MODULE-2

LESSON STRUCTURE

2.1 Stress-cycle

2.2 Endurance limit

2.3 Effect of Loading on Endurance Limit—Load Factor

2.4 Low cycle Fatigue

2.5 High cycle Fatigue

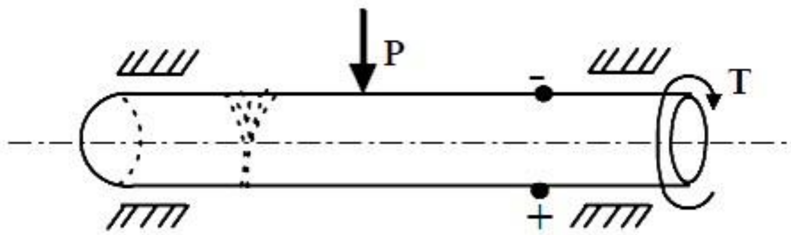
Objectives

- *Mean and variable stresses and endurance limit.*
- *S-N plots for metals and non-metals and relation between endurance limit and ultimate tensile strength.*
- *Low cycle and high cycle fatigue with finite and infinite lives.*
- *Endurance limit modifying factors and methods of finding these factors*
- *Design of components subjected to low cycle fatigue; concept and necessary formulations.*
- *Design of components subjected to high cycle fatigue loading with finite life; concept and necessary formulations.*
- *Fatigue strength formulations; Gerber, Goodman and Soderberg equations*.

DESIGN FOR FATIGUE STRENGTH

2.Introduction

Conditions often arise in machines and mechanisms when stresses fluctuate between a upper and a lower limit. For example in **figure-3.3.1.1**, the fiber on the surface of a rotating shaft subjected to a bending load, undergoes both tension and compression for each revolution of the shaft.



3.3.1.1F- *Stresses developed in a rotating shaft subjected to a bending load.*

Any fiber on the shaft is therefore subjected to fluctuating stresses. Machine elements subjected to fluctuating stresses usually fail at stress levels much below their ultimate strength and in many cases below the yield point of the material too. These failures occur due to very large number of stress cycle and are known as fatigue failure. These failures usually begin with a small crack which may develop at the points of discontinuity, an existing subsurface crack or surface faults. Once a crack is developed it propagates with the increase in stress cycle finally leading to failure of the component by fracture. There are mainly two characteristics of this kind of failures:

(a) Progressive development of crack.

(b) Sudden fracture without any warning since yielding is practically absent.

Fatigue failures are influenced by

(i) Nature and magnitude of the stress cycle.

(ii) Endurance limit.

(iii) Stress concentration.

(iv) Surface characteristics.

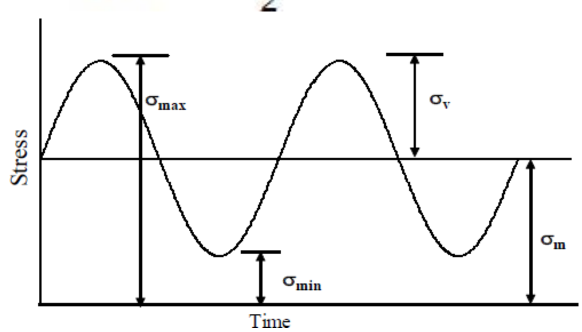
These factors are therefore interdependent. For example, by grinding and polishing, case hardening or coating a surface, the endurance limit may be improved. For machined steel endurance limit is approximately half the ultimate tensile stress. The influence of such parameters on fatigue failures will now be discussed in sequence.

2.1Stress Cycle

A typical stress cycle is shown in **figure- 3.3.2.1** where the maximum, minimum, mean and variable stresses are indicated. The mean and variable stresses are given by

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_{variable} = \frac{\sigma_{max} - \sigma_{min}}{2}$$



- A typical stress cycle showing maximum, mean and variable stresses.

2.2Endurance Limit

It has been found experimentally that when a material is subjected to repeated stresses; it fails at stresses below the yield point stresses. Such type of failure of a material is known as **fatigue.** The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. The fatigue of material is effected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals.

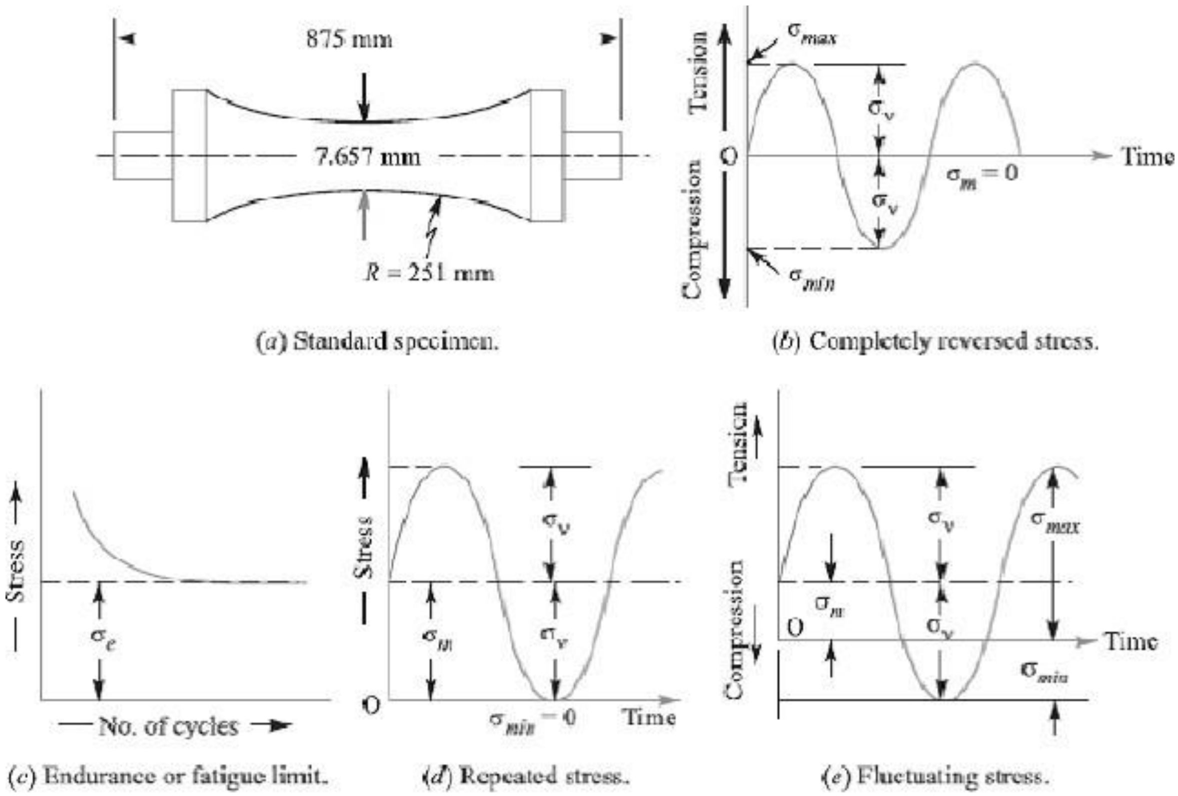


Fig.2. Time-stress diagrams.

In order to study the effect of fatigue of a material, a rotating mirror beam method is used. In this method, a standard mirror polished specimen, as shown in Fig.2 (*a*), is rotated in a fatigue testing machine while the specimen is loaded in bending. As the specimen rotates, the bending stress at the upper fibres varies from maximum compressive to maximum tensile while the bending stress at the lower fibres varies from maximum tensile to maximum compressive. In other words, the specimen is subjected to a completely reversed stress cycle. This is represented by a time-stress diagram as shown in Fig.2 (*b*). A record is kept of the number of cycles required to produce failure at a given stress, and the results are plotted in stress-cycle curve as shown in Fig.2 (*c*). A little consideration will show that if the stress is kept below a certain value as shown by dotted line in Fig.2 (*c*), the material will not fail whatever may be the number of cycles. This stress, as represented by dotted line, is known as endurance or ***fatigue limit*** (σ_e). It is defined as maximum value of the completely reversed bending stress which a polished standard specimen can withstand without failure, for infinite number of cycles (usually 107 cycles).

It may be noted that the term endurance limit is used for reversed bending only while for other types of loading, the term ***endurance strength*** may be used when referring the fatigue strength of the material. It may be defined as the safe maximum stress which can be applied to the machine part working under actual conditions.

We have seen that when a machine member is subjected to a completely reversed stress, the maximum stress in tension is equal to the maximum stress in compression as shown in Fig.2 (*b*). In actual practice, many machine members undergo different range of stress than the completely reversed stress. The stress verses ***time*** diagram for fluctuating stress having values σ_{min} and σ_{max} is shown in Fig.2 (*e*). The variable stress, in general, may be considered as a combination of steady (or mean or average) stress and a completely reversed stress component σ_v. The following relations are derived from Fig. 2 (*e*):

1. Mean or average stress,

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

2. Reversed stress component or alternating or variable stress,

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

For repeated loading, the stress varies from maximum to zero (*i.e.* $\sigma_{min} = 0$) in each cycle as shown in Fig.2 (*d*).

$$\sigma_m = \sigma_v = \frac{\sigma_{max}}{2}$$

3. Stress ratio, $R = \sigma_{max}/\sigma_{min}$. For completely reversed stresses, $R = -1$ and for repeated stresses, $R = 0$. It may be noted that R cannot be greater than unity.

4. The following relation between endurance limit and stress ratio may be used

$$\sigma'_e = \frac{3\sigma_e}{2 - R}$$

2.3 Effect of Loading on Endurance Limit—Load Factor

The endurance limit (σe) of a material as determined by the rotating beam method is for reversed bending load. There are many machine members which are subjected to loads other than reversed bending loads. Thus the endurance limit will also be different for different types of loading. The endurance limit depending upon the type of loading may be modified as discussed below:

Let, Kb = Load correction factor for the reversed or rotating bending load. Its value is usually taken as unity.

K_a = Load correction factor for the reversed axial load. Its value may be taken as 0.8.

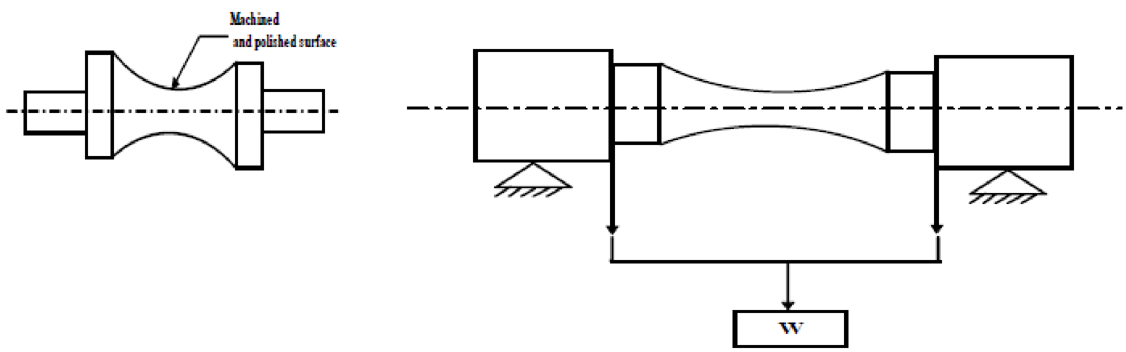
Ks = Load correction factor for the reversed torsional or shear load. Its value may be taken as 0.55 for ductile materials and 0.8 for brittle materials.

∴ Endurance limit for reversed bending load, $\sigma_{eb} = \sigma_e K_b = \sigma_e$

Endurance limit for reversed axial load, $\sigma_{ea} = \sigma_e K_a$

and endurance limit for reversed torsional or shear load, $\tau_e = \sigma_e K_s$

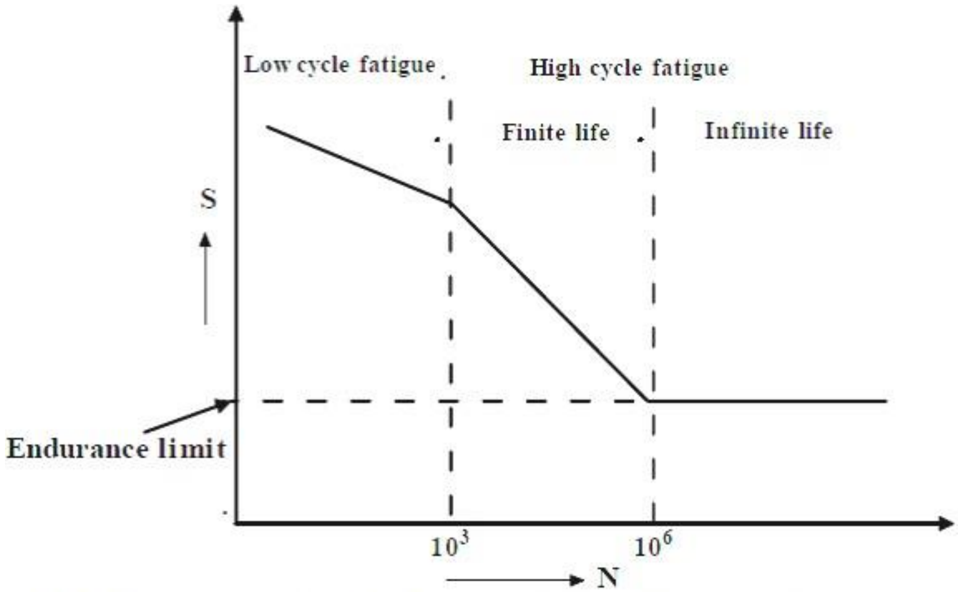
Figure- 3.3.3.1 shows the rotating beam arrangement along with the specimen.



(a) Beam specimen (b) Loading arrangement

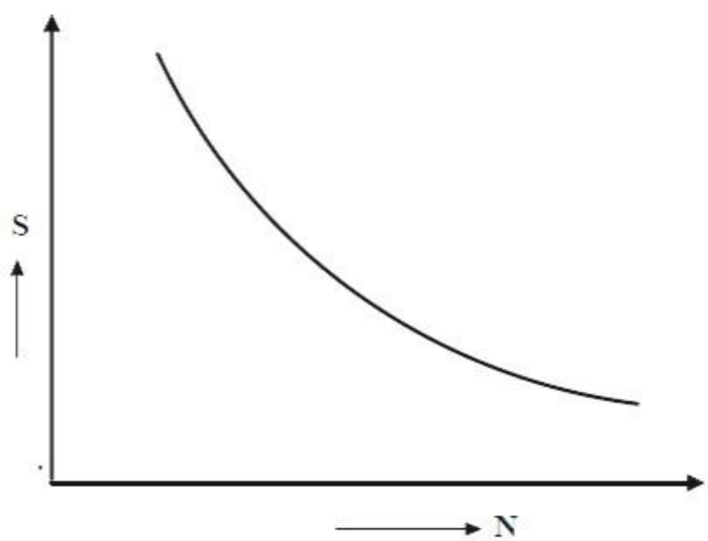
3.3.3.1F- A typical rotating beam arrangement.

The loading is such that there is a constant bending moment over the specimen length and the bending stress is greatest at the center where the section is smallest. The arrangement gives pure bending and avoids transverse shear since bending moment is constant over the length. Large number of tests with varying bending loads are carried out to find the number of cycles to fail. A typical plot of reversed stress (S) against number of cycles to fail (N) is shown in **figure-3.3.3.2**. The zone below 10^3 cycles is considered as low cycle fatigue, zone between 10^3 and 10^6 cycles is high cycle fatigue with finite life and beyond 10^6 cycles, the zone is considered to be high cycle fatigue with infinite life.



3.3.3.2F- A schematic plot of reversed stress (S) against number of cycles to fail (N) for steel.

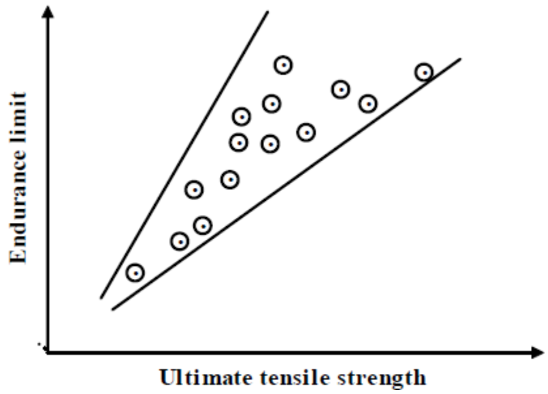
The above test is for reversed bending. Tests for reversed axial, torsional or combined stresses are also carried out. For aerospace applications and non-metals axial fatigue testing is preferred. For non-ferrous metals there is no knee in the curve as shown in **figure- 3.3.3.3** indicating that there is no specified transition from finite to infinite life.



3.3.3.3F- A schematic plot of reversed stress (S) against number of cycles to fail (N) for non-metals, showing the absence of a knee in the plot.

A schematic plot of endurance limit for different materials against the ultimate tensile strengths (UTS) is shown in **figure- 3.3.3.4**. The points lie within a narrow band and the following data is useful:

Steel	Endurance limit	~	35-60 % UTS
Cast Iron	Endurance limit	~	23-63 % UTS



3.3.3.4 F- A schematic representation of the limits of variation of endurance limit with ultimate tensile strength.

The endurance limits are obtained from standard rotating beam experiments carried out under certain specific conditions. They need be corrected using a number of factors. In general the modified endurance limit σ_e' is given by

$$\sigma_e' = \sigma_e C_1C_2C_3C_4C_5 / K_f$$

C_1 is the size factor and the values may roughly be taken as

$C_1 = 1, \quad d \leq 7.6$ mm

$= 0.85, \quad 7.6 \leq d \leq 50$ mm

$= 0.75, \quad d \geq 50$ mm

For large size C_1= 0.6. Then data applies mainly to cylindrical steel parts. Some authors consider 'd' to represent the section depths for non-circular parts in bending.

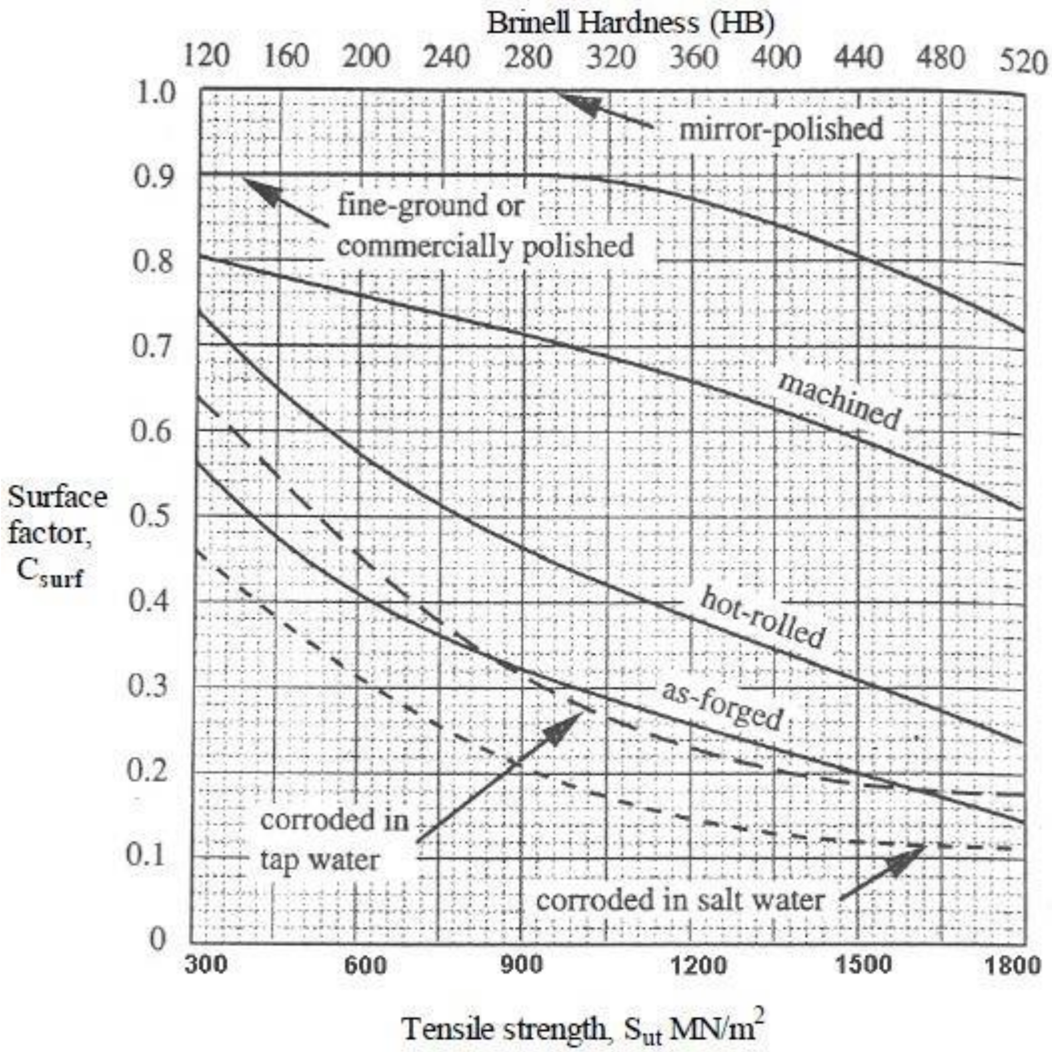
C_2 is the loading factor and the values are given as

$C_2 = 1$, for reversed bending load.

= 0.85, for reversed axial loading for steel parts

= 0.78, for reversed torsional loading for steel parts.

C_3 is the surface factor and since the rotating beam specimen is given a mirror polish the factor is used to suit the condition of a machine part. Since machining process rolling and forging contribute to the surface quality the plots of C_3 versus tensile strength or Brinnel hardness number for different production process, in **figure- 3.3.3.5**, is useful in selecting the value of C_3.



Variation of surface factor with tensile strength and Brinnel hardness for steels with different surface conditions

C_4 is the temperature factor and the values may be taken as follows:

$$C_4 = 1, \qquad \text{for } T \le 450^\circ C.$$

$$= 1-0.0058(T-450) \qquad \text{for } 450^\circ C < T \le 550^\circ C.$$

C_5 is the reliability factor

K_f is the fatigue stress concentration factor

Effect of Size on Endurance Limit—Size Factor

A little consideration will show that if the size of the standard specimen as shown in Fig.2 (*a*) is increased, then the endurance limit of the material will decrease. This is due to the fact that a longer specimen will have more defects than a smaller one.

Let K_{sz} = Size factor.

Then, Endurance limit,

$$\sigma_{e2} = \sigma_{e1} \times K_{sz} \quad \text{...(Considering surface finish factor also)}$$
$$= \sigma_{eb}.K_{sur}.K_{sz} = \sigma_e.K_b.K_{sur}.K_{sz} = \sigma_e.K_{sur}.K_{sz} \quad (\because K_b = 1)$$
$$= \sigma_{ea}.K_{sur}.K_{sz} = \sigma_e.K_a.K_{sur}.K_{sz} \quad \text{...(For reversed axial load)}$$
$$= \tau_e.K_{sur}.K_{sz} = \sigma_e.K_s.K_{sur}.K_{sz} \quad \text{... (For reversed torsional or shear load)}$$

The value of size factor is taken as unity for the standard specimen having nominal diameter of 7.657 mm. When the nominal diameter of the specimen is more than 7.657 mm but less than 50 mm, the value of size factor may be taken as 0.85. When the nominal diameter of the specimen is more than 50 mm, then the value of size factor may be taken as 0.75.

Effect of Miscellaneous Factors on Endurance Limit

In addition to the surface finish factor (K_{sur}), size factor (K_{sz}) and load factors K_b, K_a and K_s, there are many other factors such as reliability factor (K_r), temperature factor (K_t), impact factor (K_i) etc. which has effect on the endurance limit of a material. Considering all these factors, the endurance limit may be determined by using the following expressions:

1. For the reversed bending load, endurance limit,

$$\sigma'_e = \sigma_{eb}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

2. For the reversed axial load, endurance limit,

$$\sigma'_e = \sigma_{ea}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

3. For the reversed torsional or shear load, endurance limit,

$$\sigma'_e = \tau_e.K_{sur}.K_{sz}.K_r.K_t.K_i$$

In solving problems, if the value of any of the above factors is not known, it may be taken as unity.

Relation between Endurance Limit and Ultimate Tensile Strength

It has been found experimentally that endurance limit (σ_e) of a material subjected to fatigue loading is a function of ultimate tensile strength (σ_u).

For steel, $\sigma_e = 0.5\ \sigma_u$;
For cast steel, $\sigma_e = 0.4\ \sigma_u$;
For cast iron, $\sigma_e = 0.35\ \sigma_u$;
For non-ferrous metals and alloys, $\sigma_e = 0.3\ \sigma_u$

Factor of Safety for Fatigue Loading

When a component is subjected to fatigue loading, the endurance limit is the criterion for failure. Therefore, the factor of safety should be based on endurance limit. Mathematically,

$$\text{Factor of safety } (F.S.) = \frac{\text{Endurance limit stress}}{\text{Design or working stress}} = \frac{\sigma_e}{\sigma_d}$$

For steel, $\sigma_e = 0.8 \text{ to } 0.9\ \sigma_y$
σ_e = Endurance limit stress for completely reversed stress cycle, and
σ_y = Yield point stress.

Fatigue Stress Concentration Factor

When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor. Since the determination of fatigue stress concentration factor is not an easy task, therefore from experimental tests it is defined as

Fatigue stress concentration factor,

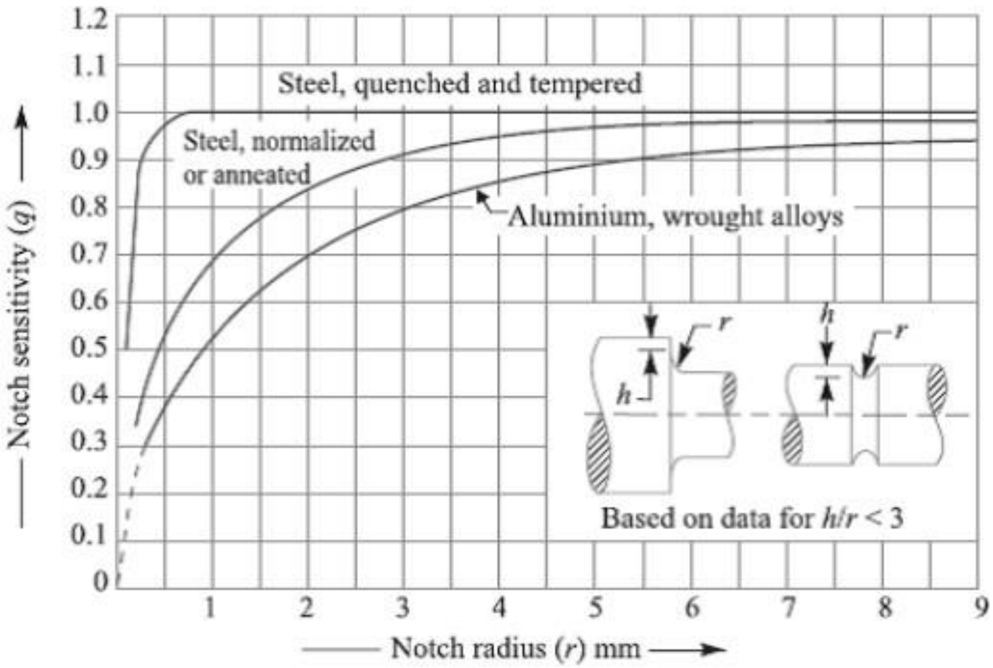
$$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

Notch Sensitivity

In cyclic loading, the effect of the notch or the fillet is usually less than predicted by the use of the theoretical factors as discussed before. The difference depends upon the stress gradient in the region of the stress concentration and on the hardness of the material. The term ***notch sensitivity*** is applied to this behaviour. It may be defined as the degree to which the theoretical effect of stress concentration is actually reached. The stress gradient depends mainly on the radius of the notch, hole or fillet and on the grain size of the material. Since the extensive data for estimating the notch sensitivity factor (q) is not available, therefore the curves, as shown in Fig., may be used for determining the values of q for two steals. When the notch sensitivity factor q is used in cyclic loading, then fatigue stress concentration factor may be obtained from the following relations:

$$q = \frac{K_f - 1}{K_t - 1}$$

Or



$$K_f = 1 + q\,(K_t - 1) \qquad \text{...[For tensile or bending stress]}$$

And

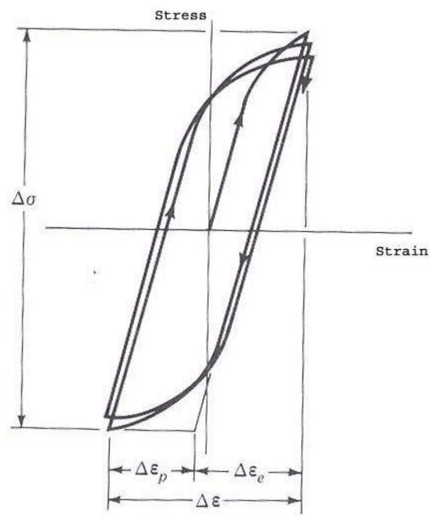
$$K_{fs} = 1 + q\,(K_{ts} - 1) \qquad \text{...[For shear stress]}$$

Where K_t = Theoretical stress concentration factor for axial or bending loading, and

K_{ts} = Theoretical stress concentration factor for torsional or shear loading.

2.4 Low Cycle Fatigue

This is mainly applicable for short-lived devices where very large overloads may occur at low cycles. Typical examples include the elements of control systems in mechanical devices. A fatigue failure mostly begins at a local discontinuity and when the stress at the discontinuity exceeds elastic limit there is plastic strain. The cyclic plastic strain is responsible for crack propagation and fracture. Experiments have been carried out with reversed loading and the true stress strain hysteresis loops are shown in **figure-3.4.1.1**. Due to cyclic strain the elastic limit increases for annealed steel and decreases for cold drawn steel. Low cycle fatigue is investigated in terms of cyclic strain. For this purpose we consider a typical plot of strain amplitude versus number of stress reversals to fail for steel as shown in **figure-3.4.1.2**.



***3.4.1.1F-** A typical stress-strain plot with a number of stress reversals (Ref.[4]).*

Here the stress range is $\Delta\sigma$. $\Delta\varepsilon_p$ and $\Delta\varepsilon_e$ are the plastic and elastic strain ranges, the total strain range being $\Delta\varepsilon$. Considering that the total strain amplitude can be given as

$$\Delta\varepsilon = \Delta\varepsilon_p + \Delta\varepsilon_e$$

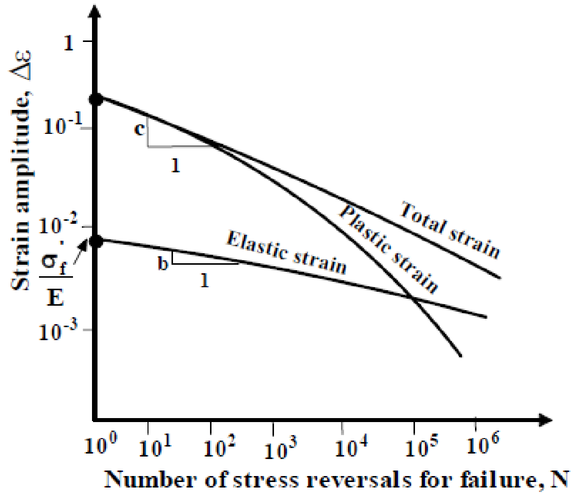
A relationship between strain and a number of stress reversals can be given as

$$\Delta\varepsilon = \frac{\sigma_f'}{E}(N)^a + \varepsilon_f'(N)^b$$

where σ_f and ε_f are the true stress and strain corresponding to fracture in one cycle and a, b are systems constants. The equations have been simplified as follows:

$$\Delta\varepsilon = \frac{3.5\sigma_u}{EN^{0.12}} + \left(\frac{\varepsilon_p}{N}\right)^{0.6}$$

In this form the equation can be readily used since σ_u, ε_p and E can be measured in a typical tensile test. However, in the presence of notches and cracks determination of total strain is difficult.



***3.4.1.2F-** Plots of strain amplitude vs number of stress reversals for failure.*

2.5 High Cycle Fatigue

This applies to most commonly used machine parts and this can be analyzed by idealizing the S-N curve for, say, steel, as shown in **figure- 3.4.2.1** .

The line between 10^3 and 10^6 cycles is taken to represent high cycle fatigue with finite life and this can be given by

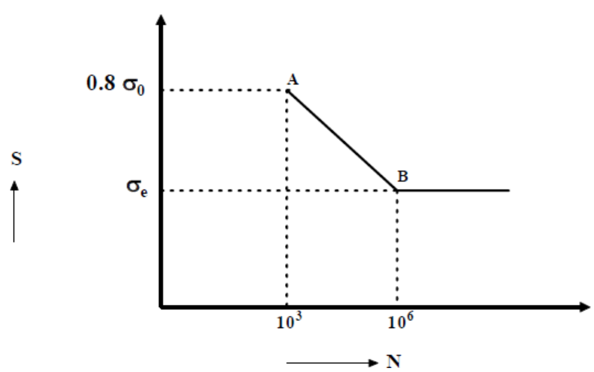
$$\log S = b \log N + c$$

where S is the reversed stress and b and c are constants.

At point A $\log(0.8\sigma_u) = b\log 10^3 + c$ where σ_u is the ultimate tensile stress

and at point B $\log\sigma_e = b\log 10^6 + c$ where σ_e is the endurance limit.

This gives $b = -\frac{1}{3}\log\frac{0.8\sigma_u}{\sigma_e}$ and $c = \log\frac{(0.8\sigma_u)^2}{\sigma_e}$



***3.4.2.1F-** A schematic plot of reversed stress against number of cycles to fail.*

Goodman Method for Combination of Stresses:

A straight line connecting the endurance limit (σe) and the ultimate strength (σu), as shown by line *AB* in figure given below follows the suggestion of Goodman. A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.

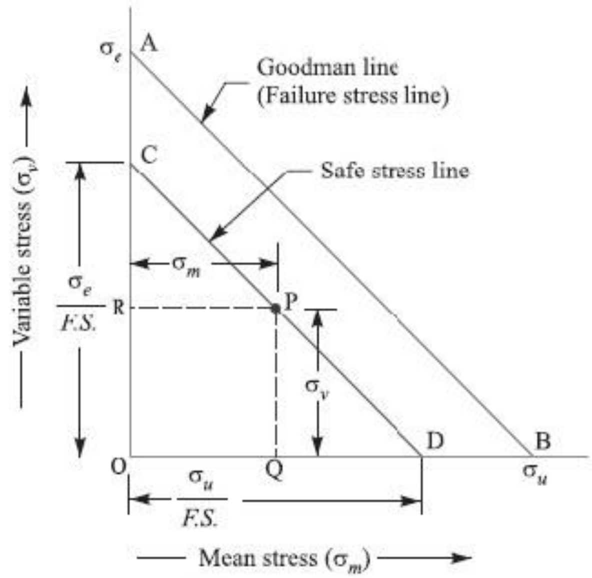


Figure 2.8

Now from similar triangles COD and PQD,

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD} = 1 - \frac{OQ}{OD} \qquad ...(\because QD = OD - OQ)$$

$$\therefore \qquad \frac{\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_u / F.S.}$$

$$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_u / F.S.}\right] = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_u}\right]$$

or

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e} \qquad ...(i)$$

This expression does not include the effect of stress concentration. It may be noted that for ductile materials, the stress concentration may be ignored under steady loads. Since many machine and structural parts that are subjected to fatigue loads contain regions of high stress concentration, therefore equation (*i*) must be altered to include this effect. In such cases, the fatigue stress concentration factor (*Kf*) is used to multiply the variable stress (σv). The equation **(*i*)** may now be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e} \qquad ...(ii)$$

where $F.S.$ = Factor of safety,

σ_m = Mean stress,

σ_u = Ultimate stress,

σ_v = Variable stress,

σ_e = Endurance limit for reversed loading, and

K_f = Fatigue stress concentration factor.

Considering the load factor, surface finish factor and size factor, the equation (*ii*) may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_b \times K_{sur} \times K_{sz}} \qquad ...(iii)$$

$$= \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}} \qquad ...(\because \sigma_{eb} = \sigma_e \times K_b \text{ and } K_b = 1)$$

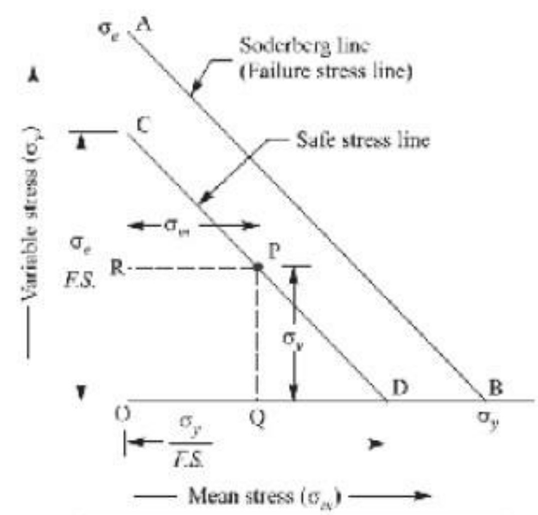
where K_b = Load factor for reversed bending load,

K_{sur} = Surface finish factor, and

K_{sz} = Size factor.

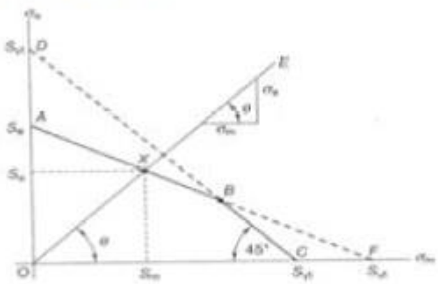
Soderberg Method for Combination of Stresses

A straight line connecting the endurance limit (σe) and the yield strength (σy), as shown by the line *AB* in following figure, follows the suggestion of Soderberg line. This line is used when the design is based on yield strength. the line *AB* connecting σe and σy, as shown in following figure, is called ***Soderberg's failure stress line.*** If a suitable factor of safety (*F.S.*) is applied to the endurance limit and yield strength, a safe stress line *CD* may be drawn parallel to the line *AB*. Let us consider a design point *P* on the line *CD*. Now from similar triangles *COD* and *PQD*,

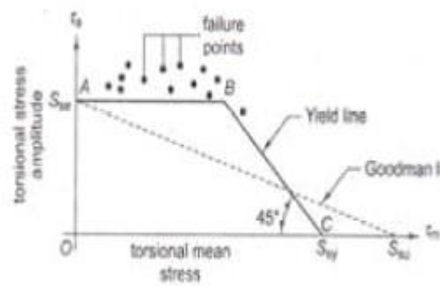


Modified Goodman Diagram:

In the design of components subjected to fluctuating stresses, the Goodman diagram is slightly modified to account for the yielding failure of the components, especially, at higher values of the mean stresses. The diagram known as modified Goodman diagram and is most widely used in the design of the components subjected to fluctuating stresses. There are two modified Goodman diagrams for the axial, normal or bending stresses and shear or torsion shear stresses separately as shown below. In the following diagrams the safe zones are ABCOA.



Modified Goodman Diagram for the Axial and Bending Stresses



Modified Goodman Diagram for the Torsional Shear Stresses

Figure 2.10

DESIGN APPROACH FOR FATIGUE LOADINGS

Design for Infinite Life

It has been noted that if a plot is made of the applied stress amplitude verses the number of reversals to failure to (S-N curve) the following behaviour is typically observed.

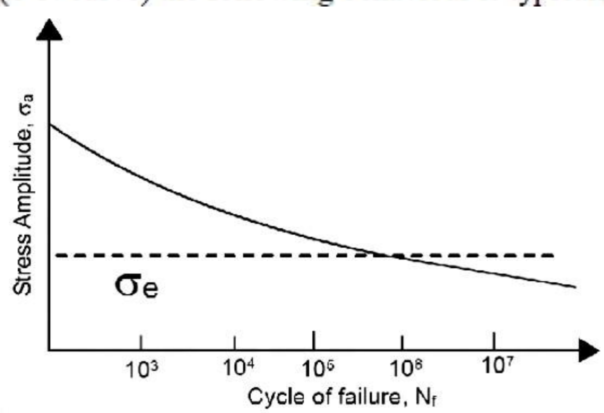


Figure 2.11

Completely Reversible Loading

If the stress is below the (the endurance limit or fatigue limit), the component has effectively infinite life. for the most steel and copper alloys. If the material does not have a well defined σ_e, Then, endurance limit is arbitrarily defined as Stress(0.35- 0.50) that gives For a known load (Moment) the section area/(modulus) will be designed such that the resulting amplitude stress will be well below the endurance limit.

Design approach can be better learnt by solving a problem.

Stress Concentration Factor

Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighborhood of the discontinuity is different. His irregularity in the stress distribution caused by abrupt changes of form is called ***stress concentration.*** It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc. In order to understand fully the idea of stress concentration, consider a member with different cross-section under a tensile load as shown in Fig. A little consideration will show that the nominal stress in the right and left hand sides will be uniform but in the region where the cross-section is changing, a re-distribution of the force within the member must take place. The material near the edges is stressed considerably higher than the average value. The maximum stress occurs at some point on the fillet and is directed parallel to the boundary at that point.

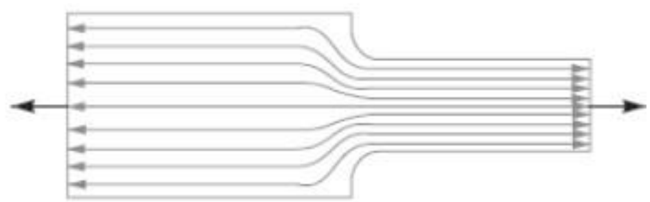


Fig. Stress concentration

Theoretical or Form Stress Concentration Factor

The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon net area. Mathematically, theoretical or form stress concentration factor,

$$K_t = \text{Maximum stress/ Nominal stress}$$

The value of K_t depends upon the material and geometry of the part. In static loading, stress concentration in ductile materials is not so serious as in brittle materials, because in ductile materials local deformation or yielding takes place which reduces the concentration. In brittle materials, cracks may appear at these local concentrations of stress which will increase the stress over the rest of the section. It is, therefore, necessary that in designing parts of brittle materials such as castings, care should be taken. In order to avoid failure due to stress concentration, fillets at the changes of section must be provided.

In cyclic loading, stress concentration in ductile materials is always serious because the ductility of the material is not effective in relieving the concentration of stress caused by cracks, flaws, surface roughness, or any sharp discontinuity in the geometrical form of the member. If the stress at any point in a member is above the endurance limit of the material, a crack may develop under the action of repeated load and the crack will lead to failure of the member.

Stress Concentration due to Holes and Notches

Consider a plate with transverse elliptical hole and subjected to a tensile load as shown in Fig.1(a). We see from the stress-distribution that the stress at the point away from the hole is practically uniform and the maximum stress will be induced at the edge of the hole. The maximum stress is given by

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{b}\right)$$

And the theoretical stress concentration factor,

$$K_t = \frac{\sigma_{max}}{\sigma} = \left(1 + \frac{2a}{r}\right)$$

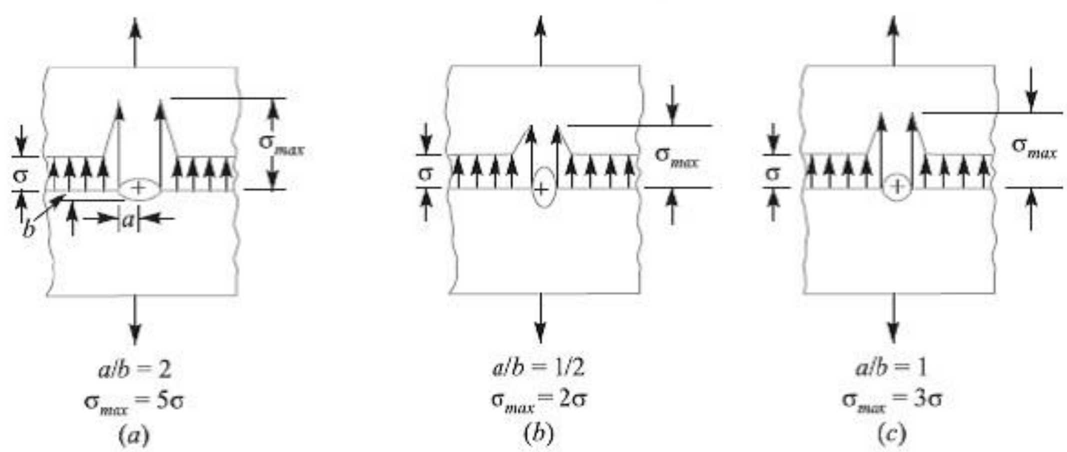


Fig.1. Stress concentration due to holes.

The stress concentration in the notched tension member, as shown in Fig. 2, is influenced by the depth a of the notch and radius r at the bottom of the notch. The maximum stress, which applies to members having notches that are small in comparison with the width of the plate, may be obtained by the following equation,

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{r}\right)$$

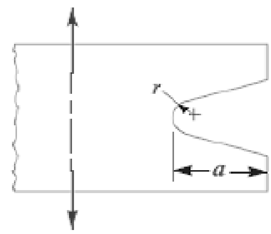


Fig.2. Stress concentration due to notches.

Methods of Reducing Stress Concentration

Whenever there is a change in cross-section, such as shoulders, holes, notches or keyways and where there is an interference fit between a hub or bearing race and a shaft, then stress concentration results. The presence of stress concentration can not be totally eliminated but it may be reduced to some extent. A device or concept that is useful in assisting a design engineer to visualize the presence of stress concentration and how it may be mitigated is that of stress flow lines, as shown in Fig.3. The mitigation of stress concentration means that the stress flow lines shall maintain their spacing as far as possible.

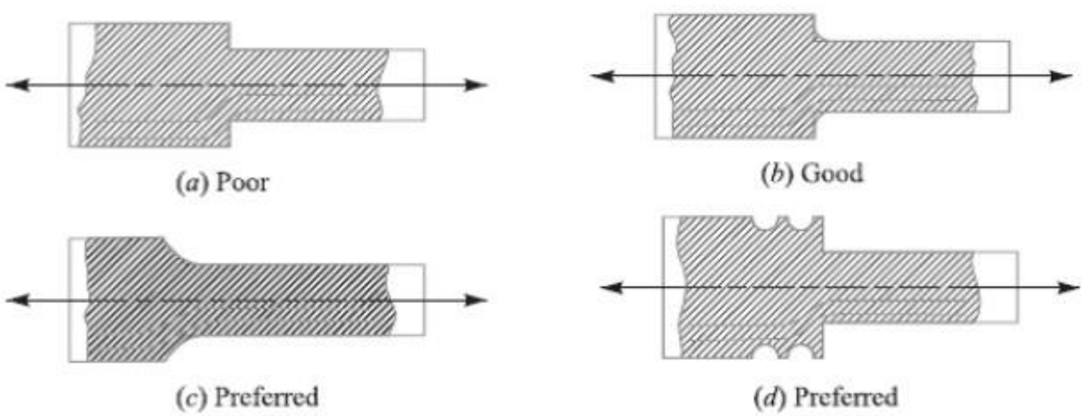


Fig.3

In Fig. 3 (a) we see that stress lines tend to bunch up and cut very close to the sharp re-entrant corner. In order to improve the situation, fillets may be provided, as shown in Fig. 3 (b) and (c) to give more equally spaced flow lines.

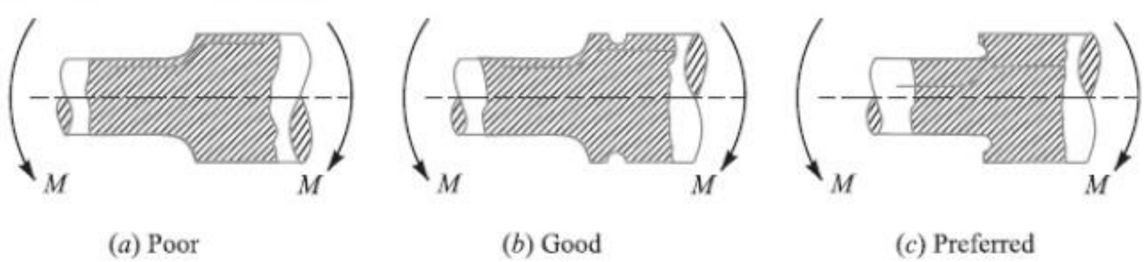
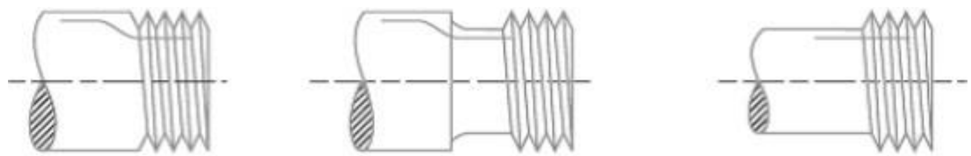


Fig. reducing stress concentration in cylindrical members with shoulders

(a) Poor (b) Preferred

Fig. Reducing stress concentration in cylindrical members with holes.



(a) Poor (b) Good (c) Preferred

Fig. Reducing stress concentration in cylindrical members with holes

Problems:

Q. A machine component is subjected to bending stress which fluctuates between 300 N/mm^2 tensile and 150 N/mm^2 compressive in cyclic manner. Using the Goodman and Soderberg criterion, calculate the minimum required ultimate tensile strength of the material. Take the factor of safety 1.5 and the endurance limit in reversed bending as 50% of ultimate tensile strength.

Solution:

Assuming the yield strength $S_{yt} = 0.55 \times$ ultimate strength S_{ut}

Mean stress " $\sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2} = \dfrac{300 + (-150)}{2} = 75\text{MPa};$

Amplitude stress " $\sigma_a = \dfrac{\sigma_{max} - \sigma_{min}}{2} = \dfrac{300 - (-150)}{2} = 225\text{MPa};$

As per Goodman Relation :

$$\frac{1}{\text{f.o.s}} = \frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e}$$

As given $S_e = 0.5 S_{ut}$

$$\frac{1}{1.5} = \frac{75}{S_{ut}} + \frac{225}{0.5S_{ut}} \Rightarrow \frac{1}{1.5} = \frac{525}{S_{ut}} \Rightarrow S_{ut} = 787.5\text{MPa};$$

As per Soderberg Relation :

$$\frac{1}{\text{f.o.s}} = \frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_e}$$

As given $S_{yt} = 0.55 S_{ut}$

$$\frac{1}{1.5} = \frac{75}{0.55S_{ut}} + \frac{225}{0.5S_{ut}} \Rightarrow \frac{1}{1.5} = \frac{586.36}{S_{ut}} \Rightarrow S_{ut} = 879.545\text{MPa};$$

Q. A circular bar is subjected to a completely reversed axial load of 150 kN. Determine the size of the bar for infinite life, if it is made of plain carbon steel having ultimate tensile strength of 800 N/mm^2 and yield point in tension of 600 N/mm^2. Assuming the surface finish factor as 0.80, size factor 0.85, reliability as 90%, and modifying factor for the stress concentration as 0.9.

Solution:

GIVEN:

Maximum Axial Load "P_{max}" $= +150kN$; *Minimum Axial Load* "P_{min}" $= -150kN$;

Ultimate tensile strength of the material of the bar "S_{ut}" $= 800\ N/mm^2$;

Yield point in tension of the material of the bar "S_{yt}" $= 600\ N/mm^2$;

Surface finish factor "k_a" $= 0.80$; *Size factor* "k_b" $= 0.85$; *Reliability factor* "k_c" $= 0.90$; *Modifying the stress concentration factor* "k_e" $= 0.90$.

ASSUMING:

The temperature factor $k_d = 1.0$; & miscelleneous factor $k_g = 1.0$;

Factor of safety $= 1.0$;

Endurance Limit of the material:

$$S_e' = 0.5 \times S_{ut} = 0.5 \times 800 = 400\text{MPa}$$

Modified Endurance Limit of the Material of the Bar:

$$S_e = k_a \times k_b \times k_c \times k_d \times k_e \times k_g \times S_e' = 0.80 \times 0.85 \times 0.90 \times 1.0 \times 0.9 \times 1.0 \times 400 = 220.32\text{ MPa};$$

Amplitude and Mean Normal Stresses:

$$\text{Amplitude Load "}P_a\text{"} = \frac{P_{max} - P_{min}}{2} = \frac{150 - (-150)}{2} = 150\text{ kN};$$

$$\text{Mean Load "}P_m\text{"} = \frac{P_{max} + P_{min}}{2} = \frac{150 + (-150)}{2} = 0\text{ kN};$$

$$\text{Amplitude Stress "}\sigma_a\text{"} = \frac{4 \times P_a}{\pi \times d^2} = \frac{4 \times 150}{\pi \times d^2} = \frac{190.9859}{d^2}\text{ kN/mm}^2 = \frac{190985.9}{d^2}\text{ N/mm}^2;$$

$$\text{Mean Stress "}\sigma_m\text{"} = 0\ \text{N/mm}^2;$$

Using Modified Goodman Diagram:

The Load Line becomes the amplitude axis.

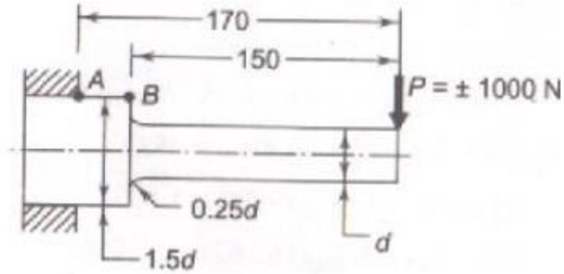
Hence the design equation may be written for infinite life as:

$$S_e \geq \sigma_a \Rightarrow 220.32 = \frac{190985.9}{d^2};$$

$$d \geq \sqrt{\frac{190985.9}{220.32}} \Rightarrow d \geq 29.44$$

$$d = 30\text{mm}.$$

(b) A cantilever beam made of cold drawn steel 20C8 (S_{ut} =540 N/mm^2) is subjected to a completely reversed load of 1000 N as shown in below figure. The corrected endurance limit for the material of the beam may be taken as 123.8 N/mm^2. Determine the diameter "d" of the beam for a life of 10000 cycles.



Solution:

GIVEN:

Maximum Axial Load "P_{max}" = +1000 N;

Minimum Axial Load "P_{min}" = −1000 N;

Ultimate tensile strength of the material of the bar "S_{ut}" = 540 N/mm^2;

Corrected Endurance Limit "S_e" = 123.8N / mm^2

USING THE S-N DIAGRAM:

The values of various points

$0.9S_{ut} = 0.9 \times 540 = 486\ \text{N/mm}^2;$

$\log_{10}(0.9S_{ut}) = \log_{10}(486) = 2.6866;$

$\log_{10}(S_e) = \log_{10}(123.8) = 2.0927;$

$\log_{10}(N) = \log_{10}(10000) = 4.0$

From the S-N Diagram:

$$\overline{AE} = \frac{\overline{AD} \times \overline{EF}}{\overline{DB}} = \frac{(2.6866 - 2.0927) \times (4-3)}{(6-3)} = 0.198$$

Therefor

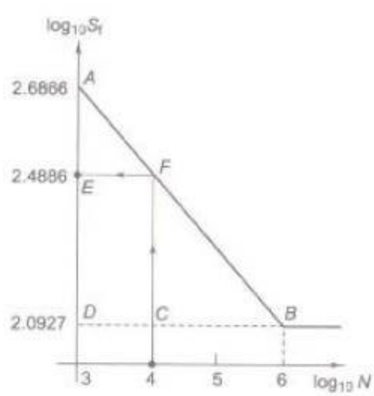
$\log_{10} S_f = 2.6866 - \overline{AE} = 2.6866 - 0.198 = 2.4886;$

$S_f = 308.03\ \text{N/mm}^2;$

And

$$S_f = \sigma_b = \frac{32M_b}{\pi d^3} \Rightarrow d^3 = \frac{32 \times M_b}{\pi \times S_f} = \frac{32 \times (1000 \times 150)}{\pi \times 308.3}$$

$d = 17.05$ mm.



Q. **A flat bar as shown in the figure 1 is subjected to an axial load F equal to 500 N. Assuming that the stress in the bar is limited to 200 MPa, determine the thickness of the bar. All dimensions are in mm.**

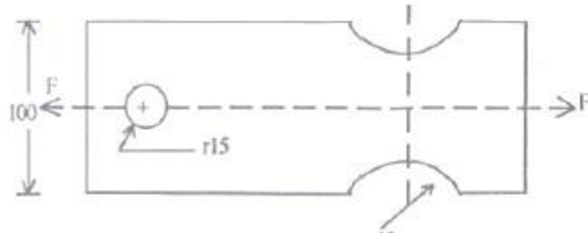


Figure 1

Solution: The stress concentration factor at circular hole k_t =2.35

The stress concentration factor at circular hole k_t =1.78

Hence the critical section is at the section of circular hole. The stress magnitude induced at this location

$$\sigma = \frac{F}{t \times (100-30)} = \frac{500}{t \times 70} = \frac{7.143}{\mathrm{t}} N/mm^2;$$

For successful design

$$\frac{7.143}{\mathrm{t}} \le 200;$$
$$t \ge 0.036mm;$$

However the minimum cross section area "A" =62t mm^2;

$$\sigma = \frac{F}{62t} = \frac{500}{t \times 62} = \frac{8.06452}{\mathrm{t}} N/mm^2;$$

For successful design

$$\frac{8.06452}{\mathrm{t}} \le 200;$$
$$t \ge 0.0403mm;$$

Hence the thickness of the plate = 0.0403 mm.

Q. A forged steel bar 50 mm in diameter is subjected to a reversed bending stress of 300 MPa. The bar is made of 40C8. Calculate the life of the bar for a reliability of 90%

Given: ***The material 40C8,***

The diameter of the shaft = d=50mm;

Reversed bending stress = 300MPa.

Reliability = 90%

Assuming: *σ_{ut}= 600MPa ; σ_{yt}=380 MPa Fatigue stress concentration factor=1.612*

Assuming the size factor $Ksz = 0.85$;

Surface finish factor $Ksur = 0.89$

Reliability factor K_{re}=0.892

Endurance Limit:

$$S_e' = 0.5\sigma_{ut} \qquad \because \sigma_{ut} < 1400MPa;$$
$$S_e' = 0.5 \times 600 = 300MPa;$$

Modified Endurance Limit:

$$S_e = K_{sz} \times K_{sur} \times K_{re} \times \frac{1}{K_f} \times S_e'$$
$$= 0.85 \times 0.89 \times 0.892 \times \frac{1}{1.612} \times 300 = 125.583\,\text{MPa}$$

Using the S-N diagram

$$\log_{10}(S_{ut}) = \log_{10}(600) = 2.778;$$
$$0.9 \times \log_{10}(S_{ut}) = 2.500$$
$$\log_{10}(S_e) = \log_{10}(125.583) = 2.099;$$
$$\log_{10}(\sigma_a) = \log_{10}(300) = 2.477;$$
$$\frac{2.50 - 2.099}{6 - 3} = \frac{2.477 - 2.099}{6 - \log_{10} N}$$
$$6 - \log_{10} N = 2.82793$$
$$\log_{10} N = 6 - 2.82793 = 3.17207$$
$$N = 1486.1752\,\text{reversals}$$

Q. A shaft subjected to bending moment varying from -200 N m to +500 N m and a varying torque from 50 N m to 175 N m. If material of the shaft is 30C8, stress concentration factor is 1.85, notch sensitivity is 0.95 reliability 99.9% and factor of safety is 1.5, find the diameter of the shaft.

Solution: Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{500+(-200)}{2} = 150N-m\ ;$$

Amplitude or variable bending moment

$$M_a = \frac{M_{max} - M_{min}}{2} = \frac{500-(-200)}{2} == 350N-m$$

Mean or average torque,

$$T_m = \frac{T_{max} + T_{min}}{2} = \frac{175+(50)}{2} = 112.5N-m\ ;$$

Amplitude or variable torque

$$T_a = \frac{T_{max} - T_{min}}{2} = \frac{175-(50)}{2} == 62.5N-m$$

Equivalent mean and amplitude bending moments

$$M_{em} = \frac{1}{2}\left[M_m + \sqrt{M_m^2 + T_m^2}\right] = \frac{1}{2}\left[150 + \sqrt{150^2 + 112.5^2}\right] = 168.75\text{N-m} = 168750\text{ N-mm};$$

$$M_{am} = \frac{1}{2}\left[M_a + \sqrt{M_a^2 + T_a^2}\right] = \frac{1}{2}\left[350 + \sqrt{350^2 + 62.5^2}\right] = 352.77\text{N-m} = 352770\text{ N-mm};$$

Mean or average bending stress,

$$\sigma_m = \frac{32M_{em}}{\pi \times d^3} = \frac{32 \times 168750}{\pi \times d^3} = \frac{1718874}{d^3};\ ;$$

Amplitude or variable bending moment

$$\sigma_a = \frac{32M_a}{\pi \times d^3} = \frac{32 \times 352.77}{\pi \times d^3} = \frac{3593286}{d^3};$$

Material properties: σ_{ut}= 490MPa ; σ_{yt}=270 MPa (assumed)

Given Notch sensitivity $q = 0.95$;

Assuming the size factor $Ksz = 0.85$;

Surface finish factor $Ksur = 0.89$

Reliability factor K_{re}=0.75 corresponding to 99.9% reliability (Assumed)

The fatigue stress concentration factor

$$K_f = 1 + q(K_t - 1) = 1 + 0.95(1.85 - 1) = 1.8075$$

Endurance Limit:

$$S'_e = 0.5\sigma_{ut} \qquad \because \sigma_{ut} < 1400MPa;$$
$$S'_e = 0.5 \times 490 = 245MPa;$$

Modified Endurance Limit:

$$S_e = K_{sz} \times K_{sur} \times K_{re} \times \frac{1}{K_f} \times S'_e = 0.85 \times 0.89 \times 0.75 \times \frac{1}{1.8075} \times 245 = 76.91\text{MPa}$$

We know that according to Soderberg formula;

$$\frac{1}{f.o.s.} = \frac{\sigma_m}{\sigma_{yt}} + \frac{\sigma_a}{S_e};$$
$$\frac{1}{1.5} = \frac{\frac{1718874}{\text{d}^3}}{270} + \frac{\frac{3593286}{\text{d}^3}}{76.91};$$
$$\frac{d^3}{1.5} = \frac{1718874}{270} + \frac{3593286}{76.91};$$
$$d^3 = 1.5 \times [100519 + 6083178] = 79630.291$$
$$\text{d} = 43.022 \text{ mm} \approx 45 \text{ mm};$$

We know that according to Goodman's formula

$$\frac{1}{f.o.s.} = \frac{\sigma_m}{\sigma_{ut}} + \frac{\sigma_a}{S_e};$$
$$\frac{1}{1.5} = \frac{\frac{1718874}{\text{d}^3}}{490} + \frac{\frac{3593286}{\text{d}^3}}{76.91};$$
$$\frac{d^3}{1.5} = \frac{1718874}{490} + \frac{3593286}{76.91};$$
$$d^3 = 75342.85$$
$$\text{d} = 42.235\text{mm} \approx 45 \text{ mm};$$

Hence the shaft diameter is 45 mm. Ans.

Outcomes

- Analyze the behaviour of machine components under static, impact, fatigue loading.

Questions

1. Derive Soderberg Equation.
2. Explain the various forms variable stresses
3. Define Endurance strength.
4. State and explain the factors for modifying endurance limit.
5. Explain with a neat graph the S-N curve

Further Reading

1. Design of Machine Elements, V.B. Bhandari, Tata McGraw Hill Publishing Company Ltd., New Delhi, 2nd Edition 2007.
2. Mechanical Engineering Design, Joseph E Shigley and Charles R. Mischke. McGraw Hill International edition, 6th Edition, 2009.