2) \pm (\Delta V1 + \Delta V2)$

Product of quantities:
When a calculated quantity is the product of two or more quantities, the percentage error is the sum of the percentage errors in each quantity [consider Figure]



Percentage error in product or quotient of quantities equals sum of percentage errors.

P = EI

P = (E ± ΔE) × (I ± ΔI)

% error in P = (E * I) ± [(% error in E) + (% error in I]

Quotients of quantities:

Here again it can be shown that the percentage error is the sum of the percentage errors in each quantity.

R = (E ± ΔE)/ (I ± ΔI)

% error in R= (E/I) ± [(% error in E) + (% error in I)]

Raised to a power of quantity:

When a quantity A is raised to a power B. the percentage error in A B can be shown to be

% error A^B = B (% error in A)

AMMETERS

Introduction:

Ammeter is measuring instrument to measure current in circuit. It uses PMMC galvanometer as a basic meter. As the name suggests it has permanent magnets which are employed in this kind of measuring instruments. It is particularly suited for DC measurement because here deflection is proportional to the current. This type of instrument is called D' Arnsonval type instrument. It has major advantage of having linear scale, low power consumption, high accuracy.

An ammeter can measure a wide range of current values because at high values only a small portion of the current is directed through the meter mechanism, a shunt in parallel with the basic meter carries the major portion as shown in Fig 1. The value of shunt can be determined as follows:

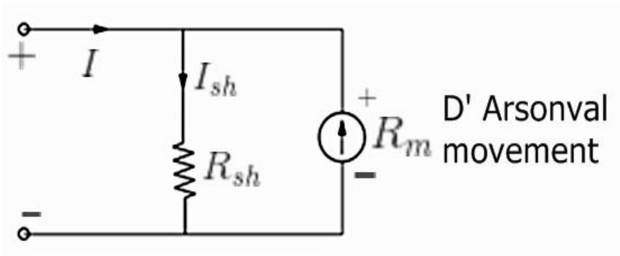


Fig. 1 Basic Ammeter

Rsh = resistance of the shunt
Rm = internal resistance of the meter movements (movable coil)
Ish = shunt current
Im = full scale deflection current of the meter movement
I = full-scale deflection current for the ammeter

As shunt is parallel with the basic meter, the drop across shunt and basic meter will be same and it is given by,
Vm = Im*Rm and Vsh = Ish*Rsh
Vsh = Vm
IshRsh = ImRm

$$\text{Rsh} = \frac{\text{ImRm}}{\text{Ish}} \quad (\Omega)$$

But I = Ish + Im
Thus Ish = I – Im

Therefore, $$\text{Rsh} = \frac{\text{ImRm}}{(\text{I} - \text{Im})} \quad (\Omega)$$

This determines the value of shunt resistance for full scale meter current.

Multirange Ammeter:

- The range of the basic d.c. ammeter can be extended by using number of shunts and a selector switch. Such ammeter is called multirange ammeter as shown in the Fig 2
- R1, R2, R3 and R4 are four shunts. When connected in parallel with the meter, they can give four different ranges I1 ,I2 , I3 and I4.

- The selector switch S is multiposition switch, having low contact resistance and high current carrying capacity.
- This uses a make before break type switch for the range changing.
- If the ordinary switch is used, while range changing the switch remains open and full current passes through the meter damaging the meter due to high current. So make before break switch is used.
- While using the multirange ammeter, highest range should be used first and the current range should be decreased till good upscale reading is obtained.
- All the shunts are very precise resistance and hence cost of such multirange ammeter is high.

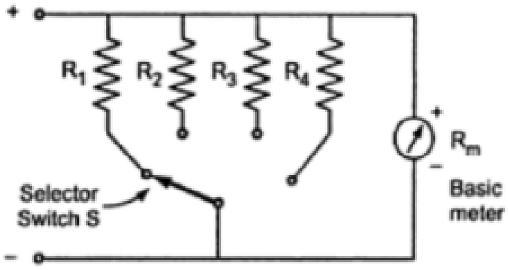


Fig 2. Multirange Ammeter

The Ayrton Shunt or Universal Shunt:

The Ayrton shunt or universal shunt is another configuration of ammeter which eliminates the possibility of having a meter without a shunt. The meter with Ayrton shunt is shown in the Fig. 3.

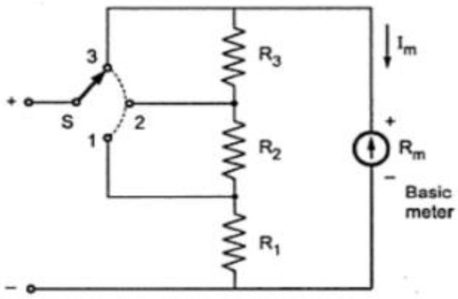


Fig. 3 Ayrton shunt or Universal shunt

- The selector switch S, selects the appropriate shunt required to change the range of the meter.
- When the position of the switch is '1' then the resistance R1 is in parallel with the series combination of R2, R3 and Rm. Hence current through the shunt is more than the current through the meter, thus protecting the basic meter.
- The voltage drop across the two parallel branches is always equal.

  Thus, Ish Rsh = Im Rm ,

  In position 1, R1 is in parallel with R2 + R3 + Rm

  Thus I1[R1] = Im[R2+R3+Rm]
- When the switch is in the position '2', then the series resistance of R1 and R2 is in parallel with the series combination of R3 and Rm.

  In position 2, R1+R2 is in parallel with R3+Rm

Thus I2[R1+R2] = Im[R3+Rm]

- In the position '3', the resistances R1, R2 and R3 are in series and acts as the shunt. In this position, the maximum current flows through the meter. This increases the sensitivity of the meter.

  In position 3, R1 + R2 + R3 is in parallel with Rm

  I3[R1+R2+R3] = ImRm

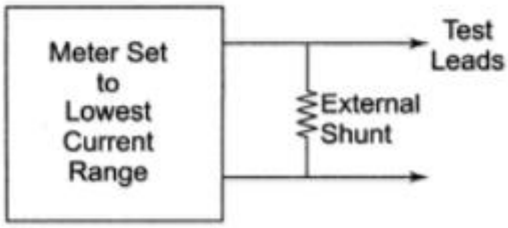
Requirements of Shunts:

- The electrical resistance of these shunts should not differ at higher temperature,
- They should have very low value of temperature coefficient.
- The resistance should not vary with time.
- They should be able to carry high value of current without much rise in temperature.
- The material used to join the shunts should have low thermo dielectric voltage drop i.e soldering of joins should not cause voltage drop.
- Solderability: The shunt resistances can be of different values and size and while soldering the change in value should be minimum

Usually ‘manganin’ is used as shunt for DC instruments as it gives low thermal emf and ‘constantan’ is useful material for AC instruments.

Extending of Ammeters:

The range of ammeter can be extended by using external shunts connected to the basic meter movement as shown in the figure below.



Extending of ammeters

RF Ammeter (Thermocouple)

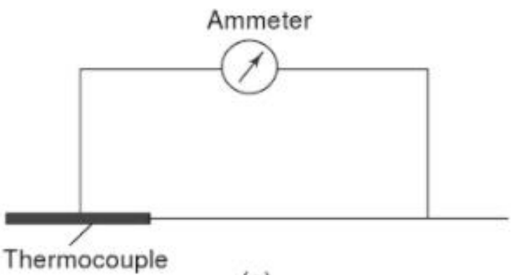
- Basically thermocouple consists of two different metals which are placed in contact with each other.
- First part is called the heater element because when the current will flow through this, a heat is produced and the temperature is increased at the junction.
- At this junction an emf is produced, the emf produced is a DC voltage which is directly proportional to root mean square value of electric current or voltage proportional to heating effect. This DC voltage generation by heating effect is called as thermoelectric effect.
- A permanent magnet moving coil instrument is connected with the second part to read the current passing through the heater.

- Usually a permanent magnet coil instrument is used because it has greater accuracy and sensitivity towards the measurement of value.
- The thermocouple type instruments employ thermocouple in their construction and have greater accuracy in measuring the current and voltages at very high frequency accurately. Thermocouple type instruments can be used for both ac and dc applications.

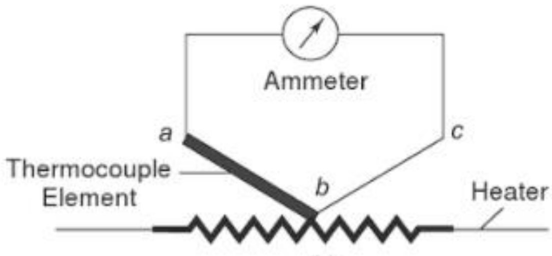
Types of Thermocouples:

Mutual Type: In this type, the current is passed directly to the thermocouple itself and through any heater wire as shown in fig (a). But the problem seen is the meter shunts the thermocouple and may not be very accurate. The sensitivity of this is very high.

Contact Type: In this it has a separate heater which is shown in fig (b). The current to be measured is passed to the heater and not to the thermocouple. Thus it is less sensitive compared to mutual type.



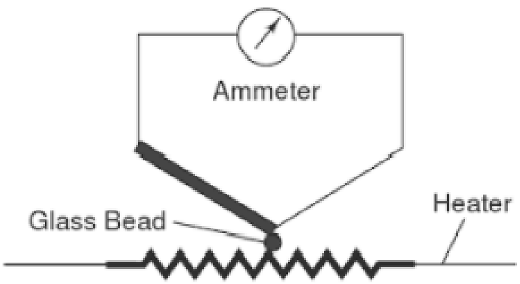
(a)



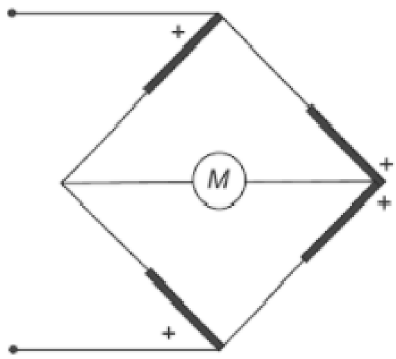
(b)

Separate Heater Type: It is also called as a non-contact type. There is insulation between the heating element and the thermocouple i.e. there no direct contact between two. The thermocouple is held near heater but insulated using a glass bead. This is shown in fig (c). Due to this the instruments are not much sensitive as compared contact type and also sluggish. The separate type is useful for certain applications.

Bridge type: This has high sensitivity as that of mutual type and also eliminates the shunting effect. This is seen in bridge configuration as shown in fig (d).



(c)



(d)

In the bridge configuration all 4 arms have similar thermocouple and to increase the sensitivity the instrument is placed in vacuum.

- Materials (metal combinations) used commonly for thermocouple are copper-constantan, iron-constantan, chromel-constantan, chromel-alumel and platinum-rhodium
- The heating element usually for open air heaters is a platinum alloy, which is non-corroding and in vacuum type heaters carbon filament is used.

Limitations of Thermocouple:

- Thermocouple heaters can withstand only small overloads.
- With rise in temperature there is change in resistance of the heater.
- There are harmonics present which changes the meter readings due to heating effect.

Advantages of Thermocouple:

- Accurate r.m.s value of current or voltage can be measured.
- Have very high sensitivity.
- Not affected by stay magnetic fields.
- In comparison with other instruments have high accuracy and frequency range

VOLTMETERS AND MULTIMETERS

Voltmeter is used measure potential difference between two points of an electric circuit. The analog voltmeters gives indication by moving pointer across the scale proportional to the voltage in the circuit.

With basic meter and by adding various elements different instruments can be formed.

I. Basic meter movement can be made D.C instrument to measure

(i) DC current: adding a shunt resistance it results in forming a microammeter, miliammeter or an ammeter.

(ii) DC voltage: adding series resistance called multiplier it results in forming a milivoltmeter, voltmeter or kilovoltmeter.

(iii) Resistance: with a battery and resistive network, resistance can be measured. The instrument is ohmmeter.

II. Basic meter movement can be made A.C instruments to measure

(i) AC voltage or current: with a rectifier circuit it forms a rectifier meter which measures power and audio frequencies.

(ii)RF voltage or current: Using a thermocouple type meter radio frequency (RF) voltage or current can be measured.

(iii)Expanded scale for power line voltage: Using a thermistor in a resistive bridge network, expanded scale for power line voltage can be obtained. This can be used for power line monitoring.

Basic meter as dc voltmeter:

The basic d.c voltmeter is nothing but a PMMC D' Arsonval movement meter. To this a resistance is required to be connected in series to use it as a voltmeter. This series resistance is called a multiplier. The multiplier resistance limits the current through the basic meter so that the meter current does not exceed the full scale deflection value. The voltmeter measures the voltage across the two points of a circuit or a voltage across circuit component. The basic d.c. voltmeter is shown in the Fig.3.1

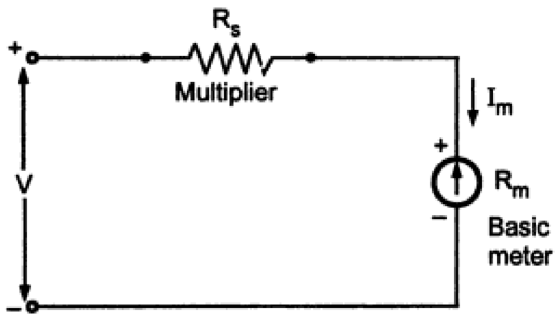


Fig. 3.1. Basic d.c voltmeter

The voltmeter must be connected across the two points or a component/load, to measure the potential difference, with the proper polarity.

The multiplier resistance can be calculated as:

Let Rm = Internal resistance of coil i.e. meter

Rs = series multiplier resistance

Im = full scale deflection current (can also be represented as I_{fsd})

V = full range voltage to be measured

From the Fig. 3.1 using KVL,

$$V = Im\ (Rm + Rs)$$

$$V = ImRm + Im\ Rs$$

$$ImRs = V - Im\ Rm$$

$$\text{Thus } Rs = \frac{V - ImRm}{Im}$$

or $$Rs = \frac{V}{Im} - Rm$$

Multirange Voltmeter:

As we have seen in multirange ammeter, the range of the basic d.c. voltmeter can also be extended by using number of multipliers and a selector switch. Such type of meter is called multirange voltmeter. Fig. 3.2 shows multirange voltmeters with 3 multipliers R1, R2 and R3

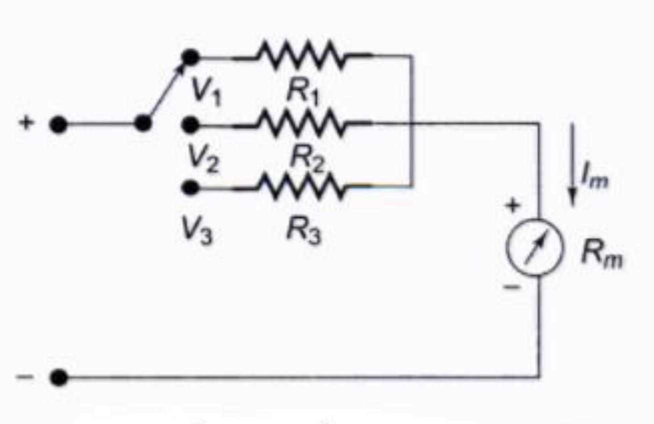


Fig. 3.2 Multirange voltmeter

This can further be modified which gives a more practical multiplier arrangement in multirange voltmeter. The arrangement is shown in Fig 3.3. The multipliers R1, R2, R3 and R4 are connected in series along with the selector switch.

This configuration is advantageous as all resistors expect R4 are all standard resistor values.

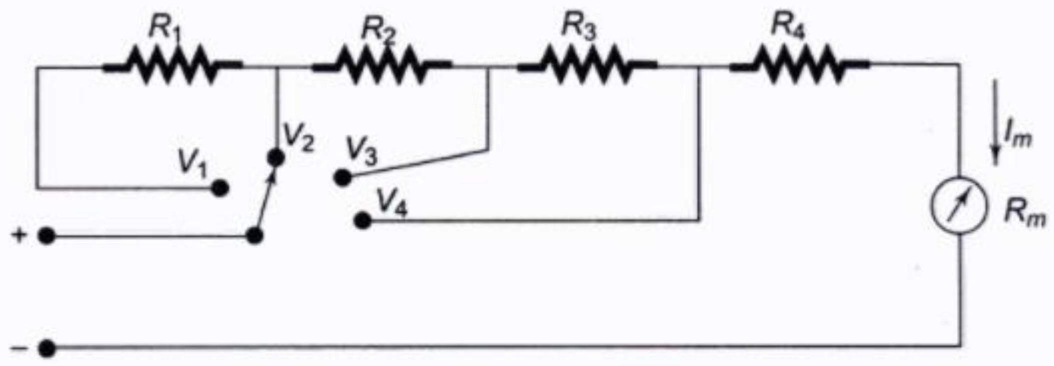


Fig. 3.3 Multipliers connected in series string

Extending voltmeter ranges:

The range of voltmeters can be extended to measure high voltages using an external multiplier resistor as shown in Fig 3.4. The basic meter can be used to measure low voltages and care must be taken to see that the voltage does not exceed the full scale deflection.

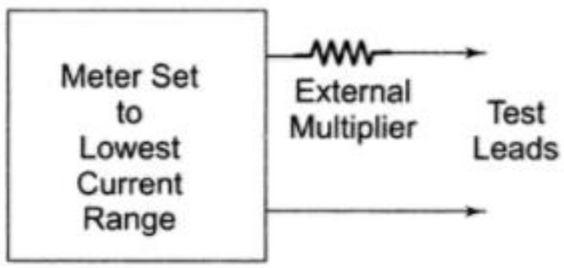


Fig.3.4 Extending voltage range

Sensitivity:

The sensitivity of a voltmeter is given in ohms per volt. It is determined by dividing the sum of the resistance of the meter (Rm), plus the series resistance (Rs), by the full-scale reading in volts. In other words sensitivity can be defined as the ratio of total resistance to voltage to be measured (i.e voltage range). In equation form, sensitivity is expressed as follows:

Sensitivity = (Rm+Rs)/E = R/V

This is the same as saying the sensitivity is equal to the reciprocal of the full-scale deflection current.

$$\text{Sensitivity} = \frac{1}{I_{fsd}}$$

Loading effect:

- While selecting the voltmeter, the voltmeter consideration of sensitivity is very important.
- A low resistance voltmeter may give correct reading when measuring voltage in low resistance circuit but the Voltmeter produces unreliable and erroneous reading when connected in high resistance circuit.
- This is because the resistant of the meter acts as shunt and the equivalent resistance at that portion reduces.
- This results in showing lower reading indication than the actual value that existed before connecting of the meter. This is calling as loading effect.
- Thus ideally the resistance of a Voltmeter should be infinite so that voltmeter does not alter circuit current and gives correct readings.

Transistor voltmeter (TVM):

- Figure 3.5 gives a simplified schematic diagram of a dc coupled amplifier with an indicating meter.

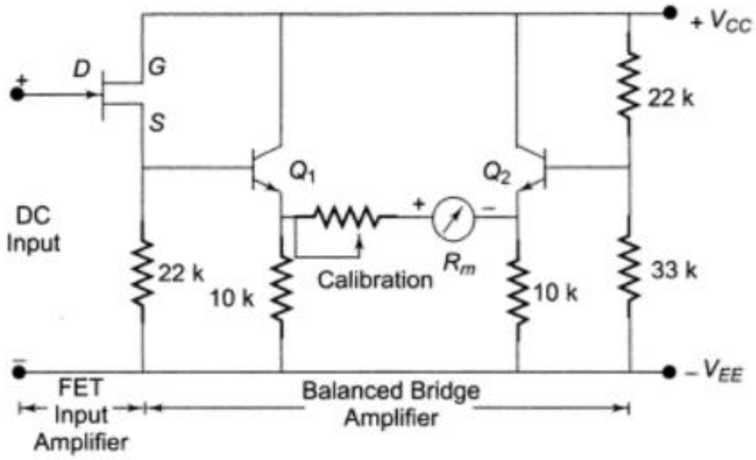


Fig. 3.5 Transistor voltmeter

- The input stage consists of a FET which provides high input impedance to effectively isolate the meter circuit from the circuit under measurement. This forms the input amplifier. The input impedance of a FET is greater than 10 MΩ.
- It has two transistors, Q1 and Q2 forms a dc coupled amplifier driving the meter movement, along with resistors forms the bridge. The bridge is balanced, so that for zero input the dial indicates zero. If not, balance can be obtained through calibration resistance.
- Within the dynamic range of the amplifier, the meter deflection is proportional to the magnitude of the applied input voltage.
- The input exceeds then it does not burn the meter because the amplifier saturates, limiting the maximum current through the meter.
- The gain of the dc amplifier allows the instrument to be used for measurement of voltages in the mV range.

- Instruments in the μV range of measurement require a high gain dc amplifier to supply sufficient current for driving the meter movement.

<u>Differential Voltmeters:</u>

- The differential voltmeter provides extremely accurate voltage measurements and it is highly reliable piece of precision test equipment. The function is to compare an unknown voltage with a known internal reference voltage and to indicate the difference in their values.
- Figure 3.6 shows a basic circuit of a basic differential voltmeter which is based on the potentiometric method. Hence it is also called a potentiometric voltmeter.

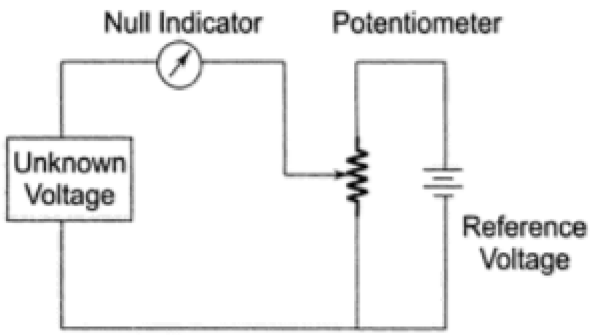


Fig. 3.6 Basic Differential Voltmeter

- In this, upon the application of unknown voltage the potentiometer is varied until the voltage across it equals the unknown voltage.
- At this point the null indicator reads zero. Under null conditions, potential across either side of potentiometer is same and the meter draws current from neither the reference source nor the unknown known voltage source
- This shows that unknown voltage equals to the reference voltage.
- Thus the differential voltmeter presents an infinite impedance to the unknown source.
- The null meter serves as an indicator and does not measure any voltages.
- To detect small differences the meter movement must be sensitive, but it need not be calibrated, since only zero has to be indicated.
- The reference source used is usually a 1 V dc standard source or a Zener controlled precision supply. For measuring high voltages a high voltage reference supply can be used but this increases the cost and also loading effect is seen.
- Alternate to this voltage dividers or attenuators across an unknown source can be used to reduce the voltage. But even this has low input impedance and loading effect respectively.

In order to measure ac voltages, the ac voltage must be converted into dc by incorporating a precision rectifier circuit. A block diagram of an ac differential voltmeter is shown in Fig. 3.7.

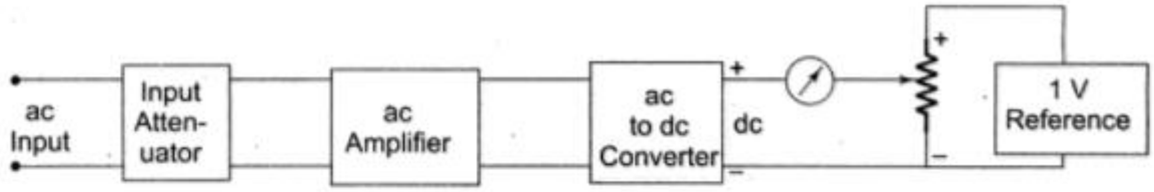


Fig. 3.7 Block diagram of an ac differential voltmeter

AC Voltmeters using Rectifiers:

- The PMMC movement along with rectifier arrangement is used in rectifier type ac instruments. The rectifier is used to convert a.c voltage to be measured, to d.c.
- This d.c if required is amplified and then given to the PMMC movement.
- The PMMC movement gives the deflection proportional to the quantity to be measured. For this silicon diodes are preferred as they exhibit low reverse current and high forward current ratting.
- Fig 3.8 (a) shows ac voltmeter having a multiplier, a bridge rectifier and basic meter movement.

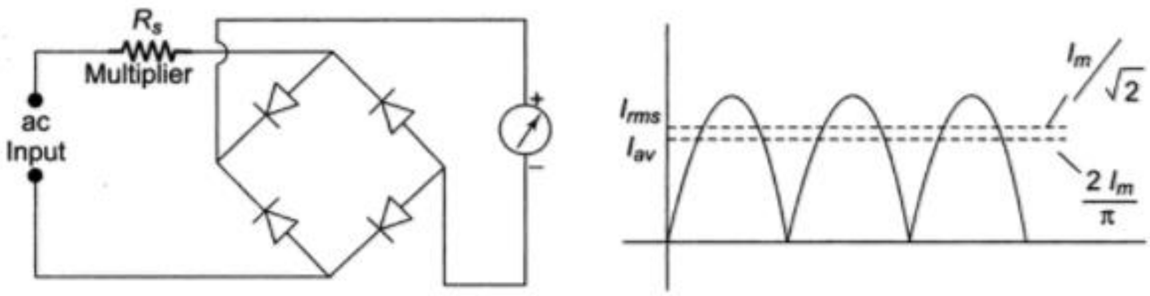


Fig. 3.8 (a) ac voltmeter (b) Average and RMS value of current

- Bridge rectifier gives a full wave pulsating dc and meter indicates steady deflection proportional to the average value of the current as shown in Fig 3.8 (b). However the meter can be calibrated to give rms value of the input signal.

rms value and average value: The rms. value of an alternating quantity is given by that steady current (d.c.) which when flowing through a given circuit for a given time produces the same amount of heat as produced by the a.c current which when flowing through the same circuit for the same time.

The rms value is calculated by measuring the quantity at equal intervals for one complete cycle. Then squaring each quantity, the average of squared values is obtained. The square root of this average value is the rms. value. The rms means root-mean square i.e. squaring, finding the mean i.e. average and taking the root.

For continuous signal the rms value is obtained by integrating the signal over the period of time T. It is given by,

Vrms = $\sqrt{\frac{1}{T}\int_0^T Vin^2\,dt}$ and 1/T represents the average value.

For pure sinusoidal signal it is given by

$$V_{rms} = 0.707\ V_m,$$

where Vm = peak value of the sine wave.

Similarly the average value of a continuous a.c signal can be calculated by taking the average value over half period of the signal. It is given by

$$V_{av} = \frac{2}{T}\int_0^{T/2} Vin\, dt$$

T/2 represents the average value over half cycle.

For pure sinusoidal signal it is given by

$$V_{av} = \frac{2}{\pi} Vm = 0.636\ Vm$$

Where Vm = peak value of the sine wave.

General rectifier type ac voltmeter:

Practical rectifiers are non-linear devices particularly at low forward current and hence the meter scale is non-linear at lower values. This can be observed in the diode characteristics shown in Fig 3.9

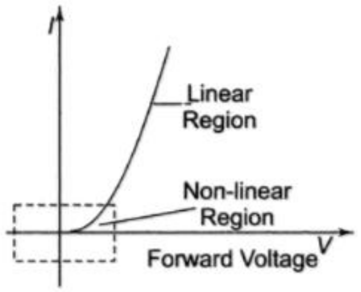


Fig. 3.9 Diode characteristics (Forward)

A general rectifier type voltmeter is shown in Fig. 3.10

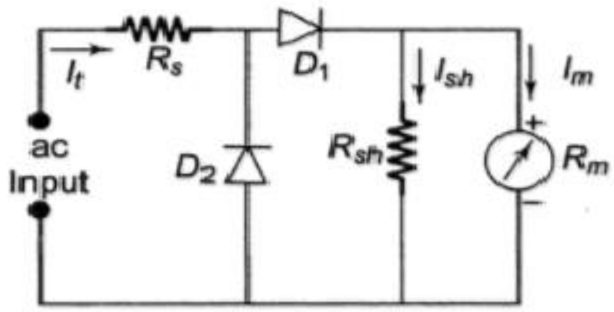


Fig. 3.10 General rectifier type ac voltmeter

- Two diodes D1 and D2 are used in the rectifier circuit. When the a.c. input is applied, for the positive half cycle, the diode D1 conducts and causes the meter deflection proportional to the average value of that half cycle.
- As the diodes exhibit nonlinear behavior for the low currents and to increase the current through diode D1, the meter is shunted with a resistance Rsh. This helps in moving the diode operation into linear region of the characteristic curve.
- In the negative cycle, the diode D2 conducts and D1 is reverse biased. The current through the meter is in opposite direction and hence meter movement is bypassed.
- Thus due to diodes, the rectifying action produces pulsating d.c. and the meter indicates the average value of the input.

AC voltmeter using half wave rectifier:

To the ac voltmeter if a diode D1 is added as shown in Fig. 3.11, we get an half wave rectifier circuit capable of measuring ac voltages.

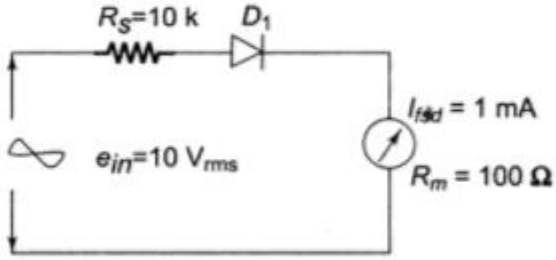


Fig. 3.11 ac voltmeter using half wave rectifier

Considering an example of the basic meter having full scale deflection current of 1mA and assuming D1 to be an ideal diode with negligible forward bias resistance, if the sensitivity of the dc voltmeter is given by

Sdc = $1/I_{fsd}$ = 1/1mA = 1KΩ/V

For this if dc input is replaced by a 10 V rms sine wave input. The voltages appearing at the output is due to the +ve half cycle due to rectifying action. The peak value of 10 V rms sine wave is given by,

Ep = 0.707 × $\sqrt{2}$ × Vrms = 0.707 × $\sqrt{2}$ × 10 = 14.41V

The dc will respond to the average value of the ac input, therefore

Eav = 0.636 Ep = 0.636 ×14.41 = 8.99V = 9 V

Since the diode conducts only during the positive half cycle, the average value over the entire cycle is one half the average value of 8.99 V, i.e. about 4.5 V.

Thus, the pointer will deflect for a full scale if 10 V dc is applied and 4.5 V when a 10 Vrms sinusoidal signal is applied. This indicates that an a.c voltmeter is not as sensitive as a dc voltmeter.

Thus we can say that **Edc = 0.45 Eac**

With rectifier in the voltmeter, the multiplier resistance can be calculated as

$$Rs = \frac{Edc}{Idc} - Rm \text{ or } Rs = = \frac{0.45Eac}{Idc} - Rm$$

AC voltmeter using Full Wave Rectifier:

The full wave rectifier circuit uses a bridge to convert a.c to d.c as shown in the Fig. 3.12. During both half cycles the diodes will be conducting.

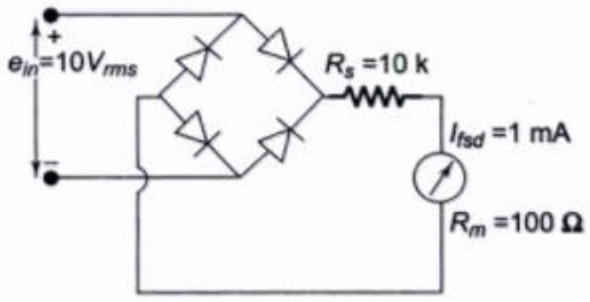


Fig. 3.12 ac voltmeter using full wave rectifier

To this now 10 V rms signal is applied then the peak value is given by,

Ep = 0.707 × $\sqrt{2}$ × Vrms = 0.707 × $\sqrt{2}$ × 10 = 14.41V

The average value is given by,

Eav = 0.636 Ep = 0.636 ×14.41 = 8.99V = 9 V

As the diode conducts for both the half cycle the average value over one entire cycle is 9 V only.

Therefore, we can see that a 10 V rms voltage is equal to a 9 V dc for full scale deflection, i.e. the pointer will deflect to 90% of full scale. Thus we can say that,

Edc = 0.9 Eac

With full wave rectifier in the voltmeter, the multiplier resistance can be calculated as

$$Rs = \frac{Edc}{Idc} - Rm \text{ or } Rs = = \frac{0.9\, Eac}{Idc} - Rm$$

- With sensitivity we can have for both half wave and full wave as

Sensitivity (ac) = 0.45 Sensitivity (dc) -----Half wave rectifier

Sensitivity (ac) = 0.9 Sensitivity (dc) -----Full wave rectifier

True RMS Voltmeter:

- RMS value of the sinusoidal waveform can be measured by the average reading voltmeter and if can be calibrated to read the rms value.
- This method is quite simple and less expensive. But sometimes rms value of the non-sinusoidal or complex waveform may be required to be measured. For such a measurement a true rms reading voltmeter is required.
- True rms reading voltmeter gives meter reading based on heating power of waveform which is proportional to the square of the rms value of the voltage.

- Thermocouple is used to measure the heating power of the input waveform and it is given to the heater by the amplified version of the input waveform.
- Output voltage of the thermocouple is proportional to the square of the rms value of the input waveform.
- One more thermocouple, called the balancing thermo-couple, is used in the same thermal environment in order to eliminate the difficulty arising due to non-linear behavior of the thermo-couple.
- Thus the non-linearity of the input circuit thermo-couple is cancelled by the similar non-linear effects of the balancing thermocouple.
- These thermocouples form part of a bridge in the circuit of a dc amplifier, as shown in block diagram in Fig.3.13.

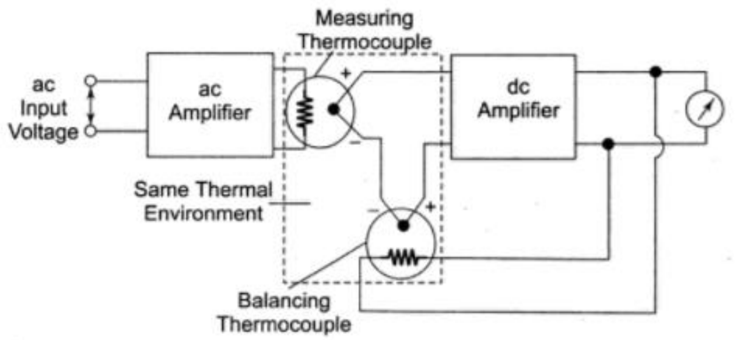


Fig.3.13 True RMS voltmeter (Block diagram)

- AC waveform to be measured is applied to the heating element of the measuring thermo-couple through an a.c amplifier. Under the absence of input waveform, output of both thermo-couples are equal, therefore the input to dc amplifier is zero indicating meter connected to the output of dc amplifier reads zero.
- But on the application of input waveform, output of measuring thermo-couple upsets the balance and an error signal is produced, which gets amplified by the dc amplifier and is fedback to the heating element of the balancing thermo-couple.
- This feedback current reduces the value of error signal and ultimately makes it zero to obtain the balanced bridge condition.
- In this balanced condition, feedback current supplied by the dc amplifier to the heating element of the balance thermo-couple is equal to the ac current flowing in the heating element of main thermo-couple.
- Hence this direct current is directly proportional to the rms value of the input ac voltage and is indicated by the meter connected in the output of the dc amplifier. The PMMC meter may be calibrated to read the rms voltage directly.

Considerations while choosing an analog voltmeter:

Input Impedance: The input impedance or resistance of the voltmeter should be as high as possible so as to avoid the loading effect. Input impedance should always be higher than the impedance of the circuit under measurement.

Voltage Ranges: The voltage ranges on the meter scale should have same dB separation (may be in a 1-3-10 sequence with 10 dB or a 1.5-5-15 sequence) or in a single scale calibrated in decibels. In any case, the scale division should be compatible with the accuracy of the instrument.

Decibels: For measurements covering a wide range of voltages, the use of the decibel scale can be very effective, e.g., in the frequency response curve of an amplifier, where the output voltage is measured as a function of the frequency of the applied input voltage.

Sensitivity v/s Bandwidth: Noise consists of unwanted frequencies. Since noise is a function of the bandwidth, a voltmeter with a narrow bandwidth picks up less noise than a large bandwidth voltmeter. Lesser the noise higher is the sensitivity of the meter.

Battery Operation: A voltmeter (VTVM) powered by an internal battery is essential for field work.

To summarize, the general guidelines are as follows:

- For dc measurement, select the meter with the widest capability meeting the requirements of the circuit.
- For ac measurements involving sine waves with less than 10% distortion, the average responding voltmeter is most sensitive and provides the best
- For high frequency measurement (> 10 MHz), the peak responding voltmeter with a diode probe input is best. Peak responding circuits are acceptable if inaccuracies caused by distortion in the input waveform are allowed (tolerated).
- For measurements where it is important to find the effective power of waveforms that depart from the true sinusoidal form, the rms responding voltmeter is the appropriate choice.

Multimeter:

A multimeter has ammeter, voltmeter and ohmmeter together with a function switch to connect appropriate circuit to Basic meter or D'Arsonval movement. It is also known as Voltage-Ohm Meter (VOM) or multimeter.

Multimeter as a voltmeter:

To get different ranges of voltages, different multiplier resistances are connected in series (as this configuration is more practical than the parallel configuration of multiplier resistance) which can be put in the circuit with the range selector switch. We can get different ranges to measure the d.c. voltages by selecting the proper resistance in series with the basic meter. This is shown in the fig 3.14. To measure a.c voltages rectifiers are included in the circuit.

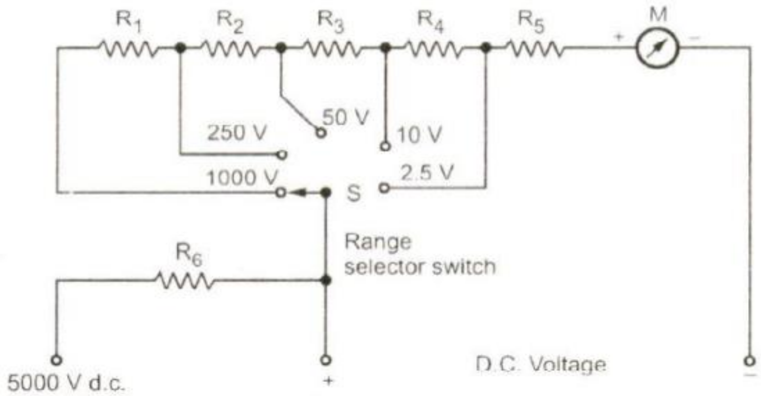


Fig. 3.14. Multirange voltmeter within multimeter

Multimeter as an ammeter:

To get different current ranges for ammeter, different shunts are connected across the meter with the help of range selector switch. This is shown in fig 3.15.

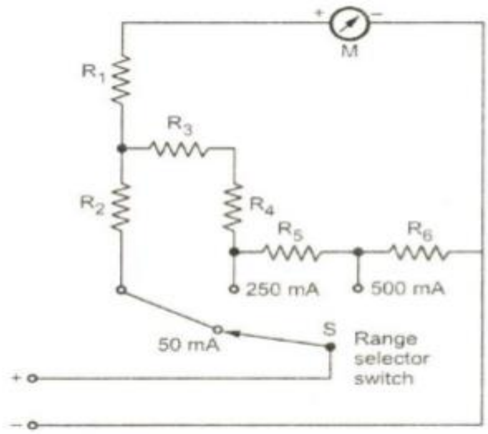


Fig. 3.15 Multirange ammeter within Multimeter

Multimeter as ohmmeter: As mentioned earlier with a battery and resistive network, resistance can be measured.

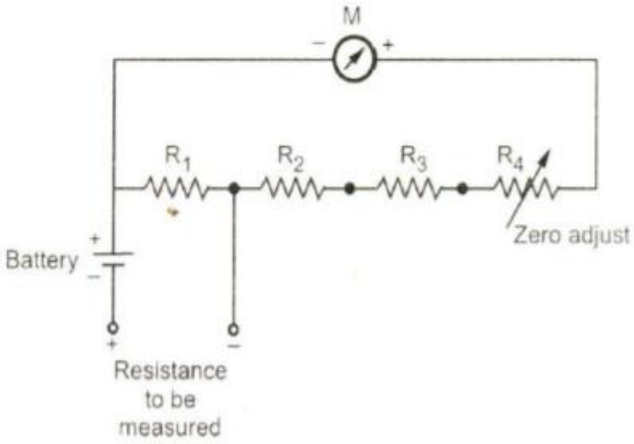


Fig. 3.16. ohmmeter within multimeter

The Fig.3.16 shows ohmmeter section of multimeter. Before any measurement is made, the instrument is to be calibrated for zero adjustments. This is done by short circuiting the instrument and "zero adjust" control is varied until the meter reads zero resistance i.e. it shows full scale current. With resistor network the circuit takes the form of a variation of the shunt type ohmmeter. Scale multiplications of 100 and 10,000 can also be used for measuring high resistances. Voltage is supplied to the circuit with the help of battery.

Measurement and Errors.

Problem related to absolute and relative error.

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Problem: The expected value of the voltage across the resistor is 80V. However the measurement gives value of 79V. Calculate. (1.) Absolute error (2) % error (3) relative accuracy (4) % of accuracy

→ Given $A_a = 80V$
$A_m = 79V$.

(1) Absolute error ⇒ $e = A_a - A_m$.
$e = 80V - 79V$ $\boxed{e = 1V}$

(2) % error $= \dfrac{\text{Absolute error}}{A_a} \times 100$.

$= \dfrac{1}{80} \times 100 = 1.25\%$.

$\boxed{\%\ \text{error} = 1.25\%}$

(3) relative accuracy.

$A = 1 - \left|\dfrac{e}{A_a}\right| = 1 - \dfrac{1}{80}$

$\boxed{A = 0.9875}$

(4) % of accuracy $= A \times 100\%$.

$\boxed{a = 98.75\%}$

Ex: The expected value of the current thr' a resistor is 20mA. However the measurement yields current value of 18mA. Calculate (1) Absolute error (2) % error (3) relative accuracy (iv) % accuracy.

→ $e = A_a - A_m$
$= 20mA - 18mA$

$e = 2mA$

$\%\ \text{error} = \dfrac{e}{A_a} \times 100 = \dfrac{2mA}{20mA} \times 100 = 10\%$

relative accuracy $= 1 - \dfrac{e}{A_a} = 1 - \dfrac{2}{20} = 0.9$

% Accuracy = 90%

Ex:- Manufacturer constructs resistance b/w 1.14kΩ & 1.26kΩ & classifies them to be 1.2kΩ. what tolerance should be stated? If the resistance values are specified at 25°C & resistor have temp coefficient of +500ppm/°c. Calculate max resistance that one of these components might have at 75°C.

given & the change will be /°c

→ Absolute error = $1.26k\Omega - 1.2k\Omega$

$= 1.14k\Omega - 1.2k\Omega$

$= \pm 0.06 k\Omega$.

Given : True value of resistor = $1.2k\Omega$

Tolerance $= \frac{0.06k\Omega}{1.2k\Omega} \times 100 = \pm 5\%$.

The largest possible resistance at 25°c

$R = 1.2k\Omega + 0.06k\Omega$

$= 1.26k\Omega$

least possible resistance at 25°c is

$R = 1.2k\Omega - 0.06k\Omega$

$R = 1.14k\Omega$.

Resistance change /°c

500 ppm of R $= \frac{R}{1000000} \times x\,ppm$

$R = 1.2k\Omega$

$x\,ppm = 500ppm$

$= \frac{1.2k\Omega \times 500}{1000000}$

$= 0.63\ \Omega/°c$.

Temperature increase $= \Delta T = 75°C - 25°C$

$= 50°C$.

Total resistance increase $= \Delta R$ = Resistance change /°c × °c

= Resistance change

$= 0.63\Omega/°c \times 50°c$

$\boxed{\Delta R = 31.5\ \Omega}$

Ex:- An ammeter reads 6.7A & the true value of current is 6.54A. Find the absolute error & the correction for this instrument.

→ Measured value = 6.7A

True value = 6.54V

Absolute error = 6.7 − 6.54 = 0.16A.

correction for the instrument is $\underline{-0.16A}$ since the measu value is 0.16 A excess of the true value.

Ex:- Current thr' resistor is 2.5A, but measurement yields value of 2.45A, find the % error of measurement.

→ True value = 2.5A

Measured value = 2.45A

Absolute error = 2.45 − 2.5

= 0.05A.

$\%$ error = Relative error $= \pm \frac{0.05}{2.5} \times 100$

$= \pm 2\%$

Problem related to combination of errors.

Ex:- If 2 capacitors $100 \pm 1.4\mu F$ & $80 \pm 1.5\mu F$ are connected in parallel. Determine the error of resultant capacitance in μF & in %

→ $C_1 = 100 \pm 1.4\mu F$ $\quad C_2 = 80 \pm 1.5\mu F$

$C_T = C_1 + C_2$

$= (100 \pm 1.4\mu F) + (80 \pm 1.5\mu F)$

$= 180\mu F \pm 2.9\mu F$

error in % is given by

$\%$ error $= \frac{2.9\mu F}{180\mu F} \times 100$

$\boxed{\%\ \text{error} = \pm 1.61\%}$

Ammeter

Problem related to DC Ammeter.

Ex:- A 2mA meter movement with internal resistance of 100 Ω is to be converted to a 0 - 200mA. Determine the value of shunt resistance required.

→ Given $R_m = 100\Omega$, $I_m = 2mA$ $I = 200mA$, $R_{sh} = ?$

$$R_{sh} = \frac{I_m R_m}{I - I_m} = \frac{2mA\,(100)}{200mA - 2mA} = \underline{1.01\,\Omega}.$$

Problem related to Multi Range DC Ammeter.

Ex:- A 1mA meter movement having an internal resistance of 100Ω is used to convert toan multirange ammeter having the 0-10mA, 0-20mA & 0-50mA. Find the value of shunt resistance

→ $R_m = 100\Omega$

$I_m = 1mA$

For 0-10mA range $I = 10mA$

$$R_{sh} = \frac{I_m R_m}{I - I_m}$$

$$= \frac{1mA \times 100\Omega}{(10mA - 1m)} = \frac{100\,m\Omega}{9mA} = 11.11\Omega$$

IIIly for 0-20mA range i.e. $I = 20mA$

$$R_{sh} = \frac{1mA \times 100\Omega}{20mA - 1mA} = \frac{100mA}{19mA} = 5.2\ \Omega.$$

IIIly for 0-50mA range $I = 50mA$

$$R_{sh} = \frac{1mA \times 100\Omega}{50mA - 1mA} = 2.041\Omega$$

Ex: Design multirange ammeter with range 0-1A, 5A & 10A respectively employing individual shunt in each A D'Arsonval movement with an internal resistance of 500Ω & a full scale deflection of 10mA is available

→ $I_m = 10mA$, $R_m = 500\Omega$

Case 1 : Range 0-1A ; $R_{sh} = \frac{I_m R_m}{I - I_m} = \frac{10mA \times 500\Omega}{1A - 10mA} = 5.05\Omega.$

se 2 : Range 0-5A $R_{sh} = 1.002\,\Omega$

ase 3 : Range 0-10A $R_{sh} = \cancel{0.050\Omega}\ 0.50\,\Omega$.

Hence the values of shunt resistances are $5.05\,\Omega$, $1.002\,\Omega$ & $0.50\,\Omega$

Ex:- A 100 μA meter movement with an internal resistance of 500 Ω is to be used in a 0-100mA Ammeter. Find the values of the shunt required.

→ ✱ The shunt can also be determined by considering current I to be 'n' times larger than I_m. This is called a multiplying factor & relates total current & meter current.

$$I = n I_m$$

Thus the equation for shunt $R_s = \dfrac{I_m R_m}{I - I_m} = \dfrac{I_m R_m}{n I_m - I_m} = \dfrac{I_m R_m}{I_m(n-1)}$

$$R_{sh} = \frac{R_m}{n-1}$$

Given parameters $I_m = 100\,\mu A$, $R_m = 500\,\Omega$, $I = 100\,mA$

$$\text{now } n = \frac{I}{I_m} = \frac{100\,mA}{100\,\mu A} = 1000$$

$$R_{sh} = \frac{500}{1000-1} = 0.5\,\Omega \qquad \boxed{R_{sh} = 0.5\,\Omega}$$

OR

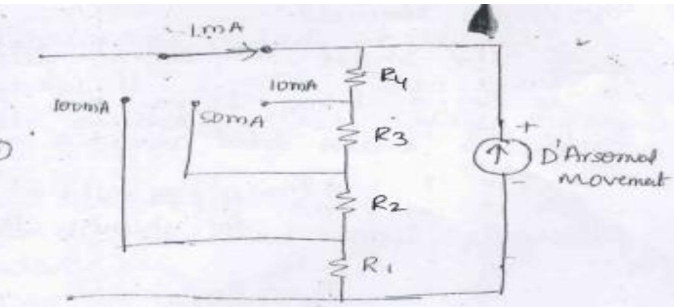
$$R_{sh} = \frac{I_m R_m}{I - I_m} = \frac{100\,\mu A \times 500\,\Omega}{100\,mA - 100\,\mu A} = 0.50\,\Omega \qquad \boxed{R_{sh} = 0.5\,\Omega}$$

Thus both gives same value.

Ex:- Design an Asyton shunt to provide an ammeter with current range of 0-1mA, 10mA, 50mA & 100mA. A D'Arsonval movement with an internal resistance of 100Ω & full scale current of 50μA is used.

→ Given $I_m = 50\mu A$, $R_m = 100\Omega$.

Case 1:
For 0-1mA range.



$I_{sh} R_{sh} = I_m R_m$.

$R_{sh} = \frac{I_m R_m}{I_{sh}} = \frac{I_m R_m}{I - I_m}$ — ①

$R_{sh} = R_4 + R_3 + R_2 + R_1$

$I = 1mA$

Now ① becomes

$R_4 + R_3 + R_2 + R_1 = \frac{50\mu A \times 100}{1mA - 50\mu A}$

$R_1 + R_2 + R_3 + R_4 = 5.26\ \Omega$ — (A)

Case 2:
For 0-10mA range.

$R_3 + R_2 + R_1 = \frac{50\mu A \times (100 + R_4)}{10mA - 50\mu A}$ ∵ R_4 is in series with R_m.

$9950\mu A\,(R_1 + R_2 + R_3) = 50\mu A\,(100 + R_4)$ — (B)

case 3: For 0-50mA range.

$I_{sh} R_{sh} = I_m R_m \Rightarrow (I - I_m) R_{sh} = I_m R_m$.

$49.95mA\,(R_1 + R_2) = 50\mu A\,(R_4 + R_3 + 100)$ — (C)

Case 4: For 0-100mA range.

$I_{sh} R_{sh} = I_m R_m = (I - I_m) R_{sh} = I_m R_m$.

$99.95mA\,(R_1) = 50\mu A\,(R_4 + R_3 + R_2 + 100)$ — (D)

Rearranging (A) $R_1 + R_2 + R_3 = 5.26\ \Omega - R_4$.

Using the above eqn in (B)

$9950\mu A\,(5.26\Omega - R_4) = 50\mu A\,(100 + R_4)$

solving for R_4.

$9950\mu A \times 5.26 - 9950\mu A R_4 = 50\mu A \times 100 + 50\mu A \times R_4$

$\boxed{R_4 = 4.733\ \Omega}$

Using R_4 in (A), we get

$R_1 + R_2 + R_3 = 5.26 - 4.73 = 0.53$

$R_1 + R_2 + R_3 = 0.53$

$R_1 + R_2 = 0.53 - R_3$.

Using this in (C)

$49.95mA(R_1+R_2) = 50\mu A(R_4+R_3+100)$

$49.95mA(0.53-R_3) = 50\mu A(4.73+100+R_3)$

solving for R_3.

$\boxed{R_3 = 0.424\Omega} = \boxed{R_3 = 0.42\Omega}$

we have $R_1+R_2 = 0.53-R_3$

$R_1+R_2 = 0.53-0.42$

$R_1+R_2 = 0.11$

$R_2 = 0.11-R_1$

We have eqn (D) as

$99.95mA(R_1) = 50\mu A(R_4+R_3+R_2+100)$

But $R_2+R_3+R_4 = 5.26-R_1$ (using (A))

$\therefore 99.95mA(R_1) = 50\mu A((5.26-R_1)+100)$

$99.95mA\,R_1 = 50\mu A\times 5.26 - 50\mu A\,R_1 + 50\mu A\times 100$

$\therefore R_1 = 0.05263$

$\boxed{R_1 = 0.053\Omega}$

we have

$R_2 = 0.11-R_1$

$= 0.11-0.053$

$\boxed{R_2 = 0.057\Omega}$

Thus, the value of shunts for universal ammeter are

$R_1 = 0.053\Omega$

$R_2 = 0.057\Omega$

$R_3 = 0.42\Omega$

$R_4 = 4.73\Omega$

Voltmeter and Multimeter

Problems related to DC Voltmeter

<u>Ex:-</u> Basic D'Arsonval movement with full scale deflection of 50μA & internal resistance of 500Ω is used as volt-meter. Determine the value of multiplier resistance needed to measure vtg range of 0-10V. (2)

→ Given $I_{fsd} = 50\mu A = I_m$. Rang = 0-10V

$R_m = 500\Omega$ $\therefore V = 10V$.

$$R_s = \frac{V}{I_m} - R_m$$

$$= \frac{10}{50\mu A} - 500$$

$$\boxed{R_s = 199.5k\Omega}$$

<u>Ex:-</u> Calculate value of multiplier resistance on the 50V range of a dc voltmeter that uses a 500μA meter movement with internal resistance of 1kΩ.

Given $I_{fsd} = 500\mu A$

$V = 50V$

$R_m = 1k\Omega$.

→ The sensitivity $S = 1/I_{fsd}$

$$S = \frac{1}{500\mu A} = 2k\Omega/V$$

The value of multiplier $= R_s = S \times range - R_m$.

$$= 2k\Omega/V \times 50V - 1k.$$

$$= 100k - 1k = 99k\Omega$$

$$\boxed{R_s = 99k\Omega}$$

Problems related to Multirange Voltmeter.

Ex:- A D'Arsonval movement with full scale deflection curre of 10mA & internal resistance of 500Ω is to be converte into a multirang voltmeter. Determine the value of multiplier required for 0-20V, 0-50V & 0-100V.

→ Given: $I_m = 10mA$

$R_m = 500\Omega$

Case 1: Range 0-20V

$$R_s = \frac{V}{I_m} - R_m$$

$$= \frac{20}{10mA} - 500\Omega$$

$$\boxed{R_s = 1{,}500\Omega}$$

Case 2: Range 0-50V

$$R_s = \frac{50}{10m} - 500\Omega$$

$$\boxed{R_s = 4.5k\Omega}$$

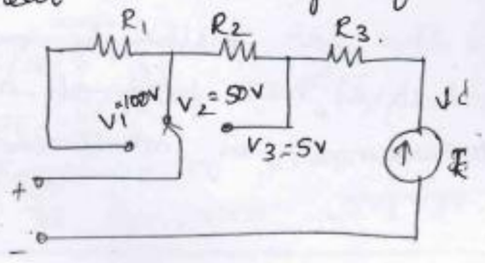
Case 3: Range 0-100V

$$R_s = \frac{V}{I_m} - R_m$$

$$= \frac{100}{10m} - 500$$

$$\boxed{R_s = 9.5k\Omega}$$

Ex: Convert a basic D'Arsonval movement with an internal resistance of 100Ω & full scale deflection of 10mA into multirange dc voltmeter with ranges from 0-5V, 0-50V & 0-100V.



→ Given: $I_m = 10mA$

$R_m = 100\Omega$

For 5V ie V_3 position, the total resistance is. (3)

$$R_t = \frac{V_3}{I_{fsd}} = \frac{5}{10mA} = 500\,\Omega$$

$\therefore\ R_3 = R_t - R_m$

$= 500 - 100$

$\boxed{R_3 = 400\,\Omega}$

$S = \Omega/V = R_t/V$

$R_t = S \cdot V$

$= V/I_{fsd}$

or

$V = IR$ or $R = \frac{V}{I}$

For 50V ie V_2 position, the total resistance is

$$R_t = \frac{V_2}{I_{fsd}} = \frac{50}{10mA} = 5\,k\Omega$$

$R_t = R_2 + R_3 + R_m$.

$\therefore\ R_2 = R_t - (R_3 + R_m)$

$= 5k - (400 + 100)$

$= 5k - 500\,\Omega$

$\boxed{R_2 = 4.5\,k\Omega}$

For 100V ie V_1 position, total resistance is -

$$R_t = \frac{V_1}{I_{fsd}} = \frac{100}{10mA} = 10\,k\Omega.$$

$\therefore\ R_t = R_1 + R_2 + R_3 + R_m$.

$R_1 = R_t - (R_2 + R_3 + R_m)$

$= 10k - (4.5k + 400 + 100)$.

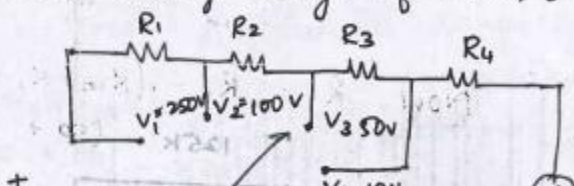
$\boxed{R_1 = 5\,k\Omega}$

Only R_3 is nonstandard value.

Q: Convert a basic D'Arsonval movement with an internal resistance of $50\,\Omega$ & a full scale deflection current of 2mA into multirange dc voltmeter with voltage range of 0-10V, 0-50V, 0-100V, & 0-250V.

Given: $R_m = 50\,\Omega$

$I_m = 2mA$.



i) for 10V range i.e :- V_4 position of switch, the total resistance of the circuit is

$$R_t = \frac{V}{I_{fsd}} = \frac{10V}{2mA} = 5k\Omega.$$

$$\therefore R_4 = R_t - R_m = 5k\Omega - 50\Omega = 4.95k$$

$$\boxed{R_4 = 4.95k\Omega}$$

(ii) For 50V range ie V_3 position of switch, the total resistance of the circuit is:

$$R_t = \frac{V}{I_{fsd}} = \frac{50V}{2mA} = 25k\Omega.$$

$$\text{Now, } R_3 = R_t - (R_m + R_4) = 25k\Omega - (50\Omega + 4.95k\Omega)$$

$$\therefore \boxed{R_3 = 20k\Omega}$$

(iii) For 100V range, ie V_2 position of switch, the total resistance of the circuit is:

$$R_t = \frac{V}{I_{fsd}} = \frac{100}{2mA} = 50k\Omega.$$

$$\& \quad R_2 = R_t - (R_m + R_3 + R_4) = 50k - (50\Omega + 4.95\Omega + 20$$

$$\boxed{R_2 = 25k\Omega}$$

(iv) For 250V range ie V_1 position of switch, the tota circuit resistance is

$$R_t = \frac{V}{I_{fsd}} = \frac{250}{2mA} = 125k\Omega.$$

$$\text{Now } R_1 = R_t - (R_m + R_2 + R_3 + R_4)$$

$$= 125k - (50 + 25k + 20k + 4.95k).$$

$$\boxed{R_1 = 75k\Omega}$$

Ex:- Calculate the value of multiplier resistance on 50V range of a dc voltmeter, that uses a 200μA meter movement with an internal resistance of 100Ω.

→ $R_s = ?$, $I_m = 200\mu A$, $R_m = 100\Omega$, $V = 50V$ (Range).

$\because R_s = S \times V - R_m$.

$S = 1/I_{fsd} = \frac{1}{200\mu A} = 5000$

$\therefore R_s = 5000 \times 50 - 100$

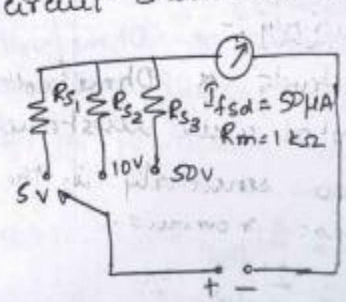
$\boxed{R_s = 249.9k\Omega}$

or $R_s = \frac{V}{I_m} - R_m$

$= \frac{50}{200 \times 10^{-6}} - 100$

$= 249.9k\Omega$

Ex:- Calculate the value of multiplier resistance for the multiple range dc voltmeter circuit shown in fig.



→ The sensitivity of the meter movement is given by.

$S = 1/I_{fsd} = 1/50\mu A$

$S = 20,000$

{Given $I_{fsd} = 50\mu A$, $R_m = 1k\Omega$.}

Now the value of multiplier resistances are

(i) for range 5V,

$R_{s_1} = S \times V - R_m$
$= 20,000 \times 5 - 1k$

$\boxed{R_{s_1} = 99k\Omega}$

(ii) for range 10V

$R_{s2} = S \times V - R_m$
$= 20,000 \times 10 - 1k$

$\boxed{R_{s2} = 199k\Omega}$

(iii) For range 50V

$R_{s_3} = S \times V - R_m$
$= 20,000 \times 50 - 1k$

$\boxed{R_{s_3} = 999k\Omega}$

Ex:- A moving coil instrument gives a full scale deflection of 20 mA when the potential difference across its terminals is 100 mV. Calculate

(i) shunt resistance for full scale deflection corresponding of 50A

(ii) The series resistance for a full scale reading with 500 V. Also calculate the power dissipation in each case.

→ Given $I_m = 20mA$ & voltage = 100mV

To find $R_m = \frac{\text{voltage}}{I_m} = \frac{100mV}{20mA} = 5$

$$\boxed{R_m = 5\Omega}$$

(i) To find shunt,

$$R_{sh} = \frac{I_m R_m}{I - I_m} = \frac{20m \times 5}{50 - 20m}$$

$$R_{sh} = 2.0 m\Omega \quad \text{or} \quad R_{sh} = 0.002\Omega$$

$$\boxed{R_{sh} = 0.002\Omega}$$

(ii) To find series resistance or voltage multiplier.

$$R_s = \frac{V}{I_m} - R_m$$

$$= \frac{500}{20mA} - 5$$

$$\boxed{R_s = 24.999\,k\Omega}$$

Power $= V_m \times I_m$

$= 500 \times 20 \times 10^{-3}$

$$\boxed{P = 10\,W}$$

Power $= I_m^2 R_m$

$= (20mA)^2 \times 5$

$$\boxed{\text{Power} = 2mW}$$

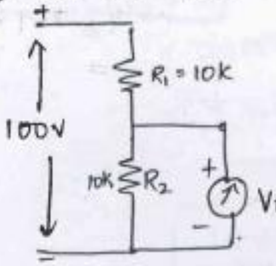
Problems related to Loading effect.

Ex:- For the ckt shown, the series resistors R_1 & R_2 are connected to a 100V dc source. If the voltage across R_2 is to be measured by voltmeters having.

(i) a sensitivity of $1000\,\Omega/V$ &

(ii) a sensitivity of $20{,}000\,\Omega/V$, find

which voltmeter will read the accurate value of volt across R_2. Both meters are used on the 50V range.



→ From voltage divider rule,

the voltage across R_2

$$V_2 = \frac{R_2 \times V}{R_1 + R_2} = \frac{10k \times 100}{20k}$$

$V_2 = 50V$ → This is the true voltage across R_2.

(i) Using voltmeter with sensitivity of $1000\,\Omega/V$ (6)

It has resistance of $1000 \times 50 = 50k\Omega$ on 50 V range.

Now the equivalent resistance when meter is connected

$$R_{eq} = \frac{10k \times 50k}{10k + 50k} = 8.33k\Omega$$

$R_t = S \times V$

The voltage across total combination is given by

$$V_2 = \frac{R_{eq} \times V}{R_{eq} + R_1}$$

Using voltage divider rule

$$= \frac{8.33k \times 100}{8.33k + 10k} =$$

$$\boxed{V_2 = 45.46V}$$

(ii) Using voltmeter with sensitivity of 20,000 Ω/V.

It has resistance of $20{,}000 \times 50 = 1M\Omega$ on 50V range.

Now the equivalent resistance, when meter is connected

$$R_{eq} = \frac{10k \times 1M\Omega}{10k + 1M\Omega} = 9.9k\Omega$$

The voltage across total combination is given by.

$$V_2 = \frac{R_{eq} \times V}{R_{eq} + R_1} = \frac{9.9k \times 100}{9.9k + 10k}$$

$$\boxed{V_2 = 49.74V}$$

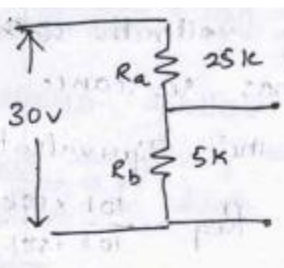
Observing both o/ps, the meter with high sensitivity gives accurate readings.

x:- Two different voltmeters are used to measure the voltage across R_b in the ckt shown. The meters are as follows

Meter 1: $S = 1k\Omega/V$ $R_m = 0.2K$; range $= 10V$.

Meter 2: $S = 20k\Omega/V$ $R_m = 1.5K$, range $- 10V$

calculate (i) voltage across R_b without any meter across it.
(ii) voltage across R_b with meter 1
(iii) voltage across R_b with meter 2
(iv) errors in voltmeters.



→ Given: $R_a = 25K$, $R_b = 5k\Omega$ $V = 30V$.

(i) Vtg across R_b is given by:

$$V_{Rb} = \frac{R_b \times V}{R_b + R_a} = \frac{5k \times 30}{30K} = 5V$$

$$\boxed{V_{Rb} = 5V}$$

(ii) vtg across R_b with meter 1 :

Meter 1's sensitivity = $1k\Omega$, $R_m = 0.2K$ & range = 10V.

∴ Total resistance in the ckt is given by.

$$R_{m_1} = S \times V$$
$$R_{m_1} = 1k\Omega \times 10V = 10k\Omega.$$

with meter 1 across R_b , the equivalent resistance

$$R_{eq} = \frac{R_b \times R_{m1}}{R_b + R_{m1}} = \frac{5K \times 10K}{15K} = 3.33k\Omega.$$

Now the voltage across R_b with meter 1 gives.

$$V_{Rb} = \frac{3.33k \times 30}{3.33k + 25k} = 3.53V$$

$$\boxed{V_{Rb} = 3.53V}$$

(iii) vtg across R_b with meter 2: $R_m = 1.5K$ & range = 10V.

Meter 2's sensitivity = $20k\Omega/V$

Total resistance in the ckt is given by.

$$R_{m2} = S \times V = 20k\Omega/V \times 10V = 200k\Omega.$$

with meter 2 across R_b, the equivalent resistance will be.

$$R_{eq} = \frac{R_b \times R_{m2}}{R_b + R_{m2}} = \frac{5K \times 200K}{205K} = 4.88k\Omega.$$

Now voltage across R_b with meter 2 gives

$$V_{Rb} = \frac{4.88K \times 30}{4.88K + 25K} = 4.89V$$

$$\boxed{V_{Rb} = 4.9V}$$

(iv) Error in reading of voltmeter is given by.

$$\% \text{ Error} = \frac{\text{Actual voltage} - \text{Measured vtg}}{\text{Actual voltage}}$$

For meter 1 :

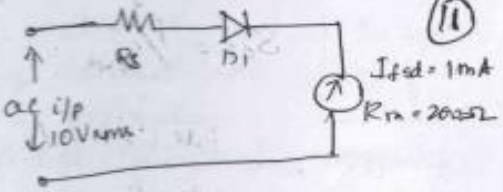
$$\% \text{ error} = \frac{5 - 3.53}{5} = 29.4\%.$$

For meter 2 :

$$\% \text{ error} = \frac{5 - 4.9}{5} = 2\%.$$

Problems related to AC Voltmeter using Rectifiers.

Ex:- Calculate value of multiplier resistor for 10V range on the voltmeter shown. (11)



→ Given : $I_{fsd} = 1mA$
$R_m = 200\Omega$
$R_S = ?$

Sensitivity of meter is given by

$$S_{dc} = \frac{1}{I_{fsd}} = \frac{1}{1mA} = 1k\Omega$$

$\boxed{S_{dc} = 1k\Omega/V}$

we have expression for R_s

$$R_s = S_{dc} \times V - R_m$$

$= \cancel{1K\Omega \times 10V - 200\Omega} = \_\_\_$

$R_t = S \times V$
$R_s + R_m = S \times V$
$\therefore R_s = S \times V - R_m.$

~~Suppose~~ Here V → voltage range is the average dc value.

$$\therefore R_s = S \times 0.45 \times E_{rms} - R_m$$
$$= 1k \times 0.45 \times 10 - 200.$$

$V_{dc} = 0.45\, V_{rms}.$
$E_{dc} = 0.45\, E_{rms}$

$\boxed{R_s = 4.3k\Omega}$

OR

Another method.

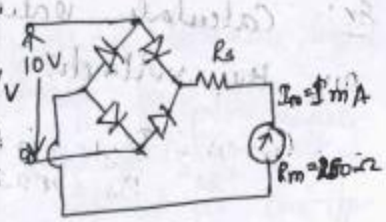
$$R_s = \frac{0.45\, E_{rms}}{I_m} - R_m.$$
$$= \frac{0.45 \times 10}{1mA} - 200$$

$R_s = \frac{V}{I_m} - R_m$
$V = 0.45\, E_{rms}.$
$= 0.45 \times 10$

$\boxed{R_s = 4.3k\Omega}$

Ex:- Calculate value of multiplier resistor for a 10 V rms range on the voltmeter using a full wave bridge rectifier and basic meter with scale deflection current of 1mA & meter resistance is of 2500Ω.

→ dc sensitivity is given by



$$S_{dc} = 1/I_{fsd} = 1/1mA = 1k\Omega/V$$

$$S_{dc} = 1k\Omega/V$$

ac sensitivity is given by

$$S_{ac} = 0.9 \times S_{dc}$$
$$= 0.9 \times 1k\Omega/V$$

$$S_{ac} = 0.9\ k\Omega/V$$

Now multiplier resistor is given by

$$R_s = S_{ac} \times range - R_m$$

$$= 0.9 k\Omega/V \times 10V - 250$$

$$= 9k - 250$$

$$\boxed{R_s = 8.75k\Omega}$$

$\boxed{OR}$

→ Full wave rectifier,

$$E_{dc} = 0.9\ E_{rms}$$

$$\text{Now } R_s = \frac{E_{dc}}{I_{dc}} - R_m$$

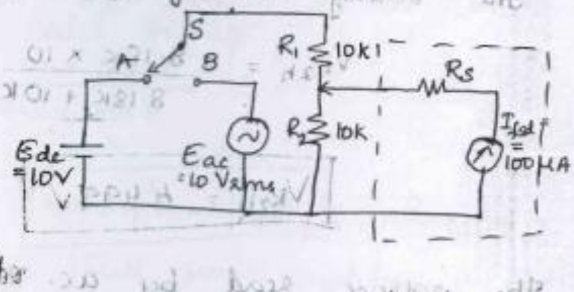
$$= \frac{0.9 E_{rms}}{1 \times 10^{-3}} - 250.$$

$$= \frac{9}{1 \times 10^{-3}} - 250$$

$$\boxed{R_s = 8.75k\Omega}$$

Ex:- Determine the reading obtained with a dc voltmeter in the ckt, when the switch is set to position A, then set the switch to position B & determine the reading obtained with a half wave rectifier and a full wave rectifier ac voltmeter.
All meters uses a 100μA full scale deflection meter movement and are set to on 10Vdc or rms ranges.

→ The sensitivity of dc voltmeter is



$S_{dc} = 1/I_{fsd} = 1/100\mu A$

$\boxed{S_{dc} = 10k\Omega/V}$

The Rs resistance value is given by

$R_s = S_{dc} \times \text{range} = 10k\Omega/V \times 10V$

$\boxed{R_s = 100\,k\Omega}$

Now Rs acts as shunt across 10kΩ (R_2) resistance thus the equivalent resistance is given by

$$R_2 = \frac{R_2 \times R_s}{R_2 + R_s} = \frac{10k \times 100k}{10k + 100k} = 9.09k\Omega$$

The voltage a/c R_2 now is (Edc = 10V = Range)

$$V_{R2} = \frac{R_2 \times \text{Range}}{R_1 + R_2} = \frac{9.09k \times 10}{10k + 9.09k}$$

$\boxed{V_{R2} = 4.76V}$

$V_{R2} = 4.76V$ is the voltage read by dc voltmeter.

Using half wave rectifier, the voltage read by the ac voltmeter is determined as follows:

we have $S_{hw} = 0.45 \times S_{dc}$ | Sensitivity of ac meter = 0.45 × sensitivity of dc

$= 0.45 \times 10\,k\Omega/V$

$S_{hw} = 4.5\,k\Omega/V$

Now $R_{sH} = S_{hw} \times \text{Range} = 4.5k\Omega/V \times 10V = 45k\Omega$. | R_{sh} ⇒ series resistance with half wave rectifier

$$R_{2h} = \frac{R_{sh} \times R_2}{R_{sh} + R_2} = \frac{45k \times 10k}{45k + 10k} = 8.18\ k\Omega$$

$$\boxed{R_{2h} = 8.18k}$$

The voltage across R_{2h}

$$V_{R_{2h}} = \frac{8.18k \times 10}{8.18k + 10k}$$

$$\boxed{V_{R_{2h}} = 4.499V}$$

The voltage read by ac voltmeter using full wave rectifier is determined as follows.

$$S_{fw} = 0.9\ S_{dc}$$
$$= 0.9 \times 10k\Omega/V$$

$$\boxed{S_{fw} = 9k\Omega/V.}$$ | sensitivity of ac meter with full wave rectifier.

$$R_{sf} = S_{fw} \times Range$$
$$= 9k\Omega/V \times 10\ V$$

$$\boxed{R_{sf} = 90k\Omega}$$ | Rs series resistance of ac meter with full wave rectifier.

Now with R_{sf}, it acts as shunt across R_2, thus eff resistance is given by.

$$R_{2f} = \frac{R_{sf} \times R_2}{R_{sf} + R_2} = \frac{90k \times 10k}{100k} = 9k\Omega$$

Now the voltage read across R_{2f} is given by.

$$V_{R_{2f}} = \frac{R_{2f} \times Voltage\ range}{R_{2f} \times R_1} = \frac{9k \times 10V}{9k + 10k}$$

$$\boxed{V_{R_{2f}} = 4.736V}$$

Thus ac voltmeter using half & full wave rectifier has more loading effect than dc voltmeter.

Ex:- A 25mA full scale current meter with an internal resistance of 100 Ω is available for constructing an ac voltmeter with a voltage range of 200 Vrms. The meter uses the bridge configuration for the rectifier of the instrument. If each diode has forward resistance of 500 Ω & infinite reverse resistance, calculate the value of series resistance, to limit the current to the rated value at the rated voltage.

→ Given $I_{fsd} = 25mA$.

$R_m = 100 \Omega$.

Voltage range = $V_{rms} = E_{rms} = 200V$, $R_s = ?$

We have $E_{dc} = 0.9 E_{rms}$ | Full wave rectifier.

$$\& \; R_s = \frac{E_{dc}}{I_{dc}} - R_m$$

$$R_s \Rightarrow \frac{0.9 E_{rms}}{I_{dc}} - R_m.$$

But in the rectifier ckt, the diode forward resistance = 500 Ω. Since bridge configuration is used, 2 diodes will be conducting & will be series. ~~ckt~~ Thus the diode resistance will be

$500 \Omega + 500 \Omega = 1 k\Omega$.

Thus the total meter resistance will be

$R_m = 100 \Omega + 1 k\Omega = 1100 \Omega$.

Now $\cdot R_s = \frac{0.9 \times 200}{25mA} - 1100$

$$\boxed{R_s = 6.1 k\Omega}$$