MODULE-2

Balancing of Rotating Masses

CONTENTS

OBJECTIVES

- To study Importance of Balancing of rotating masses.
- To solve Various problems on Balancing of rotating masses.

2.1Balancing of Rotating Masses

INTRODUCTION:

When man invented the wheel, he very quickly learnt that if it wasn't completely round and if it didn't rotate evenly about it's central axis, then he had a problem!

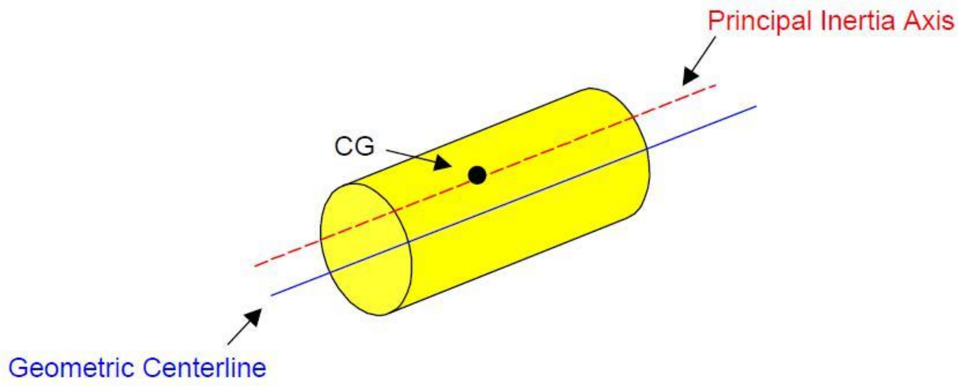
What the problem he had?

The wheel would vibrate causing damage to itself and it's support mechanism and in severe cases, is unusable.

A method had to be found to minimize the problem. The mass had to be evenly distributed about the rotating centerline so that the resultant vibration was at a minimum.

UNBALANCE:

The condition which exists in a rotor when vibratory force or motion is imparted to its bearings as a result of centrifugal forces is called unbalance or the uneven distribution of mass about a rotor's rotating centerline.



Rotating centerline:

The rotating centerline being defined as the axis about which the rotor would rotate if not constrained by its bearings. (Also called the Principle Inertia Axis or PIA).

Geometric centerline:

The geometric centerline being the physical centerline of the rotor.When the two centerlines are coincident, then the rotor will be in a state of balance. When they are apart, the rotor will be unbalanced.

Different types of unbalance can be defined by the relationship between the two centerlines. These include:

Static Unbalance – where the PIA is displaced parallel to the geometric centerline. (Shown above)

Couple Unbalance – where the PIA intersects the geometric centerline at the center of gravity. (CG)

Dynamic Unbalance – where the PIA and the geometric centerline do not coincide or touch.

The most common of these is dynamic unbalance.

Causes of Unbalance:

In the design of rotating parts of a machine every care is taken to eliminate any out of balance or couple, but there will be always some residual unbalance left in the finished part because of

i) slight variation in the density of the material or

ii) inaccuracies in the casting or

iii) inaccuracies in machining of the parts.

Why balancing is so important?

iii) A level of unbalance that is acceptable at a low speed is completely unacceptable at a higher speed.

iv) As machines get bigger and go faster, the effect of the unbalance is much more severe.

v) The force caused by unbalance increases by the square of the speed.

vi) If the speed is doubled, the force quadruples; if the speed is tripled the force increases by a factor of nine!

Identifying and correcting the mass distribution and thus minimizing the force and resultant vibration is very very important

2.2 Static and dynamic balancing.

BALANCING:

Balancing is the technique of correcting or eliminating unwanted inertia forces or moments in rotating or reciprocating masses and is achieved by changing the location of the mass centers.

The objectives of balancing an engine are to ensure:

That the centre of gravity of the system remains stationery during a complete revolution of the crank shaft and .That the couples involved in acceleration of the different moving parts balance each other.

Types of balancing:

Static Balancing:

Static balancing is a balance of forces due to action of gravity.

A body is said to be in static balance when its centre of gravity is in the axis of rotation.

Dynamic balancing:

Dynamic balance is a balance due to the action of inertia forces.

A body is said to be in dynamic balance when the resultant moments or couples, which involved in the acceleration of different moving parts is equal to zero.

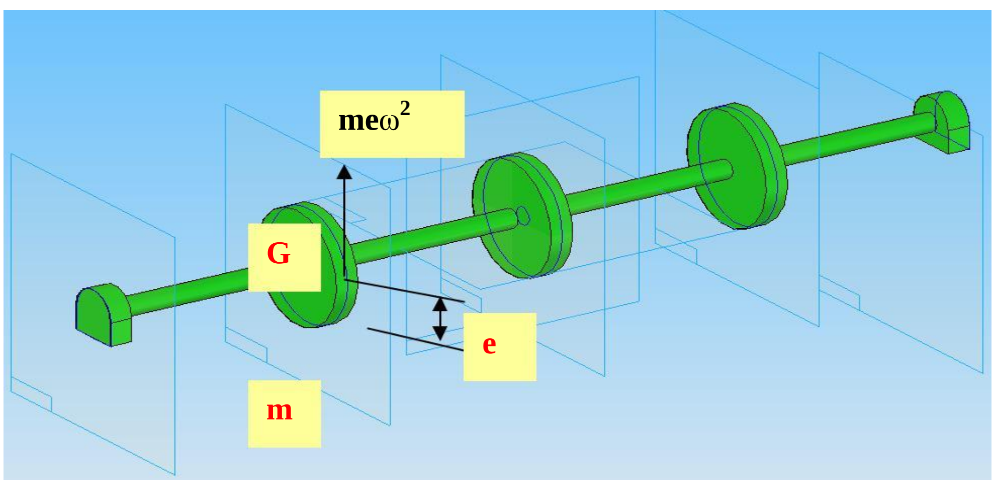
The conditions of dynamic balance are met, the conditions of static balance are also met.

In rotor or reciprocating machines many a times unbalance of forces is produced due to inertia forces associated with the moving masses. If these parts are not properly balanced, the dynamic forces are set up and forces not only increase loads on bearings and stresses in the various components, but also unpleasant and dangerous vibrations.
Balancing is a process of designing or modifying machinery so that the unbalance is reduced to an acceptable level and if possible eliminated entirely.

BALANCING OF ROTATING MASSES

When a mass moves along a circular path, it experiences a centripetal acceleration and a force is required to produce it. An equal and opposite force called centrifugal force acts radially outwards and is a disturbing force on the axis of rotation. The magnitude of this remains constant but the direction changes with the rotation of the mass.

In a revolving rotor, the centrifugal force remains balanced as long as the centre of the mass of rotor lies on the axis of rotation of the shaft. When this does not happen, there is an eccentricity and an unbalance force is produced. This type of unbalance is common in steam turbine rotors, engine crankshafts, rotors of compressors, centrifugal pumps etc.



The unbalance forces exerted on machine members are time varying, impart vibratory motion and noise, there are human discomfort, performance of the machine deteriorate and detrimental effect on the structural integrity of the machine foundation.

Balancing involves redistributing the mass which may be carried out by addition or removal of mass from various machine members

Balancing of rotating masses can be of

1) Balancing of a single rotating mass by a single mass rotating in the same plane.
2) Balancing of a single rotating mass by two masses rotating in different planes.
3) Balancing of several masses rotating in the same plane
4) Balancing of several masses rotating in different planes

STATIC BALANCING

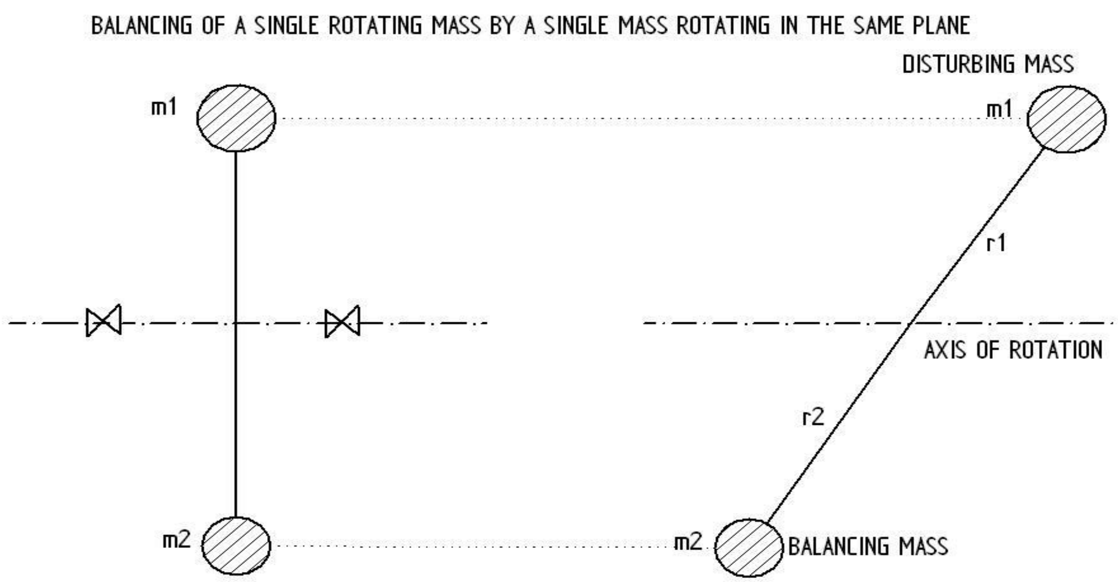
A system of rotating masses is said to be in static balance if the combined mass centre of the system lies on the axis of rotation

DYNAMIC BALANCING

When several masses rotate in different planes, the centrifugal forces, in addition to being out of balance, also form couples. A system of rotating masses is in dynamic balance when there does not exist any resultant centrifugal force as well as resultant couple.

2.3 Balancing of single rotating mass by balancing masses in same plane and in different planes.

CASE 1.
BALANCING OF A SINGLE ROTATING MASS BY A SINGLE MASS ROTATING IN THE SAME PLANE



Consider a disturbing mass m_1 which is attached to a shaft rotating at ω rad/s. Let

r_1 = radius of rotation of the mass m_1

=distance between the axis of rotation of the shaft and the centre of gravity of the mass m_1

The centrifugal force exerted by mass m_1 on the shaft is given by,

$$\mathbf{F_{c1} = m_1 \omega^2 r_1} \text{ ------------------------------ } \mathbf{(1)}$$

This force acts radially outwards and produces bending moment on the shaft. In order to counteract the effect of this force F_{c1} , a balancing mass m_2 may be attached in the same plane of rotation of the disturbing mass m_1 such that the centrifugal forces due to the two masses are equal and opposite.

Let,

r_2 = radius of rotation of the mass m_2

= distance between the axis of rotation of the shaft and the centre of gravity of the mass m_2

Therefore the centrifugal force due to mass m_2 will be,

$F_{c2} = m_2 \omega^2 r_2$ −−−−−−−−−−−−−−−−−−−−(2)

Equating equations (1) and (2), we get

$^{F}c1 \; ^{=F}c2$

$m \omega^2 r = m \omega^2 r$ or $m r = m r$ −−−−−−−−−−−−−−−−−−(3)

The product $\mathbf{m_2 \; r_2}$ can be split up in any convenient way. As for as possible the radius of rotation of mass m_2 that is r_2 is generally made large in order to reduce the balancing mass m_2.

CASE 2:

BALANCING OF A SINGLE ROTATING MASS BY TWO MASSES ROTATING IN DIFFERENT PLANES.

There are two possibilities while attaching two balancing masses:

1. The plane of the disturbing mass may be in between the planes of the two balancing masses.

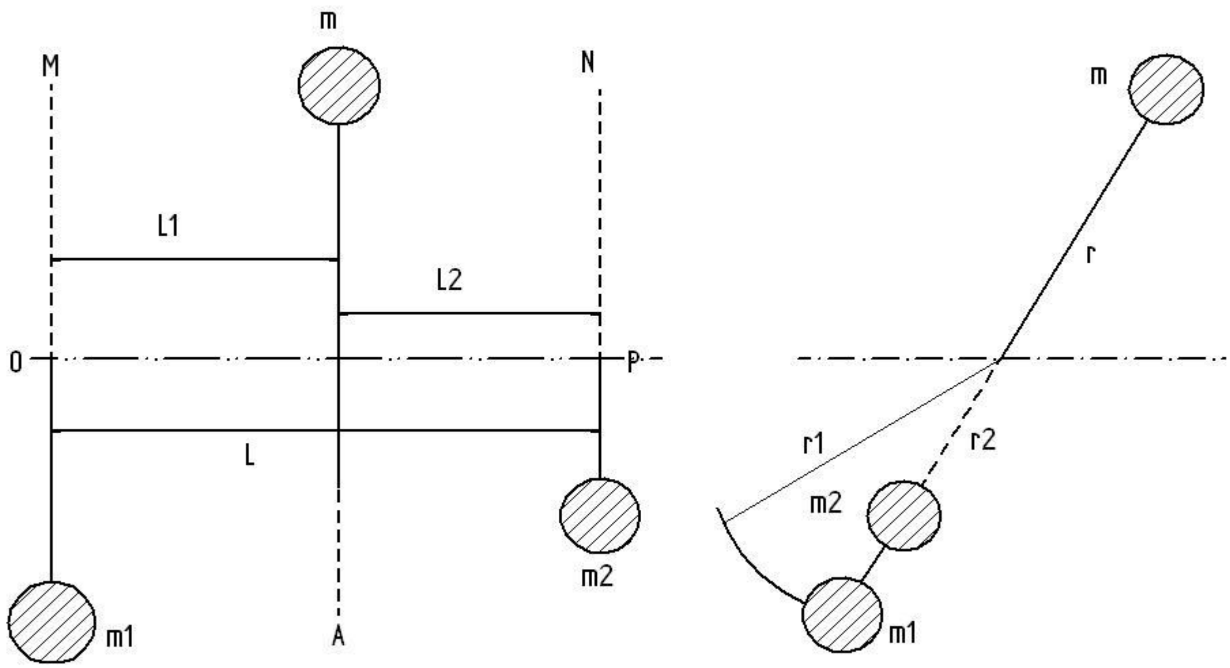
2. The plane of the disturbing mass may be on the left or right side of two planes containing the balancing masses.

In order to balance a single rotating mass by two masses rotating in different planes which are parallel to the plane of rotation of the disturbing mass i) the net dynamic force acting on the shaft must be equal to zero, i.e. the centre of the masses of the system must lie on the axis of rotation and this is the condition for static balancing ii) the net couple due to the dynamic forces acting on the shaft must be equal to zero, i.e. the algebraic sum of the moments about any point in the plane must be zero. The conditions i) and ii) together give dynamic balancing.

CASE 2(I):

THE PLANE OF THE DISTURBING MASS LIES IN BETWEEN THE PLANES OF THE TWO BALANCING MASSES.

The plane of the disturbing mass lies inbetween the planes of the two balancing masses



Consider the disturbing mass m lying in a plane A which is to be balanced by two rotating masses m_1 and m_2 lying in two different planes M and N which are parallel to the plane A as shown.

Let r, r_1 and r_2 be the radii of rotation of the masses in planes A, M and N respectively.

Let L_1, L_2 and L be the distance between A and M, A and N, and M and N respectively.

Now,

The centrifugal force exerted by the mass m in plane A will be,

$$F_c = m\,\omega^2\, r \text{ ---------------------- } (1)$$

Similarly,

The centrifugal force exerted by the mass m_1 in plane M will be,

$$F_{c1} = m_1\,\omega^2\, r_1 \text{ ---------------------- } (2)$$

And the centrifugal force exerted by the mass m_2 in plane N will be,

$$F_{c2} = m_2 \omega^2 r_2 \text{ ------------------- } (3)$$

For the condition of static balancing,

$$F_c = F_{c1} + F_{c2}$$

$$\text{or } m\omega^2 r = m\,\omega^2 r + m\,\omega^2 r$$

$$\text{i.e. } mr = m_1 r_1 + m_2 r_2 \text{ ----------------- } (4)$$

Now, to determine the magnitude of balancing force in the plane ‘M’ or the dynamic force at the bearing ‘O’ of a shaft, take moments about ‘ P ’ which is the point of intersection of the plane N and the axis of rotation.

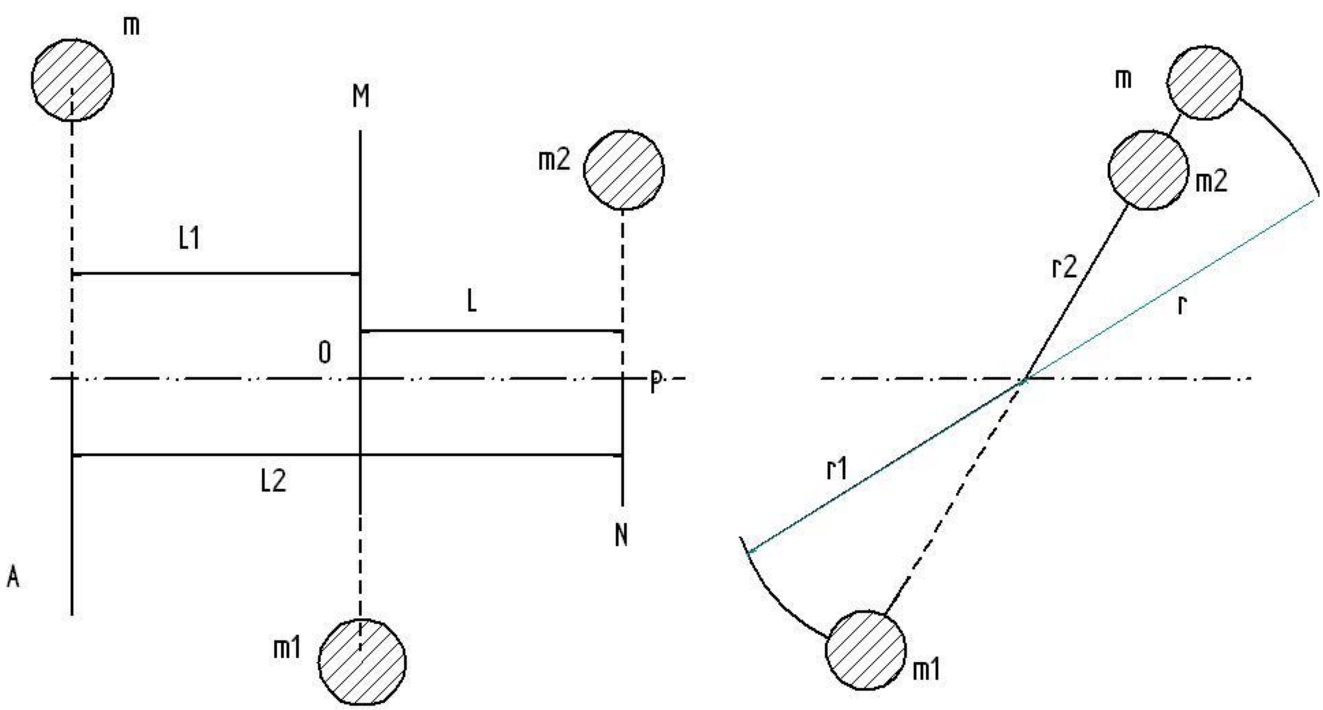
Similarly, in order to find the balancing force in plane ‘N’ or the dynamic force at the bearing ‘P’ of a shaft, take moments about ‘ O ’ which is the point of intersection of the plane M and the axis of rotation

For dynamic balancing equations (5) or (6) must be satisfied along with equation (4).

CASE 2(II):

WHEN THE PLANE OF THE DISTURBING MASS LIES ON ONE END OF THE TWO PLANES CONTAINING THE BALANCING MASSES.

When the plane of the disturbing mass lies on one end of the planes of the balancing masses



For static balancing,

$$F_{c1} = F_c + F_{c2}$$

$$\text{or } m_1\,\omega^2\,r_1 = m\omega^2\,r + m_2\,\omega^2\,r_2$$

$$\text{i.e. } m_1\,r_1 = mr + m_2\,r_2 \text{ ------------------ (1)}$$

For dynamic balance the net dynamic force acting on the shaft and the net couple due to dynamic forces acting on the shaft is equal to zero.

To find the balancing force in the plane ‘M’ or the dynamic force at the bearing ‘O’ of a shaft, take moments about ‘P’. i.e.

$F_{c1} \times L = F_c \times L_2$

or $m_1 \omega^2 r_1 \times L = m\omega^2 r \times L_2$

Therefore,

$$m_1 r_1 L = mrL_2 \qquad \text{or } m_1 r_1 = mr \frac{L_2}{L} \quad \text{---------(2)}$$

Similarly, to find the balancing force in the plane 'N' , take moments about 'O', i.e.,

$F_{c2} \times L = F_c \times L_1$

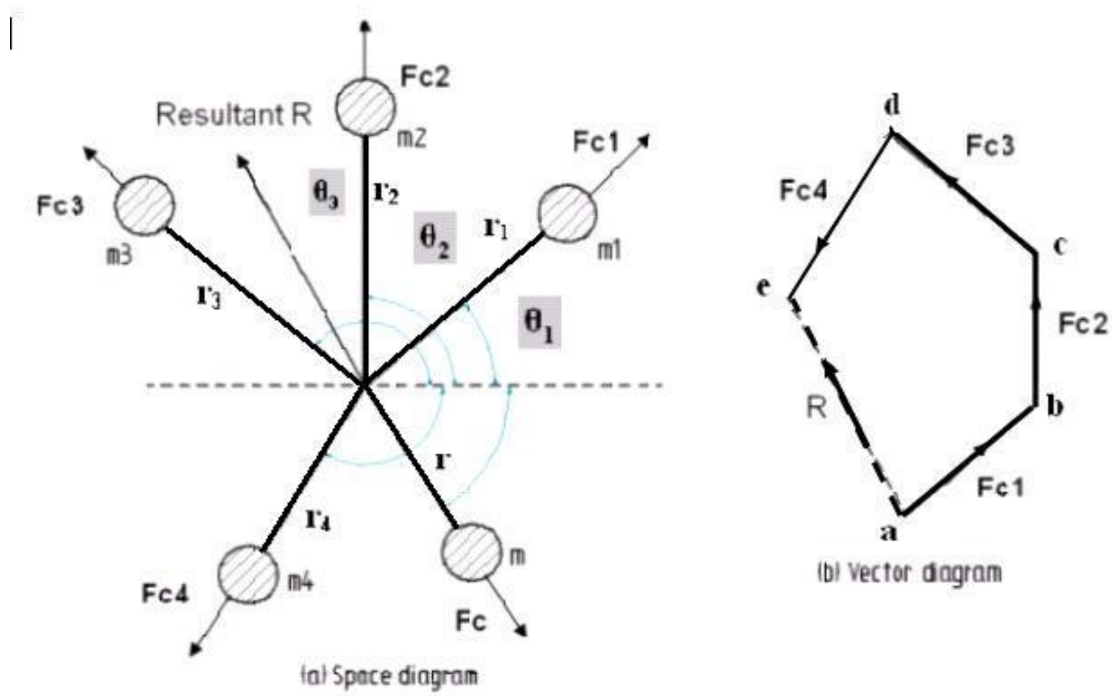
or $m_2 \omega^2 r_2 \times L = m\omega^2 r \times L_1$

Therefore,

$$m_2 r_2 L = mrL_1 \qquad \text{or } m_2 r_2 = mr \frac{L_1}{L} \quad \text{---------(3)}$$

2.4 Balancing of several rotating masses by balancing masses in same plane and in different planes.

CASE 3:
BALANCING OF SEVERAL MASSES ROTATING IN THE SAME PLANE



BALANCING OF SEVERAL MASSES ROTATING IN THE SAME PLANE

Consider a rigid rotor revolving with a constant angular velocity ω rad/s. A number of masses say, four are depicted by point masses at different radii in the same transverse plane.

If m_1, m_2, m_3 and m_4 are the masses revolving at radii r_1, r_2, r_3 and r_4 respectively in the same plane.

The centrifugal forces exerted by each of the masses are F_{c1}, F_{c2}, F_{c3} and F_{c4} respectively.

Let F be the vector sum of these forces. i.e.

1. $= F_{c1} + F_{c2} + F_{c3} + F_{c4}$

$$= m_1 \omega^2 r_1 + m_2 \omega^2 r_2 + m_3 \omega^2 r_3 + m_4 \omega^2 r_4 \text{ ---------- (1)}$$

The rotor is said to be statically balanced if the vector sum F is zero. If the vector sum F is not zero, i.e. the rotor is unbalanced, then introduce a counterweight (balance weight) of mass 'm' at radius 'r' to balance the rotor so that,

$$m_1 \omega^2 r_1 + m_2 \omega^2 r_2 + m_3 \omega^2 r_3 + m_4 \omega^2 r_4 + m \omega^2 r = 0 \text{ ---------- (2)}$$

or

$$m_1 r_1 + m_2 r_2 + m_3 r_3 + m_4 r_4 + m r = 0 \text{ ----------------- (3)}$$

The magnitude of either 'm' or 'r' may be selected and the other can be calculated.
In general, if $\sum \mathbf{m_i r_i}$ is the vector sum of $\mathbf{m_1 r_1}$, $\mathbf{m_2 r_2}$, $\mathbf{m_3 r_3}$, $\mathbf{m_4 r_4}$ etc, then,

$$\sum m_i r_i + mr = 0 \text{ --------- (4)}$$

The above equation can be solved either analytically or graphically.

1. Analytical Method:

2. Resolve the centrifugal forces horizontally and vertically and find their sums, *i.e.* ΣH and ΣV . We know that

   Sum of horizontal components of the centrifugal forces,

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + \ldots\ldots$$

and sum of vertical components of the centrifugal forces,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + \ldots\ldots$$

3. Magnitude of the resultant centrifugal force,

$$F_C = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

4. If θ is the angle, which the resultant force makes with the horizontal, then

$$\tan\theta = \Sigma V / \Sigma H$$

5. The balancing force is then equal to the resultant force, but in ***opposite direction.***
6. Now find out the magnitude of the balancing mass, such that

$$F_C = m \cdot r$$

where m = Balancing mass, and

r = Its radius of rotation.

2. Graphical Method:

Step 1:

Draw the space diagram with the positions of the several masses, as shown.

Step 2:

Find out the centrifugal forces or product of the mass and radius of rotation exerted by each mass.

Step 3:

Now draw the vector diagram with the obtained centrifugal forces or product of the masses and radii of rotation. To draw vector diagram take a suitable scale.

Let ab, bc, cd, de represents the forces F_{c1}, F_{c2}, F_{c3} and F_{c4} on the vector diagram.

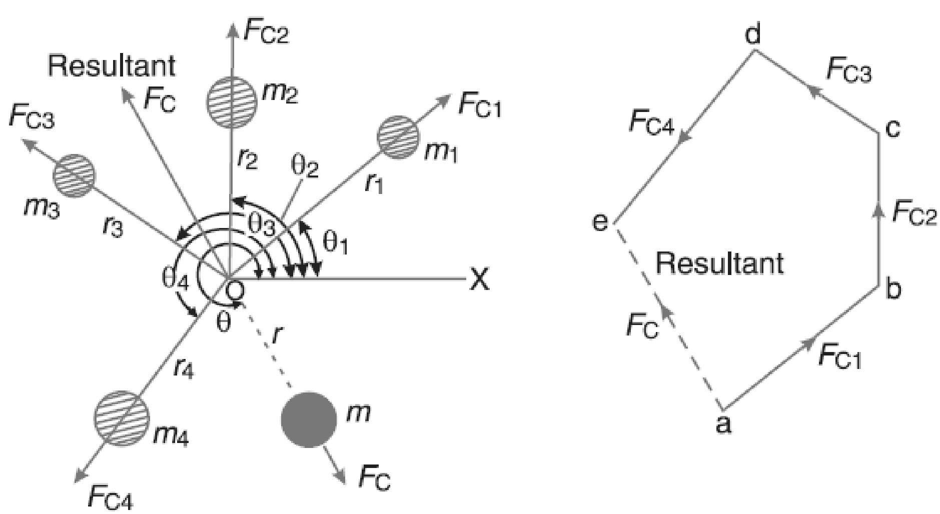
Draw 'ab' parallel to force F_{c1} of the space diagram, at 'b' draw a line parallel to force F_{c2}. Similarly draw lines cd, de parallel to F_{c3} and F_{c4} respectively.

Step 4:

As per polygon law of forces, the closing side 'ae' represents the resultant force in magnitude and direction as shown in vector diagram.

Step 5:

The balancing force is then , equal and opposite to the resultant force.



(a) Space diagram. (b) Vector diagram.

Step 6:

Determine the magnitude of the balancing mass (m) at a given radius of rotation (r), such that,

$$F_c = m\omega^2 r$$

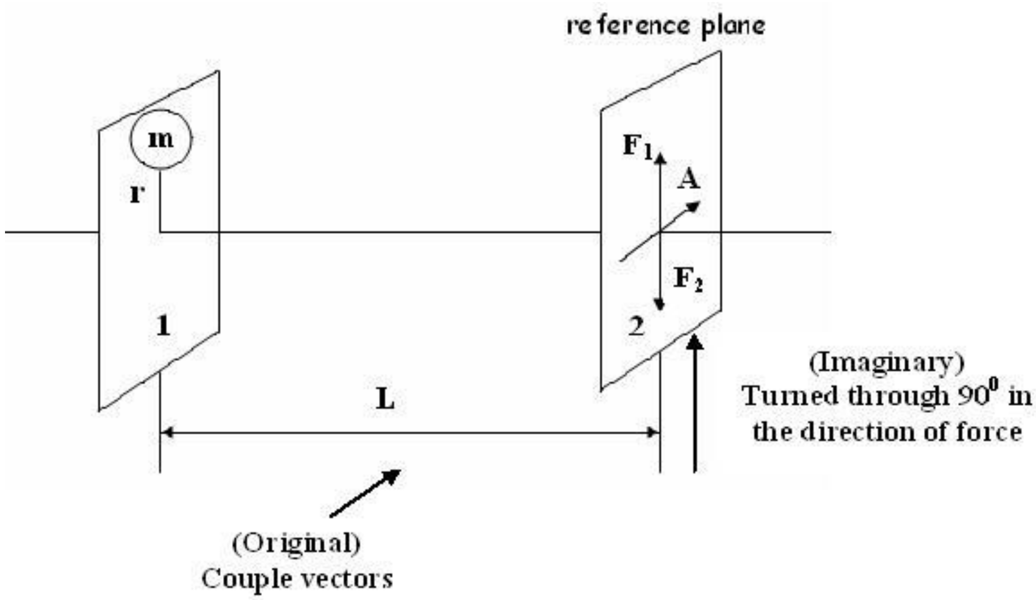
or

$$Mr = \text{resultant of } m_1 r_1, m_2 r_2, m_3 r_3 \text{ and } m_4 r_4$$

CASE 4:

BALANCING OF SEVERAL MASSES ROTATING IN DIFFERENT PLANES

When several masses revolve in different planes, they may be transferred to a reference plane and this reference plane is a plane passing through a point on the axis of rotation and perpendicular to it.



When a revolving mass in one plane is transferred to a reference plane, its effect is to cause a force of same magnitude to the centrifugal force of the revolving mass to act in the reference plane along with a couple of magnitude equal to the product of the force and the distance between the two planes.

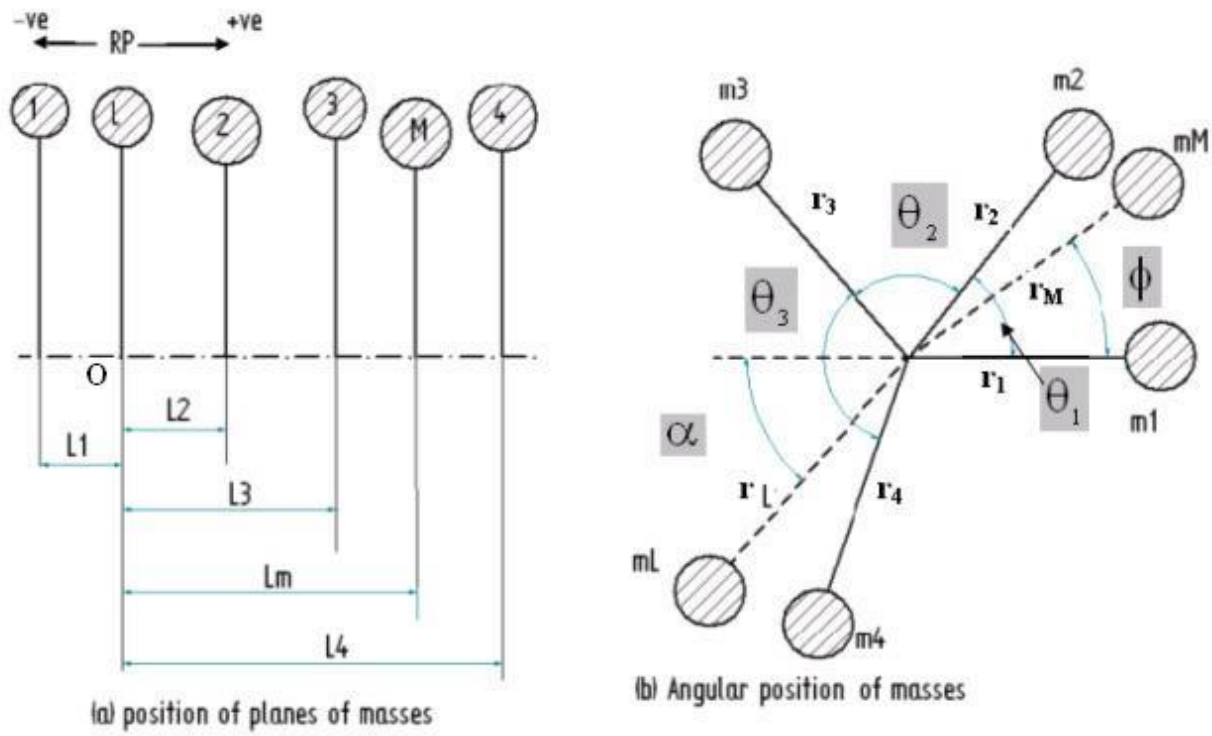
In order to have a complete balance of the several revolving masses in different planes,

1. the forces in the reference plane must balance, i.e., the resultant force must be zero and
2. the couples about the reference plane must balance i.e., the resultant couple must be zero.

A mass placed in the reference plane may satisfy the first condition but the couple balance is satisfied only by two forces of equal magnitude in different planes. Thus, in general, two planes are needed to balance a system of rotating masses.

Example:

Consider four masses m_1, m_2, m_3 and m_4 attached to the rotor at radii r_1, r_2, r_3 and r_4 respectively. The masses m_1, m_2, m_3 and m_4 rotate in planes 1, 2, 3 and 4 respectively.



a) Position of planes of masses

Choose a reference plane at 'O' so that the distance of the planes 1, 2, 3 and 4 from 'O' are L_1, L_2 , L_3 and L_4 respectively. The reference plane chosen is plane 'L'. Choose another plane 'M' between plane 3 and 4 as shown.

Plane 'M' is at a distance of L_m from the reference plane 'L'. The distances of all the other planes to the left of 'L' may be taken as negative(-ve) and to the right may be taken as positive (+ve).

The magnitude of the balancing masses m_L and m_M in planes L and M may be obtained by following the steps given below.

Step 1:

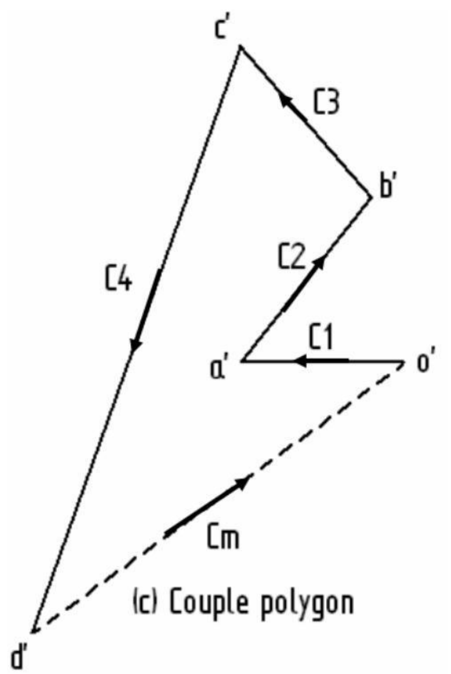
Tabulate the given data as shown after drawing the sketches of position of planes of masses and angular position of masses. The planes are tabulated in the same order in which they occur from left to right.

Plane 1	Mass (m) 2	Radius (r) 3	Centrifugal force/ω^2 (m r) 4	Distance from Ref. plane 'L' (L) 5	Couple/ ω^2 (m r L) 6
1	m1	r1	m1 r1	$-L_1$	$-m_1 r_1 L_1$
L	mL	rL	mL rL	0	0
2	m_2	r_2	$m_2 r_2$	L_2	m2 r2 L2
3	m_3	r_3	$m_3 r_3$	L_3	m3 r3 L3
M	mM	rM	mM rM	LM	$m_M r_M L_M$
4	m_4	r_4	$m_4 r_4$	L_4	$m_4 r_4 L_4$

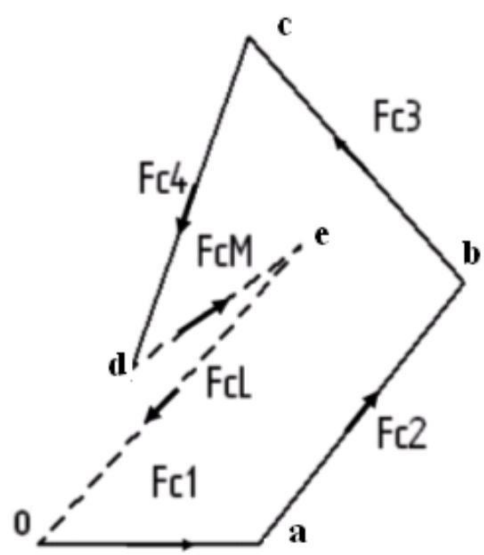
Step 2:

Construct the couple polygon first. (The couple polygon can be drawn by taking a convenient scale)

Add the known vectors and considering each vector parallel to the radial line of the mass draw the couple diagram. Then the closing vector will be '$m_M r_M L_M$'.



(c) Couple polygon



(d) Force polygon

The vector d 'o' on the couple polygon represents the balanced couple. Since the balanced couple C_M is proportional to $m_M r_M L_M$, therefore,

$$C_M = m_M r_M L_M = \text{vector } d'o'$$

$$\text{or} \quad m_M = \frac{\text{vector } d'o'}{r_M L_M}$$

From this the value of m_M in the plane M can be determined and the angle of inclination φ of this mass may be measured from figure (b).

Step 3:

Now draw the force polygon (The force polygon can be drawn by taking a convenient scale) by adding the known vectors along with 'm_M r_M'. The closing vector will be 'm_L r_L'. This represents the balanced force. Since the balanced force is proportional to 'm_L r_L' ,

$$m_L \; r_L = \text{vector eo}$$

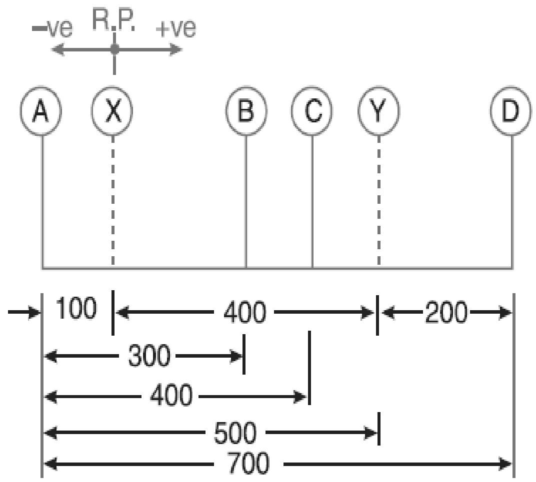
$$\text{or} \quad m_L = \frac{\text{vector eo}}{r_L}$$

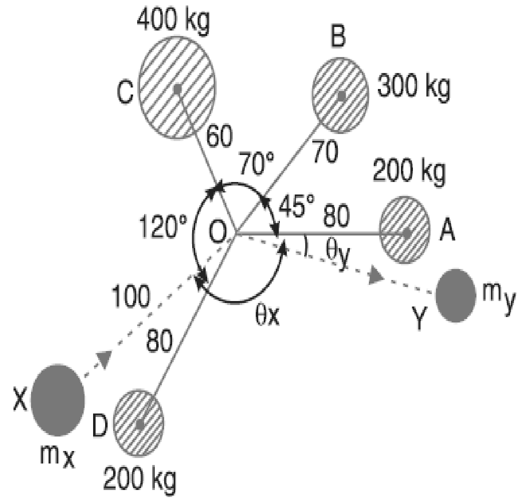
From this the balancing mass m_L can be obtained in plane 'L' and the angle of inclination of this mass with the horizontal may be measured from figure (b).

Problems and solutions

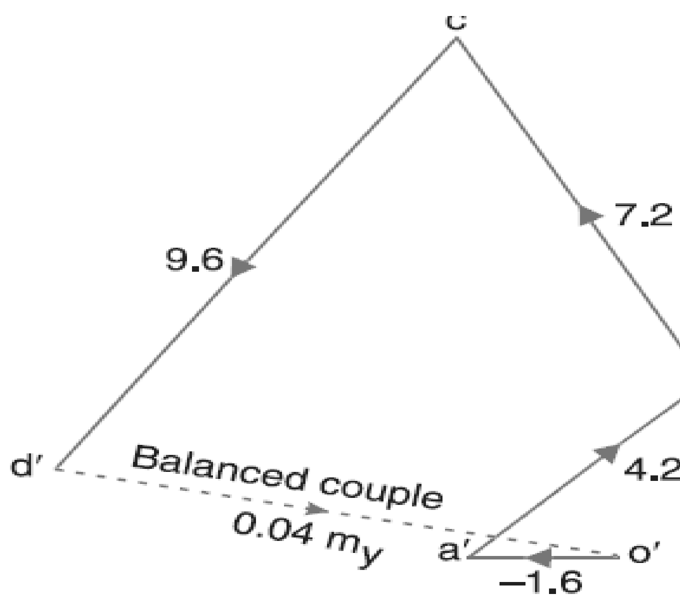
1. A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.

Given : m_A = 200 kg ; m_B = 300 kg ; m_C = 400 kg ;m_D = 200 kg ,r_A = 80 mmv= 0.08m ; r_B = 70 mm = 0.07 m ; r_C = 60 mm = 0.06 m ; r_D = 80 mm = 0.08 m ; r_X = r_Y = 100 mm = 0.1 m





Plane *(1)*	*Mass (m)* *kg* *(2)*	*Radius (r)* *m* *(3)*	*Cent.force* $\div \omega^2$ *(m.r) kg-m* *(4)*	*Distance from* *Plane x(l) m* *(5)*	*Couple* $\div \omega^2$ *(m.r.l) kg-m*^{2} *(6)*
A	200	0.08	16	– 0.1	– 1.6
X(R.P.)	m_X	0.1	0.1 m_X	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_Y	0.1	0.1 m_Y	0.4	0.04 m_Y
D	200	0.08	16	0.6	9.6

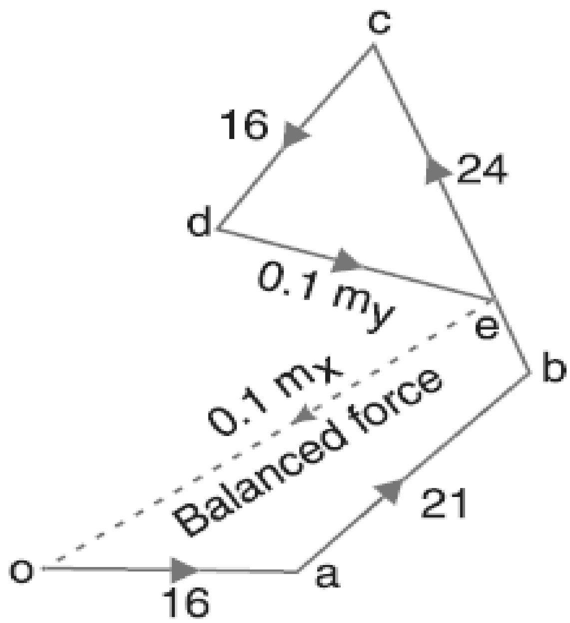


(*c*) Couple polygon.

By measurement, the angular position of m_Y is $\theta_Y = 12°$ in the clockwise direction from mass m_A (*i.e.* 200 kg).

$$0.04\, m_Y = \text{vector } d'o' = 7.3 \text{ kg-m}^2$$

$$m_Y = 182.5 \text{ kg}$$



(*d*) Force polygon.

$$0.1\,m_X = \text{vector } eo = 35.5 \text{ kg-m}$$

$m_X = 355$ kg

By measurement, the angular position of m_X is $\theta_X = 145°$ in the clockwise direction from mass m_A (*i.e.* 200 kg).

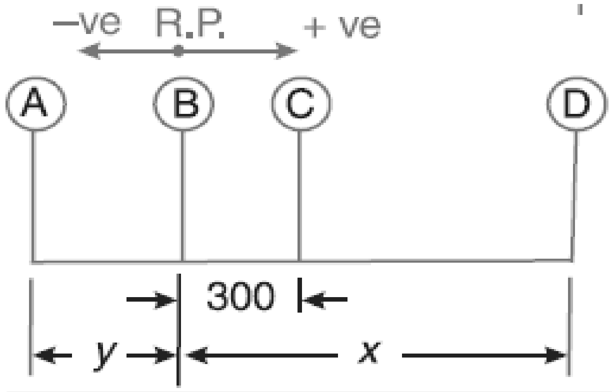
2. Four masses A, B, C and D as shown below are to be completely balanced. The planes containing masses B and C are 300 mm apart. The angle between planes containing B and C is 90°. B and C make angles of 210° and 120° respectively with D in the same sense. Find :

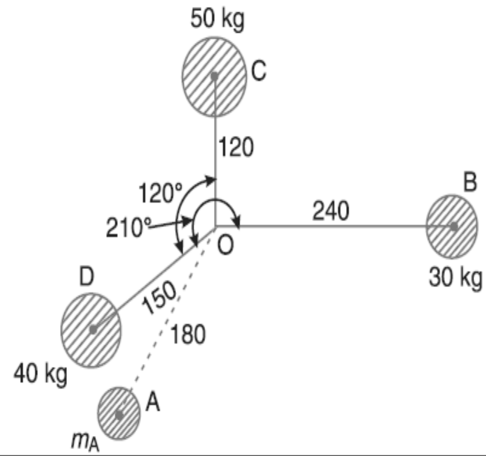
1. The magnitude and the angular position of mass A ; and

2. The position of planes A and D.

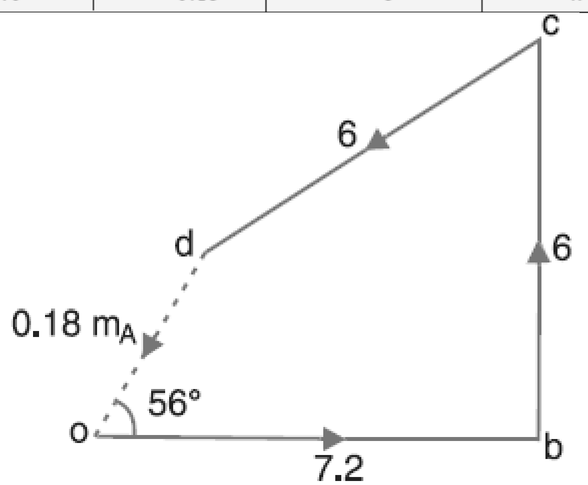
Given r_A = 180 mm = 0.18 m ; m_B = 30 kg ; r_B = 240 mm = 0.24 m ; m_C = 50 kg ; r_C = 120 mm = 0.12 m ; m_D = 40 kg ; r_D = 150 mm = 0.15 m ; ∠BOC = 90° ; ∠BOD = 210° ; ∠COD = 120°

	A	*B*	*C*	*D*
Mass (kg)	—	*30*	*50*	*40*
Radius (mm)	*180*	*240*	*120*	*150*





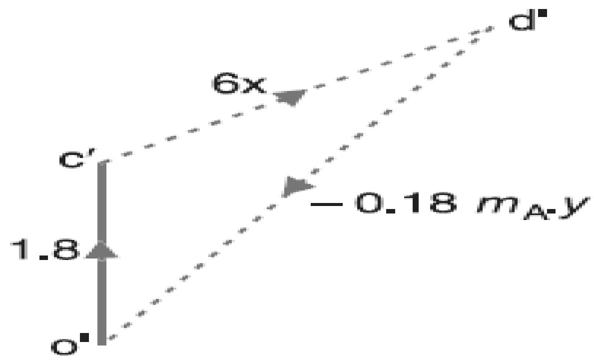
Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent.force ÷ ω^2 (m.r) kg-m (4)	Distance from plane B (l) m (5)	Couple ÷ ω^2 (m.r.l) kg-m^2 (6)
A	m_A	0.18	0.08 m_A	– y	– 0.18 $m_A y$
B (R.P)	30	0.24	7.2	0	0
C	50	0.12	6	0.3	1.8
D	40	0.15	6	x	6x



(c) Force polygon.

$0.18\ m_A$ = Vector do = 3.6 kg-m $m_A = 20$ kg

the angular position of mass A from mass B in the anticlockwise direction is $\angle AOB = 236°$



(d) Couple polygon.

$6\,x$ = vector $c'\,d'$ = 2.3 kg-m^2

$x = 0.383$ m

$-0.18\,m_A.y$ = vector $o'\,d'$ = 3.6 kg-m^2

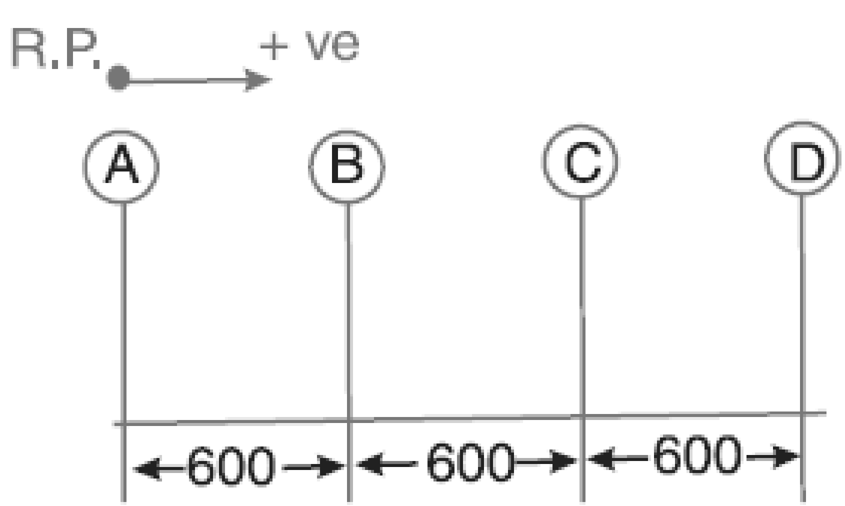
$-0.18 \times 20\,y = 3.6$

$y = -1$ m

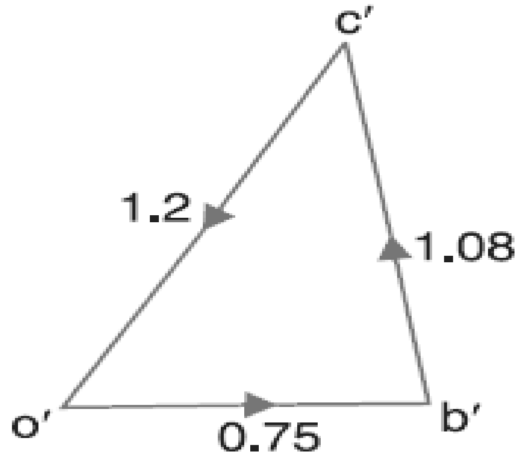
The negative sign indicates that the plane A is not towards left of B as assumed but it is 1000 mm towards right of plane B.

3. A, B, C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the mass of B, C and D are 10 kg, 5 kg, and 4 kg respectively. Find the required mass A and the relative angular settings of the four masses so that the shaft shall be in complete balance.

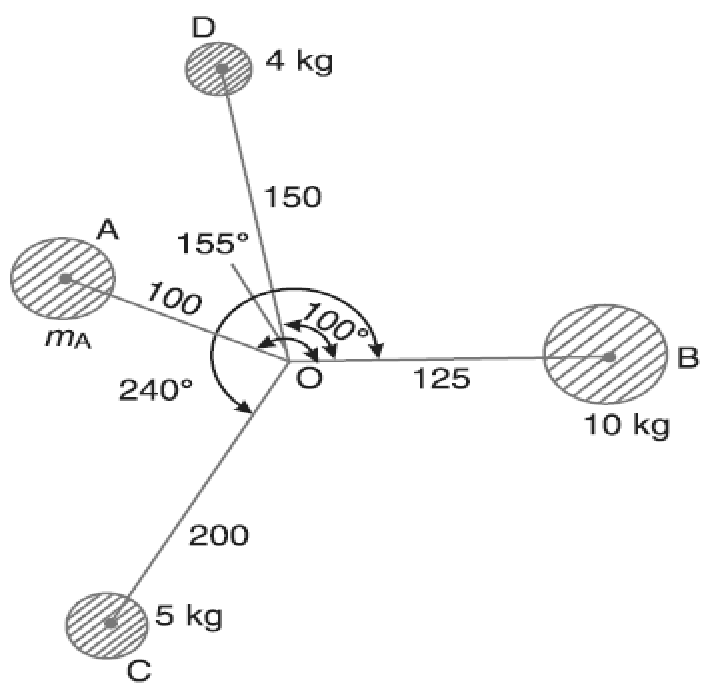
Given : r_A = 100 mm = 0.1 m ; r_B = 125 mm = 0.125 m ; r_C = 200 mm = 0.2 m ;
r_D = 150 mm = 0.15 m ; m_B = 10 kg ; m_C = 5 kg ; m_D = 4 kg



Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. Force ÷ ω^2 (m.r)kg-m (4)	Distance from plane A (l)m (5)	Couple ÷ ω^2 (m.r.l) kg-m^2 (6)
A(R.P.)	m_A	0.1	0.1 m_A	0	0
B	10	0.125	1.25	0.6	0.75
C	5	0.2	1	1.2	1.2
D	4	0.15	0.6	1.8	1.08

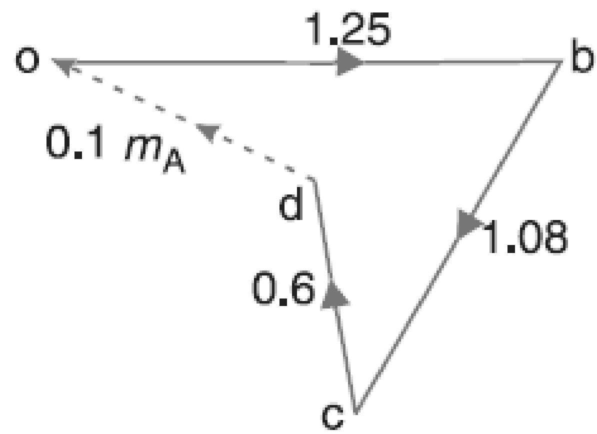


(c) Couple polygon.



$$\angle BOC = 240°$$

$$\angle BOD = 100°$$



(*d*) Force polygon.

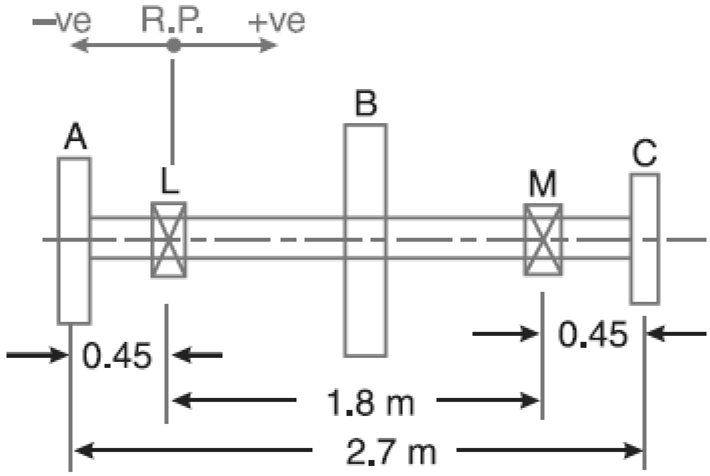
$$0.1\ m_A = 0.7\ \text{kg-m}^2$$

$$m_A = 7\ \text{kg}$$

$$\angle BOA = 155°$$

4. A shaft is supported in bearings 1.8 m apart and projects 0.45 m beyond bearings at each end. The shaft carries three pulleys one at each end and one at the middle of its length. The mass of end pulleys is 48 kg and 20 kg and their centre of gravity are 15 mm and 12.5 mm respectively from the shaft axis. The centre pulley has a mass of 56 kg and its centre of gravity is 15 mm from the shaft axis. If the pulleys are arranged so as to give static balance, determine : 1. relative angular positions of the pulleys, and 2. dynamic forces produced on the bearings when the shaft rotates at 300 r.p.m.

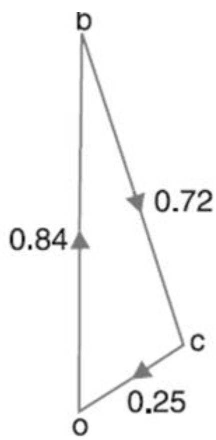
Given : m_A = 48 kg ; m_C = 20 kg ; r_A = 15 mm = 0.015 m ; r_C = 12.5 mm = 0.0125 m ; m_B = 56 kg ; r_B = 15 mm = 0.015 m ; N = 300 r.p.m. or $\omega = 2\pi \times 300/60 = 31.42$ rad/s



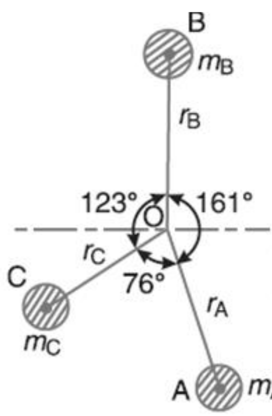
(*a*) Position of shaft and pulleys.

1. *Relative angular position of the pulleys*

Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent. force* $\div \omega^2$ *(m.r) kg-m* (4)	*Distance from plane L(l)m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m*^{2} (6)
A	48	0.015	0.72	– 0.45	– 0.324
L(R.P)	m_L	r_L	$m_L.r_L$	0	0
B	56	0.015	0.84	0.9	0.756
M	m_M	r_M	$m_M.r_M$	1.8	1.8 $m_M.r_M$
C	20	0.0125	0.25	2.25	0.5625

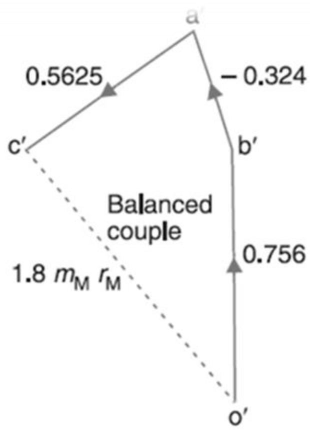


(*c*) Force polygon



Angle between pulleys *B* and *A* = 161°
Angle between pulleys *A* and *C* = 76°
Angle between pulleys *C* and *B* = 123°

(*b*) Angular position of pulleys.



(*d*) Couple polygon.

$$1.8\ m_M.r_M = \text{vector } c'o' = 0.97 \text{ kg-m}^2$$

$$m_M.r_M = 0.54 \text{ kg-m}^2$$

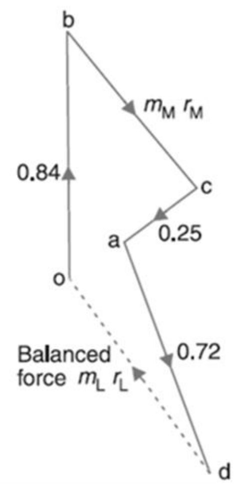
Dynamic force at the bearing *M*

$$= m_M.r_M.\omega^2 = 0.54\ (31.42)^2 = 533 \text{ N}$$

$$m_L.r_L = 0.54 \text{ kg-m}$$

Dynamic force at the bearing *L*

$$= m_L.r_L.\omega^2 = 0.54\ (31.42)^2 = 533 \text{ N}$$



(*e*) Force polygon

OUT COMES

1. Students will be able Check static and Dynamic balancing for Rotating systems.
2. Students able to solve problems on balancing of rotating masses

Exercise

1. What is meant by balancing of rotating masses?
2. Why rotating masses are to be dynamically balanced?
3. Define static balancing.
4. Define dynamic balancing.

FURTHER READING

1. Theory of Machines by S.S.Rattan, Third Edition, Tata McGraw Hill Education Private Limited.
2. Kinematics and Dynamics of Machinery by R. L. Norton, First Edition in SI units, Tata McGraw Hill Education Private Limited.
3. Primer on Dynamic Balancing “Causes, Corrections and Consequences” By Jim Lyons International Sales Manager IRD Balancing Div. EntekIRD International