Module 3: TRANSISTOR and FET FREQUENCY RESPONSE

The frequency response of an amplifier is the plot of the magnitude of voltage gain as a function of frequency. In transistor amplifier the low frequency response is governed by coupling and bypass capacitors. The high frequency response is affected by the transistor parasitic capacitances and stray wiring capacitances. The mid frequency response is unaffected by these capacitances.

General frequency considerations:

The response of a single stage or multistage amplifier depends on the frequency of the applied signal. The coupling and bypass capacitors affect the low frequency response since the reactance of these capacitors decreases with increase in frequency. The internal capacitances of the active devices and the stray wiring capacitances will limit the high frequency response of the system. An increase in the number of stages of a cascaded system will also limit the low and high frequency responses.

Frequency response of RC Coupled amplifier:

Fig. (1) shows the frequency response of RC coupled amplifier. The horizontal scale is a logarithmic scale to permit a plot extending from low to high frequency regions. The frequency range is divided into 3 regions.

(i) Low frequency region.
(ii) Mid frequency region.
(iii) High frequency region.

The drop in the gain at low frequencies is due to the coupling capacitors (C_C,C_S) and bypass capacitors (C_E). At high frequencies the drop in gain is due to the internal device capacitances and the stray wiring capacitances. In the mid frequency range the gain is almost independent of the frequency. This is due to the fact that at mid frequencies the coupling and bypass capacitors act as short circuits and the device and stray wiring capacitances act as open circuits due to their low capacitance. The mid band gain is denoted as Av_{mid}.

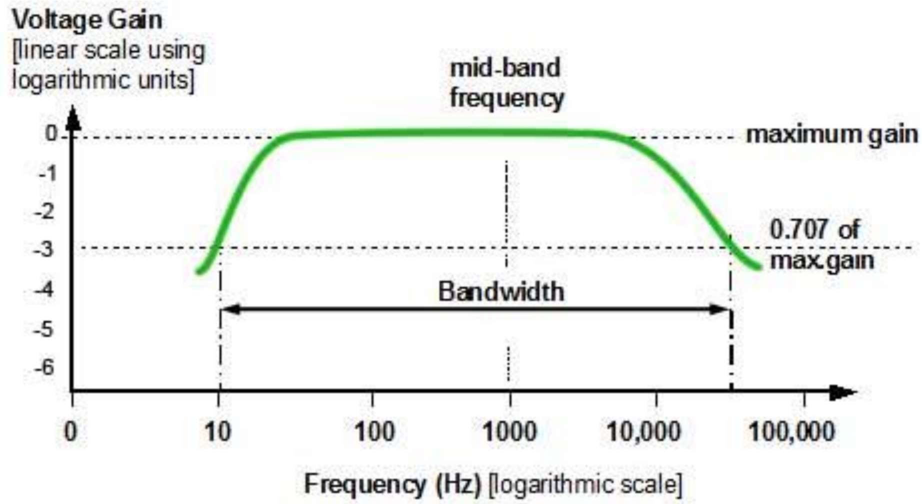

Fig. (1) Frequency response of RC coupled amplifier.

Frequency response of transformer coupled amplifier:

Fig.(2) shows frequency response of transformer coupled amplifier. The magnetizing inductive reactance of the transformer winding is $X_L=2\pi fl$. At low frequencies the gain drops due to small value of X_L. At f=0 (DC) there is no change in flux in the core. As a result the secondary induced voltage or output voltage is zero and hence the gain. At high frequencies the gain drops due to stray capacitance between the turns of primary and secondary windings.

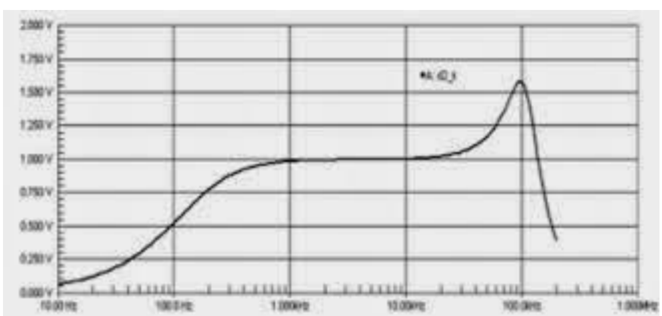

Fig. (2) Frequency response of transformer coupled amplifier.

Frequency response of direct coupled amplifier:

Fig. (3) shows frequency response of direct coupled amplifier. Since there are no coupling or bypass capacitors, there is no drop in gain at low frequencies. It has a flat response to the upper cut-off frequency. Gain drops at high frequencies due to device internal capacitances and the stray wiring capacitances.

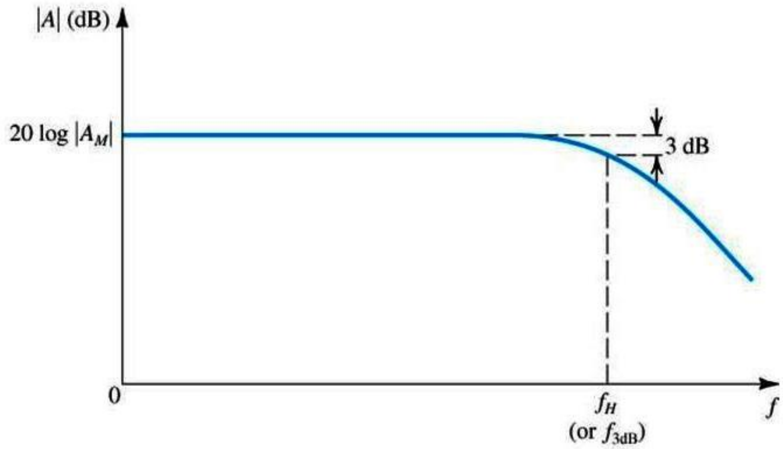

Fig. (3) Frequency response of direct coupled amplifier.

Half power frequencies and bandwidth:

The frequencies f_1 and f_2 at which the gain is 0.707 Av_{mid} are called cut-off frequencies or corner frequencies or break frequencies. f_1 is called the lower cut-off frequency and f_2 is called the upper cut-off frequency.

Bandwidth or pass band of the amplifier is

BW= f_2- f_1----- (1)

The output voltage in the mid band is $|V_O|=|Av_{mid}|\,|V_i|$

Output power in the mid band is

$$P_{O(mid)}=\frac{|V_O|^2}{R_O}$$

$$=\frac{|Av_{mid}|^2|V_i|^2}{R_O} \text{ ----- (2)}$$

The output voltage at cut-off frequencies is

$|V_O|= |0.707Av_{mid}|\,|V_i|$

The output power at cut-off frequencies is

$$P_{O\,(cut\text{-}off)}=\frac{|0.707Av_{mid}|^2|V_i|^2}{R_O}$$

$= \frac{0.5\ |Av_{mid}|^2 |V_i|^2}{R_O}$

$= 0.5\ P_{O\ (mid)}$ ----- (3)

Thus, the output power at cut-off frequencies is half the mid band power output. f_1 is called the lower half power frequency and f_2 is called the upper half power frequency.

Normalized gain V/s Frequency plot:

The normalized gain is obtained by dividing the gain Av at each frequency by the mid band gain Av_{mid}.

Therefore normalized gain = $\frac{A_V}{Av_{mid}}$ ----- (4)

Fig. (4) shows the normalized gain V/s frequency plot for an RC coupled amplifier.

The normalized mid band gain is $\frac{Av_{mid}}{Av_{mid}} = 1$

The normalized gain at cut-off frequencies is $\frac{0.707 Av_{mid}}{Av_{mid}} = 0.707$

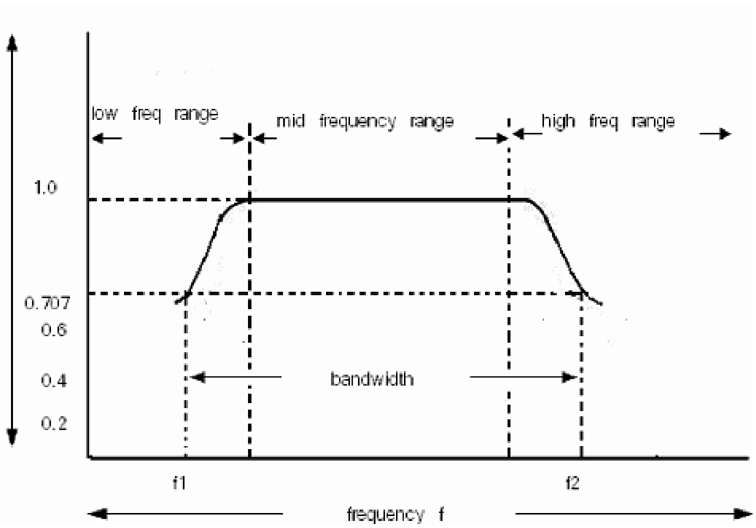

Fig. (4) Normalized gain V/s frequency plot

Normalized decibel gain is $\left.\frac{A_V}{Av_{mid}}\right|_{dB} = 20 \log_{10} \left[\frac{A_V}{Av_{mid}}\right]$ -----(5)

Normalized decibel voltage gain in mid band is

$$20\log_{10}\left[\frac{A_V}{Av_{mid}}\right]=0$$

Normalized decibel voltage gain at cut-off frequencies is

$$20\log_{10}\left[\frac{0.707A_V}{Av_{mid}}\right]=-3\text{dB}$$

Since normalized decibel voltage gain at cut-off frequencies is 3dB less than the normalized decibel mid band voltage gain. f_1 and f_2 are also called 3dB frequencies.

f_1 →lower 3dB frequency

f_2→upper 3dB frequency

Fig. (5) shows the plot of normalized dB voltage gain V/s frequency for an RC coupled amplifier.

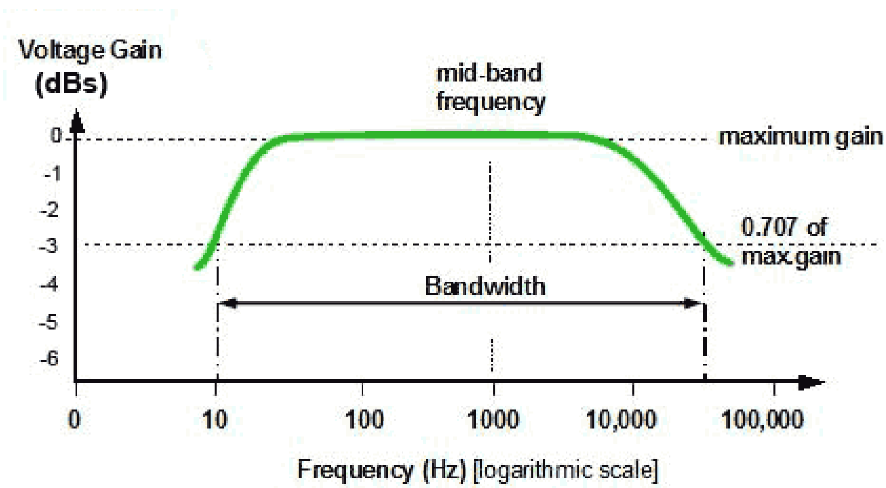

Fig. (5) Plot of normalized decibel voltage gain V/s frequency.

Phase angle plot:

A single stage RC coupled amplifier introduces a 180^0 phase shift between input and output signals in the mid band region. At low frequencies the output voltage V_O lags V_i by an additional angle θ_1. Therefore, the total phase phase shift between V_O and V_i is more than 180^0. At high frequencies, V_O leads V_i by an additional angle θ_2. As a result, the total phase shift drops below 180^0. Fig. (6) shows the phase plot for a single stage RC coupled amplifier.

Fig. (6) Phase plot of single stage RC coupled amplifier.

Low frequency analysis:

In the low frequency region of a single stage BJT amplifier, the amplifier gain increases with frequency. Hence it can be modelled as a high pass RC circuit as shown in Fig. (7).

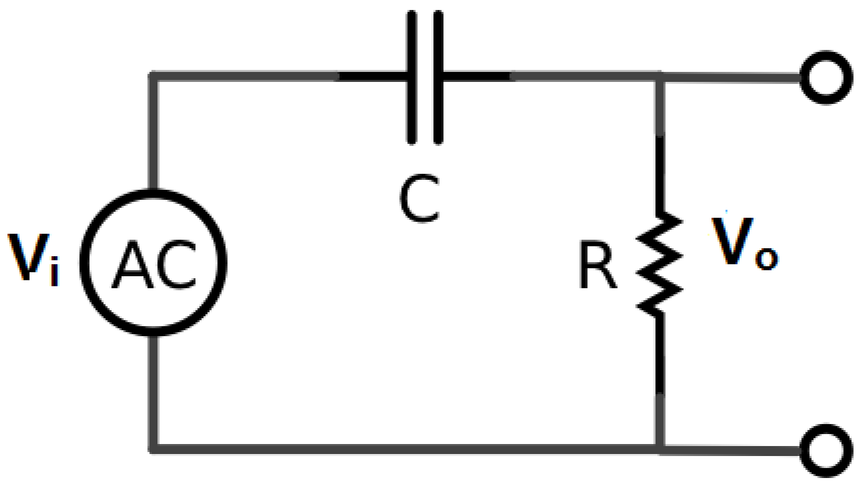

Fig. (7) Amplifier modelled as high pass RC circuit.

The capacitor C represents the combined effect of coupling and bypass capacitors and the resistance R represents the combined effect of resistive elements of the amplifier network.

The capacitive reactance is

$X_C = \frac{1}{2\pi f c}$ ----- (6)

At f=0, $X_C = \infty\Omega$. i.e. at low frequencies, the capacitor acts as a open circuit as shown in Fig. (8)

From Fig. (8), V_O=0.

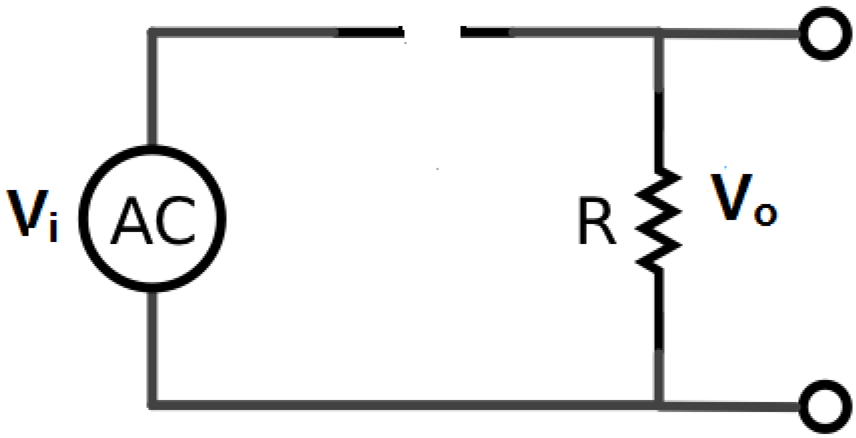

Fig. (8) at f=0

At high frequencies, $X_C \approx 0\Omega$ i.e. at high frequencies, capacitor acts as a short circuit as shown in Fig. (9).

From Fig. (9), $V_O \approx V_L$

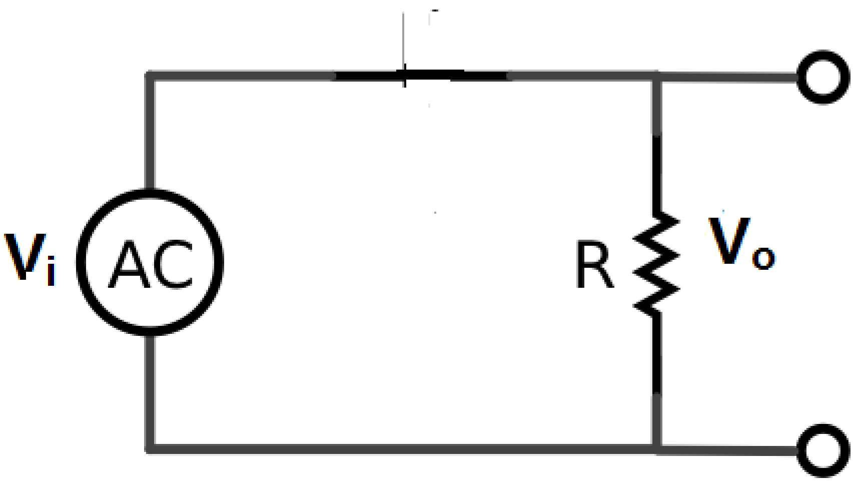

Fig. (9) at high frequencies

Hence as the input signal frequency increases from zero to mid band value, the output voltage rises from zero to V_i and hence the gain from zero to 1.

Mathematical analysis:

Apply Voltage division rule to circuit in Fig. (7),

$V_O = \frac{V_i R}{R - jX_C}$

Voltage gain is given by

$A_V = \frac{V_o}{V_i} = \frac{R}{R - jX_C} = \frac{R}{R[1 - j\frac{X_C}{R}]}$

$A_V = \frac{1}{1 - j[\frac{X_C}{R}]}$ ------ (7)

The magnitude of voltage gain is

$|A_V| = \frac{1}{\sqrt{1 + [\frac{X_C}{R}]^2}}$ ----- (8)

(i) At f=0, $X_C = \frac{1}{2\pi f c} = \infty\Omega$ therefore $|A_V|=0$

(ii) At high frequencies, $f \rightarrow \infty$, therefore $X_C \rightarrow 0$. Hence $|A_V| \rightarrow 1 = |A_V|_{mid}$

$|A_V|_{mid\ (dB)} = 20 \log_{10}(1) = 0dB$

(iii) When $X_C = R$ ----- (9)

$|A_V| = \frac{1}{\sqrt{2}} \Rightarrow \frac{V_o}{V_i} = \frac{1}{\sqrt{2}}$ or $V_O = 0.707V_i$

The corresponding decibel gain is

$20 \log_{10} \frac{1}{\sqrt{2}} = -3dB$

From Eq. (9), $\frac{1}{2\pi f c} = R$

$f = \frac{1}{2\pi R c}$

The frequency given by the above Eq. is the lower cut-off frequency or lower 3dB cut-off frequency denoted by f_1.

Therefore $f_1 = \frac{1}{2\pi R c}$ ----- (10)

$\frac{X_C}{R} = \frac{1}{2\pi fcR} = [\frac{1}{2\pi Rc}] \times \frac{1}{f} = \frac{f_1}{f}$

Using in Eq. (7) and (8)

$A_V = \frac{1}{1-j[\frac{f_1}{f}]}$ ------ (11)

$|A_V| = \frac{1}{\sqrt{1+[\frac{f_1}{f}]^2}}$ ----- (12)

From Eq. (11), phase angle of A_V is

$\theta_1 = \tan^{-1}\left[\frac{f_1}{f}\right]$ ----- (13)

Since θ_1 is +Ve, V_O leads V_i by an angle θ_1

In magnitude and phase form, Eq. (11) can be written as

$A_V = |A_V| \Delta \theta_1$

$= \frac{1}{\sqrt{1+[\frac{f_1}{f}]^2}} \Delta \tan^{-1}\left[\frac{f_1}{f}\right]$ ----- (14)

Fig. (10) shows the plot of $|A_V|$ V/s frequency.

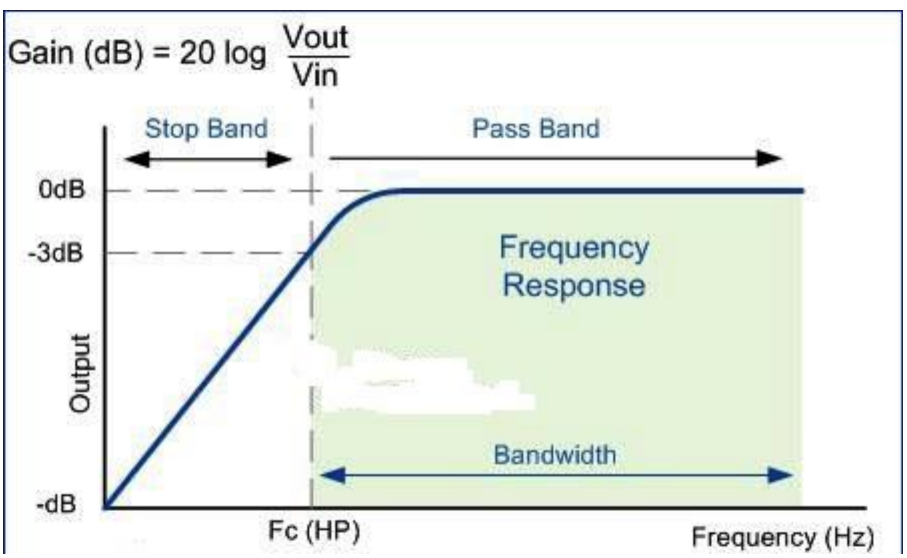

Fig. (10) Low frequency response of high pass RC circuit.

Bode plot of low frequency response:

From Eq. (12), we have

$$|A_V| = \frac{1}{\sqrt{1+[\frac{f_1}{f}]^2}}$$

Voltage gain in dB is

$$|A_V|_{dB} = 20 \log_{10}\left[\frac{1}{\sqrt{1+[\frac{f_1}{f}]^2}}\right]$$

$$|A_V|_{dB} = -20 \log_{10}\sqrt{1+[\frac{f_1}{f}]^2} \text{ ----- (15)}$$

To construct the plot of $|A_V|_{dB}$ V/s frequency using straight line segments, we consider the following frequency ranges.

(a) For frequencies f << f_1 or $\frac{f_1}{f}$ >> 1

Eq. (15) can be approximated as

$$|A_V|_{dB} = -20 \log_{10}\sqrt{[\frac{f_1}{f}]^2}$$

$$|A_V|_{dB} = -20 \log_{10}[\frac{f_1}{f}] \text{ ----- (16)}$$

$|A_V|_{dB}$ is calculated at different values of $\frac{f_1}{f}$ and tabulated in table (1).

f	$\frac{f_1}{f}$	$\lvert A_V\rvert_{dB} = -20 \log_{10}[\frac{f_1}{f}]$
$\frac{f_1}{10}$	10	-20dB
$\frac{f_1}{4}$	4	-12dB

$\frac{f_1}{2}$	2	-6dB
f_1	1	0dB

Table (1): $|A_V|_{dB}$ at different frequencies.

Following conclusions can be drawn from table (1).

(i) A change in frequency by a factor of 2 is equal to one octave. When the frequency changes from $\frac{f_1}{4}$ to $\frac{f_1}{2}$ or $\frac{f_1}{2}$ to f_1(one octave), the gain increases by 6dB.

(ii) A change in frequency by a factor of ten is equal to one decade. When the frequency changes from $\frac{f_1}{10}$ to f_1(one decade), the gain increases by 20dB.

(iii) If a plot of $|A_V|_{dB}$ against log scale in the frequency range $\frac{f_1}{10} < f < f_1$ is plotted, a straight line with slope 6dB/octave or 20dB/decade is obtained as shown in Fig. (11).

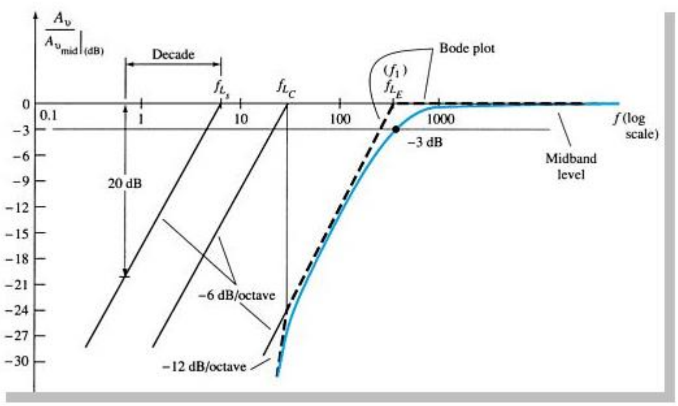

Fig. 11: Bode plot for low frequency region.

(b) For frequencies, $f >> f_1$ or $\frac{f_1}{f} << 1$

Eq. (15) can be approximated by

$|A_V|_{dB} \approx -20 \log_{10} 1 = 0dB$.

The plot of $|A_V|_{dB}$ against log scale for the frequency range f >>f_1, is a straight line on the frequency axis as shown in Fig. (11). The slope of this line is zero, since the gain is constant at 0dB. The plot in Fig. (11) is made of 2 straight line segments called asymptotes with a break point at f_1. Hence f_1 is called break frequency or corner frequency. This peicewise linear plot is also called bode magnitude plot or simply bode plot.

From bode plot, at f= f_1 , $|A_V|_{dB}$ =0.

From bode plot, at f= f_1 , $|A_V|_{dB}$ =-3.

Thus, at f= f_1 , the gain read from bode plot differs from the actual gain by 3dB.

Phase Plot:

At low frequencies, V_O leads V_i by an angle θ_1 given by

$$\theta_1 = \tan^{-1}\left[\frac{f_1}{f}\right] \text{ ----- } (17)$$

The value of θ_1 is calculated at different values of $\frac{f_1}{f}$ as shown in table (2).

Table (2): Phase angle between V_O and V_i

f	$\frac{f_1}{f}$	$\theta_1 = \tan^{-1}\left[\frac{f_1}{f}\right]$	Total phase shift θ=180 + θ_1
0	∞	90^0	270^0
$\frac{f_1}{100}$	100	89.4^0	269.4^0
f_1	1	45^0	225^0
$100f_1$	0.01	0.572^0	180.572^0
∞	0	0^0	180^0

The total phase shift θ_1 between V_O and V_i is the sum of the phase shift of RC network and inherent phase shift (180^0) introduced by the amplifier.

From table (2),

(i) The phase shift θ_1 due to RC network decreases from 90^0 to 0^0. The plot of θ_1 V/s frequency is shown in Fig. (12).

(ii) The total phase θ, decreases from 270^0 to 180^0.

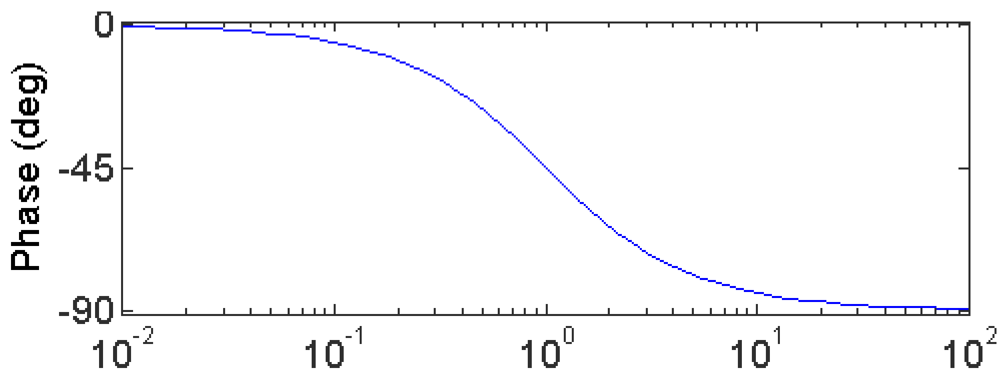

Fig. (12) Phase response of RC network.

Low frequency Response of BJT amplifier:

Fig. (13) shows the circuit of single stage BJT amplifier. The coupling capacitors C_S and C_C and bypass capacitor C_E determines the low frequency response.

Effect of C_S on low frequency response:

The input coupling capacitor C_S couples the source signal to BJT. First, we will neglect the effects of C_C and C_E i.e. they are treated as short circuits.

The AC equivalent circuit is obtained by reducing VCC to zero and C_C and C_E by their short circuit equivalent as shown in Fig. (14).

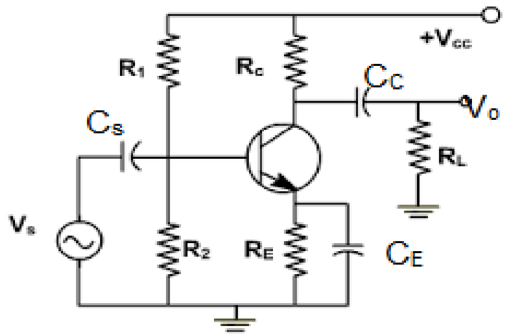

Fig. (13) Single stage BJT amplifier

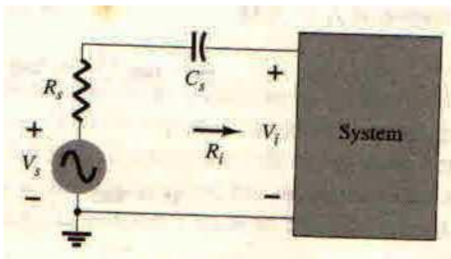

Fig. (14) AC equivalent circuit

The resistance of the transistor between base-emitter is h_{ie}. The input AC equivalent circuit is shown in Fig. (15).

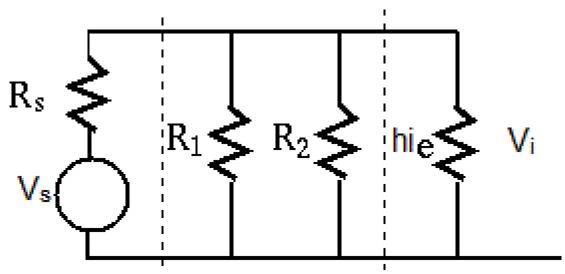

Fig. (15) Input AC equivalent

Let $R_i = R_1 \parallel R_2 \parallel h_{ie}$ ----- (18)

Where $h_{ie} = \beta r_e$ ----- (19)

Using voltage division rule in the circuit of Fig. (15) the voltage applied to the amplifier is

$V_i = \frac{V_S R_i}{(R_{S+}R_i) - fXc_S}$ ----- (20)

Where $Xc_S = \frac{1}{2\pi f C_S}$ ----- (21)

$$V_i = \frac{V_S\left[\frac{R_i}{R_S+R_i}\right]}{1-j\left[\frac{Xc_S}{R_i+R_S}\right]}$$

$$V_i = \frac{|V_S|\left[\frac{R_i}{R_S+R_i}\right]}{\sqrt{1+j\left[\frac{Xc_S}{R_i+R_S}\right]^2}} \text{ -----(22)}$$

In the mid frequency band, f is large. As a result, $Xc_S \rightarrow 0$.

Therefore from Eq. (22),

$$|V_i|_{mid} = \frac{|V_S| R_i}{(R_{S+}R_i)} \text{ ----- (23)}$$

Therefore Eq. (22) becomes,

$$|V_i| = \frac{|Vi|mid}{\sqrt{1+\left[\frac{Xc_S}{R_i+R_S}\right]^2}} \text{ ----- (24)}$$

The lower 3dB cut-off occurs when $|V_i| = \frac{|Vi|mid}{\sqrt{2}} = 0.707\ |V_i|_{mid}$

Therefore Eq. (24) becomes,

$$0.707\ |V_i|_{mid} = \frac{|Vi|mid}{\sqrt{1+\left[\frac{Xc_S}{R_i+R_S}\right]^2}}$$

This condition is satisfie, if $\frac{Xc_S}{R_i+R_S} = 1$ or $Xc_S = R_i + R_S$

$$\frac{1}{2\pi f C_S} = R_i + R_S$$

Therefore $f = \frac{1}{2\pi(R_S+R_i)C_S}$ ----- (25)

Eq. (25) gives the lower 3dB cut-off frequency due to C_S.

Therefore $f_{Ls} = \frac{1}{2\pi(R_S+R_i)C_S}$ ----- (26)

Effect of output coupling capacitor C_C on low frequency response :

The output coupling capacitor C_C couples the output of the BJT to the load. The equivalent circuit on the output side by neglecting the effect of C_S and C_E by treating them as short circuits is as shown in Fig. (16).

Fig. (16) (a) AC equivalent circuit of output side (b) Simplified AC equivalent circuit

Let $R_O = r_O \| R_C$ ----- (27)

V_C= output voltage of BJT

V_O=load voltage

Using voltage division rule in circuit of Fig. (16) (a), the load voltage is,

$V_O = \frac{V_C R_L}{(R_O + R_L) - jXc_C}$ -----(28)

Where $Xc_C = \frac{1}{2\pi f C_C}$ ----- (29)

$$V_O = \frac{V_C\left[\frac{R_L}{R_O+R_L}\right]}{1-j\left[\frac{Xc_C}{R_O+R_L}\right]}$$

$$|V_O| = \frac{|V_C|\left[\frac{R_L}{R_O+R_L}\right]}{\sqrt{1+j\left[\frac{Xc_C}{R_O+R_L}\right]^2}} \text{ ----- (30)}$$

In the mid frequency band, $Xc_C \rightarrow 0$

Therefore $|V_O|_{mid} = \frac{|V_C|R_L}{(R_O+R_L)}$ ----- (31)

Substitute Eq. (31) in (30)

$|V_O| = \frac{|Vo|mid}{\sqrt{1+\left[\frac{Xc_C}{R_O+R_L}\right]^2}}$ ----- (32)

The lower 3dB cut-off occurs when $|V_O| = \frac{|Vo|mid}{\sqrt{2}} = 0.707\ |V_O|_{mid}$

This is possible iff,

$\frac{Xc_C}{R_O+R_L} = 1$ or $Xc_C = R_O + R_L$

Therefore $f = \frac{1}{2\pi(R_O+R_L)C_C}$ ----- (33)

Eq. (33) gives the lower 3dB cut-off frequency due to C_C.

Therefore $f_{Lc} = \frac{1}{2\pi(R_O+R_L)C_C}$ ----- (34)

Effect of Emitter bypass capacitor C_E on low frequency response :

The equivalent circuit (13) considering the effect of C_E is as shown in Fig. (17). Hence the effect of C_S and C_C are neglected.

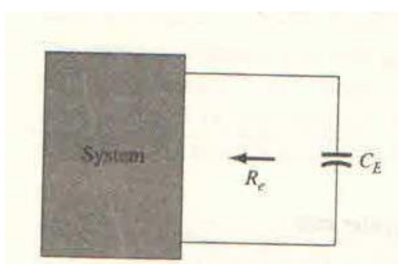

Fig. (17) AC equivalent circuit

Replacing the transistor by its low frequency small signal hybrid model the AC equivalent circuit is as shown in Fig. (18).

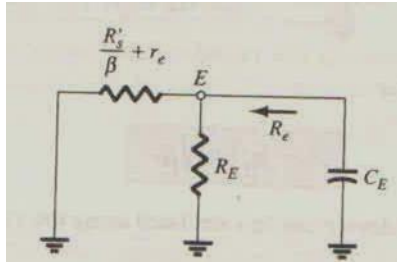

Fig. (18) AC equivalent circuit using hybrid model

R$_e$ is the AC equivalent resistance seen by C$_e$. To find Re, V$_S$ is reduced to 0 as shown in Fig. (18).

Let $\acute{Z}_i$= R$_S$∥ R$_1$ ∥ R$_2$ ----- (35)

The AC equivalent circuit is redrawn as shown in Fig. (20)

Fig. (18) AC equivalent circuit to find R$_e$

$\acute{R_S} = \beta r_e$ is in base circuit. When it is transformed to emitter circuit, it is divided by β. Therefore $I_E \approx I_C = \beta I_B$.

The resulting circuit is shown in Fig. (21).

$R_e = R_E \parallel \frac{\acute{R_S}}{\beta} + r_e$ ----- (36)

From Fig. (18), the lower cut-off frequency due to C$_E$ is

$f_{L_E} = \frac{1}{2\pi R_e C_E}$ ----- (37)

Effect of C$_E$ on voltage gain:

The mid band voltage gain of amplifier of Fig. (13) without C$_E$ is given by,

$A_{V_{mid}} = -\frac{R_O||R_L}{r_e+R_E}$ ----- (38)

Where $R_O = R_C||r_O$

If C$_E$ is connected in parallel with R$_E$, then voltage gain becomes a function of frequency. The voltage gain at any frequency is

$A_V = -\frac{R_O||R_L}{r_e+R_E||X_{C_E}}$ ----- (39)

Where $X_{C_E} = \frac{1}{2\pi f C_E}$ ----- (40)

As the frequency increases:

(i) X_{C_E} decreases.
(ii) $R_E \parallel X_{C_E}$ decreases.
(iii) A_V increases in magnitude.

As the frequency approaches the mid band value

(i) X_{C_E} approaches zero.
(ii) $R_E \parallel X_{C_E}$ approaches zero. (i.e. R_E is shorted out)
(iii) A_V approaches maximum value or mid band value.

$A_{V_{mid}} = -\frac{R_O||R_L}{r_e}$ ----- (41)

Overall lower cutoff frequency:

The low frequency response of the amplifier is influenced by the capacitors C_S, C_C and C_E. The lower cut-off frequencies due to C_S, C_C and C_E are f_{L_S}, f_{L_C} and f_{L_E} respectively. If these cut-off frequencies are relatively apart (i.e. one is greater than the other by 4 times or more) the higher of the 3 is approximately the lower cut-off frequency for the amplifier stage.

Ex: $f_{L_S} = 6Hz$, $f_{L_C} = 25Hz$ and $f_{L_E} = 320Hz$

Then the lower cut-off frequency of amplifier is $f_{L_E} = 320Hz$ Because, $f_{L_E} > 4f_{L_S}$

$f_{L_E} > 4f_{L_C}$.

Miller Effect Capacitance:

Fig. (19) shows an inverting amplifier with a capacitance C_f between the input and output nodes. WKT, A_V is –Ve for inverting amplifier since V_O and V_i are

180^0 out of phase. Using Millers theorem we can find the loading effect of C_f on the input and output circuits of the amplifier.

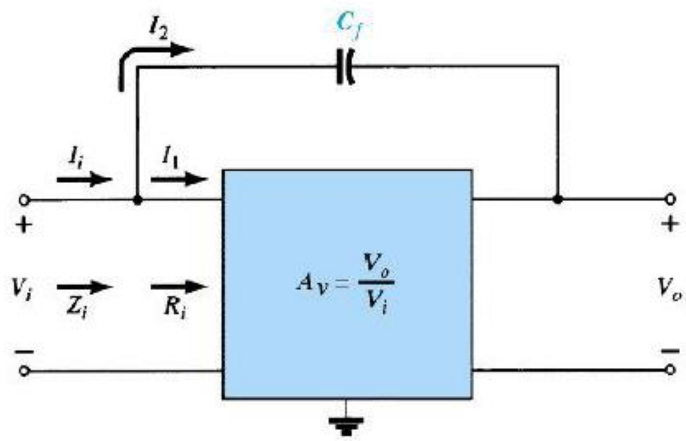

Fig. (19) Inverting amplifier with capacitance between input and output nodes.

To find Miller-Input Capacitance (C_{mi}) :

From Fig. (19),

$R_i = \frac{V_i}{I_1} \Rightarrow I_1 = \frac{V_i}{R_i}$

$Z_i = \frac{V_i}{I_i} \Rightarrow I_i = \frac{V_i}{Z_i}$

Apply KCL at input node A,

$I_i = I_1 + I_2$ ----- (42)

From Fig. (23),

$I_2 = \frac{V_i - V_o}{X_{C_f}}$

But $V_0 = A_V V_i$

Therefore $I_2 = \frac{V_i - A_V V_i}{X_{C_f}} = \frac{V_i[1 - A_V]}{X_{C_f}}$

Substitute for I_i , I_1 and I_2 in Eq. (42), we get

$\frac{V_i}{Z_i} = \frac{V_i}{R_i} + \frac{V_i[1-A_V]}{X_{C_f}}$

Eliminating V_i throught,

$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{[1-A_V]}{X_{C_f}} = \frac{1}{R_i} + \frac{1}{\frac{X_{C_f}}{1-A_V}}$

Let $X_{C_{mi}} = \frac{X_{C_f}}{1-A_V}$ ----- (43)

$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{X_{C_{mi}}}$ ----- (44)

But $X_{C_f} = \frac{1}{2\pi f C_f}$

$X_{C_{mi}} = \frac{1}{[1-A_V]2\pi f C_f}$ ----- (45)

Where $C_{mi} = [1 - A_V]C_f$ = miller input capacitance ----- (46)

From Eq. (44), Z_f can be interpreted as the impedance resulting from the parallel combination of R_i and C_{mi} as shown in Fig. (24).

To find Miller output capacitance (C_{mo}) :

From Fig. (19),

$R_O = \frac{V_o}{V_i}$ ➔ $I_1 = \frac{V_o}{R_O}$

$Z_O = \frac{V_o}{I_O}$ ➔ $I_O = \frac{V_o}{Z_O}$

Apply KCL at node B,

$I_0 = I_1 + I_2$ ----- (47)

From Fig. (19)

$I_2 = \frac{V_O - V_i}{X_{C_f}}$

But $V_0 = A_V + V_i$ or $V_i = \frac{V_o}{A_V}$

Therefore $I_2 = \frac{V_O - \frac{V_o}{A_V}}{X_{C_f}} = \frac{V_O\left[1 - \frac{1}{A_V}\right]}{X_{C_f}}$

Usually, R_O is large therefore $\frac{V_o}{R_0}$ can be neglected

Eq. (47), $\frac{V_o}{Z_0} = \frac{V_o}{R_0} + \frac{V_O\left[1 - \frac{1}{A_V}\right]}{X_{C_f}}$

$$I_0 \approx \frac{V_O\left[1 - \frac{1}{A_V}\right]}{X_{C_f}}$$

$Z_0 = \frac{V_0}{I_0} = \frac{X_{C_f}}{1 - \frac{1}{A_V}} = \frac{1}{\left[1 - \frac{1}{A_V}\right] 2\pi f C_f}$

$Z_0 = \frac{1}{2\pi f C_{m_o}}$ ----- (48)

Where, $C_{m_o} = \left[1 - \frac{1}{A_V}\right] C_f$ ----- (49)

C_{m_o} is called Miller output Capacitance.

Statement of Millar's theorem:

A capacitance C_f connected between the input and output nodes of an inverting amplifier can be replaced by

(i) Miller input capacitance, C_{m_i}= $= [1 - A_V]C_f$ connected between input node and ground.

(ii) Miller output capacitance, $C_{m_o} = \left[1 - \frac{1}{A_V}\right] C_f$ connected between output node and ground.

For an non-inverting amplifier A_V is positive. In order to obtain positive values for $C_{m_i}$and C_{m_o}. Eq. (46) and Eq. (49) should be modified as follows

$C_{m_i} = = [1 - A_V]C_f$ ----- (50)

$C_{m_o} = \left[1 + \frac{1}{A_V}\right] C_f$ ----- (51)

Applications of Miller's theorem to the amplifier of Fig. (19) results in network shown in Fig. (20).

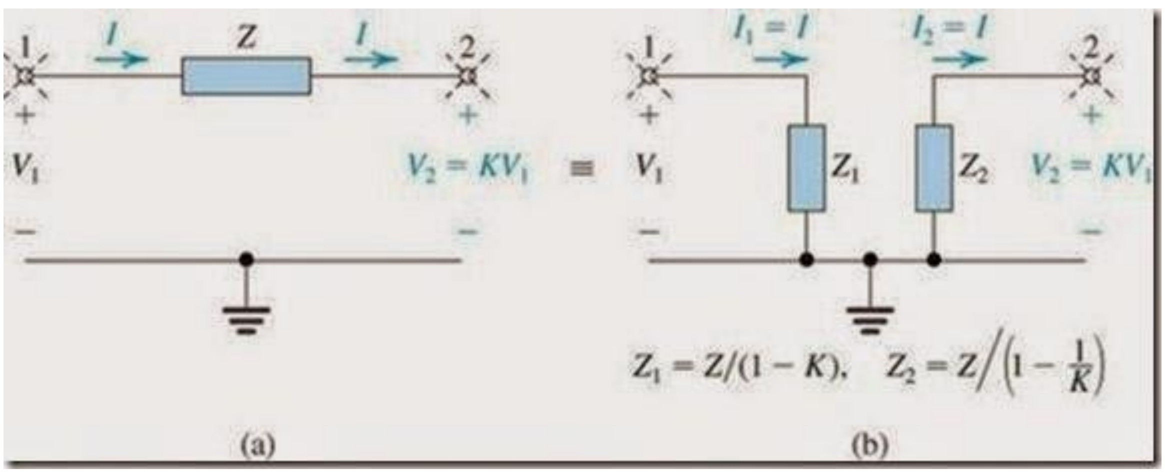

Fig. (20) Amplifier with C_f replaced by Miller's capacitances.

High Frequency Response of BJT amplifier:

In the high frequency response of BJT amplifier, the upper 3dB cut-off point is defined by the following factors.

(i) The network capacitance which includes the parasitic capacitances of the transistor and the wiring capacitances.
(ii) The frequency dependence of short circuit $C_E$current gain h_{f_e} or β.

Network Parameters:

Fig. (21) shows the RC coupled amplifier with parasitic and wiring capacitances. C_{be}, C_{bc} and C_{ce} are the parasitic capacitances of the transistor. C_{wi} and C_{wo} are input and output wiring capacitances which are introduced during the construction of the amplifier circuit.

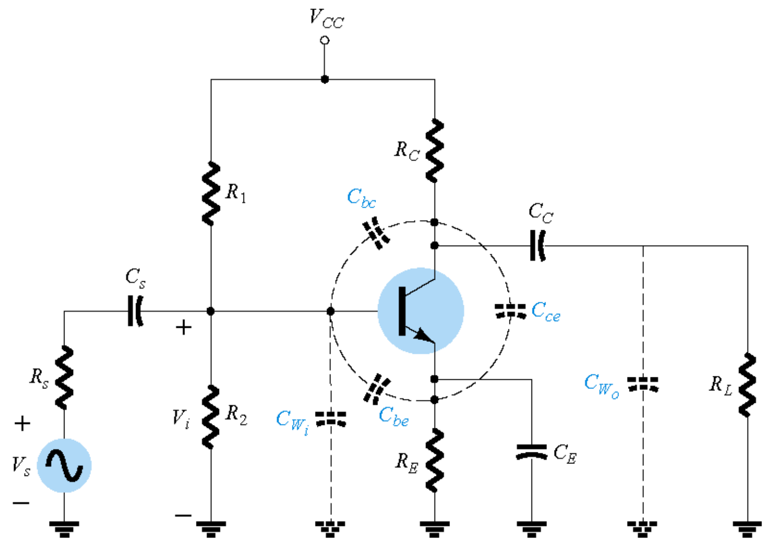

Fig. (21) RC- coupled amplifier with parasitic and wiring capacitances.

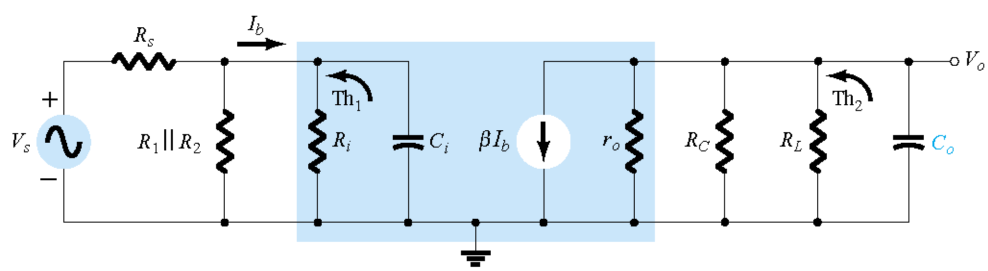

Fig. (22) shows the high frequency AC equivalent circuit of RC-coupled amplifier.

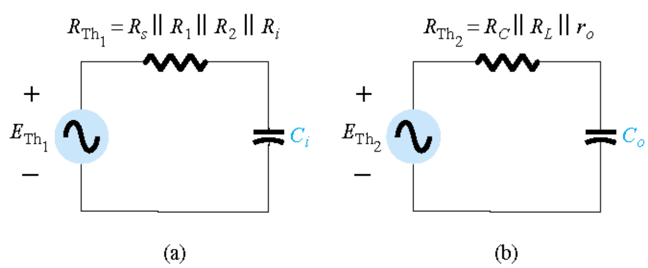

Fig. (23) High frequency AC equivalent circuit of amplifier of Fig. (22).

(i) Using Miller's theorem, the transit capacitance, C_{bc} can be replaced by two capacitances; C_{mi} at the input and C_{mo} at output.

(ii) The total capacitance C_i is the sum of C_{mi}, C_{be} and C_{wi}.
i.e. C_i = C_{mi}+C_{be}+C_{wi} ----- (52)
where C_{mi}=$[1 - A_V]C_{bc}$ ----- (53)

(iii) The total output capacitance is the sum of C_{mo}, C_{ce} and C_{wo}.
i.e. C_0 = C_{wo}+C_{ce}+C_{mo} ----- (54)
where C_{m_o}= $\left[1 + \frac{1}{A_V}\right] C_f$

Upper cut-off frequency due to C_i:

Apply voltage division rule to circuit of Fig. (22),

E_{Thi}= $V_s \left[\frac{R_1 \| R_2 \| \acute{R}_l}{R_S + R_1 \| R_2 \| \acute{R}_l}\right]$ ----- (56)

From circuit in Fig. (21);

$R_{Thi} = R_S + R_1 \| R_2 \| \acute{R}_l$ ----- (57)

Where $\acute{R}_l = \beta r_e$

From Fig. (29) (b), Apply V_g division rule,

$|V_i| = |E_{Thi}| \left[\frac{X_{C_i}}{\sqrt{(R_{Thi})^2 + (X_{Ci})^2}}\right]$

$|V_i| = |E_{Thi}| \frac{|E_{Thi}|}{\sqrt{1+\left(\frac{R_{Thi}}{X_{Ci}}\right)^2}}$ ----- (58)

Where $X_{Ci}= \frac{1}{2\pi f C_i}$ ----- (59)

In the mid band, the effect of C_i is negligible. As a result, X_{Ci} can be treated as open circuit i.e. $X_{Ci} = \infty$.

Therefore $|V_i|_{mid} \approx |E_{Thi}|$

At high frequencies, C_i cannot be neglected with increase in f, X_{Ci} decreases, $\frac{R_{Thi}}{X_{Ci}}$ increases, $|V_i|$ decreases and hence the voltage gain decreases.

3dB cut-off occurs at a frequency at which

$|V_i| = \frac{|\text{Vi}|\text{mid}}{\sqrt{2}} = \frac{|E_{Thi}|}{\sqrt{2}}$

From (58), this condition occurs, when

$R_{Thi} = X_{Ci}$

$R_{Thi} = \frac{1}{2\pi f C_i}$

Or f= $f_{Hi} = \frac{1}{2\pi R_{Thi} C_i}$

$= \left[\frac{f}{f_{Hi}}\right]$

Therefore Eq. (58) becomes,

$|V_i| = \frac{|E_{Thi}|}{\sqrt{1+\left(\frac{f}{f_{Hi}}\right)^2}}$ ----- (62)

Thus, due to C_i, V_g gain decreases at the rate of 20dB/decade.

Upper cut-off frequency due to output capacitance C_O :

Consider the output circuit of Fig.(22) which is shown in Fig. (23).

βI_b, r_o and $R_C||R_L$ is connected to voltage source as shown in Fig. (22).

$R_{Tho} = r_o \; ||R_C||R_L$ ----- (63)

$E_{Tho} = [-\beta I_b] \; [r_o \; ||R_C||R_L]$ ----- (64)

Using the same procedure as listed above, we have

$|V_o| = \frac{|E_{Tho}|}{\sqrt{1+\left(\frac{R_{Tho}}{X_{Co}}\right)^2}}$ ----- (65)

Where $X_{Co} = \frac{1}{2\pi f C_o}$ ----- (66)

The output voltage in mid band is

$|V_o|_{mid} \approx |E_{Tho}|$ ----- (67)

The Upper 3dB cutoff frequency due to C_o is

$f_{HO} = \frac{1}{2\pi R_{Tho} C_o}$ ----- (68)

and magnitude of voltage gain is

$|A_V| = \frac{1}{\sqrt{1+\left(\frac{f}{f_{Ho}}\right)^2}}$ ----- (69)

Thus due to Co, Vg gain decreases at the rate of 20dB/ Decade.

Combined effect of C_i and C_O on high frequency response:

(i) The input capacitance C_i, defines upper cut-off frequency f_{Hi}.
(ii) The output capacitance C_o, defines another upper cut-off frequency f_{Ho}.
(iii) The lowest of these 2 frequencies will be taken as overall upper cut-off frequency.
(iv) If the variation of h_{fe} with frequency is considered then the actual cut-off frequency may be lower than f_{Hi} or f_{Ho}.

Variation of h_{fe} with frequency:

Fig. (24) shows hybrid-π high frequency small signal model of BJT.

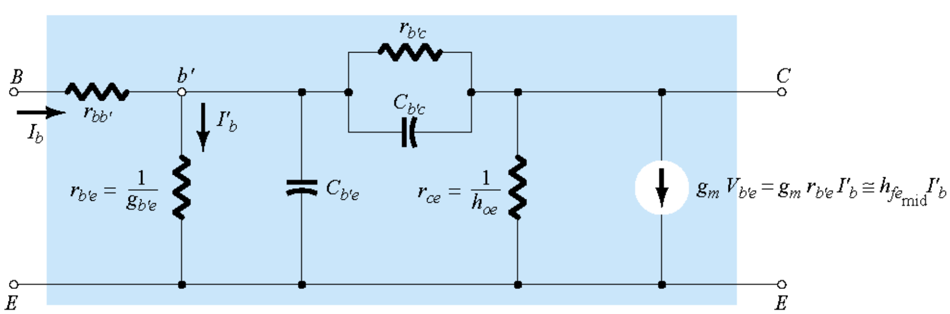

Fig. (24) Hybrid-π high frequency small signal model of BJT.

The B-E input capacitance C_π (C_{be}) and B-C depletion capacitance C_u (C_{bc}) makes the short circuit current gain h_{fe} to vary with frequency in high frequency region.

Expression for h_{fe} as a function of frequency:

Following assumptions are made:

(i) r_b is few tens of Ω. Hence it is treated as short circuit.
(ii) r_u is few tens of $M\Omega$. Hence it is treated as open circuit.

To find h_{fe}, short the output terminals. The resulting circuit is shown in Fig. (24).

Fig. (24) circuit to find h_{fe}.

WKT, $h_{fe}=\frac{I_C}{I_b}\Big|V_{ce}=0$ ----- (70)

The current $g_m V_\pi$ flows into short circuit.

Therefore $I_C=g_m V_\pi$ ----- (71)

to find I_b FROM Fig 24

Let Z= $r_\pi || \frac{1}{j\omega(C_\pi+C_u)}$ ----- (72)

$Z=\frac{r_\pi}{1+j\omega(C_\pi+C_u)\, r_\pi}$

Now, $V_\pi = I_b$ Z

$V_\pi = \frac{I_b r_\pi}{1+j\omega(C_\pi+C_u)\, r_\pi}$ ----- (73)

Substitute Eq. (73) in Eq. (71), we have

$I_C = \frac{g_m r_\pi}{1+j2\pi f(C_\pi+C_u)\, r_\pi}$ ----- (74)

Let $f_\beta = \frac{1}{2\pi(C_\pi+C_u)\, r_\pi}$ ----- (75)

Substitute Eq. (75) in Eq. (74), we have

$$h_{fe} = \frac{g_m r_\pi}{1+j\left(\frac{f}{f_\beta}\right)} \text{ ----- (76)}$$

$$|h_{fe}| = \frac{g_m r_\pi}{\sqrt{1+j\left(\frac{f}{f_\beta}\right)^2}} \text{ ----- (77)}$$

In the mid band, f << f_β, As a result $\left(\frac{f}{f_\beta}\right)^2$ <<1

From Eq. (77), we have

$|h_{fe}|_{\text{mid}} = g_m r_\pi = h_{fe\,\text{mid}}$ ----- (78)

$h_{fe\,\text{mid}}$ is also denoted by In β_{mid}.

Substitute Eq. (78) in Eq. (77),

$$|h_{fe}| = \frac{|h_{fe}|\text{mid}}{\sqrt{1+j\left(\frac{f}{f_\beta}\right)^2}} \text{ ----- (79)}$$

Eq. (79) gives the variation of $|h_{fe}|$ withfrequency.

(i) As f increases, $\left(\frac{f}{f_\beta}\right)^2$ increases and hence $|h_{fe}|$ decreases.

(ii) When f=f_β, =$|h_{fe}|$ =$\frac{h_{fe}\text{mid}}{\sqrt{2}}$ = $\frac{\beta_{mid}}{\sqrt{2}}$

f_β defines the upper 3dB cut-off point for short circuit current gain h_{fe}. f_β is also denoted by fh_{fe}. Eq. (75) can be written as

$$f_\beta = fh_{fe} = \frac{1}{2\pi(C_\pi+C_u)\, r_\pi} \text{ ----- (80)}$$

But, $r_\pi = \beta_{re} = \beta_{mid}\, r_e$

$$f_\beta = fh_{fe} = \frac{1}{2\pi\beta_{mid}\, r_e(C_\pi+C_u)} \text{ ----- (80)}$$

f_β is called β cut-off frequency. f_β is also the bandwidth for the short circuit current gain h_{fe}. Fig. (38) shows the variation of $|h_{fe}|$ with frequency. $|h_{fe}|$ decreases from its mid band value $h_{fe\,\mathrm{mid}}$ with a slope of 20dB/decade.

Expression for gain band width product f_T:

f_T is the frequency at which $|h_{fe}|$ =1 or $|h_{fe}|_{\mathrm{dB}}$ =0dB.

Using this in Eq. (79),

$$\left.\frac{h_{fe}\mathrm{mid}}{\sqrt{1+j\left(\frac{f}{f_\beta}\right)^2}}\right|_{f=ft} = 1$$

$$\frac{h_{fe}\mathrm{mid}}{\sqrt{1+j\left(\frac{f_T}{f_\beta}\right)^2}} \text{----- (82)}$$

Since $f_T >> f_\beta$, $\left(\frac{f_T}{f_\beta}\right)^2 >>1$

Therefore $\sqrt{1+j\left(\frac{f_T}{f_\beta}\right)^2} \approx \frac{f_T}{f_\beta}$

Therefore Eq. (82) becomes

$$\frac{h_{fe}\mathrm{mid}}{\frac{f_T}{f_\beta}} = 1$$

Or $f_T = f_\beta\ h_{fe}\mathrm{mid} = \beta_{mid}\ f_\beta$----- (83)

Since h_{fe}mid is the mid band short circuit current gain and f_β is the bandwidth, f_T is called gain-bandwidth product.

Eq. (81) in Eq. (83),

$$f_T = \beta_{mid} \times \frac{1}{2\pi\beta_{mid}\, r_e(C_\pi + C_u)} = \frac{1}{2\pi(C_\pi + C_u)\, r_e} \text{----- (84)}$$

Low frequency response of FET amplifier:

Consider a coomon source amplifier as shown in Fig. 25

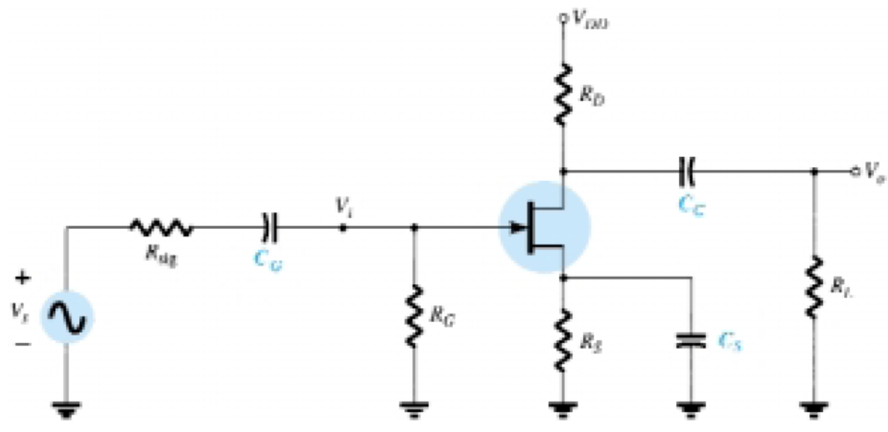

Fig 25 Capacitive elements that affect the low-frequency response of a JFET amplifier.

Effect of C_G on Low frequency response:

The lower cut-off frequency of this network is shown in fig 26

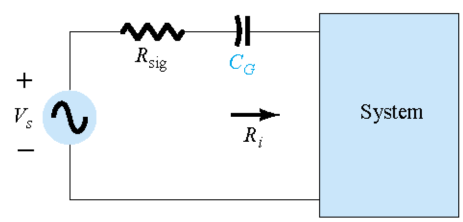

Fig 26 Determining the effect of *CG* on the low-frequency response.

$$f_{C_G} = \frac{1}{2\pi(R_{sig}+R_i)C_G}$$

Where $R_i = R_G \| R_{in(gate)}$

$$R_{in(gate)} = \left|\frac{V_{GS}}{I_{GSS}}\right|$$

I_{GSS} = gate reverse current

$R_G >> R_{sig}$ and $R_{in(gate)} >> R_G$

$R_i \approx R_G$

Therefore $f_{C_G} = \frac{1}{2\pi R_G C_G}$

Effect of C_C on Low frequency response:

Consider output part of equivalent circuit as shown in Fig 27

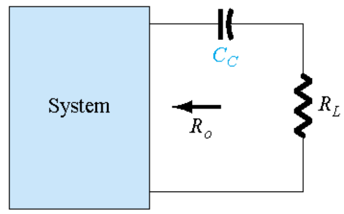

Fig 27 Determining the effect of *Cc* on the low-frequency response.

$R_o = r_d \parallel R_D$

Therefore $f_{L_C} = \frac{1}{2\pi(R_o+R_L)C_G}$

Effect of C_S on Low frequency response:

Consider the RC network shown in Fig 28 formed by C_S and let R_{eq} be the resistance looking in at the source.

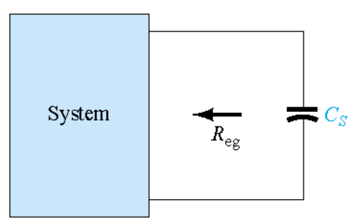

Fig 28 Determining the effect of *CS* on the low-frequency response.

$f_{L_S} = \frac{1}{2\pi(R_{eq})C_S}$

$R_{eq} = \frac{R_S}{1+R_S(1+g_m r_d)|\,(r_d+R_D||R_L)}$

$=R_S \parallel \frac{1}{g_m}$

Therefore $r_d >> R_D$

High Frequency Response FET amplifier:

Fig. 29 shows CS JFET amplifier with inter electrode capacitances and wiring capacitances. The capacitors C_{gs} and C_{gd} vary from 1pF, whereas the capacitance C_{ds} is smaller ranging from 0.1pF to 1pF.

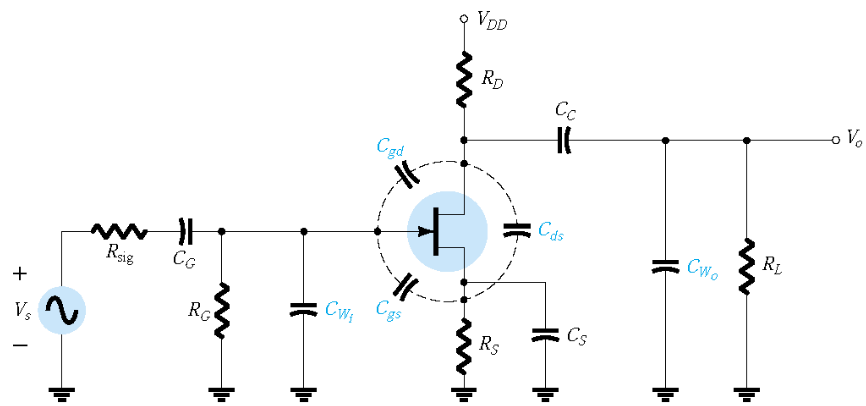

Fig 29 Capacitive elements that affect the high frequency response of a JFET amplifier

Fig. 30 shows high-frequency AC equivalent circuit. At high frequencies, C_i will approach a short circuit equivalent and V_{gs} will drop in value and reduce the overall gain. At frequencies where C_o approaches its short circuit equivalent, the parallel output voltage Vo will drop in magnitude.

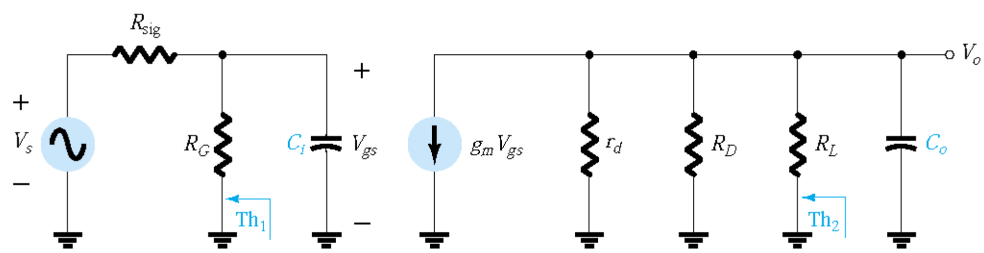

Fig 30 High-frequency ac equivalent circuit for Fig. 29.

Effect of C_i on high frequency response:

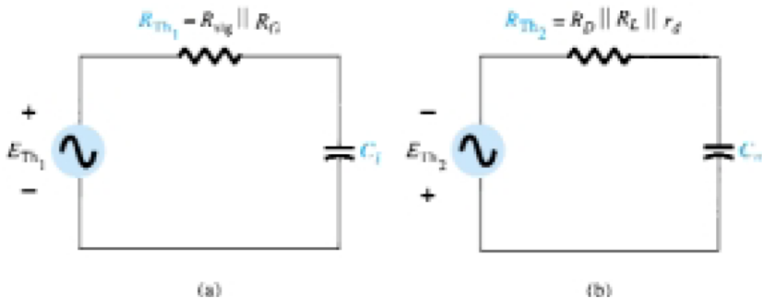

Fig 31 The Thévenin equivalent circuits for the (a) input circuit and (b) output circuit.

$$f_{H_i} = \frac{1}{2\pi(R_{Thi})C_i}$$

Where $R_{Thi} = R_{sig} \parallel R_G$

$C_i = C_{wi} + C_{gs} + (1 - A_V)C_{gd}$

Effect of C_o on high frequency response:

$$f_{H_o} = \frac{1}{2\pi(R_{Tho})C_o}$$

Where $R_{Tho} = r_d \parallel R_D \parallel R_L$

$C_o = C_{wo} + C_{ds} + (1 - 1/A_V)C_{gd}$

Multistage Frequency Effects:

For a second transistor stage connected directly to the output of a first stage, there will be a significant change in the overall frequency response. In the high frequency region, the output capacitance C_O must include the wiring capacitance (C_{W1}), parasitic capacitance (C_{be}) and miller capacitance (C_{mi}) of the following stage. There will be additional low frequency cut-off levels due to the 2nd stage, which will further reduce the overall gain of the system in the region. For each additional stage, the upper cutoff frequency will be determined by the stage having the lowest cutoff frequency. The low frequency cutoff is determined by the stage having the highest low-frequency cutoff frequency. The multistage amplifier frequency response is shown in Fig 32.

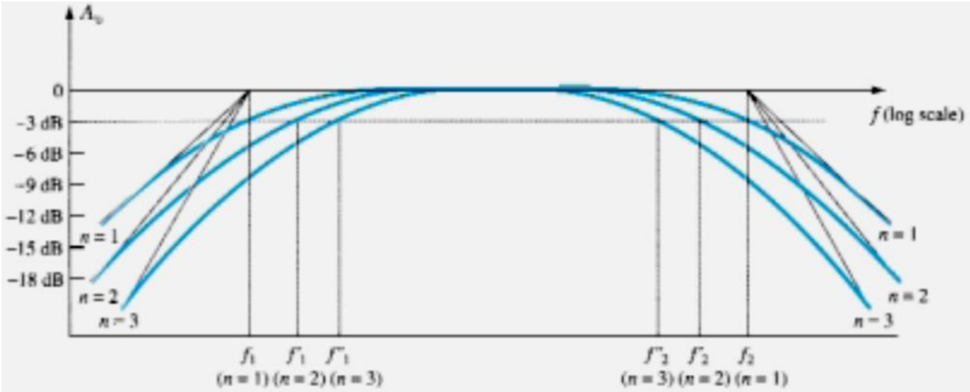

Fig 32 Effect of an increased number of stages on the cutoff frequencies and the bandwidth.

Assuming identical stage, for low frequency region

$A_{V\,low_{(overall)}} = A_{V1\,low} \quad A_{V2\,low}$ -- $A_{Vn\,low}$

Since all stages are identical, $A_{V1\,low} = A_{V2\,low} =$ etc.

Therefore $A_{V\,low_{(overall)}} = \left(A_{V1\,low}\right)^n$

Or $\frac{A_{V\,low_{(overall)}}}{A_{V\,mid\,(overall)}} = \left(\frac{A_{V\,low}}{A_{V\,mid}}\right)^n = \frac{1}{\left(1-j\frac{f1}{f}\right)^n}$

At 3dB frequency

$$\frac{1}{\sqrt{\left(1+j\left(\frac{f1}{f}\right)^2\right)^n}} = \frac{1}{\sqrt{2}}$$

$$\left(1+\left(\frac{f1}{f}\right)^{1/2}\right)^n = 2^{1/2}$$

$$f1' = \frac{f1}{\sqrt{2^{1/n}-1}}$$

Similarly $f2' = \sqrt{2^{1/n}-1}$ f2