NETWORK ANALYSIS (15EC34)

Syllabus:-

Module -1

Basic Concepts: Practical sources, Source transformations, Network reduction using Star – Delta transformation, Loop and node analysis With linearly dependent and independent sources for DC and AC networks, Concepts of super node and super mesh.

Module -2

Network Theorems: Superposition, Reciprocity, Millman's theorems, Thevinin's and Norton's theorems, Maximum Power transfer theorem and Millers Theorem.

Module -3

Transient behavior and initial conditions: Behavior of circuit elements under switching condition and their Representation, evaluation of initial and final conditions in RL, RC and RLC circuits for AC and DC excitations.

Laplace Transformation & Applications: Solution of networks, step, ramp and impulse responses, waveform Synthesis.

Module -4

Resonant Circuits: Series and parallel resonance, frequency- response of series and Parallel circuits, Q–Factor, Bandwidth.

Module -5

Two port network parameters: Definition of z, y, h and transmission parameters, modeling with these parameters, relationship between parameters sets.

Text Books:

1. M.E. Van Valkenberg (2000), “Network analysis”, Prentice Hall of India, 3rd edition, 2000, ISBN: 9780136110958.

2. Roy Choudhury, “Networks and systems”, 2nd edition, New Age International Publications, 2006, ISBN: 9788122427677.

Reference Books:

1. Hayt, Kemmerly and Durbin “Engineering Circuit Analysis”, TMH 7th Edition, 2010.

2. J. David Irwin /R. Mark Nelms, “Basic Engineering Circuit Analysis”, John Wiley, 8th edition, 2006.

3. Charles K Alexander and Mathew N O Sadiku, “Fundamentals of Electric Circuits”, Tata McGraw-Hill, 3rd Ed, 2009.

Module 1: Basic Circuit Concepts

Network: Any interconnection of network or circuit elements (R, L, C, Voltage and Current sources).

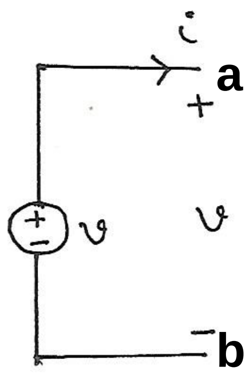
Circuit: Interconnection of network or circuit elements in such a way that a closed path is formed and an electric current flows in it.

Active Circuit elements deliver the energy to the network (Voltage and Current sources)

Passive Circuit elements absorb the energy from the network (R, L and C).

Active elements:

Ideal Voltage Source is that energy source whose terminal voltage remains constant regardless of the value of the terminal current that flows. Fig.1a shows the representation of Ideal voltage source and Fig.1b, it's V-I characteristics.



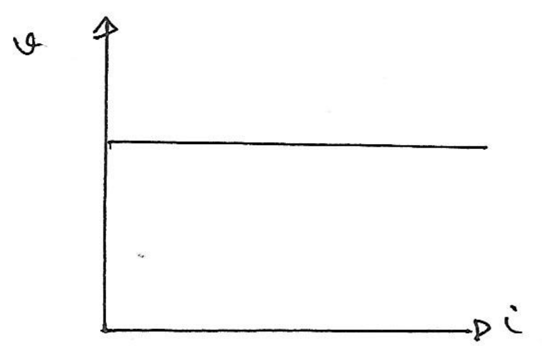


Fig.1a: Ideal Voltage source Representation **Fig. 1b: V-I characteristics**

Practical Voltage source: is that energy source whose terminal voltage decreases with the increase in the current that flows through it. The practical voltage source is represented by an ideal voltage source and a series resistance called internal resistance. It is because of this resistance there will be potential drop **within the source and with the increase in terminal current or load current, the drop across resistor increases, thus**

reducing the terminal voltage. Fig.2a shows the representation of practical voltage source and Fig.2b, it's V-I characteristics.

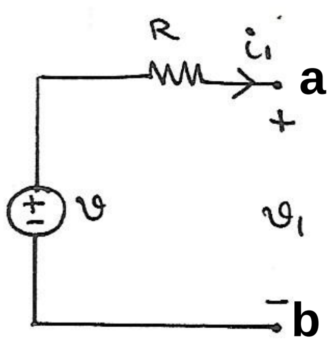


Fig. 2a: Practical Current Source

Fig. 2b: V-I characteristics

Here, $i_1 = i - v_1/R$ …… (2)

Dependent or Controlled Sources: These are the sources whose voltage/current depends on voltage or current that appears at some other location of the network. We may observe 4 types of dependent sources.

i) Voltage Controlled Voltage Source (VCVS)

ii) Voltage Controlled Current Source (VCCS)

iii) Current Controlled Voltage Source (CCVS)

iv) Current Controlled Current Source (CCCS)

Fig.3a, 3b, 3c and 3d represent the above sources in the same order as listed.

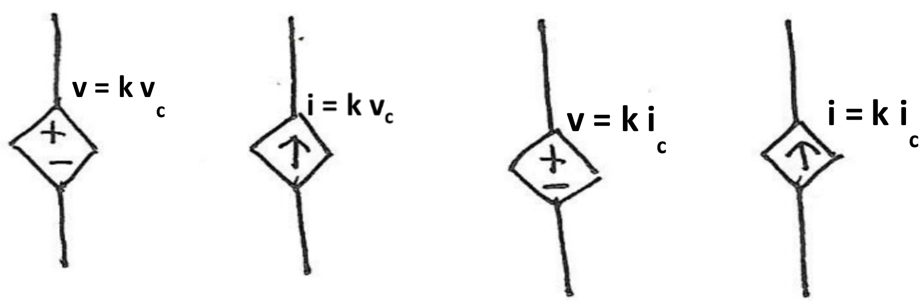


Fig. 3 a) VCVS b) VCCS c) CCVS d) CCCS

Kirchhoff’s Voltage Law (KVL)

It states that algebraic sum of all branch voltages around any closed path of the network is equal to zero at all instants of time. Based on the law of conservation of energy.

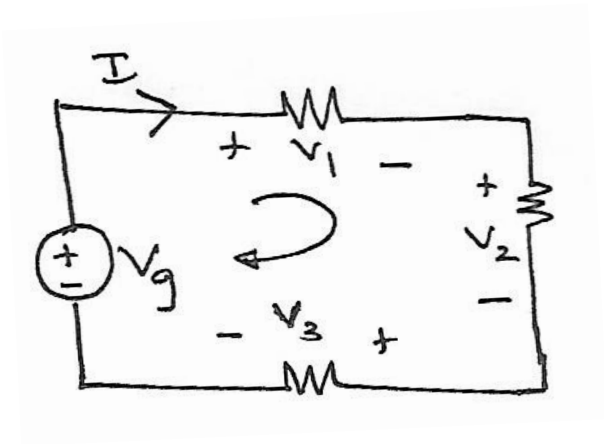


Fig. 4: Example illustrating KVL

Applying KVL clockwise, $+ V_1 + V_2 + V_3 – V_g = 0$ …… (3)

=> $V_g = V_1 + V_2 + V_3$ …… (4), indicative of energy delivered

= energy absorbed

Kirchhoff’s Current Law (KCL)

The algebraic sum of branch currents that leave a node of a network is equal to zero at all instants of time. Based on the law of conservation of charge.

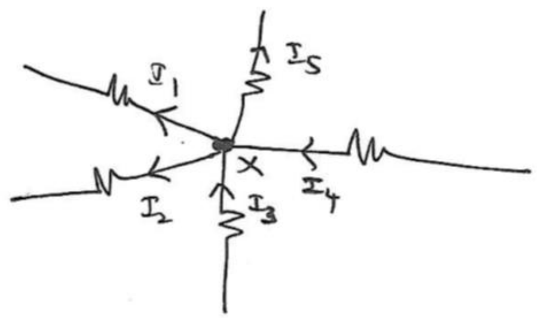


Fig. 5: Example illustrating KCL

Applying KCL at node X, + I_1 + I_2 - I_3 - I_4 + I_5 = 0 ……. (5)

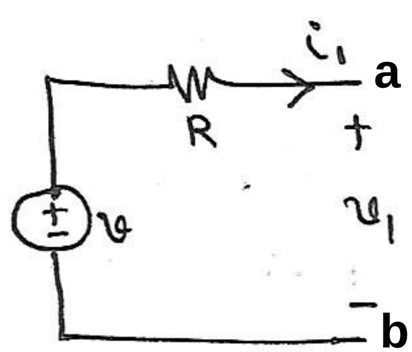
=> $I_3 + I_4 = I_1 + I_2 + I_5$ …… (6), indicative of sum of incoming currents

= sum of outgoing currents at a node.

<u>Source Transformation</u>

Source Transformation involves the transformation of voltage source to its equivalent current source and vice-versa.

Consider a voltage source with a series resistance R, in Fig. 6a and a current source with the same resistance R connected across, in Fig.6b.



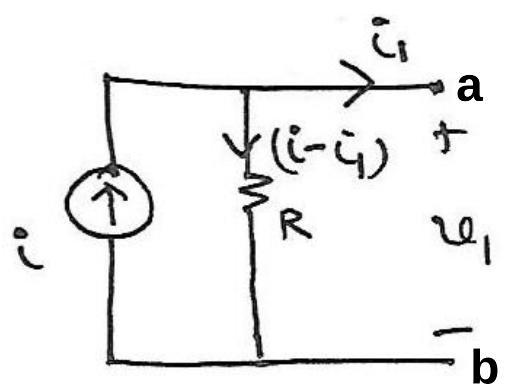


Fig.6a Voltage Source **Fig.6b Current Source**

The terminal voltage and current relationship in the case of voltage source is;

$v_1 = v - i_1 R$ …… (7)

The terminal voltage and current relationship in the case of current source is;

$i_1 = i - v_1/R$, which can be written as, $v_1 = iR - i_1R$ …… (8)

If the voltage source above has to be equivalently transformed to or represented by, a current source then the terminal voltages and currents have to be same in both cases.

This means eqn. (7) should be equal to eqn. (8). This implies, $v = iR$ **or** $i = v / R$…(9). If eqn.(9) holds good, then the voltage source above can be equivalently transformed to or represented by, the current source shown above and vice-versa.

Problems:

1) For the network shown below in Fig.7, find the current through 2Ω resistor, using source transformation technique.

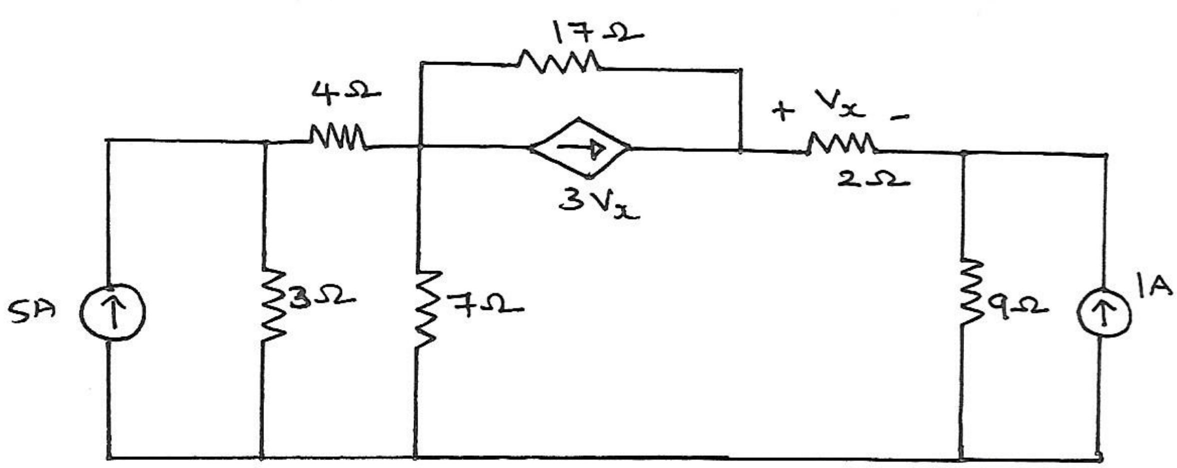
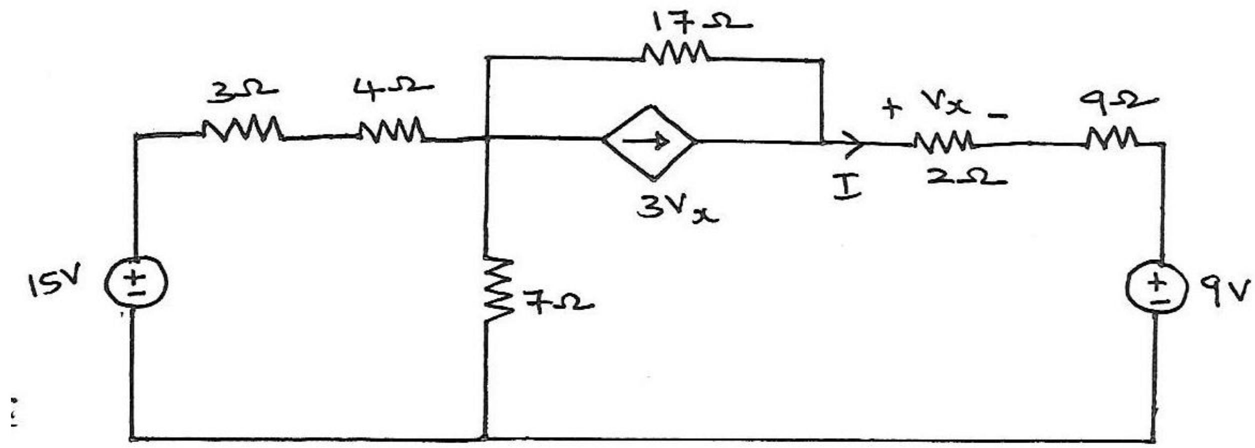
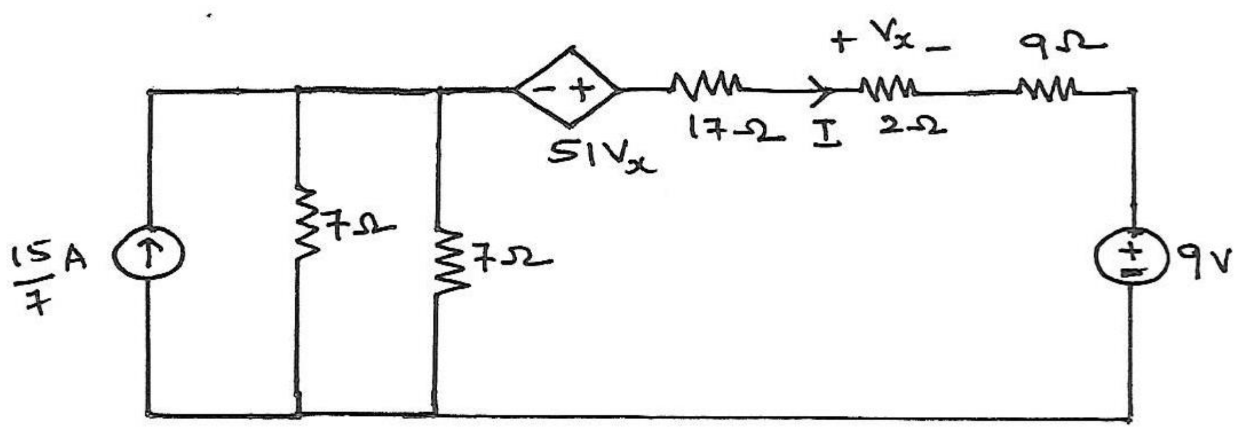


Fig.7

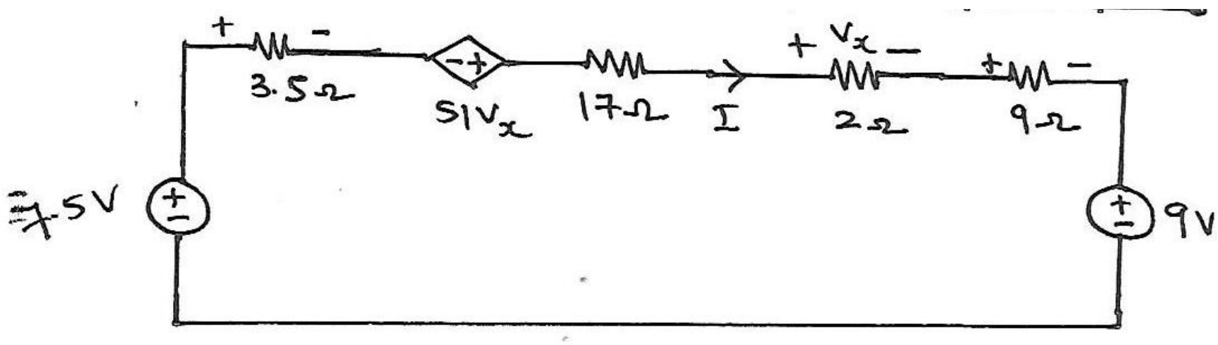
Solution: In the given circuit, Converting 5A source to voltage source so that resistor 4Ω comes in series with source resistor 3Ω and equivalent of them can be found. Also converting 1A source to voltage source, we obtain the circuit as below;



Converting 15V source above to current source and converting $3V_x$ dependent current source to dependent voltage source, we get the following;



Taking equivalent of the parallel combination of 7Ω resistors and converting 15/7 A current source to voltage source, we get as shown below;



Applying KVL to the loop above clockwise, we get;

3.5 I - 51 V_x+ 17 I +2I + 9I + 9 -7.5=0

From the circuit above, V_x =2I, substitute in above eqn, then we get;

-70.5 I = -1.5

=> I = 0.02127 A = 21.27mA

2) Represent the network shown below in Fig.8, by a single voltage source in series with a resistance between the terminals A and B, using source transformation techniques

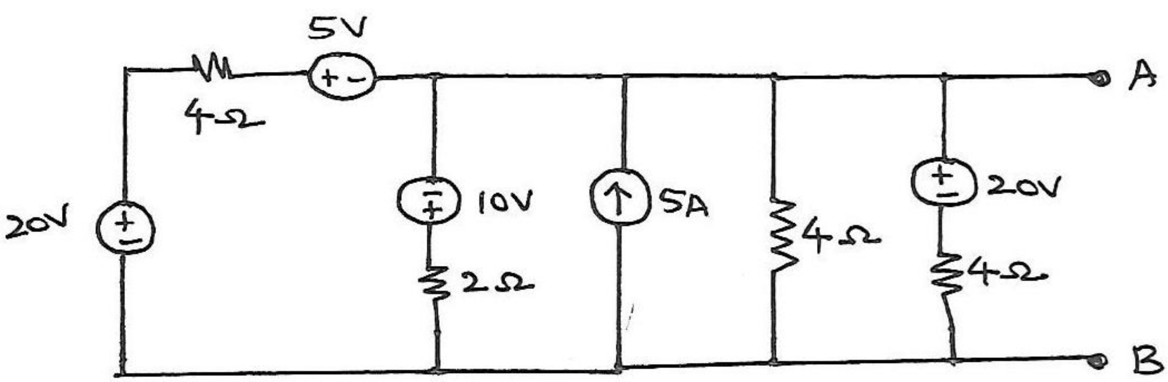
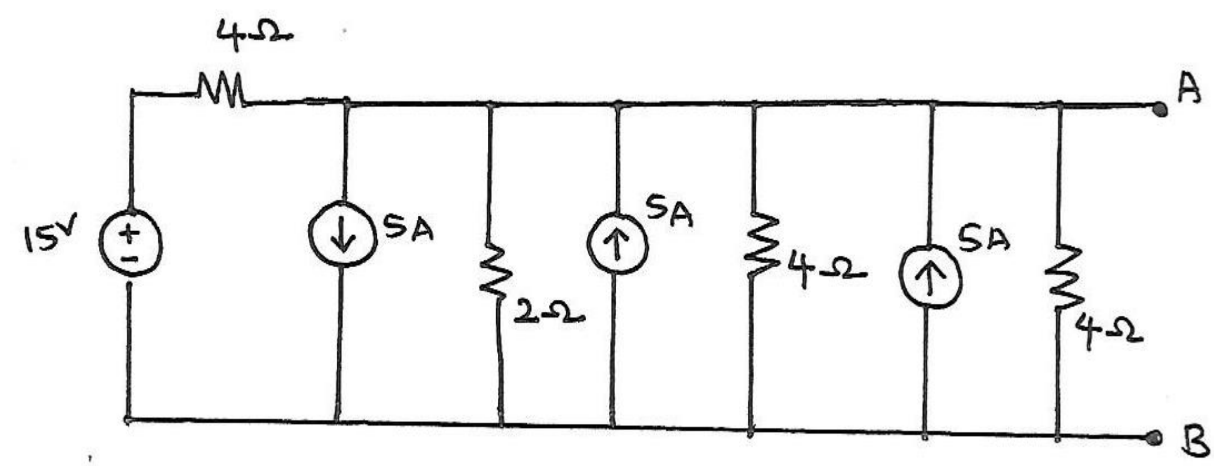


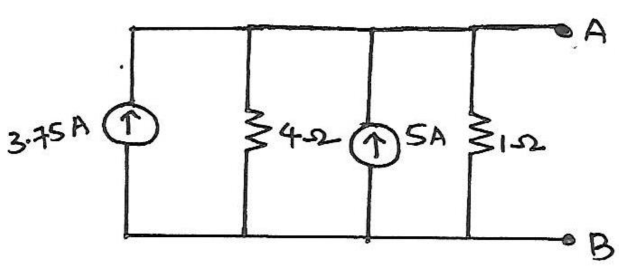
Fig.8

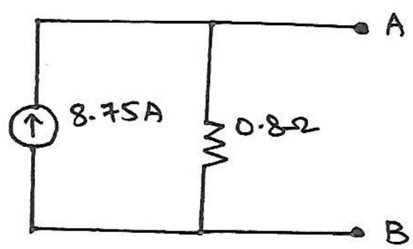
Solution: In the circuit above, 5V and 20 V sources are present in series arm and they are series opposing.

So, the sources are replaced by single voltage source which is the difference of two (as they are opposing, if series aiding then sum has to be considered). The polarity of the resulting voltage source will have same as that of higher value voltage source. Multiple current sources in parallel, can be added if they are in same direction and if they are in opposite direction, then difference is taken and resulting source will have same direction as that of higher one.

Taking source transformation, such that we get all current sources in parallel and all resistances in parallel, between the terminals. This leads to finding of equivalent current source and equivalent resistance between A-B. The source transformation leads to single voltage source in series with a resistance. These are shown below;







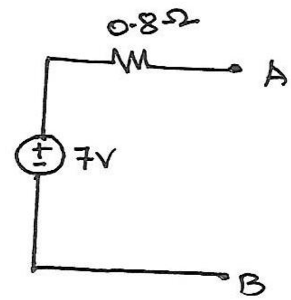


Illustration of Mesh Analysis:

3) Find the mesh currents in the network shown in fig.9

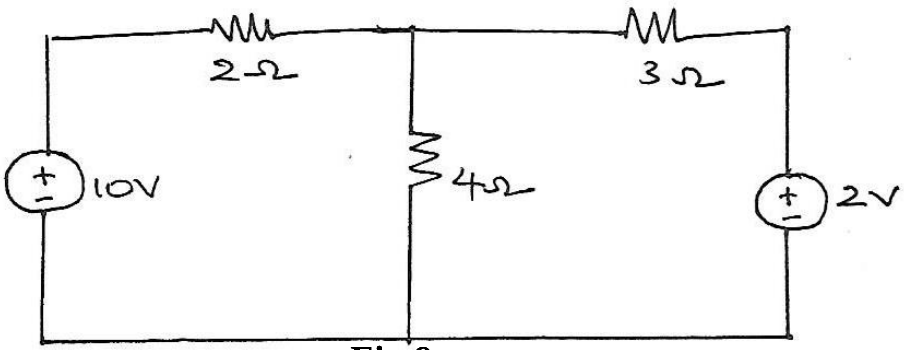


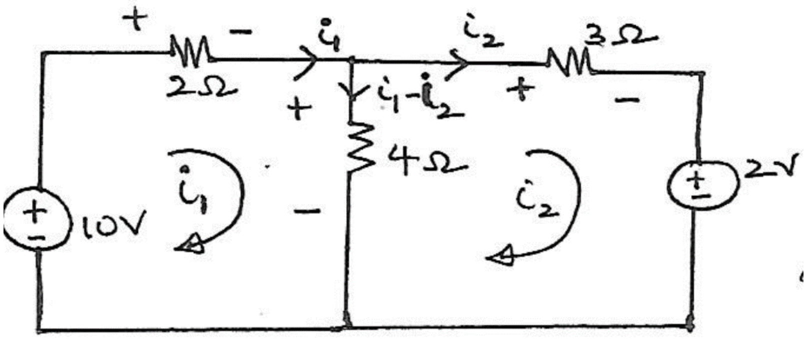
Fig.9

We identify two meshes; 10V-2Ω-4 Ω called as mesh 1 and 3Ω-2V-4 Ω called as mesh2. We consider i_1 to flow in mesh1 and i_2 to flow in mesh2. Their directions are always considered to be clockwise. If they are in opposite direction in actual, we get negative values when we calculate them, indicative of actual direction to be opposite.

10V-2Ω branch only belongs to mesh1 and so current through it is i_1 and 3Ω-2V branch only belongs to mesh2 and so current through it is always i_2. Also, 4Ω belongs to both meshes and so, the current through it will be the resultant of i_1 and i_2. These are shown below;

Next we will apply KVL to each of the meshes; As a result, In this case, we get two equations in terms of i_1 and i_2 and when we solve them we get i_1 and i_2. And when we know the mesh current values, we can find the response at any point of network.

The polarities of the potential drops across passive circuit elements are based on the directions of the current that flows through them



Applying KVL to mesh1;

$+2\ i_1 + 4\ (i_1 - i_2)\ -10 = 0$

=> $+6\ i_1 - 4\ i_2 = 10$…… (1)

Applying KVL to mesh2;

$+3\ i_2 + 2 - 4\ (i_1 - i_2) = 0$

Above equation can be rewritten as

$+3\ i_2 + 2 + 4\ (i_2 - i_1) = 0$

=> $-4\ i_1 + 7\ i_2 = -2$ …… (2)

Also observing the bold equations above, we may say that easily the potential drops across passive circuit elements can be considered to take +ve signs. From now onwards, we will not specifically identify polarities of potential drops across **passive circuit elements**. They are considered to take positive signs. For the case of shared element, like 4Ω above, which is shared between mesh1 and mesh2, the potential drop across it , is considered to be $+4(i_1 - i_2)$, when we apply KVL to mesh1 and $+4(i_2 - i_1)$, when we apply KVL to mesh2. Now eqn1 and eqn2 above can be represented in matrix form as shown;

$$\begin{pmatrix} 6 & -4 \\ -4 & 7 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} 10 \\ -2 \end{pmatrix}$$

Using cramer's rule;

$$\Delta = \begin{vmatrix} 6 & -4 \\ -4 & 7 \end{vmatrix} = 26$$

$$\Delta i_1 = \begin{vmatrix} 10 & -4 \\ -2 & 7 \end{vmatrix} = 62$$

$$\Delta i_1 = \begin{vmatrix} 6 & 10 \\ -4 & -2 \end{vmatrix} = 28$$

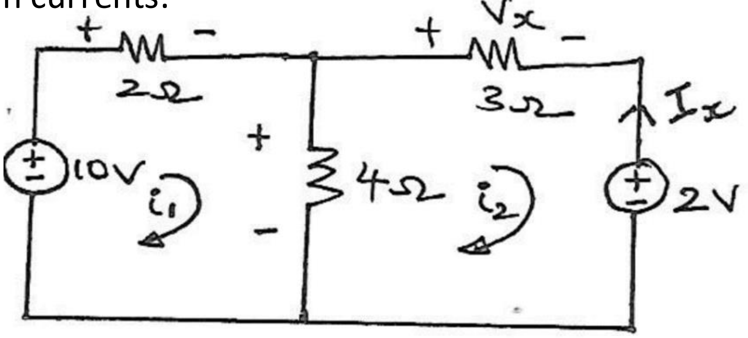
=> $i_1 = \Delta i_1 / \Delta = 2.384$ A

=> $i_2 = \Delta i_2 / \Delta = 1.076$ A

As already told, if we know the mesh current values, we can find the response at any point of network. And so, V_x and I_x identified, can be easily obtained using the mesh currents.

$I_x = -i_2 = -1.076$ A

$V_x = 3i_2 = 3.228$ A



4) Find the power delivered or absorbed by each of the sources shown in the network in Fig.10.Use mesh analysis

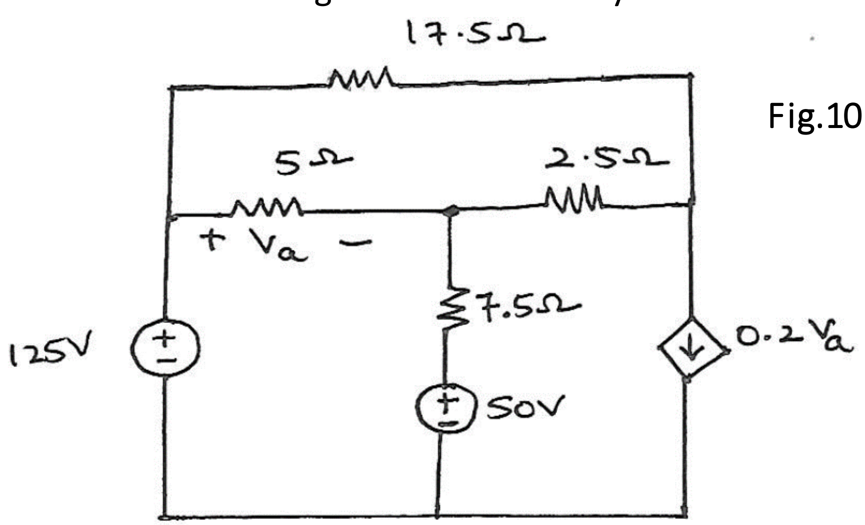
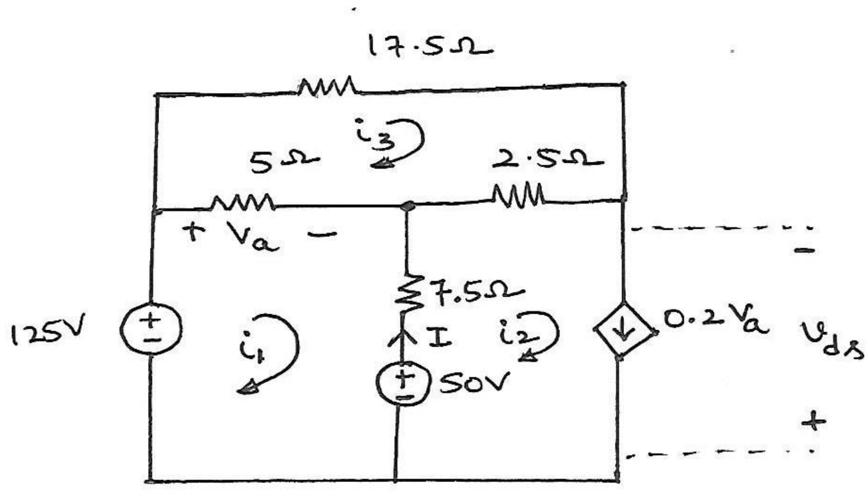


Fig.10



Solution:-

Power delivered by 125 V source, $P_{125} = 125\ i_1$

Power delivered by 50V source, $P_{50} = 50\ I = 50\ (i_2 - i_1)$

Power delvd. by dependent current source, $P_{ds} = (0.2V_a)\ (v_{ds}) = (i_1 - i_3)\ (v_{ds})$

{Because $V_a = 5\ (i_1 - i_3)$}

From the circuit; $V_a = 5\ (i_1 - i_3)$

Also; $i_2 = 0.2\ V_a = i_1 - i_3$ (it is as good as specifying the value of i_2 or we can say we have obtained equation from mesh2, so no need of applying KVL to mesh2)

Applying KVL to mesh1;

$5\ (i_1 - i_3) + 7.5(i_1 - i_2) + 50 - 125 = 0$

$12.5\ i_1 - 7.5\ i_2 - 5\ i_3 = 75$; substituting $i_2 = i_1 - i_3$; we have;

$5\ i_1 + 2.5\ i_3 = 125$ …… (1)

Applying KVL to mesh3;

$17.5\ i_3 + 2.5\ (i_3 - i_2) + 5(i_3 - i_1) = 0$

$-5\ i_1 - 2.5\ i_2 + 25\ i_3 = 0$; substituting $i_2 = i_1 - i_3$; we have;

$-7.5\ i_1 + 27.5\ i_3 = 0$ …… (2)

Solving (1) and (2), we get; i_1=13.2 A and i_3=3.6 A

So, $i_2 = i_1 - i_3$ = 13.2 -3.6 =9.6 A

P_{125} = 125 i_1= 125 (13.2) =1650 W (power delivered)

P_{50} =50 I =50 ($i_2 - i_1$) = 50 (9.6 -13.2) = -180 W, here negative value of power delivered is the indicative of the fact that power is actually absorbed by 50V source.

To find v_{ds} in the network shown, we apply KVL to the outer loop 17.5Ω→0.2V_a→125V;

+17.5 i_3 - v_{ds} -125 =0 {when **applying KVL, the potential drop across passive circuit element is taken as, + (resistance or impedance value) x (that particular current which is in alignment with KVL direction), if clockwise direction is considered, then clockwise current**)}

=> v_{ds} = - 62V

P_{ds} = (0.2 V_a)(v_{ds}) =($i_1 - i_3$) v_{ds} = - 595.2W => Dependent source absorbs power of 595.2 W

5) Find the power delivered by dependent source in the network shown in Fig.11.Use mesh analysis

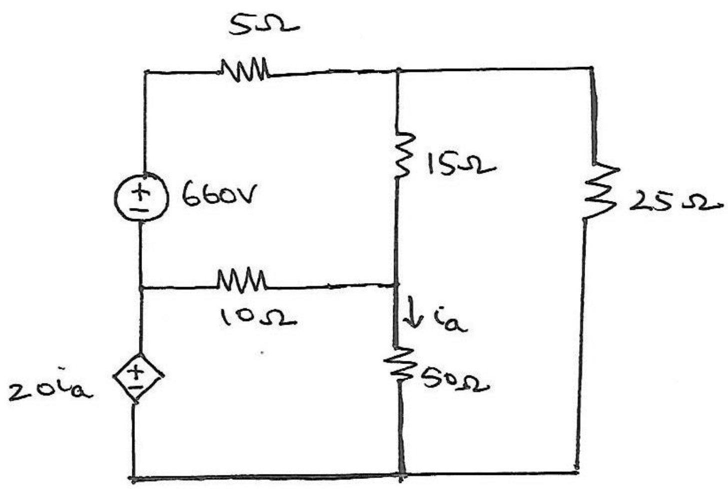
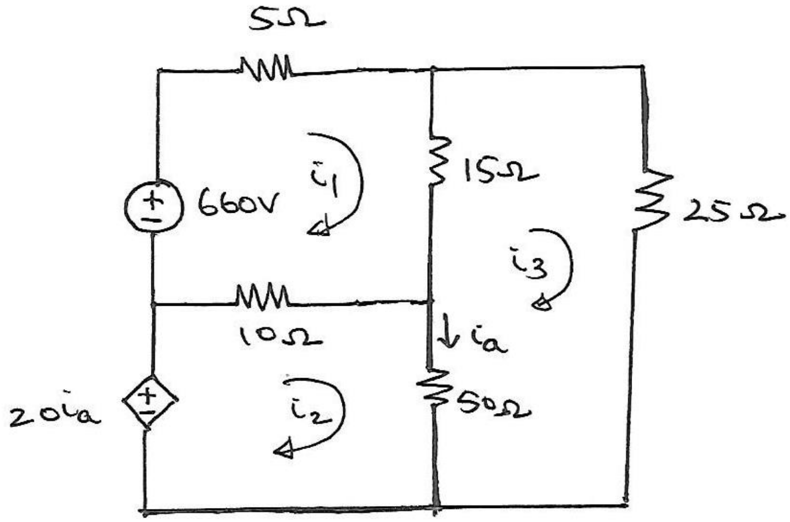


Fig.11

Solution:-



From the circuit,

$i_a = i_2 - i_3$

Power delivered by dependent source, $P_{ds} = (20\ i_a)\ (i_2) = 20\ (i_2 - i_3)\ i_2$

Apply KVL to mesh1

$5\ i_1 + 15\ (i_1 - i_3) + 10\ (i_1 - i_2) - 660 = 0$

$30\ i_1 - 10\ i_2 - 15\ i_3 = 660$…… (1)

Apply KVL to mesh2

$10\ (i_2 - i_1) + 50\ (i_2 - i_3) - 20\ i_a = 0$

$10\ (i_2 - i_1) + 50\ (i_2 - i_3) - 20\ (i_2 - i_3)$

$-10\ i_1 + 40\ i_2 - 30\ i_3 = 0$ …… (2)

Apply KVL to mesh3

$25\ i_3 + 50\ (i_3 - i_2) + 15\ (i_3 - i_1) = 0$

$-15\ i_1 - 50\ i_2 + 90\ i_3 = 0$ …… (3)

Solving (1), (2) and (3), we get i_2= 27 A and i_3 =22A

$P_{ds} = (20)\ (i_2 - i_3)\ i_2 = 20(5)27) = 2700W$, power delivered.

AC Circuits

These circuits consist L and C components along with R. Here we consider the excitation of the circuits by sinusoidal sources. Consider an AC circuit shown below;

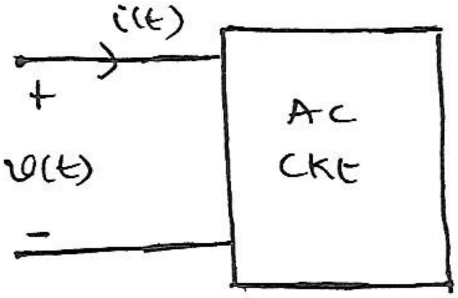


Fig.12

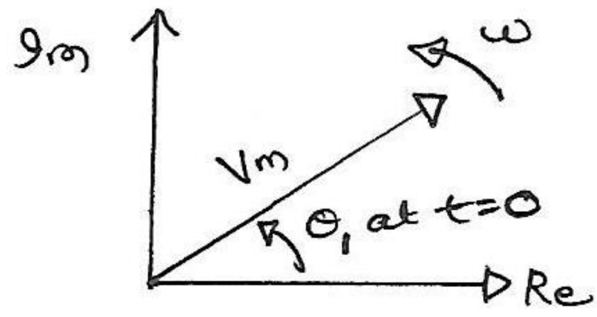


Fig.13

Let the applied voltage, $v(t) = V_m \sin(\omega t + \theta_1)$, the circuit current that flows is i(t) and is given as; $i(t) = I_m \sin(\omega t + \theta_2)$. These two sinusoidal quantities can be represented by phasors; a phasor is a rotating vector in the complex plane. This is shown in Fig.13, which is a voltage phasor. The phasor has a magnitude of V_m and rotates at an angular frequency of ω with time.

The voltage phasor is given by $V_m \angle\theta_1$ (Also referred as polar form of phasor). The rectangular form is $V_m \cos\theta_1 + j V_m \sin\theta_1$.

Similarly, the current phasor is given by $I_m \angle\theta_2$ (Also referred as polar form of phasor). The rectangular form is $I_m \cos\theta_2 + j I_m \sin\theta_2$.

The ratio of voltage phasor to the current phasor is called as impedance. $Z = (V_m \angle\theta_1)/(I_m \angle\theta_2) = (V_m/I_m) \angle(\theta_1 - \theta_2) = (V_m/I_m) \angle\theta$

The impedance although a complex quantity but is not a phasor, as with respect to time, the angle of impedance do not change

- If the AC circuit above is represented equivalently by single resistance, then $Z = (V_m \angle\theta_1)/(I_m \angle\theta_1)$ {since in resistance there is no phase difference between voltage and current and so $\theta_2 = \theta_1$}.

So, $Z = (V_m/I_m) \angle 0°$

= $(V_m/I_m) \cos 0° + j (V_m/I_m) \sin 0°$

= $V_m/I_m = R$.

- If the AC circuit above is represented equivalently by single inductance, then $Z = (V_m \angle\theta_1)/(I_m \angle(\theta_1 - 90°))$ { since in inductance, current lags the voltage in phase by 90°}

So, $Z = (V_m/I_m) \angle 90°$

= $(V_m/I_m) \cos 90° + j (V_m/I_m) \sin 90°$

= $j (V_m/I_m)$

= $j\omega L$ {in inductance, the ratio of peak value of voltage to peak value of current is always the reactance which is given by ωL}. Now we can say, any inductance of L henry can be equivalently represented by impedance of $j\omega L$ Ohms.

- If the AC circuit above is represented equivalently by single capacitance, then $Z = (V_m \angle\theta_1)/(I_m \angle(\theta_1 + 90°))$ { since in capacitance, current leads the voltage in phase by 90°}

So, $Z = (V_m/I_m) \angle -90°$

= $(V_m/I_m) \cos 90° - j (V_m/I_m) \sin 90°$

= $-j (V_m/I_m)$

= $-j(1/\omega C)$

= $-j/\omega C$ {in capacitance, the ratio of peak value of voltage to peak value of current is always the reactance which is given by $1/\omega c$. Now we can say, any capacitance of C farad can be equivalently represented by impedance of $-j/\omega C$ Ohms.

6) Find the current through the capacitor in the circuit shown in Fig.14. Use mesh Analysis.

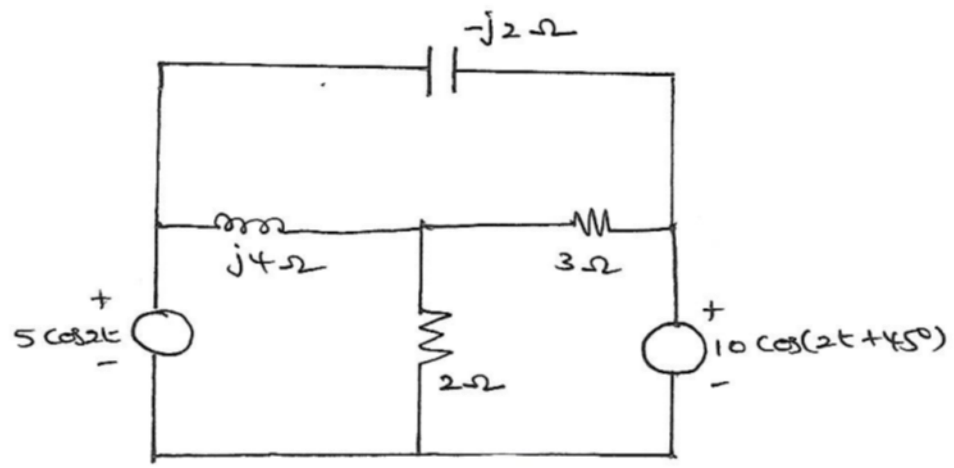
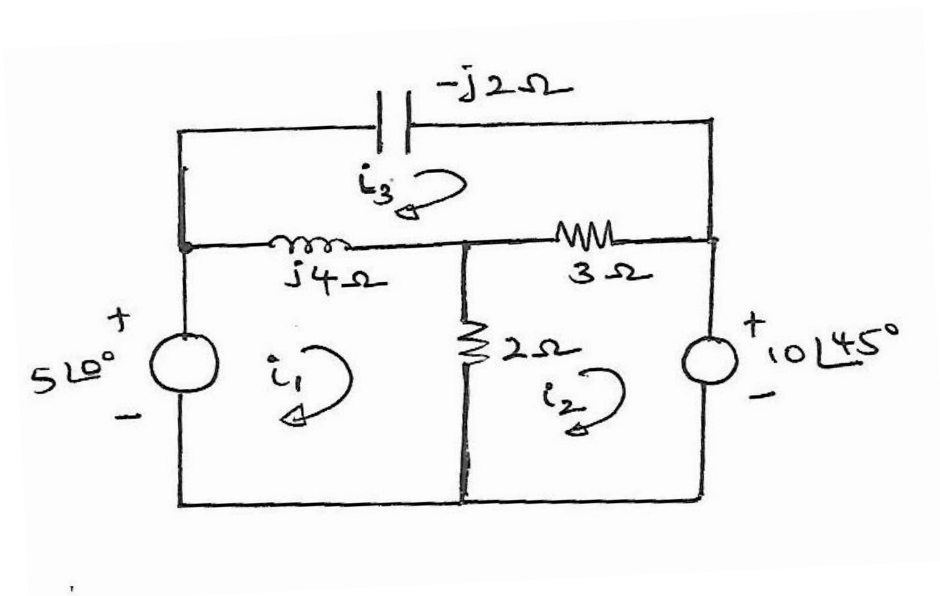


Fig.14

Solution:

The sources are represented by phasors. The mesh currents are identified. The current through the capacitor is i_3. So, i_3 needs to be found using mesh analysis.



Apply KVL to mesh1;

$j4(i_1 - i_3) + 2(i_1 - i_2) - (5\angle 0°) = 0$

$(2+j4)\, i_1 - 2\, i_2 - j4\, i_3 = 5$ ……(1)

Apply KVL to mesh2;

$3(i_2 - i_3) + (10\angle 45°) + 2(i_2 - i_1) = 0$

$-2\, i_1 + 5\, i_2 - 3\, i_3 = -(10\angle 45°) = -7.07 - j\,7.07$ ……(2)

Apply KVL to mesh3;

$-j2\, i_3 + 3(i_3 - i_2) + j4(i_3 - i_1) = 0$

$-j4\, i_1 - 3\, i_2 + (3+j2)\, i_3 = 0$ …… (3) **Mesh equations in matrix form;**

$$\begin{pmatrix} 2+j4 & -2 & -j4 \\ -2 & 5 & -3 \\ -j4 & -3 & 3+j2 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 5 \\ -7.07 - j\,7.07 \\ 0 \end{pmatrix}$$

Using Cramer's rule to find i_3 .

$$\Delta = \begin{vmatrix} 2+j4 & -2 & -j4 \\ -2 & 5 & -3 \\ -j4 & -3 & 3+j2 \end{vmatrix}$$

$= (2+j4)[5(3+j2)-9] + 2[-2(3+j2)- (-3)(-j4)] - j4[6+j20]$

$= 40-j12$

$$\Delta i_3 = \begin{vmatrix} 2+j4 & -2 & 5 \\ -2 & 5 & -7.07-j7.07 \\ -j4 & -3 & 0 \end{vmatrix}$$

$= (2 + j4)[+3(-7.07 – j\,7.07)] + 2[+j4(-7.07-j7.07)] +5[6+ j\,20]$

$=128.98 – j83.82$

Therefore, $i_3 = \Delta i_3 / \Delta = (128.98 – j83.82)/(40-j12)$

$= 3.535-j1.035$

$= 3.68\angle -16.31°$ A.

The above result represents the phasor of capacitor current. From this we can easily write the steady state expression of capacitor current, as,

$i_3(t) = 3.68 \cos(2t -16.31°)$ A

Node analysis

Here, we identify nodes of the given network and consider one node as ground node, which is considered to be zero potential point. We then identify the voltage at each of the remaining nodes which is nothing but potential difference between a node of interest and ground node, with ground node as reference. Node analysis involves the computation of node voltages, and when once these are found, we can find the response at any point of network.

Illustration

7) Find the node voltages in the network shown in Fig. 15;

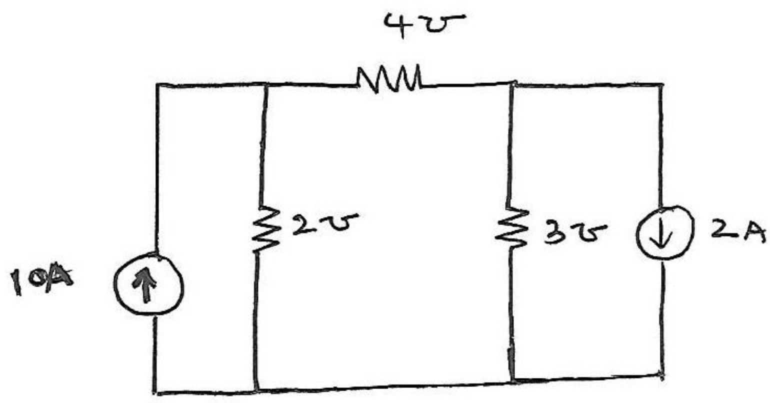
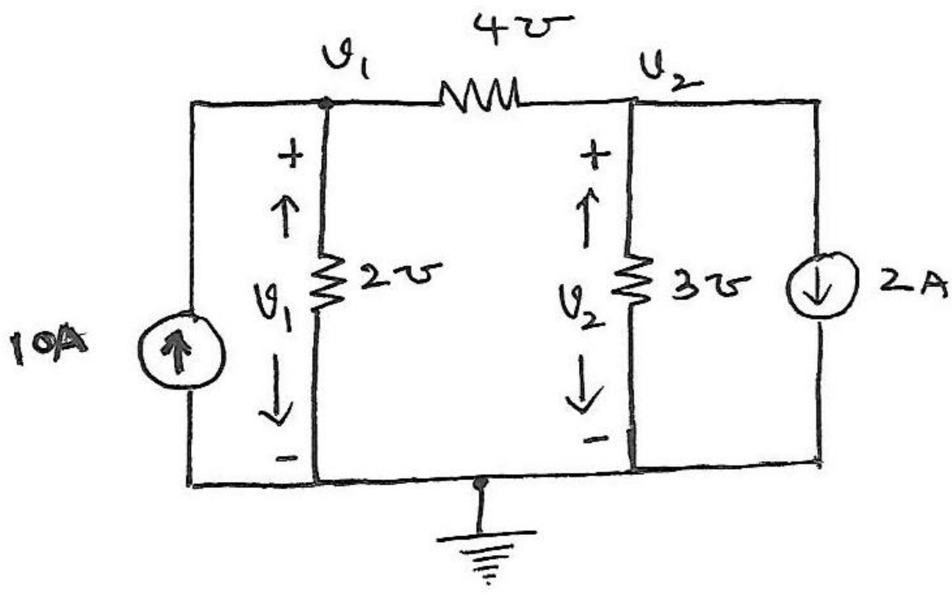


Fig.15

Solution:

There are 3 nodes in the network. The bottom node is selected as ground node. The voltage at node1 is identified as v_1 and it is the potential difference between the node1 and the ground, with ground as reference. The voltage at node2 is identified as v_2 and it is the potential difference between node2 and the ground, with ground as reference.



Recall KCL statement that "the algebraic sum of branch currents leaving a node of a network is zero at all instants of time".

Apply KCL at node1;

$-10 + 2v_1 + 4(v_1 - v_2) = 0$

$\Rightarrow 6v_1 - 4v_2 = 10$ ......(1)

Apply KCL at node2;

$+4(v_2 - v_1) + 3v_2 + 2 = 0$

$\Rightarrow -4v_1 + 7v_2 = -2$ ...... (2)

Node equations in Matrix form

$$\begin{pmatrix} 6 & -4 \\ -4 & 7 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 10 \\ -2 \end{pmatrix}$$

Using Cramer's rule;

$$\Delta = \begin{vmatrix} 6 & -4 \\ -4 & 7 \end{vmatrix} = 26$$

$$\Delta v_1 = \begin{vmatrix} 10 & -4 \\ -2 & 7 \end{vmatrix} = 62$$

$$\Delta v_2 = \begin{vmatrix} 6 & 10 \\ -4 & -2 \end{vmatrix} = 28$$

$v_1 = \Delta v_1 / \Delta = 62/26$

$v_1 = 2.384V$

$v_2 = \Delta v_2 / \Delta = 28/26$

$v_2 = 1.076V$

Node Analysis Contd.

8) Use Node analysis to find the voltage V_x in the circuit shown in Fig. 16

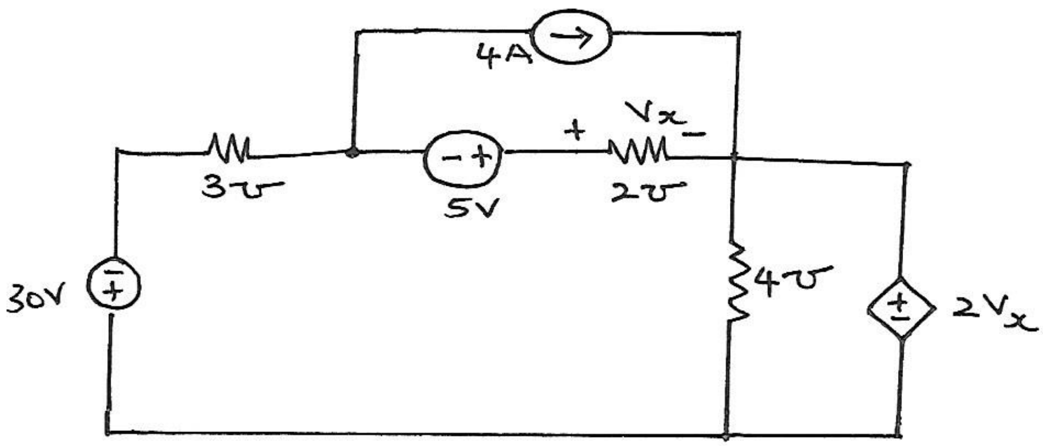
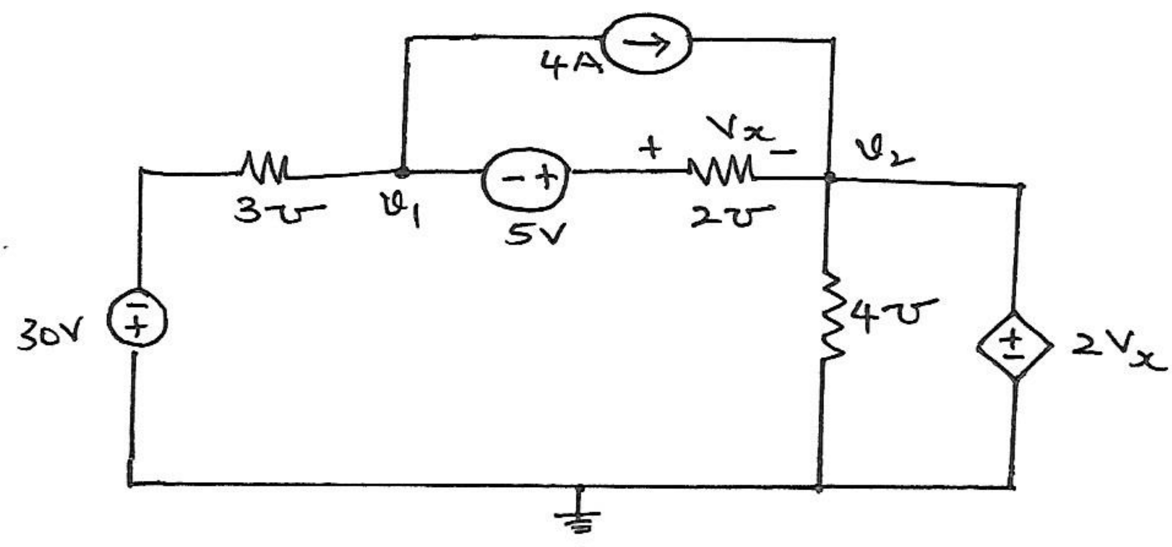


Fig.16

The ground node and other nodes with their voltages are identified as shown;



Although that point where two circuit elements join is referred as node (like 30V and 3 mho joining point above), we do not consider voltage there or apply KCL, because it will simply contribute for redundancy, as without considering the above, still the solution can be obtained. Therefore, we consider voltages or apply KCL to those nodes where three or more circuit elements join.

From the circuit; $V_x = v_1 + 5 - v_2$ and $v2 = 2V_x$

$v_2 = 2 (v_1 + 5 - v_2)$

- $\Rightarrow$ $2 v_1 - 3 v_2 = -10$ …… (1), now we have an equation expressing v_2 or an equation associated with node 2. So no need of applying KCL at node2.
- $\Rightarrow$ Apply KCL at node1;

$3 (v_1 - (-30)) + 4 + 2(v_1 + 5 - v_2) = 0$

- $\Rightarrow$ $5 v_1 - 2 v_2 = -104$ ……….(2)
- $\Rightarrow$ Solving (1) and (2), we get;
- $\Rightarrow$ $v_1 = -26.545V$ and $v_2 = -14.363V$
- $\Rightarrow$ Therefore, $V_x = v_1 + 5 - v_2$
- $\Rightarrow$ $-26.545 + 5 + 14.363 = -7.182$ V.

9) Find the power delivered by dependent source using node analysis in the circuit shown in Fig. 17.

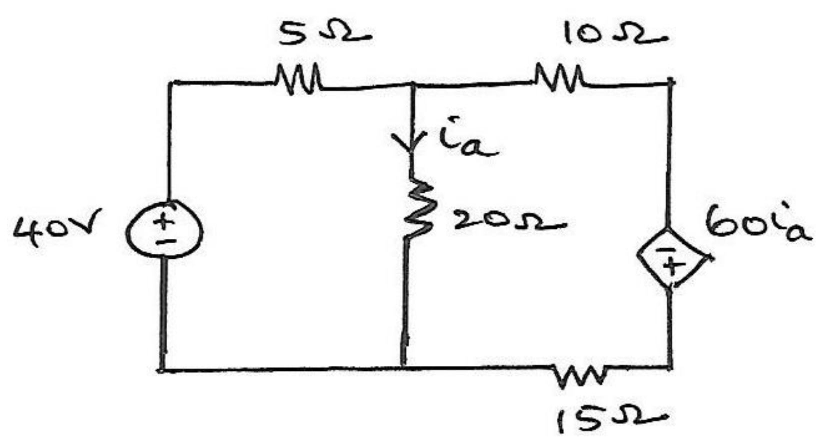
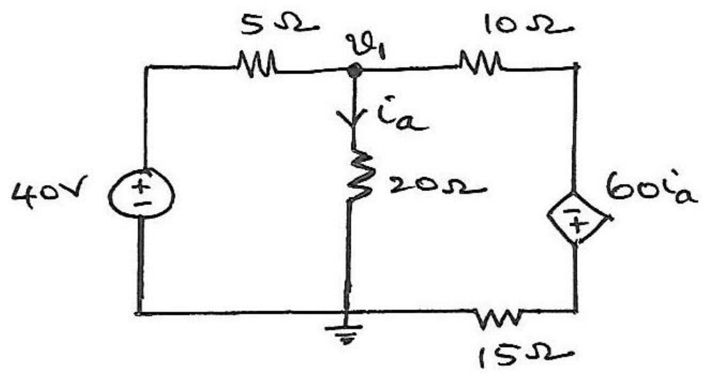


Fig.17

Solution: Identify ground node and other node with its voltage as shown;



From the circuit;

$i_a = v_1/20$ and

P_{ds} = (60 i_a) x (current that comes out of +ve polarity of 60i_a)

= (60 i_a) [(v_1-(-60i_a))/(10 +15)]

= (60 i_a) (v_1 + 60 i_a)/25

10) Find the current i_1 in the network shown in Fig. 18. Use node Analysis.

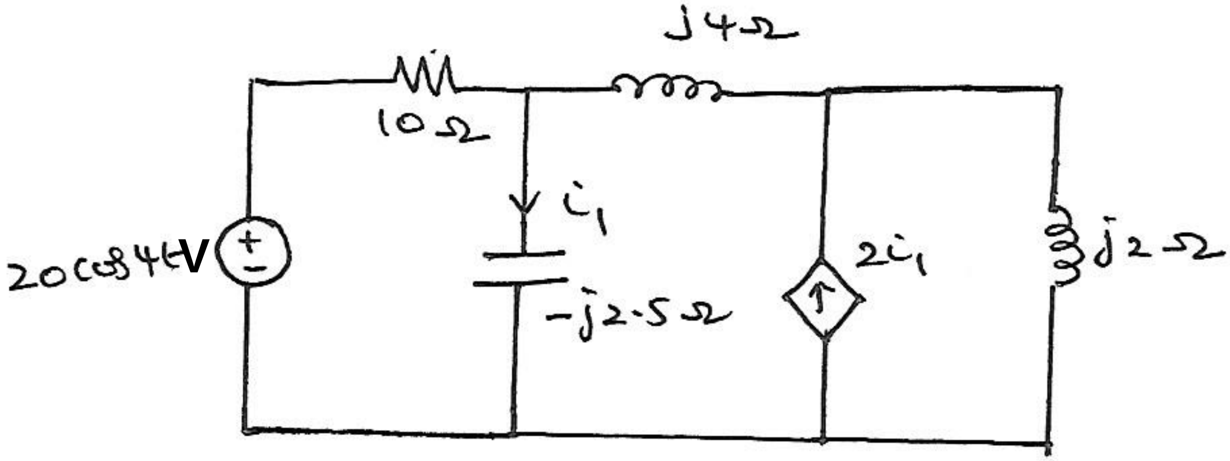
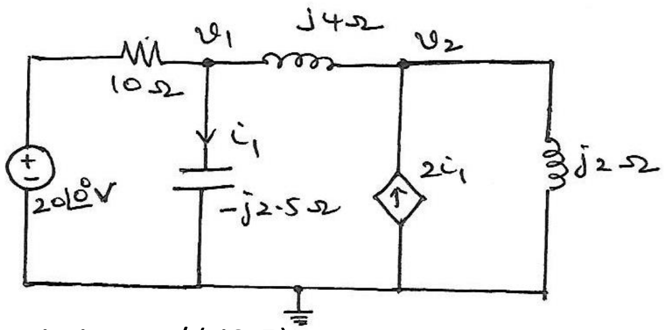


Fig.18

Identify ground node and other node voltages as shown. Also writing source using phasor representation.



From the circuit; $i_1 = v_1 / (-j2.5)$

Apply KCL at node1;

$v_1 / (-j2.5) + (v_1 - (20\angle 0°))/10 + (v_1 - v_2) / j4 = 0$

$\Rightarrow$ $j\,0.4\, v_1 + 0.1\, v_1 - j\,0.25 v_1 + j0.25 v_2 = 2$

$\Rightarrow$ $(0.1 + j0.15)\, v_1 + j\,0.25\, v_2 = 2$ ……(1)

Apply KCL at node 2;

$-2i_1 + v_2 / j2 + (v_2 - v_1)/j4 = 0$

$\Rightarrow$ $-2(v_1 / (-j2.5)) + v_2 / j2 + (v_2 - v_1)/j4 = 0$

$\Rightarrow$ $-j0.8\, v_1 \; -j\,0.5\, v_2 - j0.25 v_2 + j0.25 v_1 = 0$

$\Rightarrow$ $-j0.55\, v_1 - j\,0.75\, v_2 = 0$ ………..(2)

Using Cramer's rule;

$$\Delta = \begin{vmatrix} 0.1 + j\,0.15 & j\,0.25 \\ -j0.55 & -j0.75 \end{vmatrix} \begin{aligned} &= (0.1 + j0.15)(-j0.75) - 0.25(0.55) \\ &= -0.025 - j0.075 \end{aligned}$$

$$\Delta v_1 = \begin{vmatrix} 2 & j\,0.25 \\ 0 & -j0.75 \end{vmatrix} \quad = -\,j\,1.5$$

$v_1 = \Delta v_1 / \Delta = (-j1.5) / (-0.025 - j0.075) = 18 + j6 = 18.97\angle 18.43° V$

Therefore, $i_1 = v_1 / (-j2.5) = -2.4 + j7.2 = 7.58 \angle 108.43°$ A.

$i_1(t) = 7.58 \cos(4t + 108.43°)$ A

Concept of Supermesh:

Supermesh concept is considered whenever a current source appears in common to two meshes.

Consider the Network Below;

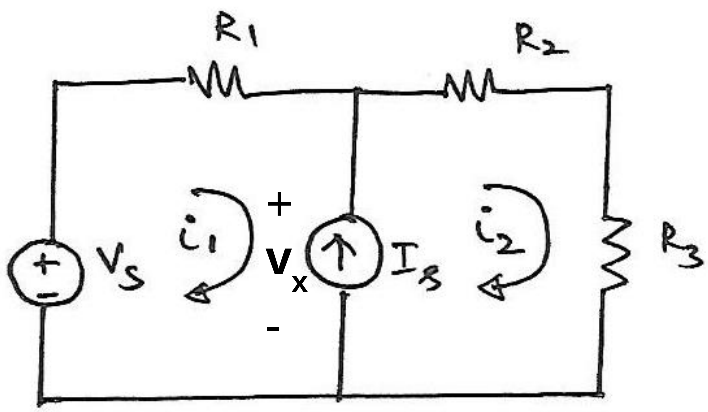


Fig.19

To know the advantage of applying supermesh concept; first consider usual way;

Applying KVL to mesh 1;

$R_1 i_1 + v_x - V_s = 0$

$R_1 i_1 + v_x = V_s$ .....(1)

Applying KVL to mesh 2;

$(R_2 + R_3)i_2 - v_x = 0$

$v_x = (R_2 + R_3)i_2$ .....(2)

Substituting (2) in (1), we get;

$R_1 i_1 + (R_2 + R_3)i_2 = V_s$ .....(3)

Also from the circuit;

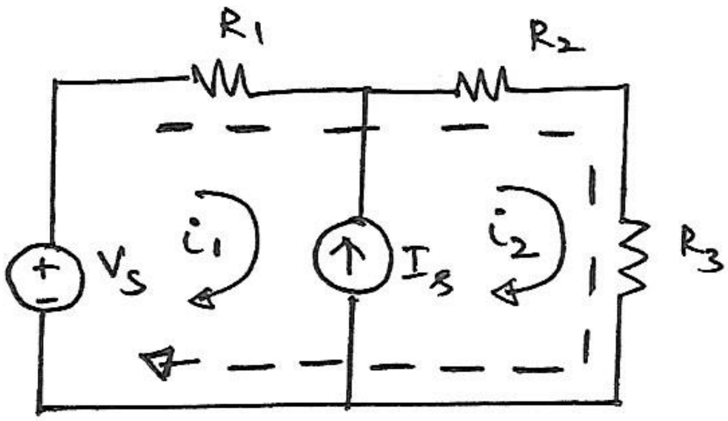
$i_2 - i_1 = I_s$

$\Rightarrow i_2 = I_s + i_1$ ......(4)

$\Rightarrow$ Substituting (4) in (3) we get, i_1;

$\Rightarrow$ Substituting i_1 in (4), we get i_2.

Applying the concept of supermesh;



Here, after identifying a current source common to two meshes; we first write constraint equation which relates corresponding mesh currents and the current source value.

$i_2 - i_1 = I_s$

Or $i_2 = I_s + i_1$..... (1)

We then club those two meshes and call it as supermesh; shown by dashed lines in the figure; Now we apply KVL to supermesh;

$R_1 i_1 + R_1 i_2 + R_3\, i_2 - V_s = 0$

$R_1 i_1 + (R_1 + R_3)\, i_2 = V_s$ ......(2), this equation is exactly the same as (3) in previous case. In this case, it was easily obtained thus reducing the steps. Now, substituting (1) in (2), we get i_1. Then substituting i_1 in (1) we get i_2. Therefore, mesh currents were easily obtained using supermesh concept.

11) Use mesh analysis to find V_x in the circuit shown in fig. 20

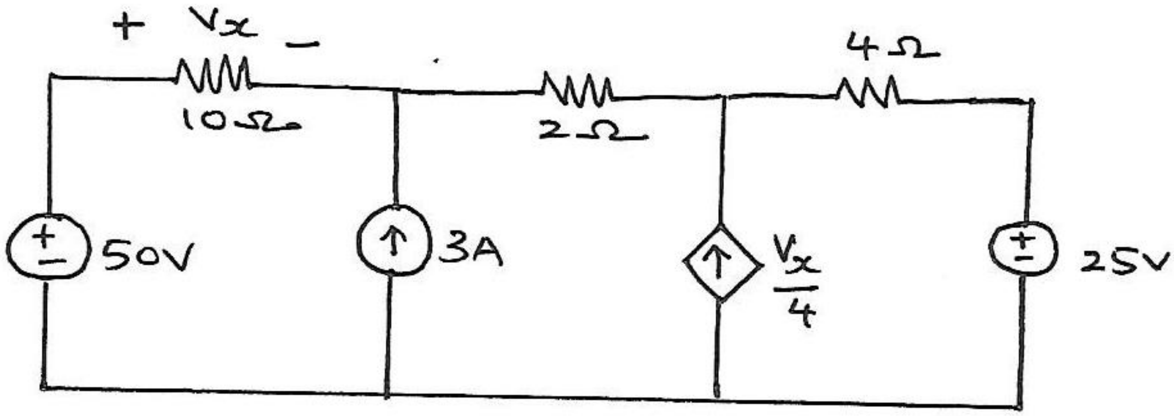
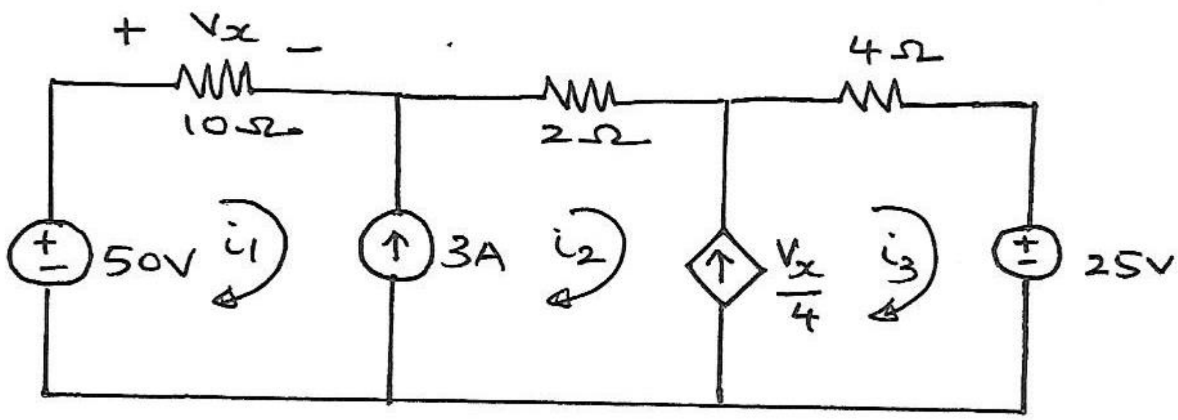


Fig.20



Solution: From the circuit; $V_x = 10i_1$

Identifying 3A and $V_x/4$ current sources appearing in common to mesh-1&2 and mesh-2&3 respectively; the constraint equations are written as; $i_2 - i_1 = 3$

=> $i_2 = 3 + i_1$ Also $i_3 - i_2 = V_x/4$,

wkt, $V_x = 10\, i_1$

Substituting in above equation we get $i_3 - i_2 = 10\, i_1/4$, wkt $i_2 = 3 + i_1$ substituting this => $4\, i_3 - 4(3+i_1) - 10i_1 = 0$

$-14\, i_1 + 4\, i_3 = 12$ …….(1)

Apply KVL to supermesh

formed by 10Ω→2 Ω→4Ω→25V→50V→10Ω

$10\,i_1 + 2\,i_2 + 4\,i_3 + 25 - 50 = 0$

$\Rightarrow 10\,i_1 + 2\,i_2 + 4\,i_3 = 25$

$\Rightarrow 10\,i_1 + 2\,(3+i_1) + 4\,i_3 = 25$

$\Rightarrow 12\,i_1 + 4\,i_3 = 19$ ....(2)

$\Rightarrow$ Solving (1) and (2), we get $i_1 = 0.2692$ A and $i_3 = 3.9423$ A

$\Rightarrow i_2 = 3 + i_1 = 3.2692$ A.

$\Rightarrow V_x = 10\,i_1 = 2.692$V

12) Find v_x in the circuit shown in fig. 21, using mesh analysis;

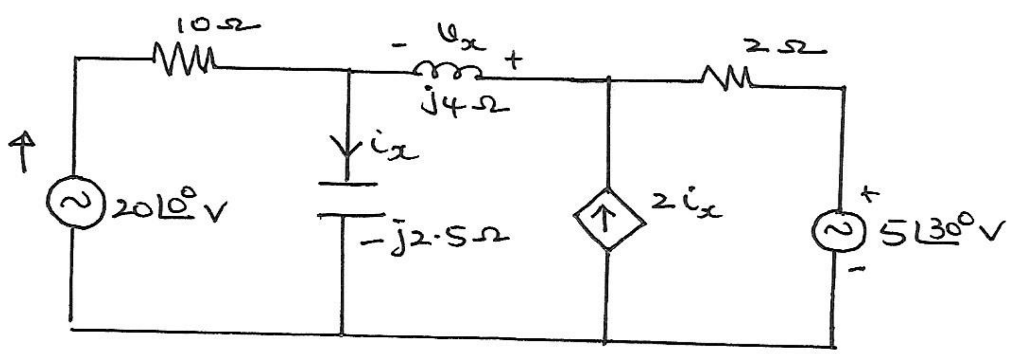
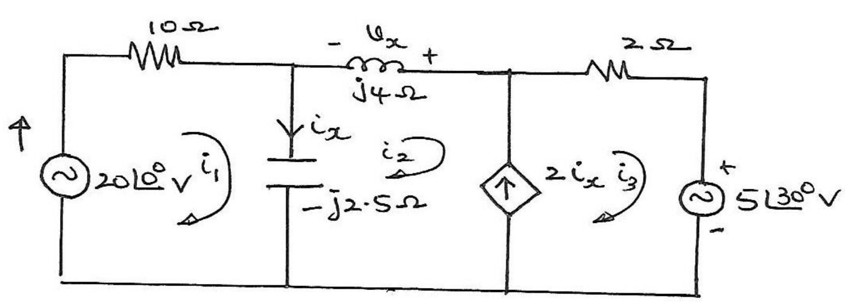


Fig.21

Solution:-



From the circuit; $v_x = -\,j4\,i_2$

$i_x = i_1 - i_2$

$i_3 - i_2 = 2\, i_x$ (current source $2i_x$ appears in common to two meshes)

$i_3 - i_2 = 2(i_1 - i_2)$

$i_3 = 2i_1 - i_2$

Apply KVL to mesh 1;

$10\, i_1 - j\, 2.5(i_1 - i_2) - (20\angle 0°) = 0$

$(10 - j2.5)\, i_1 + j\, 2.5\, i_2 = 20$ ....(1)

Apply KVL to supermesh formed by

$j4\Omega \rightarrow 2\Omega \rightarrow 5\angle 30° \rightarrow -j2.5\,\Omega \rightarrow j4\Omega$, we have,

$j4\, i_2 + 2\, i_3 + (5\angle 30°) - j\, 2.5\, (i_2 - i_1) = 0$

wkt $i_3 = 2i_1 - i_2$, subs in above eqn;

$j4\, i_2 + 2\,(2i_1 - i_2) + (5\angle 30°) - j2.5\,(i_2 - i_1) = 0$

$(4 + j2.5)\, i_1 + (-2 + j1.5)\, i_2 = -(5\angle 30°) = -4.33 - j2.5$ ....(2)

Using cramer's rule;

$$\Delta = \begin{vmatrix} 10 - j2.5 & j2.5 \\ 4 + j2.5 & -2 + j1.5 \end{vmatrix} = (10-j2.5)(-2+j1.5) - j2.5(4+j2.5) = -10 + j10$$

$$\Delta i_2 = \begin{vmatrix} 10-j2.5 & 20 \\ 4+j2.5 & -4.33-j2.5 \end{vmatrix} = (10-j2.5)(-4.33-j2.5) - 20(4+j2.5)$$
$$= -129.55 - j64.175$$

$i_2 = \Delta i_2 / \Delta = (-129.55 - j64.175) / (-10 + j10)$

$i_2 = 3.268 + j9.686$

$i_2 = 10.22 \angle 71.35° A$

Therefore, $v_x = -j4\, i_2 = 38.74 - j13.07 = 40.89 \angle -18.64° V$

Concept of Supernode:

Supernode concept is applied whenever a voltage source appears in common to two nodes.

Consider the network below;

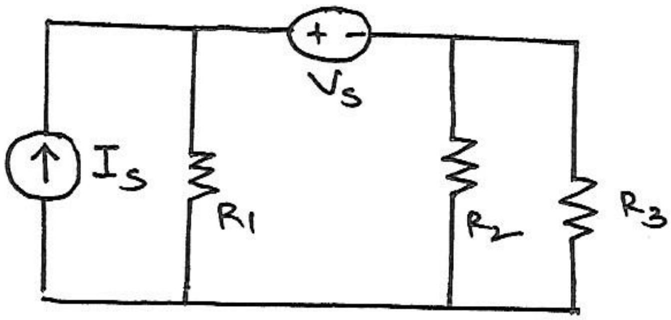
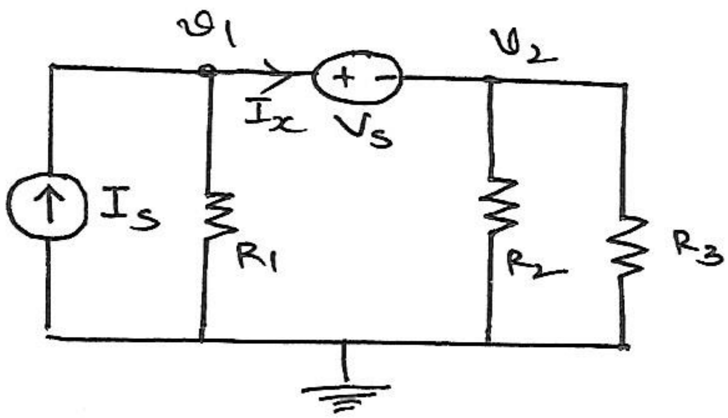


Fig.22

To illustrate the advantage of supernode concept; we first find the node voltages of the network by the usual way;



Apply KCL at node 1;

$v_1/R_1 - I_S + I_X = 0$

$v_1/ R_1 + I_X = I_S$ .... (1)

Apply KCL at node 2;

$v_2/R_2 + v_2/R_3 - I_X = 0$

$v_2/R_2 + v_2/R_3 = I_X$ .....(2)

Subs (2) in (1), we get;

$v_1/R_1 + v_2/R_2 + v_2/R_3 = I_S$ ...... (3)

Also from the circuit; $v_1 - v_2 = V_S$

=> $v_1 = V_S + v_2$ .... (4)

Substituting (4) in (3) will give the value of v_2

Substituting the value of v_2 in (4) will give the value of v_1.

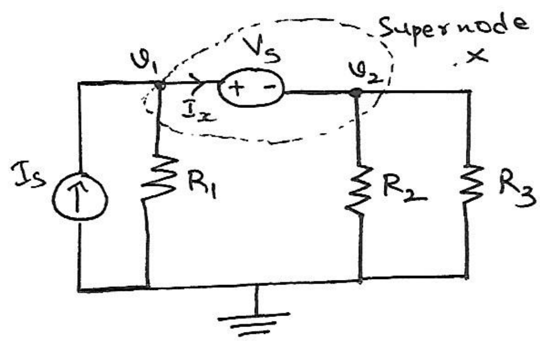
Applying the concept of supernode;

After identifying the voltage source appearing in common to two nodes;

We first write constraint equation; which relates the voltage source value with the corresponding node voltages; here it is; $v_1 - v_2 = V_S$

$v_1 = v_2 + V_S$ ...... (1)

After this, we club the corresponding nodes to become one node and call it as a supernode. Then we apply KCL to supernode. Here, we apply KCL at supernode X as shown;



$v_1/R_1 - I_S + v_2/R_2 + v_2/R_3 = 0$

$v_1/R_1 + v_2/R_2 + v_2/R_3 = I_S$ ......(2)

The above equation is same as eqn 3 in previous method, but the above equation was easily obtained in just one step. Therefore, when a voltage source is appearing in common to two nodes, it is always advantageous to consider the concept of supermesh.

Now, substituting (1) in (2), we get v_2.

Substituting v_2 in (2) we get v_1.

13) Find i_a and v_a in the network shown in fig. 23 using node analysis.

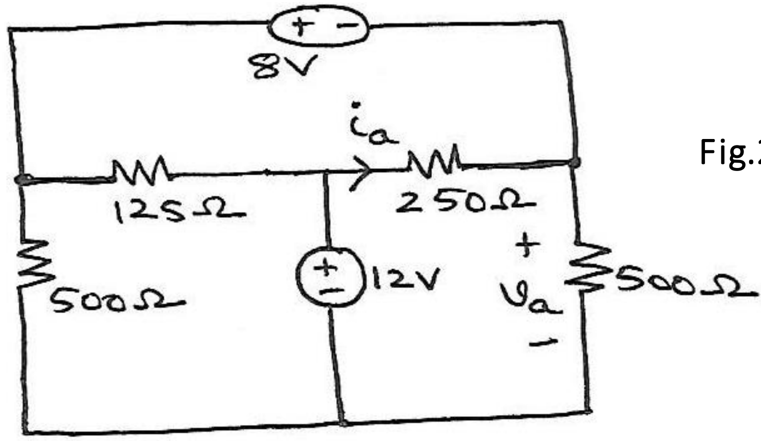
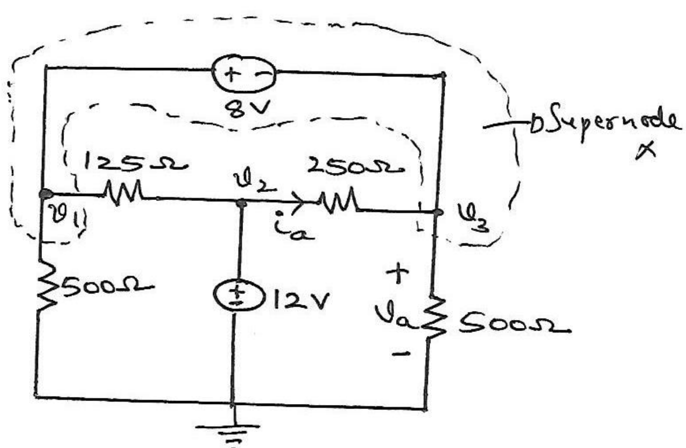


Fig.23

Solution:-



From the circuit;

$i_a = (v_2 - v_3)/250$

$v_a = v_3$

Also; $v_2 = 12$ V

$v_1 - v_3 = 8$

$\Rightarrow v_1 = 8 + v_3$

Apply KCL at supernode X;

$V1/500 + (v_1 - v_2)/125 + (v_3 - v_2)/250 + v_3/500 = 0$

$v_1 + 4v_1 - 4v_2 + 2v_3 - 2v_2 + v_3 = 0$

$5v_1 - 6v_2 + 3v_3 = 0$

Substituting $v_1 = 8 + v_3$ in above equation, we get; $5(8+v_3) - 6v_2 + 3v_3 = 0$

$-6v_2 + 8v_3 = -40$

Wkt $v_2 = 12$ V

Therefore, $v_3 = (-40+6(12))/8 = 4V$

Now, $i_a = (v_2 - v_3)/250 = 0.032 = 32$ mA.

$v_a = v_3 = 4V$.

14) Find all the node voltages in the network shown in fig.24

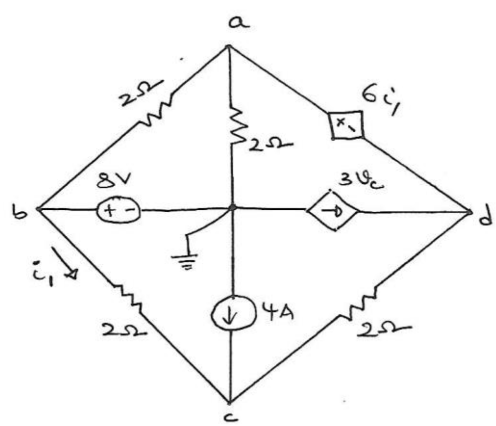
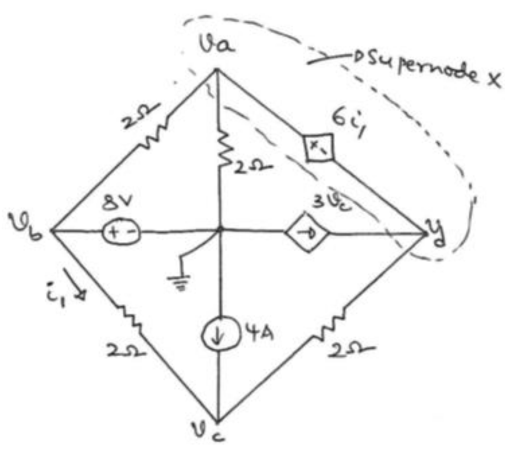


Fig.24

Solution:



From the circuit;

$v_b = 8$ V

Also, $v_a - v_d = 6\, i_1$

$i_1 = (v_b - v_c)/2$ subs in above eqn. we get;

$v_a - v_d = 6\,(v_b - v_c)\,/2$

$\Rightarrow 2v_a - 2v_d = 6\,v_b - 6\,v_c$

$\Rightarrow 2v_a + 6v_c - 2v_d = 6\,v_b = 6(8) = 48$ ....(1)

Apply KCL at supernode X as shown;

$(v_a - v_b)/2 + v_a/2 - 3v_c + (v_d - v_c)/2 = 0$

$(v_a - 8)/2 + v_a/2 - 3v_c + (v_d - v_c)/2 = 0$

$\Rightarrow v_a - 8 + v_a - 6\,v_c + v_d - v_c = 0$

$\Rightarrow 2v_a - 7v_c + v_d = 8$ ....(2)

Apply KCL at node C

$-4 + (v_c - v_d)/2 + (v_c - v_b)/2 = 0$

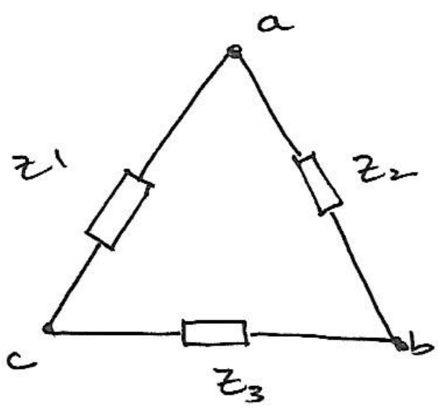
$\Rightarrow -8 + v_c - v_d + v_c - v_b = 0$

$\Rightarrow 2v_c - v_d = v_b + 8 = 16$ ....(3)

Solving (1),(2) and (3), we get; $v_a = 9.142V$, $v_c = -1.142\ V$, $v_d = -18.28V$

and $v_b = 8V$ (given)

Star- delta (Δ) and delta (Δ) to star transformations



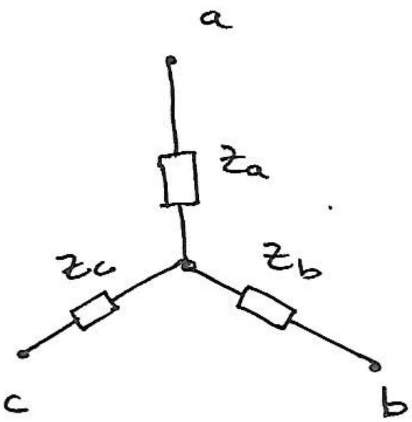


Fig25a delta arrangement **Fig.25bStar arrangement**

(The positions of Z_1, Z_2 and Z_3 should be noted. Z_1 will appear between a and c; from there, going clockwise we see Z_2 and Z_3. The positions of Z_a, Z_b and Z_c should be noted. Z_a connected to vertex-a and centroid. Z_b connected to vertex-b and centroid. Z_c connected to vertex-c and centroid.)

Consider the above arrangements are equivalent; then;

$Z_{ac} = Z_1(Z_2+ Z_3) / (Z_1+Z_2+ Z_3) = Z_a +Z_c$ ....(1)

Also,

$Z_{ab} = Z_2(Z_3+ Z_1) / (Z_1+Z_2+ Z_3) = Z_a + Z_b$ ....(2)

$Z_{bc} = Z_3(Z_1+ Z_2) / (Z_1+Z_2+ Z_3) = Z_b + Z_c$ ....(3)

Eqn. (1) –Eqn.(3)

$(Z_1Z_2 – Z_2Z_3)/(Z_1+Z_2+Z_3) = Z_a - Z_b$ ....(4)

Solving (2) and (4), we get, $Z_a = Z_1 Z_2 /(Z_1 +Z_2 +Z_3)$.........(5)

Substituting (5) in (2), solving for Z_a, we get;

$Z_b = Z_2 Z_3 /(Z_1 +Z_2 +Z_3)$.........(6)

Substituting (5) in (1), solving for Z_c, we get;

$Z_c = Z_1 Z_3 /(Z_1 + Z_2 + Z_3)$………(7)

Consider

$Z_a Z_b + Z_b Z_c + Z_a Z_c = (Z_1 Z_2^2 Z_3 + Z_1 Z_2 Z_3^2 + Z_1^2 Z_2 Z_3) / (Z_1 + Z_2 + Z_3)^2$

$Z_a Z_b + Z_b Z_c + Z_a Z_c = Z_1 Z_2 Z_3 / (Z_1 + Z_2 + Z_3)$ ………(8)

Eqn(8) / Z_b gives

$Z_1 = (Z_a Z_b + Z_b Z_c + Z_a Z_c) / Z_b$ …………………(9)

Eqn(8) / Z_c gives

$Z_2 = (Z_a Z_b + Z_b Z_c + Z_a Z_c) / Z_c$ …………………(10)

Eqn(8) / Z_a gives

$Z_3 = (Z_a Z_b + Z_b Z_c + Z_a Z_c) / Z_a$ …………………(11)

15) Reduce the network shown in fig.26 to a single resistor between terminals a-b.

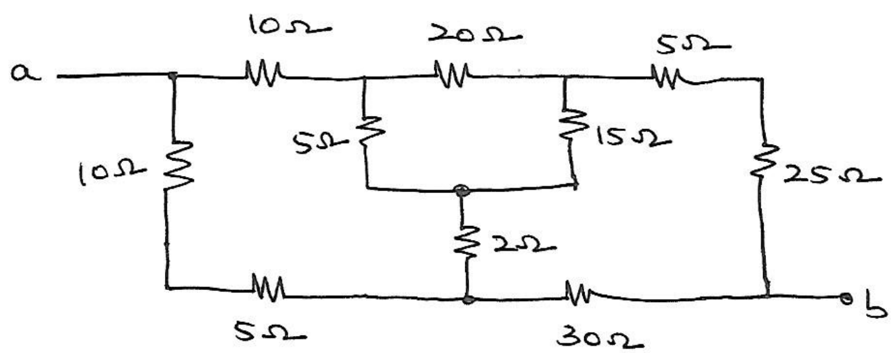
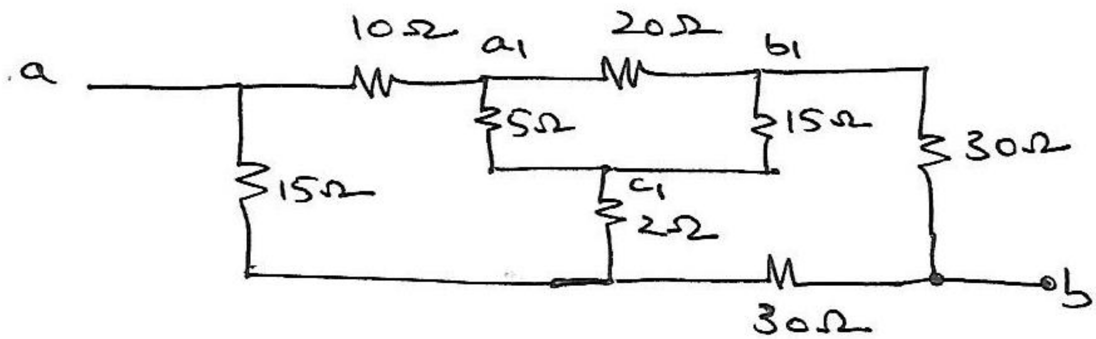


Fig.26

Solution:-



From the network above, we observe, 10Ω and 5Ω are in series and also 5Ω and 25Ω are in series. Therefore they are equivalently replaced by

15 Ω and 30 Ω as shown.

Identifying delta between the vertices a1-b1-c1;

We have $R_1 \rightarrow R_2 \rightarrow R_3$

as, 5Ω→20Ω→15Ω

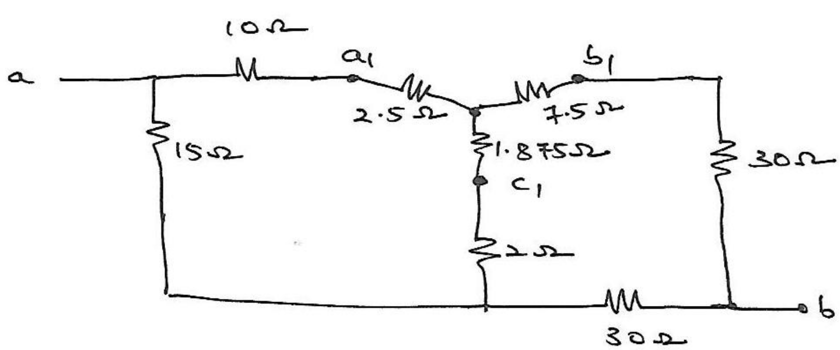
Corresponding star will have;

$R_a = R_1 R_2 /(R_1 + R_2 + R_3)$ = 100/40 =2.5 Ω (resistance connected to vertex a1)

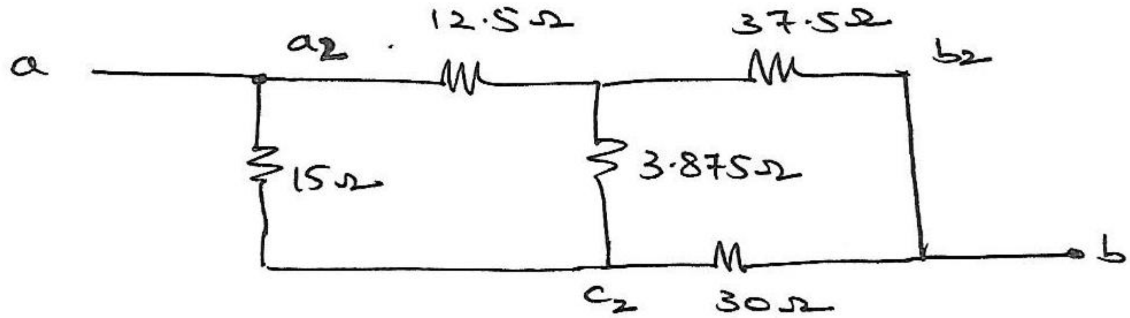
$R_b = R_2 R_3 /(R_1 + R_2 + R_3)$ = 300/40 =7.5 Ω (resistance connected to vertex b1)

$R_c = R_1 R_3 /(R_1 + R_2 + R_3)$ = 75/40 =1.875 Ω (resistance connected to vertex c1)

After replacing delta elements by corresponding star elements;



10Ω and 2.5 Ω appear in series. 30Ω and 7.5Ω appear in series. 2Ω and 1.875Ω appear in series. They are replaced by their equivalent resistances.



Identifying star between the vertices a2-b2-c2;

We have $R_a \rightarrow R_b \rightarrow R_c$

as, 12.5Ω→37.5Ω→3.875Ω

Corresponding delta will have;

$R_1 = (R_a R_b + R_b R_c + R_a R_c)/R_b$

$= [(12.5)(37.5) + (37.5)(3.875) + (3.875)(12.5)]/37.5$

$=662.5/37.5= 17.66\ \Omega$ (resistance connected b/n vertex a2 and c2)

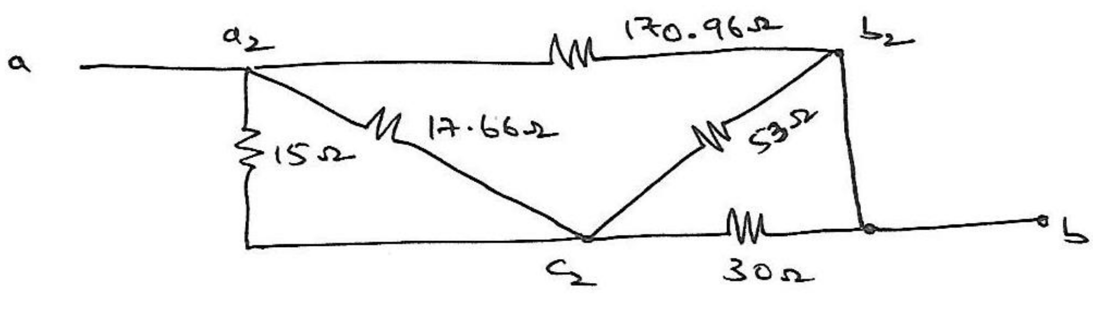
$R_2 = (R_a R_b + R_b R_c + R_a R_c)/R_c$

$=662.5/3.875= 170.96\ \Omega$ (resistance connected b/n vertex a2 and b2)

$R_3 = (R_a R_b + R_b R_c + R_a R_c)/R_a$

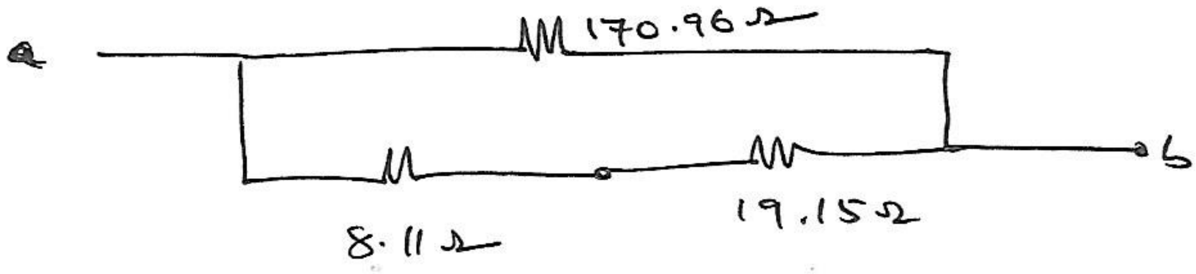
$=662.5/12.5= 53\ \Omega$ (resistance connected b/n vertex b2 and c2)

After replacing star elements by corresponding delta elements;

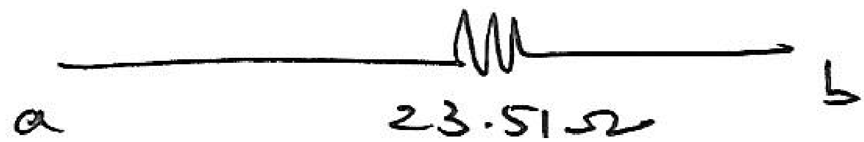


$15||17.66 = 8.11\Omega$

$53||30 = 19.15\Omega$



Therefore, $R_{ab} = (19.15 + 8.11)||170.96 = 23.51\Omega$



Q16) Find the current I in the network shown in fig.27, by reducing the network to contain a source and and a single series impedance.

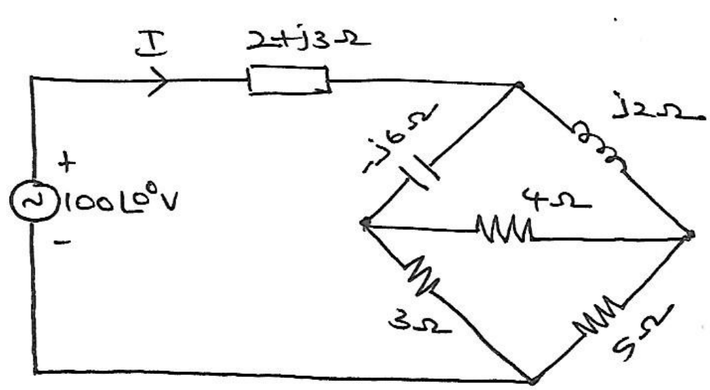
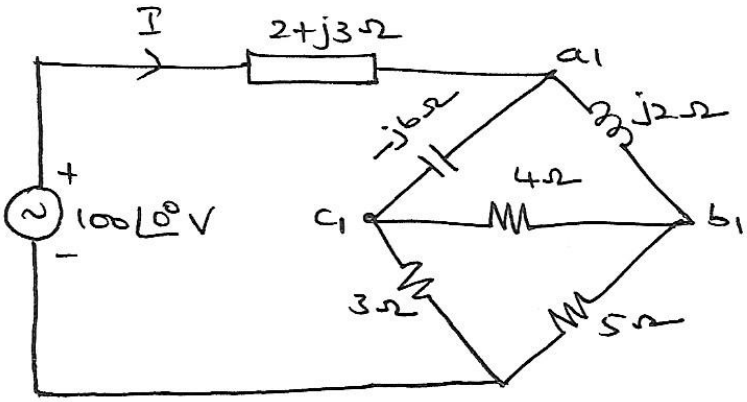


Fig.27

Solution:-



Identifying delta between the vertices a1-b1-c1;

We have $Z_1 \rightarrow Z_2 \rightarrow Z_3$

as, -j6Ω→j2Ω→4Ω

Corresponding star will have;

$Z_a = Z_1 Z_2 /(Z_1 + Z_2 + Z_3)$ = (-j6)(j2)/(4-j4) = 1.5 + j1.5Ω

(Impedance connected to vertex a1)

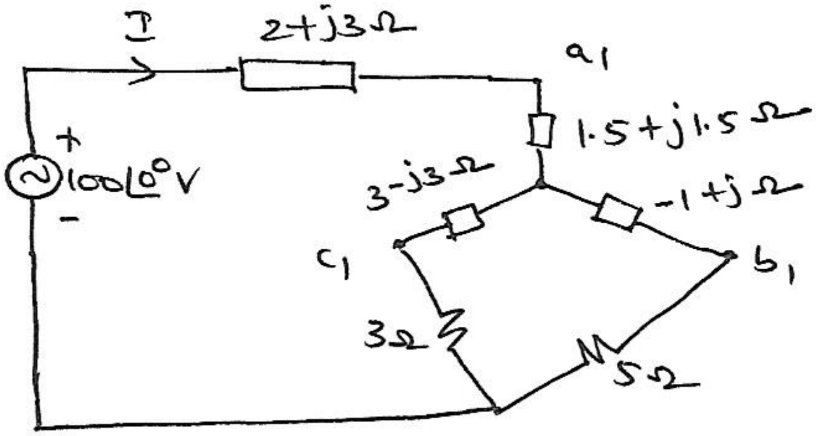
$Z_b = Z_2 Z_3 /(Z_1 + Z_2 + Z_3)$ = (j2)(4)/(4-j4) = -1 + j Ω

(Impedance connected to vertex b1)

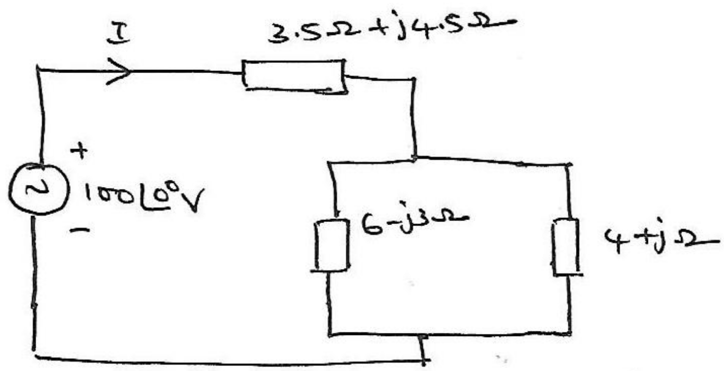
$Z_c = Z_1 Z_3 /(Z_1 + Z_2 + Z_3)$ = (-j6)(4)/(4-j4) = 3-j3 Ω

(Impedance connected to vertex c1)

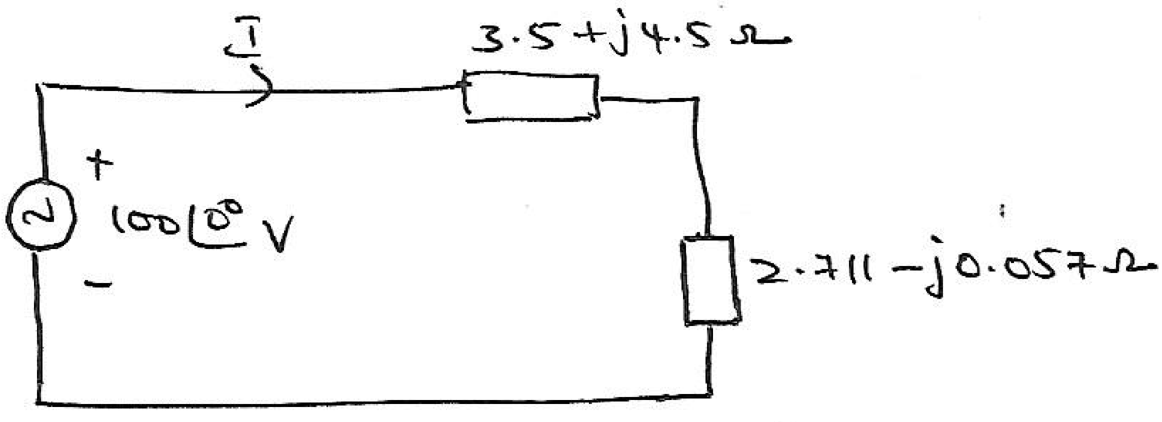
After replacing delta elements by corresponding star elements;



The series impedances are replaced by equivalent impedances



(6-j3) // (4+j) = 2.711 - j 0.057Ω



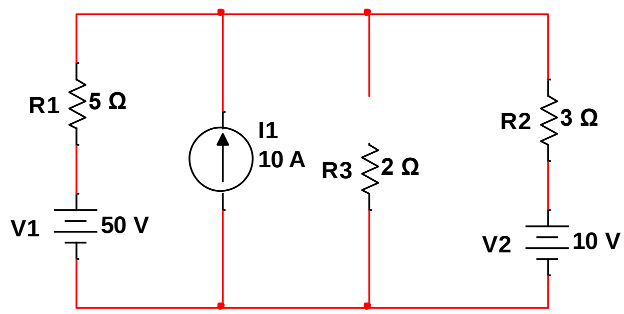
The single series impedance value , Z = (3.5 + j4.5) + (2.711 - j 0.057)

$$Z = 6.211 + j\,4.443\ \Omega$$

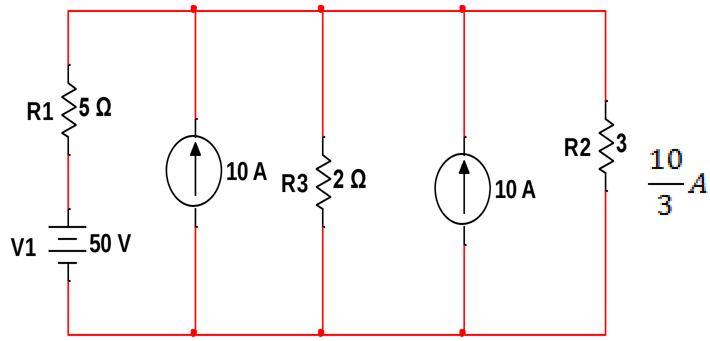
Therefore, $I = 100/Z = 100/(6.211 + j4.443) = 13.09\angle -35.57^\circ$ A

Additional Problems and Solutions

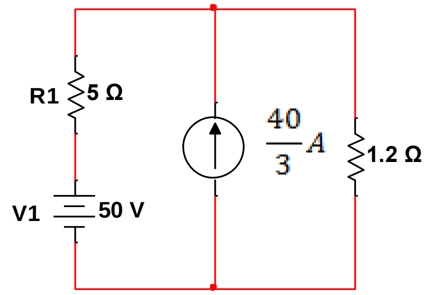
1) Using source transform, find the power delivered by the 50V source in the circuit shown:-



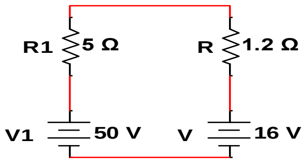
Solution: - Using source transformation for the pair V2 and R2, we get,



Adding the parallel current sources and obtaining equivalent resistance of R3 and R2, we have,



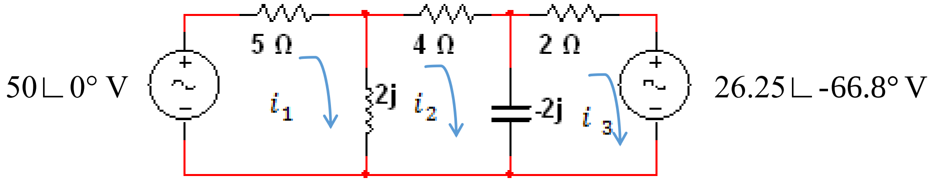
Converting the current source back to voltage source,



If I is the current in the circuit, $I = \frac{50-16}{6.2} = 5.48A$

Therefore Power delivered by 50V source is $P = I \times 50 = 5.48 \times 50 = 274.19W$.

2) Find the current through 4Ω in the network shown:



Solution: - Applying KVL to mesh 1 (mesh with i_1)

$$5i_1 + 2j(i_1 - i_2) - 50 = 0$$
$$\Rightarrow (5+2j)i_1 - (2j)i_2 = 50$$

Applying KVL to mesh 2

$$4i_2 - 2j(i_2 - i_3) + 2j(i_2 - i_1) = 0$$
$$\Rightarrow (-2j)i_1 + (4)i_2 + 2j(i_2 - i_1) = 0$$

Applying KVL to mesh 3

$$(2j)i_3 + (26.25\angle{-66.8^\circ}) - (2j)(\ i_3 - i_2) = 0$$
$$\Rightarrow (2-2j)i_3 + (2j)i_2 = (26.25\angle{-66.8^\circ}) = -10.39 + (24.12)\,j$$

Matrix form

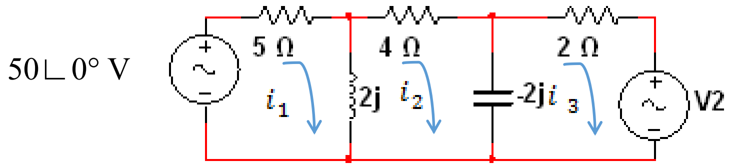
$$\begin{bmatrix} 5+2j & -2j & 0 \\ -2j & 4 & j-2 \\ 0 & 2j & 2-2j \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \\ -10.39+24.12j \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 5+2j & -2j & 0 \\ -2j & 4 & 2j \\ 0 & 2j & 2-2j \end{vmatrix} = 84 - 24j$$

$$\Delta i_2 = \begin{vmatrix} 5+2j & 50 & 0 \\ -2j & 0 & 2j \\ 0 & -10.39+24.12j & 2-2j \end{vmatrix} = 399.64 + 400.38j$$

$\boldsymbol{i_2} = \frac{\Delta i_2}{\Delta} = \mathbf{6.47\angle 60.99}°\text{A}$

3) Find the value of V2 if the current through 4Ω is zero.



Solution: - Given i_2=0

Applying KVL to mesh 3 (mesh with i_3), we get

$2i_3 + V2 - 2j(i_3) = 0$

$\Rightarrow \text{V2} = (-2 + 2\text{j})i_3$

Applying KVL to mesh 2,

$4i_2 - 2j(i_2 - i_3) + 2j(i_2 - i_1) = 0$

$\Rightarrow i_3 = i_1$

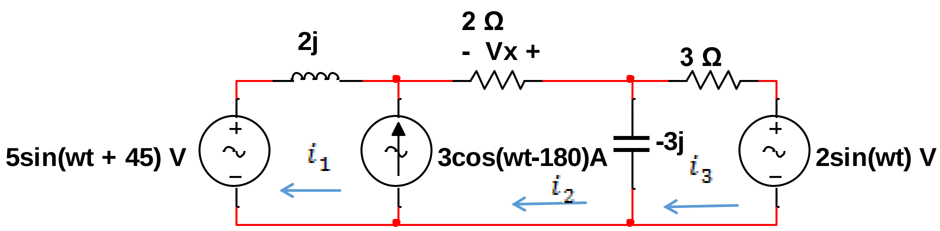
Applying KVL to mesh 1,

$5i_1 + 2j(i_1) = 50$

$\Rightarrow i_1 = 9.28\angle -21.8°\ \text{A} = i_3$

Therefore, **V2 =** $\boldsymbol{i_3(-2 + 2j) = 26.26\angle 113.19°}$ **V**

4) Find V_x using mesh analysis for the circuit shown



Solution: - From the circuit $V_x = -2i_2$

Applying concept of super mesh, $i_2 - i_1 = 3\angle -90°$

Therefore, $i_1 = -3\angle -90° + i_2$

Remove the arm of the current source and apply kvl,

$(2j)i_1 - V_x - (3j)(i_2 - i_3) - 5\angle 45° = 0$

$\Rightarrow (2 - j)i_2 + (3j)i_3 = 9.535 + j3.535$

Applying KVL to mesh with i_3

$$(3-3j)i_3 + (3j)i_2 = -2$$

Therefore $\Delta = \begin{vmatrix} 2-j & j3 \\ j3 & 3-j3 \end{vmatrix} = 12 - 9j$

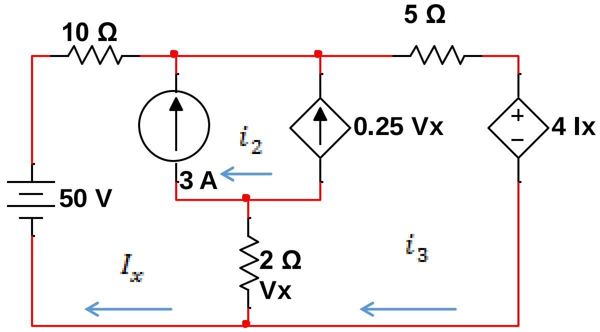
$$\Delta i_2 = \begin{vmatrix} 9.535 + j3.535 & j3 \\ -2 & 3-j3 \end{vmatrix} = 39.21 - j12$$

$$i_2 = \frac{\Delta i_2}{\Delta} = 2.73\angle 19.85^\circ \text{ A}$$

$$V_x = -2(2.73\angle 19.85^\circ)\text{v}$$

Therefore, $\boldsymbol{V_x = 5.49\angle -160.15^\circ}$ **V**

5) Find V_x and I_x in the circuit shown using mesh analysis



Solution: - From the circuit $V_x = 2(I_x - i_3) \ldots..(1)$

Also from the circuit $i_2 - I_x = 3 \ldots \ldots.(2)$; $i_3 - i_2 = 0.25\, V_x \ldots \ldots.(3)$

Substituting equations 1 and 2 in 3, we get

$6(i_3 - I_x) = 12 \Rightarrow i_3 - I_x = 2$......(4)

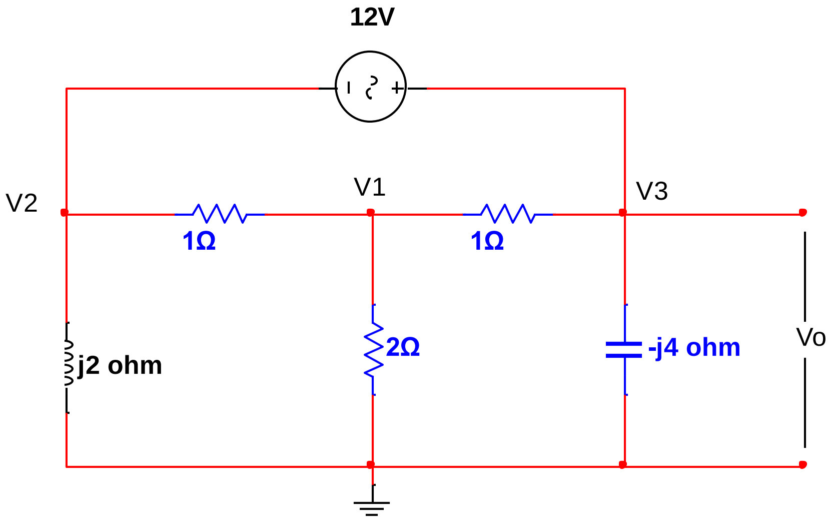
Removing the arm containing common current source and applying KVL, we get

$$14I_x + 5i_3 = 50 \ldots \ldots(5)$$

Solving equations 4 and 5, we get $\boldsymbol{I_x = 2.1A}$

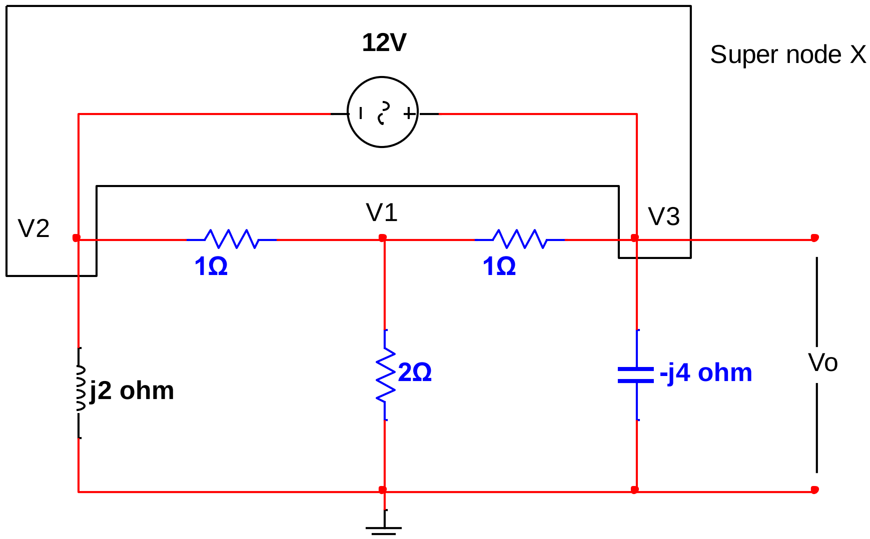
Therefore, $\boldsymbol{V_x = -4V}$.

6) Use node analysis to find V_0 in the circuit shown below



From the circuit,

$$V_0 = V_3;\ V_0 - V_2 = 12V \text{ ----- (1);}$$



Applying KCL to super node X,

$$\Rightarrow \frac{V_2}{j2} + \frac{V_2 - V_1}{1} + \frac{V_0 - V_1}{1} + \frac{V_0}{-j4} = 0$$

$$\Rightarrow \frac{-jV_2}{2} + V_2 - V_1 + V_0 - V_1 + \frac{jV_0}{4} = 0$$

$$\Rightarrow -2jV_2 + 4V_2 - 4V_1 + 4V_0 - 4V_1 + jV_0 = 0$$

$\Rightarrow\ (4+j)V_0 - 8V_1 + (4-j2)(V_0 - 12) = 0$ (From (1))

$\Rightarrow\ 4V_0 + jV_0 - 8V_1 + 4V_0 - j2V_0 - 48 + j24 = 0$

$\Rightarrow\ (8-j)V_0 - 8V_1 = 48 - j24$ --------- (2)

Applying KCL at V_1,

$\Rightarrow\ \frac{V_1}{2} + \frac{V_1 - V_0}{1} + \frac{V_1 - V_2}{1} = 0$

$\Rightarrow\ V_1 + 2V_2 - 2V_0 + 2V_1 - 2V_2 = 0$

$\Rightarrow\ -2V_0 + 5V_1 - 2V_2 = 0$

$\Rightarrow\ -2V_0 + 5V_1 - 2(V_0 - 12) = 0$

$\Rightarrow\ -4V_0 + 5V_1 = -24$ ------------- (3)

Using Cramer's rule,

$\Delta = \begin{vmatrix} 8-j & -8 \\ -4 & 5 \end{vmatrix}$

$\Delta = 5(8-j) - 32$

$\Delta = -5j + 8$

$\Delta V_0 = \begin{vmatrix} 48-j24 & -8 \\ -24 & 5 \end{vmatrix}$

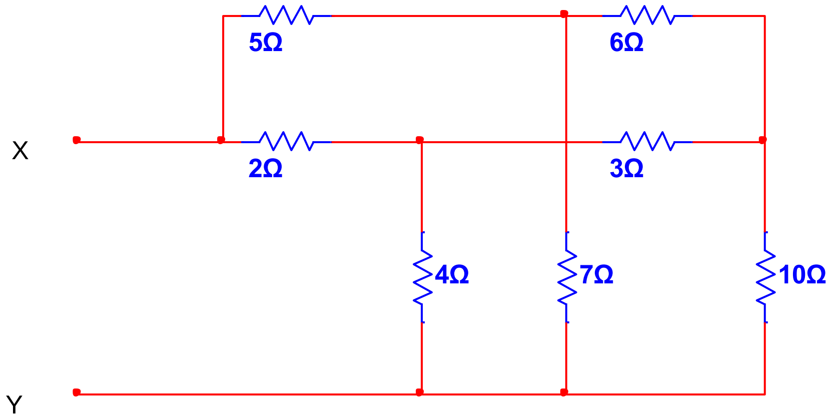
$\Delta V_0 = (48 - j24)5 - 192$

$\Delta V_0 = -j120 + 48$

W.K.T, $V_0 = \frac{\Delta V_0}{\Delta}$

$\therefore\ V_0 = 13.69V\ @ - 36.19°$

7) Find the equivalent resistance between the terminals X and Y



Solution:-

Star 1:- $R_a = 2;\ R_b = 3;\ R_c = 4$;

Corresponding Delta will have,

$$R_1 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_b}$$

$\therefore\ R_1 = 8.66\Omega$

Similarly,

$R_2 = \frac{26}{4}$ = 6.5 Ω

$R_3 = \frac{26}{3}$ = 13 Ω

Now consider star 2:- $R_a = 5;\ R_b = 6;\ R_c = 7$;

Corresponding Delta will have,

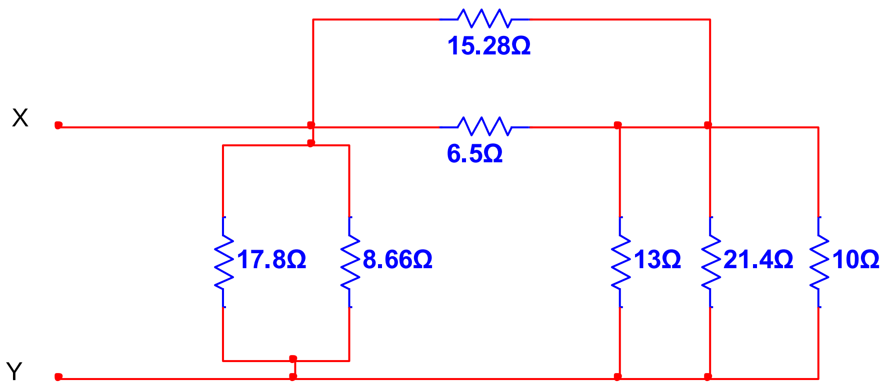
$$R_1 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_b}$$

$\therefore\ R_1 = 17.8\Omega$

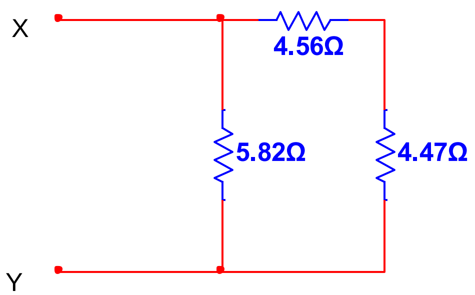
Similarly,

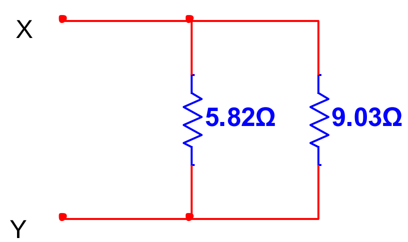
$R_2 = \frac{107}{7}$ = 15.28 Ω

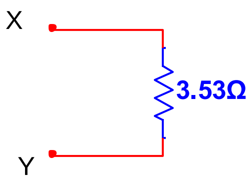
$R_3 = \frac{107}{5}$ = 21.4 Ω



This circuit can be reduced now using parallel and series combination of resistors as show below.

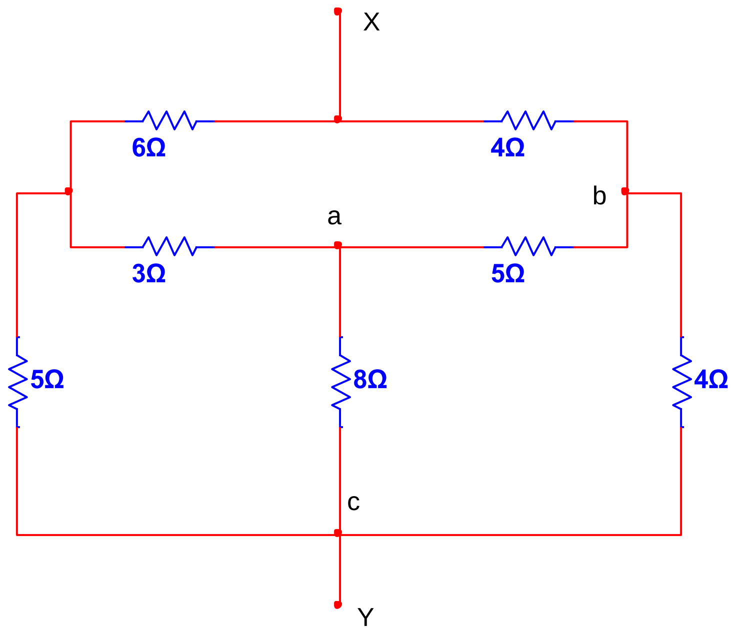






Therefore the equivalent resistance between X & Y = 3.53 Ω

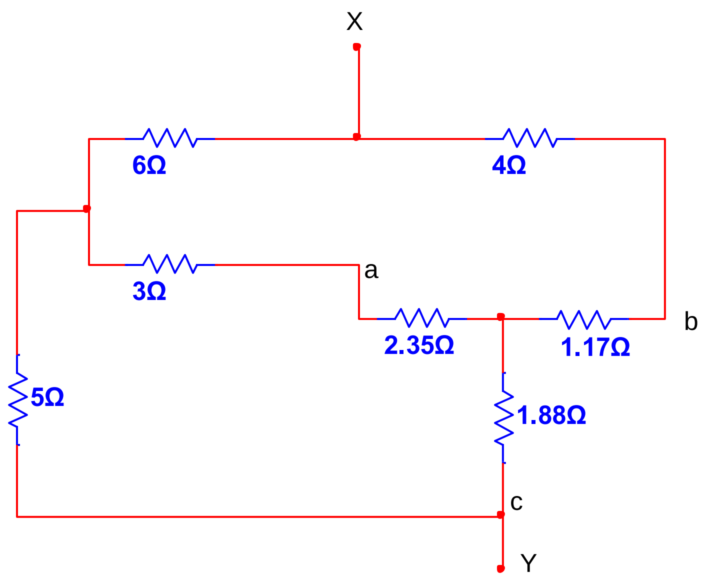
8) Determine the equivalent resistance between the terminals X & Y



Solution:

Consider the Delta $R_1 = 8; R_2 = 5; R_3 = 4;$

It can be replaced with the circuit shown below



Where, $R_a = \frac{R_1R_2}{R_1+R_2+R_3}$

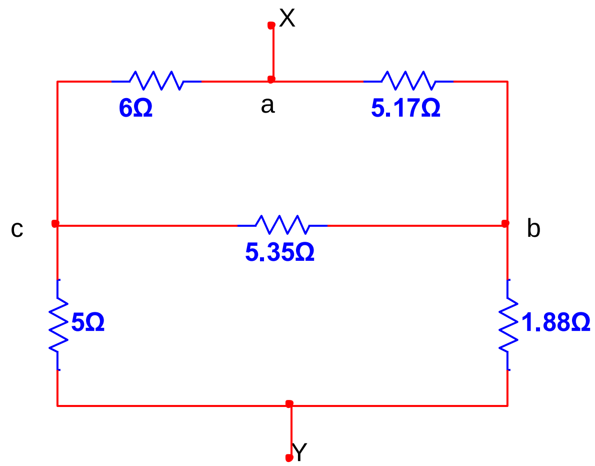
$\therefore R_a = 2.35\Omega$

Similarly,

$R_b = 1.17\Omega$

$R_c = 1.88\Omega$

The above circuit can be written as,

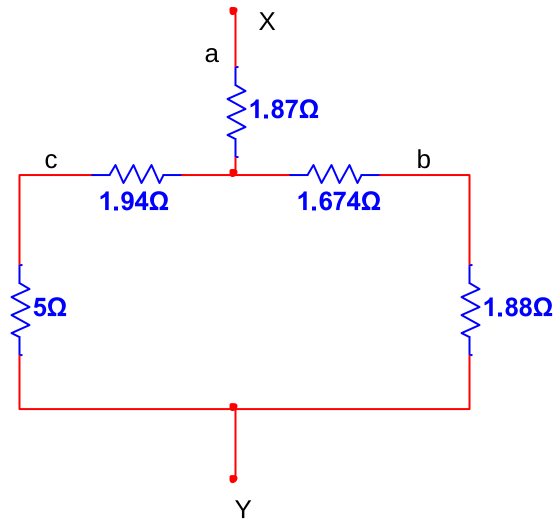


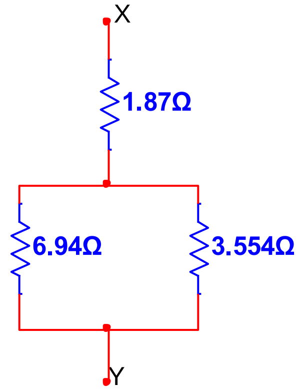
Consider the Delta, $R_1 = 6;\ R_2 = 5.17;\ R_3 = 5.35;$

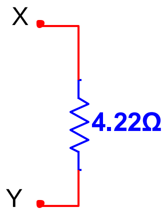
$\therefore R_a = 1.877\Omega$

$\therefore R_b = 1.674\Omega$

$\therefore R_c = 1.94\Omega$



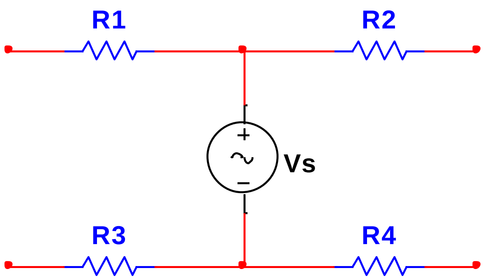




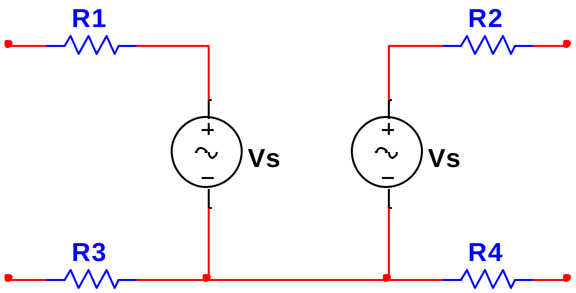
Therefore the equivalent resistance between X & Y = 4.22 Ω

Source Shifting:

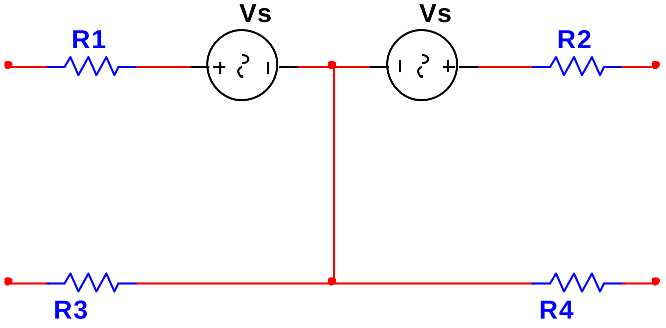
(i) Voltage Source Shifting:-



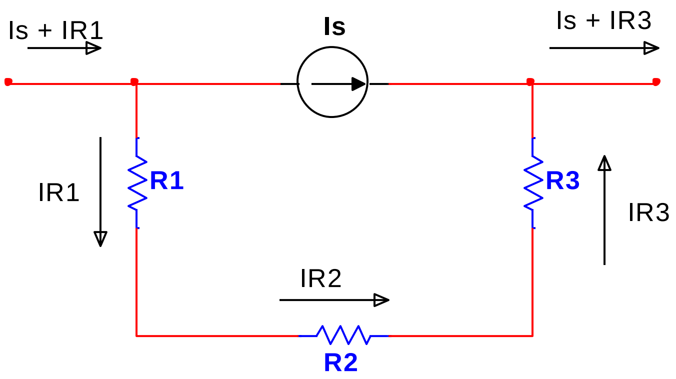
The above circuit can be written as,



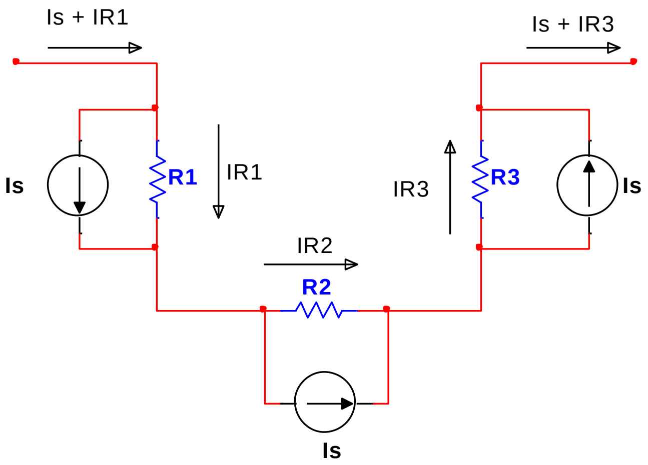
Which is equivalent to,



(ii) Current Source Shifting:-

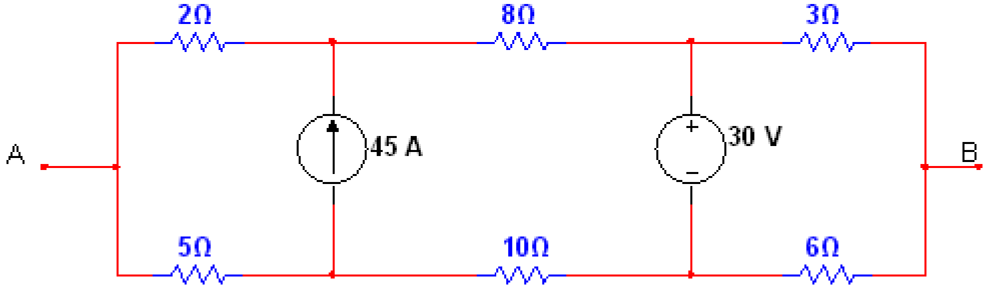


The above circuit can be redrawn as,



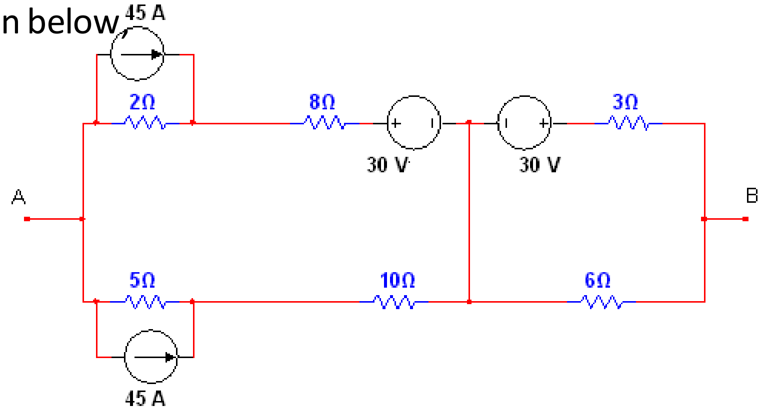
Problems on Source Shifting & Source Transformation:-

1) Reduce the network shown to a single voltage source in series with a resistance using source shifting and source transformation.

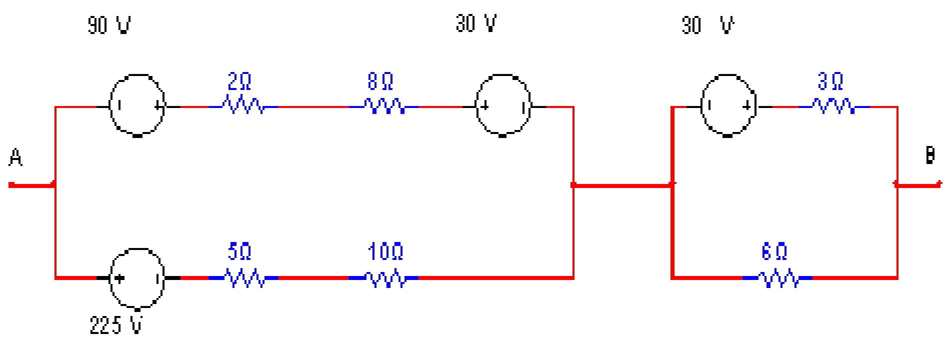


Solution:-

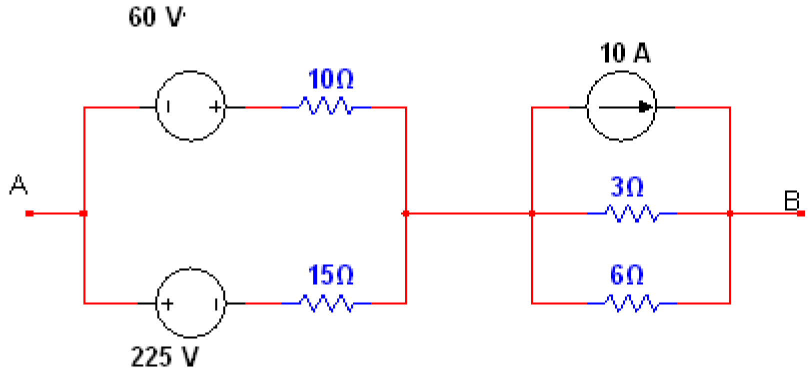
Use Source shifting property on both the sources and rewrite the circuit a shown below,



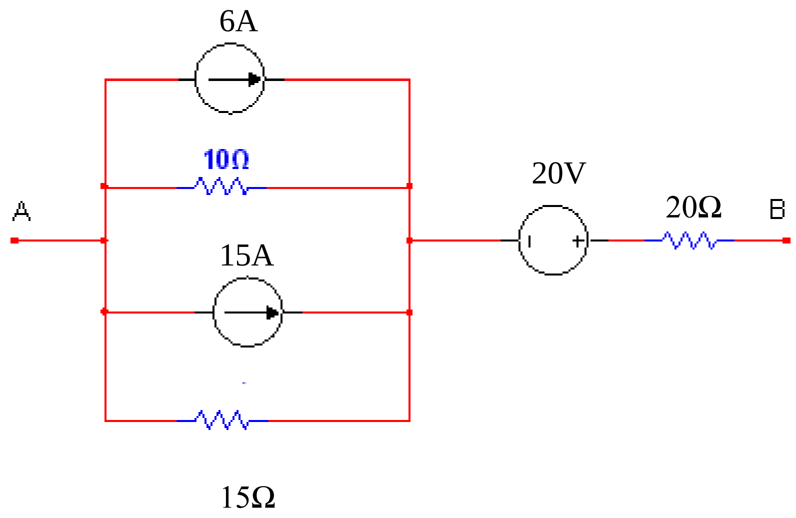
Now using Source transformation we get,

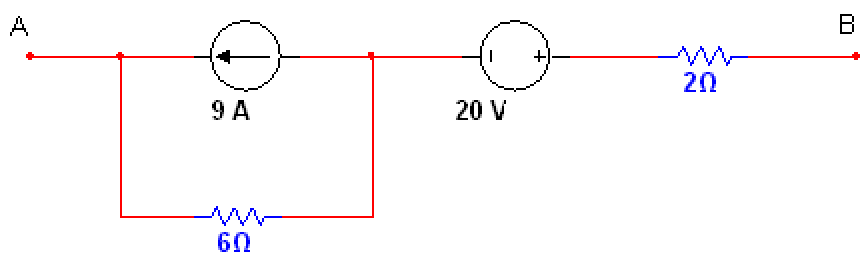


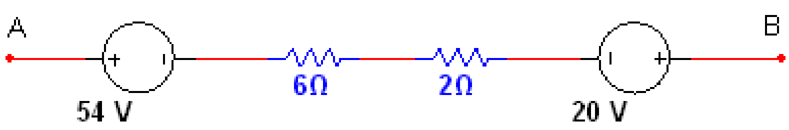
After simplifying the above circuit and applying Source transformation again, we get,

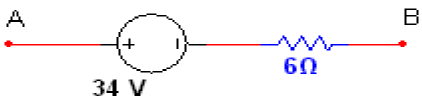


Which can be further simplified using Source transformation yet again,

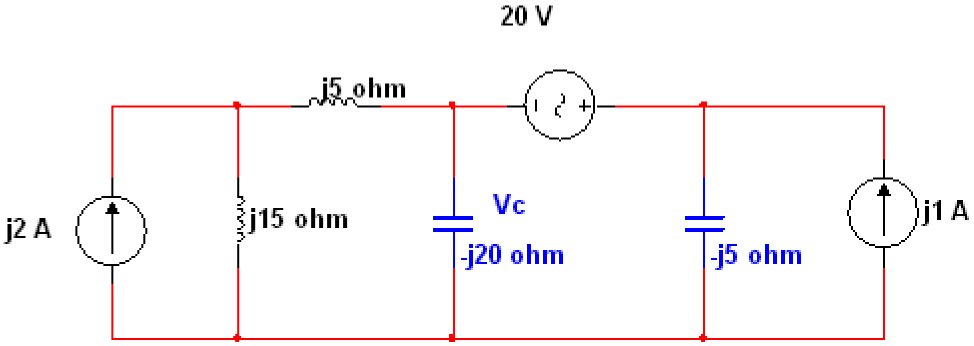




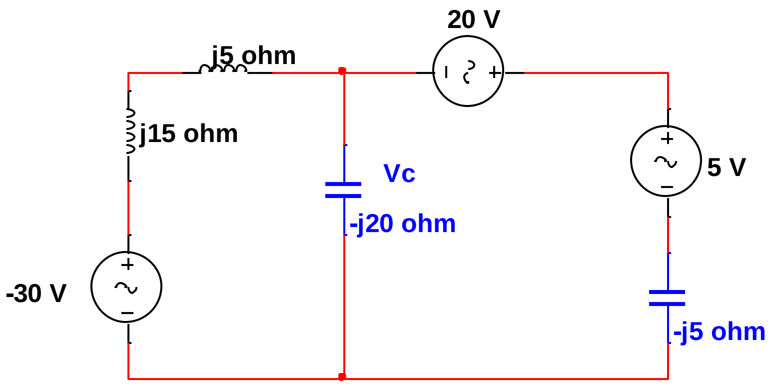


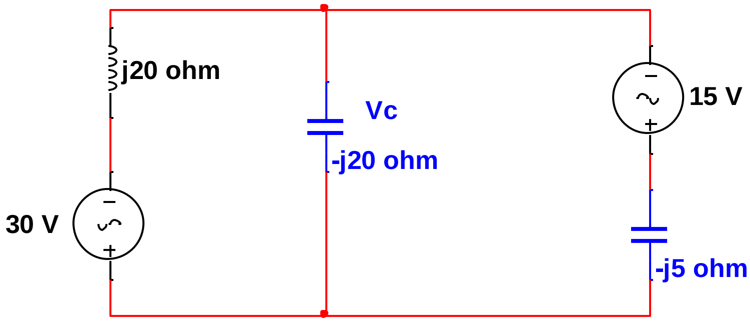


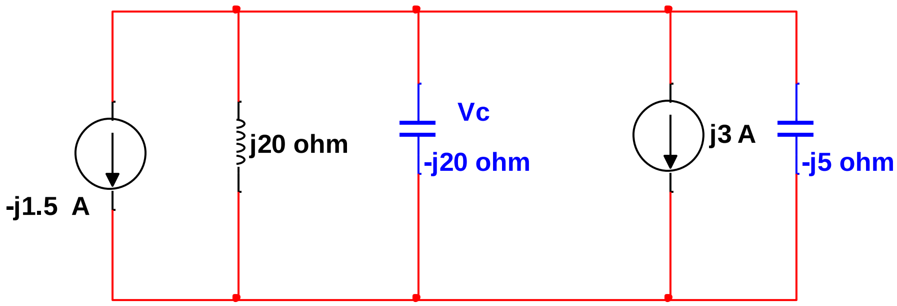
2) Find the voltage across the capacitor of 20Ω reactance of the network.

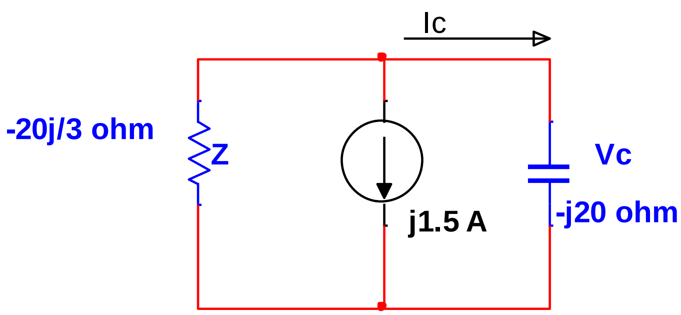


Solution:- Using Source Transformation,









From the above circuit,

$$I_c = \frac{(-j1.5)(-j6.67)}{(-j26.67)}$$

$$\therefore\ I_c = -j(0.375)\ A$$

$$\therefore\ V_c = I_c(-j20)$$

$$\therefore\ V_c = \ -7.5\ V$$

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