Module 1:
Introduction

1.1.1 Signal definition

A **signal** is a function representing a physical quantity or variable, and typically it contains information about the behaviour or nature of the phenomenon.

For instance, in a RC circuit the signal may represent the voltage across the capacitor or the current flowing in the resistor. Mathematically, a signal is represented as a function of an independent variable **'t'**. Usually **'t'** represents time. Thus, a signal is denoted by **x(t)**.

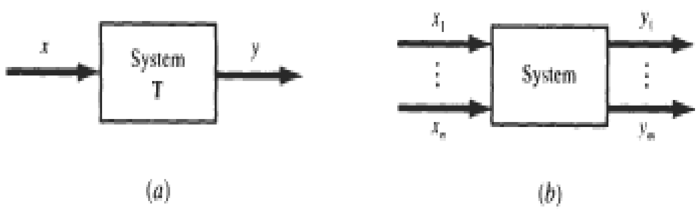
1.1.2 System definition

A system is a mathematical model of a physical process that relates the input (or excitation) signal to the output (or response) signal.

Let x and y be the input and output signals, respectively, of a system. Then the system is viewed as a transformation (or mapping) of **x** into y. This transformation is represented by the mathematical notation

y= Tx ---(1.1)

where **T** is the operator representing some well-defined rule by which **x** is transformed into y. Relationship (1.1) is depicted as shown in Fig. 1-1(a). Multiple input and/or output signals are possible as shown in Fig. 1-1(b). We will restrict our attention for the most part in this text to the single-input, single-output case.



1.1 System with single or multiple input and output signals

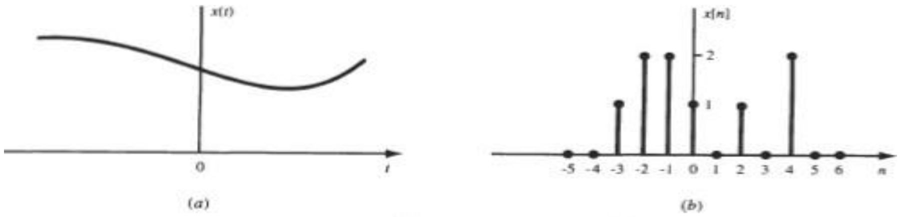
1.2 Classification of signals

Basically seven different classifications are there:

- Continuous-Time and Discrete-Time Signals
- Analog and Digital Signals
- Real and Complex Signals
- Deterministic and Random Signals
- Even and Odd Signals
- Periodic and Nonperiodic Signals
- Energy and Power Signals

Continuous-Time and Discrete-Time Signals

A signal x(t) is a continuous-time signal if t is a continuous variable. If t is a discrete variable, that is, x(t) is defined at discrete times, then x(t) is a discrete-time signal. Since a discrete-time signal is defined at discrete times, a discrete-time signal is often identified as a sequence of numbers, denoted by {x,) or x[n], where n = integer. Illustrations of a continuous-time signal x(t) and of a discrete-time signal x[n] are shown in Fig. 1-2.



1.2 Graphical representation of (a) continuous-time and (b) discrete-time signals

Analog and Digital Signals

If a continuous-time signal x(t) can take on any value in the continuous interval (a, b), where a may be $-\infty$ and b may be $+\infty$ then the continuous-time signal x(t) is called an analog signal. If a discrete-time signal x[n] can take on only a finite number of distinct values, then we call this signal a digital signal.

Real and Complex Signals

A signal x(t) is a real signal if its value is a real number, and a signal x(t) is a complex signal if its value is a complex number. A general complex signal x(t) is a function of the form

$$x(t) = x_1(t) + jx_2(t) \text{ ------------------------------ } 1.2$$

where $x_1(t)$ and $x_2(t)$ are real signals and $j = \sqrt{-1}$

Note that in Eq. (1.2) **'t'** represents either a continuous or a discrete variable.

Deterministic and Random Signals:

Deterministic signals are those signals whose values are completely specified for any given time. Thus, a deterministic signal can be modelled by a known function of time **'t'.**

Random signals are those signals that take random values at any given time and must be characterized statistically.

Even and Odd Signals

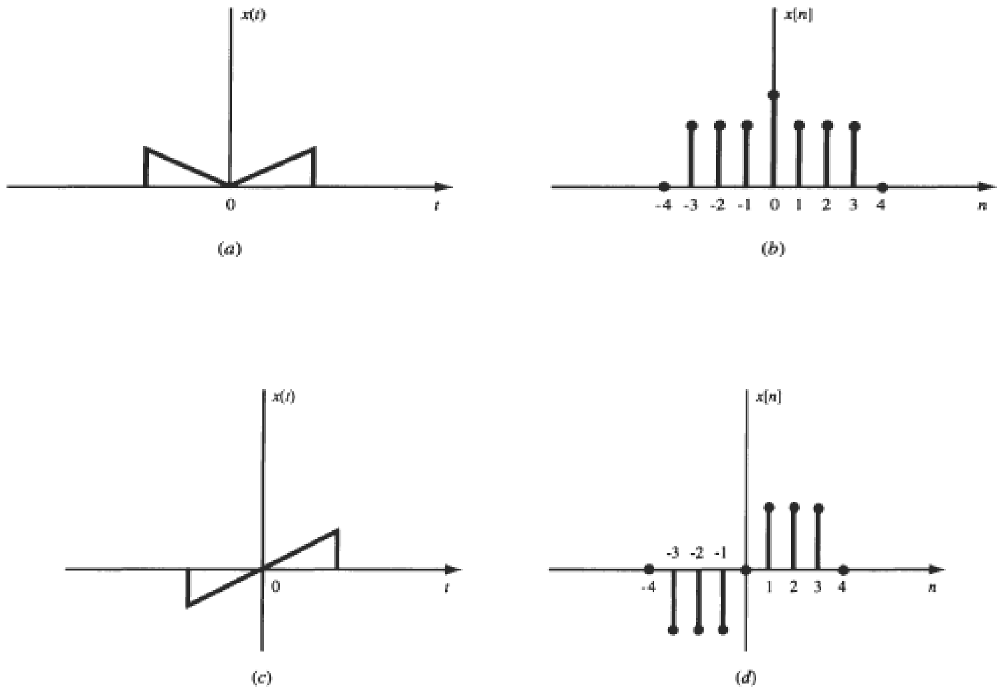
A signal ***x (t)*** or ***x[n]*** is referred to as an ***even*** signal if

$$x(-t) = x(t)$$
$$x[-n] = x[n] \text{ -------------(1.3)}$$

A signal ***x (t)*** or ***x[n]*** is referred to as an ***odd*** signal if

$$x(-t) = -x(t)$$
$$x[-n] = -x[n] \text{--------------(1.4)}$$

Examples of even and odd signals are shown in Fig. 1.3.



1.3 Examples of even signals (a and b) and odd signals (c and d)**.**

Any signal x(t) or x[n] can be expressed as a sum of two signals, one of which is even and one of which is odd. That is,

$$x(t) = x_o(t) + x_e(t) \quad \text{-------(1.5)}$$

Where,

$$x_e(t) = \frac{1}{2}\left(x(t) + x(-t)\right)$$

$$x_o(t) = \frac{1}{2}\left(x(t) - x(-t)\right) \quad \text{-----(1.6)}$$

Similarly for x[n],

$$x[n] = x_o[n] + x_e[n] \quad \text{-------(1.7)}$$

Where,

$$x_e[n] = \frac{1}{2}\left(x[n] + x[-n]\right)$$

$$x_o[n] = \frac{1}{2}\left(x[n] - x[-n]\right) \quad \text{--------(1.8)}$$

Note that the product of two even signals or of two odd signals is an even signal and that the product of an even signal and an odd signal is an odd signal.

Periodic and Nonperiodic Signals

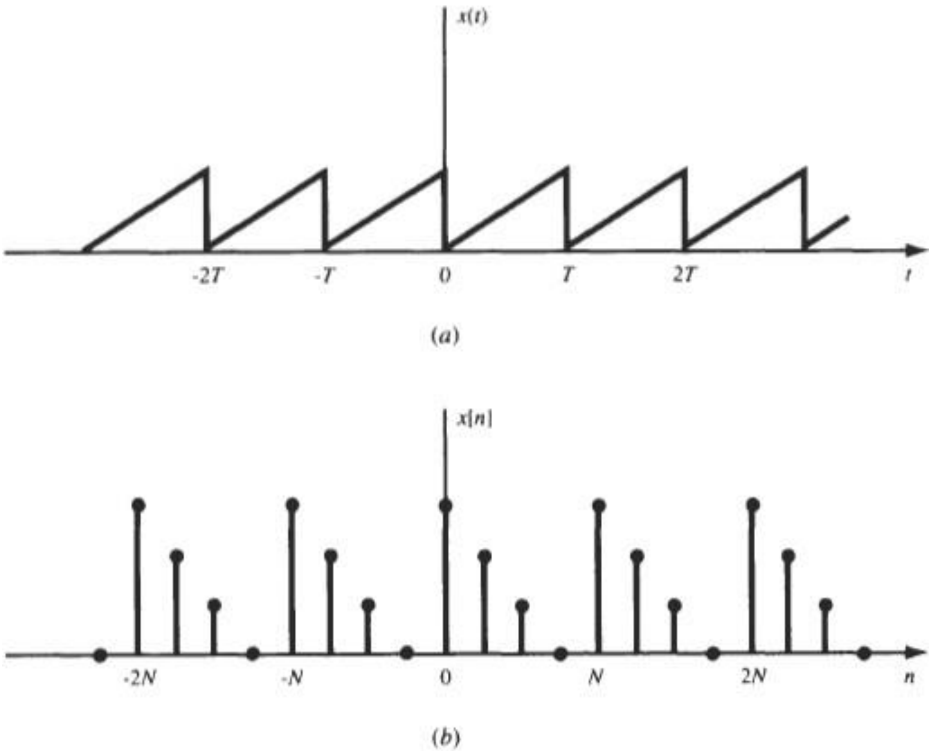
A continuous-time signal x (t) is said to be periodic with period T if there is a positive nonzero value of T for which

$$x(t+T) = x(t) \qquad \text{all } t \quad \text{...........(1.9)}$$

An example of such a signal is given in Fig. 1-4(a). From Eq. (1.9) or Fig. 1-4(a) it follows that

$$x(t + mT) = x(t) \quad \text{---------------------------(1.10)}$$

for all t and any integer m. The fundamental period T, of x(t) is the smallest positive value of T for which Eq. (1.9) holds. Note that this definition does not work for a constant



1.4 Examples of periodic signals.

signal x(t) (known as a dc signal). For a constant signal x(t) the fundamental period is undefined since x(t) is periodic for any choice of T (and so there is no smallest positive value). Any continuous-time signal which is not periodic is called a nonperiodic (or aperiodic) signal.

Periodic discrete-time signals are defined analogously. A sequence (discrete-time signal) x[n] is periodic with period N if there is a positive integer N for which

$$x[n + N] = x[n] \qquad \text{all } n \quad \text{……….(1.11)}$$

An example of such a sequence is given in Fig. 1-4(b). From Eq. (1.11) and Fig. 1-4(b) it follows that

$$x[n + mN] = x[n] \quad \text{……………………..(1.12)}$$

for all n and any integer m. The fundamental period N_o of x[n] is the smallest positive integer N for which Eq.(1.11) holds. Any sequence which is not periodic is called a nonperiodic (or aperiodic sequence.

Note that a sequence obtained by uniform sampling of a periodic continuous-time signal may not be periodic. Note also that the sum of two continuous-time periodic signals may not be periodic but that the sum of two periodic sequences is always periodic.

Energy and Power Signals

Consider v(t) to be the voltage across a resistor R producing a current i(t). The instantaneous power p(t) per ohm is defined as

$$p(t) = \frac{v(t)i(t)}{R} = i^2(t)$$ …………(1.13)

Total energy E and average power P on a per-ohm basis are

$$E = \int_{-\infty}^{\infty} i^2(t)\,dt \quad \text{joules}$$

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} i^2(t)\,dt \quad \text{watts}$$ ……(1.14)

For an arbitrary continuous-time signal x(t), the normalized energy content E of x(t) is defined as

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$ …………………(1.15)

The normalized average power P of x(t) is defined as

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\,dt$$ (1.16)

Similarly, for a discrete-time signal x[n], the normalized energy content ***E*** of x[n] is defined as

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$ (1.17)

The normalized average power P of x[n] is defined as

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$ (1.18)

Based on definitions (1.15) to (1.18), the following classes of signals are defined:

1. x(t) (or x[n]) is said to be an energy signal (or sequence) if and only if 0 < E < m, and so P = 0.
2. x(t) (or x[n]) is said to be a power signal (or sequence) if and only if 0 < P < m, thus implying that E = m.
3. Signals that satisfy neither property are referred to as neither energy signals nor power signals.

Note that a periodic signal is a power signal if its energy content per period is finite, and then the average power of this signal need only be calculated over a period

1.3 Basic Operations on signals

The operations performed on signals can be broadly classified into two kinds

- Operations on dependent variables
- Operations on independent variables

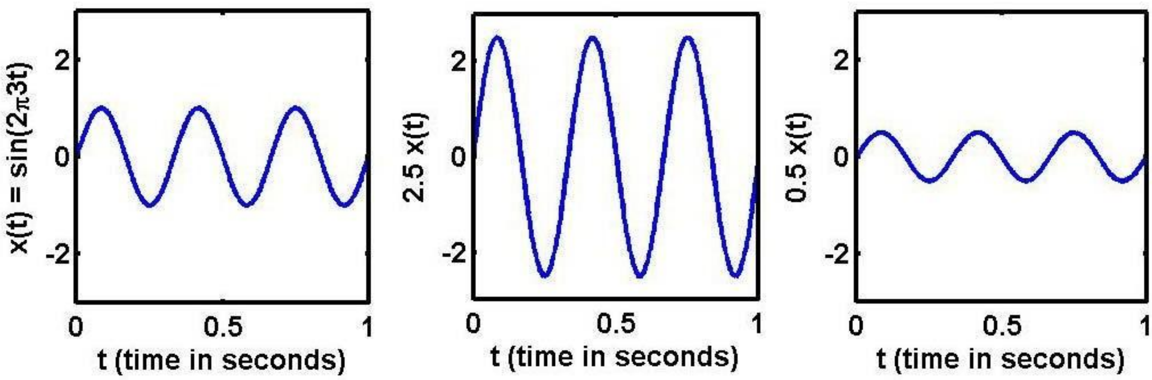
Operations on dependent variables

The operations of the dependent variable can be classified into five types: amplitude scaling, addition, multiplication, integration and differentiation.

Amplitude scaling

Amplitude scaling of a signal $x(t)$ given by equation 1.19, results in amplification of $x(t)$ if $a>1$, and attenuation if $a<1$.

$$y(t) = ax(t) \ldots\ldots..(1.20)$$

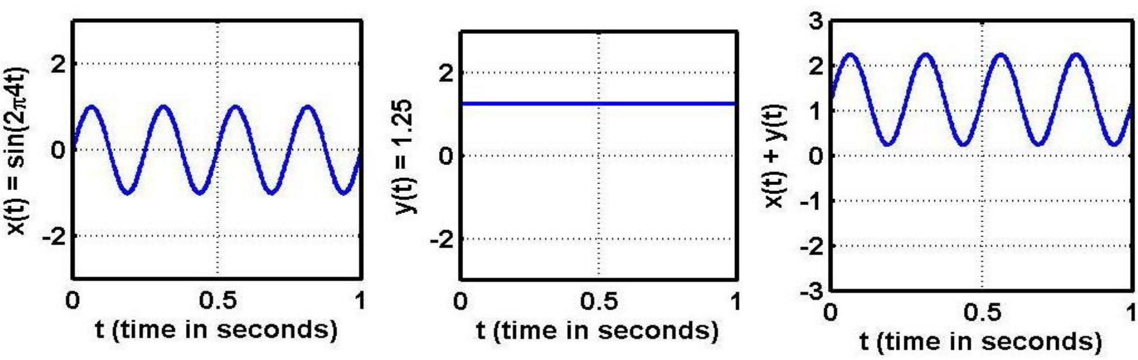


1.5 Amplitude scaling of sinusoidal signal

Addition

The addition of signals is given by equation of 1.21.

$$y(t) = x1(t) + x2\,(t) \ldots\ldots(1.21)$$



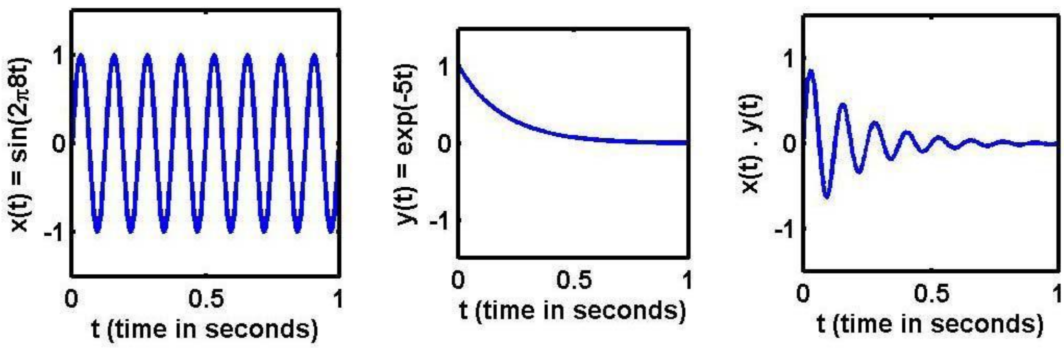
1.6 Example of the addition of a sinusoidal signal with a signal of constant amplitude (positive constant)

Physical significance of this operation is to add two signals like in the addition of the background music along with the human audio. Another example is the undesired addition of noise along with the desired audio signals.

Multiplication

The multiplication of signals is given by the simple equation of 1.22.

$$y(t) = x1(t).x2\ (t)\ldots\ldots..(1.22)$$



1.7 Example of multiplication of two signals

Differentiation

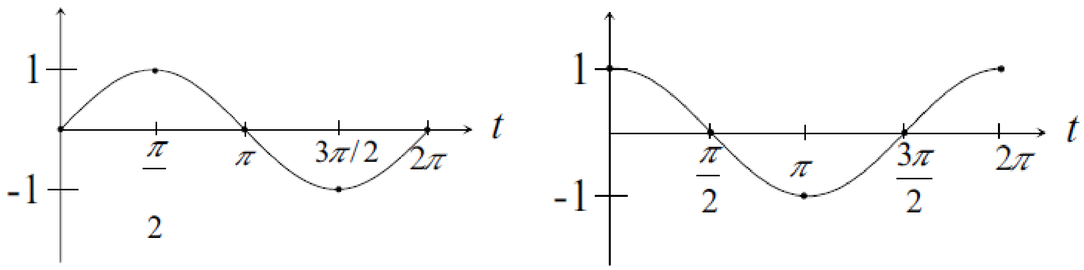
The differentiation of signals is given by the equation of 1.23 for the continuous.

$$y(t) = \frac{d}{dt}x(t) \quad \ldots..1.23$$

The operation of differentiation gives the rate at which the signal changes with respect to time, and can be computed using the following equation, with Δt being a small interval of time.

$$\frac{d}{dt}x(t) = \underset{\Delta t \to 0}{Lim}\frac{x(t+\Delta t)-x(t)}{\Delta t} \quad \ldots.1.24$$

If a signal doesn‟t change with time, its derivative is zero, and if it changes at a fixed rate with time, its derivative is constant. This is evident by the example given in figure 1.8.

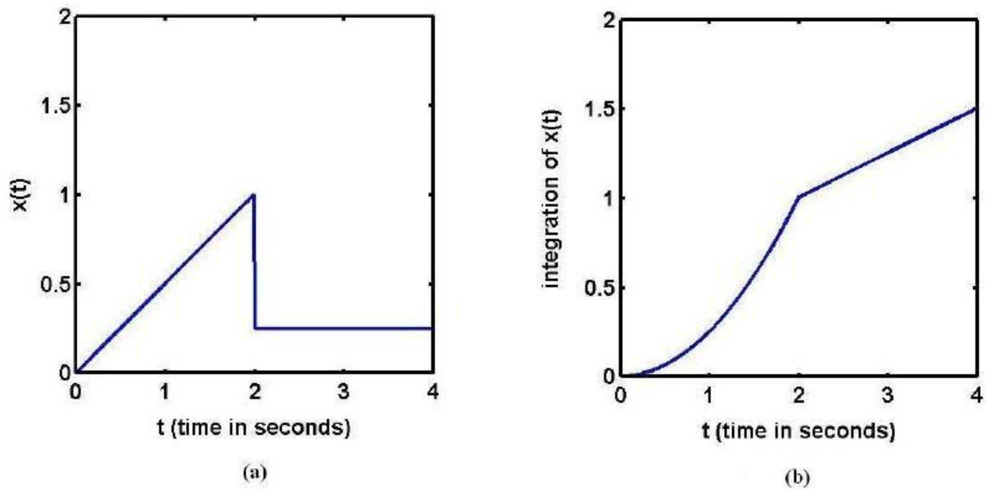


1.8 Differentiation of Sine - Cosine

Integration

The integration of a signal $x(t)$, is given by equation 1.25

$$y(t) = \int_{-\infty}^{t} x(\tau) d\tau \quad \text{……1.25}$$



1.9 Integration of x(t)

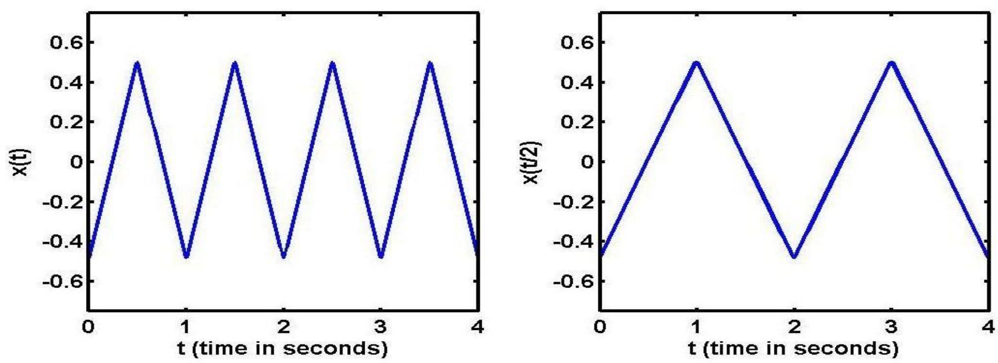
Operations on independent variables

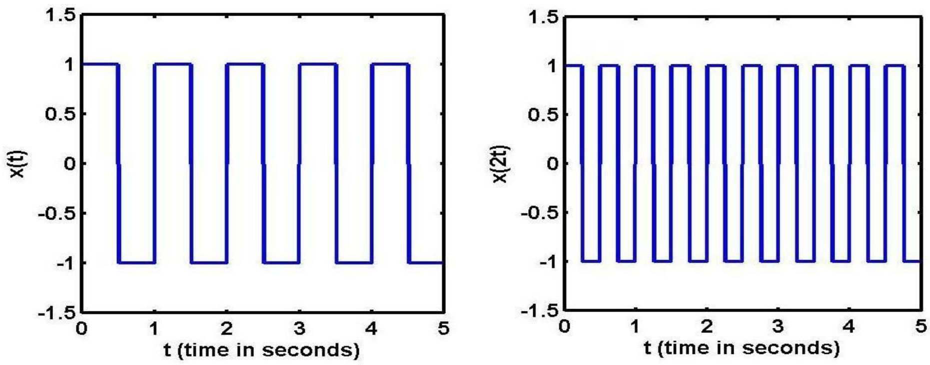
Time scaling

Time scaling operation is given by equation 1.26

$$y(t) = x(at) \text{ .... } 1.26$$

This operation results in expansion in time for a<1 and compression in time for a>1, as evident from the examples of figure 1.10.





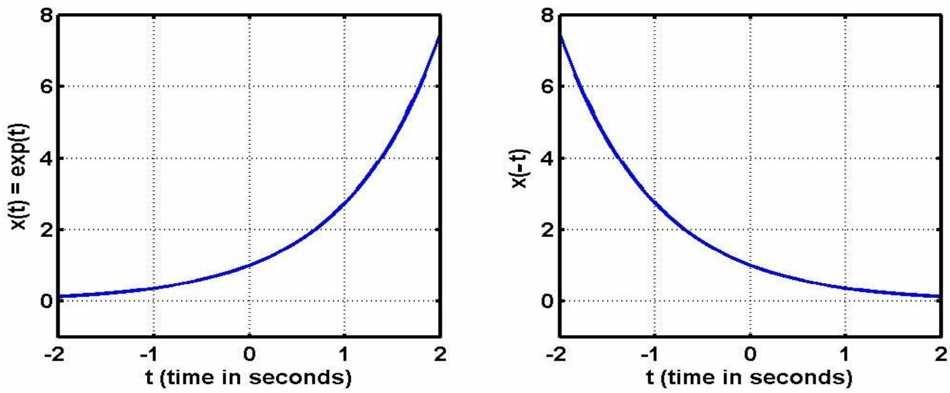
1.10 Examples of time scaling of a continuous time signal

An example of this operation is the compression or expansion of the time scale that results in the „*fast-forward'* or the „*slow motion'* in a video, provided we have the entire video in some stored form.

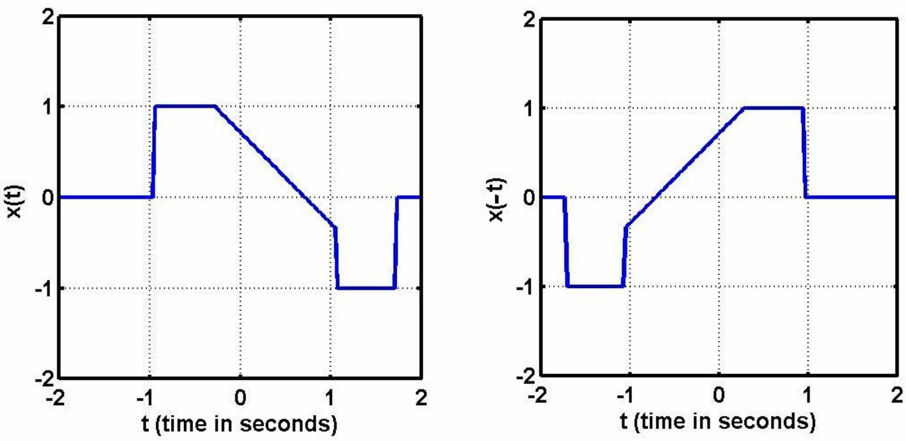
Time reflection

Time reflection is given by equation (1.27), and some examples are contained in fig1.11.

$$y(t) = x(-t) \ldots\ldots\ldots\ldots. 1.27$$



(a)



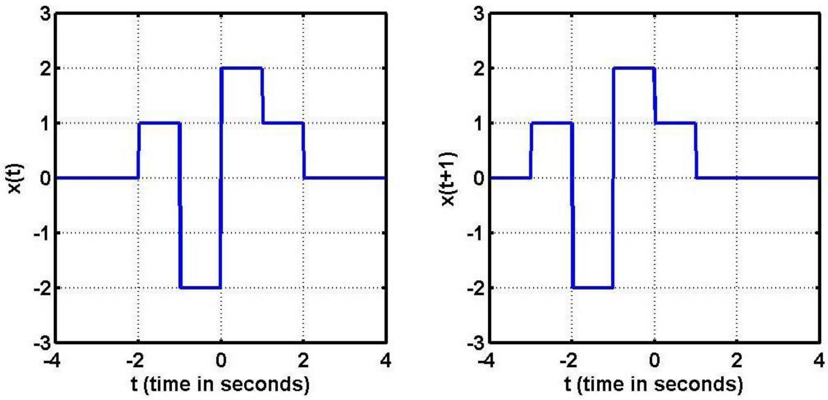
(b)

1.11 Examples of time reflection of a continuous time signal

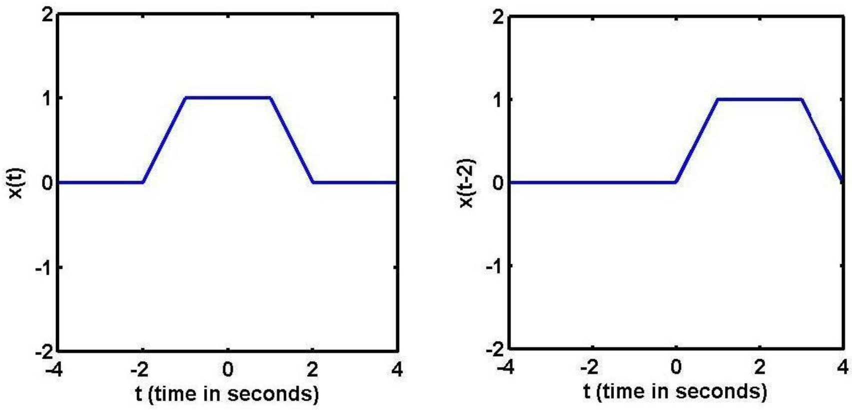
Time shifting

The equation representing time shifting is given by equation (1.28), and examples of this operation are given in figure 1.12.

$y(t) = x(t - t0)$…............. 1.28



(a)



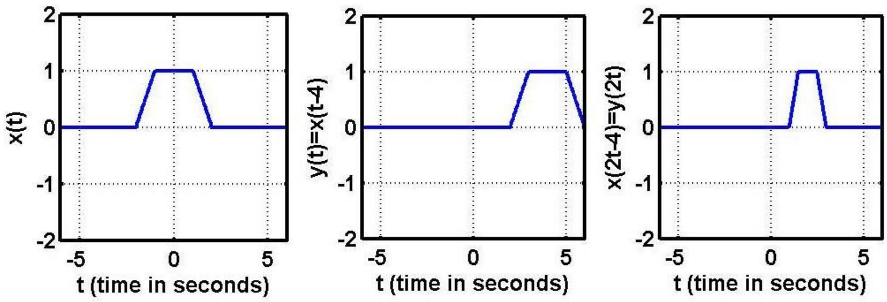
(b)

1.12 Examples of time shift of a continuous time signal

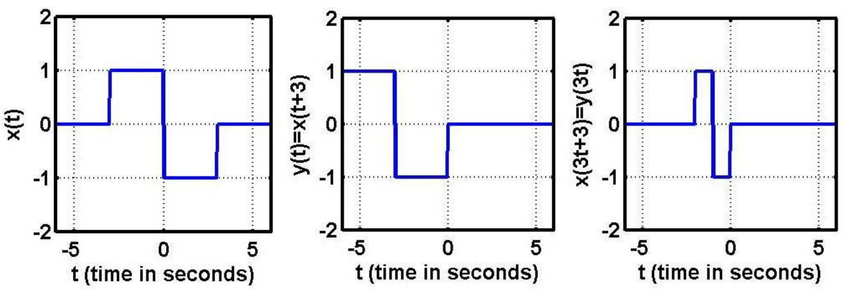
Time shifting and scaling

The combined transformation of shifting and scaling is contained in equation (1.29), along with examples in figure 1.13. Here, time shift has a higher precedence than time scale.

$y(t) = x(at - t0)$.......................1.29



(a)



(b)

1.13 Examples of simultaneous time shifting and scaling. The signal has to be shifted first and then time scaled.

1.4 Elementary signals

Exponential signals:

The exponential signal given by equation (1.29), is a monotonically increasing function if $a > 0$, and is a decreasing function if $a < 0$.

$$x(t) = e^{at} \qquad \text{......................(1.29)}$$

It can be seen that, for an exponential signal,

$$x(t + a^{-1}) = e.x(t)$$

$$x(t - a^{-1}) = e^{-1}.x(t) \qquad \text{.....................(1.30)}$$

Hence, equation (1.30), shows that change in time by $\pm 1/a$ seconds, results in change in magnitude by $e\pm1$. The term $1/a$ having units of time, is known as the time-constant. Let us consider a decaying exponential signal

$$x(t) = e^{-at} \quad \text{for } t \geq 0. \qquad \text{..............(1.31)}$$

This signal has an initial value $x(0) = 1$, and a final value $x(\infty) = 0$. The magnitude of this signal at five times the time constant is,

$$x(5/a) = 6.7x10^{-3} \qquad \text{.....................(1.32)}$$

while at ten times the time constant, it is as low as,

$$x(10/a) = 4.5x10^{-5} \qquad \text{..............(1.33)}$$

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It can be seen that the value at ten times the time constant is almost zero, the final value of the signal. Hence, in most engineering applications, the exponential signal can be said to have reached its final value in about ten times the time constant. If the time constant is 1 second, then final value is achieved in 10 seconds!! We have some examples of the exponential signal in figure 1.14.

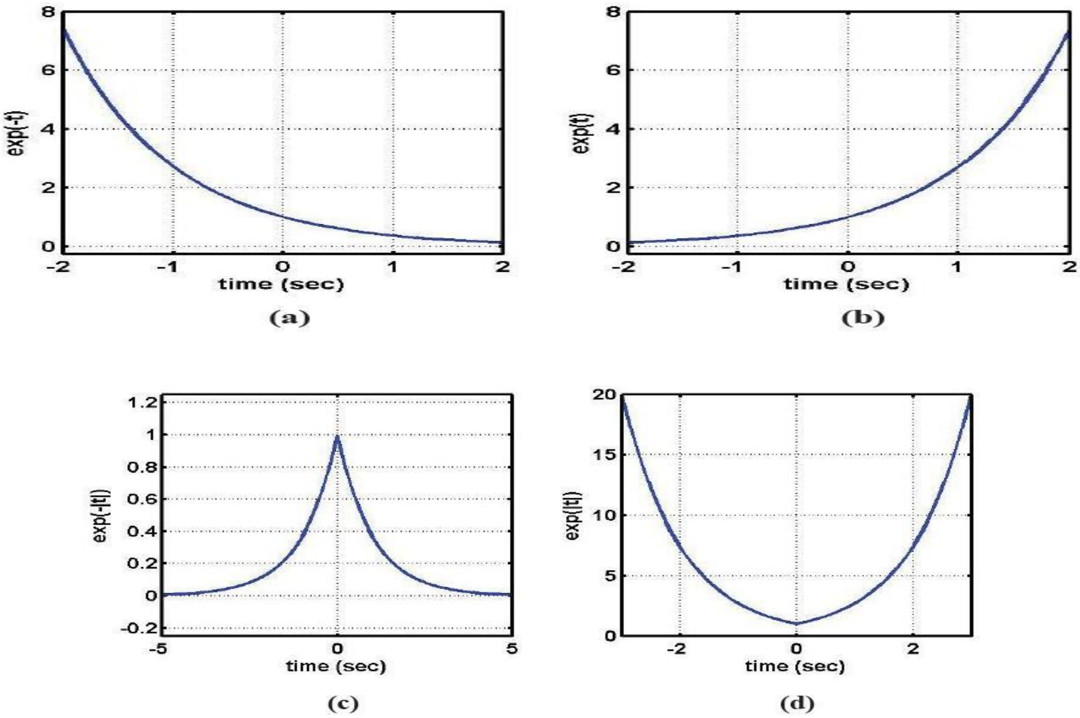


Fig 1.14 The continuous time exponential signal (a) e^{-t} , (b) e^{t} , (c) $e^{-|t|}$, and (d) $e^{|t|}$

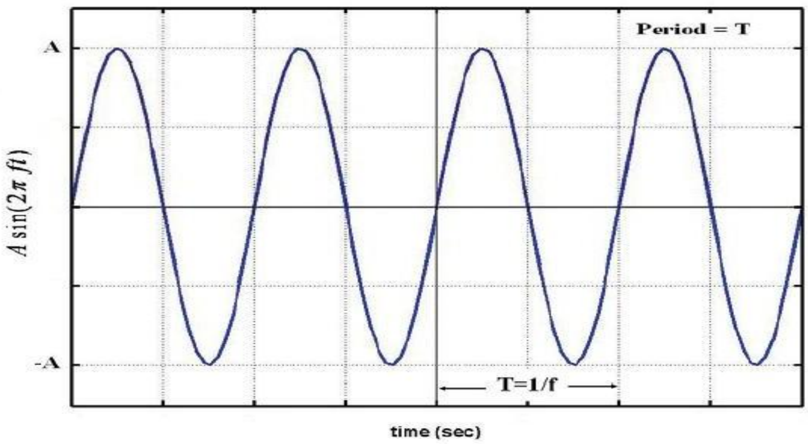
The sinusoidal signal:

The sinusoidal continuous time periodic signal is given by equation 1.34, and examples are given in figure 1.15

$$\boldsymbol{x(t) = A\sin(2\pi ft)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.34)$$

The different parameters are:

Angular frequency $\omega = 2\pi f$ in radians,
Frequency f in Hertz, (cycles per second)
Amplitude A in Volts (or Amperes)
Period T in seconds



The complex exponential:

We now represent the complex exponential using the Euler"s identity (equation (1.35)),

$$e^{j\theta} = \left(\cos\theta + j\sin\theta\right) \quad \ldots\ldots\ldots\ldots\ldots(1.35)$$

to represent sinusoidal signals. We have the complex exponential signal given by equation (1.36)

$$e^{j\omega t} = \left(\cos(\omega t) + j\sin(\omega t)\right)$$
$$e^{-j\omega t} = \left(\cos(\omega t) - j\sin(\omega t)\right)$$

………(1.36)

Since sine and cosine signals are periodic, the complex exponential is also periodic with the same period as sine or cosine. From equation (1.36), we can see that the real periodic sinusoidal signals can be expressed as:

$$\cos(\omega t) = \left(\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right)$$

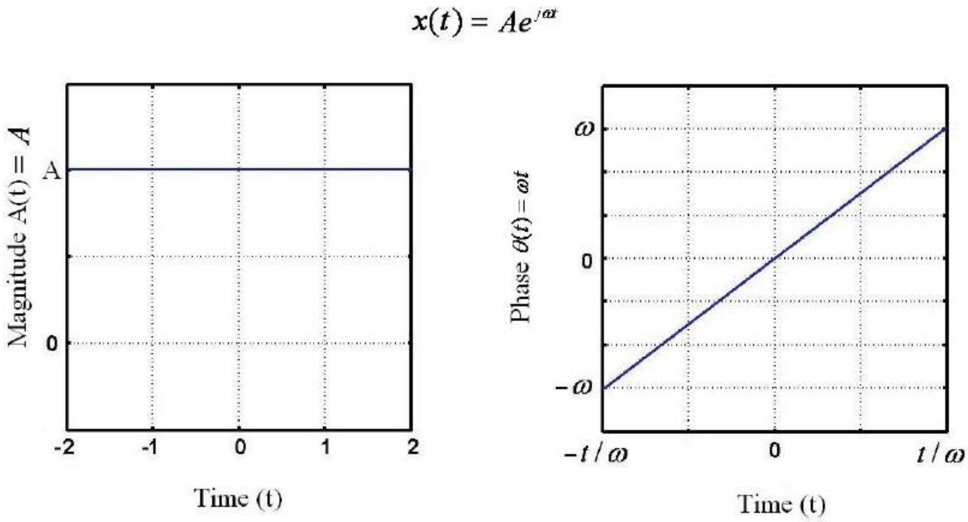
$$\sin(\omega t) = \left(\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right)$$

………………..(1.37)

Let us consider the signal $x(t)$ given by equation (1.38). The sketch of this is given in fig 1.15

$$x(t) = A(t)e^{j\theta(t)}$$

……………………..(1.38)



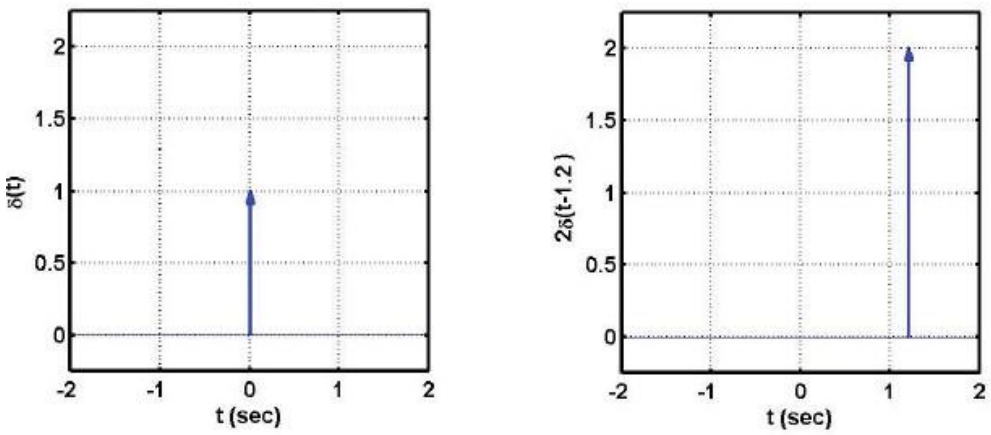
The unit impulse:

The unit impulse usually represented as δ (t) , also known as the dirac delta function, is given by,

$$\delta(t) = 0 \quad for \quad t \neq 0; \quad and \quad \int_{-\infty} \delta(t)dt = 1$$

…….(1.38)

From equation (1.38), it can be seen that the impulse exists only at $t = 0$, such that its area is 1. This is a function which cannot be practically generated. Figure 1.16, has the plot of the impulse function



The unit step:

The unit step function, usually represented as $u(t)$, is given by,

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

……………….(1.39)

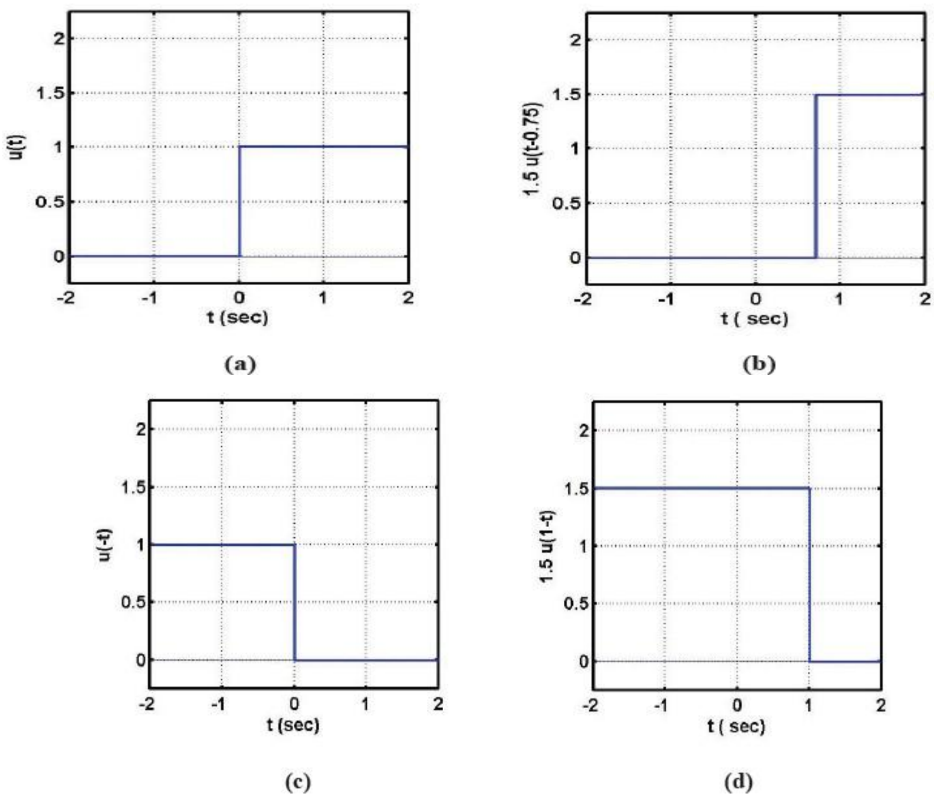


Fig 1.17 Plot of the unit step function along with a few of its transformations

The unit ramp:

The unit ramp function, usually represented as $r(t)$, is given by,

$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

…………….(1.40)

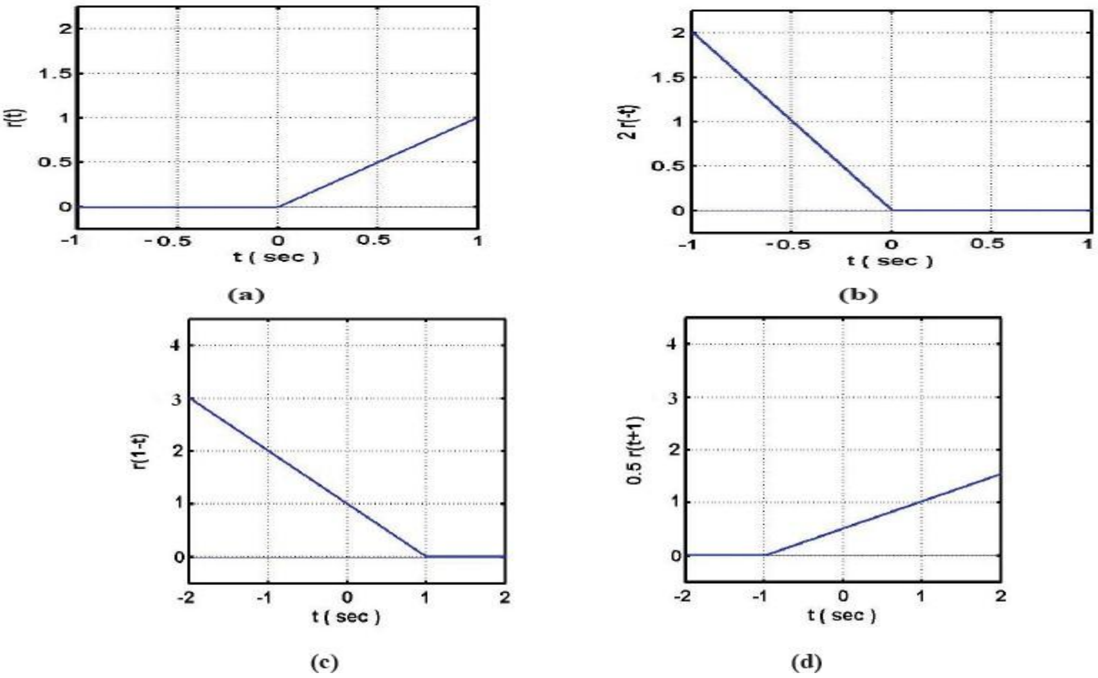


Fig 1.18 Plot of the unit ramp function along with a few of its transformations

The signum function:

The signum function, usually represented as sgn(t) , is given by

$$\operatorname{sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$

…………………………….(1.41)

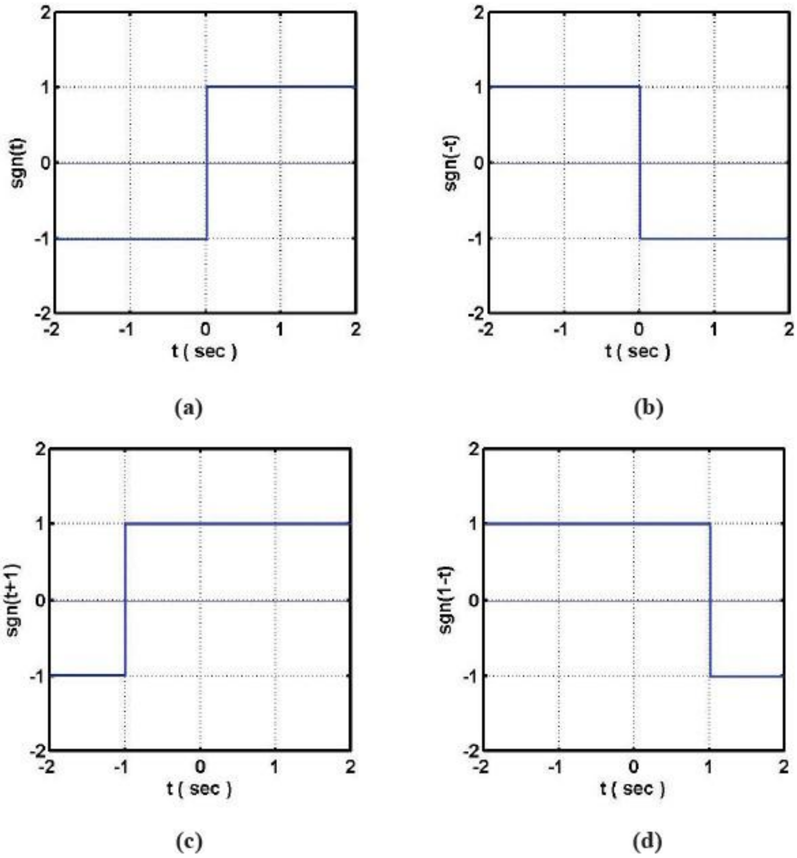


Fig 1.19 Plot of the unit signum function along with a few of its transformations

1.5 System viewed as interconnection of operation:

This article is dealt in detail again in chapter 2/3. This article basically deals with system connected in series or parallel. Further these systems are connected with adders/subtractor, multipliers etc.

1.6 Properties of system:

In this article discrete systems are taken into account. The same explanation stands for continuous time systems also.

The discrete time system:
The discrete time system is a device which accepts a discrete time signal as its input, transforms it to another desirable discrete time signal at its output as shown in figure 1.20

Fig 1.20 DT system

Stability
A system is stable if „bounded input results in a bounded output". This condition, denoted by BIBO, can be represented by:

$$\sum_{n=-\infty}^{\infty} |x[n]| < \infty \quad implies \quad \sum_{n=-\infty}^{\infty} |y[n]| < \infty \quad for \quad all \quad n \qquad ....(1.42)$$

Hence, a finite input should produce a finite output, if the system is stable. Some examples of stable and unstable systems are given in figure 1.21

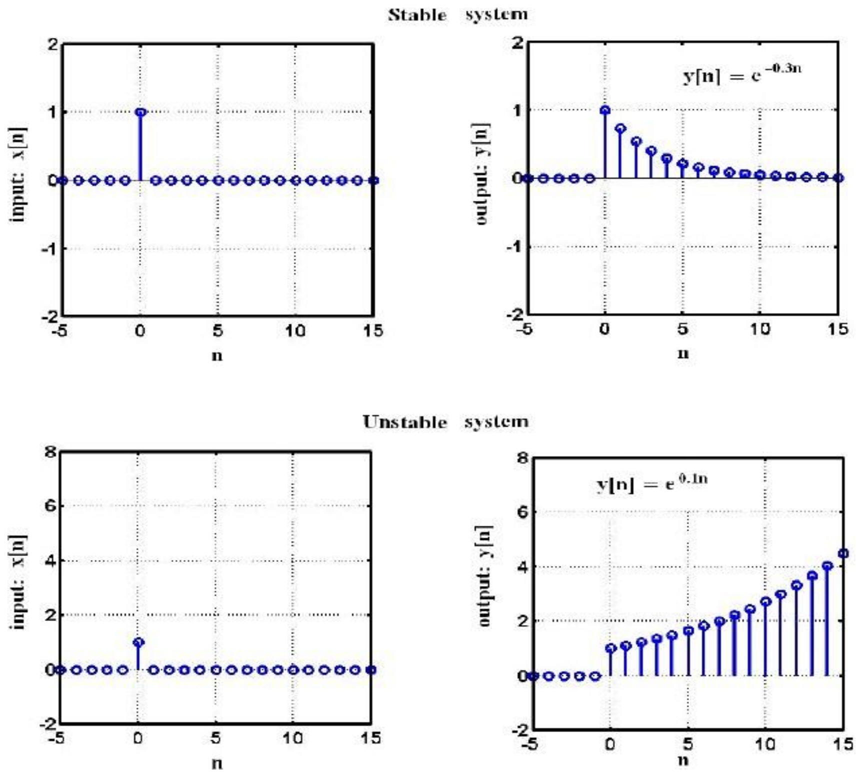


Fig 1.21 Examples for system stability

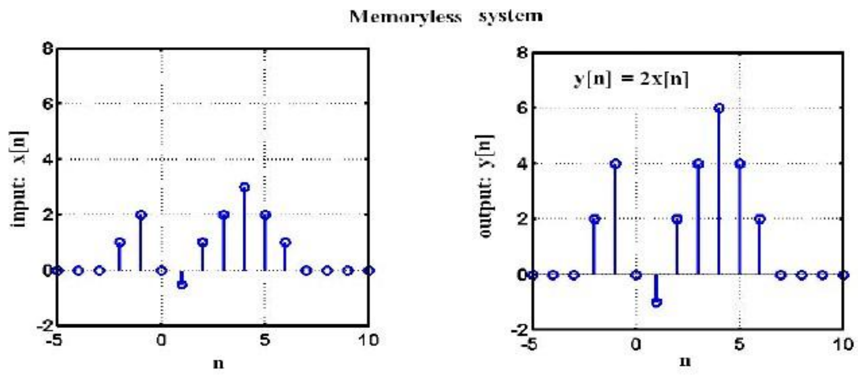
Memory
The system is memory-less if its instantaneous output depends only on the current input. In memory-less systems, the output does not depend on the previous or the future input.

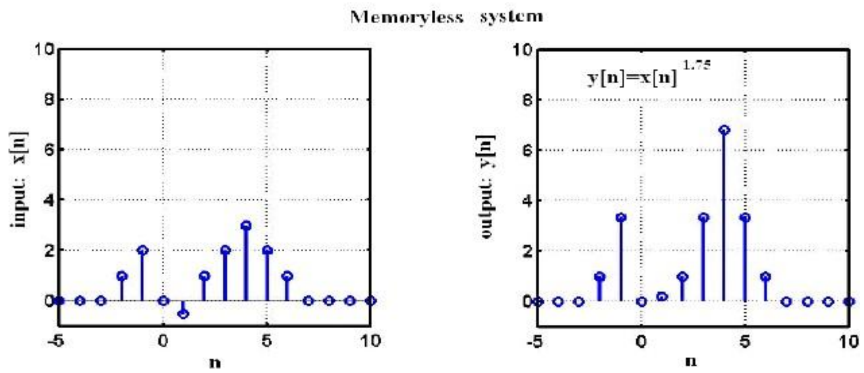
Examples of memory less systems:

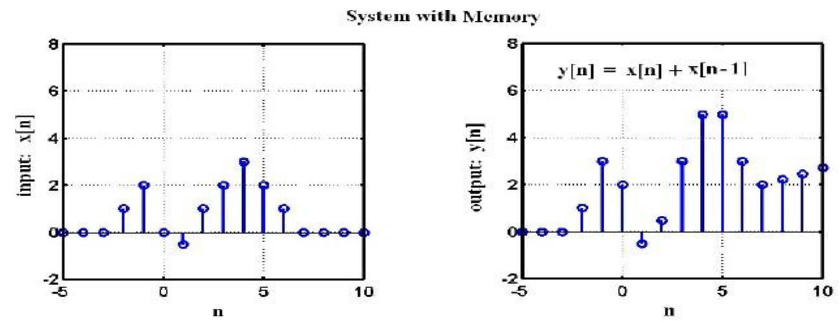
$$y[n] = ax[n]$$

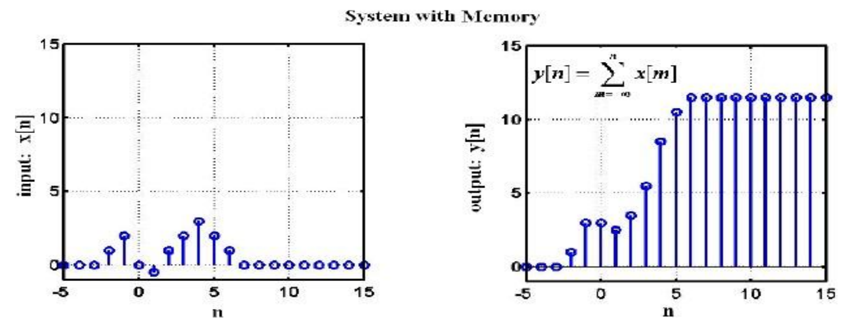
$$y[n] = ax^2[n]$$

$$i[n] = a_0 + a_1 v[n] + a_2 v^2[n] + a_3 v^3[n] + ...$$









Causality:

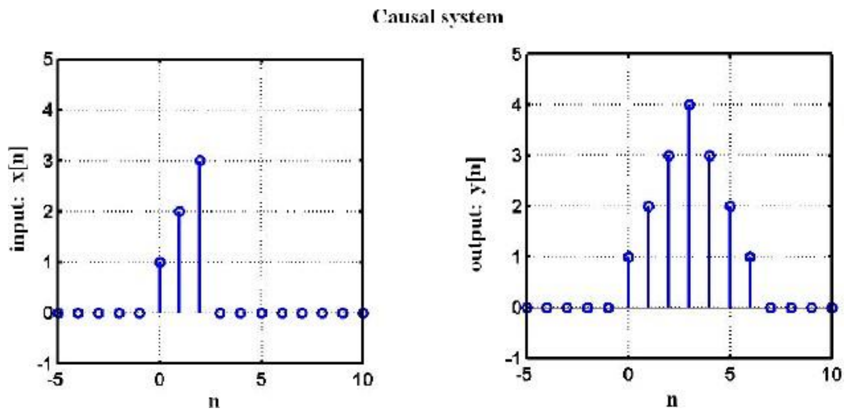
A system is causal, if its output at any instant depends on the current and past values of input. The output of a causal system does not depend on the future values of input. This can be represented as:

$$y[n] = F\{x[m]\} \quad for\ m \le n$$

For a causal system, the output should occur only after the input is applied, hence,

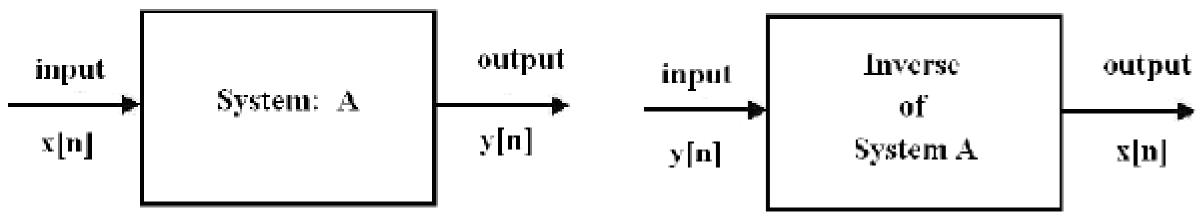
$$x[n] = 0 \; for\ n < 0 \text{ implies } \quad y[n] = 0 \; for\ n < 0$$

All physical systems are causal (examples in figure 7.5). Non-causal systems do not exist. This classification of a system may seem redundant. But, it is not so. This is because, sometimes, it may be necessary to design systems for given specifications. When a system design problem is attempted, it becomes necessary to test the causality of the system, which if not satisfied, cannot be realized by any means. **Hypothetical examples** of non-causal systems are given in figure below.



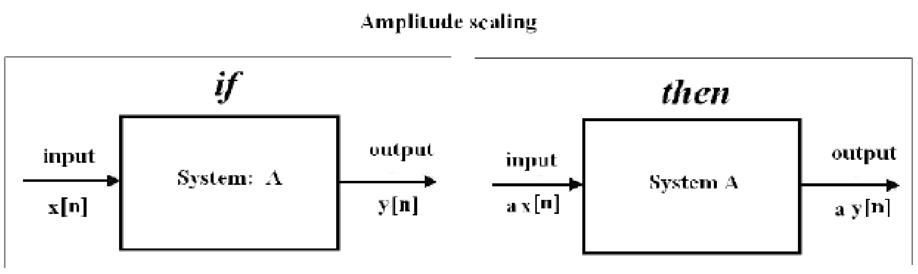
Invertibility:

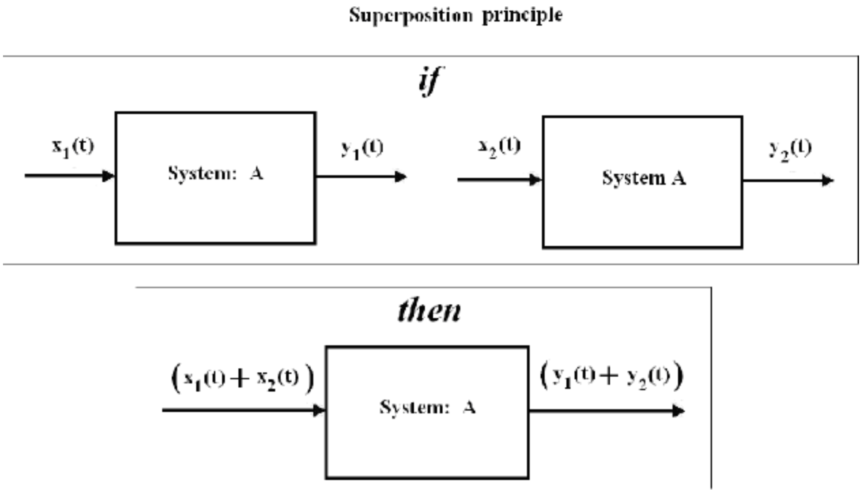
A system is invertible if,



Linearity:

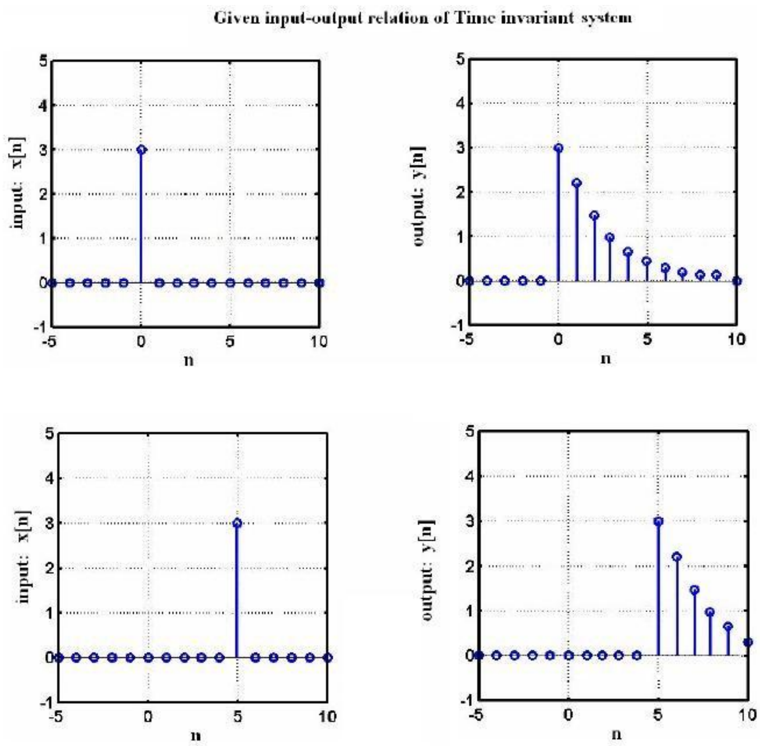
The system is a device which accepts a signal, transforms it to another desirable signal, and is available at its output. We give the signal to the system, because the output is s





Time invariance:

A system is time invariant, if its output depends on the input applied, and not on the time of application of the input. Hence, time invariant systems, give delayed outputs for delayed inputs.



Recommended Questions

1. What are even and Odd signals
2. Find the even and odd components of the following signals
   a $x(t) = \cos t + \sin t + \sin t \cos t$
   b. $x(t) + 1 + 3t^2 + 5t^3 + 9t^4$
   c. $x(t) + (1 + t^3)\cos t^3 10t$
3. What are periodic and A periodic signals. Explain for both continuous and discrete cases.
4. Determine whether the following signals are periodic. If they are periodic find the fundamental period.
   a. $x(t) = (\cos(2\pi t))^2$
   b. $x(n) = \cos(2n)$
   c. $x(n) = \cos 2\pi n$
5. Define energy and power of a signal for both continuous and discrete case.
6. Which of the following are energy signals and power signals and find the power or energy of the signal identified.
   a. $x(t) = \begin{cases} t, & 0 \le t \le 1 \\ 2 - t, & 1 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$
   b. $x(n) = \begin{cases} n, & 0 \le n \le 5 \\ 10 - n, & 5 \le n \le 10 \\ 0 & \text{otherwise} \end{cases}$
   c $x(t) = \begin{cases} 5\cos \pi t & -0.5 \le t \le 0.5 \\ 0 & \end{cases}$
   d $x(n) = \begin{cases} \sin \pi n, & -4 \le n \le 4 \\ 0 & \text{otherwise} \end{cases}$