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SAI VIDYA INSTITUTE OF TECHNOLOGY

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PRINCIPLES OF COMMUNICATION SYSTEMS (15EC45)

IV SEMESTER ECE

MODULE 1 SYLLABUS

AMPLITUDE MODULATION:

Introduction, Amplitude Modulation: Time & Frequency – Domain description, Switching modulator, Envelop detector

DOUBLE SIDE BAND-SUPPRESSED CARRIER MODULATION:

Time and Frequency – Domain description, Ring modulator, Coherent detection, Costas Receiver, Quadrature Carrier Multiplexing (QAM).

SINGLE SIDE–BAND AND VESTIGIAL SIDEBAND METHODS OF MODULATION:

SSB Modulation, VSB Modulation, Frequency Translation, Frequency- Division Multiplexing, Theme Example: VSB Transmission of Analog and Digital Television

MODULE-1 AMPLITUDE MODULATION

INTRODUCTION:-

- Message signals are incompatible for direct transmission. For such a signal, to travel longer distances, its strength has to be increased by modulating with a high frequency carrier wave.
- Modulation is the process of altering any one parameter (amplitude/frequency/phase) of the carrier signal, in accordance with the instantaneous values of the message signal by keeping other parameters of carrier constant.
- Figure1, shows the general schematic representation of modulation process. It consists of 3-types of signals as follows,

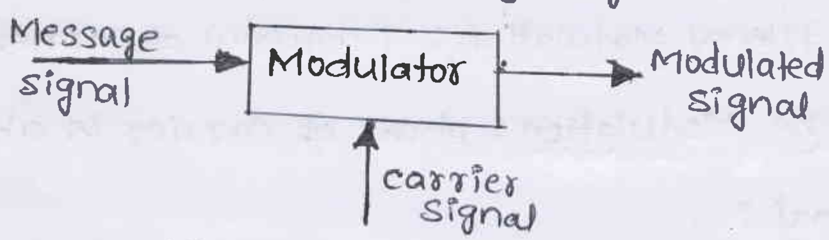


Figure1: Modulation process

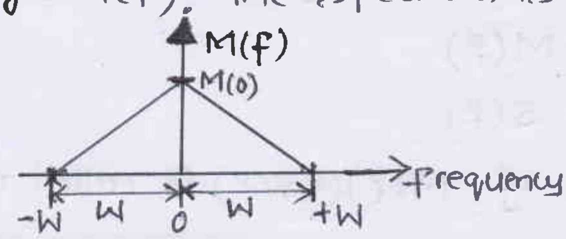
1. Message signal : The signal which contains a message to be transmitted, is called as message signal. It is also known as base-band signal, which has to undergo the process of modulation. Hence it is also named as "Modulating signal". Mathematically, it is denoted by $m(t)$.

Ex: $m(t) = A_m \cos 2\pi f_m t$

where, A_m = Amplitude of message signal in Volts.

f_m = frequency of message signal in Hz.

**In frequency domain the spectrum of $m(t)$ is denoted by $M(f)$. The spectrum is bandlimited to $2W$ Hz.



- Where $\boxed{W = f_m}$
- $M(0)$ = Amplitude of $M(f)$ at frequency $\boxed{f=0}$.

2. Carrier signal: It is a high frequency signal, used to carry the signal to the receiver after modulation.
Mathematically it is denoted by $c(t)$.

Ex: $c(t) = A_c \cos(2\pi f_c t + \phi_c)$, its 3-parameters are

A_c = Amplitude of carrier signal in Volts

f_c = frequency of carrier signal in Hz

ϕ_c = phase of carrier signal in degrees.

↳ Depending on the altering parameter of carrier signal, there are 3-types of Modulation techniques namely.

i) Amplitude Modulation : Amplitude of carrier is altered.

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ii) Frequency Modulation : Frequency of carrier is altered.

iii) Phase Modulation : phase of carrier is altered.

3. Modulated signal :-

The resultant signal after the process of modulation is called as "Modulated signal". This signal consists of modulating signal and carrier signal.
Mathematically it is denoted by $s(t)$.

Example: The Amplitude modulated signal for any $m(t)$ is given by,

$$s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

where, K_a = Amplitude Sensitivity parameter (Discussed in Next Section).

Note: • In this module the time domain and Frequency domain analysis is discussed

• Fourier transformation : It is a technique used to convert time domain signal to frequency domain signal.

Ex: i.e., $m(t) \xrightarrow{F.T} M(f)$

$s(t) \xrightarrow{F.T} S(f)$

* Graphical representation of $M(f)$ (or) $S(f)$ is called as "Spectrum".

* Frequency spectrum gives the details of frequency components.

* Table 1 gives the Fourier transformation of few standard time domain signals, which are repeatedly used in this module.

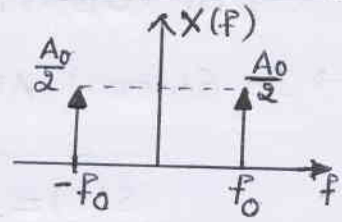
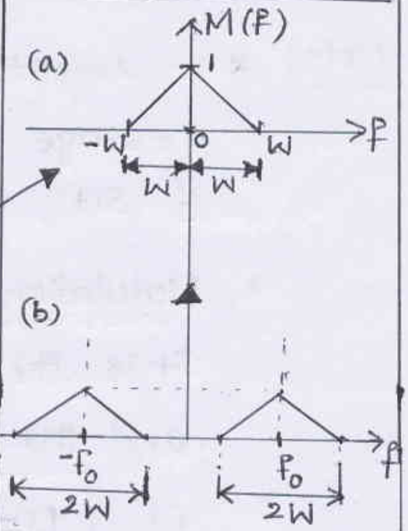
Time Domain Signal	Fourier Transformation (F.T)	Frequency Spectrum
$x(t) = A_0 \cos 2\pi f_0 t$ → Any General Cosine Signal with amplitude A_0 and frequency 'f_0'.	$X(f) = \frac{A_0}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$ ($\delta(f-f_0)$ → Impulse at $f=f_0$; $\delta(f+f_0)$ → Impulse at $f=-f_0$) * $\delta(f-f_0)$ and $\delta(f+f_0)$ are "Impulse"-signals.	 ↳ spectrum of $x(t)$ consists of two impulse signals at frequencies $f=f_0$ and $f=-f_c$ with equal amplitudes $\frac{A_0}{2}$.
$\underbrace{m(t) \cdot \cos 2\pi f_0 t}_{\text{Product of}}$ Any $m(t)$ with Cosine signal, having frequency 'f_0'.	$\frac{1}{2}\left[M(f-f_0) + M(f+f_0)\right]$ * $M(f)$ → spectrum of $m(t)$ shown in fig (a) Centered at $f=0$ * $M(f-f_0)$ → $M(f)$ at center frequency $f=f_0$ * $M(f+f_0)$ - ↳ $M(f)$ at Center frequency $f=-f_0$	

Table 1

* **Advantages of Modulation:**

Advantages of using modulation process in Communication systems are as follows.

1. Reduces the height of Antenna
2. Avoids mixing of signals
3. Allows Multiplexing of signals
4. Allows Adjustments of Bandwidth
5. Increases the range of Communication
6. Improves Quality of Reception.

1.2. Amplitude Modulation: (A.M)

Definition: It is a process of altering the amplitude of carrier signal in accordance with the instantaneous values of message signal by keeping frequency and phase of carrier signal constant.

↳ In General AM signal, standard equation is given by

$$\boxed{s(t) = A_c[1 + K_a m(t)] \cos 2\pi f_c t} \text{ ——— } *$$

K_a = Amplitude sensitivity parameter

$m(t)$ = Message signal

A_c = Amplitude of carrier signal

f_c = frequency of carrier signal.

Note: * In Amplitude Modulation, Information present in the message signal $m(t)$, resides only in the amplitude of $s(t)$.

* Modulation Index :- (μ)

It is the product of amplitude sensitivity parameter and the maximum value of message signal.

i.e., $\boxed{\mu = K_a |m(t)|_{max}}$ No units (or) $\boxed{\mu = K_a A_m}$

K_a = Amplitude sensitivity

$|m(t)|_{max}$ = Maximum value (or) Amplitude of $m(t)$.

↳ The Maximum Value of Modulation Index, $\mu = 1$.

↳ If $\mu > 1$, the carrier wave becomes over modulated.

↳ If $\mu < 1$, the carrier wave becomes under modulated

↳ If $\mu = 1$, the carrier wave becomes critically modulated.

* In AM, the carrier frequency, $\boxed{f_c \gg f_m}$; f_m = frequency of $m(t)$

** Time and Frequency domain description of AM-signal:

Q) Define Amplitude Modulation. Obtain the expression for AM by both time domain and frequency domain representation with necessary waveforms.

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↳ Amplitude Modulation :-

Defn :- It is a process of altering the amplitude of carrier signal in accordance with the instantaneous values of message signal by keeping frequency and phase of carrier signal constant.

Expression for AM signal :-

- The instantaneous value of message signal is given by,

$$m(t) = A_m \cos(2\pi f_m t) \quad \text{——— (1)}$$

where, $A_m \Rightarrow$ Amplitude of message signal.

$f_m \Rightarrow$ frequency (or) Bandwidth of message signal.

- The instantaneous value of carrier signal is given by,

$$c(t) = A_c \cos(2\pi f_c t) \quad \text{——— (2)}$$

where, $A_c \Rightarrow$ Amplitude of carrier signal.

$f_c \Rightarrow$ frequency of carrier signal.

- We know that the standard equation of AM signal is given by,

$$s(t) = A_c\left[1 + K_a m(t)\right] \cos(2\pi f_c t) \quad \text{——— (3)}$$

where, K_a = Amplitude Sensitivity parameter.

Substitute $m(t) = A_m \cos 2\pi f_m t$ in equation (3)

$$s(t) = A_c\left[1 + \underbrace{K_a \cdot A_m}_{} \cos(2\pi f_m t)\right] \cos(2\pi f_c t)$$

$$\therefore \quad s(t) = A_c\left[1 + \mu \cos(2\pi f_m t)\right] \cos(2\pi f_c t) \quad \text{——— (4)}$$

Where $\mu = K_a A_m \Rightarrow$ Modulation Index for AM-signal

$$s(t) = [A_c + \mu A_c \cos 2\pi f_m t] \cos 2\pi f_c t$$

$$s(t) = A_c \cos 2\pi f_c t + \mu A_c \cos 2\pi f_c t \cdot \cos 2\pi f_m t$$

We know that, $\cos A \cdot \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

$$\therefore s(t) = A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2} \cos 2\pi (f_c - f_m) t + \frac{\mu A_c}{2} \cos 2\pi (f_c + f_m) t$$

$$\therefore \boxed{s(t) = \underbrace{A_c \cos 2\pi f_c t}_{\text{carrier}} + \underbrace{\frac{\mu A_c}{2} \cos 2\pi (f_c - f_m) t}_{\text{LSB}} + \underbrace{\frac{\mu A_c}{2} \cos 2\pi (f_c + f_m) t}_{\text{USB}}} \longrightarrow (5)$$

Equation (5) gives the simplified expression of AM-signal. It consists of three frequency components

- $f_c \longrightarrow$ carrier frequency with amplitude 'A_c', which does not contains any message signal
- $f_c - f_m \rightarrow$ Lower side band (LSB) with amplitude $\frac{\mu A_c}{2}$
- $f_c + f_m \rightarrow$ upper side band (USB) with amplitude $\frac{\mu A_c}{2}$

Taking Fourier transformation on both sides of equation (5), we get

$$\boxed{\begin{aligned} S(f) = {} & \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \\ & \frac{\mu A_c}{4}[\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))] + \\ & \frac{\mu A_c}{4}[\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))] \end{aligned}} \longrightarrow (6)$$

Equation (6) gives the fourier transform of $s(t)$.

Figure 1(a) shows the spectrum of message signal $m(t)$. and Figure 1(b) shows the spectrum of AM wave $S(t)$.

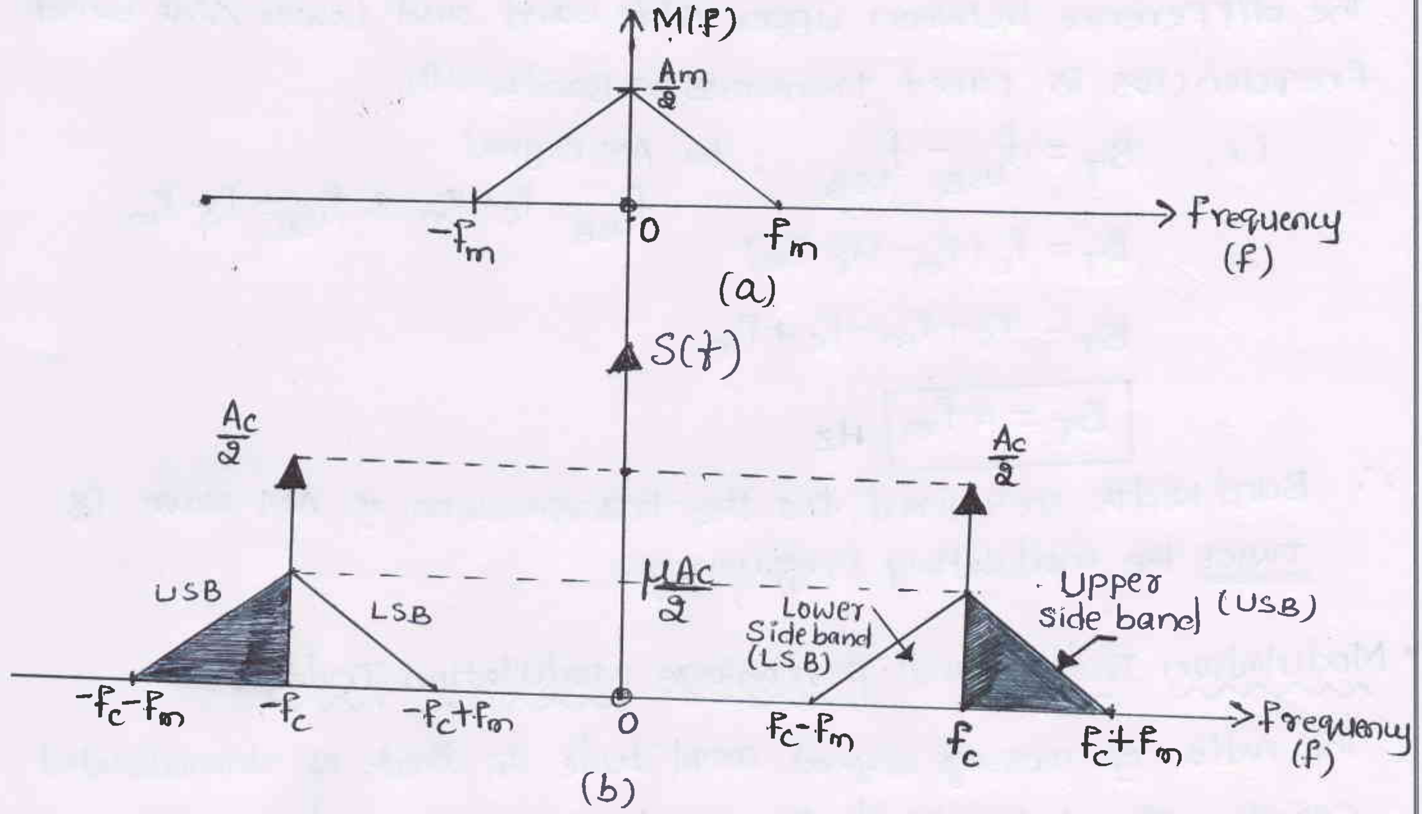

Figure 1. (a) Spectrum of $m(t)$ (b) spectrum of AM signal.

Figure 2, shows the ~~time~~ domain signal waveforms.

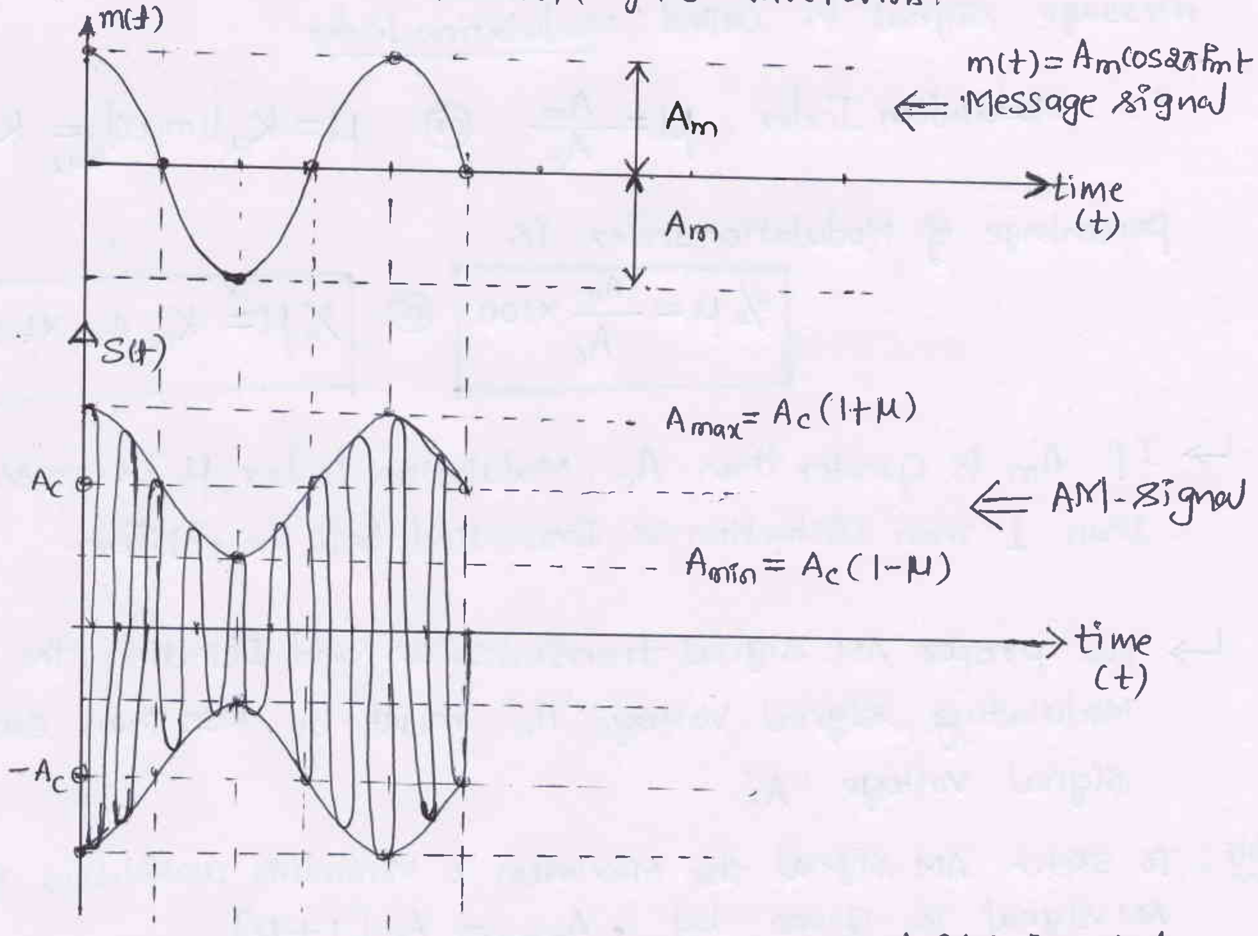

Figure 2: (a) Message signal $m(t)$ (b) AM-Signal $S(t)$ for $\mu < 1$

- **Transmission Bandwidth of AM signal :- (B_T)**

The difference between upper side band and Lower side band frequencies is called transmission Bandwidth.

i.e., $B_T = f_{USB} - f_{LSB}$ ∴ for AM signal $f_{USB} = f_c + f_m$ & $f_{LSB} = f_c - f_m$

∴ $B_T = f_c + f_m - (f_c - f_m)$

$B_T = f_c + f_m - f_c + f_m$

$$\boxed{B_T = 2 f_m} \text{ Hz}$$

∴ Band width required for the transmission of AM-wave is TWICE the modulating frequency.

- **Modulation Index and percentage Modulation Index :-**

The ratio of message signal amplitude to that of unmodulated carrier signal amplitude is called "Modulation Index".

(or)

The product of amplitude Sensitivity parameter and amplitude of message signal is called 'Modulation Index'.

i.e., Modulation Index, $\mu = \frac{A_m}{A_c}$ (or) $\mu = K_a |m(t)|_{max} = K_a A_m$

percentage of Modulation Index is

$$\boxed{\% \mu = \frac{A_m}{A_c} \times 100} \text{ (or) } \boxed{\% \mu = K_a \cdot A_m \times 100}$$

↳ If A_m is Greater than A_c, Modulation index μ becomes greater than 1 then Distortion is Introduced into the system.

↳ For proper AM-signal transmission and Detection, the Modulating signal Voltage 'A_m' must be less than carrier signal voltage 'A_c'.

Note: To sketch AM signal the Maximum & Minimum amplitudes of AM signal is given by
- $A_{max} = A_c [1 + \mu]$
- $A_{min} = A_c [1 - \mu]$

- Expression for AM-signal Modulation Index in terms of its Maximum and Minimum amplitudes:-

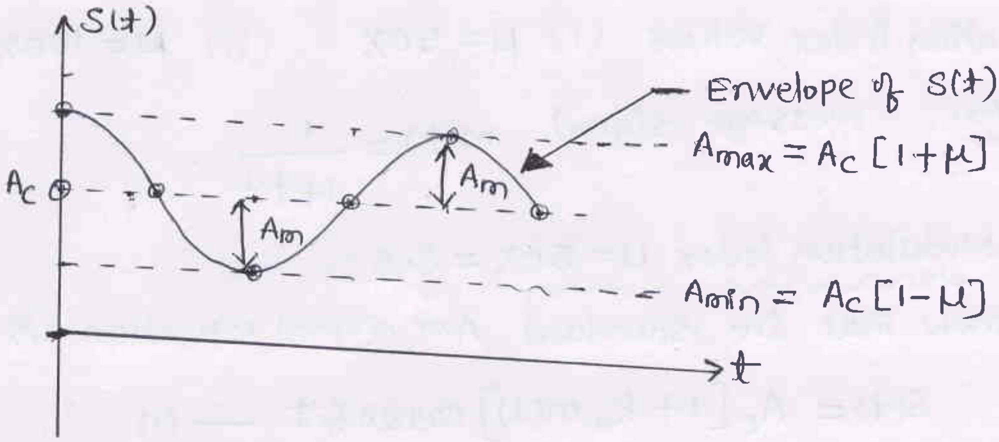

Figure 3: Envelope of AM signal S(t) [(positive Envelope)]

We know that the standard AM-signal is given by

$$S(t) = A_c [1 + K_a m(t)] \cos 2\pi f_c t \quad \text{--- (1)}$$

Let $A_{max} \Rightarrow$ Maximum amplitude of S(t)

$A_{min} \Rightarrow$ Minimum amplitude of S(t)

Figure 3. shows the Envelope of S(t) [i.e., $A_c[1 + K_a m(t)]$] sketch.

AM signal reaches its maximum value when $m(t) = A_m$.

$$\therefore A_{max} = A_c (1 + K_a \cdot A_m) = A_c (1 + \mu) \quad \text{--- (1)} \qquad \because \mu = K_a A_m$$

Similarly, AM signal reaches minimum value when $m(t) = -A_m$

$$\therefore A_{min} = A_c (1 - K_a A_m) = A_c (1 - \mu) \quad \text{--- (2)}$$

Dividing equation (1) with equation (2) we get

$$\frac{A_{max}}{A_{min}} = \frac{A_c(1+\mu)}{A_c(1-\mu)} = \frac{1+\mu}{1-\mu}$$

$$A_{max}(1-\mu) = A_{min}(1+\mu)$$

$$A_{max} - \mu \cdot A_{max} = A_{min} + \mu A_{min}$$

$$A_{max} - A_{min} = \mu A_{max} + \mu A_{min} = \mu (A_{max} + A_{min})$$

$\therefore$ Modulation Index : $\boxed{\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}}$

Example 1.1: Using the message signal $m(t)=\frac{1}{1+t^2}$, Determine and sketch the amplitude modulated wave for the following modulation index values (i) $\mu = 50\%$ (ii) $\mu = 100\%$

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↳ Given: Message signal $m(t) = \frac{1}{1+t^2}$

(i) Modulation index $\mu = 50\% = 0.5$:-

We know that the standard AM signal equation is

$$S(t) = A_c\left[1 + K_a\, m(t)\right]\cos 2\pi f_c t \quad \text{— (1)}$$

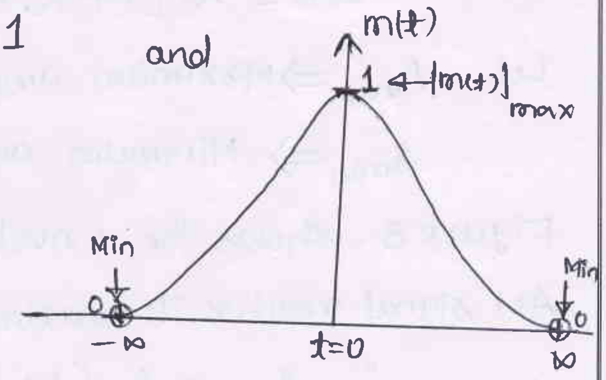
for $\mu = 0.5$: We know that, $\mu = K_a\,|m(t)|_{max}$

- The given signal $m(t)$ is maximum at $t=0$.

$\therefore \quad |m(t)|_{max} = \left.\frac{1}{1+t^2}\right|_{t=0} = \frac{1}{1+0} = 1$ and

- $m(t)$ is Minimum at $t = \pm\infty$.

$$|m(t)|_{min} = \left.\frac{1}{1+t^2}\right|_{t=\pm\infty} = 0.$$



— ! Sketch of $m(t)$:-

$\therefore \quad \mu = K_a\,|m(t)|_{max} = K_a \times 1$

$\therefore \quad \boxed{\mu = K_a = 0.5}$

$\therefore \quad S(t) = A_c\left[1 + \frac{0.5}{1+t^2}\right]\cos 2\pi f_c t$

To Sketch $S(t)$:- $A_{max} = A_c\left[1 + \underbrace{K_a}_{0.5}\underbrace{|m(t)|_{max}}_{1}\right] = A_c\left[1+0.5(1)\right] = 1.5A_c$

$A_{min} = A_c\left[1 + K_a\underbrace{|m(t)|_{min}}_{0}\right] = A_c\left[1+0\right] = A_c$

$\therefore$ The Envelope of $m(t)$ appears between $A_{min} = A_c$ and $A_{max} = 1.5A_c$ in $S(t)$ signal as shown in figure 1.

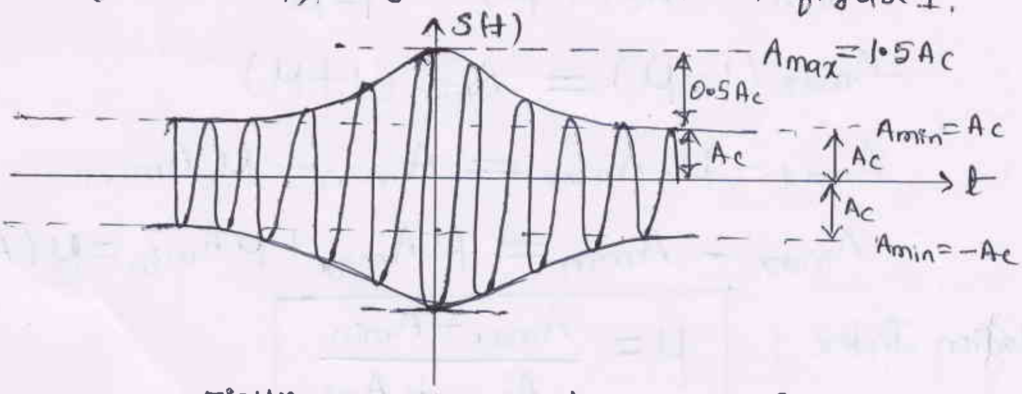


Figure 1: AM Signal for $\mu = 0.5$

ii) When $\mu = 100\% = 1$:- $K_a = \frac{\mu}{|m(t)|_{max}} = \frac{1}{1} = 1.$ $\left(\because \mu = K_a |m(t)|_{max}\right)$

$\therefore$ AM equation is,

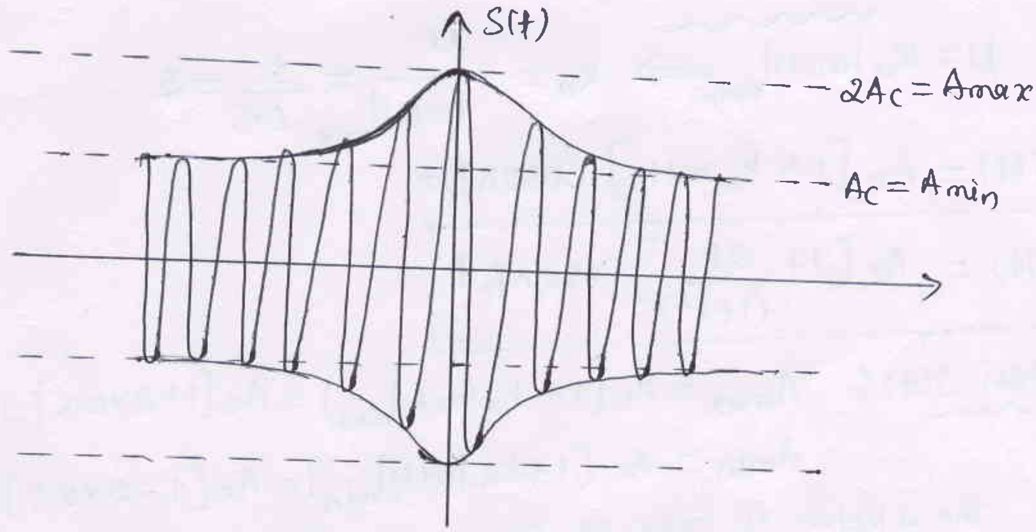
$$S(t) = A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t) \quad , \quad m(t) = \frac{1}{1+t^2}$$

$$\Longrightarrow S(t) = A_c\left[1 + \frac{1}{(1+t^2)}\right]\cos(2\pi f_c t)$$

To Sketch $S(t)$:-

$$A_{max} = A_c\left[1 + K_a \cancel{|m(t)|_{max}}^{1}\right] = A_c\left[1+1\right] = 2A_c$$

$$A_{min} = A_c\left[1 + K_a \cancel{|m(t)|_{min}}^{0}\right] = A_c\left[1+0\right] = A_c$$



Sketch of AM wave $S(t)$ for $\mu = 1$

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Example 1.2 :- Using the Message signal,

$$m(t) = \frac{t}{1+t^2}$$

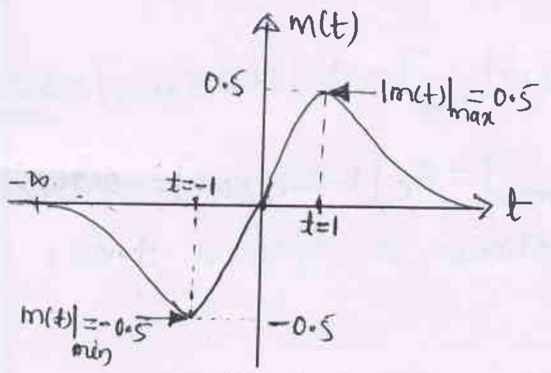
, Determine and Sketch AM signal for (i) $\mu = 50\%$ (ii) $\mu = 100\%$ (iii) $\mu = 125\%$

$\longrightarrow$ Given data : Message signal $m(t) = \frac{t}{1+t^2}$

t	$-\infty$	$\dots$	-3	-2	-1	0.5	0	0.5	1	2	3	$\dots$	∞
$m(t)$	0	$\dots$	-0.3	-0.4	-0.5	-0.4	0	0.4	0.5	0.4		$\dots$	0

$|m(t)|_{min} = -0.5$ at $t = -1$

$|m(t)|_{max} = 0.5$ at $t = 1$



Sketch of $m(t)$

$\therefore$ For this message signal, AM signal $S(t)$ becomes Maximum at $t = 1$ and reaches minimum at $t = -1$.

Case(i) $\mu = 50\% = 0.5$:- W.K.T. $K_a =$

The standard equation of $s(t)$, AM signal is

$$s(t) = A_c[1 + K_a m(t)]\cos 2\pi f_c t$$

W.K.T; $K_a = \frac{\mu}{|m(t)|_{max}} = \frac{0.5}{0.5} = 1$ & $m(t) = \frac{t}{1+t^2}$

$$\therefore \boxed{s(t) = A_c\left[1 + \left(\frac{t}{1+t^2}\right)\right]\cos 2\pi f_c t}$$ and is shown in figure 1.2(a).

To Sketch $s(t)$:-

$A_{max} = A_c[1 + K_a|m(t)|_{max}]$

$A_{max} = A_c[1 + 1\times 0.5]$

$\boxed{A_{max} = 1.5A_c}$

$A_{min} = A_c[1 - 0.5]$

$\boxed{A_{min} = 0.5A_c}$

Case (ii) :- for $\mu = 100\% = 1$:-

$$\mu = K_a|m(t)|_{max} \Longrightarrow K_a = \frac{\mu}{|m(t)|_{max}} = \frac{1}{0.5} = 2$$

$$\therefore s(t) = A_c[1 + K_a m(t)]\cos 2\pi f_c t$$

$$\boxed{s(t) = A_c\left[1 + \frac{2t}{(1+t^2)}\right]\cos 2\pi f_c t}$$

To Sketch $s(t)$:- $A_{max} = A_c[1 + K_a|m(t)|_{max}] = A_c[1 + 2\times 0.5] = 2A_c$.

$A_{min} = A_c[1 + k_a[m(t)]_{min}] = A_c[1 - 2\times 0.5] = 0$

The sketch of $s(t)$ is shown in figure 1.2.(b). (over Modulated)

Case (iii) :- for $\mu = 125\% = 1.25$:-

$$\mu = K_a|m(t)|_{max} \Longrightarrow K_a = \frac{\mu}{|m(t)|_{max}} = \frac{1.25}{0.5} = 2.5$$

$$\therefore s(t) = A_c[1 + K_a m(t)]\cos 2\pi f_c t$$

$$s(t) = A_c\left[1 + \frac{2.5\times t}{1+t^2}\right]\cos 2\pi f_c t.$$

$$\therefore s(t) = A_c\left[1 + \frac{2.5t}{(1+t^2)}\right]\cos 2\pi f_c t$$

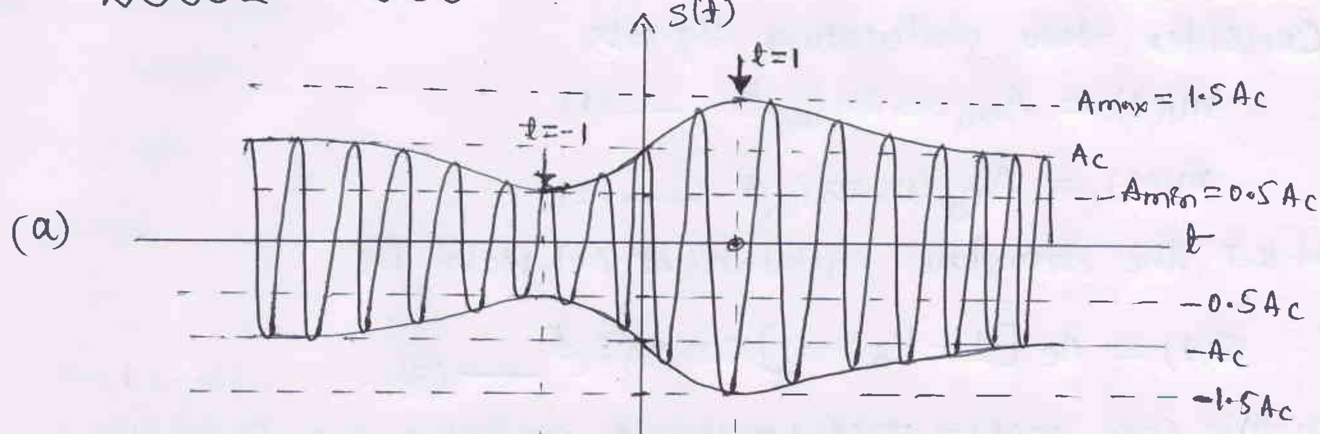
To Sketch $s(t)$:-

$A_{max} = A_c[1 + K_a|m(t)|_{max}] = A_c[1 + 2.5\times 0.5] = 2.25A_c$

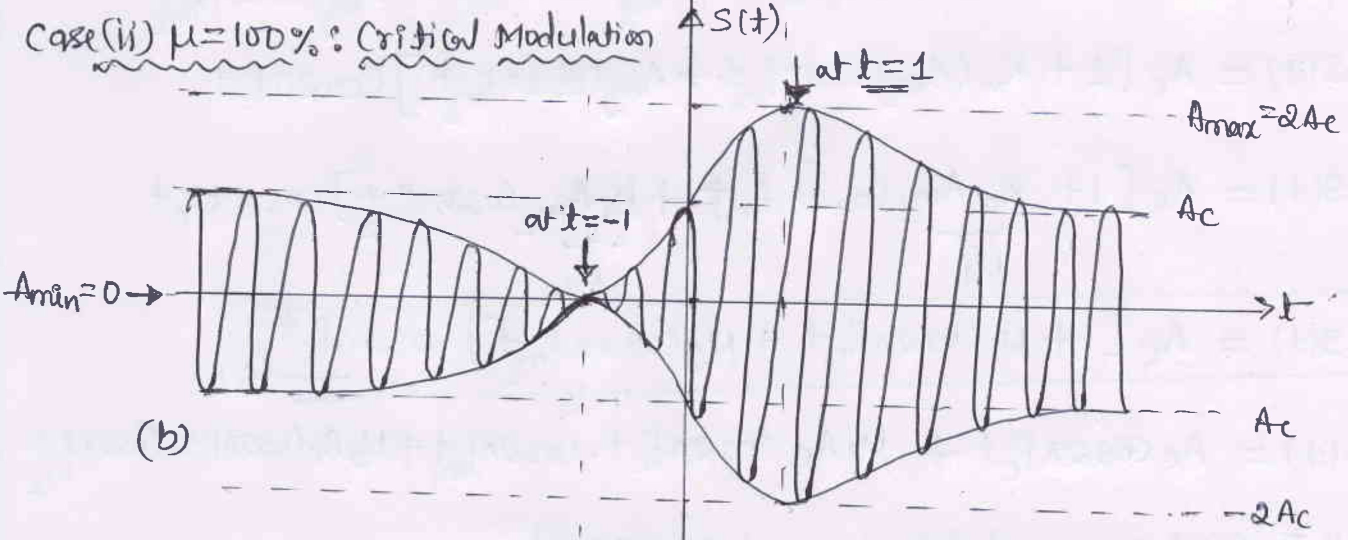
$A_{min} = A_c[1 + K_a|m(t)|_{min}] = A_c[1 - 2.25\times 0.5] = -0.25A_c$

The sketch of $s(t)$ for $\mu = 1.25$ is shown in figure 1.2(c) and is over Modulated.

Case(i) : $\mu = 50\%$: Under Modulation :-



Case(ii) $\mu = 100\%$: Critical Modulation



Case(iii): $\mu = 125\%$: Over Modulation :-

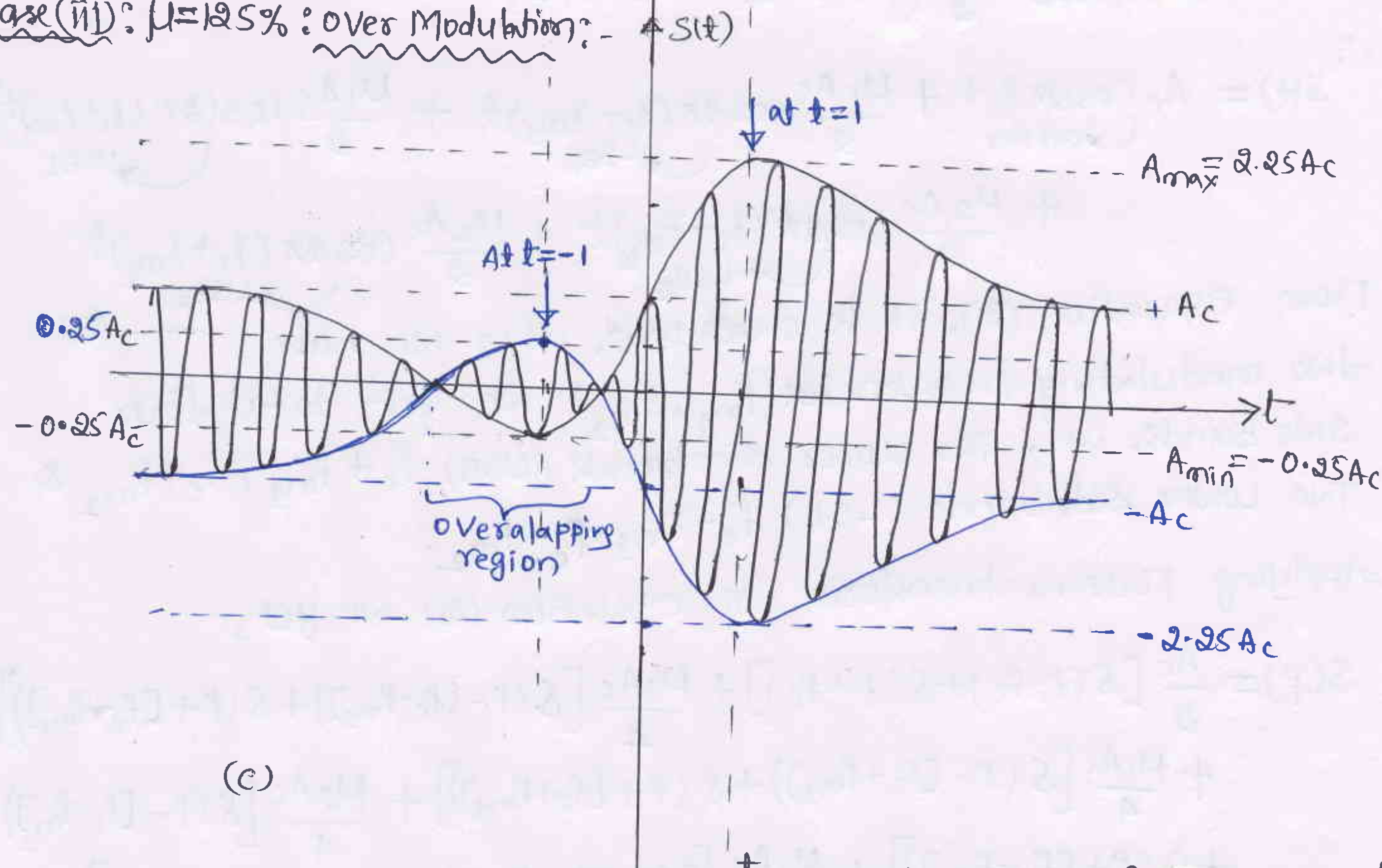


Figure 1.2: AM signal for $m(t) = \frac{t}{1+t^2}$ for (a) $\mu = 50\%$ (b) $\mu = 100\%$ and (c) $\mu = 125\%$

* Expression for Multitone Amplitude Modulation:-

Consider two modulating signals

$$m_1(t) = A_{m_1} \cos 2\pi f_{m_1} t \quad \text{—— (1)}$$

$$m_2(t) = A_{m_2} \cos 2\pi f_{m_2} t \quad \text{—— (2)}$$

W.K.T the standard equation of AM-wave is

$$S(t) = A_c [1 + K_a m(t)] \cos 2\pi f_c t \quad \text{—— (3)}$$

In this case $m(t) = m_1(t) + m_2(t) = A_{m_1} \cos 2\pi f_{m_1} t + A_{m_2} \cos 2\pi f_{m_2} t$

$$\therefore S(t) = A_c [1 + K_a (A_{m_1} \cos 2\pi f_{m_1} t + A_{m_2} \cos 2\pi f_{m_2} t)] \cos 2\pi f_c t$$

$$S(t) = A_c [1 + \underbrace{K_a A_{m_1}}_{\mu_1} \cos 2\pi f_{m_1} t + \underbrace{K_a A_{m_2}}_{\mu_2} \cos 2\pi f_{m_2} t] \cos 2\pi f_c t$$

$$\therefore \boxed{S(t) = A_c [1 + \mu_1 \cos 2\pi f_{m_1} t + \mu_2 \cos 2\pi f_{m_2} t] \cos 2\pi f_c t} \quad \text{—— (4)}$$

$$S(t) = A_c \cos 2\pi f_c t + \mu_1 A_c \cos 2\pi f_c t \cdot \cos 2\pi f_{m_1} t + \mu_2 A_c \cos 2\pi f_c t \cdot \cos 2\pi f_{m_2} t$$

W.K.T. $\cos A \cdot \cos B = \frac{1}{2} \{\cos(A-B) + \cos(A+B)\}$

$$\therefore S(t) = \underbrace{A_c \cos 2\pi f_c t}_{\text{carrier}} + \frac{\mu_1 A_c}{2} \underbrace{\cos 2\pi (f_c - f_{m_1}) t}_{\text{LSB1}} + \frac{\mu_1 A_c}{2} \underbrace{\cos(2\pi (f_c + f_{m_1}) t)}_{\text{USB1}}$$

$$+ \frac{\mu_2 A_c}{2} \underbrace{\cos 2\pi (f_c - f_{m_2}) t}_{\text{LSB}_2} + \frac{\mu_2 A_c}{2} \underbrace{\cos 2\pi (f_c + f_{m_2}) t}_{\text{USB}_2} \quad \text{—— (5)}$$

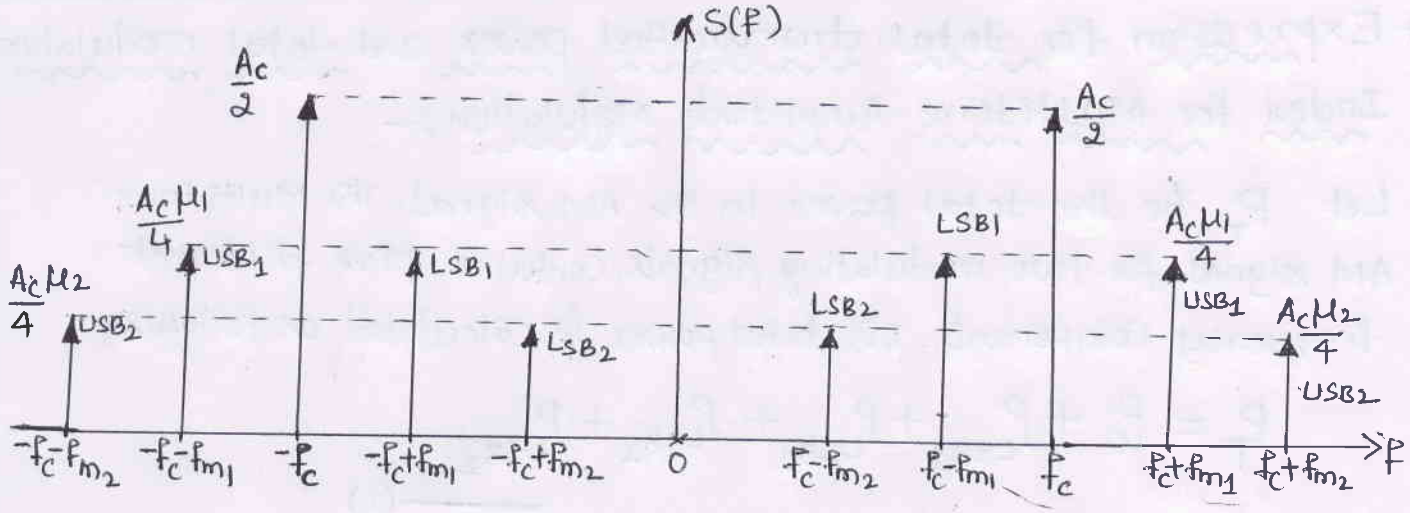
From equation (5), it is clear that, when we have two modulating frequencies (f_{m_1}, f_{m_2}) we get total four side bands. i.e., Two upper sidebands (USB) $f_c + f_{m_1}$, $f_c + f_{m_2}$ & Two Lower sidebands (LSB) $f_c - f_{m_1}$, $f_c - f_{m_2}$.

Applying Fourier transform to equation (5) we get,

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{\mu_1 A_c}{4}[\delta(f - [f_c - f_{m_1}]) + \delta(f + [f_c - f_{m_1}])]$$

$$+ \frac{\mu_1 A_c}{4}[\delta(f - [f_c + f_{m_1}]) + \delta(f + [f_c + f_{m_1}])] + \frac{\mu_2 A_c}{4}[\delta(f - [f_c - f_{m_2}])$$

$$+ \delta(f + [f_c - f_{m_2}])] + \frac{\mu_2 A_c}{4}[\delta(f - [f_c + f_{m_2}]) + \delta(f + [f_c + f_{m_2}])] \quad \text{—— (6)}$$



— : Spectrum for Multitone Amplitude Modulation :—

Total Band width :- The total transmission Band width for Multitone Amplitude modulated wave is

$$\boxed{BW_T = 2W} \text{ Hz.}$$

Where $W = \text{Maximum}\,(f_{m_1}, f_{m_2}, \ldots, f_{m_N})$

Note: * In Multitone Amplitude Modulated Wave shown in equation (5) has five different frequencies as follows

- $f_c \longrightarrow$ carrier signal with amplitude 'A_c'.
- $f_c - f_{m_1} \rightarrow LSB_1$ with amplitude $\Big\}\ \dfrac{\mu_1 A_c}{2}$
- $f_c + f_{m_1} \rightarrow USB1$ with amplitude
- $f_c - f_{m_2} \longrightarrow LSB_2$ with amplitude $\Big\}\ \dfrac{\mu_2 A_c}{2}$
- $f_c + f_{m_2} \longrightarrow USB_2$ with amplitude

* power present in any alternating signal (voltage signal) is given by

$$P = \frac{(V_{RMS})^2}{R} = \frac{(V_M/\sqrt{2})^2}{R} = \frac{V_m^2}{2R} = \frac{(\text{Amplitude})^2}{2R}$$

Where R = Load Resistance in Ω,

* Expression for total transmitted power and total modulation Index for Multitone Amplitude Modulation :-

Let P_T, be the total power in the AM-signal. The Multitone AM signal for two modulating signals contains five different frequency components. Its total power is calculated as follows,

$$P_T = P_c + P_{LSB1} + P_{USB1} + P_{LSB_2} + P_{USB_2} \quad \text{——(1)}$$

$$P_c = \frac{A_c^2}{2R}$$

$$P_{LSB_1} = P_{USB_1} = \frac{\left(\frac{\mu_1 A_c}{2}\right)^2}{2R} = \frac{\mu_1^2 A_c^2}{8R} = \frac{A_c^2}{2R}\left(\frac{\mu_1^2}{4}\right) = P_c\frac{\mu_1^2}{4}$$

$$P_{LSB_2} = P_{USB_2} = \frac{\left(\frac{\mu_2 A_c}{2}\right)^2}{2R} = \frac{\mu_2^2 A_c^2}{8R} = \frac{A_c^2}{2R}\left(\frac{\mu_2^2}{4}\right) = P_c\frac{\mu_2^2}{4}$$

$$\therefore \quad P_T = P_c + P_c\frac{\mu_1^2}{4} + P_c\frac{\mu_1^2}{4} + P_c\frac{\mu_2^2}{4} + \frac{P_c\mu_2^2}{4}$$

$$P_T = P_c\left[1 + \frac{\mu_1^2}{4} + \frac{\mu_1^2}{4} + \frac{\mu_2^2}{4} + \frac{\mu_2^2}{4}\right]$$

$$P_T = P_c\left[1 + \frac{\mu_1^2}{2} + \frac{\mu_2^2}{2}\right] = P_c\left[1 + \frac{(\mu_1^2 + \mu_2^2)}{2}\right]$$

$$\boxed{P_T = P_c\left[1 + \frac{\mu_t^2}{2}\right]} \Longrightarrow \text{Total power in AM signal}$$

Where $\mu_t^2 = \mu_1^2 + \mu_2^2 = N$

$$\therefore \quad \boxed{\mu_t = \sqrt{\mu_1^2 + \mu_2^2}} \Longrightarrow \text{Net Modulation Index}$$

In General, Net Modulation index for N-message signals is given by

$$\boxed{\mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2 + \cdots + \mu_N^2}}$$

Note: For Single Tone AM signal,

- Total power, $P_T = P_c\left(1+\frac{\mu^2}{2}\right)$

- Total Side band power : $P_{SB} = P_{LSB} + P_{USB} = \frac{P_c\mu^2}{4} + \frac{P_c\mu^2}{4}$

$$\therefore \boxed{P_{SB} = P_c\frac{\mu^2}{2}} \leftarrow \text{Total power in side bands}$$

GATE

* **Example 1.3 :** Determine the ratio of Maximum average ~~total~~ power to unmodulated carrier power in AM-signal.

→ W.K.T. total power in AM signal is given by

$$P_T = P_c\left[1+\frac{\mu^2}{2}\right] \quad \text{—— (1)}$$

Where, P_c = unmodulated carrier power

μ = Modulation Index. Varies from 0 to 1

$\therefore$ P_T becomes Maximum at $\mu = 1$

$$\therefore \quad P_{T_{max}} = P_c\left[1+\frac{\mu^2}{2}\right]\Bigg|_{\mu=1}$$

$$\therefore \boxed{\frac{P_{T_{max}}}{P_c} = \left(1+\frac{1^2}{2}\right) = 1+\frac{1}{2} = \frac{3}{2} = 1.5}$$

$\therefore$ The ratio of Maximum transmitted power to that of unmodulated Carrier power in AM signal is <u>1.5</u>.

Note: • The Maximum power radiated from an AM-broadcasting station is $\boxed{P_{t_{max}} = 1.5P_c}$

- In AM-signal
  - ↳ carrier power does not Contains any message signal.
    - $\therefore$ The presence of Carrier power 'P_c' in total power P_t, reduces the Efficiency of AM-signal.
  - ↳ LSB & USB Carries equal power "$P_c\mu^2/4$".

* Efficiency of AM signal: (η)

It is the ratio of total side band power to that of total transmitted power.

i.e., $$\eta = \frac{P_{SB}}{P_t} = \frac{P_{LSB} + P_{USB}}{P_t}$$

W.K.T. $$P_t = P_c\left[1 + \frac{\mu^2}{2}\right]$$

$$P_{SB} = P_{LSB} + P_{USB} = P_c \frac{\mu^2}{2}$$

$$\therefore \eta = \frac{P_{SB}}{P_t} = \frac{P_c \frac{\mu^2}{2}}{P_c\left(1 + \frac{\mu^2}{2}\right)} = \frac{\frac{\mu^2}{2}}{\frac{2+\mu^2}{2}} = \frac{\mu^2}{2+\mu^2}$$

$$\therefore \boxed{\%\eta = \frac{\mu^2}{2+\mu^2} \times 100}$$

Note: * the Maximum Efficiency of AM signal is

$$\eta_{max} = \left.\frac{\mu^2}{2+\mu^2} \times 100\right|_{\mu=1} = \frac{1^2}{2+1} \times 100 = \frac{1}{3} \times 100$$

$$\therefore \boxed{\eta_{max} = 33.33\%}$$

***
1.3: SWITCHING MODULATOR :-

It is a Diode circuit used to generate AM-Signal.

Q) Explain the operation of switching Modulator with circuit diagram and waveforms. Dec/Jan 2017 2016

↳ Switching Modulator is used to generate AM Signal.

Circuit diagram :-

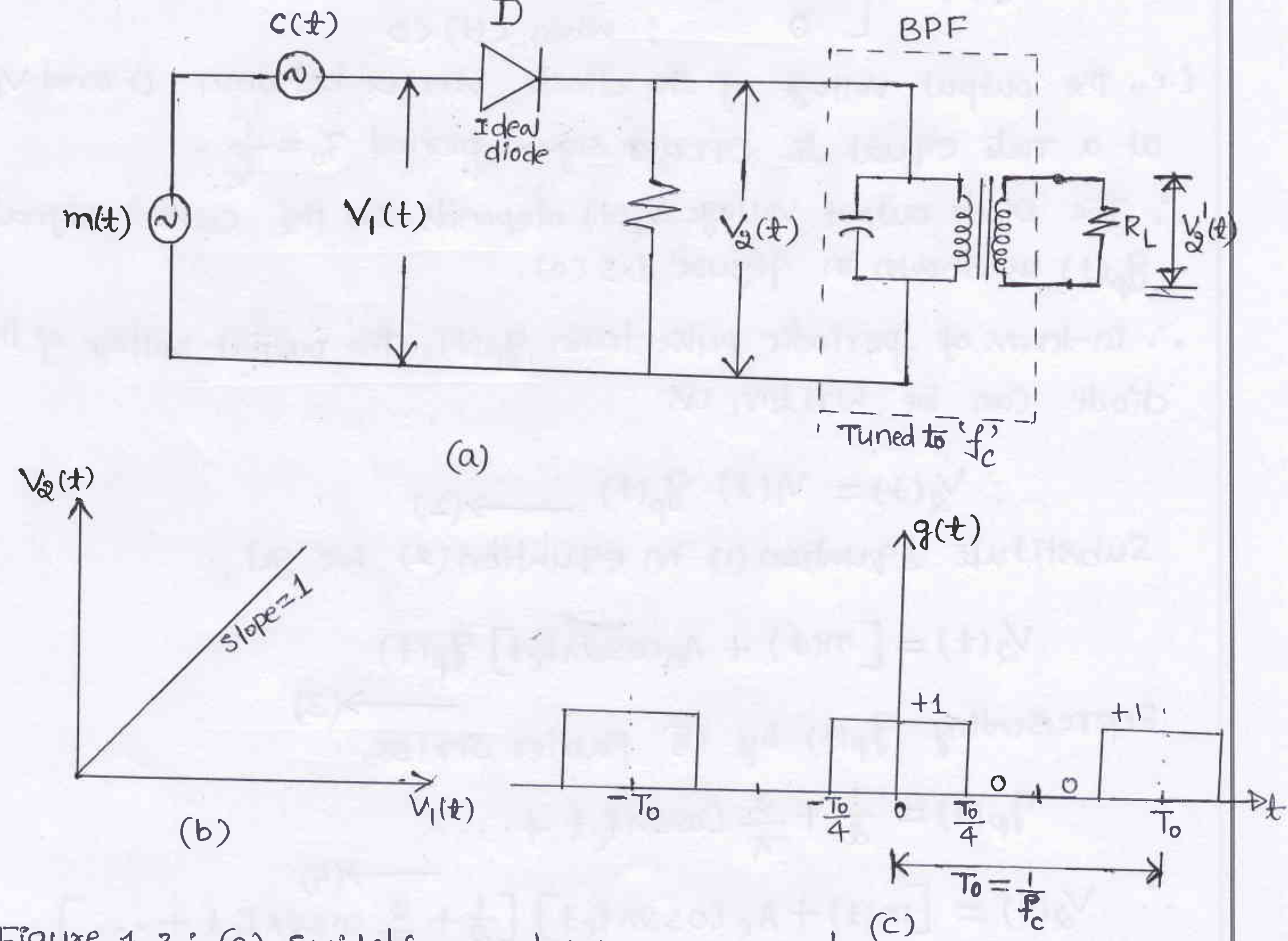


Figure 1.3 : (a) Switching Modulator circuit diagram
(b) Idealized input ($V_1(t)$) and output $V_2(t)$ relation of Diode
(c) periodic pulse-train of C(t).

Explanation :- Switching modulator consists of an Ideal diode which is used as a switch, followed by Band pass Filter (BPF) tuned to frequency 'f_c' as shown in figure 1.3 (a).

↳ Message signal m(t) and Carrier signal c(t) are simultaneously applied as input signal for ideal diode 'D', as shown in figure. 1.3(a).

∴ The ~~total~~ input '$V_1(t)$' to the diode is given by

$$V_1(t) = m(t) + c(t)$$

$$\therefore \boxed{V_1(t) = m(t) + A_c \cos 2\pi f_c t} \longrightarrow (1)$$

It is assumed that $|m(t)| \ll A_c$. Therefore ON & OFF of Diode 'D' is Controlled by $c(t)$.

∴ the output voltage of Diode 'D' is,

$$V_2(t) = \begin{cases} V_1(t) & ; \text{ when } c(t) > 0 \Rightarrow \text{shown in figure 1.3(b)} \\ 0 & ; \text{ when } c(t) < 0 \end{cases}$$

i.e., the output voltage of the diode varies between 0 and $V_1(t)$ at a rate equal to carrier signal period $T_0 = \frac{1}{f_c}$.

∴ The Diode output voltage $V_2(t)$ depends on the control signal $g_p(t)$ as shown in figure 1.3 (c).

∴ In terms of periodic pulse train $g_p(t)$, the output voltage of the diode can be written as

$$V_2(t) = V_1(t) \cdot g_p(t) \longrightarrow (2)$$

Substitute equation (1) in equation (2) we get,

$$V_2(t) = [m(t) + A_c \cos 2\pi f_c t]\, g_p(t) \longrightarrow (3)$$

Representing $g_p(t)$ by its Fourier Series,

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \cos 2\pi f_c t + \cdots \longrightarrow (4)$$

$$\therefore V_2(t) = [m(t) + A_c \cos 2\pi f_c t]\left[\frac{1}{2} + \frac{2}{\pi}\cos 2\pi f_c t + \cdots\right]$$

$$V_2(t) = \frac{1}{2} m(t) + \frac{2}{\pi} m(t) \cdot \cos 2\pi f_c t + \frac{A_c}{2} \cos 2\pi f_c t + \frac{2A_c}{4}\Big[\underbrace{\cos^2 2\pi f_c t}_{\left(\frac{1+\cos 2\theta}{2}\right)}\Big] + \cdots$$

$$V_2(t) = \frac{1}{2} m(t) + \frac{2}{\pi} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \cos 2\pi f_c t + \underbrace{\frac{A_c}{4}}_{(DC)} + \frac{A_c}{4} \cos 4\pi f_c t + \cdots \longrightarrow (5)$$

The required AM wave centered at 'f_c' is obtained by passing $V_2(t)$ through an ideal BPF having center frequency 'f_c' and $BW = 2f_m$ Hz

∴ The output of the BPF is

$$V_2'(t) = \frac{2}{\pi} m(t) \cos(2\pi f_c t) + \frac{A_c}{2} \cos 2\pi f_c t$$

$$V_2'(t) = \frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} \cdot m(t)\right] \cos 2\pi f_c t$$

$$V_2'(t) = \frac{A_c}{2}\left[1 + k_a m(t)\right] \cos 2\pi f_c t \quad \Leftarrow \text{AM-Wave.} \longrightarrow (6)$$

where $K_a = \frac{4}{\pi A_c}$ = Amplitude Sensitivity parameter

Equation (6) is the standard AM signal produced by the Switching Modulator with carrier amplitude scaled down to '$\frac{A_c}{2}$'

* * * * *

*** 1.4. ENVELOPE DETECTOR ***

Q) Explain the operation of envelope detector with neat diagrams and waveforms. Also mention the Significance of RC-time Constant.

June-July 2017

→ Demodulation (or) Detection is the process of recovering the original message signal from the modulated wave at the receiver.

Envelope Detector: It is a simple and highly effective diode circuit which is commonly used for demodulation of AM-signal.

Circuit diagram:-

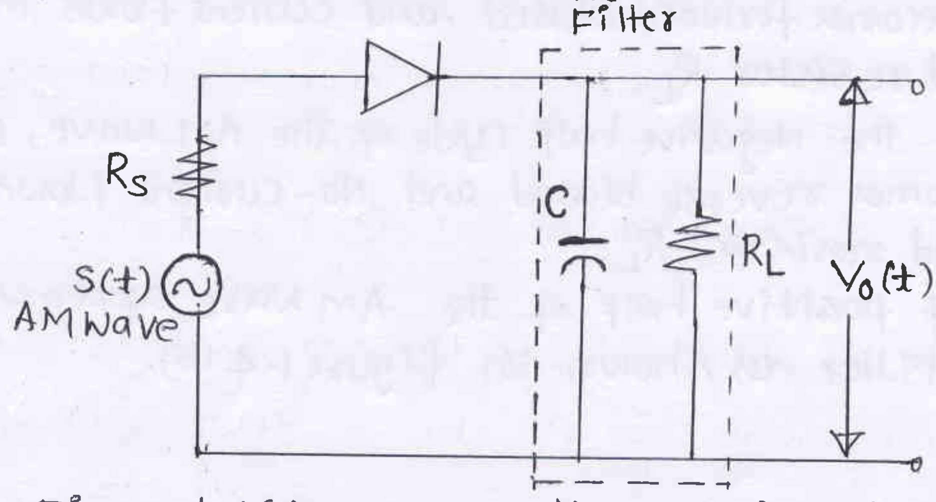


Figure 1.4(a): Circuit diagram of Envelope Detector:-

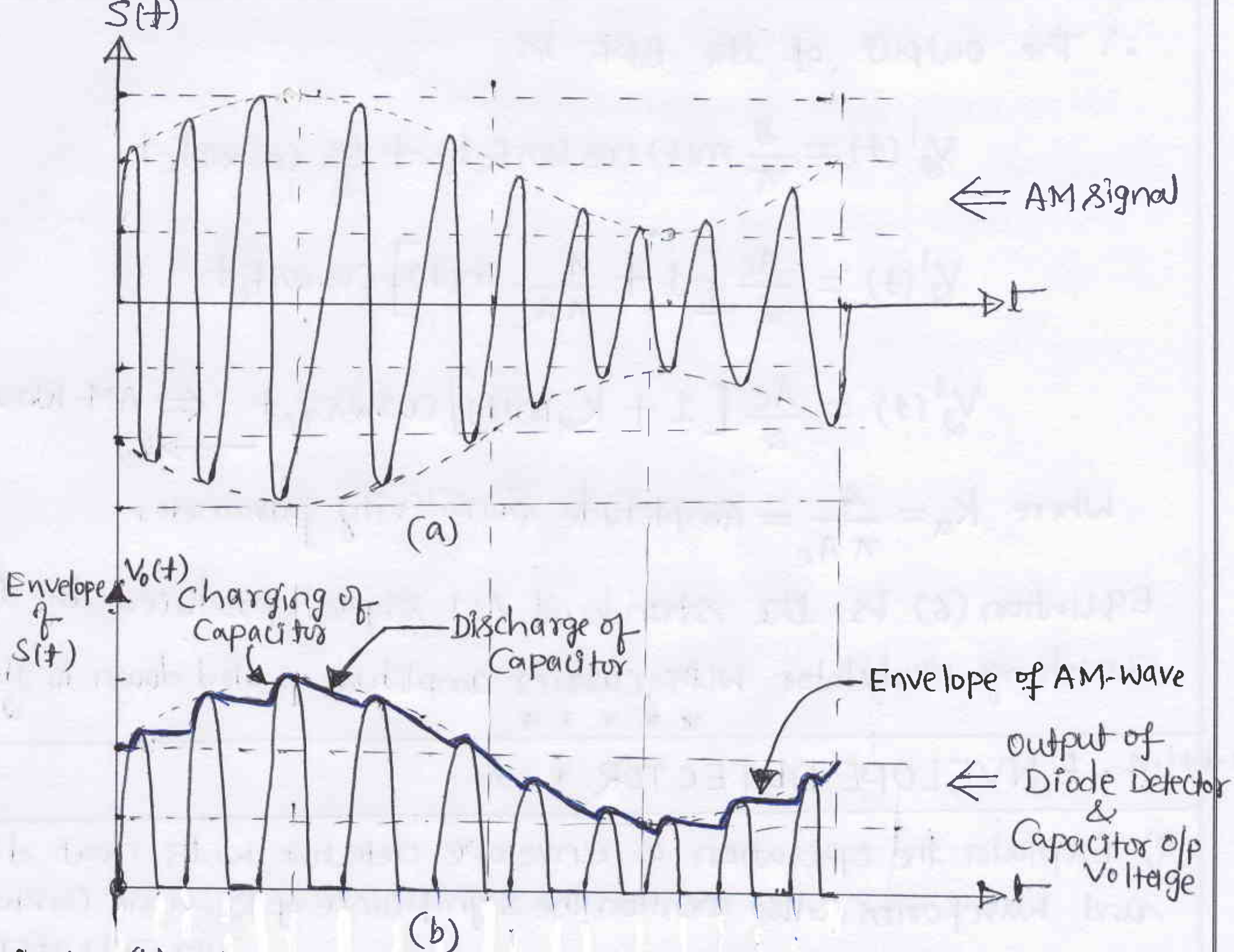


Figure 1.4(b): (a) AM Signal S(t), input to the Envelope Detector
(b) Envelope of AM Signal and Voltage across Capacitor

Figure 4 (a) shows the envelope detector circuit. It Consists of a diode and a RC-filter. This circuit is also known as "Diode Detector".

Circuit operation:-

↳ In the positive half cycle of the AM-Signal, Diode 'D' becomes forward biased and current flows through load resistor 'R_L'.

↳ In the Negative half cycle of the AM-Wave, Diode 'D' becomes reverse biased and No-current flows through load resistor 'R_L'.

∴ Only positive half of the AM wave appears across RC-Filter as shown in figure 1.4 (b).

Working of RC-Filter:-

↳ During the +ve half cycle of AM wave, the capacitor 'C' charges up rapidly towards the peak value of the input signal. When the input signal falls below this value, the Diode becomes Reverse biased and the Capacitor 'C' discharges slowly through the load resistor 'R_L'.

↳ The Discharging process Continues until the next positive half cycle of AM-wave. When the input signal becomes greater than the voltage across capacitor, the diode starts conducting again and the process is repeated.

↳ This Continuous process of charging and Discharging of Capacitor, gives the Envelope of AM signal as shown in figure 1.4(b). which is in same shape as that of message signal.

Selection of RC Constant:-

* The charging time constant '$R_S C$' must be very much less than the carrier period '$1/f_c$'.

$$\therefore \quad R_S C \ll \frac{1}{f_c} \; ; \Longrightarrow$$ To ensure Capacitor Charges up rapidly.

* The Discharging Time Constant '$R_L C$' should be long enough to ensure that the capacitor discharges slowly through the load resistor 'R_L' between positive peaks of the Carrier Wave.

i.e.,
$$\boxed{\frac{1}{f_c} \ll R_L C \ll \frac{1}{f_m}} \Longrightarrow$$ To ensure Slow Discharge of Capacitor

——— * ** ——— *** ———

* Advantages of Amplitude Modulation :-

1. Low Bandwidth
2. AM - Waves can travel longer distance.
   i.e., AM waves covers large area
3. AM transmitters are less complex
4. AM receivers are simple, Detection is easy and Cost Efficient.

** Disadvantages (or) Limitations of AM :

1. power is wasted in the transmitted signal.
2. AM needs Larger Bandwidth ($BW_T = 2W$)
3. AM-signal gets affected due to Noise.

* Applications of AM :

The Major applications of AM are

↳ Radio broadcasting.

↳ picture Transmission in a TV-system.
(Television Broadcasting).

DOUBLE SIDE BAND SUPPRESSED CARRIER (DSBSC) MODULATION

↳ To overcome the drawback of power wastage in AM wave DSBSC-Modulation is used.

↳ DSBSC is a method of transmission of message signal, where only two side bands are transmitted without the carrier signal

↳ Amplitude Modulated wave in which the carrier is suppressed is called "DSBSC-Modulation"

1.5. Time and Frequency domain description of DSBSC-signal:

Let $m(t) = A_m \cos 2\pi f_m t$ —— (1) : Modulating signal and

$c(t) = A_c \cos 2\pi f_c t$ —— (2) : Carrier signal.

Then the Time domain Expression for DSBSC-signal is given by,

$$S(t) = m(t) \cdot c(t)$$

$$\boxed{S(t) = A_c \cos 2\pi f_c t \cdot m(t)} \quad \text{—— (3)}$$

Equation (3) is the general expression of DSBSC-signal for any message signal m(t).

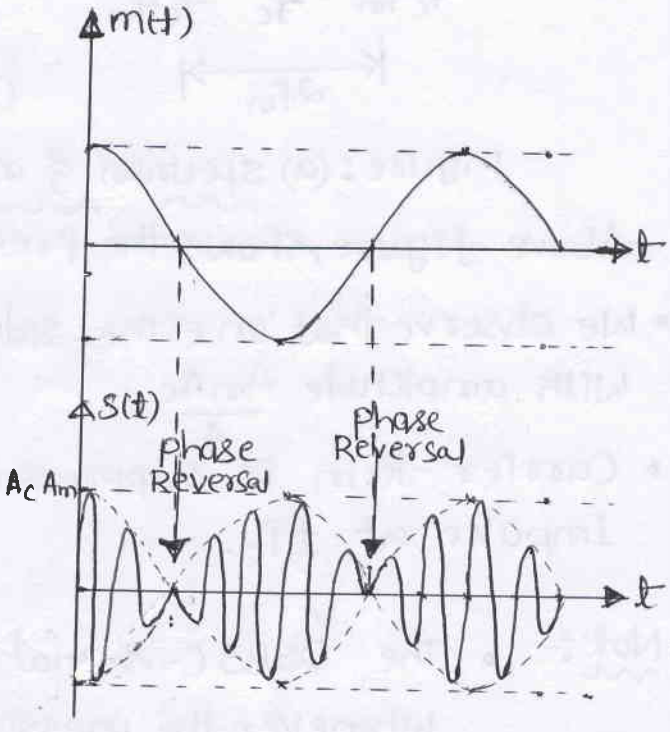


Figure 1.5: DSBSC Modulated signal

for $m(t) = A_m \cos 2\pi f_m t$

$$S(t) = A_c \cos 2\pi f_c t \times A_m \cos 2\pi f_m t$$

$$S(t) = A_m A_c \cos 2\pi f_c t \cdot \cos 2\pi f_m t$$

W.K.T. $\cos A \cdot \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

$$\therefore S(t) = \frac{A_m A_c}{2}\left[\cos 2\pi (f_c - f_m)t + \cos 2\pi (f_c + f_m)t\right] \longrightarrow (4)$$

Take Fourier Transform on both sides of equation (4), we get

$$S(f) = \frac{A_m A_c}{4}\left[\delta(f-[f_c-f_m]) + \delta(f+[f_c-f_m])\right] + \frac{A_m A_c}{4}\left[\delta(f-[f_c+f_m]) + \delta(f+[f_c+f_m])\right] \longrightarrow (5)$$

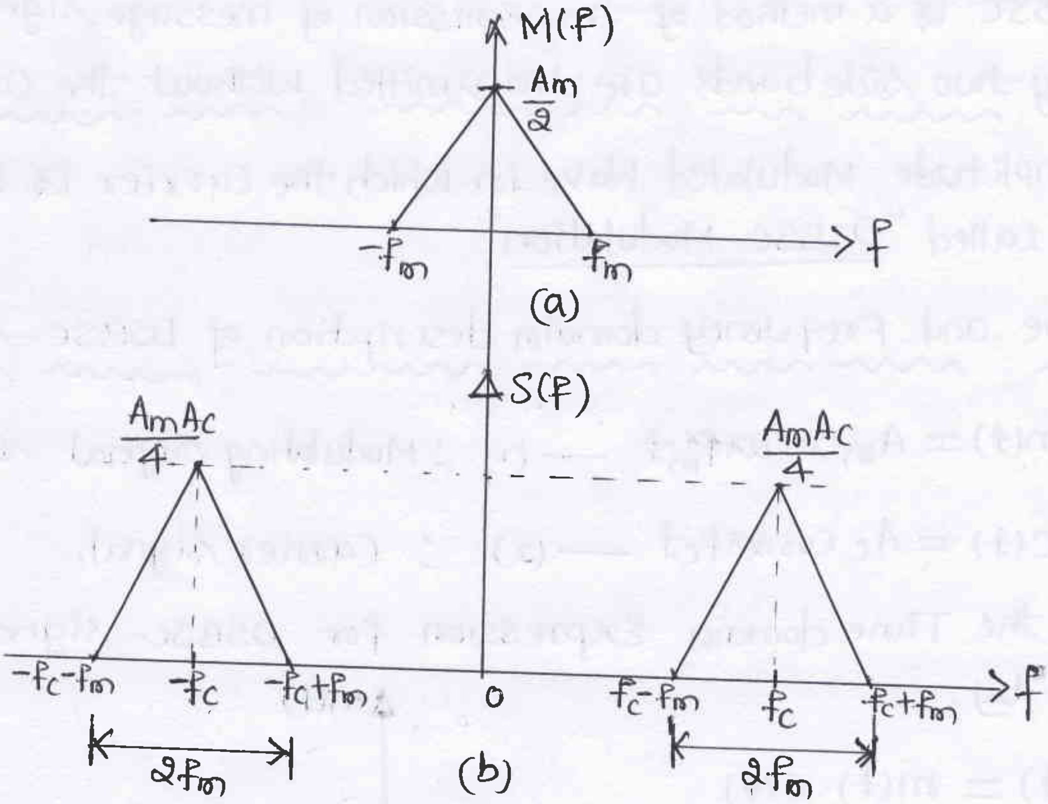


Figure: (a) spectrum of m(t) (b) spectrum of DSBSC signal

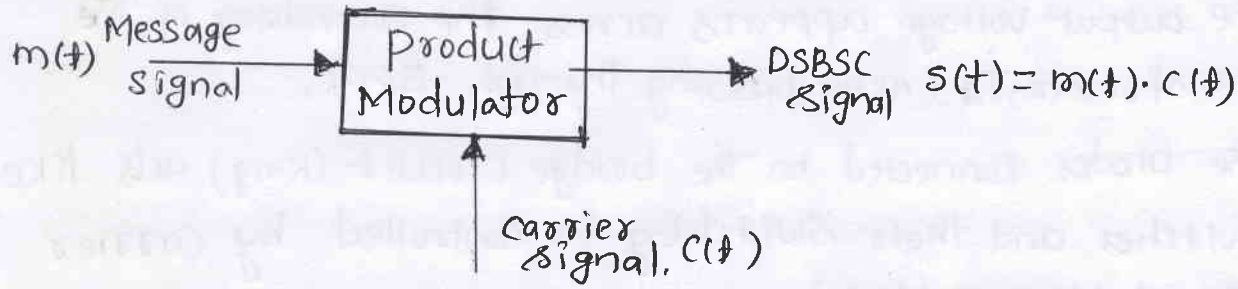
Above figure shows the frequency spectrum of a DSBSC signal.

- We observe that on either side of $\pm f_c$, we have LSB and USB with amplitude $\frac{A_m A_c}{4}$.
- Carrier term is suppressed in the spectrum as there is no Impulse at $\pm f_c$.

Note:-
- The DSBSC-signal $s(t)$, undergoes a phase Reversal whenever the message signal crosses zero, as shown in figure 5.1.
- A DSBSC signal can be generated by a Multiplier (or also called product Modulator)

Generation of DSB-SC Wave:-

The devices used to generate DSBSC waves are known as the product Modulators.



The most commonly used product Modulator to generate DSBSC signal is "Ring Modulator."

1.6. RING MODULATOR ***

Q) Explain the generation of DSBSC wave using Ring Modulator and also sketch the necessary waveforms.

↳ Ring Modulator is a product Modulator used for Generating DSBSC-Modulated signal.

Circuit diagram:

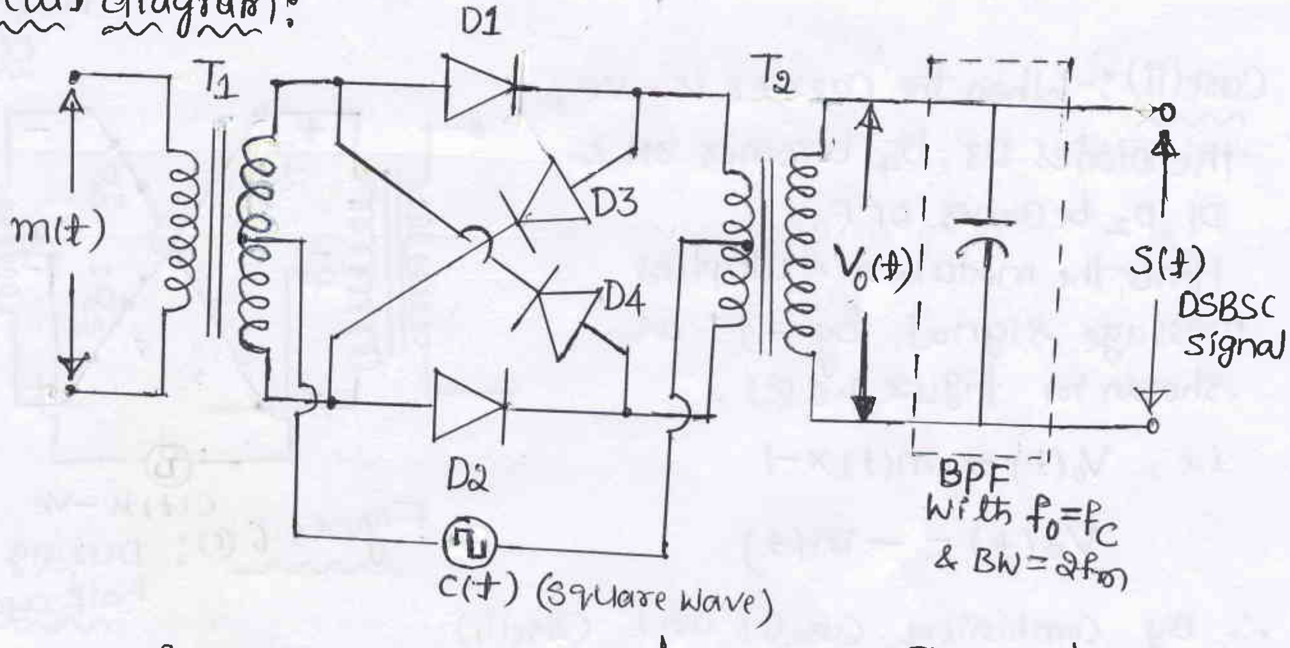


Figure 1.6(a): circuit diagram of Ring Modulator

↳ The circuit diagram of Ring modulator is shown in figure 1.6(a) consists of two center-tapped transforms T1, T2 and Four diodes D1, D2, D3 and D4 connected in bridge circuit and a BPF with center frequency 'f_c', BW=$2f_m$.

↳ The carrier signal is applied to the Center taps of the input (T_1) and output (T_2) transformers. Modulating signal is applied to the input transformer T_1.

↳ The output voltage appears across the secondary of the transformer, T_2 (After passing through BPF).

↳ The Diodes connected in the bridge circuit (Ring) acts like switches and their switching is controlled by carrier signal (square wave).

Circuit operation:-

Case(i): When the Carrier is +ve, the Diodes D1, D2 becomes ON & Diodes D3, D4 becomes OFF.
Hence the Modulator Multiplies message signal m(t) by +1.

i.e., $V_0(t) = m(t) \times (+1) = m(t)$

Equivalent circuit is shown in Figure 1.6(b)

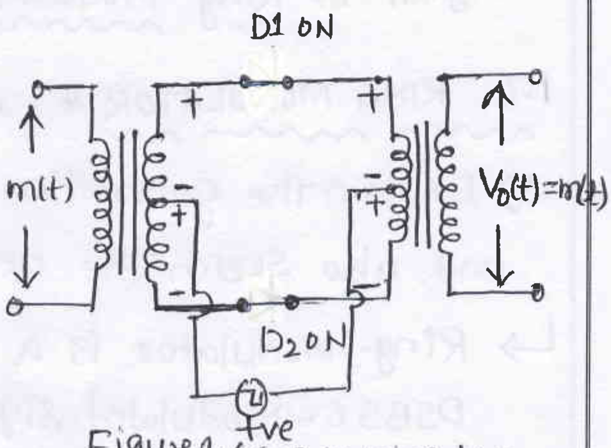


Figure 1.6(b): During +ve half cycle of C(t)

Case(ii):- When the Carrier is −ve, the Diodes D3, D4 becomes ON & D1, D2 becomes OFF.
Hence the modulator multiplies message signal by "−1" as shown in figure 1.6(c).

i.e., $V_0(t) = m(t) \times -1$

$V_0(t) = -m(t)$

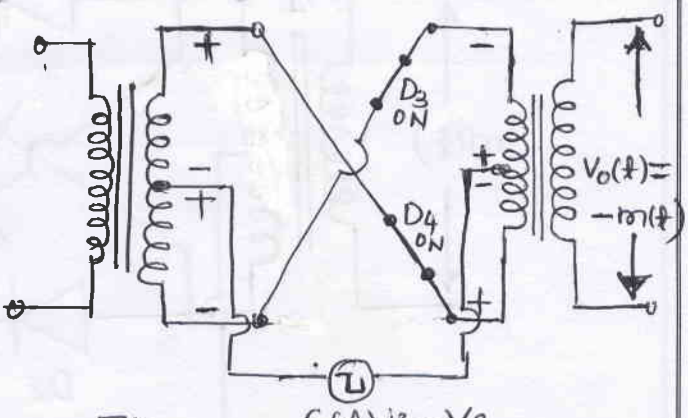


Figure 1.6 (c): During −ve half cycle of C(t)

∴ By combining Case(i) and Case(ii)
The Ring Modulator output at the Secondary of transformer T_2 is given by

$$V_0(t) = m(t) \times c(t) \quad \text{——(1)}$$

The square wave Carrier $c(t)$ can be represented by a Fourier Series as:

$$c(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c t(2n-1)]$$

$$\therefore c(t) = \frac{4}{\pi}\left[\cos 2\pi f_c t - \frac{1}{3}\cos 6\pi f_c t + \cdots\right] \quad \text{—— (2)}$$

$\therefore$ Substitute equation (2) in $V_o(t)$ equation (1) we get

$$V_o(t) = m(t) \times \frac{4}{\pi}\left[\cos 2\pi f_c t - \frac{1}{3}\cos 6\pi f_c t + \cdots\right] \quad \text{—— (3)}$$

When $V_o(t)$ is passed through BPF having Center frequency 'f_c' and Bandwidth '$2f_m$' we get DSBSC Signal,

$$\boxed{S(t) = \frac{4}{\pi} m(t) \cos 2\pi f_c t}$$ ⟸ DSBSC wave generated from RING Modulator

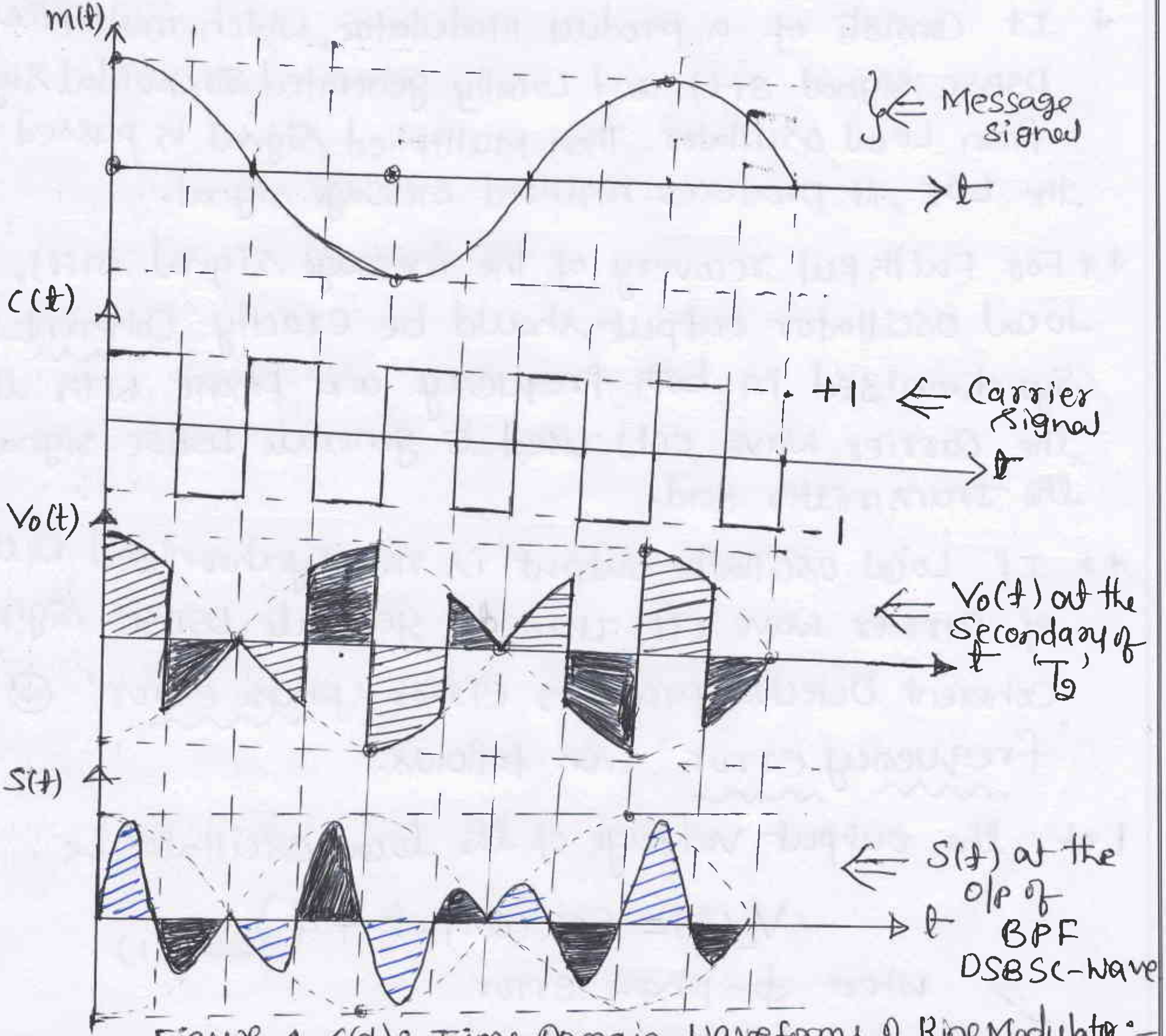


Figure 1.6(d): Time Domain Waveforms of Ring Modulator:-

1.7. Coherent Detection :-

Q) With relevant diagram Explain the operation of the coherent detection of DSBSC Modulated Waves. Also Explain phase error and frequency error.

↳

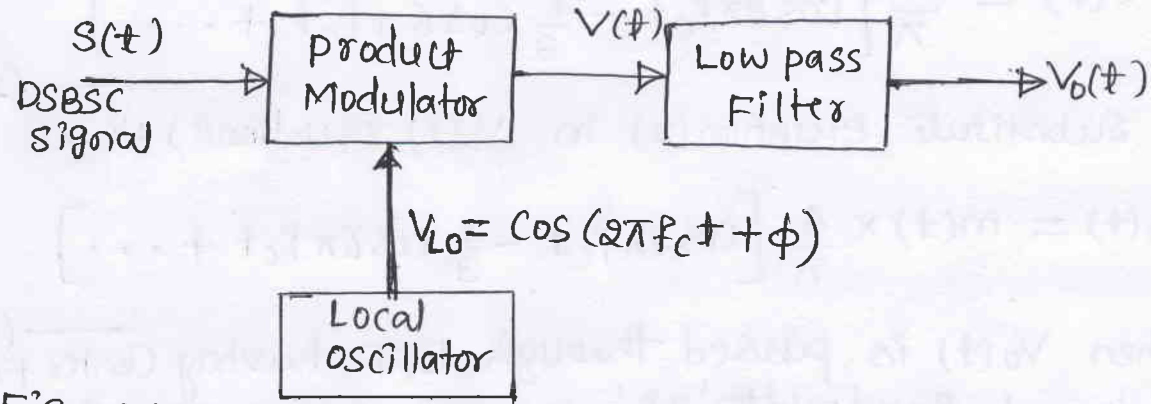


Figure 1.7: Coherent Detector for DSBSC Signal

* The Modulating signal m(t) is recovered from a DSBSC Wave S(t) by using Coherent Detector shown in figure 1.7.

* It Consists of a product Modulator, which multiplies DSBSC Signal S(t) and Locally generated Sinusoidal Signal from Local oscillator. Then Multiplied Signal is passed through the LPF, it produces required message signal.

** For Faithful recovery of the message signal m(t), the local oscillator output should be exactly Coherent (or) Synchronized in both frequency and phase with that of the Carrier Wave c(t) used to generate DSBSC signal at the transmitter end.

** If Local oscillator output is not synchronized with that of carrier wave c(t) used to generate DSBSC-signal, Coherent Detector produces either 'phase error' (or) 'frequency error' as follows.

Let the output voltage of the local oscillator is

$$V_L(t) = \cos(2\pi f_c t + \phi) \quad \text{---(1)}$$

where ϕ = phase error

↳ Then the product Modulator output,

$$V(t) = S(t) \times V_L(t) \longrightarrow (2)$$

W.K.T, DSBSC-signal $S(t) = A_C\, m(t) \cos 2\pi f_c t$

$$\therefore V(t) = A_C\, m(t) \underbrace{\cos 2\pi f_c t}_{B} \times \underbrace{\cos(2\pi f_c t + \phi)}_{A}$$

$$\therefore V(t) = \frac{A_C\, m(t)}{2}\left[\cos(2\pi f_c t + \phi - 2\pi f_c t) + \cos(2\pi f_c t + \phi + 2\pi f_c t)\right]$$

$$\therefore V(t) = \frac{A_c}{2} \cdot m(t)\left[\cos\phi + \cos(4\pi f_c t + \phi)\right] \longrightarrow (3)$$

When $V(t)$ is passed through LPF having Bandwidth "$\pm f_m$" we get

$$\boxed{V_0(t) = \frac{A_c}{2} \cdot \cos\phi \cdot m(t)}$$ output of LPF

$\therefore$ The Demodulated signal $V_0(t)$ is proportional to $m(t)$.

and $\cos\phi \Rightarrow$ phase error.

* When $\phi = 0^\circ$: $V_0(t) = \frac{A_c}{2} \cdot m(t) \Rightarrow$ output Voltage is Maximum

* When $\phi = 90^\circ$: $V_0(t) = 0$ ($\because \cos 90^\circ = 0$). Then the output Voltage $V_0(t)$ is minimum (zero). This effect is called Quadrature Null Effect of the Coherent Detector. —— ** —— ** ——

Note:-

↳ Similarly if frequencies are not synchronized, i.e., Local oscillator frequency $f_c' \neq$ carrier frequency 'f_c'. then the frequency difference is $\boxed{(f_c' - f_c) = \Delta f}$

↳ The presence of frequency difference $\boxed{\Delta f = f_c' - f_c}$, Results in shift in the modulating signal frequency.

i.e., frequency of the output voltage signal of Coherent Detector is : $f_0 = f_m + \Delta f \rightarrow$ upward shift ($f_c' < f_c$) if Δf is Negative

$f_0 = f_m - \Delta f \rightarrow$ Down ward shift ($f_c' > f_c$) (if Δf is positive).

Problems on Coherent Detector:-

** Example 1.4: Consider a Message Signal m(t) Containing frequency Components at 100, 200 and 400 Hz. This signal is applied to an SSB Modulator together with a carrier at 100 KHz, With only the upper side band retained. In the Coherent detector used to recover m(t), The Local oscillator supplies a sine wave of frequency 100.02 KHz.

** June/July-2017

(i) Determine the frequency Components of the detector output

(ii) Repeat the analysis Assuming that only the lower side band is transmitted.

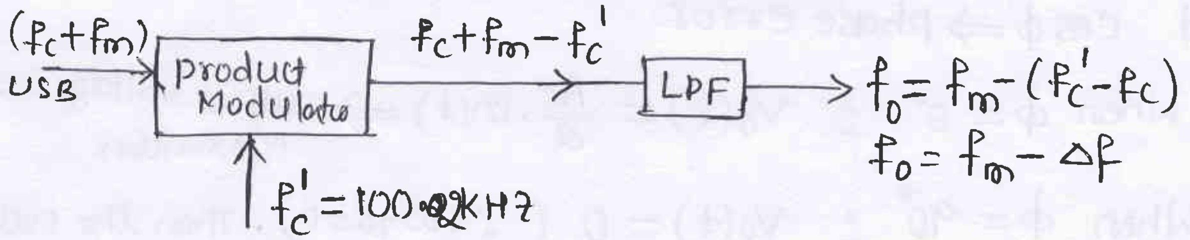
(4 Marks)

July 2016 (8 Marks)

↳ Given data: carrier frequency used at SSB Modulator $\Rightarrow f_C = 100\,KHz$

frequency components of $m(t) \Rightarrow$ 100 HZ, 200 HZ & 400 HZ

Case(i): When the upper side band is retained:-



∴ output frequencies of Coherent Detector are down shifted by

$\Delta f = f_c' - f_c = 100.02\,KHz - 100\,KHz = 20\,Hz.$

∴ output frequencies are $(\underset{f_{m_1}}{100} - \underset{\Delta f}{20}) = \underline{80\,Hz}$, $\underset{f_{m_2}}{200} - \underset{\Delta f}{20} = \underline{180\,Hz}$ & $\underset{f_{m_3}}{400} - \underset{\Delta f}{20} = \underline{380\,Hz}$

Case(ii):- When lower side band is transmitted, output frequencies of Coherent detector is $\boxed{f_0 = f_m + \Delta f}$. i.e., all the message signal frequencies are shifted upward by $\Delta f = 20\,Hz$.

∴ Frequency Components of the detector output are $\underline{120\,Hz}$, $\underline{220\,Hz}$ and $\underline{420\,Hz}$.

****

1.8. Costas Receiver :-

Q) Explain how Costas receiver can be used for demodulating DSB-SC signal. June/July - 2017 (6M)

↳ The costas receiver is a practical synchronous receiver system, suitable for demodulating DSBSC-Waves. It is also named as Costas loop ⓞ practical synchronous receiving system.

Block diagram :-

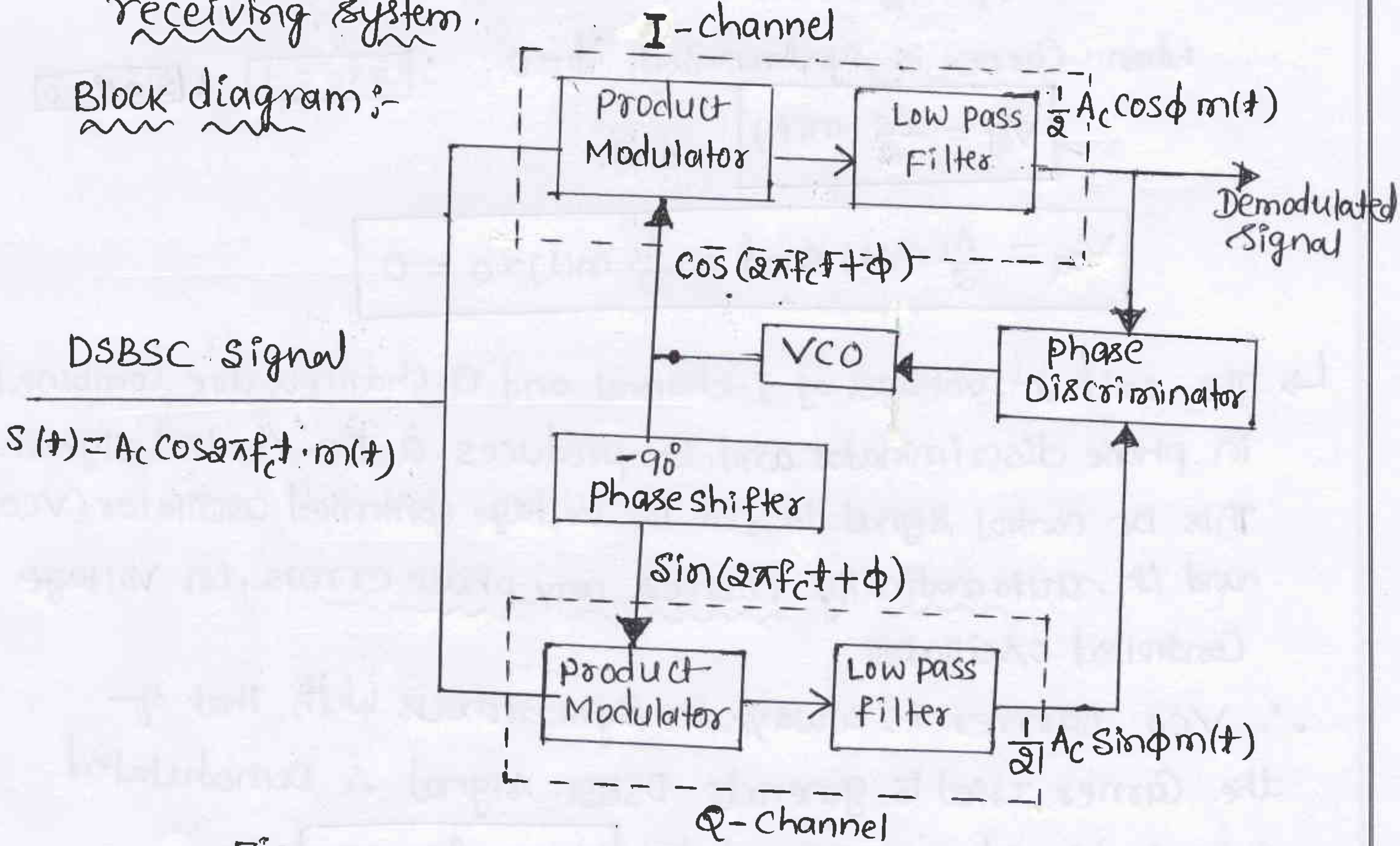


Figure 1.8 : Costas receiver ⓞ Costas loop

↳ The Costas receiver consists of two coherent detectors supplied with same input signal (DSBSC-Wave $s(t)$) but with individual local oscillator signals that are phase Quadrature with respect to each other. (i.e., the local oscillator signals supplied to the product modulators are 90° out of phase).

↳ The coherent detector in the upper path is referred to as the In-phase detector [ⓞ I-channel] and that in the lower path is referred to as Quadrature-phase Detector [ⓞ Q-channel] as shown in figure 1.8.

Operation:

↳ When local oscillator signal is of the same phase and frequency as that of carrier wave $A_c \cos 2\pi f_c t$ used to generate the incoming DSBSC wave. Then the I-channel output contains the desired demodulating signal $m(t)$ and Q-channel output is zero.

W.K.T, $V_{OI} = \frac{A_c}{2} \cdot m(t) \cos\phi$

When carrier is synchronized $\phi = 0°$ $\therefore \boxed{\cos 0° = 1}$ & $\boxed{\sin 0° = 0}$

$$\boxed{V_{OI} = \frac{A_c}{2} \cdot m(t)}$$

and

$$\boxed{V_{OQ} = \frac{A_c}{2} m(t) \sin\phi = \frac{A_c}{2} m(t) \times 0 = 0}$$

↳ The output voltages of I-channel and Q-channels are combined in phase discriminator and it produces a DC-control signal. This DC control signal is fed to voltage controlled oscillator (VCO) and it automatically corrects any phase errors in voltage controlled oscillator.

$\therefore$ VCO carrier is always in synchronous with that of the carrier used to generate DSBSC-signal. $\therefore$ Demodulated output is always equal to $\boxed{V_{OI} = \frac{A_c}{2} m(t)}$

1.9. Quadrature Carrier Multiplexing:

⟨Q⟩ With relevant diagrams, explain the operation of the quadrature carrier multiplexing transmitter scheme and receiver scheme. Dec 2016/Jan 2017 8M.

↳ Quadrature Carrier Multiplexing is a technique in which we can transmit more number of signals (DSBSC-wave) within the same channel Bandwidth. This technique is also named as Quadrature Amplitude Modulation (QAM).

QAM transmitter:

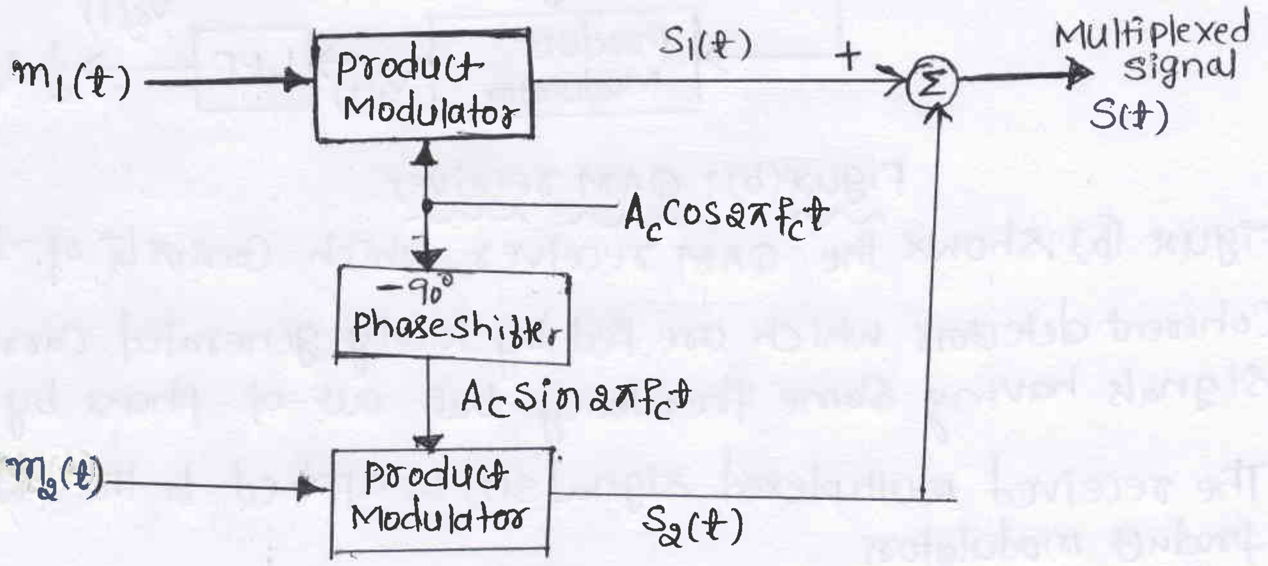


Fig (a) Quadrature Carrier Multiplexing (or) QAM Transmitter

- Figure (a) shows QAM transmitter. It consists of two product modulators that are supplied with carriers which differ in phase by 90° (phase quadrature)
- The output of the two product modulators are summed to produce multiplexed signal. S(t).

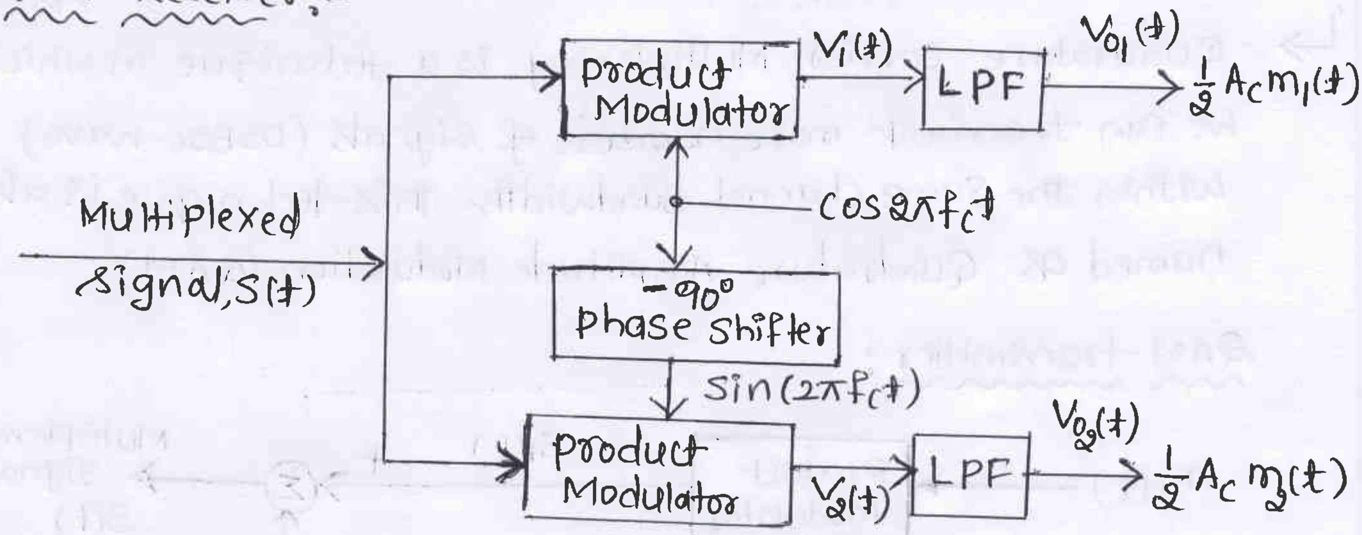
i.e., $S(t) = S_1(t) + S_2(t)$ -

$$S(t) = A_c m_1(t) \cos 2\pi f_c t + A_c m_2(t) \sin 2\pi f_c t$$

∴ QAM-transmitter allows two modulated (DSBSC) waves to occupy the same transmission channel Bandwidth.

∴ The Multiplexed Signal $s(t)$, occupies a channel Bandwidth of $BW = 2W$; $W = \text{Maximum}(f_{m_1}, f_{m_2})$ Centered at the Carrier frequency 'f_c'.

QAM - Receiver:-



Figure(b): QAM receiver

- Figure (b) shows the QAM receiver, which Consists of two Coherent detectors which are fed by locally generated Carrier signals having same frequency but out-of-phase by 90°.
- The received multiplexed signal $s(t)$ is applied to the two product modulators.

↳ The output of top product Modulator is

$$V_1(t) = s(t) \times \cos 2\pi f_c t.$$

↳ The top LPF removes the high frequency terms and allows only $\frac{A_c}{2} m_1(t)$.

$$\therefore V_{01}(t) = \frac{A_c}{2} m_1(t)$$

Similarly the output of bottom product modulator is

$$V_2(t) = s(t) \times \sin 2\pi f_c t$$

↳ The bottom LPF removes the high Frequency terms and allows only $\frac{A_c}{2} \cdot m_2(t)$

$$\therefore V_{02}(t) = \frac{A_c}{2} m_2(t)$$

Application: • used in color TV

• It is Bandwidth - Conservation Scheme.

* Advantages of DSBSC-Modulation :-

1. Carrier signal is supressed
2. Low power Consumption.
3. Efficiency is more than AM.
4. The Modulation system is simple.
5. Linear Modulation.

* Disadvantages (or) Limitations of DSBSC-Modulation :-

1. Design of receiver is complex.
2. Bandwidth required is same as that of AM

   i.e., $BW_{AM} = BW_{DSBSC} = 2f_m$

* Applications of DSBSC :-

↳ point to point Communication.

↳ Analog TV systems to transmit Color Information.

1.10: SINGLE SIDE BAND SUPRESSED CARRIER (SSBSC) MODULATION:

↳ Standard AM and DSBSC Modulation requires a transmission bandwidth of $BW_T = 2f_m$.

↳ ∴ In both AM & DSBSC-Modulation, half of the transmission bandwidth is occupied by the upper side band of the modulated wave and other half of the transmission BW is occupied by LSB.

↳ The USB and LSB are uniquely related to each other by virtue of their symmetry about the carrier frequency "f_c".

∴ only one sideband is necessary for transmission of Message signal.

↳ When only one side band is transmitted, the modulation is referred to as "Single Side band Modulation"

*** SSB-Modulation:-

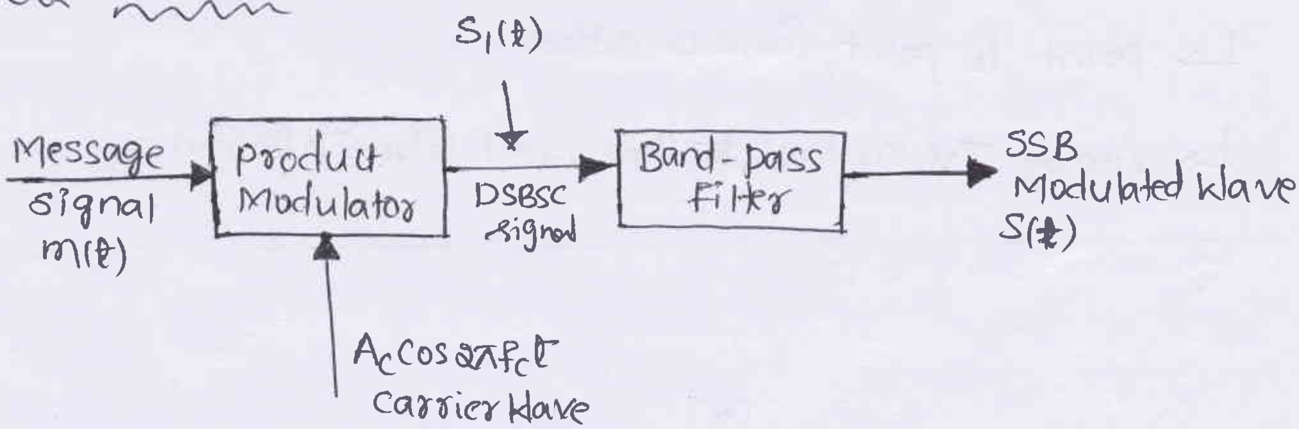


Figure : Frequency descrimination scheme for the generation of SSB Modulated wave.

- Frequency descrimination scheme for the generation of SSB Modulated wave is shown in above figure.
- It Consists of product Modulator followed by BPF.
- The output of the product Modulator is DSBSC signal

$$S_1(t) = m(t) \times A_c \cos 2\pi f_c t$$

$\therefore \quad s_1(t) = A_C\, m(t) \cdot \cos 2\pi f_C t$ ——(1)

Taking Fourier-transform on both sides of equation (1) we get

$$S_1(f) = \frac{A_C}{2}\left[M(f-f_C) + M(f+f_C)\right]$$

$$S_1(f) = \frac{A_C}{2}\underbrace{M(f-f_C)}_{LSB} + \frac{A_C}{2}\underbrace{M(f+f_C)}_{USB} \quad \text{——(2)}$$

- Where $M(f)$ is frequency spectrum of message signal $m(t)$ as shown in figure 2(a).
- The spectrum of DSBSC signal $S_1(f)$, shown in figure 2(b) consists of both the side bands (LSB & USB) centered with respect to carrier frequency.
- When DSBSC signal is passed through BPF with its center frequency 'f_C' and bandwidth 'W' Hz. If it transmits USB then we get spectrum of SSB modulated signal.

$$S(f) = \frac{A_C}{2} M(f+f_C) \quad \text{——(3)}$$; SSB Modulated signal with only USB.

- If BPF allows only LSB, then

$$S(f) = \frac{A_C}{2}\left(M(f-f_C)\right) \quad \text{——(4)}$$; SSB Modulated signal with only LSB

$\therefore$ The single side band suppressed carrier modulated signal $S(t)$ produced at the output of BPF contains only one side band and carrier signal, other side band are eliminated (or) Suppressed

The Frequency domain description of SSB-Modulated signal is clearly represented in Figure 2.

Figure 2(c) shows the spectrum of SSB-signal with USB only.

Figure 2(d) shows the spectrum of SSB-signal with LSB only.

$\therefore$ Transmission Bandwidth of SSB Modulated signal is $\Big\} = B_T = W \Leftarrow$ Half of that of AM & DSBSC-signals.

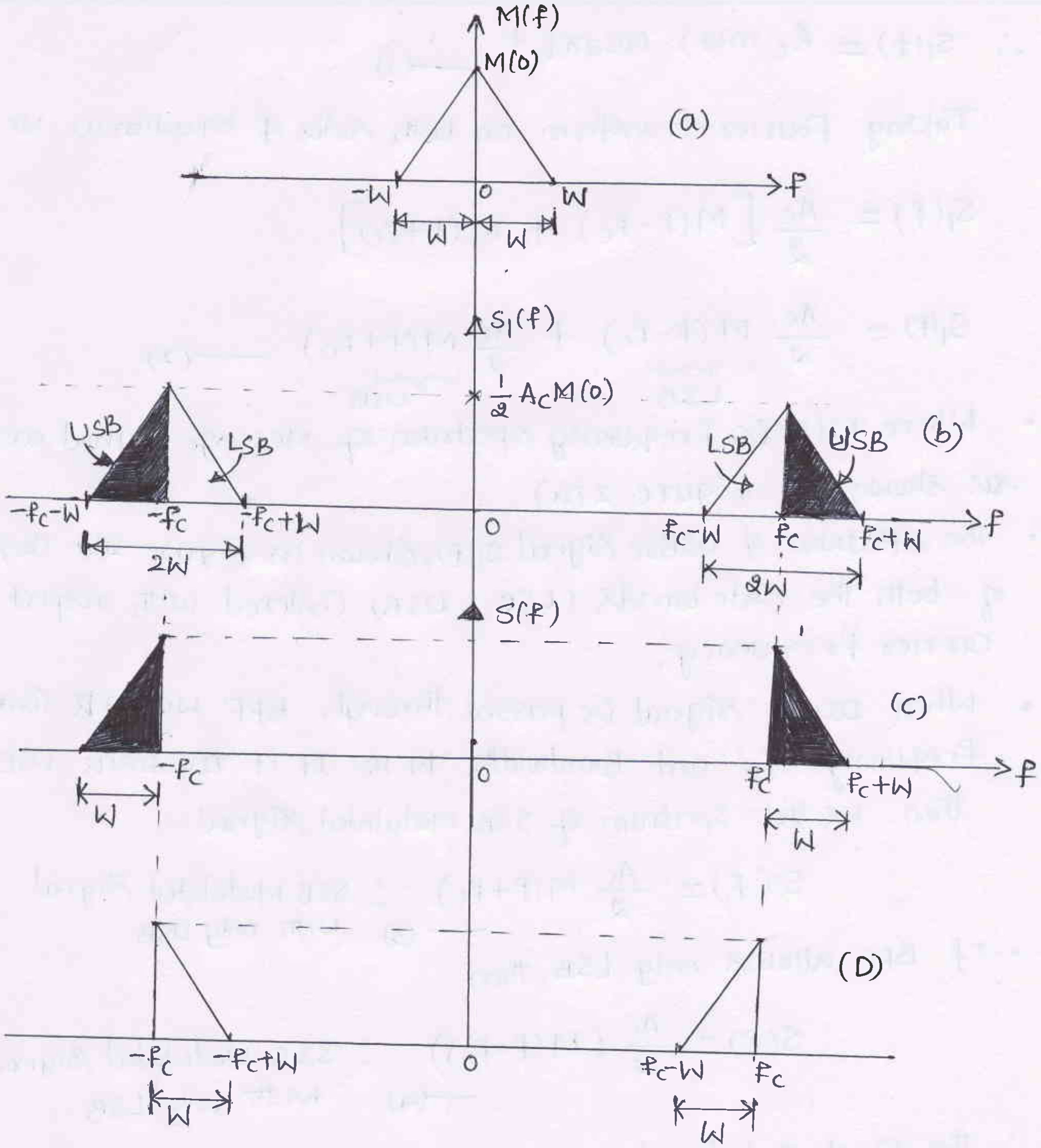


Figure 2: (a) Spectrum of m(t) (b) Spectrum of DSBSC-signal $S_1(t)$.
(c) Spectrum of SSB Mod. signal having only USB S(t) (d) Spectrum of SSB Modulated Signal having only LSB. S(t)

** Advantages of SSB-Modulation :-

1. SSB-Modulation requires half of the Bandwidth required for AM and DSBSC-signal.
2. Due to suppression of carrier and one side band, power is saved
3. Reduced interference of Noise.
4. Fading does not occur in SSB-transmission.

Disadvantages of SSB:-

1. The Generation and reception of SSB signal is a Complex process.
2. SSB-Modulation system is Expensive and highly Complex to implement.
3. Since the carrier and one side band is suppressed, the SSB transmitter and receiver need to have Excellent frequency stability.

Applications of SSB:-

1. Mobile Communication system: (Since SSB is power saving modulation)
2. SSB is also used in applications in which bandwidth requirements are Low.

   Ex: • point to point Communication
   • Telemetry
   • Military applications.
   • Radio Navigation

1.11

* VESTIGIAL SIDEBAND MODULATION (VSB) :-

Necessity (or) Need for VSB-Modulation:

↳ The SSB modulation is not appropriate way of modulation. Because the upper side band and lower side band meet at the Carrier frequency 'f_c' and it is very difficult to isolate one side band. Therefore generating SSB-signal is challenging.

↳ To overcome this difficulty, the modulation technique known as "Vestigial side Band (VSB) - Modulation" is used.

↳ Vestigial sideband Modulated signal (VSB-signal) consists of

- Almost one complete side band and
- Vestige (or Trace) (or part) of the other side band.

* Generation of VSB Modulated Wave:

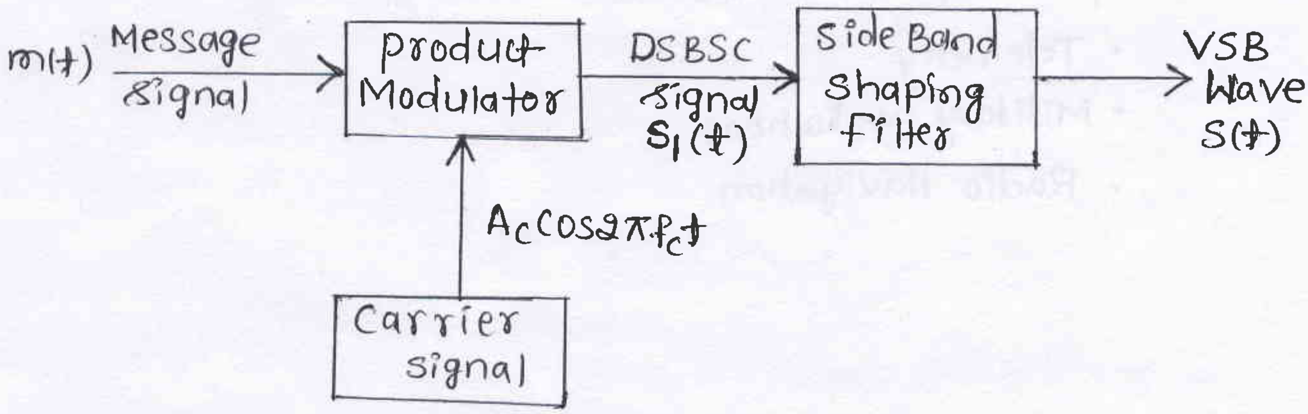


Figure 1: VSB Generator

- VSB signal generator consists of a product Modulator and a sideband shaping filter as shown in figure 1.
- product Modulator generates a DSBSC signal and then pass it through a side band shaping filter.
- Let H(f) be the transfer function of side band shaping filter. This filter will pass one complete sideband along with a Vestige (or) trace (or) a part of unwanted (other) side band.

↳ The relation between the transfer function $H(f)$ of the filter and the spectrum $S(f)$ of the VSB-Modulated wave $s(t)$ is defined by, $S(f) = S_1(f) \times H(f)$.

$$\therefore \quad \boxed{S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] H(f)}$$

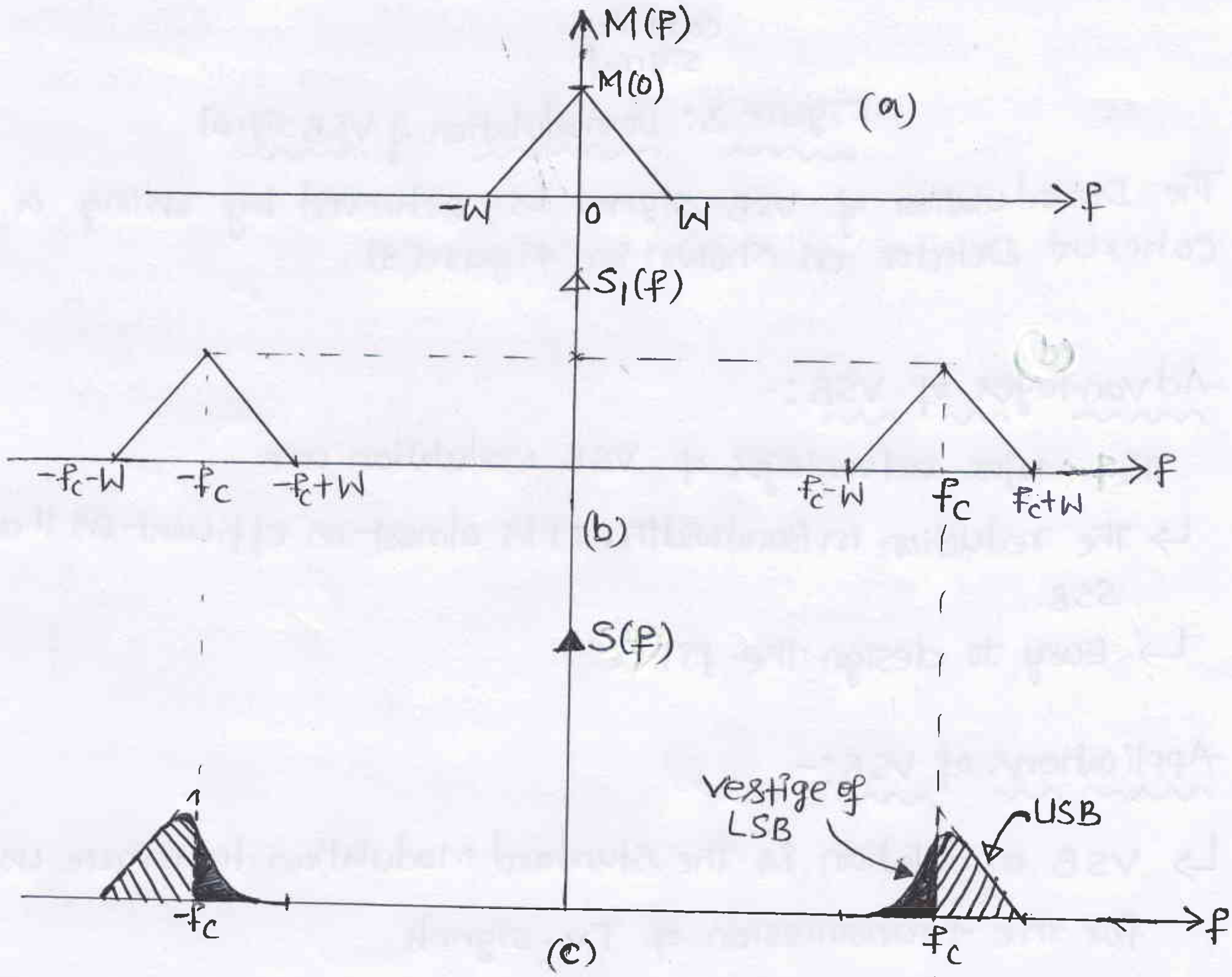


Figure 2: (a) Spectrum of $m(t)$ (b) Spectrum of DSBSC signal $S_1(t)$ (c) Spectrum of VSB-Modulated Signal $s(t)$

Frequency domain description of VSB modulated wave is shown in figure 2. Figure 2(b) is the spectrum of DSBSC signal produced at the output of product Modulator. Figure 2(c) shows the spectrum of VSB-modulated signal $s(t)$.

From figure 2(c) it is evident that the Total transmission Bandwidth of VSB-Modulated signal is higher than that of SSB and Lower than that of DSBSC-signal.

i.e., $W < BW_{T(VSB)} < 2W$

* Demodulation of VSB-Modulated Wave:-

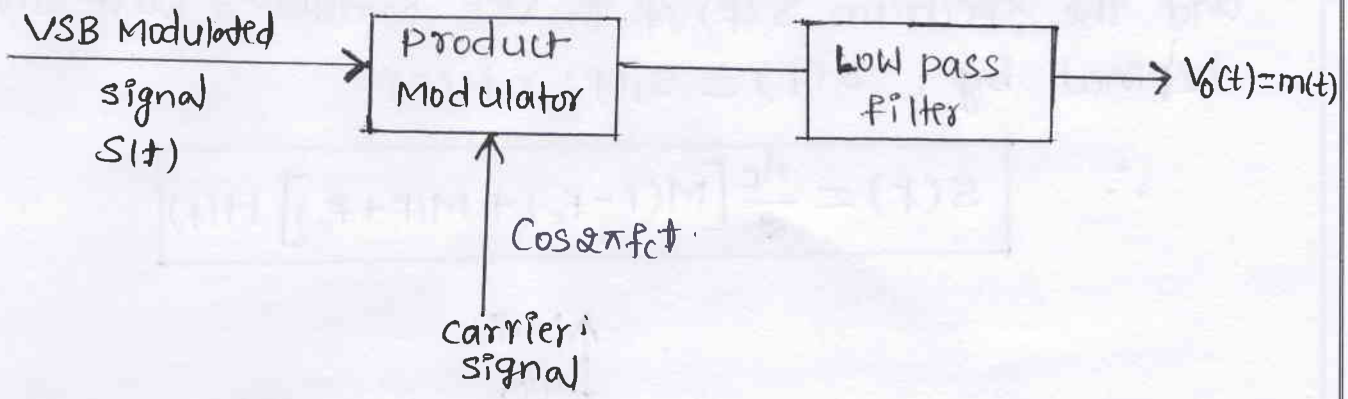


Figure 3: Demodulation of VSB signal

The Demodulation of VSB-signal is achieved by using a coherent Detector as shown in figure(3).

-Advantages of VSB:-

The Major advantages of VSB Modulation are

- The reduction in Bandwidth. It is almost as efficient as that of SSB.
- Easy to design the filter.

-Applications of VSB:-

- VSB modulation is the standard Modulation technique used for the transmission of TV-signals.

1.12: Frequency Translation : [Mixing (or) Heterodyning]

↳ In the communication systems, it is necessary to translate modulated wave $s(t)$, frequency Upward (or) Downward in frequency so that it occupies a new frequency band.

↳ i.e., If 'f_c' is the frequency of Modulated wave $s(t)$ then frequency translator changes modulated wave frequency 'f_c' to either $f_c+f_L \Rightarrow$ Upward translation (or)

$f_c-f_L \Rightarrow$ Downward translation.

↳ Frequency translation is achieved by using a Frequency Multiplier (product Modulator) followed by a Band pass filter as shown in figure 1.

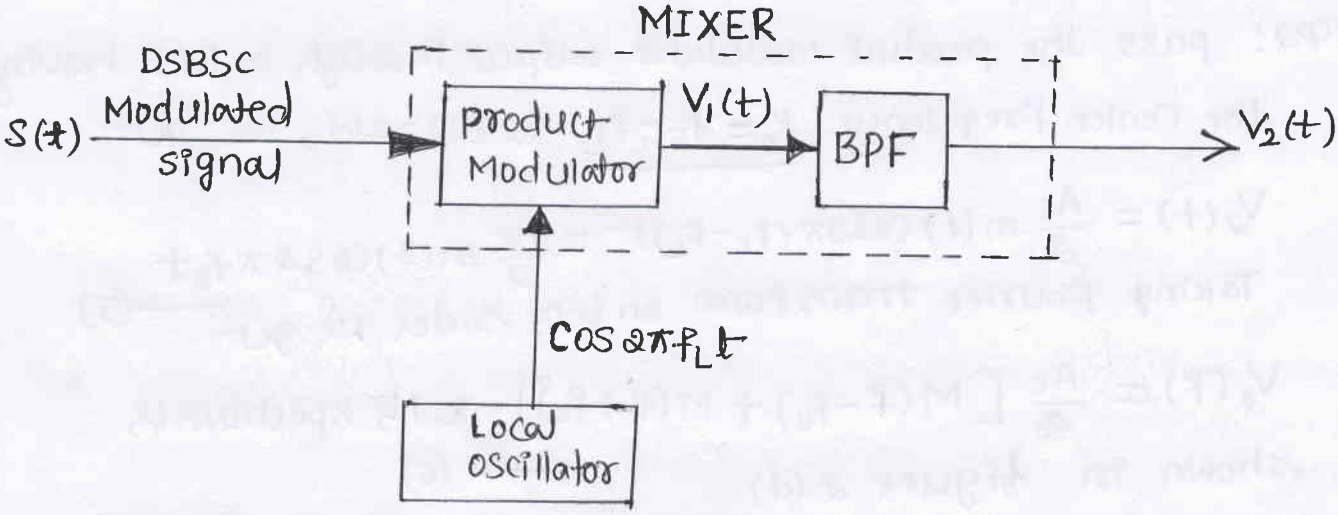


Figure 1: Frequency Translation (or) Mixing (or) Heterodyning

Consider a DSBSC Modulated signal,

$$s(t) = A_c m(t) \cdot \cos 2\pi f_c t \quad \text{—— (1)}$$

• Where $m(t)$ is limited to the frequency band $-W \le f \le W$ shown in figure2(a)

The fourier transform of equation (1) is

$$S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] \quad \text{—— (2)}$$

• The spectrum of $s(t)$ occupies the band of frequencies (f_c-W) to (f_c+W) and $-(f_c-W)$ to $-(f_c+W)$ as shown in figure2(b)

Case(i): Frequency translation to Lower frequency (or)

i.e, $\left(f_c \longrightarrow f_0 = f_c - f_L\right)$ (Downward frequency)

step1: The output of the product modulator is given by

$$V_1(t) = s(t) \times \cos 2\pi f_L t$$

$$V_1(t) = A_c m(t) \cos 2\pi f_c t \times \cos 2\pi f_L t$$

$$\therefore V_1(t) = \frac{A_c m(t)}{2} \cos 2\pi (f_c - f_L)t + \frac{A_c m(t)}{2} \cos 2\pi (f_c + f_L)t \longrightarrow (3)$$

Taking Fourier transformation on both sides we get

$$V_1(f) = \frac{A_c}{4}\left[M(f-(f_c-f_L)) + M(f+(f_c-f_L))\right] + \frac{A_c}{4}\left[M(f-(f_c+f_L)) + M(f+(f_c+f_L))\right] \longrightarrow (4)$$

The spectrum of $V_1(f)$ is shown in figure 2.(c)

step2: pass the product modulator output through a BPF having the center frequency $f_0 = f_c - f_L$ & $BW = 2W$, we get

$$V_2(t) = \frac{A_c}{2} \cdot m(t) \cos 2\pi (f_c - f_L)t = \frac{A_c}{2} m(t) \cos 2\pi f_0 t \longrightarrow (5)$$

Taking Fourier transform on both sides we get

$$V_2(f) = \frac{A_c}{4}\left[M(f-f_0) + M(f+f_0)\right] \longrightarrow (6)$$

& its spectrum is shown in figure 2.(d).

$\therefore$ By comparing equation (2) $[S(f)]$ and output of BPF $V_2(f)$ it is clear that the center frequency 'f_c' of DSBSC modulated signal $s(t)$ is translated to a new frequency band $\boxed{f_0 = f_c - f_L}$

Case(ii): Frequency translation to Higher Frequency (or)

i.e, $f_c \longrightarrow f_0 = f_c + f_L$ (Upward Frequency)

↳ pass the product modulator output $V_1(t)$ shown in eqn (3) through a BPF having Center frequency $f_0 = f_c + f_L$ & $BW = 2W$, we get, the output of BPF

$$V_2(t) = \frac{A_c}{2} m(t) \cos 2\pi (f_c + f_L) t$$

$$V_2(t) = \frac{A_c}{2} m(t) \cos 2\pi f_0 t \quad ; \quad f_0 = f_c + f_L \qquad \text{—— (7)}$$

Taking Fourier transformation on both sides we get

$$V_2(f) = \frac{A_c}{4}\left[M(f - f_0) + M(f + f_0)\right] \qquad \text{—— (8)}$$

∴ The Center frequency of DSBSC Modulated signal is (f_c) translated to upward frequency $\boxed{f_0 = f_c + f_L}$

The spectrum of $V_2(f)$ is shown in figure 2.(e).

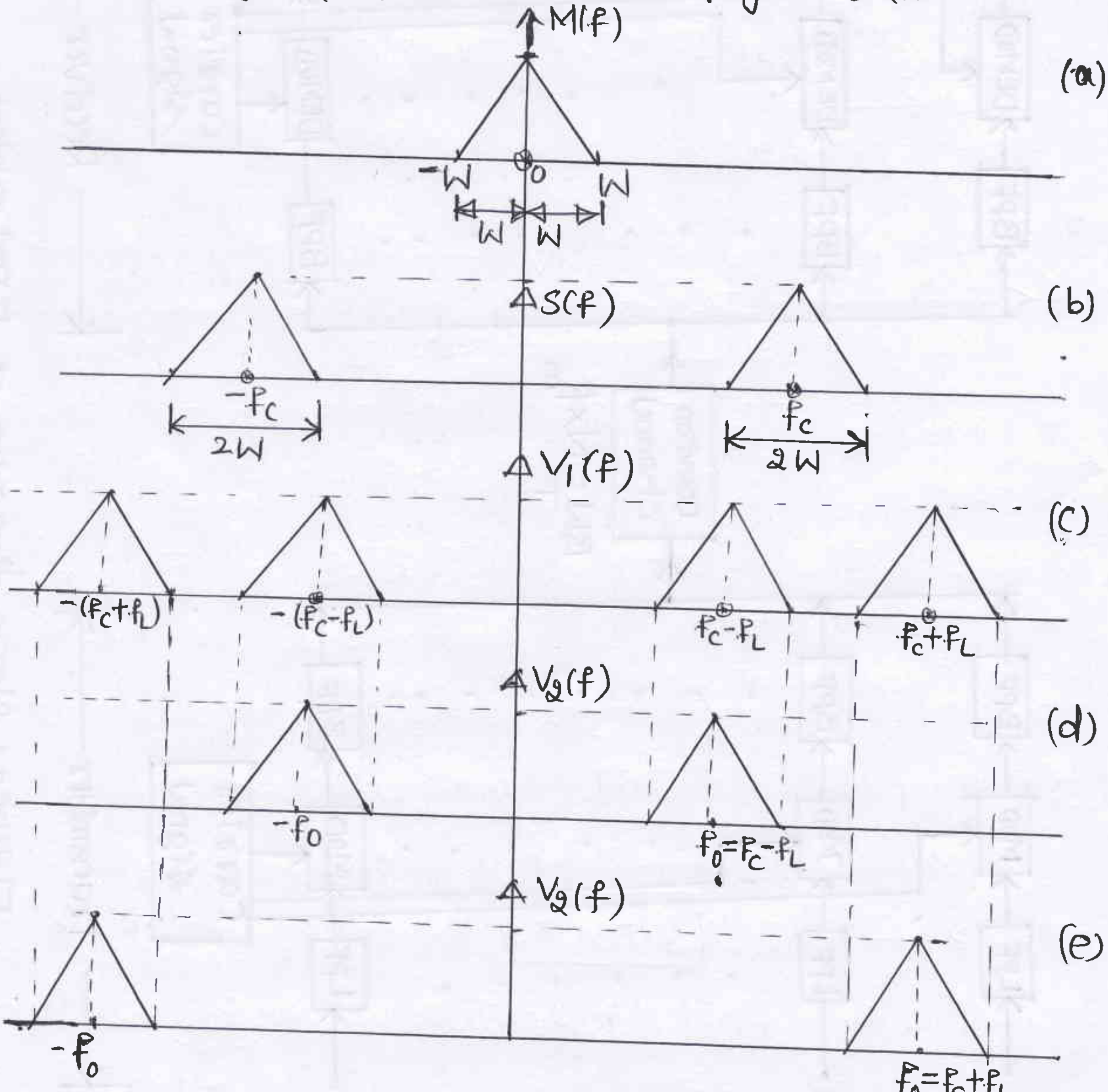


Figure 2: (a) spectrum of m(t) (b) spectrum of DSBSC signal s(t)
(c) Spectrum of product modulator output $V_1(t)$
(d) Spectrum of downward translated signal $V_2(t)$
(e) Spectrum of upward translated signal $V_2(t)$

1.13 * Frequency Division Multiplexing <FDM>

- Multiplexing is a process of combining N-independent message signals into a composite signal suitable for transmission over a common channel
- Multiplexing is accomplished by seperating the signals either in frequency (or) time.
- The technique of separating the signals in frequency domain is referred to as "Frequency Division Multiplexing" (FDM)

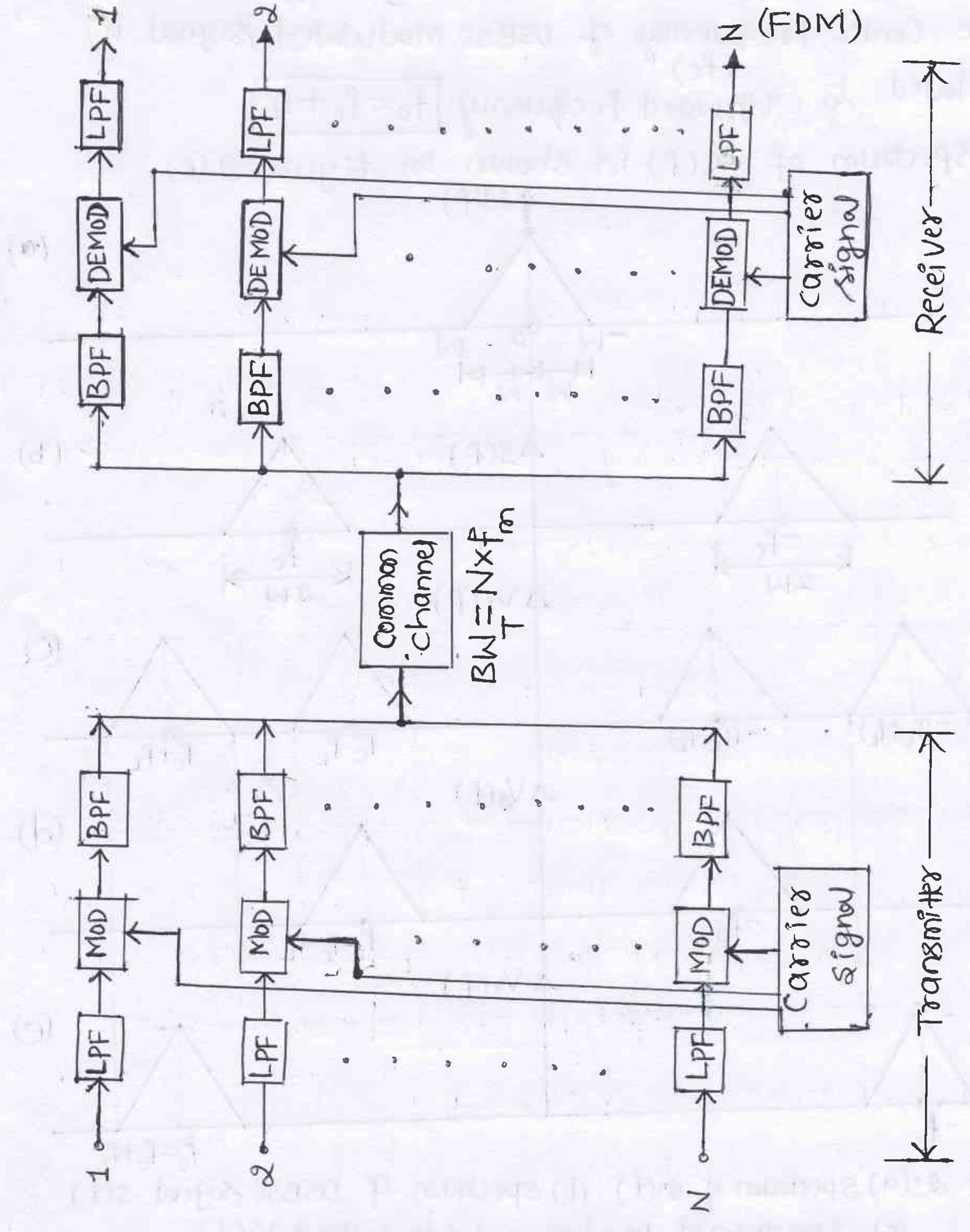


Figure 1: Block diagram of FDM system

The block diagram of FDM-system is shown in figure1.

↳ N-Incoming independent message signals are modulated by mutually Exclusive carriers supplied from carrier source at each modulator. The Modulated signals are passed through the BPF to select any one side band. Therefore BPF's produces SSB-signals and are separated in frequency and combined into a composite signal. and this process is called Frequency division multiplexing.

↳ Multiplexed signal is transmitted over the communication channel.

↳ Total Bandwidth required to N-SSB Modulated signals without any Guard band is

$$BW_T = N \times f_m \;;\; N = \text{number of Input signals}$$

↳ At the receiver side N-independent message signals are recovered by passing the composite signal through the BPF followed by Demodulator and LPF.

Advantages of FDM:-

1. A Large Number of signals can be transmitted simultaneously.
2. FDM does not requires synchronization between Transmitter & receiver.
3. Demodulation of FDM is easy

Dis advantages of FDM:-

1. Communication channel must have Large Bandwidth i.e., $BW_T = N \times f_m$
2. Large Numbers of Modulators & Filters are required.
3. Cross talk occurs in FDM

Theme Example 1.14: VSB Transmission of Analog and Digital TV:

-6M-

↳ Vestigial side band modulation plays a key role in commercial television.

↳ The exact details of modulation format used to transmit the video signal characterizing a TV system are influenced by two factors:

i) The video signal exhibits a large bandwidth and significant low frequency information, which requires the use of vestigial side band modulation.

ii) The circuitry used for demodulation in the receiver must be simple and inexpensive. This suggests the use of envelope detection, which requires the addition of a carrier wave to the VSB modulated wave.

Figure (a) shows the ideal spectrum of a transmitted TV-signal. It consists of the upper side band, 25% of the LSB and the picture carrier are transmitted.

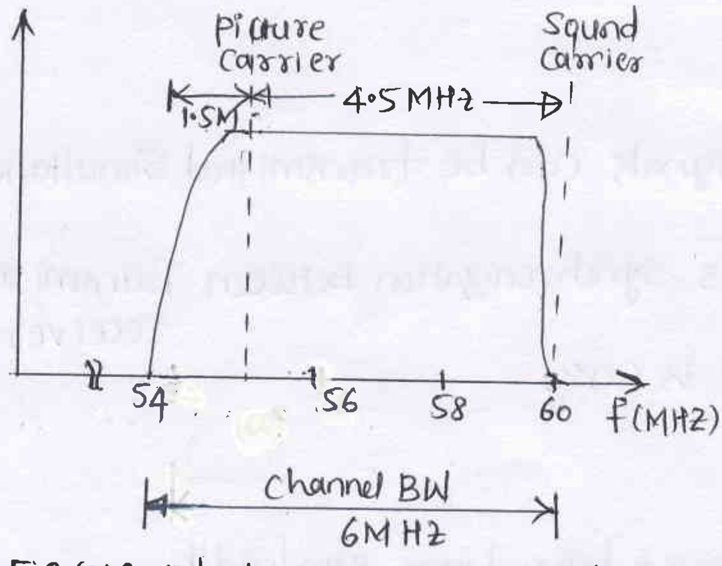


Fig (a): Ideal Transmission spectrum of TV signal

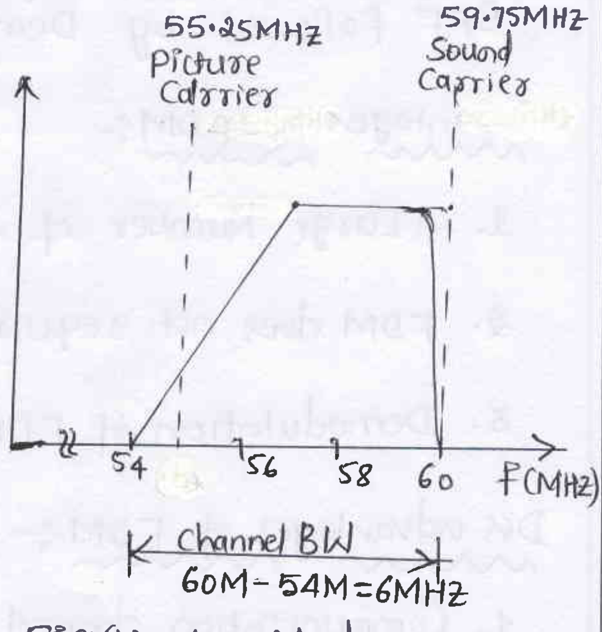


Fig (b): Amplitude response of a VSB-shaping filter at the receiver

- Figure (b) shows the amplitude response of a VSB shaping filter used at the receiver
- The channel bandwidth used for TV broadcasting in North America is 6MHz as shown in fig (a) & fig (b).

Module-1 :-

Amplitude Modulation

(Numerical problems) V.T.U Q.Papers

List of Formulae :-

1. Amplitude Modulation Index (or) Depth of Modulation:

$\mu = K_a A_m$, where A_m = Amplitude of message Signal in Volts.
K_a = Amplitude Sensitivity parameter

(or) $\boxed{\mu = K_a A_m}$

Note: • The Maximum Value of Modulation index is "1"

- When, $\mu < 1 \Rightarrow$ Results in under Modulation
- When, $\mu = 1 \Rightarrow$ Results in critical Modulation
- when, $\mu > 1 \Rightarrow$ Results in over Modulation.

2. AM Wave equation:

↳ Single-tone AM: (considering single message signal)

- $s(t) = A_c [1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$

↳ Multitone AM: (considering multiple message signals)

- $s(t) = A_c [1 + \mu_1 \cos(2\pi f_{m_1} t) + \mu_2 \cos(2\pi f_{m_2} t) + \dots] \cos(2\pi f_c t)$

Where, $\mu_1 = K_a A_{m_1}$; $\mu_2 = K_a A_{m_2}$

3. Net Modulation Index :-

For Multitone Modulation, Net Modulation Index μ_t is

$$\mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2 + \dots + \mu_n^2}$$

4. The Maximum and Minimum amplitudes of AM:-

- $A_{max} = A_c(1+\mu)$; • $A_{min} = A_c(1-\mu)$

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→ A_{max} & A_{min} are required to sketch resulting AM Signal for any given Modulation index 'μ'.

5. Total power in AM-wave:-

↳ In terms of carrier power 'P_c' and 'μ'

$$P_t = P_c\left(1+\frac{\mu^2}{2}\right) \;;\; P_c = \frac{A_c^2}{2R} \;:\; R = \text{Load Resistance}$$

↳ power in side bands: $P_{LSB} = P_{USB} = P_c \frac{\mu^2}{4}$ Watts

↳ Total power in side bands: $P_{SB} = P_{LSB} + P_{USB} = P_c \frac{\mu^2}{2}$

$$P_t = P_c + P_{LSB} + P_{USB} = P_c + P_{SB} \Longrightarrow P_{SB} = P_t - P_c$$

↳ In terms of Antenna RMS currents: $P_t = I_t^2 R$ and $P_c = I_c^2 R$

Note:- When $\mu = 1$: $P_t\Big|_{max} = P_c\left(1+\frac{\mu^2}{2}\right)\Big|_{\mu=1} = \frac{3}{2}P_c = 1.5P_c$.

∴ The Max. transmitted power is 1.5 times Carrier power

6. Efficiency of AM:-

$$\eta = \frac{P_{LSB} + P_{USB}}{P_t} = \frac{\mu^2}{2+\mu^2}$$

Note: Max. Efficiency of AM is $\eta_{max} = \frac{\mu^2}{2+\mu^2}\Big|_{\mu=1} = \frac{1}{3} = \underline{\underline{0.3333}}$

7. Total transmission Band Width of AM:-

$$BW_T = 2W = f_{USB} - f_{LSB}$$

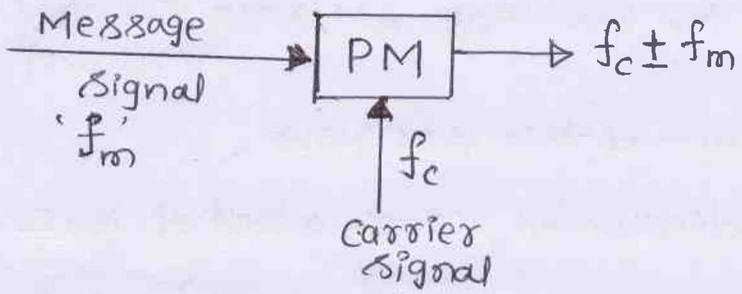
where $W = \text{Maximum}(f_{m_1}, f_{m_2}, \ldots, f_{m_n})$; for Multitone AM.

$W = f_m$, for single tone AM

f_{USB} = frequency of upper side Band = $f_c + f_m$

f_{LSB} = frequency of Lower side Band = $f_c - f_m$

8. Product Modulator :- <DSBSC - Generator>



↳ product Modulator produces two output frequencies

(i) $f_c + f_m = f_{USB}$

(ii) $f_c - f_m = f_{LSB}$

9. Band pass Filter :- It is used to select any one side band frequency.

- $f_c \pm f_m \longrightarrow \boxed{BPF} \longrightarrow f_c + f_m$: if Center frequency of BPF is $(f_c + f_m)$

- $f_c \pm f_m \longrightarrow \boxed{BPF} \longrightarrow f_c - f_m$: if Center frequency of BPF is '$f_c - f_m$'

10. Amplitude of each side band in AM $= \mu \frac{A_c}{2}$

11. General formulas :-

↳ $\cos A \cdot \cos B = \frac{1}{2}\{\cos(A-B) + \cos(A+B)\}$.

↳ $\cos(2\pi f_0 t) \xrightleftharpoons{F \cdot T} \frac{1}{2}\left[\delta(f - f_0) + \delta(f + f_0)\right]$

↳ $A_c\, m(t) \cdot \cos(2\pi f_c t) \xrightleftharpoons{F \cdot T} \frac{A_c}{2}\left[M(f - f_0) + M(f + f_0)\right]$

Where, • $\delta(f - f_c) = \begin{cases} 1 & : \text{only at } f = f_c \\ 0 & : ew \end{cases}$ ⇐ Impulse signal (or) Delta function

• $\delta(f + f_c) = \begin{cases} 1 & : \text{only at } f = -f_c \\ 0 & : ew \end{cases}$

• $M(f)$ is Spectrum of $m(t)$.

12. Band width of SSBSC is : $BW_{SSB} = W = f_m$

1. Consider a message signal $m(t) = 20\cos(2\pi t)$ Volts and a carrier signal $c(t) = 50\cos(100\pi t)$ Volts.
   (i) Find and sketch the resulting AM wave for 75% modulation. (VTU.Q.P)
   (ii) Sketch the spectrum of this AM wave.
   (iii) Find the power dissipated across a load of 100Ω.

↳ Given data : $m(t) = 20\cos(2\pi t)$ $\therefore A_m = 20$; $f_m = 1$ Hz

$c(t) = 50\cos(100\pi t)$ $\therefore A_c = 50$; $f_c = 50$ Hz

(i) Resulting AM wave for 75% Modulation :- (i.e., for $\mu = 0.75$)

W.K.T. for single Tone AM,

$$S(t) = A_c[1 + \mu\cos(2\pi f_m t)]\cos(2\pi f_c t)$$

$\therefore$ The Resulting AM wave for $\mu = 0.75$; $A_c = 50V$; $f_m = 1$ and $f_c = 50$ is

$$\boxed{S(t) = 50[1 + 0.75\cos(2\pi t)]\cos(100\pi t)}$$

- To Sketch AM signal :- • $A_{max} = A_c(1+\mu) = 50(1+0.75) = 87.5V$
  • $A_{min} = A_c(1-\mu) = 50(1-0.75) = 12.5V$

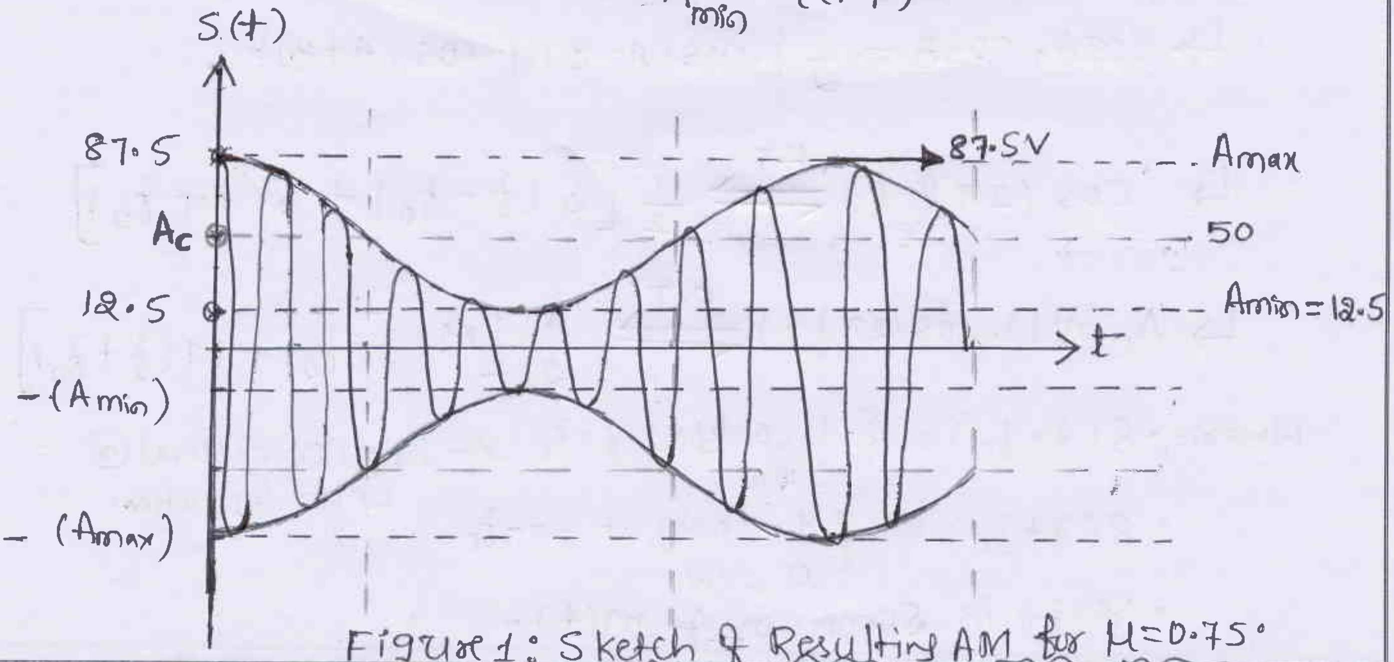


Figure 1: Sketch of Resulting AM for $\mu = 0.75$.

(ii) Spectrum of AM wave :-

W.K.T the resulting AM wave is.

$$s(t) = A_c\left[1 + \mu\cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

$$s(t) = 50\left[1 + 0.75\cos(2\pi(1)t)\right]\cos(2\pi(50)t)$$

$$\therefore s(t) = 50\cos 2\pi(50)t + 37.5\cdot\cos 2\pi(50)t\cdot\cos 2\pi(1)t$$

W.K.T $\cos A\cdot\cos B = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right)$

$$\therefore s(t) = 50\cos 2\pi(50)t + \frac{37.5}{2}\left[\cos 2\pi(50-1)t + \cos 2\pi(50+1)t\right]$$

$$\boxed{\therefore s(t) = 50\cos 2\pi(50)t + 18.75\cos 2\pi(49)t + 18.75\cos 2\pi(51)t} \quad \text{--- (1)}$$

Taking Fourier Transform of Equation (1) we get

$$S(f) = \frac{50}{2}\left[\delta(f-50) + \delta(f+50)\right] + \frac{18.75}{2}\left[\delta(f-49) + \delta(f+49)\right] + \frac{18.75}{2}\left[\delta(f+51) + \delta(f-51)\right]$$

$$\boxed{\therefore S(f) = 25\left[\delta(f-50) + \delta(f+50)\right] + 9.375\left[\delta(f-49) + \delta(f+49)\right] + 9.375\left[\delta(f+51) + \delta(f-51)\right]}$$

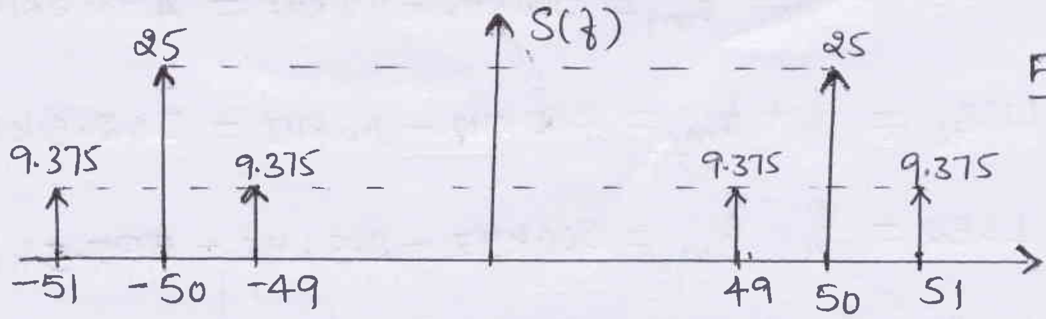


Fig.2:- Spectrum of S(t)

(iii) Power dissipated across $R = 100\Omega$:-

W.K.T. $P_t = P_c\left(1 + \frac{\mu^2}{2}\right)$; $P_c = \frac{A_c^2}{2R} = \frac{(50)^2}{2\times 100} = 12.5\,W$

$$\therefore P_t = 12.5\left(1 + \frac{(0.75)^2}{2}\right) \simeq \underline{16.016\,W}$$

2. A carrier wave $4\sin(2\pi\times500\times10^3 t)$ Volts is amplitude modulated by an audio wave $[0.2\sin 3(2\pi\times500t)+0.1\sin 5(2\pi\times500t)]$ Volts. Determine the upper and lower sidebands and sketch the complete spectrum of the modulated wave. Estimate the total power in the side band. (VTU Q.P)

↳ Given: $c(t)=4\sin(2\pi\times500\times10^3 t)$.

$\therefore A_c = 4V$ & $f_c = 500\times10^3 = 500KHZ$.

The Message signal (Audio wave),

$$m(t) = 0.2\sin 2\pi\times1500\times t + 0.1\sin 2\pi\times2500t$$

$\therefore A_{m_1} = 0.2$; $f_{m_1} = 1500 = 1.5KHZ$

$A_{m_2} = 0.1$; $f_{m_2} = 2500 = 2.5KHZ$.

$\therefore \mu_1 = \frac{A_{m1}}{A_c} = \frac{0.2}{4} = 0.05$; and $\mu_2 = \frac{A_{m2}}{A_c} = \frac{0.1}{4} = 0.025$.

Net Modulation index : $\mu_t = \sqrt{\mu_1^2+\mu_2^2} = \sqrt{0.05^2+0.025^2} = 0.056$

(*) Upper and lower side bands (USB and LSB):-

i) $USB1 = f_c + f_{m_1} = 500KHZ + 1.5KHZ = 501.5KHZ$

$LSB1 = f_c - f_{m_1} = 500KHZ - 1.5KHZ = 498.5KHZ$.

ii) $USB_2 = f_c + f_{m_2} = 500KHZ - 2.5KHZ = 502.5KHZ$

$LSB2 = f_c - f_{m_2} = 500KHZ - 2.5KHZ = 497.5KHZ$

• To Sketch the Complete spectrum of the modulated wave:-

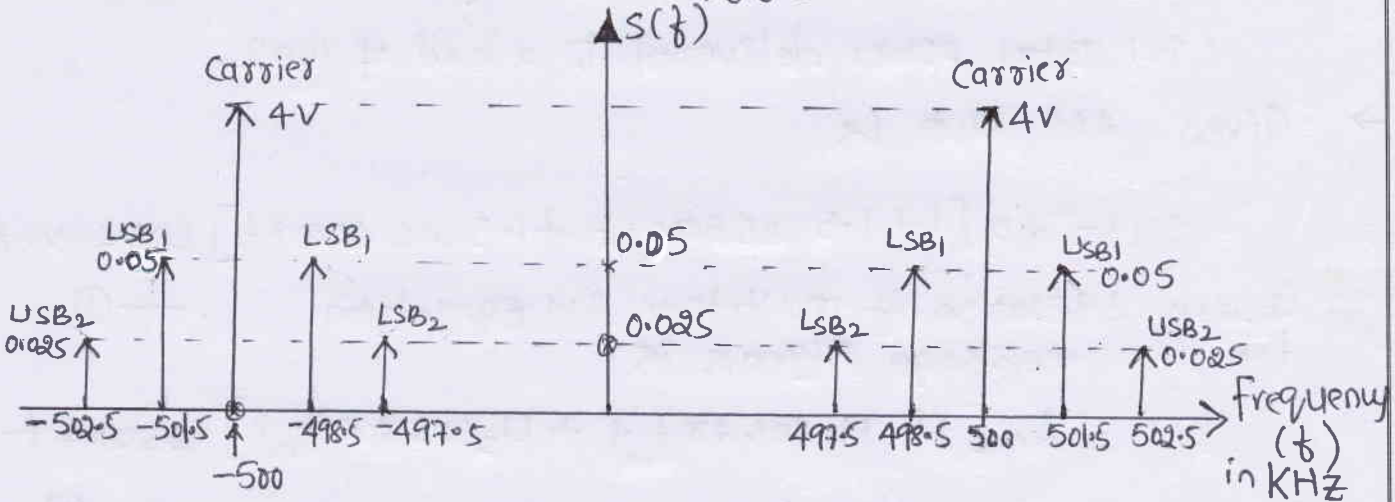
Amplitudes of upper side band and Lower side band for frequency spectrum is $\frac{\mu A_c}{4}$ & that of Carrier is '$\frac{A_c}{2}$'.

i) Amplitude of Carrier frequency 'f_c' : $\frac{A_c}{2} = \frac{4}{2} = 2V.$

ii) Amplitudes of USB_1 and LSB_1 is : $\frac{\mu_1 A_c}{4} = \frac{0.05 \times 4}{4} = 0.05$

iii) Amplitudes of USB_2 & LSB_2 is : $\frac{\mu_2 A_c}{4} = \frac{0.025 \times 4}{4} = 0.025$

∴ Complete spectrum of AM signal is



* Total power in side bands:-

$$P_{SB} = P_{LSB} + P_{USB} = P_c \frac{\mu_t^2}{2}$$

$$\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = 0.056$$

$$\therefore P_{SB} = P_c \times \frac{0.056^2}{2} \quad ; \quad P_c = \frac{A_c^2}{2R} = \frac{4^2}{2R} = \frac{8}{R} \text{ Watts}$$

$$P_{SB} = \frac{8}{R} \times 0.0125$$

$$\boxed{P_{SB} = \frac{0.0125}{R}} \text{ Watts} \quad ; \text{ Where } R = \text{load resistance}$$

if $\boxed{R = 1\Omega}$

$$\boxed{P_{SB} = 0.0125 \text{ Watts}}$$

<3> An AM wave has the form,

$$s(t) = 20\,[1 + 1.5\cos 2000\pi t + 1.5\cos 4000\pi t]\cos 40000\pi t$$

Determine,

(i) Net Modulation Index

(ii) The carrier power and side band power

(iii) $S(f)$ and Draw its frequency spectrum.

(iv) Total power delivered to a load of 100Ω.

↳ Given AM wave is

$$s(t) = 20\,[1 + 1.5\cos 2000\pi t + 1.5\cos 4000\pi t]\cos 40000\pi t \quad \text{--- ①}$$

General equation of multitone AM equation for two message signals is

$$s(t) = A_c\,[1 + \mu_1 \cos 2\pi f_{m_1} t + \mu_2 \cos 2\pi f_{m_2} t]\cos 2\pi f_c t \quad \text{--- ②}$$

∴ By comparing equations ① & ② we get

$A_c = 20V$; $\mu_1 = 1.5 = \mu_2$; $f_{m_1} = 1KHZ$; $f_{m_2} = 2KHZ$; $f_c = 20KHZ$ and Load resistance $R = 100\Omega$

(i) Net Modulation index :-

$$\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{1.5^2 + 1.5^2} = 2.12$$

(Note: $s(t)$ is over Modulated) ∵ $\mu_t > 1$

(ii) carrier power & side band power :-

↳ carrier power : $P_c = \dfrac{A_c^2}{2R} = \dfrac{20^2}{2 \times 100} = 2$ Watts.

↳ Side band power : $P_{SB} = P_c \dfrac{\mu_t^2}{2}$

$$P_{SB} = \frac{2 \times 2.12^2}{2} = 4.4944 \text{ Watts.}$$

where, P_{SB} = Total power in all side bands.

(iii) S(f) and Frequency spectrum :-

from given data

$$S(t) = 20\left[1 + 1.5\cos 2000\pi t + 1.5\cos 4000\pi t\right]\cos 40{,}000\pi t$$

i.e. $S(t) = \underset{A_c}{20}\left[1 + \underset{\mu_1}{1.5}\cos(2\pi \times \underset{f_{m1} = 1\times10^3}{1000} \times t) + \underset{\mu_2}{1.5}\cos 2\times \underset{f_{m2} = 2\times10^3}{2000}\pi t\right]\cos 2\pi \times \underset{f_c = 20\times10^3}{20{,}000}\, t$

$$\therefore S(t) = 20\cos 2\pi\times20\times10^3 t + 30\,\underbrace{\cos 2\pi\times20\times10^3 t \cdot \cos 2\pi\times1\times10^3 t}$$
$$+ 30\,\underbrace{\cos 2\pi\times20\times10^3 t \cdot \cos 2\times2\times10^3 t}.$$

W.K.T $\cos A\cdot\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

$$\therefore S(t) = 20\cos 2\pi\times20\times10^3 t + \frac{30}{2}\left[\cos 2\pi\times(20-1)\times10^3 t + \cos 2\pi\times(20+1)\times10^3 t\right]$$
$$+ \frac{30}{2}\left[\cos 2\pi\times(20-2)\times10^3 t + \cos 2\pi(20+2)\times10^3 t\right]$$

$$S(t) = 20\cos 2\pi\times20\times10^3 t + 15\left[\cos 2\pi\times19\times10^3 t + \cos 2\pi\times21\times10^3 t\right]$$
$$+ 15\left[\cos 2\pi\times18\times10^3 t + \cos 2\pi\times22\times10^3 t\right] \quad \text{—— ①}$$

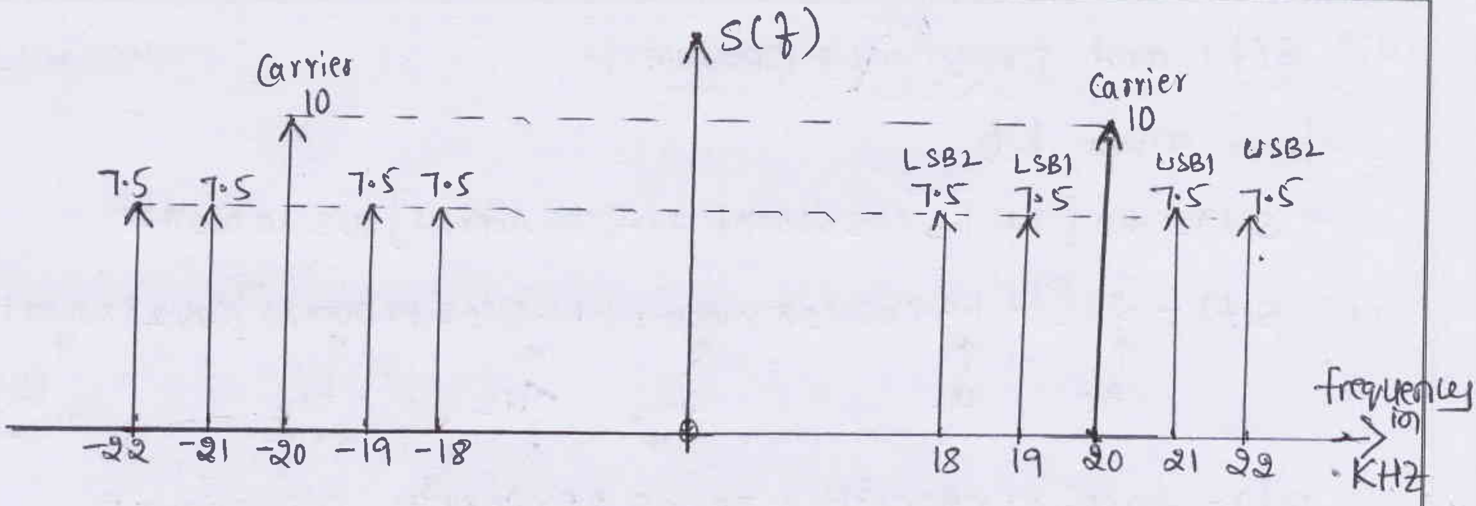
Take Fourier Transformation for Equation ①

$$S(f) = \frac{20}{2}\left[\delta(f-20K) + \delta(f+20K)\right] + \frac{15}{2}\left[\delta(f+19K) + \delta(f-19K)\right]$$
$$+ \frac{15}{2}\left[\delta(f-21K) + \delta(f+21K)\right] + \frac{15}{2}\left[\delta(f+18K) + \delta(f-18K)\right]$$
$$+ \frac{15}{2}\left[\delta(f-22K) + \delta(f+22K)\right]$$

$$\boxed{\begin{aligned}\therefore S(f) = {} & 10\left[\delta(f-20K) + \delta(f+20K)\right] + 7.5\left[\delta(f+19K) + \delta(f-19K)\right. \\ & + \delta(f-21K) + \delta(f+21K) + \delta(f-22K) + \delta(f+22K) \\ & \left. + \delta(f+18K) + \delta(f-18K)\right]\end{aligned}} \quad \text{—— ②}$$

Equation ② gives the equation of S(f) with

$\boxed{f_c = 20\text{KHZ} \quad \text{Amplitude } 10\text{V}}$; $f_{LSB_1} = 19\text{KHZ}$, $f_{USB_1} = 21\text{KHZ}$; $f_{LSB_2} = 18\text{KHZ}$, $f_{USB_2} = 22\text{KHZ}$ } Amplitude 7.5V



-: complete spectrum of s(t) :- (Plot of S(f))

(iv) Total power delivered to a load of 100Ω :-

Method 1:

W.K.T.

$$P_t = P_c\left[1+\frac{\mu_t^2}{2}\right]$$

$$P_c = 2W \; ; \; \mu_t = 2.12$$

$$\therefore P_t = 2\left[1+\frac{2.12^2}{2}\right]$$

$$\boxed{P_t = 6.4944 \text{ Watts}}$$

Method 2:-

Since we already calculated P_c & Total side bands power P_{SB}.

Total power is

$$P_t = P_c + P_{SB}$$

$$P_t = 2 + 4.4944$$

$$\boxed{P_t = 6.4944} \text{ Watts}$$

<4> An AM-Broadcasting transmitter radiates 50KW of carrier power. What will be the radiated power at 85% Modulation?

↳ Given data : carrier power, $P_c = 50KW$; $\mu = 0.85$

$\therefore$ radiated power, $P_t = P_c\left[1+\frac{\mu^2}{2}\right] = 50\times10^3\left[1+\frac{0.85^2}{2}\right]$

$$\boxed{P_t = 68.0625 \text{ KW}}$$

⟨5⟩ An audio frequency signal $10 \sin 2\pi(500)t$ is used to amplitude modulate a carrier of $50 \sin 2\pi(10^5)t$. Assume modulation index = 0.2. Determine.

i) side band frequencies

ii) Amplitude of each side band.

iii) Bandwidth required.

iv) Efficiency of AM wave.

↳ Given: $m(t) = 10 \sin 2\pi(500)t \Rightarrow A_m = 10V \ \&\ f_m = 500\,Hz$

$f_m = 0.5\,KHZ$

$c(t) = 50 \sin 2\pi(10^5)t \Rightarrow A_c = 50V \ \&\ f_c = 10^5\,Hz$

and $\mu = 0.2$ (given)

$f_c = 100\,KHZ$

i) Side band frequencies:-

- $f_{USB} = f_c + f_m = 100k + 0.5k = \underline{100.5\,KHZ}$.
- $f_{LSB} = f_c - f_m = 100k - 0.5k = \underline{99.5\,KHZ}$.

ii) Amplitude of Each side band $= \dfrac{\mu A_c}{2} = \dfrac{0.2 \times 50}{2} = \underline{5V}$

iii) Bandwidth required: $BW = 2f_m = 2(0.5\,KHz)$

$$\boxed{BW = 1\,KHZ}$$

iv) Efficiency of AM wave:-

$$\%\eta = \frac{\mu^2}{2+\mu^2} \times 100 = \frac{0.2^2}{2+0.2^2} \times 100$$

$$\boxed{\%\eta = 1.96}$$

<V.T.U> <6> An Amplitude modulated signal is given by

$s(t) = [10\cos 2\pi\times10^6 t + 5\cos 2\pi\times10^6 t \cdot \cos 2\pi\times10^3 t + 2\cos 2\pi\times10^6 t \cdot \cos 4\pi\times10^3 t]$. Determine.

i) Net Modulation index

ii) side band power

iii) Total modulated power. Assume $R = 100\Omega$

↳ Given AM signal is

$$s(t) = [10\cos 2\pi\times10^6 t + 5\cos 2\pi\times10^6 t \cdot \cos 2\pi\times10^3 t + 2\cos 2\pi\times10^6 t \cdot \cos 4\pi\times10^3 t]$$

$$s(t) = 10\cos 2\pi\times10^6 t \left[1 + \frac{5}{10}\cos 2\pi\times10^3 t + \frac{2}{10}\cos 4\pi\times10^3 t\right]$$

$$s(t) = 10[1 + 0.5\cos 2\pi\times10^3 t + 0.2\cos 4\pi\times10^3 t]\cos 2\pi\times10^6 t \quad \text{—(1)}$$

The standard AM equation for two message signals is

$$s(t) = A_c[1 + \mu_1\cos 2\pi f_{m_1} t + \mu_2\cos 2\pi f_{m_2} t]\cos 2\pi f_c t \quad \text{—(2)}$$

$\therefore A_c = 10V$; $\mu_1 = 0.5$; $\mu_2 = 0.2$; $f_{m_1} = 1\times10^3 = 1KHZ$; $f_{m_2} = 2\times10^3 = 2KHZ$; $f_c = 10^6 HZ$; $f_c = 1000KHZ$

<i> Net Modulation Index : $\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{0.5^2 + 0.2^2} = 0.538$

<ii> Sideband power :- $P_{SB} = P_{LSB} + P_{USB} = P_c \cdot \frac{\mu_t^2}{2}$

$$P_c = \frac{A_c^2}{2R} = \frac{10^2}{2\times100} = 0.5W.$$

$$\therefore P_{SB} = 0.5\times\frac{0.538^2}{2} = 0.072\,W$$

<iii> Total modulated power :- $P_t = P_c\left[1 + \frac{\mu_t^2}{2}\right] = P_c + P_{SB} = 0.572W.$

7) An AM signal has the form,

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$$S(t) = \cos(2000\pi t) + 4\cos(2400\pi t) + \cos(2800\pi t).$$

Determine the ratio of power in message signal to that of power in unmodulated Carrier signal.

↳ Given AM-equation is

$$S(t) = \cos(2000\pi t) + 4\cos(2400\pi t) + \cos(2800\pi t)$$

$$\therefore S(t) = \cos(2\pi \times \underset{\uparrow f_{LSB}}{1000} \times t) + 4\cos(2\pi \times \underset{\uparrow f_c}{1200} t) + \cos(2\pi \times \underset{\uparrow f_{USB}}{1400} t) \quad \text{—①}$$

It has 3-components, carrier signal, Lower side band (LSB) and upper side band (USB).

$\therefore$ the Amplitude of Carrier, $A_c = 4V$.

Amplitude of LSB & USB is $\frac{\mu A_c}{2} = 1 \Rightarrow \boxed{\mu = \frac{2}{A_c} = \frac{2}{4} = \frac{1}{2}}$

$\therefore$ Amplitude of message signal, $A_m = A_c \times \mu = 4 \times \frac{1}{2} = 2V \quad \left(\because \mu = \frac{A_m}{A_c}\right)$

$\therefore$ the Ratio of power in Message signal to that of Carrier power $\Bigg\} = \dfrac{\left(\frac{A_m^2}{2R}\right)}{\left(\frac{A_c^2}{2R}\right)}$ ← Power in message signal / ← power in Carrier signal

$$\frac{P_m}{P_c} = \frac{A_m^2}{A_c^2} = \left(\frac{A_m}{A_c}\right)^2$$

$$\frac{P_m}{P_c} = \left(\frac{2}{4}\right)^2 = (0.5)^2$$

$$\boxed{\frac{P_m}{P_c} = 0.25}$$

*Note:- In General for Any AM signal the Ratio of power present in Message signal to that of Carrier signal is equal to "μ^2"

- For problem 10, $\boxed{\frac{P_m}{P_c} = \mu^2 = (0.5)^2 = 0.25}$

8) Consider a 2-stage SSB-Modulator as shown in figure.1. the input signal consists of a voice signal in a frequency range of 300HZ to 3.4KHZ. The two oscillators frequencies have values $f_{c_1} = 100KHZ$ and $f_{c_2} = 10MHZ$. Determine

(i) sidebands of DSBSC modulated waves appearing at the outputs of the product modulators.

(ii) sidebands of SSB modulated wave appearing at two BPF's output.

(iii) passband and Guard band of two BPF's.

(iv) Sketch the spectrum at each stage of the

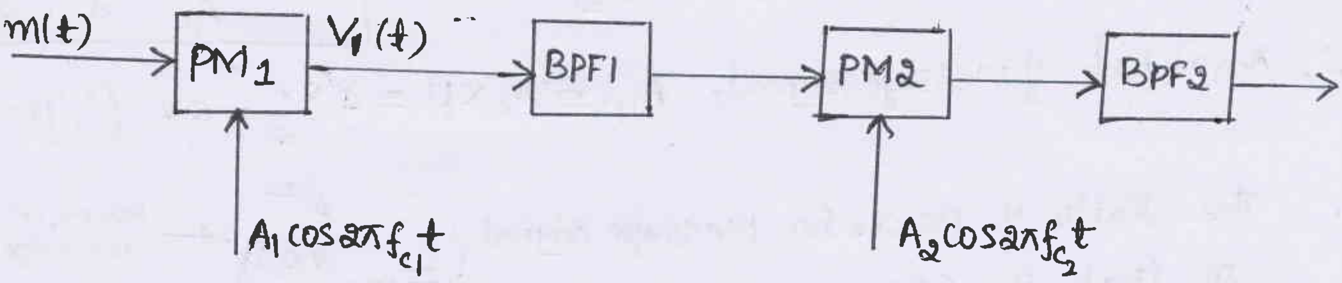


Figure1: Two Stage SSB Modulator

↳ Given :- Frequency of m(t) : $f_m = 300HZ$ to $3.4KHZ$

$f_m = 0.3KHZ$ to $3.4KHZ$

Frequency of Carrier1 : $f_{c_1} = 100KHZ$
Used for PM1

Frequency of Carrier 2 : $f_{c_2} = 10MHZ$
Used for PM2

i) The PM1 output $V_1(t)$ consists of two side bands as follows : $LSB = f_{c_1} - f_m = 100KHZ - (0.3KHZ \text{ to } 3.4KHZ)$

$$\boxed{LSB = 99.7KHZ \text{ to } 96.6KHZ}$$

and, $USB = f_{c_1} + f_m$

$USB = 100KHz + (0.3KHz \text{ to } 3.4KHz)$

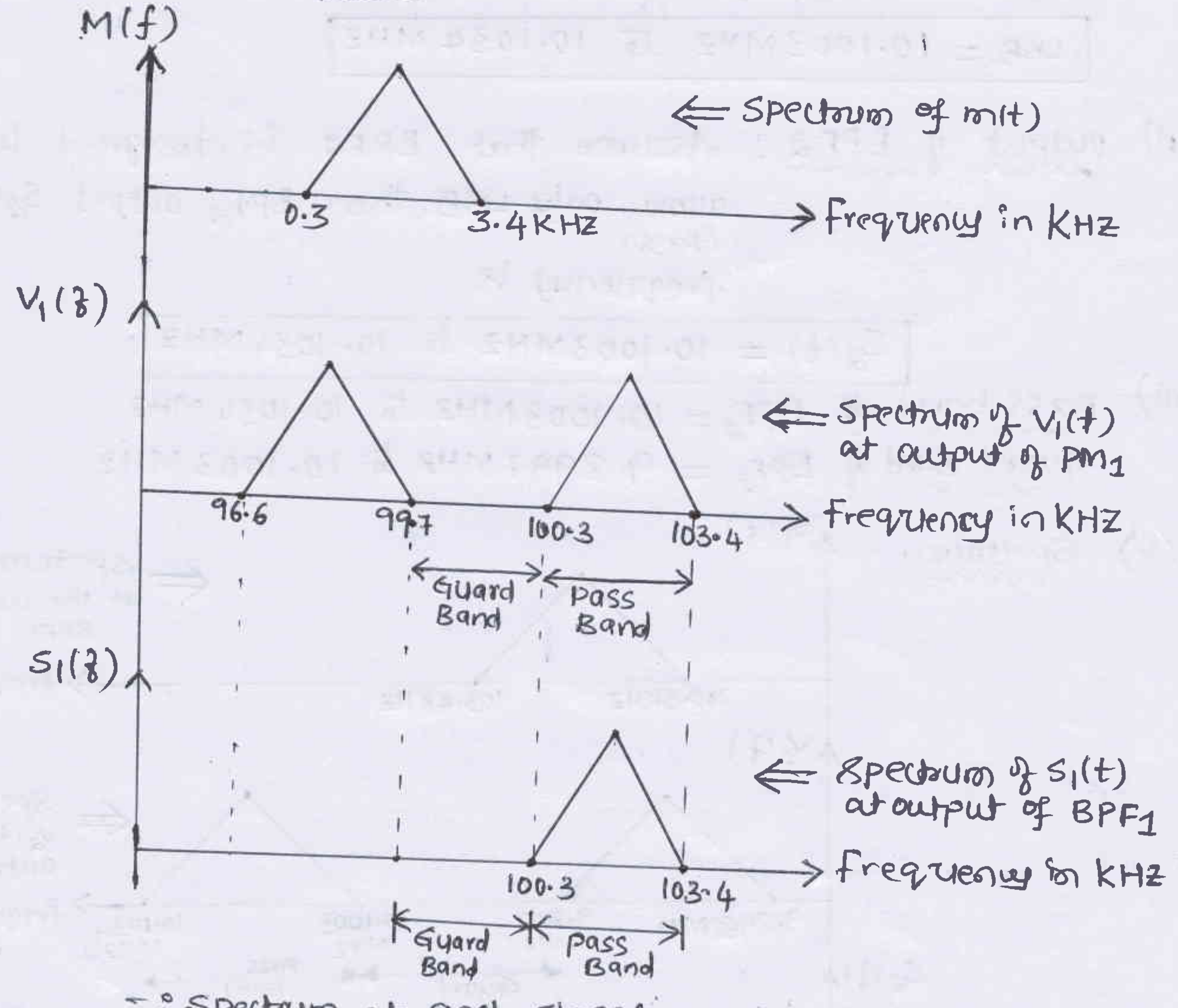
$$\boxed{USB = 100.3KHz \text{ to } 103.4\ KHz}$$

ii) output of BPF1 : Assume that BPF1 is designed to allow (pass) only USB. Then, BPF1 output $S_1(t)$ frequency is.

$$\boxed{S_1(t) = 100.3KHz \text{ to } 103.4KHz}$$

iii) ∴ Pass Band of BPF1 = 100.3 KHz to 103.4KHz.
Guard Band of BPF_1 = 99.7 KHz to 100.3 KHz
} shown in Spectrum

iv)



∴ Spectrum at each stage :-

* Similarly, the PM2 output consists of two side bands as follows. (Its input is $s_1(t)$ as message signal with frequency $f_m = f_{USB} =$ $\underbrace{100.3\,KHZ \text{ to } 103.4\,KHZ}_{m(t) \text{ frequency Range for PM2}}$ & carrier frequency $f_{c_2} = 10MHZ$)

(i) $LSB = f_{c_2} - f_m$

$= 10MHZ - (100.3\,KHZ \text{ to } 103.4\,KHZ)$

$\boxed{LSB = 9.8997\,MHZ \text{ to } 9.8966\,MHZ}$ and

$USB = f_{c_2} + f_m$

$= 10MHZ + (100.3\,KHZ \text{ to } 103.4\,KHZ)$

$\boxed{USB = 10.1003\,MHZ \text{ to } 10.1034\,MHZ}$

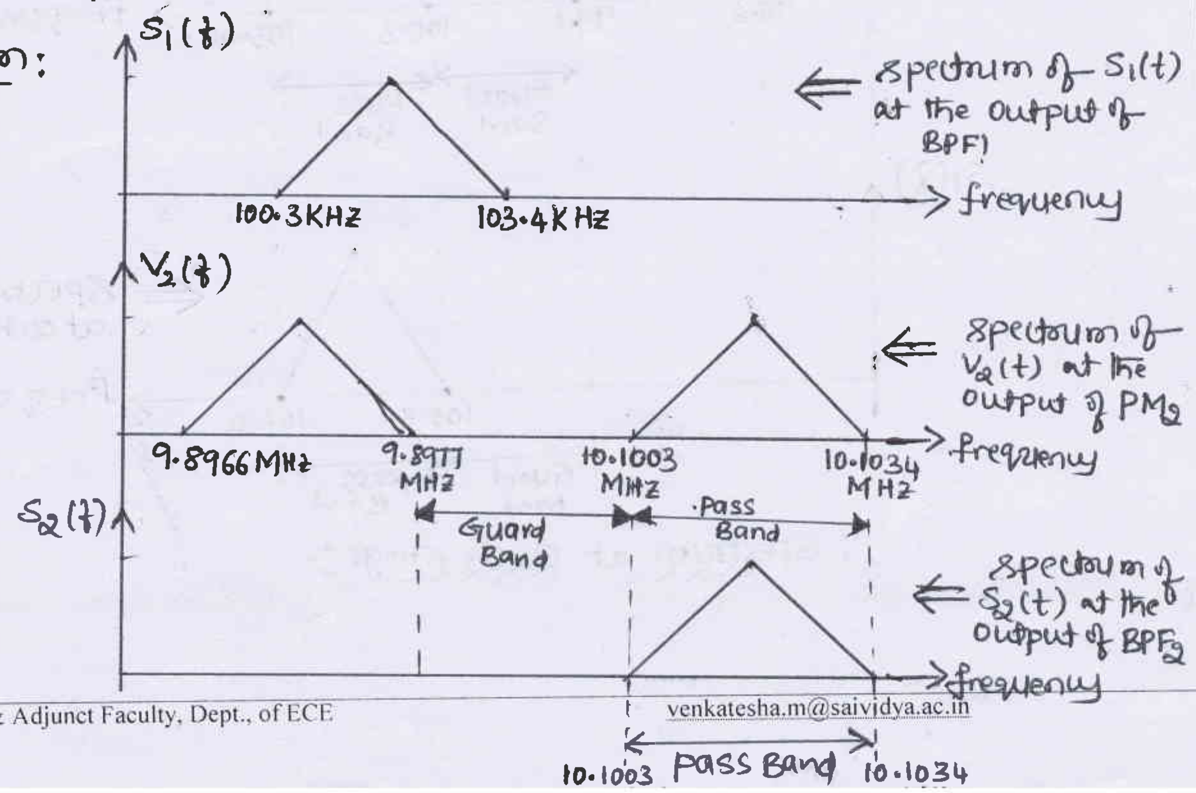
ii) output of BPF2: Assume that BPF2 is designed to allow (pass) only USB. then BPF_2 output $S_2(t)$ frequency is

$\boxed{S_2(t) = 10.1003\,MHZ \text{ to } 10.1034\,MHZ}$

iii) pass band of $BPF_2 = 10.1003\,MHZ$ to $10.1034\,MHZ$

Guard Band of $BPF_2 = 9.8997\,MHZ$ to $10.1003\,MHZ$.

iv) Spectrum:



9) A 250W carrier of 1000KHZ is simultaneously modulated by sinusoidal signals of 2KHZ, 6KHZ and 8KHZ with modulation indices of 35%, 55% and 75% respectively. What are the frequencies present in the modulated wave and what is the radiated power.

June/July 2016

5 Marks

↳ Given Data:- $P_c = 250W$; $f_c = 1000\,KHZ$

$f_{m1} = 2\,KHZ$; $f_{m2} = 6\,KHZ$; $f_{m3} = 8\,KHZ$

$\mu_1 = 0.35$; $\mu_2 = 0.55$; $\mu_3 = 0.75$

i) Frequencies present in the Modulated Wave:-

↳ carrier $f_c = 1000\,KHZ$

↳ $LSB_1 = f_c - f_{m_1} = 1000K - 2K = 998\,KHZ$

↳ $USB_1 = f_c + f_{m_2} = 1000K + 2K = 1002\,KHZ$

↳ $LSB_2 = f_c - f_{m_2} = 1000K - 6K = 994\,KHZ$

↳ $USB_2 = f_c + f_{m_2} = 1000K + 6K = 1006\,KHZ$

↳ $LSB_3 = f_c - f_{m_3} = 1000K - 8K = 992\,KHZ$

↳ $USB_3 = f_c + f_{m_3} = 1000K + 8K = 1008\,KHZ$

ii) Radiated power:-

W.K.T $P_t = P_c\left(1 + \frac{\mu_t^2}{2}\right)$

$P_c = 250W$

$\mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2} = \sqrt{0.35^2 + 0.55^2 + 0.75^2}$

$\mu_t = 0.9937$

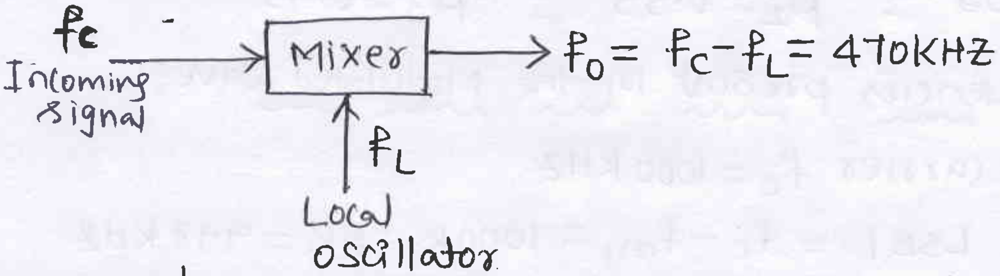
∴ Radiated power, $P_t = 250\left(1 + \frac{0.9937^2}{2}\right)$

$\boxed{P_t = 373.4}$ Watts

10) The Incoming signal has a midband frequency that may lie in the range 530KHZ to 1650KHZ. The associated bandwidth is 10KHZ. This signal is to be translated to a fixed frequency band Centered at 470KHZ. Determine the tuning range that must be provided by the local oscillator.

Given: $f_c = 530KHZ$ to $1650KHZ$; $BW = 10KHZ$

$f_0 = 470KHZ$; $f_L = ?$ ∴ It is down frequency Translator (or) Mixer.



From given data Translator (mixer) output $f_0 = 470KHZ = f_c - f_L$

∴ Local oscillator frequency, f_L is given by;

$$f_L = f_c - f_0$$

When, $f_c = 530KHZ$; $f_L = 530K - 470K = 60KHZ$

$f_c = 1650KHZ$; $f_L = 1650K - 470K = 1180KHZ$

∴ Tuning range of Local oscillator frequency f_L is 60KHZ to 1180KHZ.

11) Determine the Bandwidth of FDM-system which uses SSB modulation at the transmitter for 24 voice signals having a Bandwidth of 4KHZ each.

↳ Given $N = 24$, Voice signals with SSB Modulation.

$W = f_m = 4KHZ$

∴ Total BandWidth of FDM system is

$$BW = N \times f_m = 24 \times 4KHZ$$

$$\boxed{BW = 96KHZ}$$

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