Chapter 4

Initial Conditions in Networks

4.1 Introduction

There are many reasons for studying initial and final conditions. The most important reason is that the initial and final conditions evaluate the arbitrary constants that appear in the general solution of a differential equation.

In this chapter, we concentrate on finding the change in selected variables in a circuit when a switch is thrown open from closed position or vice versa. The time of throwing the switch is considered to be $t = 0$, and we want to determine the value of the variable at $t = 0^-$ and at $t = 0^+$, immediately before and after throwing the switch. Thus a switched circuit is an electrical circuit with one or more switches that open or close at time $t = 0$. We are very much interested in the change in currents and voltages of energy storing elements after the switch is thrown since these variables along with the sources will dictate the circuit behaviour for $t > 0$.

Initial conditions in a network depend on the past history of the circuit (before $t = 0^-$) and structure of the network at $t = 0^+$, (after switching). Past history will show up in the form of capacitor voltages and inductor currents. The computation of all voltages and currents and their derivatives at $t = 0^+$ is the main aim of this chapter.

4.2 Initial and final conditions in elements

4.2.1 The inductor

The switch is closed at $t = 0$. Hence $t = 0^-$ corresponds to the instant when the switch is just open and $t = 0^+$ corresponds to the instant when the switch is just closed.

The expression for current through the inductor is given by

$$i = \frac{1}{L}\int_{-\infty}^{t} v\,d\tau$$

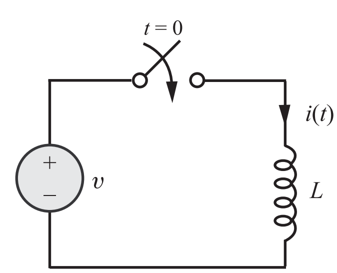

Figure 4.1 Circuit for explaining switching action of an inductor

$$\Rightarrow \qquad i = \frac{1}{L}\int_{-\infty}^{0^-} v d\tau + \frac{1}{L}\int_{0^-}^{t} v d\tau$$

$$\Rightarrow \qquad i(t) = i(0^-) + \frac{1}{L}\int_{0^-}^{t} v d\tau$$

Putting $t = 0^+$ on both sides, we get

$$i(0^+) = i(0^-) + \frac{1}{L}\int_{0^-}^{0^+} v d\tau$$

$$\Rightarrow \qquad i(0^+) = i(0^-)$$

The above equation means that the current in an inductor cannot change instantaneously. Consequently, if $i(0^-) = 0$, we get $i(0^+) = 0$. This means that at $t = 0^+$, inductor will act as an open circuit, irrespective of the voltage across the terminals. If $i(0^-) = I_o$, then $i(0^+) = I_o$. In this case at $t = 0^+$, the inductor can be thought of as a current source of I_o A. The equivalent circuits of an inductor at $t = 0^+$ is shown in Fig. 4.2.

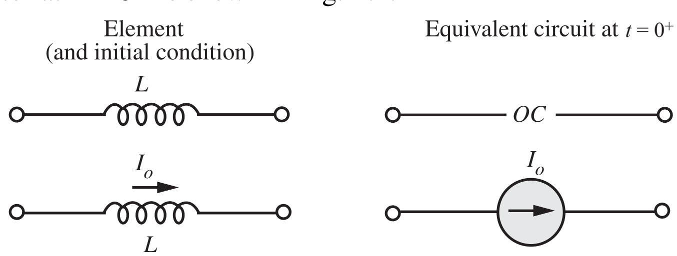

Figure 4.2 The initial-condition equivalent circuits of an inductor

The final-condition equivalent circuit of an inductor is derived from the basic relationship

$$v = L\frac{di}{dt}$$

Under steady condition, $\frac{di}{dt} = 0$. This means, $v = 0$ and hence L acts as short at $t = \infty$ (final or steady state). The final-condition equivalent circuits of an inductor is shown in Fig.4.3.

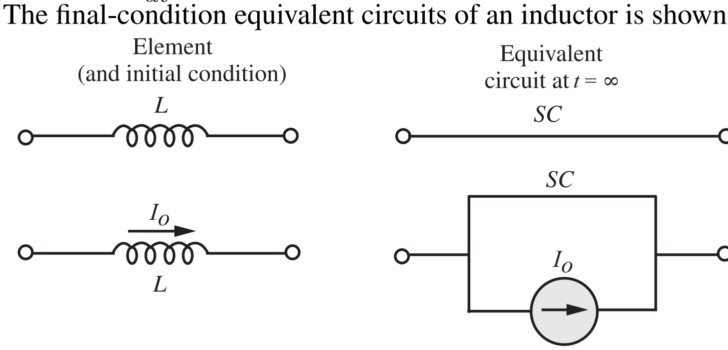

Figure 4.3 The final-condition equivalent circuit of an inductor

4.2.2 The capacitor

The switch is closed at $t = 0$. Hence, $t = 0^-$ corresponds to the instant when the switch is just open and $t = 0^+$ corresponds to the instant when the switch is just closed. The expression for voltage across the capacitor is given by

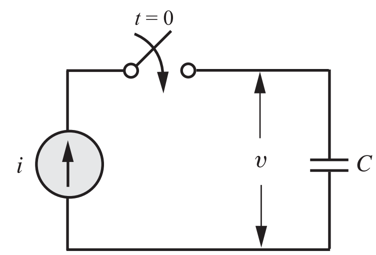

Figure 4.4 Circuit for explaining switching action of a Capacitor

$$v = \frac{1}{C}\int_{-\infty}^{t} i d\tau$$

$$\Rightarrow \qquad v(t) = \frac{1}{C}\int_{-\infty}^{0^-} i d\tau + \frac{1}{C}\int_{0^-}^{t} i d\tau$$

$$\Rightarrow \qquad v(t) = v(0^-) + \frac{1}{C}\int_{0^-}^{t} i d\tau$$

Evaluating the expression at $t = 0^+$, we get

$$v(0^+) = v(0^-) + \frac{1}{C}\int_{0^-}^{0^+} i d\tau \quad \Rightarrow \quad v(0^+) = v(0^-)$$

Thus the voltage across a capacitor cannot change instantaneously.

If $v(0^-) = 0$, then $v(0^+) = 0$. This means that at $t = 0^+$, capacitor C acts as short circuit. Conversely, if $v(0^-) = \frac{q_0}{C}$ then $v(0^+) = \frac{q_0}{C}$. These conclusions are summarized in Fig. 4.5.

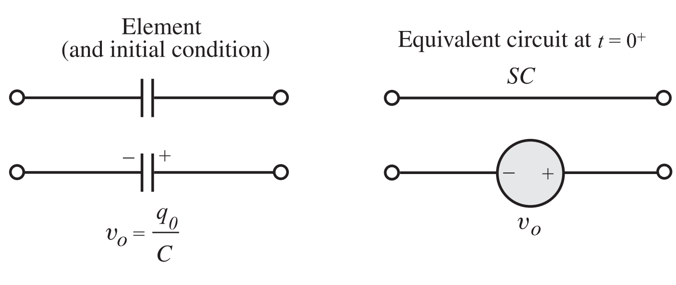

Figure 4.5 Initial-condition equivalent circuits of a capacitor

The final–condition equivalent network is derived from the basic relationship

$$i = C\frac{dv}{dt}$$

Under steady state condition, $\frac{dv}{dt} = 0$. This is, at $t = \infty$, $i = 0$. This means that $t = \infty$ or in steady state, capacitor C acts as an open circuit. The final condition equivalent circuits of a capacitor is shown in Fig. 4.6.

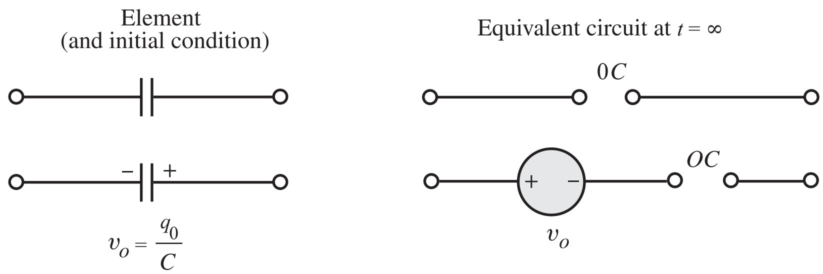

Figure 4.6 Final-condition equivalent circuits of a capacitor

4.2.3 The resistor

The cause–effect relation for an ideal resistor is given by $v = Ri$. From this equation, we find that the current through a resistor will change instantaneously if the voltage changes instantaneously. Similarly, voltage will change instantaneously if current changes instantaneously.

4.3 Procedure for evaluating initial conditions

There is no unique procedure that must be followed in solving for initial conditions. We usually solve for initial values of currents and voltages and then solve for the derivatives. For finding initial values of currents and voltages, an equivalent network of the original network at $t = 0^+$ is constructed according to the following rules:

(1) Replace all inductors with open circuit or with current sources having the value of current flowing at $t = 0^+$.

(2) Replace all capacitors with short circuits or with a voltage source of value $v_o = \frac{q_0}{C}$ if there is an initial charge.

(3) Resistors are left in the network without any changes.

EXAMPLE 4.1

Refer the circuit shown in Fig. 4.7(a). Find $i_1(0^+)$ and $i_L(0^+)$. The circuit is in steady state for $t < 0$.

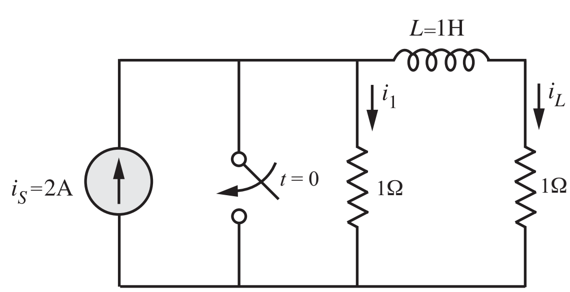

Figure 4.7(a)

SOLUTION

The symbol for the switch implies that it is open at $t = 0^-$ and then closed at $t = 0^+$. The circuit is in steady state with the switch open. This means that at $t = 0^-$, inductor L is short. Fig.4.7(b) shows the original circuit at $t = 0^-$.

Using the current division principle,

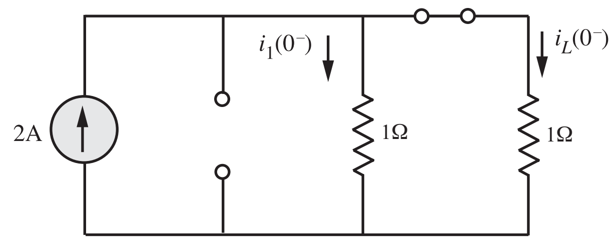

Figure 4.7(b)

$$i_L(0^-) = \frac{2 \times 1}{1+1} = 1\text{A}$$

Since the current in an inductor cannot change instantaneously, we have

$$i_L(0^+) = i_L(0^-) = 1\text{A}$$

At $t = 0^-$, $i_1(0^-) = 2 - 1 = 1\text{A}$. Please note that the current in a resistor can change instantaneously. Since at $t = 0^+$, the switch is just closed, the voltage across R_1 will be equal to zero because of the switch being short circuited and hence,

$$i_1(0^+) = 0\text{A}$$

Thus, the current in the resistor changes abruptly form 1A to 0A.

EXAMPLE 4.2

Refer the circuit shown in Fig. 4.8. Find $v_C(0^+)$. Assume that the switch was in closed state for a long time.

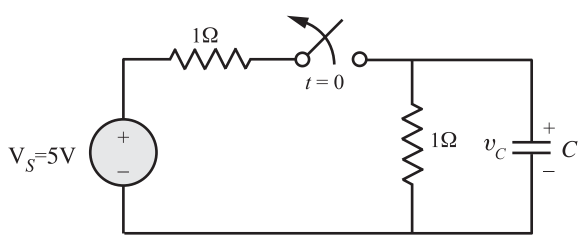

Figure 4.8

SOLUTION

The symbol for the switch implies that it is closed at $t = 0^-$ and then opens at $t = 0^+$. Since the circuit is in steady state with the switch closed, the capacitor is represented as an open circuit at $t = 0^-$. The equivalent circuit at $t = 0^-$ is as shown in Fig. 4.9.

$$v_C(0^-) = i(0^-)R_2$$

Using the principle of voltage divider,

$$v_C(0^-) = \frac{V_S}{R_1 + R_2} R_2 = \frac{5 \times 1}{1+1} = 2.5 \text{ V}$$

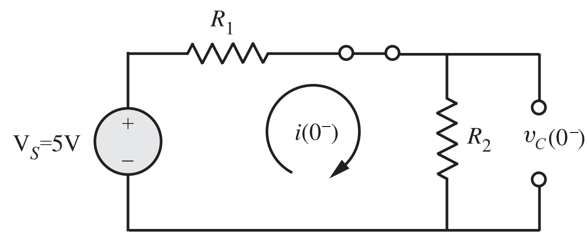

Figure 4.9

Since the voltage across a capacitor cannot change instaneously, we have

$$v_C(0^+) = v_C(0^-) = 2.5\text{V}$$

That is, when the switch is opened at $t = 0$, and if the source is removed from the circuit, still $v_C(0^+)$ remains at 2.5 V.

EXAMPLE 4.3

Refer the circuit shown in Fig 4.10. Find $i_L(0^+)$ and $v_C(0^+)$. The circuit is in steady state with the switch in closed condition.

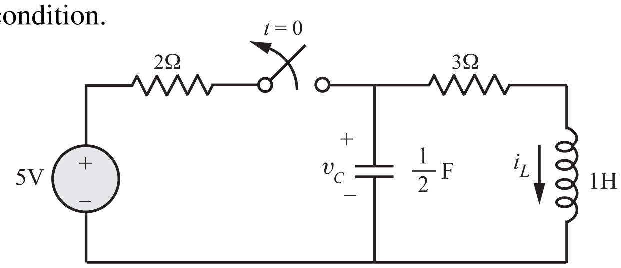

Figure 4.10

SOLUTION

The symbol for the switch implies, it is closed at $t = 0^-$ and then opens at $t = 0^+$. In order to find $v_C(0^-)$ and $i_L(0^-)$ we replace the capacitor by an open circuit and the inductor by a short circuit, as shown in Fig.4.11, because in the steady state L acts as a short circuit and C as an open circuit.

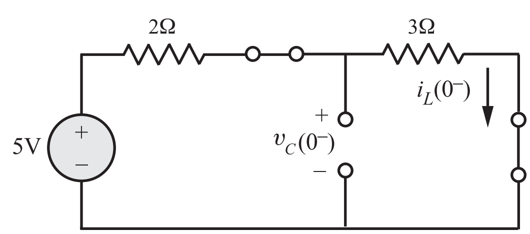

Figure 4.11

$$i_L(0^-) = \frac{5}{2+3} = 1 \text{ A}$$

Using the voltage divider principle, we note that

$$v_C(0^-) = \frac{5 \times 3}{3+2} = 3 \text{ V}$$

Then we note that:

$$v_C(0^+) = v_C(0^-) = 3 \text{ V}$$
$$i_L(0^+) = i_L(0^-) = 2 \text{ A}$$

EXAMPLE 4.4

In the given network, K is closed at $t = 0$ with zero current in the inductor. Find the values of i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t = 0^+$ if $R = 8\Omega$ and $L = 0.2$H. Refer the Fig. 4.12(a).

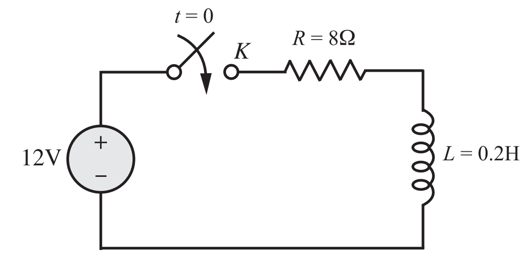

Figure 4.12(a)

SOLUTION

The symbol for the switch implies that it is open at $t = 0^-$ and then closes at $t = 0^+$. Since the current i through the inductor at $t = 0^-$ is zero, it implies that $i(0^+) = i(0^-) = 0$.

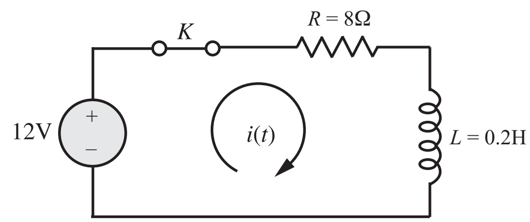

Figure 4.12(b)

To find $\frac{di(0^+)}{dt}$ and $\frac{d^2i(0^+)}{dt^2}$:

Applying KVL clockwise to the circuit shown in Fig. 4.12(b), we get

$$Ri + L\frac{di}{dt} = 12$$

$$\Rightarrow \quad 8i + 0.2\frac{di}{dt} = 12 \tag{4.1}$$

At $t = 0^+$, the equation (4.1) becomes

$$8i(0^+) + 0.2\frac{di(0^+)}{dt} = 12$$

$$\Rightarrow \quad 8 \times 0 + 0.2\frac{di(0^+)}{dt} = 12$$

$$\Rightarrow \quad \frac{di(0^+)}{dt} = \frac{12}{0.2}$$

$$= \mathbf{60\ A/sec}$$

Differentiating equation (4.1) with respect to t, we get

$$8\frac{di}{dt} + 0.2\frac{d^2i}{dt^2} = 0$$

At $t = 0^+$, the above equation becomes

$$8\frac{di(0^+)}{dt} + 0.2\frac{d^2i(0^+)}{dt^2} = 0$$

$$\Rightarrow \quad 8 \times 60 + 0.2\frac{d^2i(0^+)}{dt^2} = 0$$

Hence

$$\frac{d^2i(0^+)}{dt^2} = \mathbf{-2400\ A/sec^2}$$

EXAMPLE 4.5

In the network shown in Fig. 4.13, the switch is closed at $t = 0$. Determine i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t = 0^+$.

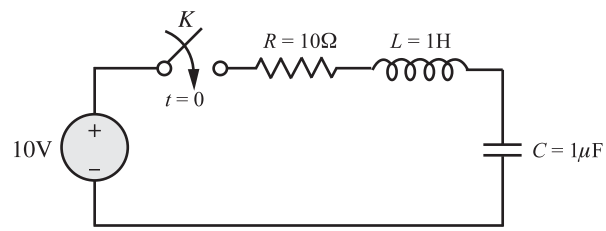

Figure 4.13

SOLUTION

The symbol for the switch implies that it is open at $t = 0^-$ and then closes at $t = 0^+$. Since there is no current through the inductor at $t = 0^-$, it implies that $i(0^+) = i(0^-) = 0$.

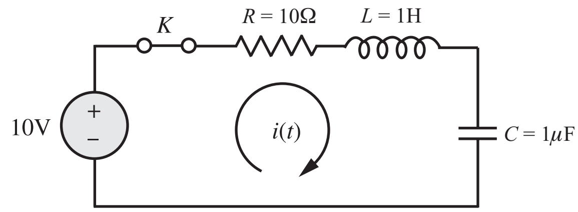

Figure 4.14

Writing *KVL clockwise* for the circuit shown in Fig. 4.14, we get

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i(\tau)d\tau = 10 \tag{4.2}$$

$$\Rightarrow \quad Ri + L\frac{di}{dt} + v_C(t) = 10 \tag{4.2a}$$

Putting $t = 0^+$ in equation (4.2a), we get

$$Ri\left(0^+\right) + L\frac{di\left(0^+\right)}{dt} + v_C\left(0^+\right) = 10$$

$$\Rightarrow \quad R \times 0 + L\frac{di\left(0^+\right)}{dt} + 0 = 10$$

$$\Rightarrow \quad \frac{di\left(0^+\right)}{dt} = \frac{10}{L} = \mathbf{10\ A/sec}$$

Differentiating equation (4.2) with respect to t, we get

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i(t)}{C} = 0$$

At $t = 0^+$, the above equation becomes

$$R\frac{di\left(0^+\right)}{dt} + L\frac{d^2i\left(0^+\right)}{dt^2} + \frac{i\left(0^+\right)}{C} = 0$$

$$\Rightarrow \qquad R \times 10 + L\frac{d^2i\left(0^+\right)}{dt^2} + \frac{0}{C} = 0$$

$$\Rightarrow \qquad 100 + \frac{d^2i\left(0^+\right)}{dt^2} = 0$$

Hence at $t = 0^+$, $$\frac{d^2i\left(0^+\right)}{dt^2} = \mathbf{-100\ A/sec^2}$$

EXAMPLE 4.6

Refer the circuit shown in Fig. 4.15. The switch K is changed from position 1 to position 2 at $t = 0$. Steady-state condition having been reached at position 1. Find the values of i, $\frac{di}{dt}$, and $\frac{d^2i}{dt^2}$ at $t = 0^+$.

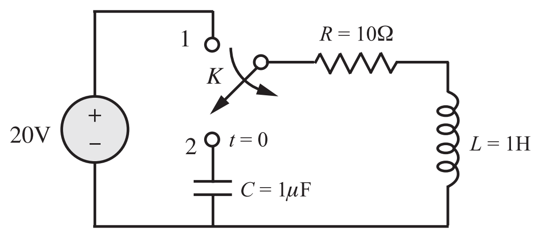

Figure 4.15

SOLUTION

The symbol for switch K implies that it is in position 1 at $t = 0^-$ and in position 2 at $t = 0^+$. Under steady-state condition, inductor acts as a short circuit. Hence at $t = 0^-$, the circuit diagram is as shown in Fig. 4.16.

$$i\left(0^-\right) = \frac{20}{10} = \mathbf{2A}$$

Since the current through an inductor cannot change instantaneously, $i\left(0^+\right) = i\left(0^-\right) = 2\text{A}$. Since there is no initial charge on the capacitor, $v_C\left(0^-\right) = 0$. Since the voltage across a capacitor cannot change instantaneously, $v_C\left(0^+\right) = v_C\left(0^-\right) = 0$. Hence at $t = 0^+$ the circuit diagram is as shown in Fig. 4.17(a).

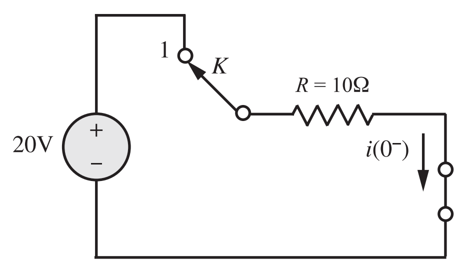

Figure 4.16

For $t \geq 0^+$, the circuit diagram is as shown in Fig. 4.17(b).

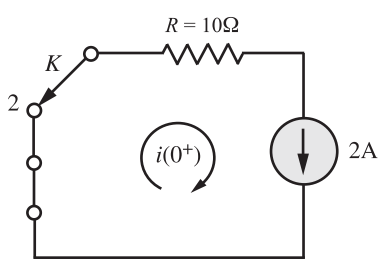

Figure 4.17(a)

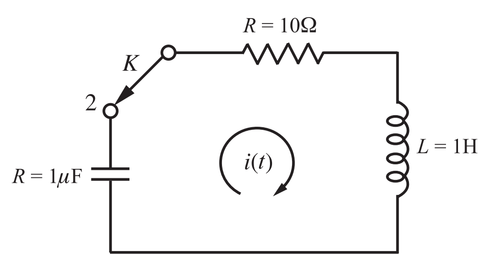

Figure 4.17(b)

Applying KVL clockwise to the circuit shown in Fig. 4.17(b), we get

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_{0^+}^{t} i(\tau)d\tau = 0 \tag{4.3}$$

$$\Rightarrow \qquad Ri(t) + L\frac{di(t)}{dt} + v_C(t) = 0 \tag{4.3a}$$

At $t = 0^+$ equation (4.3a) becomes

$$Ri\left(0^+\right) + L\frac{di\left(0^+\right)}{dt} + v_C\left(0^+\right) = 0$$

$$\Rightarrow \qquad R \times 2 + L\frac{di\left(0^+\right)}{dt} + 0 = 0$$

$$\Rightarrow \qquad 20 + \frac{di\left(0^+\right)}{dt} = 0$$

$$\Rightarrow \qquad \frac{di\left(0^+\right)}{dt} = \mathbf{-20\ A/sec}$$

Differentiating equation (4.3) with respect to t, we get

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

At $t = 0^+$, we get

$$R\frac{di\left(0^+\right)}{dt} + L\frac{d^2i\left(0^+\right)}{dt^2} + \frac{i\left(0^+\right)}{C} = 0$$

$$\Rightarrow \qquad R \times (-20) + L\frac{d^2i\left(0^+\right)}{dt^2} + \frac{2}{C} = 0$$

Hence, $$\frac{d^2i\left(0^+\right)}{dt^2} \approx \mathbf{-2 \times 10^6\ A/sec^2}$$

EXAMPLE 4.7

In the network shown in Fig. 4.18, the switch is moved from position 1 to position 2 at $t = 0$. The steady-state has been reached before switching. Calculate i, $\frac{di}{dt}$, and $\frac{d^2i}{dt^2}$ at $t = 0^+$.

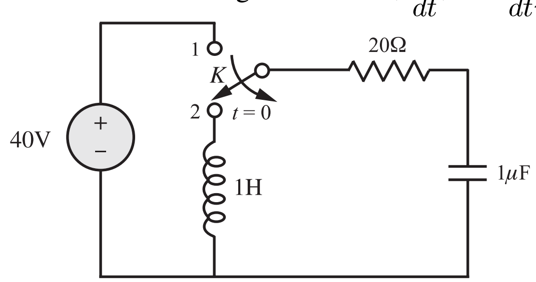

Figure 4.18

SOLUTION

The symbol for switch K implies that it is in position 1 at $t = 0^-$ and in position 2 at $t = 0^+$. Under steady-state condition, a capacitor acts as an open circuit. Hence at $t = 0^-$, the circuit diagram is as shown in Fig. 4.18(a).

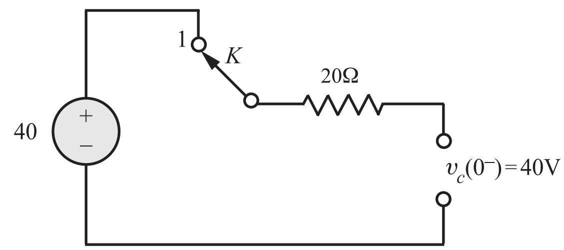

Figure 4.18(a)

We know that the voltage across a capacitor cannot change instantaneously. This means that $v_C\left(0^+\right) = v_C\left(0^-\right) = 40$ V.

At $t = 0^-$, inductor is not energized. This means that $i\left(0^-\right) = 0$. Since current in an inductor cannot change instantaneously, $i\left(0^+\right) = i\left(0^-\right) = \mathbf{0}$. Hence, the circuit diagram at $t = 0^+$ is as shown in Fig. 4.18(b).

The circuit diagram for $t \geq 0^+$ is as shown in Fig.4.18(c).

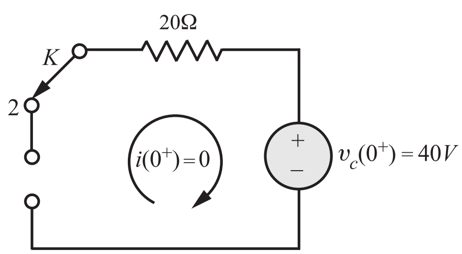

Figure 4.18(b)

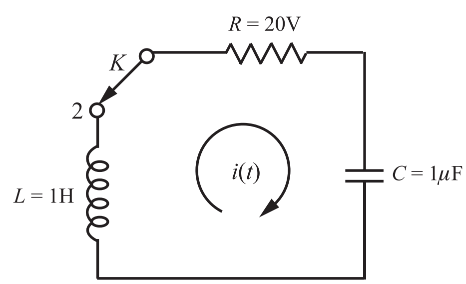

Figure 4.18(c)

Applying KVL clockwise, we get

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{0^+}^{t} i(\tau)d\tau = 0 \tag{4.4}$$

$$\Rightarrow \qquad Ri + L\frac{di}{dt} + v_C(t) = 0$$

At $t = 0^+$, we get

$$Ri(0^+) + L\frac{di(0^+)}{dt} + v_C(0^+) = 0$$

$$\Rightarrow \qquad 20 \times 0 + 1\frac{di(0^+)}{dt} + 40 = 0$$

$$\Rightarrow \qquad \frac{di(0^+)}{dt} = \mathbf{-40A/\,sec}$$

Diferentiating equation (4.4) with respect to t, we get

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

Putting $t = 0^+$ in the above equation, we get

$$R\frac{di(0^+)}{dt} + L\frac{d^2i(0^+)}{dt^2} + \frac{i(0^+)}{C} = 0$$

$$\Rightarrow \quad R \times (-40) + L\frac{d^2i(0^+)}{dt^2} + \frac{0}{C} = 0$$

Hence $$\frac{d^2i(0^+)}{dt^2} = \mathbf{800A/\,sec^2}$$

EXAMPLE 4.8

In the network shown in Fig. 4.19, $v_1(t) = e^{-t}$ for $t \geq 0$ and is zero for all $t < 0$. If the capacitor is initially uncharged, determine the value of $\frac{d^2v_2}{dt^2}$ and $\frac{d^3v_2}{dt^3}$ at $t = 0^+$.

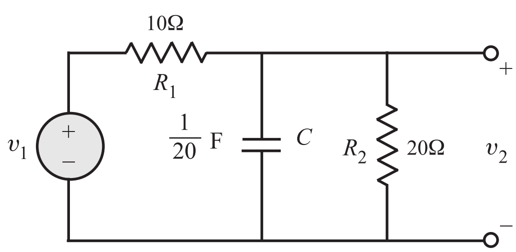

Figure 4.19

SOLUTION

Since the capacitor is initially uncharged, $v_2(0^+) = 0$

Referring to Fig. 4.19(a) and *applying KCL at node* $v_2(t)$:

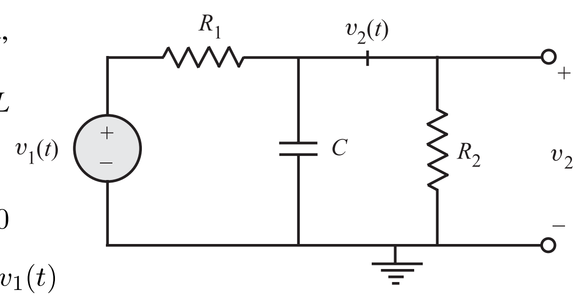

Figure 4.19(a)

$$\frac{v_2(t) - v_1(t)}{R_1} + C\frac{dv_2(t)}{dt} + \frac{v_2(t)}{R_2} = 0$$

$$\Rightarrow \quad \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_2(t) + C\frac{dv_2(t)}{dt} = \frac{v_1(t)}{R_1}$$

$$\Rightarrow \quad 0.15v_2 + 0.05\frac{dv_2}{dt} = 0.1e^{-t} \tag{4.5}$$

Putting $t = 0^+$, we get

$$0.15v_2(0^+) + 0.05\frac{dv_2(0^+)}{dt} = 0.1$$

$$\Rightarrow \quad 0.15 \times 0 + 0.05\frac{dv_2(0^+)}{dt} = 0.1$$

$$\Rightarrow \quad \frac{dv_2(0^+)}{dt} = \frac{0.1}{0.05} = 2 \text{ Volts/ sec}$$

Differentiating equation (4.5) with respect to t, we get

$$0.15\frac{dv_2}{dt} + 0.05\frac{d^2v_2}{dt^2} = -0.1e^{-t} \tag{4.6}$$

Putting $t = 0^+$ in equation (4.6), we find that

$$\frac{d^2v_2(0^+)}{dt^2} = \frac{-0.1 - 0.3}{0.05} = \mathbf{-8\ Volts/\,sec^2}$$

Again differentiating equation (4.6) with respect to t, we get

$$0.15\frac{d^2v_2}{dt^2} + 0.05\frac{d^3v_2}{dt^3} = 0.1e^{-t} \tag{4.7}$$

Putting $t = 0^+$ in equation (4.7) and solving for $\frac{d^3v_2}{dt^3}(0^+)$, we find that

$$\frac{d^3v_2(0^+)}{dt^3} = \frac{0.1 + 1.2}{0.05} = \mathbf{26\ Volts/\,sec^3}$$

EXAMPLE 4.9

Refer the circuit shown in Fig. 4.20. The circuit is in steady state with switch K closed. At $t = 0$, the switch is opened. Determine the voltage across the switch, v_K and $\frac{dv_K}{dt}$ at $t = 0^+$.

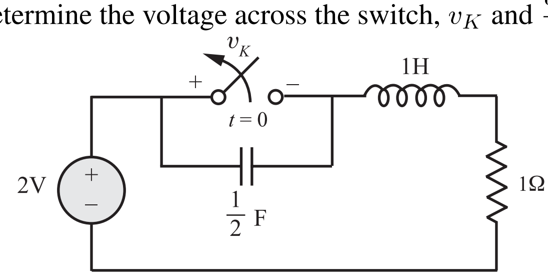

Figure 4.20

SOLUTION

The switch remains closed at $t = 0^-$ and open at $t = 0^+$. Under steady condition, inductor acts as a short circuit and hence the circuit diagram at $t = 0^-$ is as shown in Fig. 4.21(a).

$$\text{Therefore,} \quad v_K(0^+) = v_K(0^-)$$
$$= 0\text{ V}$$

For $t \geq 0^+$ the circuit diagram is as shown in Fig. 4.21(b).

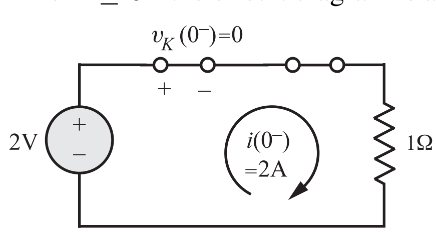

Figure 4.21(a)

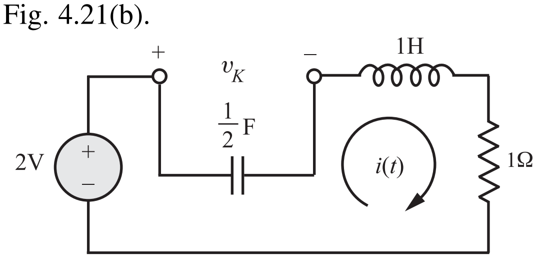

Figure 4.21(b)

$$i(t) = C\frac{dv_K}{dt}$$

At $(t) = 0^+$, we get

$$i(0^+) = C\frac{dv_K(0^+)}{dt}$$

Since the current through an inductor cannot change instantaneously, we get

$$i(0^+) = i(0^-) = 2\text{A}$$

Hence, $$2 = C\frac{dv_K(0^+)}{dt}$$

$$\frac{dv_K(0^+)}{dt} = \frac{2}{C} = \frac{2}{\frac{1}{2}} = \mathbf{4V/sec}$$

EXAMPLE 4.10

In the given network, the switch K is opened at $t = 0$. At $t = 0^+$, solve for the values of v, $\frac{dv}{dt}$ and $\frac{d^2v}{dt^2}$ if $I = 2$ A, $R = 200\ \Omega$ and $L = 1$ H

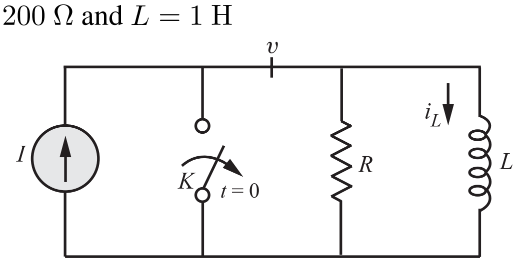

Figure 4.22

SOLUTION

The switch is opened at $t = 0$. This means that at $t = 0^-$, it is closed and at $t = 0^+$, it is open. Since $i_L(0^-) = 0$, we get $i_L(0^+) = 0$. The circuit at $t = 0^+$ is as shown in Fig. 4.23(a).

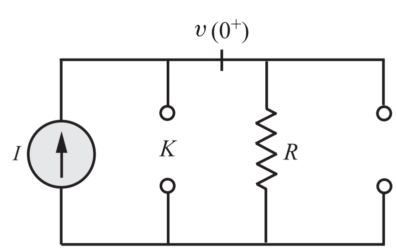

Figure 4.23(a)

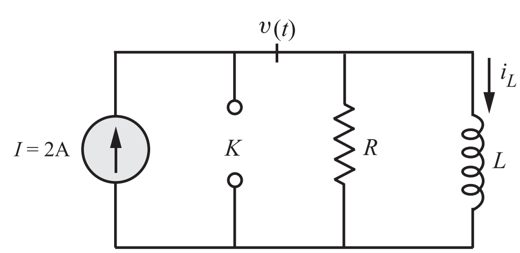

Figure 4.23(b)

$$\begin{aligned} v(0^+) &= IR \\ &= 2 \times 200 \\ &= \mathbf{400\ Volts} \end{aligned}$$

Refer to the circuit shown in Fig. 4.23(b).

For $t \geq 0^+$, the *KCL* at node $v(t)$ gives

$$I = \frac{v(t)}{R} + \frac{1}{L}\int_{0^+}^{t} v(\tau)d\tau \tag{4.8}$$

Differentiating both sides of equation (4.8) with respect to t, we get

$$0 = \frac{1}{R}\frac{dv(t)}{dt} + \frac{1}{L}v(t) \tag{4.8a}$$

At $t = 0^+$, we get

$$\frac{1}{R}\frac{dv(0^+)}{dt} + \frac{1}{L}v(0^+) = 0$$

$$\Rightarrow \quad \frac{1}{200}\frac{dv(0^+)}{dt} + \frac{1}{1} \times 400 = 0$$

$$\Rightarrow \quad \frac{dv(0^+)}{dt} = \mathbf{-8 \times 10^4\ V/\,sec}$$

Again differentiating equation (4.8a), we get

$$\frac{1}{R}\frac{d^2v(t)}{dt^2} + \frac{1}{L}\frac{dv(t)}{dt} = 0$$

At $t = 0^+$, we get

$$\frac{1}{200}\frac{d^2v(0^+)}{dt^2} + \frac{1}{1}\frac{dv(0^+)}{dt} = 0$$

$$\Rightarrow \quad \frac{d^2v(0^+)}{dt^2} = 200 \times 8 \times 10^4$$

$$= \mathbf{16 \times 10^6\ V/\,sec^2}$$

EXAMPLE 4.11

In the circuit shown in Fig. 4.24, a steady state is reached with switch K open. At $t = 0$, the switch is closed. For element values given, determine the values of $v_a(0^-)$ and $v_a(0^+)$.

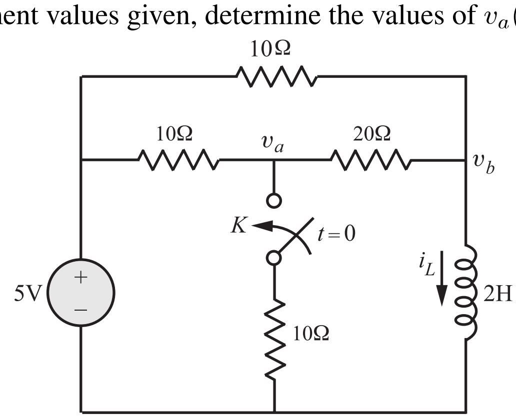

Figure 4.24

SOLUTION

At $t = 0^-$, the switch is open and at $t = 0^+$, the switch is closed. Under steady conditions, inductor L acts as a short circuit. Also the steady state is reached with switch K open. Hence, the circuit diagram at $t = 0^-$ is as shown in Fig.4.25(a).

$$i_L(0^-) = \frac{5}{30} + \frac{5}{10} = \frac{2}{3}\text{A}$$

Using the voltage divider principle:

$$v_a(0^-) = \frac{5 \times 20}{10 + 20} = \mathbf{\frac{10}{3}\ V}$$

Since the current in an inductor cannot change instantaneously,

$$i_L(0^+) = i_L(0^-) = \frac{2}{3}\ \text{A}.$$

At $t = 0^+$, the circuit diagram is as shown in Fig. 4.25(b).

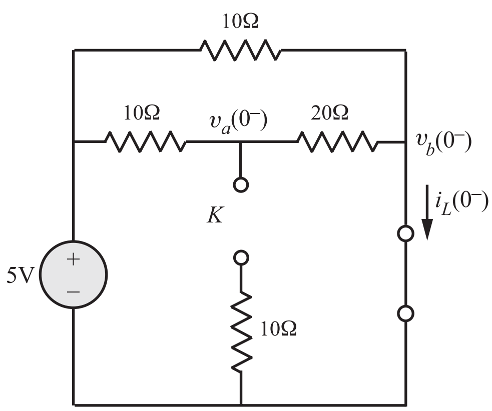

Figure 4.25(a)

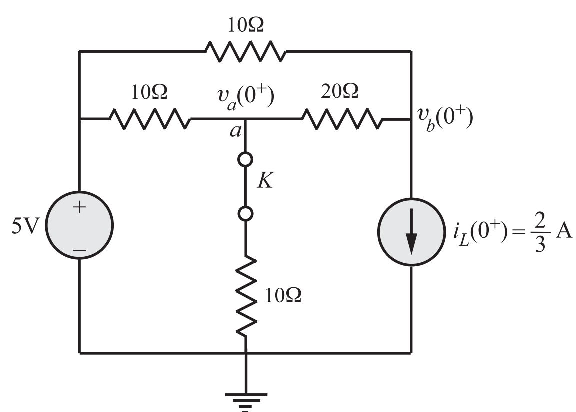

Figure 4.25(b)

Refer the circuit in Fig. 4.25(b).

KCL at node a:

$$\frac{v_a(0^+) - 5}{10} + \frac{v_a(0^+)}{10} + \frac{v_a(0^+) - v_b(0^+)}{20} = 0$$

$$\Rightarrow \quad v_a(0^+)\left[\frac{1}{10} + \frac{1}{10} + \frac{1}{20}\right] - v_b(0^+)\left[\frac{1}{20}\right] = \frac{5}{10}$$

KCL at node b:

$$\frac{v_b(0^+) - v_a(0^+)}{20} + \frac{v_b(0^+) - 5}{10} + \frac{2}{3} = 0$$

$$\Rightarrow \quad -v_a(0^+)\left[\frac{1}{20}\right] + v_b(0^+)\left[\frac{1}{20} + \frac{1}{10}\right] = \frac{5}{10} - \frac{2}{3}$$

Solving the above two nodal equations, we get,

$$v_a(0^+) = \frac{\mathbf{40}}{\mathbf{21}}\mathbf{V}$$

EXAMPLE 4.12

Find $i_L(0^+), v_C(0^+), \dfrac{dv_C(0^+)}{dt}$ and $\dfrac{di_L(0^+)}{dt}$ for the circuit shown in Fig. 4.26.

Assume that switch 1 has been opened and switch 2 has been closed for a long time and steady–state conditions prevail at $t = 0^-$.

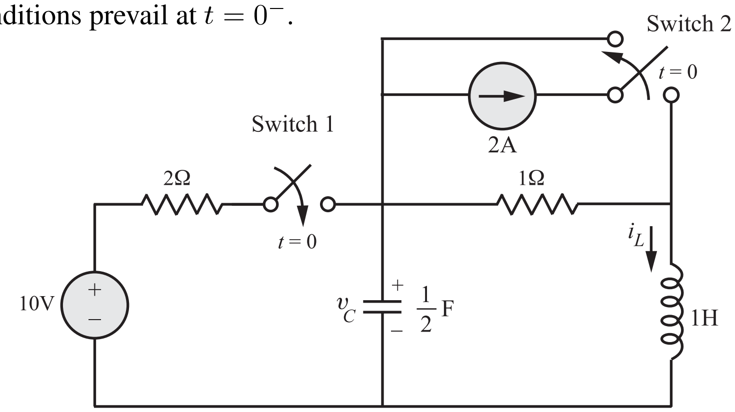

Figure 4.26

SOLUTION

At $t = 0^-$, switch 1 is open and switch 2 is closed, whereas at $t = 0^+$, switch 1 is closed and switch 2 is open.

First, let us redraw the circuit at $t = 0^-$ by replacing the inductor with a short circuit and the capacitor with an open circuit as shown in Fig. 4.27(a).

From Fig. 4.27(b), we find that $i_L(0^-) = 0$

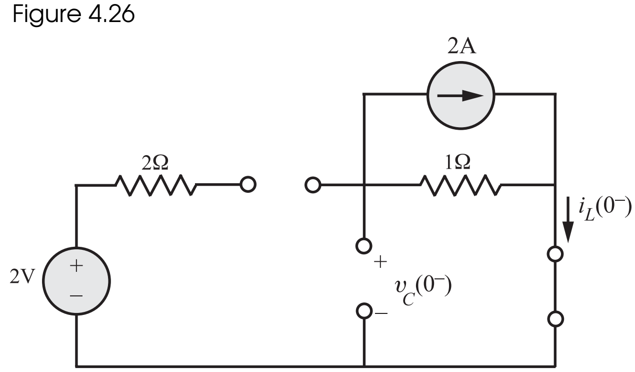

Figure 4.27(a)

$\Rightarrow$

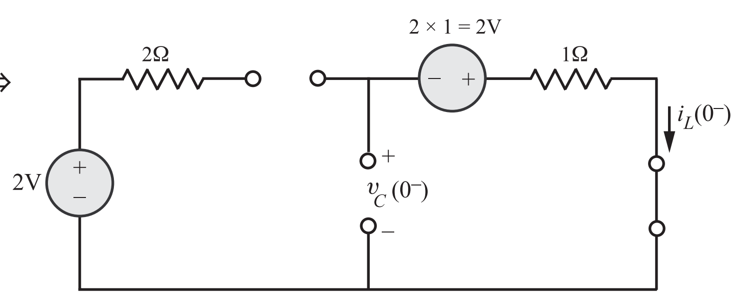

Figure 4.27(b)

Applying KVL clockwise to the loop on the right, we get

$$-v_C(0^-) - 2 + 1 \times 0 = 0$$
$$\Rightarrow \qquad v_C(0^-) = -2 \text{ V}$$

Hence, at $t = 0^+ : \ i_L(0^+) = i_L(0^-) = \mathbf{0A}$
$$v_C(0^+) = v_C(0^-) = \mathbf{-2V}$$

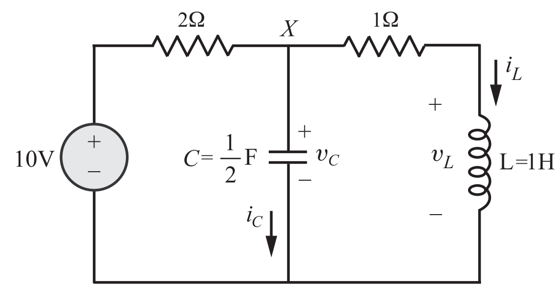

Figure 4.27(c)

The circuit diagram for $t \geq 0^+$ is shown in Fig. 4.27(c).

Applying KVL for right–hand mesh, we get

$$v_L - v_C + i_L = 0$$

At $t = 0^+$, we get

$$v_L(0^+) = v_C(0^+) - i_L(0^+)$$
$$= -2 - 0 = -2 \text{ V}$$

We know that
$$v_L = L\frac{di_L}{dt}$$

At $t = 0^+$, we get

$$\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{-2}{1} = \mathbf{-2A/\,sec}$$

Applying KCL at node X,

$$\frac{v_C - 10}{2} + i_C + i_L = 0$$

Consequently, at $t = 0^+$

$$i_C(0^+) = \frac{10 - v_C(0^+)}{2} - i_L(0^+) = 6 - 0 = 6 \text{ A}$$

Since
$$i_C = C\frac{dv_C}{dt}$$

We get,
$$\frac{dv_C(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{6}{\frac{1}{2}} = \mathbf{12V/\,sec}$$

EXAMPLE 4.13

For the circuit shown in Fig. 4.28, find:

(a) $i(0^+)$ and $v(0^+)$

(b) $\frac{di(0^+)}{dt}$ and $\frac{dv(0^+)}{dt}$

(c) $i(\infty)$ and $v(\infty)$

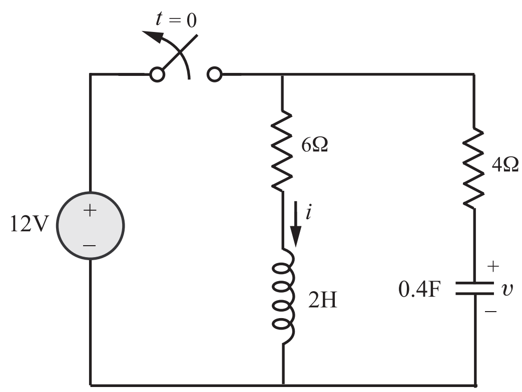

Figure 4.28

SOLUTION

(a) From the symbol of switch, we find that at $t = 0^-$, the switch is closed and $t = 0^+$, it is open. At $t = 0^-$, the circuit has reached steady state so that the equivalent circuit is as shown in Fig.4.29(a).

$$i(0^-) = \frac{12}{6} = 2\text{A}$$

$$v(0^-) = 12 \text{ V}$$

Therefore, we have

$$i(0^+) = i(0^-)$$

$$= \mathbf{2}\text{A}$$

$$v(0^+) = v(0^-) = \mathbf{12V}$$

(b) For $t \geq 0^+$, we have the equivalent circuit as shown in Fig.4.29(b).

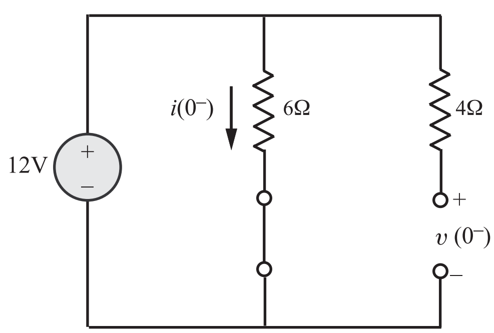

Figure 4.29(a)

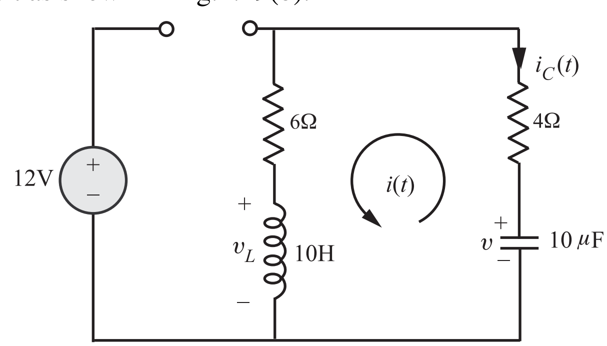

Figure 4.29(b)

Applying KVL anticlockwise to the mesh on the right, we get

$$v_L(t) - v(t) + 10i(t) = 0$$

Putting $t = 0^+$, we get

$$v_L(0^+) - v(0^+) + 10i(0^+) = 0$$

$$\Rightarrow \quad v_L(0^+) - 12 + 10 \times 2 = 0$$

$$\Rightarrow \quad v_L(0^+) = -8\text{V}$$

The voltage across the inductor is given by

$$v_L = L\frac{di}{dt}$$

$$\Rightarrow \quad v_L(0^+) = L\frac{di(0^+)}{dt}$$

$$\Rightarrow \quad \frac{di(0^+)}{dt} = \frac{1}{L}v_L(0^+)$$

$$= \frac{1}{10}(-8) = \mathbf{-0.8A/\,sec}$$

Similarly, the current through the capacitor is

$$i_C = C\frac{dv}{dt}$$

or

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{-i(0^+)}{C}$$

$$= \frac{-2}{10\times 10^{-6}} = \mathbf{-0.2\times 10^6 V/\,sec}$$

(c) As t approaches infinity, the switch is open and the circuit has attained steady state. The equivalent circuit at $t = \infty$ is shown in Fig.4.29(c).

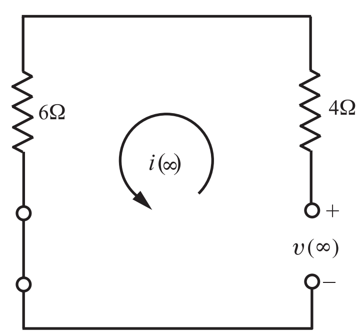

Figure 4.29(c)

$$i(\infty) = \mathbf{0}$$

$$v(\infty) = \mathbf{0}$$

EXAMPLE 4.14

Refer the circuit shown in Fig.4.30. Find the following:

(a) $v(0^+)$ and $i(0^+)$

(b) $\frac{dv(0^+)}{dt}$ and $\frac{di(0^+)}{dt}$

(c) $v(\infty)$ and $i(\infty)$

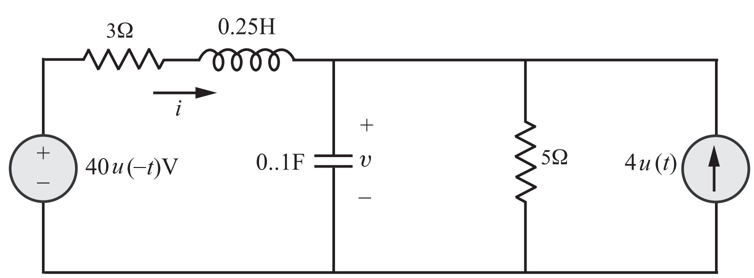

Figure 4.30

SOLUTION

From the definition of step function,

$$u(t) = \begin{cases} 1, \ t > 0 \\ 0, \ t < 0 \end{cases}$$

From Fig.4.31(a), $u(t) = 0$ at $t = 0^-$.

$$\text{Similarly,} \quad u(-t) = \begin{cases} 1, & -t > 0 \\ 0, & -t < 0 \end{cases}$$

$$\text{or} \quad u(-t) = \begin{cases} 1, & t < 0 \\ 0, & t > 0 \end{cases}$$

From Fig.4.31(b), we find that $u(-t) = 1$, at $t = 0^-$.

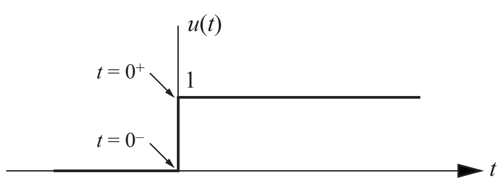

Figure 4.31(a)

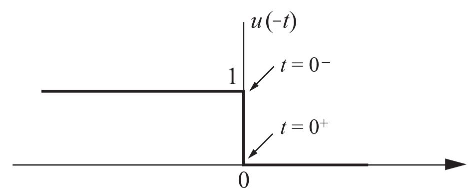

Figure 4.31(b)

Due to the presence of $u(-t)$ and $u(t)$ in the circuit of Fig.4.30, the circuit is an implicit switching circuit. We use the word implicit since there are no conventional switches in the circuit of Fig.4.30.

The equivalent circuit at $t = 0^-$ is shown in Fig.4.31(c). Please note that at $t = 0^-$, the independent current source is open because $u(t) = 0$ at $t = 0^-$ and the circuit is in steady state.

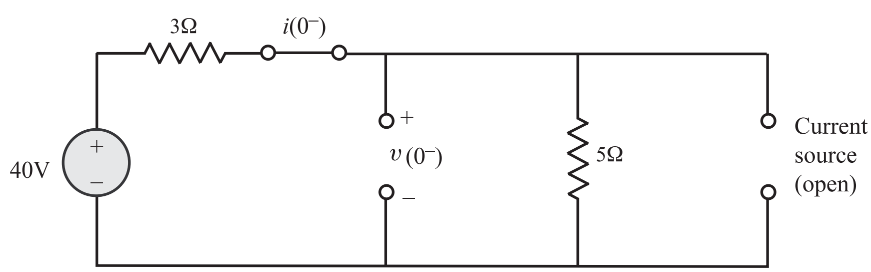

Figure 4.31(c)

$$i(0^-) = \frac{40}{3+5} = 5\text{A}$$

$$v(0^-) = 5i(0^-) = 25\text{V}$$

$$\text{Therefore} \quad i(0^+) = i(0^-) = \mathbf{5A}$$

$$v(0^+) = v(0^-) = \mathbf{25V}$$

(b) For $t \geq 0^+$, $u(-t) = 0$. This implies that the independent voltage source is zero and hence is represented by a short circuit in the circuit shown in Fig.4.31(d).

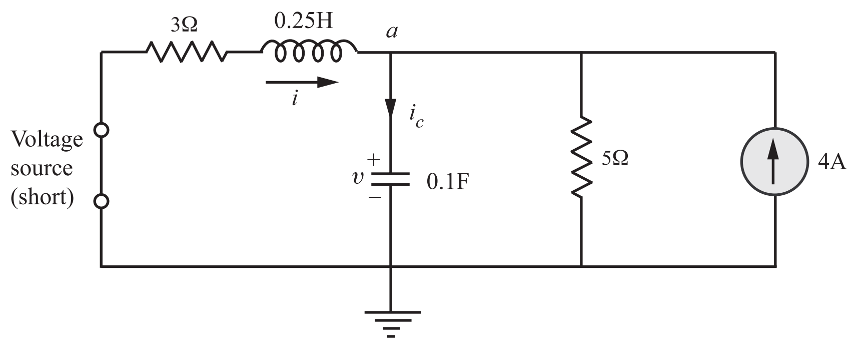

Figure 4.31(d)

Applying KVL at node a, we get

$$4 + i = C\frac{dv}{dt} + \frac{v}{5}$$

At $t = 0^+$, We get

$$4 + i(0^+) = C\frac{dv(0^+)}{dt} + \frac{v(0^+)}{5}$$

$$\Rightarrow \qquad 4 + 5 = 0.1\frac{dv(0^+)}{dt} + \frac{25}{5}$$

$$\Rightarrow \qquad \frac{dv(0^+)}{dt} = \mathbf{40V/\,sec}$$

Applying KVL to the left–mesh, we get

$$3i + 0.25\frac{di}{dt} + v = 0$$

Evaluating at $t = 0^+$, we get

$$3i(0^+) + 0.25\frac{di(0^+)}{dt} + v(0^+) = 0$$

$$\Rightarrow \qquad 3 \times 5 + 0.25\frac{di(0^+)}{dt} + 25 = 0$$

$$\Rightarrow \qquad \frac{di(0^+)}{dt} = \frac{-40}{\frac{1}{4}} = \mathbf{-160A/\,sec}$$

(c) As t approaches infinity, again the circuit is in steady state. The equivalent circuit at $t = \infty$ is shown in Fig.4.31(e).

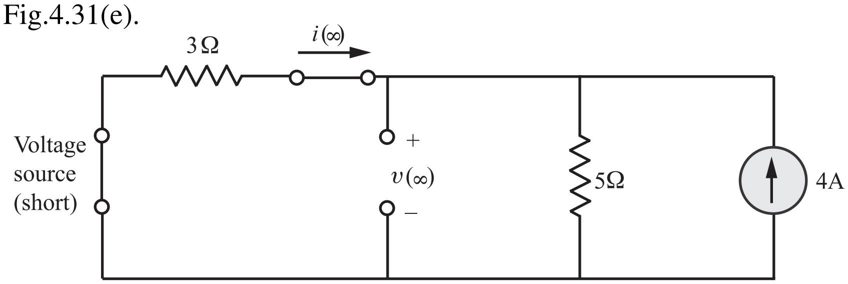

Figure 4.31(e)

Using the principle of current divider, we get

$$i(\infty) = -\left(\frac{4 \times 5}{3 + 5}\right) = \mathbf{-2.5A}$$

$$\begin{aligned} v(\infty) &= (i(\infty) + 4)\,5 \\ &= (-2.5 + 4)5 \\ &= \mathbf{7.5V} \end{aligned}$$

EXAMPLE 4.15

Refer the circuit shown in Fig.4.32. Find the following:

(a) $i(0^+)$ and $v(0^+)$

(b) $\dfrac{di(0^+)}{dt}$ and $\dfrac{dv(0^+)}{dt}$

(c) $i(\infty)$ and $v(\infty)$

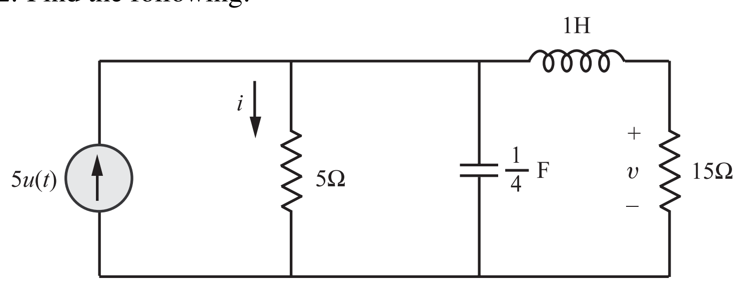

Figure 4.32

SOLUTION

Here the function $u(t)$ behaves like a switch. Mathematically,

$$u(t) = \begin{cases} 1, \ t > 0 \\ 0, \ t < 0 \end{cases}$$

The above expression means that the switch represented by $u(t)$ is open for $t < 0$ and remains closed for $t > 0$. Hence, the circuit diagram of Fig.4.32 may be redrawn as shown in Fig.4.33(a).

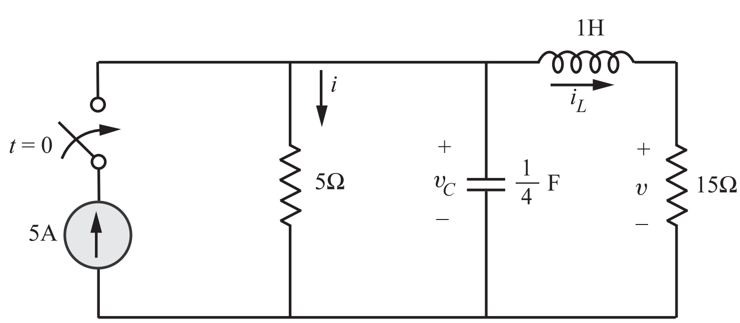

Figure 4.33(a)

For $t < 0$, the circuit is not active because switch is in open state, This implies that all the initial conditions are zero.

That is, $i_L(0^-) = 0$ and $v_C(0^-) = 0$

for $t \geq 0^+$, the equivalent circuit is as shown in Fig.4.33(b).

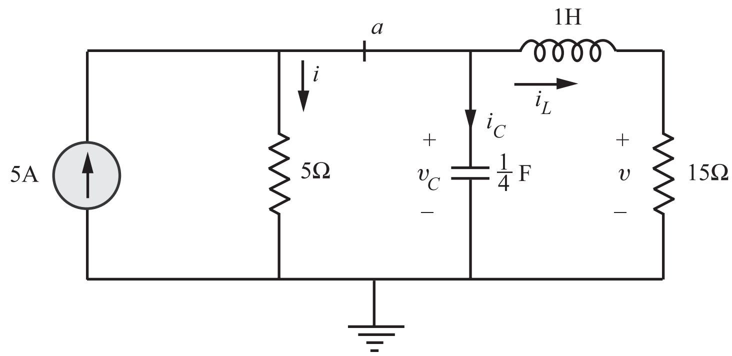

Figure 4.33(b)

From the circuit diagram of Fig.4.33(b), we find that

$$i = \frac{v_C}{5}$$

At $t = 0^+$, we get

$$i(0^+) = \frac{v_C(0^+)}{5} = \frac{v_C(0^-)}{5} = \frac{0}{5} = \mathbf{0A}$$

Also $$v = 15i_L$$

Evaluating at $t = 0^+$, we get

$$v(0^+) = 15i_L(0^+)$$
$$= 15i_L(0^-) = 15 \times 0 = \mathbf{0V}$$

(b) The equivalent circuit at $t = 0^+$ is shown in Fig.4.33(c).

We find from Fig.4.33(c) that

$$i_C(0^+) = 5A$$

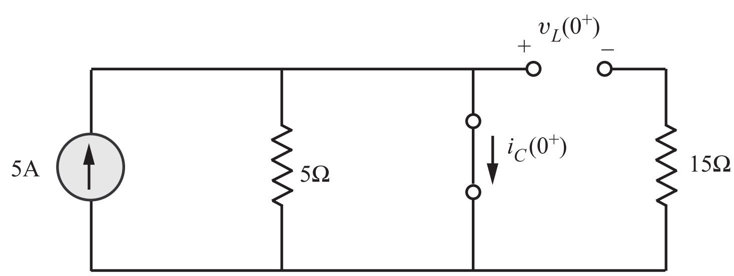

Figure 4.33(c)

From Fig.4.33(b), we can write

$$v_C = 5i$$

$$\Rightarrow \qquad \frac{dv_C}{dt} = 5\frac{di}{dt}$$

Multiplying both sides by C, we get

$$C\frac{dv_C}{dt} = 5C\frac{di}{dt}$$

$$\Rightarrow \qquad i_C = 5C\frac{di}{dt}$$

Putting $t = 0^+$, we get

$$\frac{di(0^+)}{dt} = \frac{1}{5C} i_C(0^+)$$
$$= \frac{1}{5\left(\frac{1}{4}\right)} \times 5$$
$$= \mathbf{4A/\,sec}$$

Also
$$v = 15 i_L$$
$$\Rightarrow \qquad \frac{dv}{dt} = 15\frac{di_L}{dt}$$
$$\Rightarrow \qquad \frac{dv}{dt} = 15\left[1 \times \frac{di_L}{dt}\right]$$
$$\Rightarrow \qquad \frac{dv}{dt} = 15 v_L$$

At $t = 0^+$, we find that

$$\Rightarrow \qquad \frac{dv(0^+)}{dt} = 15 v_L(0^+)$$

From Fig.4.33(b), we find that $v_L(0^+) = 0$

Hence,
$$\frac{dv(0^+)}{dt} = 15 \times 0$$
$$= \mathbf{0V/\,sec}$$

EXAMPLE 4.16

In the circuit shown in Fig. 4.34, steady state is reached with switch K open. The switch is closed at $t = 0$.

Determine: $i_1,\ i_2,\ \dfrac{di_1}{dt}$ and $\dfrac{di_2}{dt}$ at $t = 0^+$

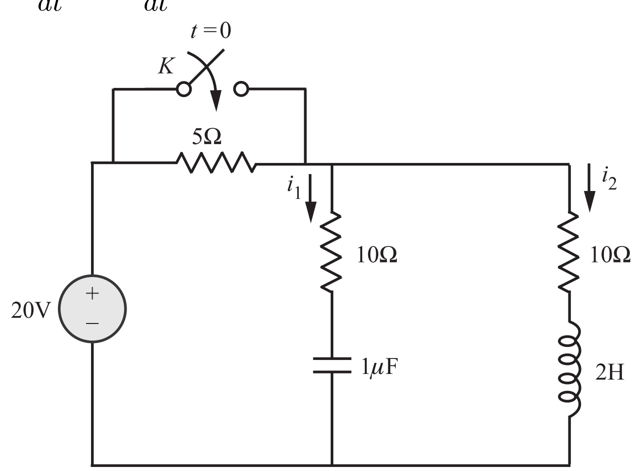

Figure 4.34

SOLUTION

At $t = 0^-$, switch K is open and at $t = 0^+$, it is closed. At $t = 0^-$, the circuit is in steady state and appears as shown in Fig.4.35(a).

$$i_2(0^-) = \frac{20}{10+5} = 1.33\text{A}$$

Hence,
$$v_C(0^-) = 10i_2(0^-) = 10 \times 1.33 = 13.3\text{V}$$

Since current through an inductor cannot change instantaneously, $i_2(0^+) = i_2(0^-) = \mathbf{1.33\ A}$.
Also, $v_C(0^+) = v_C(0^-) = 13.3\text{V}$.
The equivalent circuit at $t = 0^+$ is as shown in Fig.4.35(b).

$$i_1(0^+) = \frac{20 - 13.3}{10} = \frac{6.7}{10} = \mathbf{0.67A}$$

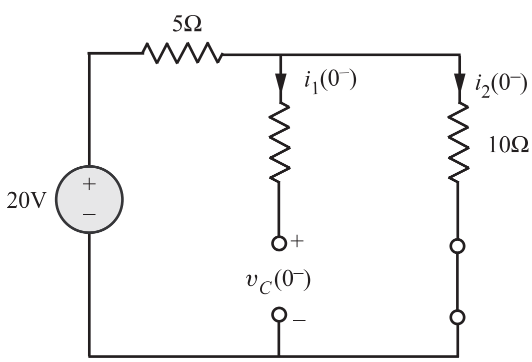

Figure 4.35(a)

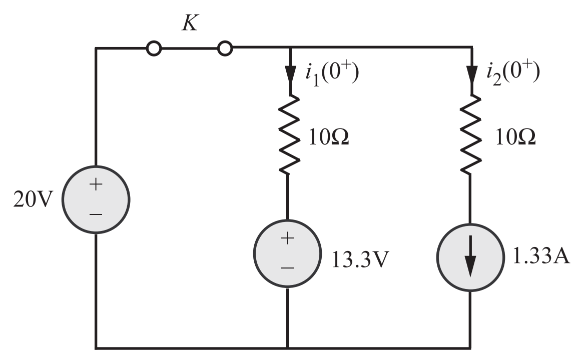

Figure 4.35(b)

For $t \geq 0^+$, the circuit is as shown in Fig.4.35(c).

Writing KVL clockwise for the left–mesh, we get

$$10i_1 + \frac{1}{C}\int_{0^+}^{t} i_1(\tau)d\tau = 20$$

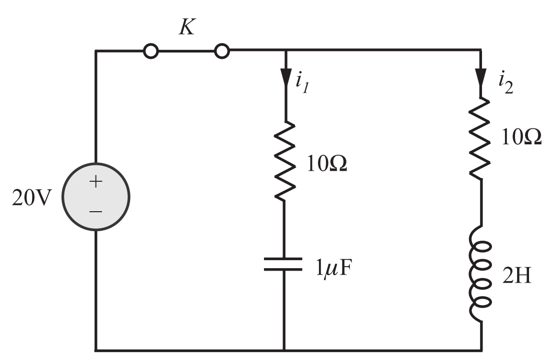

Figure 4.35(c)

Differentiating with respect to t, we get

$$10\frac{di_1}{dt} + \frac{1}{C}i_1 = 0$$

Putting $t = 0^+$, we get

$$10\frac{di_1(0^+)}{dt} + \frac{1}{C}i_1(0^+) = 0$$

$$\Rightarrow \quad \frac{di_1(0^+)}{dt} = \frac{-1}{10 \times 1 \times 10^{-6}}i_1(0^+) = \mathbf{-0.67 \times 10^5 A/\,sec}$$

Writing KVL equation to the path made of 20V $\rightarrow K \rightarrow 10\Omega \rightarrow$ 2H, we get

$$10i_2 + \frac{2di_2}{dt} = 20$$

At $t = 0^+$, the above equation becomes

$$10i_2(0^+) + \frac{2di_2(0^+)}{dt} = 20$$

$$\Rightarrow \quad 10 \times 1.33 + \frac{2di_2(0^+)}{dt} = 20$$

$$\Rightarrow \quad \frac{di_2(0^+)}{dt} = \mathbf{3.35A/\sec}$$

EXAMPLE 4.17

Refer the citcuit shown in Fig.4.36. The switch K is closed at $t = 0$. Find:

(a) v_1 and v_2 at $t = 0^+$

(b) v_1 and v_2 at $t = \infty$

(c) $\frac{dv_1}{dt}$ and $\frac{dv_2}{dt}$ at $t = 0^+$

(d) $\frac{d^2v_1}{dt^2}$ at $t = 0^+$

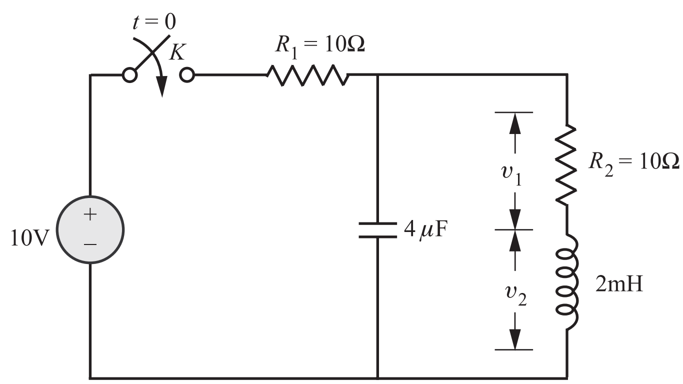

Figure 4.36

SOLUTION

(a) The circuit symbol for switch conveys that at $t = 0^-$, the switch is open and $t = 0^+$, it is closed. At $t = 0^-$, since the switch is open, the circuit is not activated. This implies that all initial conditions are zero. Hence, at $t = 0^+$, inductor is open and capactor is short. Fig 4.37(a) shows the equivalent circuit at $t = 0^+$.

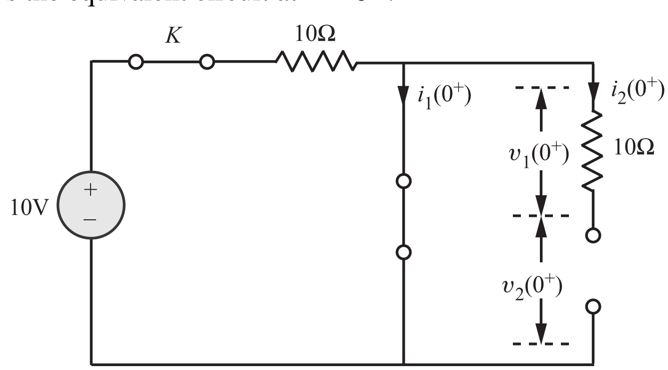

Figure 4.37(a)

$$i_1(0^+) = \frac{10}{10} = 1\text{A}$$
$$v_1(0^+) = 0, \quad i_2(0^+) = 0$$

Applying KVL to the path, 10V source → K → 10Ω → 10Ω → 2mH, we get

$$-10 + 10i_1(0^+) + v_1(0^+) + v_2(0^+) = 0$$
$$\Rightarrow \quad -10 + 10 + 0 + v_2(0^+) = 0$$
$$\Rightarrow \quad v_2(0^+) = \mathbf{0}$$

(b) At $t = \infty$, switch K remains closed and circuit is in steady state. Under steady state conditions, capacitor C is open and inductor L is short. Fig. 4.37(b) shows the equivalent circuit at $t = \infty$.

$$i_2(\infty) = \frac{10}{10 + 10} = \mathbf{0.5A}$$
$$i_1(\infty) = \mathbf{0}$$
$$v_1(\infty) = 0.5 \times 10 = \mathbf{5V}$$
$$v_2(\infty) = \mathbf{0}$$

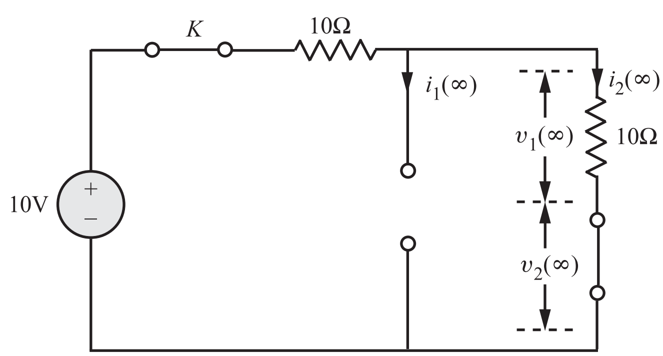

Figure 4.37(b)

(c) For $t \geq 0^+$, the circuit is as shown in Fig. 4.37(c).

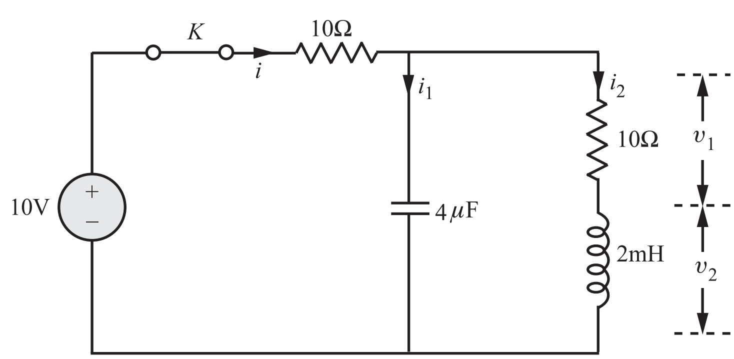

Figure 4.37(c)

$$i_2 = \frac{1}{L}\int_{0^+}^{t} v_2(\tau)d\tau = \frac{v_1(t)}{R_2}$$

Differentiating with respect to t, we get

$$\frac{v_2}{L} = \frac{1}{R_2}\frac{dv_1}{dt}$$

Evaluating at $t = 0^+$ we get

$$\frac{dv_1(0^+)}{dt} = \frac{R_2}{L_2}v_2(0^+)$$

$$\Rightarrow \qquad \frac{dv_1(0^+)}{dt} = \mathbf{0V/\,sec}$$

Applying KVL clockwise to the path 10 V source $\rightarrow K \rightarrow 10\Omega \rightarrow 4\mu$F, we get

$$-10 + 10i + \frac{1}{C}\int_{0^+}^{t} [i(\tau) - i_2(\tau)]d\tau = 0$$

Differentiating with respect to t, we get

$$10\frac{di}{dt} + \frac{1}{C}[i - i_2] = 0$$

Evaluating at $t = 0^+$, we get

$$\begin{aligned}\frac{di(0^+)}{dt} &= \frac{i_2(0^+) - i(0^+)}{C \times 10}\\ &= \frac{0-1}{10 \times 4 \times 10^{-6}} \qquad \left[\begin{aligned} \because\ i(0^+) &= i_1(0^+) + i_2(0^+)\\ &= 1 + 0\\ &= 1\text{A}\end{aligned}\right]\\ &= \mathbf{-25000A/\,sec}\end{aligned}$$

Applying KVL clockwise to the path 10 V source $\rightarrow K \rightarrow 10\Omega \rightarrow 10\Omega \rightarrow 2$ mH, we get

$$-10 + 10i + 10i_2 + v_2 = 0$$

$$\Rightarrow \qquad 10i + v_1 + v_2 = 10$$

Differentiating with respect to t, we get

$$10\frac{di}{dt} + \frac{dv_1}{dt} + \frac{dv_2}{dt} = 0$$

At $t = 0^+$, we get

$$10\frac{di(0^+)}{dt} + \frac{dv_1(0^+)}{dt} + \frac{dv_2(0^+)}{dt} = 0$$

$$\Rightarrow \qquad 10(-25000) + 0 + \frac{dv_2(0^+)}{dt} = 0$$

$$\Rightarrow \qquad \frac{dv_2(0^+)}{dt} = \mathbf{25 \times 10^4 V/\sec}$$

(d) From part (c), we have

$$\frac{1}{L}\int_{0^+}^{t} v_2(\tau)d\tau = \frac{v_1}{10}$$

Differentiating with respect to t twice, we get

$$\frac{1}{L}\frac{dv_2}{dt} = \frac{1}{10}\frac{d^2v_1}{dt^2}$$

At $t = 0^+$, we get

$$\frac{1}{L}\frac{dv_2(0^+)}{dt} = \frac{1}{10}\frac{d^2v_1(0^+)}{dt^2}$$

Hence,
$$\frac{d^2v_1(0^+)}{dt^2} = \mathbf{125 \times 10^7 V/\sec^2}$$

EXAMPLE 4.18

Refer the network shown in Fig. 4.38. Switch K is changed from a to b at $t = 0$ (a steady state having been established at position a).

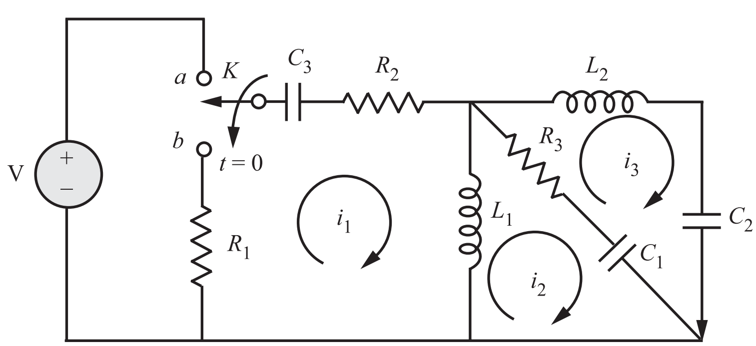

Figure 4.38

Show that at $t = 0^+$.

$$i_1 = i_2 = \frac{-V}{R_1 + R_2 + R_3}, \quad i_3 = 0$$

SOLUTION

The symbol for switch indicates that at $t = 0^-$, it is in position a and at $t = 0^+$, it is in position b. The circuit is in steady state at $t = 0^-$. Fig 4.39(a) refers to the equivalent circuit at $t = 0^-$. Please remember that at steady state C is open and L is short.

$$i_{L_1}(0^-) = 0, \;\; i_{L_2}(0^-) = 0, \;\; v_{C_2}(0^-) = 0, \;\; v_{C_1}(0^-) = 0$$

Applying KVL clockwise to the left-mesh, we get

$$-V + v_{C_3}(0^-) + 0 \times R_2 + 0 = 0$$

$$\Rightarrow \qquad v_{C_3}(0^-) = \text{V volts.}$$

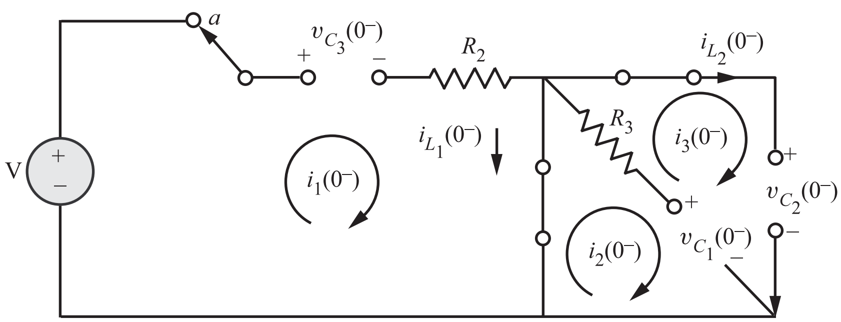

Figure 4.39(a)

Since current in an inductor and voltage across a capacitor cannot change instantaneously, the equivalent circuit at $t = 0^+$ is as shown in Fig. 4.39(b).

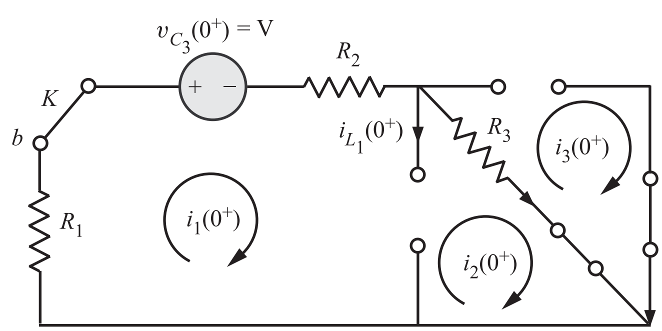

Figure 4.39(b)

$$i_1(0^+) = i_2(0^+) \text{ since } \; i_{L_1}(0^+) = 0$$
$$i_3(0^+) = 0 \text{ since } \; i_{L_2}(0^+) = 0$$

Applying KVL to the path $v_{C_3}(0^+) \rightarrow R_2 \rightarrow R_3 \rightarrow R_1 \rightarrow K$ we get,

$$V + R_2 i_1(0^+) + R_3 i_2(0^+) + R_1 i_1(0^+) = 0$$

Since $i_1(0^+) = i_2(0^+)$, the above equation becomes

$$-V = [R_1 + R_2 + R_3]\, i_1(0^+)$$

Hence,
$$i_1(0^+) = i_2(0^+) = \frac{\boldsymbol{-V}}{\boldsymbol{R_1 + R_2 + R_3}}\,\mathbf{A}$$

EXAMPLE 4.19

Refer the circuit shown in Fig. 4.40. The switch K is closed at $t = 0$.

Find (a) $\dfrac{di_1(0^+)}{dt}$ and (b) $\dfrac{di_2(0^+)}{dt}$

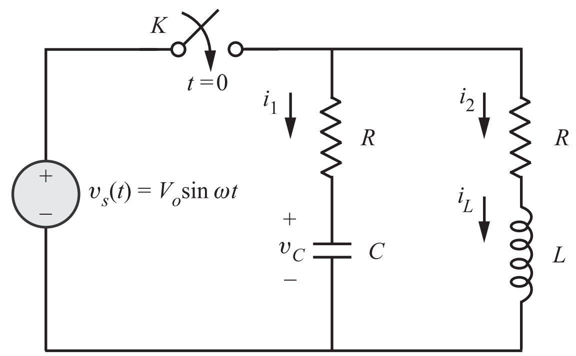

Figure 4.40

SOLUTION

The circuit symbol for the switch shows that at $t = 0^-$, it is open and at $t = 0^+$, it is closed. Hence, at $t = 0^-$, the circuit is not activated. This implies that all initial conditions are zero. That is, $v_C(0^-) = 0$ and $i_L(0^-) = i_2(0^-) = 0$. The equivalent circuit at $t = 0^+$ keeping in mind that $v_C(0^+) = v_C(0^-)$ and $i_L(0^+) = i_L(0^-)$ is as shown in Fig. 4.41 (a).

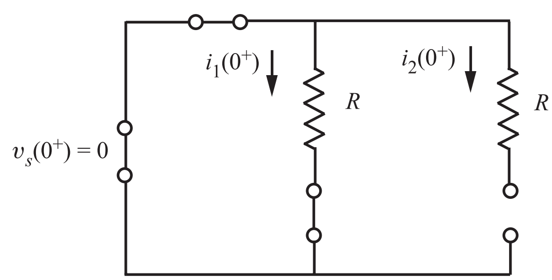

Figure 4.41(a)

$$i_1(0^+) = 0 \text{ and } i_2(0^+) = 0.$$

Figure. 4.41(b) shows the circuit diagram for $t \geq 0^+$.

$$V_o \sin \omega t = i_1 R + \frac{1}{C}\int_{0^+}^{t} i_1(\tau)d\tau$$

Differentiating with respect to t, we get

$$V_o\omega \cos \omega t = R\frac{di_1}{dt} + \frac{i_1}{C}$$

At $t = 0^+$, we get

$$V_o\omega = R\frac{di_1(0^+)}{dt} + \frac{i_1(0^+)}{C}$$

$$\Rightarrow \quad \frac{di_1(0^+)}{dt} = \frac{\boldsymbol{V_0\omega}}{\boldsymbol{R}}\mathbf{A/\,sec}$$

Also, $\quad V_o \sin\omega t = i_2 R + L\frac{di_2}{dt}$

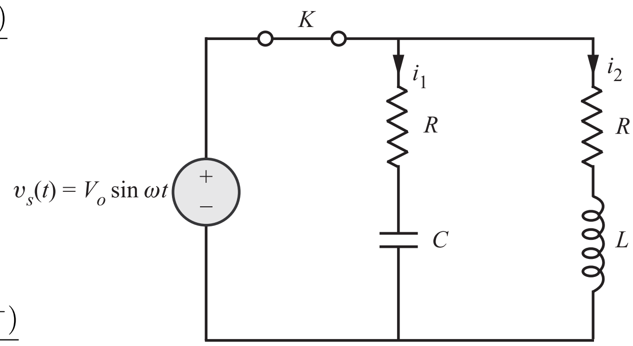

Figure 4.41(b)

Evaluating at $t = 0^+$, we get

$$0 = i_2(0^+)R + L\frac{di_2(0^+)}{dt}$$

$$\Rightarrow \quad \frac{di_2(0^+)}{dt} = \mathbf{0A/\,sec}$$

EXAMPLE 4.20

In the network of the Fig. 4.42, the switch K is opened at $t = 0$ after the network has attained steady state with the switch closed.

(a) Find the expression for v_K at $t = 0^+$.

(b) If the parameters are adjusted such that $i(0^+) = 1$, and $\frac{di(0^+)}{dt} = -1$, what is the value of the derivative of the voltage across the switch at $t = 0^+$, $\left(\frac{dv_K}{dt}(0^+)\right)$?

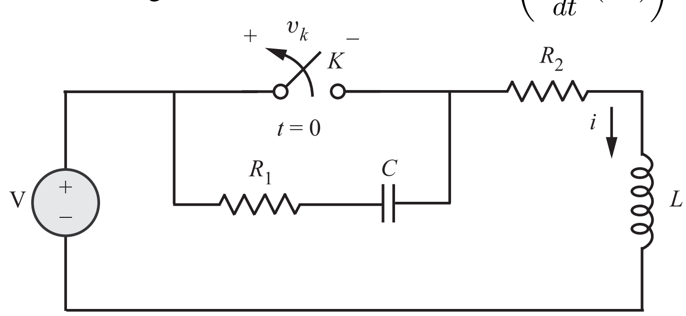

Figure 4.42

SOLUTION

At $t = 0^-$, switch is in the closed state and at $t = 0^+$, it is open. Also at $t = 0^-$, the circuit is in steady state. The equivalent circuit at $t = 0^-$ is as shown in Fig. 4.43(a).

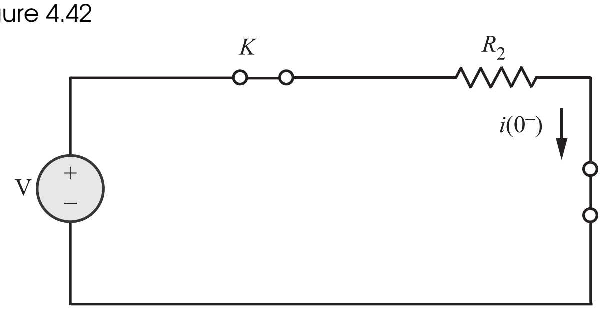

Figure 4.43(a)

$$i(0^-) = \frac{V}{R_2} \text{ and } v_C(0^-) = 0$$

For $t \geq 0^+$, the equivalent circuit is as shown in Fig. 4.43(b).
From Fig. 4.43 (b),

$$v_K = R_1 i + \frac{1}{C}\int_{0^+}^{t} i(\tau)d\tau$$

$$\Rightarrow \quad v_K = R_1 i + v_C(t)$$

At $t = 0^+$, $v_K(0^+) = R_1 i(0^+) + v_C(0^+)$

$$\Rightarrow \quad v_K(0^+) = R_1 \frac{V}{R_2} + v_C(0^-)$$

$$= \boldsymbol{R_1 \frac{V}{R_2}} \textbf{volts}$$

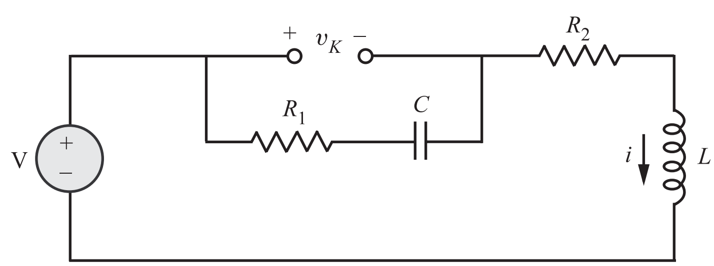

Figure 4.43(b)

(b)

$$v_K = R_1 i + \frac{1}{C}\int_{0^+}^{t} i(\tau)d\tau$$

$$\Rightarrow \quad \frac{dv_K}{dt} = R_1 \frac{di}{dt} + \frac{i}{C}$$

Evaluating at $t = 0^+$, we get

$$\frac{dv_K(0^+)}{dt} = R_1 \frac{di(0^+)}{dt} + \frac{i(0^+)}{C}$$

$$= R_1 \times (-1) + \frac{1}{C}$$

$$= \boldsymbol{\frac{1}{C} - R_1} \textbf{volts/ sec}$$

Reinforcement Problems

R.P 4.1

Refer the circuit shown in Fig RP.4.1(a). If the switch is closed at $t = 0$, find the value of $\frac{d^2 i_L(0^+)}{dt^2}$ at $t = 0^+$.

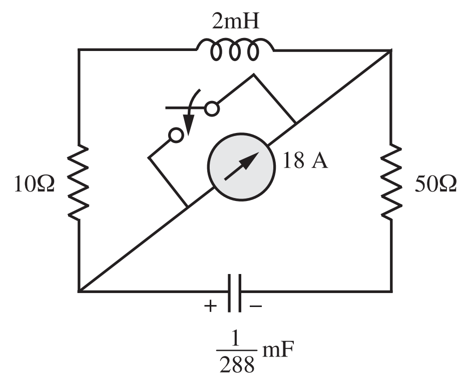

Figure R.P.4.1(a)

SOLUTION

The circuit at $t = 0^-$ is as shown in Fig RP 4.1(b).

Since current through an inductor and voltage across a capacitor cannot change instantaneously, it implies that $i_L(0^+) = 18\text{A}$ and $v_C(0^+) = -180$ V.

The circuit for $t \geq 0^+$ is as shown in Fig. RP 4.1 (c).

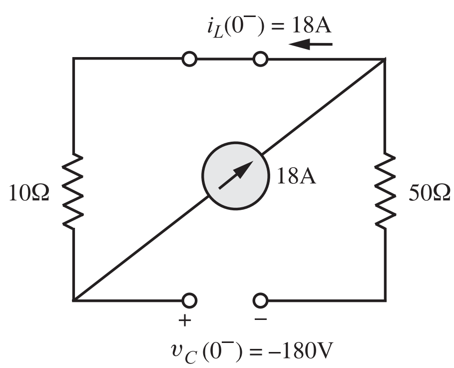

Figure R.P.4.1(b)

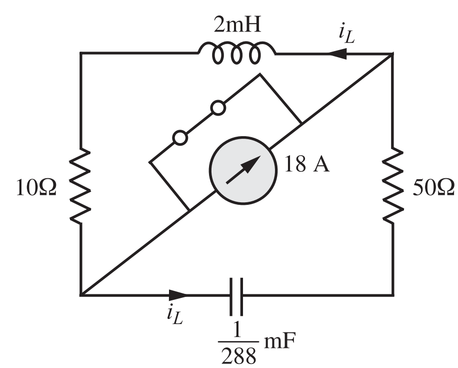

Figure R.P.4.1(c)

Referring Fig RP 4.1 (c), we can write

$$2 \times 10^{-3} \frac{di_L}{dt} + 60 i_L + 288 \times 10^3 \int_{0^+}^{t} i_L(t) dt = 0 \tag{4.9}$$

At $t = 0^+$, we get

$$\frac{di_L(0^+)}{dt} = \frac{-60 \times 18 + 180}{2 \times 10^{-3}}$$
$$= -450 \times 10^3 \text{ A/ sec}$$

Differentiating equation (4.9) with respect to t, we get

$$2 \times 10^{-3}\frac{d^2 i_L}{dt^2} + 60\frac{di_L}{dt} + 288 \times 10^3 i_L = 0$$

At $t = 0^+$, we get

$$\frac{d^2 i_L(0^+)}{dt^2} = \frac{60(450)10^3 - 288 \times 10^3(18)}{2 \times 10^{-3}}$$
$$= 1.0908 \times 10^{10} \text{ A/ sec}^2$$

R.P 4.2

For the circuit shown in Fig. RP 4.2, determine $\frac{d^2 v_C(0^+)}{dt^2}$ and $\frac{d^3 v_C(0^+)}{dt^3}$.

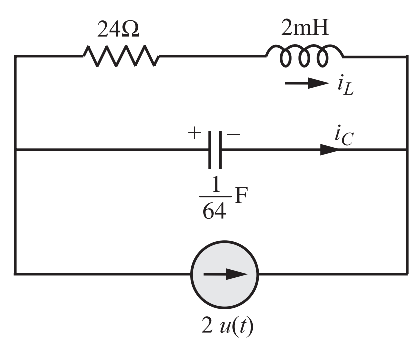

Figure R.P.4.2

SOLUTION

Given

$$i(t) = 2u(t) = \begin{cases} 2, & t \geq 0^+ \\ 0, & t \leq 0^- \end{cases}$$

Hence, at $t = 0^-$, $v_C(0^-) = 0$ and $i_L(0^-) = 0$.
For $t \geq 0^+$, the circuit equations are

$$\frac{1}{64}\frac{dv_C(t)}{dt} + \frac{1}{2}\int_{0^+}^{t} v_L(t)dt = -2 \tag{4.10}$$

$$\Rightarrow \quad \frac{1}{64}\frac{dv_C(t)}{dt} + i_L(t) = -2 \tag{4.11}$$

[**Note** : $i_C + i_L = -2$ because of the capacitor polarity]

At $t = 0^+$, equation (4.10) gives

$$\frac{1}{64}\frac{dv_C(0^+)}{dt} + i_L(0^+) = -2$$

Since, $i_L(0^+) = i_L(0^-) = 0$, we get

$$\frac{1}{64}\frac{dv_C(0^+)}{dt} + 0 = -2$$

$$\Rightarrow \qquad \frac{dv_C(0^+)}{dt} = -128 \text{ volts/ sec}$$

Differentiating equation (4.10) with respect to t we get

$$\frac{1}{64}\frac{d^2v_C(t)}{dt} + \frac{1}{2}v_L(t) = 0 \tag{4.12}$$

Also, $$\frac{v_C - v_L}{24} = \frac{1}{2}\int_{0^+}^{t} v_L dt = i_L \tag{4.13}$$

At $t = 0^+$, we get

$$\frac{v_C(0^+) - v_L(0^+)}{24} = i_L(0^+)$$

Since $v_C(0^+) = 0$ and $i_L(0^+) = 0$, we get $v_L(0^+) = 0$.

At $t = 0^+$, equation (4.12) becomes

$$\frac{1}{64}\frac{d^2v_C(0^+)}{dt^2} + \frac{1}{2}v_L(0^+) = 0$$

$$\Rightarrow \qquad \frac{1}{64}\frac{d^2v_C(0^+)}{dt^2} + \frac{1}{2} \times 0 = 0$$

$$\Rightarrow \qquad \frac{d^2v_C(0^+)}{dt^2} = 0$$

Differentiating equation (4.12) with respect to t we get

$$\Rightarrow \qquad \frac{1}{64}\frac{d^3v_C}{dt^3} + \frac{1}{2}\frac{dv_L}{dt} = 0 \tag{4.14}$$

Differentiating equation (4.13) with respect to t, we get

$$\frac{\dfrac{dv_C}{dt} - \dfrac{dv_L}{dt}}{24} = \frac{1}{2}v_L$$

At $t = 0^+$, we get

$$\Rightarrow \qquad \frac{\dfrac{dv_C(0^+)}{dt} - \dfrac{dv_L(0^+)}{dt}}{24} = \frac{1}{2}v_L(0^+)$$

$$\Rightarrow \qquad \frac{-128 - \dfrac{dv_L(0^+)}{dt}}{24} = 0$$

$$\Rightarrow \qquad \frac{dv_L(0^+)}{dt} = -128 \text{ volts/ sec}$$

At $t = 0^+$, equation (4.14) becomes

$$\frac{1}{64}\frac{d^3v_C(0^+)}{dt^3} + \frac{1}{2}\frac{dv_L(0^+)}{dt} = 0$$

$$\Rightarrow \qquad \frac{d^3v_C(0^+)}{dt^3} = 4096 \text{ volts/ sec}^3$$

R.P 4.3

In the network of Fig RP 4.3 (a), switch K is closed at $t = 0$. At $t = 0^-$ all the capacitor voltages and all the inductor currents are zero. Three node-to-datum voltages are identified as v_1, v_2 and v_3. Find at $t = 0^+$:

(i) v_1, v_2 and v_3

(ii) $\dfrac{dv_1}{dt}$, $\dfrac{dv_2}{dt}$ and $\dfrac{dv_3}{dt}$

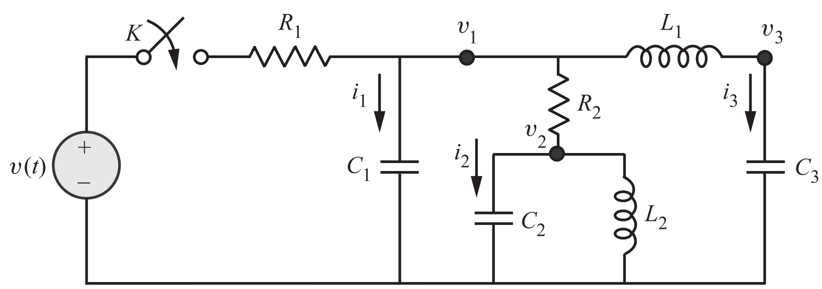

Figure R.P.4.3(a)

SOLUTION

The network at $t = 0^+$ is as shown in Fig RP-4.3 (b).

Since v_C and i_L cannot change instantaneously, we have from the network shown in Fig. RP-4.3 (b),

$$v_1(0^+) = 0$$
$$v_2(0^+) = 0$$
$$v_3(0^+) = 0$$

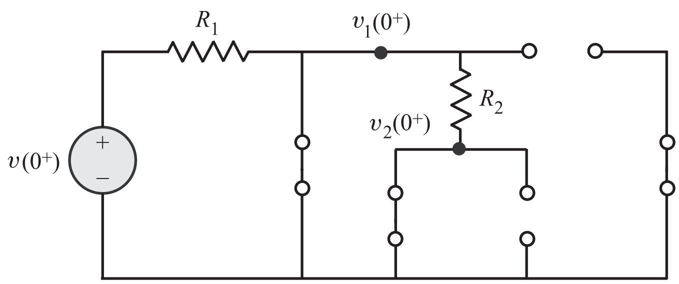

Figure R.P.4.3(b)

For $t \geq 0^+$, the circuit equations are

$$\left.\begin{aligned} v_{C_1} &= \frac{1}{C_1}\int_{0^+}^{t} i_1\,dt \\ v_{C_2} &= \frac{1}{C_2}\int_{0^+}^{t} i_2\,dt \\ v_{C_3} &= \frac{1}{C_3}\int_{0^+}^{t} i_3\,dt \end{aligned}\right\} \tag{4.15}$$

From Fig. RP-4.3 (b), we can write

$$i_1(0^+) = \frac{v(0^+)}{R_1},$$

$$i_2(0^+) = \frac{v_1(0^+) - v_2(0^+)}{R_2}$$

and $$i_3(0^+) = 0$$

Differentiating equation (4.15) with respect to t, we get

$$\frac{dv_{C_1}}{dt} = \frac{i_1}{C_1}, \frac{dv_{C_2}}{dt} = \frac{i_2}{C_2} \text{ and } \frac{dv_{C_3}}{dt} = \frac{i_3}{C_3}$$

At $t = 0^+$, the above equations give

$$\frac{dv_1(0^+)}{dt} = \frac{i_1(0^+)}{C_1} = \frac{v(0^+)}{R_1C_1}$$

$$\frac{dv_2(0^+)}{dt} = \frac{i_2(0^+)}{C_2} = \frac{v_1(0^+) - v_2(0^+)}{R_2C_2} = 0$$

and $$\frac{dv_3(0^+)}{dt} = \frac{i_3(0^+)}{C_3} = 0$$

R.P 4.4

For the network shown in Fig RP 4.4 (a) with switch K open, a steady-state is reached. The circuit papramerers are $R_1 = 10\Omega$, $R_2 = 20\Omega$, $R_3 = 20\Omega$, $L = 1$ H and $C = 1\mu$F. Take $V = 100$ volts. The switch is closed at $t = 0$.

(a) Write the integro-differential equation after the switch is closed.

(b) Find the voltage V_o across C before the switch is closed and give its polarity.

(c) Find i_1 and i_2 at $t = 0^+$.

(d) Find $\frac{di_1}{dt}$ and $\frac{di_2}{dt}$ at $t = 0^+$.

(e) What is the value of $\frac{di_1}{dt}$ at $t = \infty$?

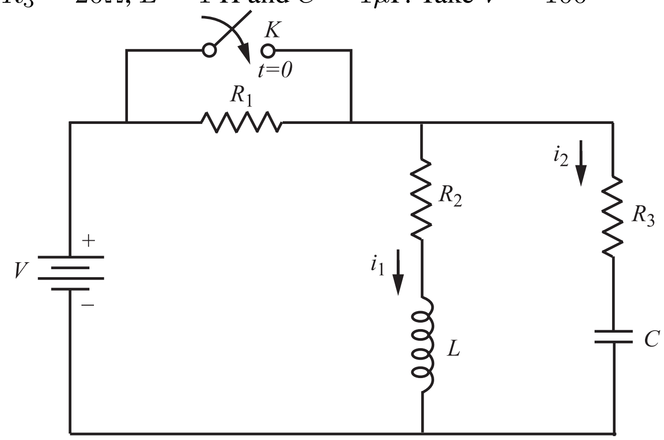

Figure R.P.4.4(a)

SOLUTION

The switch is in open state at $t = 0^-$. The network at $t = 0^-$ is as shown in Fig RP 4.4 (b).

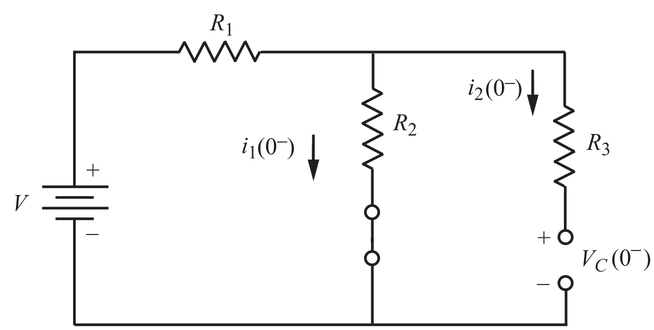

Figure R.P.4.4(b)

$$i_1(0^-) = \frac{V}{R_1 + R_2} = \frac{100}{30} = \frac{10}{3} \text{ A}$$

$$V_C(0^-) = i_1(0^-)R_2 = \frac{10}{3} \times 20 = \frac{200}{3} \text{ volts}$$

Note that L is short and C is open under steady-state condition.

For $t \geq 0^+$ (switch in closed state),

we have
$$20i_1 + \frac{di_1}{dt} = 100 \tag{4.16}$$

and
$$20i_2 + 10^6 \int_{0^+}^{t} i_2 dt = 100 \tag{4.17}$$

Also $$i_1(0^+) = i_1(0^-) = \frac{10}{3}\text{ A}$$

and $$V_C(0^+) = V_C(0^-) = \frac{200}{3}\text{ Volts}$$

From equation (4.16) at $t = 0^+$,

we have $$\frac{di_1(0^+)}{dt} = 100 - 20 \times \frac{10}{3}$$
$$= \frac{100}{3}\text{ A/sec}$$

From equation (4.17), at $t = 0^+$, we have

$$i_2(0^+) = \frac{1}{20}\left[100 - \frac{200}{3}\right] = \frac{5}{3}\text{ A}$$

Differentiating equation (4.17), we get

$$20\frac{di_2}{dt} + 10^6 i_2 = 0 \qquad (4.18)$$

From equation (4.18) at $t = 0^+$, we get

$$\frac{20di_2(0^+)}{dt} + 10^6 i_2(0^+) = 0$$

$$\Rightarrow \qquad \frac{di_2(0^+)}{dt} = \frac{-10^6 \times \frac{5}{3}}{20}$$
$$= \frac{-10^6}{12}\text{ A/ sec}$$

At $t = \infty$,

$$i_1(\infty) = \frac{100}{20} = 5\text{ A}$$
$$\frac{di_1}{dt}(\infty) = 0$$

R.P 4.5

For the network shown in Fig RP 4.5 (a), find $\dfrac{d^2 i_1(0^+)}{dt^2}$.

The switch K is closed at $t = 0$.

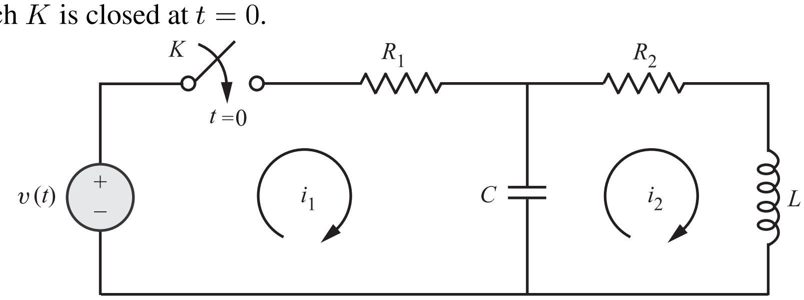

Figure R.P.4.5(a)

SOLUTION

At $t = 0^-$, we have $v_C(0^-) = 0$ and $i_2(0^-) = i_L(0^-) = 0$. Because of the switching property of L and C, we have $v_C(0^+) = 0$ and $i_2(0^+) = 0$. The network at $t = 0^+$ is as shown in Fig RP 4.5 (b).

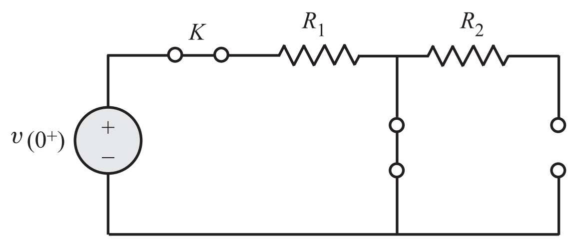

Figure R.P.4.5(b)

Referring Fig RP 4.5 (b), we find that

$$i_1(0^+) = \frac{v(0^+)}{R_1}$$

The circuit equations for $t \geq 0^+$ are

$$R_1 i_1 + \frac{1}{C}\int_{0^+}^{t}(i_1 - i_2)dt = v(t) \tag{4.19}$$

and

$$R_2 i_2 + \underbrace{\frac{1}{C}\int_{0^+}^{t}(i_2 - i_1)dt}_{v_C(t)} + L\frac{di_2}{dt} = 0 \tag{4.20}$$

At $t = 0^+$, equation (4.20) becomes

$$R_2 i_2(0^+) + v_C(0^+) + L\frac{di_2(0^+)}{dt} = 0$$

$$\Rightarrow \qquad \frac{di_2(0^+)}{dt} = 0 \tag{4.21}$$

Differentiating equation (4.19), we get

$$R_1\frac{di_1}{dt} + \frac{1}{C}(i_1 - i_2) = \frac{dv(t)}{dt} \tag{4.22}$$

Letting $t = 0^+$ in equation (4.22), we get

$$R_1\frac{di_1(0^+)}{dt} + \frac{1}{C}\left\{i_1(0^+) - i_2(0^+)\right\} = \frac{dv(0^+)}{dt}$$

$$\Rightarrow \qquad \frac{di_1(0^+)}{dt} = \frac{1}{R_1}\left\{\frac{dv(0^+)}{dt} - \frac{v(0^+)}{R_1 C}\right\} \tag{4.23}$$

Differentiating equation (4.22) gives

$$R_1 \frac{d^2 i_1}{dt^2} + \frac{1}{C}\left[\frac{di_1}{dt} - \frac{di_2}{dt}\right] = \frac{d^2 v(t)}{dt^2}$$

Letting $t = 0^+$, we get

$$R_1 \frac{d^2 i_1(0^+)}{dt^2} + \frac{1}{C}\left[\frac{di_1(0^+)}{dt} - \frac{di_2(0^+)}{dt}\right] = \frac{d^2 v(0^+)}{dt^2}$$

$$\Rightarrow \qquad R_1 \frac{d^2 i_1(0^+)}{dt^2} = -\frac{1}{C}\frac{di_1(0^+)}{dt} + \frac{d^2 v(0^+)}{dt^2}$$

$$\Rightarrow \qquad \frac{d^2 i_1(0^+)}{dt^2} = -\frac{1}{R_1 C}\left\{\frac{1}{R_1}\frac{dv(0^+)}{dt} - \frac{1}{R_1^2 C} v(0^+)\right\} + \frac{d^2 v(0^+)}{dt^2}$$

R.P 4.6

Determine $v_a(0^-)$ and $v_a(0^+)$ for the network shown in Fig RP 4.6 (a). Assume that the switch is closed at $t = 0$.

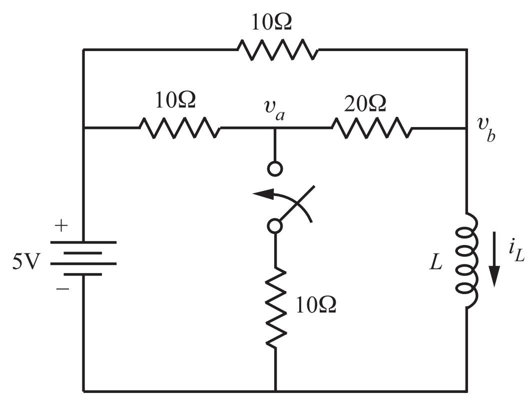

Figure R.P.4.6(a)

SOLUTION

Since L is short for DC at steady state, the network at $t = 0^-$ is as shown in Fig. RP 4.6 (b).

Applying KCL at junction a, we get

$$\frac{v_a(0^-) - 5}{10} + \frac{v_a(0^-) - v_b(0^-)}{20} = 0$$

Since $v_b(0^-) = 0$, we get

$$\frac{v_a(0^-) - 5}{10} + \frac{v_a(0^-) - 0}{20} = 0$$

$$\Rightarrow \qquad v_a(0^-) = \frac{0.5}{0.1 + 0.05} = \frac{10}{3} \text{ volts}$$

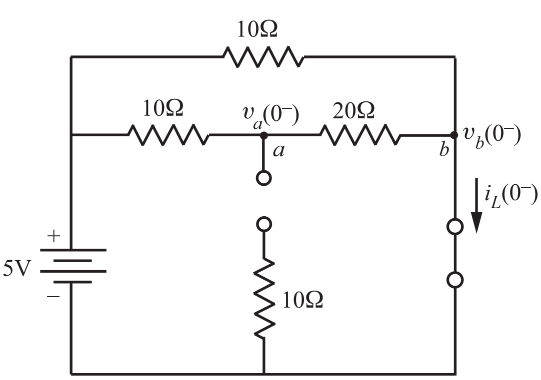

Figure R.P.4.6(b)

Also, $$i_L(0^-) = i_L(0^+) = \frac{v_a(0^-)}{20} + \frac{5}{10} = \frac{2}{3}\text{ A}$$

For $t \geq 0^+$, we can write

$$\frac{v_a - 5}{10} + \frac{v_a}{10} + \frac{v_a - v_b}{20} = 0$$

and $$\frac{v_b - v_a}{20} + \frac{v_b - 5}{10} + i_L = 0$$

Simplifying at $t = 0^+$, we get

$$\frac{1}{4}v_a(0^+) - \frac{1}{20}v_b(0^+) = \frac{1}{2}$$

and $$-\frac{1}{20}v_a(0^+) + \frac{3}{20}v_b(0^+) = \frac{-1}{6}$$

Solving we get, $v_a(0^+) = \dfrac{40}{21} = 1.905$ volts

Exercise problems

E.P 4.1

Refer the circuit shown in Fig. E.P. 4.1 Switch K is closed at $t = 0$.

Find $i(0^+)$, $\dfrac{di(0^+)}{dt}$ and $\dfrac{d^2i(0^+)}{dt^2}$.

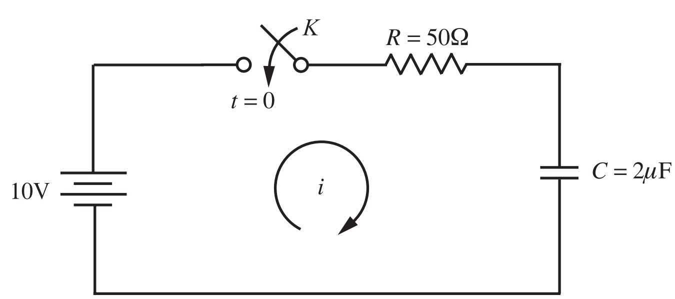

Figure E.P.4.1

Ans: $i(0^+) = 0.2\text{A}$, $\dfrac{di(0^+)}{dt} = -2 \times 10^3\text{A/sec}$, $\dfrac{d^2i(0^+)}{dt^2} = 20 \times 10^6\text{A/sec}^2$

E.P 4.2

Refer the circuit shown in Fig. E.P. 4.2. Switch K is closed at $t = 0$. Find the values of i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$.

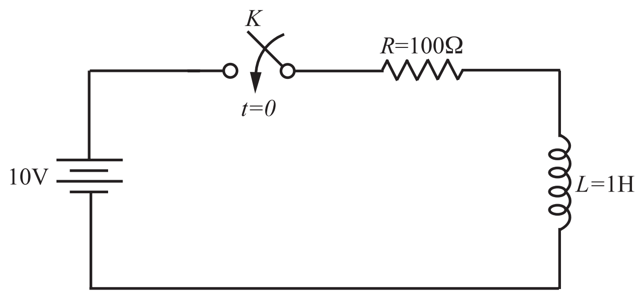

Figure E.P.4.2

Ans: $\boldsymbol{i(0^+) = 0, \quad \frac{di(0^+)}{dt} = 10 \text{ A/ sec}, \quad \frac{d^2i(0^+)}{dt^2} = -1000 \text{ A/ sec}^2}$

E.P 4.3

Refering to the circuit shown in Fig. E.P. 4.3, switch is changed from position 1 to position 2 at $t = 0$. The circuit has attained steady state before switching. Determine i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$.

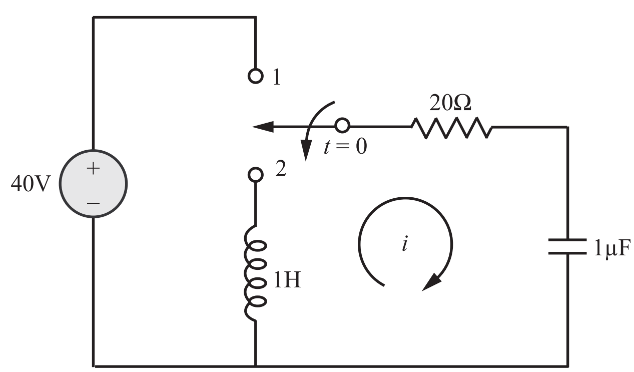

Figure E.P.4.3

Ans: $\boldsymbol{i(0^+) = 0, \quad \frac{di(0^+)}{dt} = -40 \text{ A/ sec}, \quad \frac{d^2i(0^+)}{dt^2} = 800 \text{ A/ sec}^2}$

E.P 4.4

In the network shown in Fig. E.P.4.4, the initial voltage on C_1 is V_a and on C_2 is V_b such that $v_1(0^-) = V_a$ and $v_2(0^-) = V_b$. Find the values of $\frac{dv_1}{dt}$ and $\frac{dv_2}{dt}$ at $t = 0^+$.

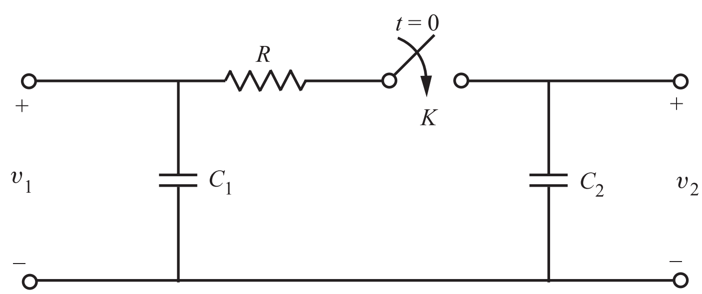

Figure E.P.4.4

Ans: $$\frac{dv_1(0^+)}{dt} = \frac{V_b - V_a}{C_1 R}\text{V/sec}, \quad \frac{dv_2(0^+)}{dt} = \frac{V_a - V_b}{C_2 R}\text{V/sec}$$

E.P 4.5

In the network shown in Fig E.P. 4.5, switch K is closed at $t = 0$ with zero capacitor voltage and zero inductor current. Find $\frac{d^2v_2}{dt^2}$ at $t = 0^+$.

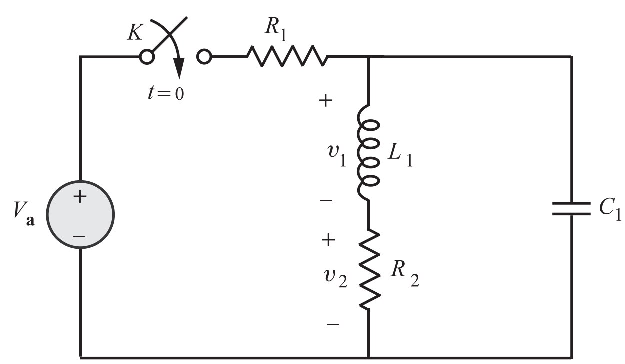

Figure E.P.4.5

Ans: $$\frac{d^2v_2(0^+)}{dt^2} = \frac{R_2 V_a}{R_1 L_1 C_1}\ \text{V/sec}^2$$

E.P 4.6

In the network shown in Fig. E.P. 4.6, switch K is closed at $t = 0$. Find $\dfrac{d^2v_1}{dt^2}$ at $t = 0^+$.

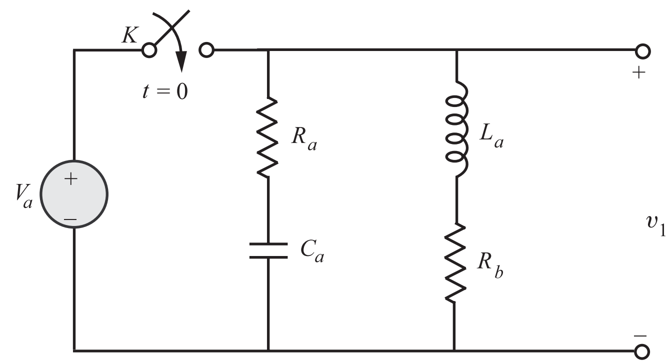

Figure E.P.4.6

Ans: $\boldsymbol{\dfrac{d^2v_1(0^+)}{dt^2} = 0\ \text{V}/\sec^2}$

E.P 4.7

The switch in Fig. E.P. 4.7 has been closed for a long time. It is open at $t = 0$. Find $\dfrac{di(0^+)}{dt}$, $\dfrac{dv(0^+)}{dt}$, $i(\infty)$ and $v(\infty)$.

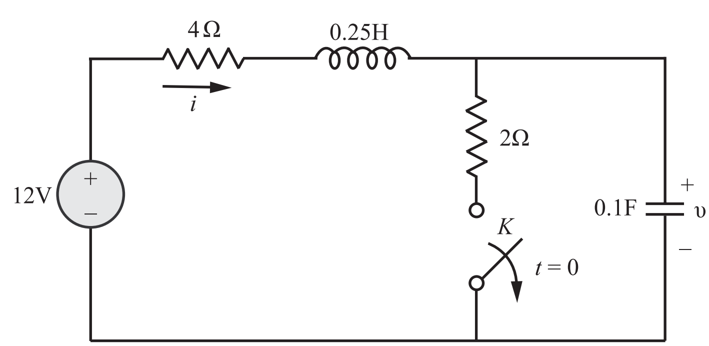

Figure E.P.4.7

Ans: $\boldsymbol{\dfrac{di(0^+)}{dt} = 0\text{A}/\sec,\quad \dfrac{dv(0^+)}{dt} = 20\text{A}/\sec,\ \ i(\infty) = 0\text{A},\ \ v(\infty) = 12\text{V}}$

E.P 4.8

In the circuit of Fig E.P. 4.8, calculate $i_L(0^+)$, $\frac{di_L(0^+)}{dt}$, $\frac{dv_C(0^+)}{dt}$, $v_R(\infty)$, $v_C(\infty)$ and $i_L(\infty)$.

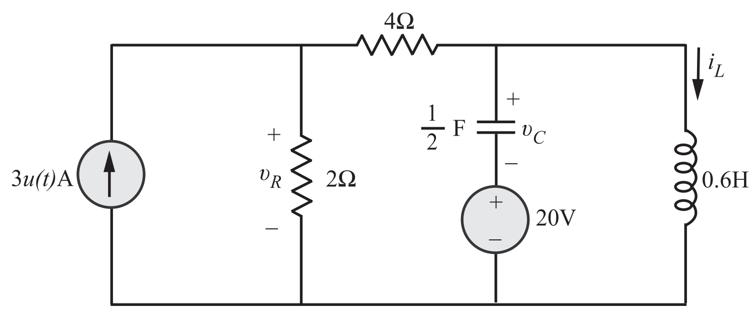

Figure E.P.4.8

Ans: $\boldsymbol{i_L(0^+) = 0\text{ A}, \frac{di_L(0^+)}{dt} = 0\text{ A/ sec}}$

$$\boldsymbol{\frac{dv_C(0^+)}{dt} = 2\text{ V/ sec}, \quad v_R(\infty) = 4\text{V}, \quad v_C(\infty) = -20\text{V}, \quad i_L(\infty) = 1\text{A}}$$

E.P 4.9

Refer the circuit shown in Fig. E.P. 4.9. Assume that the switch was closed for a long time for $t < 0$. Find $\frac{di_L(0^+)}{dt}$ and $i_L(0^+)$. Take $v(0^+) = 8$ V.

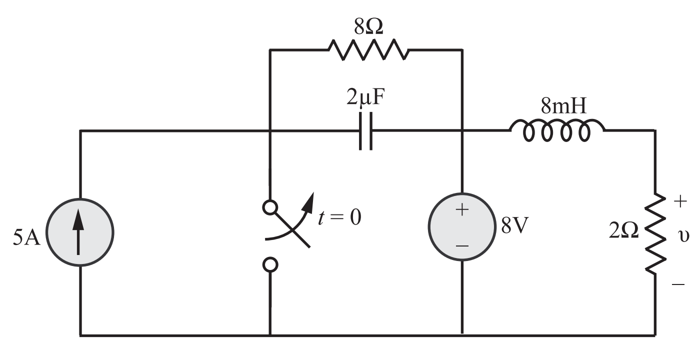

Figure E.P.4.9

Ans: $\boldsymbol{i_L(0^+) = 4\text{ A}, \frac{di_L(0^+)}{dt} = 0\text{ A/ sec}}$

E.P 4.10

Refer the network shown in Fig. E.P. 4.10. A steady state is reached with the switch K closed and with $i = 10$A. At $t = 0$, switch K is opened. Find $v_2(0^+)$ and $\frac{dv_2(0^+)}{dt}$.

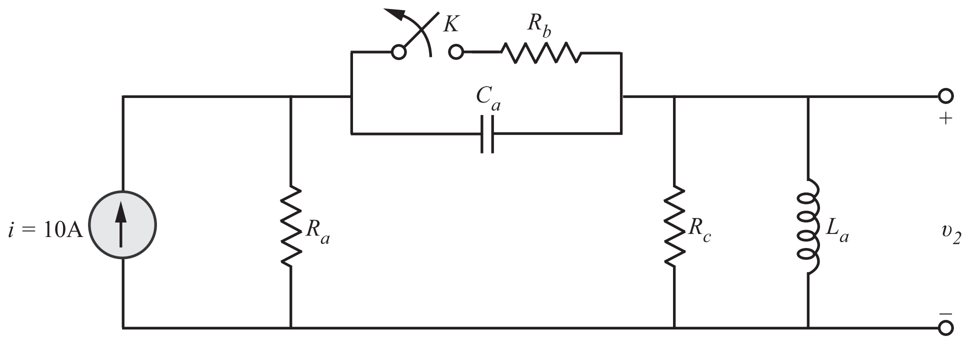

Figure E.P.4.10

Ans: $\boldsymbol{v_2(0^+) = 0,} \qquad \boldsymbol{\frac{dv_2(0^+)}{dt} = \frac{10R_aR_c}{C_a(R_a+R_b)(R_a+R_c)}}$ V/ sec.

E.P 4.11

Refer the network shown in Fig. E.P. 4.11. The network is in steady state with switch K closed. The switch is opened at $t = 0$. Find $v_k(0^+)$ and $\frac{dv_k(0^+)}{dt}$.

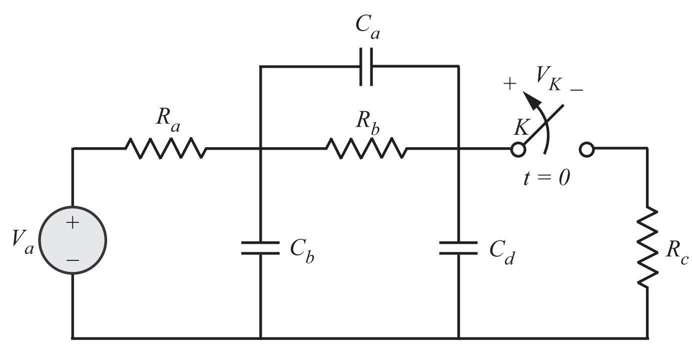

Figure E.P.4.11

Ans: $\boldsymbol{v_k(0^+) = \frac{V_aR_c}{R_a+R_b+R_c}}$ **Volts,**

$$\boldsymbol{\frac{dv_k(0^+)}{dt} = \frac{V_a(C_a+C_b)}{(R_a+R_b+R_c)(C_aC_d+C_bC_a+C_bC_d)}}\textbf{V/ sec}$$

E.P 4.12

Refer the network shown in Fig. E.P. 4.12. Find $\frac{d^2i_1(0^+)}{dt^2}$.

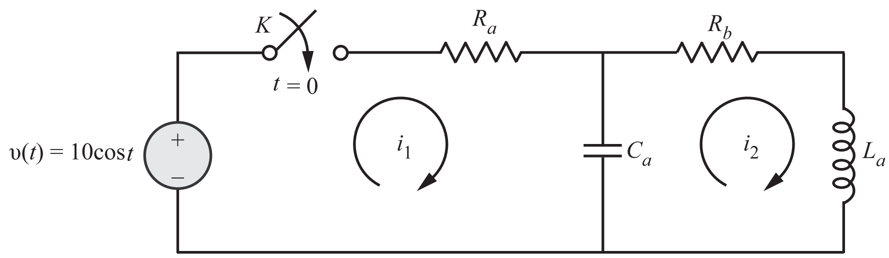

Figure E.P.4.12

Ans: $$\frac{d^2 i_1(0^+)}{dt^2} = \frac{1}{R_a}\left[-10 + \frac{10}{R_a^2 C_a^2}\right] \text{A/sec}^2$$

E.P 4.13

Refer the circuit shown in Fig. E.P. 4.13. Find $\frac{di_1(0^+)}{dt}$. Assume that the circuit has attained steady state at $t = 0^-$.

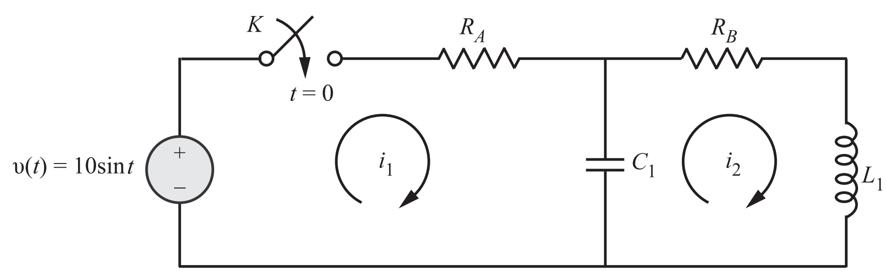

Figure E.P.4.13

Ans: $$\frac{di_1(0^+)}{dt} = \frac{10}{R_A} \text{A/sec}$$

E.P 4.14

Refer the network shown in Fig. E.P.4.14. The circuit reaches steady state with switch K closed. At a new reference time, $t = 0$, the switch K is opened. Find $\frac{dv_1(0^+)}{dt}$ and $\frac{d^2 v_2(0^+)}{dt^2}$.

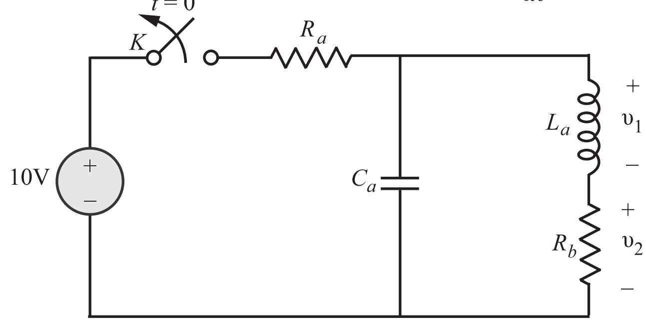

Figure E.P.4.14

Ans: $\dfrac{dv_1(0^+)}{dt} = \dfrac{-10}{C_a(R_a + R_b)}$ V/sec, $\dfrac{d^2v_2(0^+)}{dt^2} = \dfrac{-10R_b}{L_aC_a(R_a + R_b)}$ V/sec^2

E.P 4.15

The switch shown in Fig. E.P. 4.15 has been open for a long time before closing at $t = 0$. Find: $i_0(0^-)$, $i_L(0^-)$ $i_0(0^+)$, $i_L(0^+)$, $i_0(\infty)$, $i_L(\infty)$ and $v_L(\infty)$.

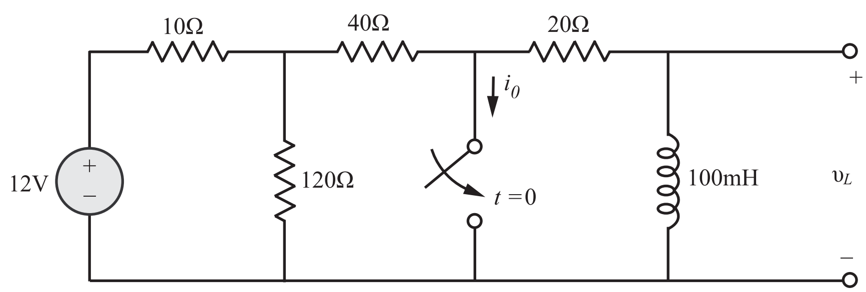

Figure E.P.4.15

Ans: $i(0^-) = 0$, $i_L(0^-) = 160\text{mA}$, $i_0(0^+) = 65\text{mA}$, $i_L(0^+) = 160\text{mA}$, $i_0(\infty) = 225\text{mA}$, $i_L(\infty) = 0$, $v_L(\infty) = 0$

E.P 4.16

The switch shown in Fig. E.P. 4.16 has been closed for a long time before opeing at $t = 0$.
Find: $i_1(0^-)$, $i_2(0^-)$, $i_1(0^+)$, $i_2(0^+)$. Explain why $i_2(0^-) \neq i_2(0^+)$.

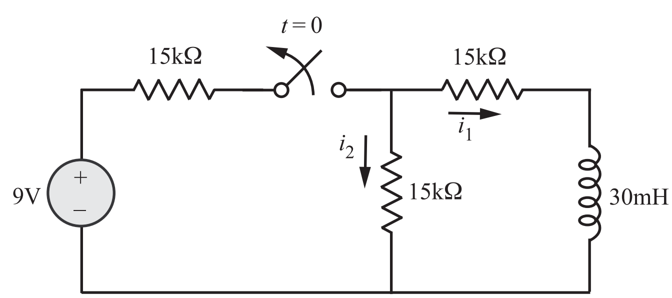

Figure E.P.4.16

Ans: $i_1(0^-) = i_2(0^-) = 0.2\text{mA}$, $i_2(0^+) = -i_1(0^+) = -0.2\text{mA}$

E.P 4.17

The switch in the circuit of Fig E.P.4.17 is closed at $t = 0$ after being open for a long time. Find:

(a) $i_1(0^-)$ and $i_2(0^-)$

(b) $i_1(0^+)$ and $i_2(0^+)$

(c) Explain why $i_1(0^-) = i_1(0^+)$

(d) Explain why $i_2(0^-) \neq i_2(0^+)$

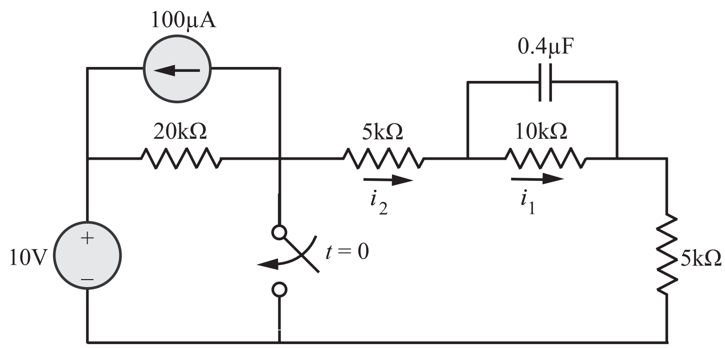

Figure E.P.4.17

Ans: $\mathbf{i_1(0^-) = i_2(0^-) = 0.2\ mA,\ i_1(0^+) = 0.2\ mA,\ i_2(0^+) = -0.2mA}$