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SAI VIDYA INSTITUTE OF TECHNOLOGY

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PRINCIPLES OF COMMUNICATION SYSTEMS (15EC45)

IV SEMESTER ECE

MODULE 5: DIGITAL REPRESENTATION OF ANALOG SIGNALS

SYLLABUS: Introduction, Why Digitize Analog Sources?, The Sampling process, Pulse Amplitude Modulation, Time Division Multiplexing, Pulse-Position Modulation, Generation of PPM Waves, Detection of PPM Waves, The Quantization Process, Quantization Noise, Pulse–Code Modulation: Sampling, Quantization, Encoding, Regeneration, Decoding, Filtering, Multiplexing, Application to Vocoder.

MODULE 5

DIGITAL REPRESENTATION OF ANALOG SIGNALS

* BYALLABUS :

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*INTRODUCTION:

- * The evolution from analog to digital transmission is the conversion of common information sources such as voice and music, which are inherently analog to digital representation.
- * In the first step from analog to digital, an analog source is sampled at discrete times. The resulting analog samples are then transmitted by means of analog pulse modulation.
- * In the second step from analog to digital, an analog source is not only sampled at discrete times but the samples themselves are also quantized to discrete levels

* WHY DIGITIZE ANALOG SOURCES:

there are many advantages that the transmission of digital information has over analog.

- i) Digital systems are less sensitive to noise than analy.
- Services. For example, video and the accompanying sound track, into the same transmission scheme.
- 3) The transmission scheme can be relatively independent of the source. For example, a digital transmission scheme that transmits voice at 10 kbps could also be used to transmit computer data at 10 kbps.
- 4) Circuitry for handling digital engrale is easier to repeat and digital Circuits are less sensitive to physical effects such as vibration and temperature.
- 5) Digital signals are simpler to characterize interms of bits I and O and do not have variability as analog signals. This makes the associated hardware easier to design.
- 6) various media sharing stratergies known as multiples ing techniques are more easily implemented with digital transmission stratergies.
- 1) Digital techniques make it easier to specify complex standarde that may be shared on a worldwide basis.

8) The techniques such as equalization, especially adaptive versions, are easier to implement with digital transmission techniques.

* COMPARISON BETWEEN ANALOG & DIGITAL COMM SYSTEMS:

S.NO	Parameter	Analog system	Digital System:
·>	Boundwidth	Less	More
&>	Error Correction and Detection	Not possible	Possible
3>.	Immune to Noise	Less	Meie
4>	System Complexity	Less .	Morea
5>	System Cost	Mosé	Lessi
6>	Quality of reconstruction	Good	Very Good
1	Synchronisation	Not required	required
8>	privacy and security to data	Not possible	possible
	Flexibility and	Lese	More
lo>	Poner required	More	Less
11>	Implementation	Difficult—	Easy
12>	Programming	Not possible	Possible

* SAMPLING PROCESS :

Statement: Sampling theorem states that any continuous time signal can be completely represented in its samples and recovered back if the sampling frequency is greater than or equal to twice the highest frequency component of base band signal.

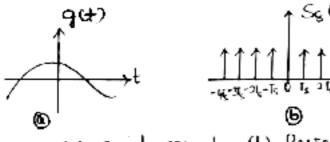
That is Sampling frequency, $f_s \ge 2W$.

Where W= Highest frequency in base band continuous time signal.

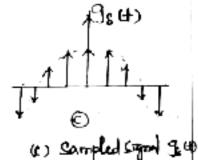
This condition is also called Nyquist condition for sampling process.

Explanation and Proof:

* consider an arbitrary signal get) of finite energy. which is specified for all time. A segment of the signal get) is shown in fig(1)(a) suppose, that we sample the signal get) instantaneously and at a uniform rate, once every Ts' seconde. Consequently, we obtain an infinte seguence of samples spaced To seconds apart and desded by {9(aTi)}, where in takes on all possible integer values. He refer to To as the sampling period, and to its reciprocal for 1/2 as the sampling rate. This ideal from of sampling is called instantaneous sampling. g(±) → 98(H) = 9(H)·S8(H) SgH)



Fig(1): (a) analogsignal (b) Periodic Signal (9,00)



* Ret 98(t) denote the signal Obtained by individually weighting the elemente of a periodic segmence spaced Te seconds. Therefore, sampled output 98(t) is given by,

 $q_{S}(t) = q(t) \cdot S_{S}(t)$

* Ret SgCt) denotes the periodic impulse train and is represented as,

$$S_s(t) = \sum_{n=-\infty}^{\infty} S(t-n\tau_s)$$

substituting Eq. 7(1) in Eq. 7(1) me get

$$g_{\varepsilon}(t) = q(t) \cdot \sum_{n=-\infty}^{\infty} S(t-nT_{\varepsilon})$$

using shifting property of impulse function

WHERE, $q(t) \cdot S(t-n\tau_s) = q(n\tau_s) \cdot S(t-n\tau_s)$

$$\therefore q_s(t) = \sum_{n=-\infty}^{\infty} q(nt_s) s(t-nt_s)$$

For frequency domain consider,

$$q_8(t) = q(t).S_8(t)$$

Taking Fourier Transform on both eider, me get

(4

where,

$$S_8(f) = t_8 \sum_{n=-\infty}^{\infty} S(f-nf_8)$$
 (5)

substituting Eq.7 (5) in Eq.3 (4) we get.

From convolution property of impulse function

wilt,
$$G(f) * S(f-nfs) = G(f-nfs)$$

$$g_s(t) = \int_{S} \sum_{n=-\infty}^{\infty} G(t-n) - G(t)$$

Eq.360 can be servitten as,

$$q_{s(f)} = f_{s} q(f) + f_{s} \sum_{n=1}^{\infty} q(f-n) - f_{s}$$

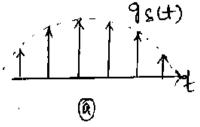
When the spectrum of $G_8(f)$ is passed through an LPF then the 2nd term in RHS of Eq. (7) is eliminated.

resulting in

$$98(f) = fs.9(f)$$

$$\therefore \quad G(t) = \frac{1}{ts} \cdot G_8(t) \qquad (8)$$

where for = 2W



Sinc Sinc D

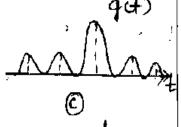


Fig : Recovering 9(4) signal from sequence of samples 9(1)

Now, we may state the sampling theorem for strictly hand limited signals of finite energy into two equivalent parts:

A bandlimited signal of finite energy, which only has frequency components less than "W" Hertz, is completely described by specifying the values of the signal at instants of time separated by Jw seconds.

A bandlimited signal of finite energy, which only has frequency components been than "W" Hertz, may be completely recovered from a knowledge of its samples taken at the rate of IN samples per second.

The sampling rate of SIN samples per second, for a signal bandwidth of 'W' Hertz is called the Nyquist rate; its reciprocal /3W (measured in seconds) is called the Nyquist interval.

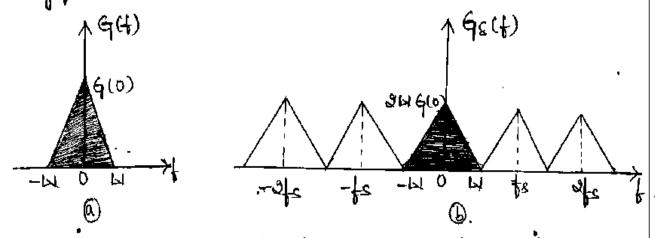
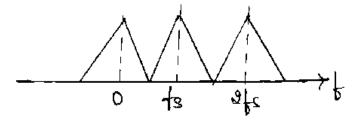


Fig : (a) 8 pectrum of a strictly bound limited 8. gnal g(t).

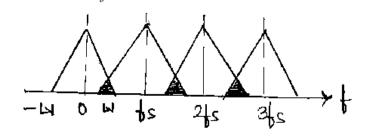
(b) 8 pectrum of a sampled version of g(t) for $T_8 = \frac{1}{3N}$.

NOTE: the concept of undersampling and over sampling is explained below.

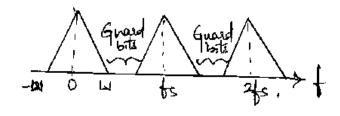
bluen bampling frequency for = 2 W then this type of sampling is called correct sampling and here there is no aliesing effect seen in this mechanism. It when for = 2 W.



will be aliasing effect induced here.



3) When to > 2 W then it is over sampling and there will no aliasing effect.



* PROBLEMS:

(1) An amalog signal is expressed by the equation 2Ct) = 3 cos sout + 10 sin 3007+ + cos 1007+. Calculate the Nyquist rate and Nyquist Interval for this signal.

Soff: Given

$$n(4) = 3 cos 500t + 10 sin 3007t + Cos 1007t _____(1)$$

comparing Eq.30 with std equation

computing w, w, and we then

$$\Omega_1 = 50 \text{ At}$$

$$\omega_3 = 100\pi$$

Nyquist Interval, Tg = 1/fs = 1/300 Hz

(2) An analog signal is expressed by the equation $\chi(t) = \frac{1}{17} \cos(4000\pi t) \cos(1000\pi t)$. Calculate the Nyquest. rate and Nyquest Interval for this signal.

到于: Given

$$\chi(t) = \frac{1}{2\pi} \cos(4000\pi t) \cos(1000\pi t)$$
. _____(1)

WIKE COSA. COSB = [COS(A-B) + COS(A+B)]

$$\Omega_0 = 5000T$$

Nyquist Rate for = 2xfm = 2x2500

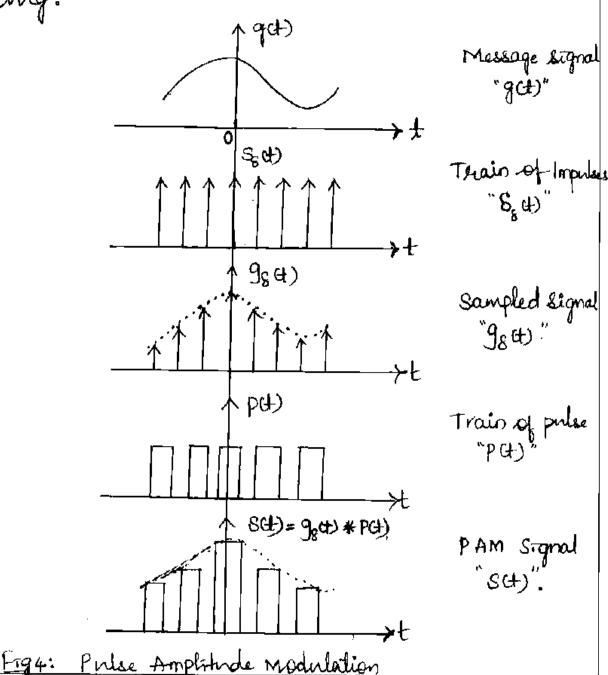
$$T_8 = 0.2 \, \text{ms}$$

* PULSE AMPLITUDE MODULATION

* It is an analog pulse Modulation Scheme in which the amplitudes of a train of rectangular carrier pulses are varied in accordance with the sample values of the modulating signal.

* In PAM, the top of each modulated rectangular pulse is maintained flat. So PAM is same as flat-top

sampling.



The waveform of a PAM signal is illustrated in fig(4). * Ret SCt) denote the sequence of flat-top samples or PAM signal, and it is expressed as

$$SCH) = \sum_{n=-\infty}^{\infty} q(n\tau_n) p(T-n\tau_n)$$
 (1)

where,

q(nte) is the sample value of q(t) obtained at time

Ts is sampling period.

P(t) is standard rectangular pulse train of duration T.

Advantages of PAM:

It is a base for all the digital modulation technique.

Disadvantages of PAM:

Due to Nyquiet Criteria, it requires high bandwidth for transmission.

since, amplitude keeps varying, so there is noise associted with it.

* Detection of PAM signal

The origeral message signal mot) is obtained by passing PAM signal to the reconstruction filter followed by equilizar.

PAM Signal Reconstruction Equilizer Message signal Message signal most Message signal most

* TIME DIVISION MULTIPLEXING : [TDM]

Time Division Multiplexing is a method of transmitting and receiving independent signals ones a common channel by means of synchronised switches at each end of transmission line so that each signal appears on the line only a fraction of time in an alternating pattern.

* Fig(5) shows the block diagram of TDM system.

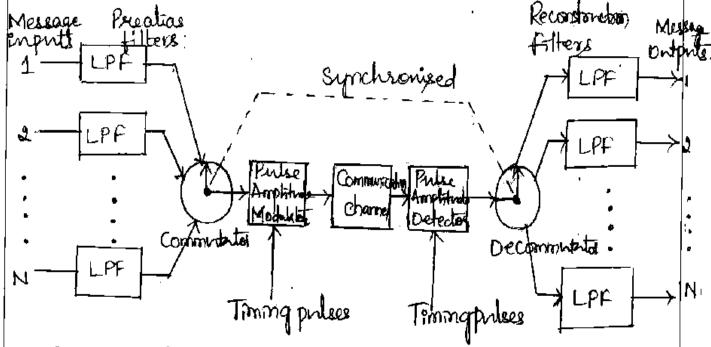


Fig 5: Block Diagram of TDM System.

* The concept of TDM is illustrated in the fig(5). The Lawpass filters are used to remove high frequency components present in the message signal. The output of the pre-alias filters are then fed to a commutator, which is usually implemented using electronic switching which is usually implemented using electronic switching

* The function of commutator is as follows:

- To take a narrow sample of each of the 'N' samples of input at a rate of to > 2W.
- is to segmentially interleane (multiplex) these 'N' samples inside a sampling interval Ts = 1/fc
- * The multiplexed signal is then applied to a pulse amplitude modulator whose purpose is to transform the multiplexed signal into a form suitable for transmoon over a common channel.
- * At the receiving end, the pulse amplitude demodulator performs the reverse operation of PAM and the decomm Intator distribute the signals to the appropriate low pass reconstruction filters. The decommitator operates in synchronisation with the communitator.

* PULSE - POSITION MODULATION

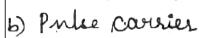
- * In pulse-dweation modulation (PDM), the samples of the message signal are used to vary the division of the individual pulses. This form of modulation is also referred to as Pulse-Width modulation os pulselength modulation.
- * In PPM, the position of a pulse relative to its unmodulated time of occurance is varied in accordance with the message signal as shown in fig(6)(d) for the case of sinusoidal modulation.

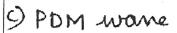
chet Te' denote the sample duration. Hong the sample $m(nT_e)$ of a message signal m(t) to modulate the position of the n^{t_7} pulse, we obtain the PPM signal

$$S(t) = \sum_{n=-\infty}^{\infty} q(t-nT_s - k_p m(nT_s))$$
 (1)

where kp is the sensitivity of the pulse-position modulator

a) Modulating signal m(t)







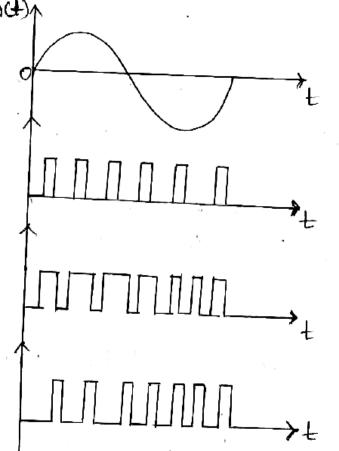


Fig (6): Illustrating two different forms of pulse-time modulation.

K GENERATION OF PPM WAVES :

The PPM signal which is generated is shown in fig(7)(a). The message signal mct) is first connected into a PAM signal by means of a sample and Hold

* DETECTION OF PPM WAVES :

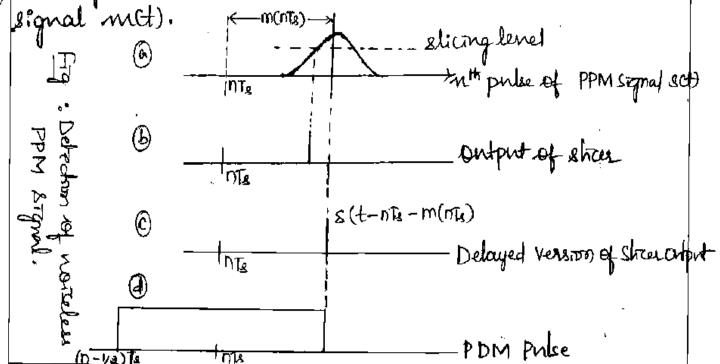
consider a PPM wave SCH) with uniform sampling, and assume that the message signal mct) is strictly bound limited. The operation of one type of PPM received may proceed as follows:

convert the received PPM wave into a PDM wave with the same modulation.

integrate this PDM wome using a donice with a finite integration time, thereby computing the area under each pulse of the PDM wave.

sample the output of the integrator at a uniform rate to produce a PAM wave, whose pulse amplitudes are proportional to the signal samples $m(n\tau_s)$ of the original PPM wave BCt).

4) Finally, demodulate the PAM wave to recover the message



* NOISE IN PULSE-POSITION MODULATION

* In a PPM system, the transmitted information is contained in the relative positions of the modulated pulses. The presence of additive noise affects the performance of such a system.

* The original to noise ratio, assuming a full-load sincepidal modulation, is therefore

$$(SNR)_{D} = \frac{T^{0}B_{T}T_{s}^{v}A^{v}}{32N_{0}} \qquad (1)$$

* The avg noise poncer in a message bandwidth H' is equal to WNo. The channel signal to noise vatio is

therefore, (SNR) = 3AX 4-T8 BTINNO

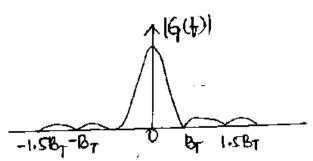


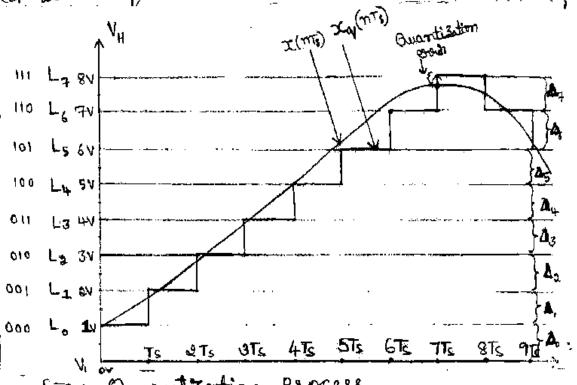
Fig: Amplitude spectrum of a raised Cosine pulse.

* Thue the figure of merit of a PPM system using a raised cosine pulse is as follows.

Figure of Merit = (SNR)0 = TOBITE A. X 4TEBILINO
(SNR)c 32NO 3A2

* THE QUANTIZATION PROCESS

The process of transforming sampled amplitude values of a message signal into a discrete amplitude value (levels) is referred to as quantization.



* The signal xCt) vohose excursion is confined to the range from VL to VH being divided into 8-equal level.

* step size is denoted by 's' and is given by

$$\Delta = \frac{V_H - V_L}{I}$$

where L = 2R and R is not of bits.

$$. \quad \Delta = \frac{VH - VL}{RR}$$

* The difference between the continuous amplitude sample level and quantized signal level is known as quantization error.

ect) = $\chi_q(n\tau_e) - \chi(n\tau_e)$

where quantization error varies from +4 to -4.

a noise at the output of the quantizer and this noise

is referred to as Quantization noise.

* Consider fig (A), the sampling, Quantizing and coding of an analog signal is as follows.

_ ·								
sampled vortnes of an analog signal	1,7	9. 7V		57	6-24	7-28	オ・ブV	7 .4V
Jeanest Quantizer Jenel	L	و_ا	لع	L4	L ₅	L	L-7	L8
gnorfizer level voltage	l	l	l l	l	ľ	7 V	8 <i>v</i>	7v
onary code	וסט	010	011	100	101	ם וו	117	110

* There are two types of quantizer they are

· Mid-tread type quantizer

· Mid-riser type quantizer

Midrice Quntizer

-case graph. * Here anontization levels are even mber.

* QUANTIZATION NOISE

as the difference between the input signal m' and the output signal V'. This error is called Quantization noise. Fig.B, illustrates a typical variation of the Quantization noise as a function of time, assuming the use of a uniform quantizer of the midtread type.

and storms of st

Fig@: Illnettation of Quantization process and Noise & Ret the random variable "Q' denotes the quantization essor and 'q' its sample value.

$$q = m - v$$
 — (i)

* Consider then an input 'm' of continuous amplitude in the range (-mmax, mmax) then, the step-size of the quantizer is given by,

$$\Delta = \frac{2 \, \text{Mmax}}{L}$$

$$\frac{3}{3} = \frac{1}{3} \left[\frac{9^{3}}{3} \right]_{-A/9}^{A/2} = \frac{1}{3A} \left[\left(\frac{A}{3} \right)^{3} - \left(\frac{-A}{3} \right)^{3} \right] = \frac{1}{3A} \left[\frac{A^{3}}{8} - \left(\frac{A}{3} \right)^{3} \right] = \frac{1}{3A} \left[\frac{A^{3}}{8} + \frac{A^{3}}{8} \right] = \frac{1}{3A} \times 2 \left(\frac{A^{3}}{84} \right) = \frac{1}{3} \cdot \frac{A^{9}}{4}$$

This is known as "Mean squared wise anotization error" or Normalized Noise power or Quantization error interms of forms.

* Ret us consider "R" which denote the number of kits per sample then the quantized level is given by,

substituting Eq. (4) in Eq. 30 me get,

$$\Delta = 2 \frac{m_{\text{max}}}{2R}$$

Now enbetitute Eq. 50 in Eq. 00 we get

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2} m_{\text{max}}}{\sqrt{2}} = \frac{\sqrt{2} m_{\text{max}}}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$60 = \frac{1}{3} \text{ minor } 2^{-2R}$$

let p' denote the ang poneer of message signal met). we may express the output signal to noise ratio of a uniform quantizer as,

$$(SNR)_0 = \frac{1}{50}$$

$$= \frac{P}{\frac{1}{3} m_{mx}^2 \sqrt{2^2 R}}$$

$$(SNR)_0 = (3P)_0 \sqrt{2^2 R}$$

$$SNR)_0 = \frac{3P}{m^2_{max}} 2^{4R}$$

K PULSE CODE MODULATION :

* In pulse code Modulation (PCM), a message signal is represented by a segnence of coded pulses, which is accomplished by representing the signal in discrete form in both time and amplitude.

* The basic operations performed in the transmitter of a PCM system are sampling, quantizing and encoding a shown in fig 6(a). The Lowpass filter prior to sampling

is included to prevent aliasing of the message signal. The quantizing and encoding operations are usually performed in the same circuit, which is called an analog to-digital converter. * The basic operations in the receives are regeneration of impaired signals, decoding and reconstruction of the train of quartized samples as shown in fig 6(1) Regeneration also occurs et intermediate points along the transmission path as necessary as indicated in figures. PCM\$qml Sourcest Samples Quartizer Encoder to channel:/p continuohs time message Bignal 1 Transmiller Distorted PCM Regenerative Regererative Repeater Repeater Regenerated PCM Signal Produced of the channel of p Zignal applied to the receiver 6 Transmission Path Fral Regeneration Decoder Reconstantin Destruction Chamel cranil-(c) Receiver F79(6): The basic elements of a PCM system.

the incoming message signal is sampled with a train

of narrow rectangular prises so as to closely appro - ximate the instantaneous sampling process. In order to ensure perfect reconstruction of the message signal at the receiver, the sampling roote must be greater than or equal to the highest frequency component is of the message signal in accordance with the sampling theorem.

ts > a M .

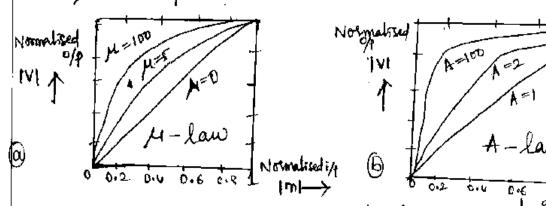
* Quantization:

The sampled version of the message signal is then quantized, thereby providing a new representation of the signal that is discrete in both time and amplitude.

* For miform quantization, me have mid-tread and mid-rise quantizer and for mon-uniform quantization, me have two compression lans ,4-low and Alaw.

* The use of a non-uniform quantizer is equivalent to passing the baseband signal through a compressor and then applying the compressed signal to a uniform quantizer. A particular form of compression law that is used in practice is the so called M-law, defined by

 $|V| = \frac{\log(1+\mu|m|)}{\log(1+\mu)}$



* Another compression law that is used in practice is the so called it-law as shown abone.

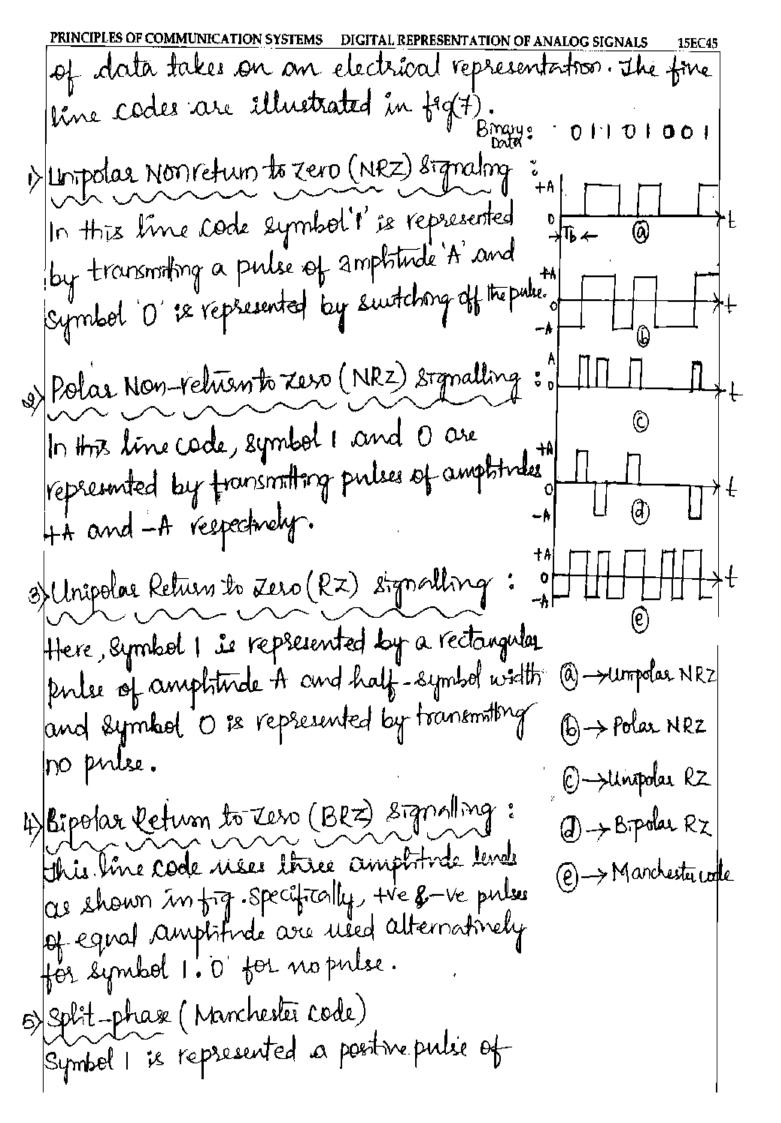
$$|V| = \begin{cases} \frac{A|m|}{1 + \log A}, & 0 \le |m| \le \frac{1}{A} \\ \frac{1 + \log (A|m|)}{1 + \log A}, & \frac{1}{A} \le |m| \le 1 \end{cases}$$

* Encoding :

* In combining the processes of sampling and quantizing the specification of a continuous message (bosseband) signal becomes limited to a discrete set of values, but not in the form best suited to transmission over a line or randio path.

* In a binary code each symbol may be either of two distinct values or kinds, such as the presence or above of a pulse. The two symbols of a binary code are cretomorily denoted as 0 and 1.

* Line code: It is a line code that a binary etream



complitude A' followed by a negative pulse of amplitude LA 10-15 bottipulses being a half-symbol wide. For good o', the polarities of these two pulses are reversed. * Differential Encoding: this method is used to encode information in terms of signal transitions. In particular, a transition is used to designate symbol 0 in the incoming binary datastream. while no transition is used to designate symbol I as shown in fig. @ Original brasydata 01101,001 6 Differentially encoded data "1" 000 1 10 @ Waveform Fig: Differential encoding. * REGENERATION: the distorted PCM wave obtained from the transmitter ie sent to the amplifier equilizer. The output of egnifizer denice is passed to the Decesion making device to decide the signal interms of 1 or 0 fooded Decerron Regenerated Making PCM wane Block diagram of a regenerative repeater.

* Decoding:

The decoding process involves generating a pulse the amplitude of which is the linear sum of all the pulse in the codeword, with each pulse being weighted by its place value (1°, 1', 2° ..., 2° -1) in the code, where k's the number of bits per sample.

* FILTERING

The final operation in the receiver is to recover the message signal wans by passing the decoder output through a lowpass reconstruction filter whose cutoff frequency is equal to the message bandwidth in.

* MULTIPLEXING :

In applications ning PCM, it is natural to multiples different message sources by time division whereby each source keeps its individually throughout the journey from the transmitter to receiver.

* This individuality accounts for the comparative ease with which message sources may be dropped or reinsexted in a time division multiplex system.

* APPLICATION TO VOCODERS :

* Linear Prediction Roding (LPC) Vocadese are model based systems. Based on the human speech model, a

