3.1 Introduction:

A symmetrical, balanced three phase system can be analysed on a single phase basis. But, an unbalanced three phase system does not permit this simplification as it involves phasors of different magnitude and phase angles in each phase. Analysis under unbalanced conditions has to be carried out on a three phase basis which is very cumbersome process. Alternatively, a more convenient method of analysing unbalanced operation is through symmetrical components.

Dr. Fortescue's theorem forms the basis of the study of symmetrical components. According to the theorem, an unbalanced system of n-related phasors can be resolved into "n" systems of balanced phasors called symmetrical components of the original phasors. The "n" phasors of each set of components are equal in length and the angles between the adjacent phasors of the set are equal. The method of symmetrical components is a general one applicable to any unbalanced polyphase system. Because of the widespread use of three phase systems, the study here is confined to three phase systems only.

3.2 Resolution of unbalanced phasors.

According to Fortescue's theorem, a set of three unbalanced phasors (voltages or currents) can be resolved into three sets of balanced phasors, each set containing three phasors. The three sets of balanced components are called positive sequence components, negative sequence components and zero sequence components. Positive sequence components consists of three balance phasors of equal magnitude, displaced from each other by 120° in phase and having the same phase sequence as the original unbalanced phasors.Negative sequence components consists of three balanced phasors of equal magnitude, displaced from each other by 120° in phase and having a phase sequence opposite to that of the original unbalanced phasors. Zero sequence components are a set of three phasors, equal to each other in all respect.

Consider three unbalanced phasors V_a, V_b and V_c as shown in fig 3.1.

Let the direction of rotation of the phasors be in the anti clockwise direction. Then, it can be observed that the phase sequence of these three unbalanced phasors is abc.

The positive sequence components V_{a1}, V_{b1} and V_{c1} shown in fig. 3.2a constituting a three phase system are equal in magnitude and are symmetrically displaced by 120°. They have the same phase sequence 'abc' as the original unbalanced phasors. The negative sequence components V_{a2}, V_{b3} and V_{c2} shown in the fig 3.2b, constituting a three phase system are equal in magnitude, symmetrically displaced by 120° and have the phase sequence acb; opposite to that of the original phasors. The zero sequence components V_{a0}, V_{b0} and V_{c0} shown in fig. 3.2c are equal in all respects. These three phasors do not constitute a three phase system. They are equivalent to three single phase phasors of equal magnitude and having zero displacement between them.

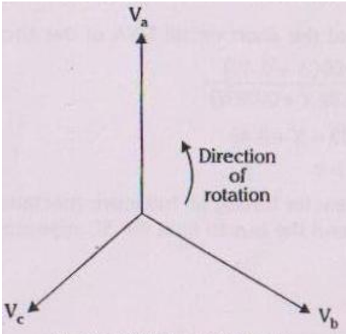


Fig. 3.1 : Unbalanced Phasor

Note:

1)Subscripts 1, 2 and 0 are used to indicate positive, negative and zero sequence components respectively.

2)The above three sets of phasors can be either voltages or currents.

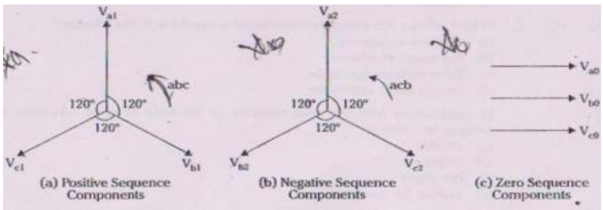


Fig. 3.2

3.3 The 'a' operator.

Because of the phase displacement of the symmetrical components of voltages and currents in a three phase system, it is convenient to have a short hand method of indicating the rotation of phasors through 120°. The letter 'a' (some books denote it as ɑ or λ also) is used to designate the operator that causes a rotation of 120° in the anticlockwise direction. This operator is a complex number of unit magnitude with an angle of 120° and is defined by the following expressions:

$a=1\angle 120° = 1.\ e^{j120°} = \cos 120°+j\sin 120° = -0.5+j0.866$

any phasor which is multiplied by 'a' remains unchanged in magnitude but is rotated by 120° in the anticlockwise direction.

Similarly, $a^2= a.a = 1\angle 240° = 1.\ e^{j240°} = \cos 240°+j\sin 240° = -0.5-j0.866$

Hence, operator 'a^2' will rotate a phasor in anticlockwise direction by 240°. This is same as rotating the phasor in clockwise direction by 120°.

It can be easily shown that,

1)$a^3 = 1$

2)$a^4 = a$

3)$1+a+a^2 = 0$

4)$a^* = a^2$, (* is conjugate)

5)$a-a^2 = j\sqrt{3}$

6)$a^2-a = -j\sqrt{3}$

These relations will be used in our discussion.

3.4 Expression for phase voltages in terms of symmetrical components.

Referring to fig. 3.2a, it can be observed that V_{b1} leads V_{a1} by 240° and the phasor V_{c1} leads V_{a1} by 120°. Since these three are also equal in magnitude we can write,

$V_{b1}=V_{a1}\angle 240°$

$V_{c1}=V_{a1}\angle 120°$

Making use of the 'a' operator, the above equations can be written as,

$V_{b1}=a^2.V_{a1}$

$V_{c1}=a.V_{a1}$ 3.1

On the same lines, referring to fig 3.2b, we get,

$V_{b2}=a.V_{a2}$

$V_{c2}=a^2.V_{a2}$ 3.2

and from fig 3.2c, it can be established that

$V_{a0}=V_{b0}=V_{c0}$ 3.3

Since three unbalanced phasors V_a, V_b and V_c can be resolved into three sets of balanced phasors, the phasor V_a, is equal to the sum of the positive sequence component V_{a1} of phase a, the negative sequence component V_{a2} of phase a and the zero sequence component V_{a0} of phase a. That is,

similarly,

$V_a=V_{a0}+V_{a1}+V_{a2}$ 3.4

$V_b=V_{b0}+V_{b1}+V_{b2}$ 3.5

$V_c=V_{c0}+V_{c1}+V_{c2}$ 3.6

Using equations 3.1, 3.2 and 3.3, the above expressions can be rewritten in terms of phase a as,

$V_a=V_{a0}+V_{a1}+V_{a2}$ 3.7

$V_b = V_{a0}+a^2.V_{a1}+a.V_{a2}$ 3.8

$V_c = V_{a0}+a.V_{a1}+a^2.V_{a2}$ 3.9

In matrix form, the above equation can be written as,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \quad ....3.10$$

The above equations establishes the relationship between the phase voltages of an unbalanced system and the symmetrical components.

3.5 Expression for symmetrical components in terms of phase voltages.

Let us denote,

$$[T] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

then eq. 3.10 becomes,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = [T] \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

or
$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = [T]^{-1} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad ....3.11$$

we determine

$[T]^{-1} = adj[T] / det[T]$

In this case,

$det[T] = 1(a^4-a^2) -1(a^2-a) +1(a-a^2) = 1(a-a^2)+(a-a^2)+(a-a^2) = 3(a-a^2)$

and,

$$adj[T] = \begin{bmatrix} +(a^4-a^2) & -(a^2-a) & +(a-a^2) \\ -(a^2-a) & +(a^2-1) & -(a-a^2) \\ +(a-a^2) & -(a-1) & +(a^2-1) \end{bmatrix} = \begin{bmatrix} (a-a^2) & (a-a^2) & (a-a^2) \\ (a-a^2) & a.(a-a^2) & a^2.(a-a^2) \\ (a-a^2) & a^2.(a-a^2) & a.(a-a^2) \end{bmatrix}$$

$$= (a-a^2) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

therefore,

$[T]^{-1} = adj[T] / det[T]$

$$= (1 / 3(a-a^2)) \times (a-a^2) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

$$= (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

Hence, eq. 3.11 becomes

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad ....3.12$$

in general form eq. 3.12 can be written as,

$V_{a0} = (1/3)\ (V_a+V_b+V_c)$ 3.13

$V_{a1} = (1/3)\ (V_a+a.V_b+a^2.V_c)$ 3.14

$V_{a2} = (1/3)\ (V_a+a^2.V_b+a.V_c)$........................3.15

The above equations gives the sequence components of voltages of phase a in terms of the phase voltages of the unbalanced system.

Equations 3.10 and 3.12 giving the transformation relationships between phase quantities and symmetrical components apply both to phase voltages and line currents of any star connected or equivalent star connected system, for line currents, the transformation is given by,

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \quad ....3.16$$

and,

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad ....3.17$$

Note:

1) Unless otherwise mentioned, symmetrical components of voltages and currents always mean phase voltages and line currents of an equivalent star connected system.

2) Also, by the sequence components of voltages and currents, it is always meant the sequence components of voltages and currents of phase a.

Example 3.1:
Prove that a balanced set of three phase voltages will have only positive sequence components of voltages only.
Solution:
A balanced three phase system of voltages is one where in all the phase voltages are of equal magnitude and symmetrically displaced by 120°. This is shown in fig 3.3

Let Va, V_b and V_c be the balanced system of three phase voltages.
From fig. 3.3, it can be observed that,

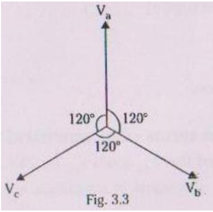


Fig. 3.3

$V_a = V_a$
$V_b = a^2.V_a$
$V_c = a.V_a$ 1

We have,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Using eq. 1 in the above matrix, we get...

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ a^2.V_a \\ a.V_a \end{bmatrix}$$

$$= (1/3) \begin{bmatrix} V_a + a^2.V_a + a.V_a \\ V_a + a^3.V_a + a^3.V_a \\ V_a + a^4.V_a + a^2.V_a \end{bmatrix}$$

putting $a^3=1$ and $a^4=a$, we get,

$$= (1/3)\begin{bmatrix} V_a + a.V_a + a^2.V_a \\ V_a + V_a + V_a \\ V_a + a.V_a + a^2.V_a \end{bmatrix}$$

$$= (1/3)\begin{bmatrix} V_a + a.V_a + a^2.V_a \\ V_a + V_a + V_a \\ V_a + a.V_a + a^2.V_a \end{bmatrix}$$

$$= (1/3)\begin{bmatrix} V_a(1+a+a^2) \\ 3V_a \\ V_a(1+a+a^2) \end{bmatrix}$$

but,

$(1+a+a^2)=0$,

$$= (1/3)\begin{bmatrix} 0 \\ 3V_a \\ 0 \end{bmatrix}$$

thus,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ V_a \\ 0 \end{bmatrix}$$

comparing the terms, we obtain,

$V_{a0}=0$

$V_{a1}=V_a$

$V_{a2}=0$

This clearly indicates that a balanced set of three phase voltages will have only positive sequence voltages. The negative and zero sequence components are always absent in a balanced system. This holds good for a balanced set of currents as well.

Example 3.2:

Determine the sequence components of the three voltages, $V_a=200\angle 0°V$, $V_b=200\angle 245°V$ and $V_c=200\ \angle 105°V$

solution:

The positive sequence components of voltage is ,

$V_{a1}=(1/3)\ (V_a + a.\ V_b + a^2.V_c)$

$=(1/3)\ (200\angle 0° + 200\angle 245°+120° + 200\ \angle 105°+240°)$

$=(1/3)\ (200+\ (199.24+j1743)+(193.19-j51.76))$

$=\ 0.9748-j11.44$

$=197.81\angle -3.3°\ V$

The negative sequence component of voltage is,

$V_{a2}=(1/3)\ (V_a + a^2.\ V_b + a.V_c)$

$=(1/3)\ (200\angle 0°+200\angle(245°+240°)+200\ \angle(105°+120°))$

$=(1/3)\ (200+(-114.72+j163.83+(-141.42-j141.42))$

$=-18.71+j7.47$

$=20.15\angle 158.2°\ V$

The zero sequence component of voltage is,

$V_{a0}=\ (1/3)\ (V_a + V_b + V_c)$

$=(1/3)\ (200\angle 0°+200\angle 245°+200\ \angle 105°)$

$=(1/3)(200+(-84.52-j181.26)+(-51.76-j193.18))$

$=21.21+j3.97$

$=21.6\angle 16.58°\ V$

Example 3.3: The positive, negative and zero sequence components of line currents are $20\angle 10°$, $6\angle 60°$ and $3\angle 30°$ A respectively. Determine the line currents.

$I_{a1}=\ 20\angle 10°$

$I_{a2}=\ 6\angle 60°$

$I_{a0}=\ \ 3\angle 30°$

we have, the line current,

$I_a=I_{a0}+I_{a1}+I_{a2}$

$=3\angle 30°+20\angle 10°\ +6\angle 60°$

$=27.25\angle 21.88°\ A$

$I_b=\ I_{a0}+a^2.I_{a1}+a.I_{a2}$

$=\ 3\angle 30°+20\angle(10°+\angle 240°\ )+6\angle(60°+120°)$

$=20.1\angle -120.7°\ A$

$I_c=\ I_{a0}+a.I_{a1}+a^2.I_{a2}$

$=3\angle 30°+20\angle(10°+120°)\ +6\angle(60°+240°)$

$=13.7\angle 122°\ A$

Example 3.4:

In a three phase system, $I_{a1}=100\angle 30°$ A, $I_{b2}=40\angle 90°$ A and $I_{c0}= 10\angle -30°$ A. Find the line currents.

Solution:

The sequence components of currents given in the problem are not of phase a only. Hence it is first required to express the sequence components in terms of phase a.

Consider fig 3.4. The negative sequence components of line currents are depicted in the sketch.

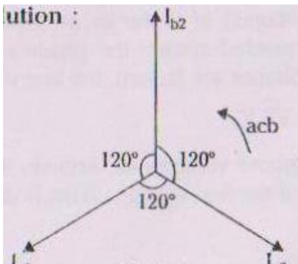


Fig. 3.4

From the fig. It can be observed that,

$I_{a2}=a^2.I_{b2}=40\angle(90°+240°)=40\angle 330°$ A

also we have

$I_{a0}=10\angle -30°$ A

$I_{a1}=100\angle 30°$ A

$I_{a2}=40\angle 330°$ A

$I_a=I_{a0}+I_{a1}+I_{a2}$

$=10\angle -30°+100\angle 30° +40\angle 330°$

$=132.24\angle 10.89°$ A

$I_b= I_{a0}+a^2.I_{a1}+a.I_{a2}$

$= 10\angle -30°+100\angle(30°+240°)+40\angle(330°+120°)$

$=65.57\angle -82.4°$ A

$I_c= I_{a0}+a.I_{a1}+a^2.I_{a2}$

$=10\angle -30°+100\angle(30°+120°)+40\angle(330°+240°)$

$=11.32\angle 167.48°$ A

3.6 Relation between sequence components of phase and line voltages in star connected systems.

It has been emphasized previously that, unless mentioned, by sequence voltages it is always meant phase voltages (line to neutral voltages) of a star or an equivalent star connected system. It is known that in a star connected system the phase voltages are different from the line voltages, But if the phase voltages are known, the line voltages can be easily determined by using the relation $V_{LL}=\sqrt{3}V_p$.

Similarly, if the sequence components of the phase voltages are known, it should be possible to determine the sequence components of the line voltages. This is discussed in the following points.

Let Va, Vb and Vc be the phase voltages having a phase sequence abc as shown in fig 3.5.

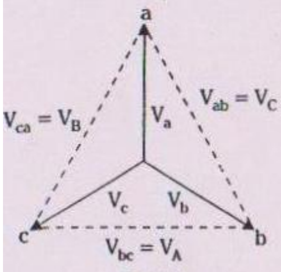


Fig. 3.5

The three line voltages of the system are Vbc, Vca and Vab. It is known from elementary vector algebra that.

$V_{bc} = V_c - V_b$

$V_{ca} = V_a - V_c$

$V_{ab} = V_b - V_a$

Let

$V_{bc} = V_A$(opposite to Vertex A)

$V_{ca} = V_B$(opposite to Vertex B)

$V_{ab} = V_C$(opposite to Vertex C)

therefore, we get

$V_A = V_{bc} = V_c - V_b$

$V_B = V_{ca} = V_a - V_c$

$V_C = V_{ab} = V_b - V_a$ 3.18

The positive sequence component of line voltage is given as

$V_{A1} = (1/3)(V_A + a.V_B + a^2.V_C)$

$= (1/3)((V_c - V_b) + a(V_a - V_c) + a^2.(V_b - V_a))$ .... in View of eq. 3.18

$= (1/3)((a(V_a + a.V_b + a^2.V_c) - a^2(V_a + a.V_b + a^2.V_c))$

$= (1/3)(a - a^2)(V_a + a.V_b + a^2.V_c)$

but, $(V_a + a.V_b + a^2.V_c) = 3.V_{a1}$

and, $(a - a^2) = j\sqrt{3}$

therefore, we get

$V_{A1} = (1/3)(j\sqrt{3})(3.V_{a1})$

Thus, $V_{A1} = j\sqrt{3}.\ V_{a1}$ 3.19

Hence, positive sequence component of line voltage is √3 times the positive sequence component of phase voltage and leads the corresponding phase voltage by 90°.

The negative sequence component of line voltage is

$V_{A2}=(1/3)(V_A+a^2.V_B+a.V_C)$

$=(1/3)((V_c-V_b)+a^2(V_a-V_c)+a(V_b-V_c))$, view of eq. 3.18

$=(1/3)\ (a^2(V_a+a^2.V_b+a.V_c)-a(V_a+a^2.V_b+a.V_c))$

$=(1/3)\ (a^2-a)\ (V_a+a^2.V_b+a.V_c)$

$=(1/3)(-j\sqrt{3})(3.V_{a2})$ [because $(V_a+a^2.V_b+a.V_c)=3.V_{a2}$ and $(a^2-a)=-j\sqrt{3}$]

Thus,

$V_{A2}=-j\sqrt{3}.V_{a2}$ 3.20

Hence, negative sequence component of line voltage is √3 times the negative sequence component of phase voltage and lags the corresponding phase voltage by 90°.

Finally, the zero sequence component of line voltage is given as,

$V_{A0}=(1/3)(V_A+V_B+V_C)$

$=(1/3)((V_c-V_b)+(V_a-V_c)+(V_a-V_b))$, in view of eq. 3.18

$=0$

Thus, $V_{A0}=0$ 3.21

Therefore, it is evident from the above equation that zero sequence component of line voltage is zero.

Note:

In similar lines as above, it can be proved that

$V_{B1}=j\sqrt{3}\ Vb1$; $V_{B2}=-j\sqrt{3}Vb2$; $V_{B0}=0$

$V_{C1}=j\sqrt{3}Vc1$; $V_{C2}=-j\sqrt{3}Vc2$; $V_{C0}=0$ 3.22

Example 3.5:

The positive and negative sequence components of phase voltages of a three phase system are Va1=230∠30° V and Va2=60∠60°V. Determine the positive and negative sequence components of line voltages and hence the line voltages.

Solution:

The positive, negative and zero sequence line voltages is given by,

$V_{A1}=j\sqrt{3}.\ V_{a1}=\sqrt{3}\ (230\angle(30°+90°))=398.37\angle120°$ V

$V_{A2}=-j\sqrt{3}.V_{a2}=\sqrt{3}\ (60\angle(60°-90°))=103.92\angle-30°$ V

It is known that zero sequence component of line voltage is zero. Thus $V_{A0}=0$.

Hence the line voltages of the system are,

$V_A=V_{A0}+V_{A1}+V_{A2}$

$=0+398.37\angle120°+103.92\angle-30°$

$=312.72\angle110°$ V

$V_B=V_{A0}+a^2.V_{A1}+a.V_{A2}$

$=0+398.37\angle(120°+240°)+103.92\angle(-30°+120°)$

$=411.7\angle 14.62°$ V

$V_C=V_{A0}+a.V_{A1}+a^2.V_{A2}$

$=0+398.37\angle(120°+120°)+103.92\angle(-30°+240°)$

$=491.13\angle -126°$ V

3.7 Relation between sequence components of phase and line currents in delta connected systems.

In a star connected system, the line currents are the same as that of the phase currents. But, this is not the case in a delta connected system. Here, the phase currents are different from the line currents. Like wise, the sequence components of line currents are different from the sequence components of phase currents.

Consider a delta connected three phase system where in the line currents Ia, Ib and Ic are entering the delta connected system as shown in fig 3.8

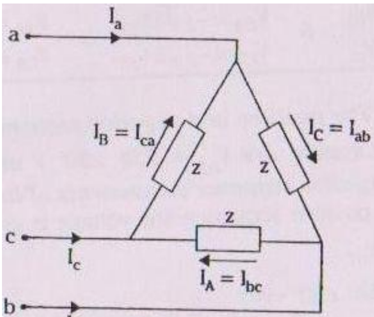


Fig. 3.8

The phase currents (currents in delta winding) are I_{ab}, I_{bc}, and I_{ca}. Let us designate $I_{ab}=I_C$, $I_{bc}=I_A$ and $I_{ca}=I_B$ (opposite to respective vertices).

Now, applying KCL to the system shown in fig 3.8, we get

$I_a=I_C-I_B$

$I_b=I_A-I_C$

$I_c=I_B-I_A$..........3.23

Then, the sequence component of line current are

$I_{a1}=(1/3)(I_a+a.I_b+a^2.I_c)$

$=(1/3)((I_C-I_B)+a(I_A-I_C)+a^2.(I_B-I_A))$

$=(1/3)((a(I_A+a.I_B+a^2.I_C)-a^2(I_A+a.I_B+a^2.I_C))$

$=(1/3)(a-a^2)(I_A+a.I_B+a^2.I_C)$

but, $(I_A+a.I_B+a^2.I_C)=3.I_{A1}$

and, $(a-a^2)=j\sqrt{3}$

therefore, we get

$I_{a1}=(1/3)(j\sqrt{3})(3.I_{A1})$

$I_{a1}= j\sqrt{3}\ .I_{A1}$ 3.24

$I_{a2}=(1/3)(I_a+a^2.I_b+a.I_c)$

$=(1/3)((I_C-I_B)+a^2(I_A-I_C)+a(I_B-I_C))$, view of eq. 3.18

$=(1/3)\ (a^2(I_A+a^2.I_B+a.I_C)-a(I_A+a^2.I_B+a.I_C))$

$=(1/3)\ (a^2-a)\ (I_A+a^2.I_B+a.I_C)$

$=(1/3)(-j\sqrt{3})(3.I_{A2})$ [because $(I_A+a^2.I_B+a.I_C)=3.I_{A2}$ and $(a^2-a)=-j\sqrt{3}$]

Thus,

$I_{a2}=-j\sqrt{3}.I_{A2}$ 3.25

From equations 3.24 and 3.25, it can be inferred that the line currents in delta system is √3 times the currents. The positive sequence line current leads the respective phase current by 90° whereas the negative sequence line current lags the negative sequence phase currents by 90°.

Finally, the zero sequence components of the line current is

$I_{a0}=(1/3)(I_a+I_b+I_c)$

$=(1/3)((I_C-I_B)+(I_A-I_C)+(I_A-I_B))$, in view of eq. 3.18

$=0$

Thus, $I_{a0}=0$ 3.26

The above result indicates that zero sequence currents are absent in the lines. In general, it can be shown that zero sequence component of line current is absent in any three wire system. This will be made clear in section 3.8. also the zero sequence line current $I_{a0}=0$, does not mean that the zero sequence phase current I_{A0} is also zero.

Note:

1)In similar lines as above, it can be proved that when the line currents are entering the data windings

$I_{b1}=j\sqrt{3}\ I_{B1}$; $I_{b2}=-j\sqrt{3}\ I_{B2}$; $I_{b0}=0$

$I_{c1}=j\sqrt{3}\ I_{C1}$; $I_{c2}=-j\sqrt{3}\ I_{C2}$; $I_{c0}=0$..................3.27

2)when the line currents I_a, I_b and I_c are leaving the delta connected windings as shown in fig.3.9, then it can be proved that

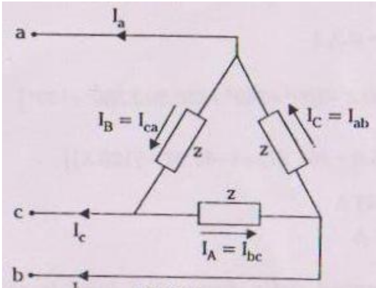


Fig. 3.9

$I_{a1}=-j\sqrt{3}\ I_{A1}$; $I_{a2}=j\sqrt{3}I_{A2}$; $I_{a0}=0$

$I_{b1}=-j\sqrt{3}\ I_{B1}$; $I_{b2}=j\sqrt{3}I_{B2}$; $I_{b0}=0$

$I_{c1}=-j\sqrt{3}\ I_{C1}$; $I_{c2}=j\sqrt{3}I_{C2}$; $I_{c0}=0$ 3.28

Example 3.7:

In a three phase, three wire system, the line currents are Ia=100∠0°A and Ib=100 ∠-100° A. Determine the sequence components of line currents.

Solution:

In three wire system always,

$I_a+I_b+I_c = 0$

$I_c= -(I_a+I_b)$

$= -(100\angle 0°+100\angle -100°)$

$=128.56\angle -130°$ A

Therefore, the sequence components of line currents are

$I_{a0}=(1/3)(I_a+I_b+I_c)$

$=(1/3)(100\angle 0°+100\angle -100°+128.56\angle -130°)$

=0 A (as expected)

$I_{a1}=(1/3)(I_a+a.I_b+a^2.I_c)$

$=(1/3)(100\angle 0°+100\angle(-100°+120°)+128.56\angle(-130°+240°)$

$=108.5\angle 10°$ A

$I_{a2}=(1/3)(I_a+a^2.I_b+a.I_c)$

$=(1/3)(100\angle 0°+100\angle(-100°+240°)+128.56\angle(-130°+120°)$

$=20.5\angle -110°$ A

Example 3.8:

A balanced delta connected load is connected to a three phase symmetrical supply. The line currents are each 10A in magnitude. If fuse in one of the lines blows out, determine the sequence components of line current.

Solution:

Let us assume that the fuse blows in line c. Then with the current in line a as reference, the diagram of the circuit is as shown in fig. 3.10

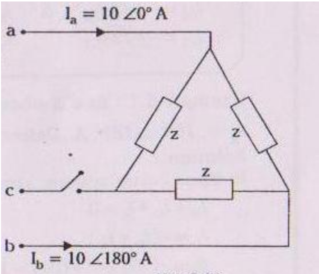


Fig. 3.10

Since this is a three wire system, we have

$I_a+I_b+I_c=0$

but,

$I_c=0$, as fuse blows out.

Therefore,

$I_b= -I_a$

if $I_a=10\angle 0°$ A, then

$I_b= -10\angle 0° = 10\angle 180°$ A

Hence, the positive sequence components of line current is

$I_{a1}=(1/3)(I_a+a.I_b+a^2.I_c)$

$=(1/3)(10\angle 0°+10\angle(180°+120°)+0)$

$=5.78\angle -30°$ A.

The negative sequence component of line current is

$I_{a2}=(1/3)(I_a+a^2.I_b+a.I_c)$

$=(1/3)(10\angle 0°+10\angle(180°+240°)+0)$

$=5.78\angle 30°$ A.

The zero sequence component of line current is absent in any three wire system. Thus $I_{a0}=0$ A

Example 3.9:

A delta connected balanced resistive load is connected across an unbalanced three phase supply as shown in fig 3.11. Find the symmetrical components of line current and delta current.

Solution:

$I_a+I_b+I_c=0$

or,

$I_c=-(I_a+I_b)$

$=-(10\angle 30° + 15\angle -60°)$
$=18\angle 154°$ A.

Hence, symmetrical components of line currents are

$I_{a1}=(1/3)(I_a+a.I_b+a^2.I_c)$
$=(1/3)(10\angle 30°+15\angle(-60°+120°)+18\angle(154°+240°))$
$=13.94\angle 41.86°$ A

$I_{a2}=(1/3)(I_a+a^2.I_b+a.I_c)$
$=(1/3)(10\angle 30°+15\angle(-60°+240°)+18\angle(154°+120°))$
$=4.65\angle 248°$ A

$I_{a0}=(1/3)(I_a+I_b+I_c)=0$ A

Here, the line currents are entering the delta connected load. Therefore, the sequence components of delta currents are,

$I_{A1}=(I_{a1} / j\sqrt{3}) = (13.94\angle(41.86°-90°) /\sqrt{3}) = 8.05\angle -48.14°$ A

$I_{A2}=(I_{a2} / -j\sqrt{3}) = (4.65\angle(248°+90°) /\sqrt{3}) = 2.68\angle 338°$ A

3.8 Effect of neutral in the system

consider a star connected system as shown in fig.3.13

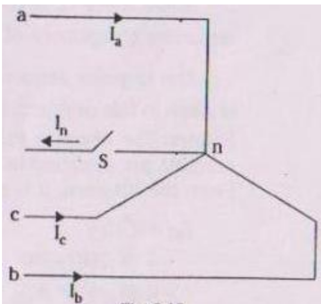


Fig. 3.13

Let the unbalanced line currents I_a, I_b and I_c. There are two possible cases here. One with the switch 's' closed i.e with the presence of the neutral wire. This forms a four wire system. The other with the switch 's' open forms a three wire system.

Let us consider both the cases independently.

Case i):

Four wire system.

Now, the current can flow through the neutral wire. Applying KCL at node 'n', we get the current through the neutral as

$I_n=I_a+I_b+I_c$ 3.20

but, we have

$I_a=I_{a0}+I_{a1}+I_{a2}$

$I_b = I_{a0} + a^2.I_{a1} + a.I_{a2}$

and,

$I_c = I_{a0} + a.I_{a1} + a^2.I_{a2}$

Using these results in eq. 3.29 yields,

$I_n = 3.I_{a0} + I_{a1}(1+a+a^2) + I_{a2}(1+a+a^2)$

$= 3I_{a0} + 0 + 0$, as $(1+a+a^2) = 0$

or $I_n = 3.I_{a0}$ 3.30

From eq. 3.30 it can be deduced that positive and negative sequence currents do not flow in the neutral wire. On the other hand, the neutral current is equal to thrice the zero sequence currents in a four wire system.

Case ii):

Three wire system

In this case, the neutral wire is not made available so that

$I_n = 0$ 3.31

Hence eq. 3.30 yields

$I_{a0} = 0$ 3.32

That is, zero sequence currents are absent in three wire system.

Note:

A delta connected system is also a three wire system. Hence, the zero sequence component of line current $I_{a0}=0$. This has been proved in section 3.7

Example 3.11:

In a three phase, three wire system, if $I_{a1}=100\angle 30°$ A, $I_{b2}=40\angle 90°$ A, find the line currents of the system.

Solution:

Since there is a three wire system, the zero sequence component of line current $I_{a0}=0$.

The negative sequence component of phase 'b' is given in this problem. Hence, we determine Ia2 as follows. The negative sequence components of line currents are depicted in fig. 3.14.

From the diagram, it is clear that

$I_{a2} = a^2.\ I_{b2}$

$= 40\angle(90° + 240°)$

$= 40\angle 330°$ A

Thus, the sequence components are

$I_{a0}=0$

$I_{a1}=100\angle 30°$ A

$I_{a2}=40\angle 330°$ A

Hence the line currents of the system are

$I_a=I_{a0}+I_{a1}+I_{a2}$

$=0+100\angle 30°+40\angle 330°$

$=124.89\angle 13.9°$ A

$I_b=I_{a0}+a^2.I_{a1}+a.I_{a2}$

$=0+100\angle(30°+240°)+40\angle(330°+120°)$

$=60\angle -90°$ A

$I_c=I_{a0}+a.I_{a1}+a^2.I_{a2}$

$=0+100\angle(30°+120°)+40\angle(330°+240°)$

$=124.89\angle 166.1°$ A

Example 3.12:

In a three phase system supplying power to a Y-load, the line currents when the neutral of the supply is not connected to the neutral of the load are Ia=20∠0° A and Ib=20∠-100° A. When the neutrals are connected, the current through the neutral wire is found to be 12∠-30° A. Determine the line currents under this situation.

Solution:

case i) when neutral of load is isolated from neutral of supply

In this case, $I_{a0}=0$

$I_a=I_{a1}+I_{a2}=20\angle 0°$ 1

$I_b=a^2.I_{a1}+a.I_{a2}=20\angle -100°$ 2

$I_c=a.I_{a1}+a^2.I_{a2}=-(I_a+I_b)= -(20\angle 0°+20\angle -100°) = 25.7\angle 130°$ 3

case ii) when the neutrals are connected

Here, it is given that I_n, the neutral current is 12∠-30°.

therefore,

$3.I_{a0} =12\angle -30°$

$I_{a0}=4\angle -30°$

Let I_a', I_b' and I_c' be the new values of line currents in this case, we get

$I_a'=I_{a0}+(I_{a1}+I_{a2})$

$= 4\angle -30°+20\angle 0°$, from result 1

$=23.53\angle -4.87°$ A

$I_b'=I_{a0}+(a^2.I_{a1}+a.I_{a2})$

$=4\angle -30°+20\angle -100°$, from result 2

$=21.69\angle-90°$ A

$I_c'=I_{a0}+(a.I_{a1}+a^2.I_{a2})$

$=4\angle-30°+25.7\angle130°$, from result 3=$22\angle126.48°$ A

3.9 Phase shift of symmetrical components in Y-Δ transformer bank.

Positive and negative sequence voltages and currents undergo a phase angle change in passing through a Y-Δ transformer (or a bank of three single phase transformers). This phenomenon is called as phase shift.

3.9.1 Voltage relations

Consider a transformer connection as shown in fig 3.15

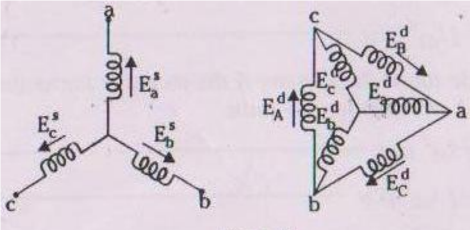


Fig. 3.15

Let E_a^s, E_b^s and E_c^s be the phase voltages on the Y-side of the transformer and E_a^d, E_b^d and E_c^d be the voltages across the windings on the Δ side of the transformer. Here, superscripts "s" and "d" stand for star side and delta side respectively.

We know from the principle of transformer, that the voltages across the various windings wound on any core will all be in phase, since these voltages are all produced due to rate of change of a common magnetic flux in the core. In this case, therefore, the phase voltage on the Y-side of the transformer should be in phase with the voltage across phase voltage on the Y-side of the transformer should be in phase with the voltage across the corresponding phase winding on the Δ side. If 'n' is the turns ratio, then we can write

$E_a^s=n.E_A^d$

$E_b^s=n.E_B^d$

$E_c^s=n.E_C^d$

Hence, the sequence components are also related as,

$E_{a1}^s=n.E_{A1}^d$ 3.33

$E_{a2}^s=n.E_{A2}^d$ 3.34

$E_{a0}^s=n.\ E_{A0}^d=0$ 3.35

The line voltages on the delta side of the transformer are equal to the voltages across the phase windings (on the delta side) of the transformer. But the phase voltages of the equivalent star on the delta side are different from the line voltages. Let E_a^d, E_b^d and E_c^d be the phase voltages of the equivalent star on the delta side. These are related to the line voltages (on the delta side) as follows.

$E_{A1}^d=j\sqrt{3}\ E_{a1}^d$ ....3.36 (refer equations 3.19 & 3.20)

and $E_{A2}^d = -j\sqrt{3}\ E_{a2}^d$ 3.37

Using these equations in 3.33 and 3.34, we get

$E_{a1}^s=n.E_{A1}^d=+j\sqrt{3}\ n.\ E_{a1}^d$ 3.38

and $E_{a2}^s = n.\ E_{A2}^d = -j\sqrt{3}.n.E_{a2}^d$ 3.39

Hence, we conclude that the positive sequence components of phase voltages on the star side of the transformer lead the corresponding positive components of the phase voltages (of the equivalent star) on the delta side by 90°. The same is true for line voltages on both sides of the transformer. The negative sequence components of the phase voltages on the star side of the transformer lags behind the corresponding negative sequence components of the equivalent phase voltages on the delta side by 90°. The relations 3.38 and 3.39 are vectorially represented in fig 3.16a and 3.16b.

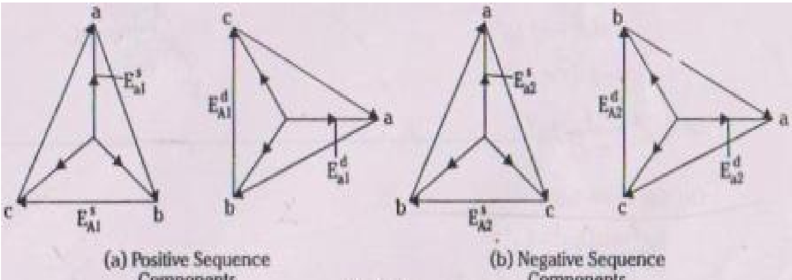


Fig. 3.16

3.9.2 Current relations

consider a star delta transformer connection as shown in fig 3.17

Let I_a^s, I_b^s and I_c^s be the line currents in the star side, I_a^d, I_b^d and I_c^d the line currents in delta side I_A^d, I_B^d and I_C^d the currents in the delta windings.

If 'n' is the turns ratio, then from the theory of transformers, we can write that, $I_A^d=n.I_a^s$ 3.40

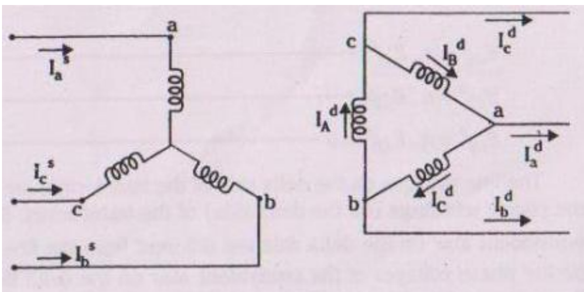


Fig. 3.17

$I_B^d = n.I_b^s$ 3.41

$I_C^d = n.I_c^s$ 3.42

Applying KCL to the nodes on the delta side, we can establish that

$I_a^d = I_B^d - I_C^d = n(I_b^s - I_c^s)$

$I_b^d = I_C^d - I_A^d = n(I_c^s - I_a^s)$, from equations 3.40 , 3.41 and 3.42

$I_c^d = I_A^d - I_B^d = n(I_a^s - I_b^s)$

Considering only positive sequence currents, the above relation becomes

$I_{a1}^d = n(I_{b1}^s - I_{c1}^s) = n(a^2.I_{a1} - a.I_{a1}^s) = n(a^2 - a)I_{a1}^s = -j\sqrt{3}\ n.I_{a1}^s$

or $I_{a1}^s = (j/n.\sqrt{3})I_{a1}^d$ 3.43

on the same lines

$I_{a2}^d = n(I_{b2}^s - I_{c2}^s) = n(a.I_{a2}^s - a2.I_{a2}^s) = n(a - a2)I_{a2}^s = j\sqrt{3}.n.I_{a2}^s$

or $I_{a2}^s = (-j/n\sqrt{3})I_{a2}^d$ 3.44

From 3.43 and 3.44, it can be inferred that the positive sequence component of the line current on the star side of the transformer leads the corresponding positive sequence component of line currents on the delta side by 90° and the negative sequence components of the line current on the star side of the transformer lags behind the corresponding negative sequence component of the line current on the delta side by 90°.

Note:

1)The turns ratio 'n' of a transformer is defined as,

n=number of primary turns / number of secondary turns

=primary voltage / secondary voltage

=secondary current / primary current

2)If each voltage is expressed in per unit with its own voltage as the base voltage, then equation 3.38 and 3.39 can be written as,

$E_{a1}^s = jE_{a1}^d$ p.u 3.45

$E_{a2}^s = -jE_{a2}^d$ p.u 3.46

3)Similarly, the per unit for a Y-Δ transformer

$I_{a1}^s = jI_{a1}^d$ p.u 3.47

$I_{a2}^s = -jI_{a2}^d$ p.u 3.48

4)If delta side forms the primary and the star side forms the secondary, that is in the case of Δ-Y transformer, we have

$I_{a1}^d = jI_{a1}^s$ p.u 3.49

$I_{a2}^d = -jI_{a2}^s$ p.u 3.50

$E_{a1}^d = jE_{a1}^s$ p.u 3.51

$E_{a2}^d = -jE_{a2}^s$ p.u 3.52

3.10 Complex power in terms of symmetrical components

If the symmetrical components of currents and voltages are known, then the power in a three phase circuit can be computed directly from the components. The total complex power flowing into a three phase circuit is given as

$S=P+jQ = V_a.I_a^*+V_b.I_b^*+V_c.I_c^*$

where,

S=Total complex power,(* indicates conjugate)

P=Active power

Q=Reactive power

In matrix from, the above equation can be expressed as,

$$S = P + jQ = [V_a \quad V_b \quad V_c] \begin{bmatrix} I_a^* \\ I_b^* \\ I_c^* \end{bmatrix}$$

..........................3.53

But,

$$[V_a \quad V_b \quad V_c] = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T = \left\{ \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \right\}^T$$

$$= \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}^T \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$ 3.54

since $([A].[B])^T = [A]^T.[B]^T$

and,

$$\begin{bmatrix} I_a^* \\ I_b^* \\ I_c^* \end{bmatrix} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \left\{ \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \right\}^*$$

since $([A].[B])^* = [A]^*.[B]^*$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}^* \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

Now $a^*=a^2$

$(a^2)^*=a$, Using these, we get

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

Substituting equation 3.52 and 3.53 in 3.51 we get,

$$S=(P+jQ)=\begin{bmatrix}V_{a0}\\V_{a1}\\V_{a2}\end{bmatrix}^T\begin{bmatrix}1&1&1\\1&a^2&a\\1&a&a^2\end{bmatrix}\cdot\begin{bmatrix}1&1&1\\1&a&a^2\\1&a^2&a\end{bmatrix}\begin{bmatrix}I_{a0}\\I_{a1}\\I_{a2}\end{bmatrix}^*$$

$$=\begin{bmatrix}V_{a0}&V_{a1}&V_{a2}\end{bmatrix}\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}\begin{bmatrix}I_{a0}\\I_{a1}\\I_{a2}\end{bmatrix}^*$$

S=P+jQ = 3{Va0.Ia0*+Va1.Ia1*+Va2.Ia2*} VA 3.56

Thus, if the symmetrical components of currents and voltages are known, then the power consumed by a three phase circuit can be computed from these components.

Note:

1)In terms of active and reactive powers, the above equation can be written as

$P=3\{|V_{a0}|\ |I_{a0}|\ \cos\theta_0 + |V_{a1}|\ |I_{a1}|\ \cos\theta_1 + |V_{a2}|\ |I_{a2}|\ \cos\theta_2\}$ W 3.57

and

$Q=3\{|V_{a0}|\ |I_{a0}|\ \sin\theta_0 + |V_{a1}|\ |I_{a1}|\ \sin\theta_1 + |V_{a2}|\ |I_{a2}|\ \sin\theta_2\}$ VAR 3.58

2)If V_B is the base voltage and I_B the base current of the system, then the complex power in pu is given as,

$S_{p.u}=3(V_{a0}\ I_{a0}^*+V_{a1}\ I_{a1}^*+V_{a2}\ I_{a2}^*)\ (1\ /\ V_B\ I_B)$ 3.59

or $S_{p.u}=S\ /S_B$ 3.60

Where S_B=base power of the system $=3V_BI_B$

3)If the symmetrical components of voltages and currents are given in pu directly, then the total 3 phase power is given as

$S_{p.u}=V_{a0}\ I_{a0}^*+V_{a1}\ I_{a1}^*+V_{a2}\ I_{a2}^*$ 3.61

Example 3.13:

The sequence components of the phase voltages are $V_{a1}=200\angle 30°$, $V_{a2}=60\angle 60°$ and $V_{a0}=20\angle -30°$ V. The line currents are $I_{a1}=20\angle 10°$, $I_{a2}=5\angle 20°$ and $I_{a0}=3\angle -10°$ A. Determine the three phase power in kVA and p.u. If the base power is 1kVA.

Solution:

The three phase complex power is given as

$S_{p.u}=3(V_{a0}\ I_{a0}^*+V_{a1}\ I_{a1}^*+V_{a2}\ I_{a2}^*)$

$=3\{(200\angle 30°)(20\angle -10°)+(60\angle 60°)(5\angle -20°)+(20\angle -30°)(3\angle 10°)\}$

$=(12.13+j4.62)$ kVA

We have $S_{p.u}=S\ /S_B$ = (12.13+j4.62) kVA / 1kVA = (12.13+j4.62) p.u

Example 3.14:

In a three phase system, the sequence quantities are V_{a1}=(0.9+j0.2)p.u; V_{a2}=(0.1+j0.1)p.u; V_{a0}=(0.1+j0.05)p.u and I_{a1}=(0.9-j0.1) p.u; I_{a2}=(0.2-j0.1)p.u; I_{a0}=(0.05-j0.02)p.u. Find the three phase complex power in p.u and in MAV on a base of 100MVA. Also compute the active and reactive powers.

Solution:

Here, the sequence components are given in p.u Hence, the total three phase power is given as,

$S_{p.u}=(V_{a0}\, I_{a0}^*+V_{a1}\, I_{a1}^*+V_{a2}\, I_{a2}^*)$

$=\{(0.1+j0.05)(0.05-j0.02)^*+(0.9+j0.2)(0.9-j0.1)^*+(0.1+j0.1)(0.2-j0.1)^*\}$

$=\{(0.1+j0.05)(0.05+j0.02)^*+(0.9+j0.2)(0.9+j0.1)^*+(0.1+j0.1)(0.2+j0.1)^*\}$

$=(0.817+j0.3126)$p.u

Next, $S=S_{p.u}\times S_B$

$=(0.817+j0.3126)\times 100$ MVA $=(81.7+j31.26)$MVA

We have, S=P+jQ=(81.7+j31.26)MVA

Therefore,

The active power is P=81.7MW

The reactive power is Q=31.26MVAR

Example 3.15:

In a three phase four wire system, the sequence voltages and currents are: $V_{a1}=0.9\angle 10°$p.u; $V_{a2}=0.25\angle 110°$p.u; $V_{a0}=0.12\angle 300°$p.u and $I_{a1}=0.75\angle 25°$ p.u; $I_{a2}=0.15\angle 170°$p.u; $I_{a0}=0.1\angle 330°$p.u. Find the complex power in p.u. If the neutral gets disconnected, find the new power.

Solution:

The total three phase power in pu is given as,

$S_{p.u}=(V_{a0}\, I_{a0}^*+V_{a1}\, I_{a1}^*+V_{a2}\, I_{a2}^*)$

$=(0.12\angle 300°)(0.1\angle 330°)^*+(0.9\angle 10°)(0.75\angle 25°)^*+(0.25\angle 110°)(0.15\angle 170°)^*$

$=(0.12\angle 300°)(0.1\angle -330°)+(0.9\angle 10°)(0.75\angle -25°)+(0.25\angle 110°)(0.15\angle -170°)$

$=(0.68-j0.212)$p.u

When the neutral gets opened, then $I_{a0}=0$. Hence the new power is,

$S_{p.u}'=(V_{a1}\, I_{a1}^*+V_{a2}\, I_{a2}^*)$

$=(0.9\angle 10°)(0.75\angle 25°)^*+(0.25\angle 110°)(0.15\angle 170°)^*$

$=(0.9\angle 10°)(0.75\angle -25°)+(0.25\angle 110°)(0.15\angle -170°)$

$=(0.67-j0.206)$p.u.

--------END--------

4.1 Introduction:

In previous chapter, we have discussed the sequence components for voltages and currents. Let us now consider the impedances offered to positive, negative and zero sequence currents in symmetrical circuits. In symmetrical circuits, currents of a given sequence produce voltage drops of the same sequence only. Before attempting a proof for this, let us define sequence impedances.

The impedances offered by the circuit for the flow of positive sequence currents is called as positive sequence impedance. Similarly, the impedance offered by the circuit for the flow of negative sequence currents is negative sequence impedance and zero sequence currents is zero sequence impedance.

4.2 Sequence impedance of a symmetrical circuit:

In this section, we show that in a symmetrical circuit, currents of a given sequence produce a voltage drop of the same sequence only.

Consider a three phase symmetrical circuit as shown in fig 4.1. Let I_a, I_b and I_c be the currents in each line. These currents return via the neutral(ground) impedance Z_n as shown.

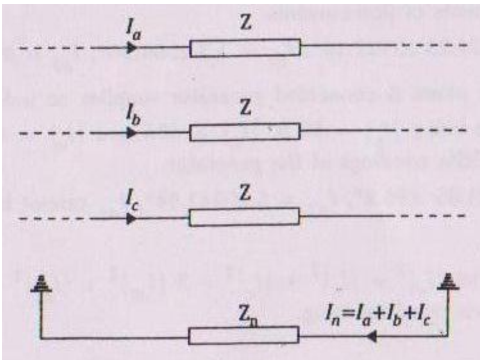


Fig. 4.1

The voltage drops are computed as follows:

$V_a=I_a.Z+I_n.Z_n$

$\quad =I_a.Z+(I_a+I_b+I_c)Z_n$

$\quad =I_a.(Z+Z_n)+I_b.Z_n+I_cZ_n$ 4.1

Similarly,

$V_b=I_a.Z_n+I_b(Z+Z_n)+I_c.Z_n$ 4.2

and $V_c=I_a.Z_n+I_b.Z_n+I_c(Z_n+Z_n)$ 4.3

In matrix form, the above equations can be expressed as:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z+Z_n & Z_n & Z_n \\ Z_n & Z+Z_n & Z_n \\ Z_n & Z_n & Z+Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad ....4.4$$

Expressing the voltages and currents by their sequence components, we get.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} Z+Z_n & Z_n & Z_n \\ Z_n & Z+Z_n & Z_n \\ Z_n & Z_n & Z+Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

or,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z+Z_n & Z_n & Z_n \\ Z_n & Z+Z_n & Z_n \\ Z_n & Z_n & Z+Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} Z+3Z_n & 0 & 0 \\ 0 & Z & 0 \\ 0 & 0 & Z \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \quad ....4.5$$

Namely, positive sequence, negative sequence and zero sequence in this manner the three sequences may be solved individually. Once the problem is solved in terms of symmetrical components. It can be transformed back to the actual circuit conditions.

This gives relationships

$V_{a0}=(Z+3Z_n)I_{a0}$

$V_{a1}=Z.I_{a1}$

$V_{a2}=Z.I_{a2}$ 4.6

The above equations indicate that in symmetrical circuits, currents of given sequence produce voltage drops of the same sequence only, i.e the sequence impedances are uncoupled in case of symmetrical circuits. Accordingly the problem can be effectively broken down into three separate systems.

Also, as per definition of sequence impedances, we have the three sequence impedances as,

$Z_0=V_{a0}/I_{a0}=Z+3Z_n$=Zero sequence impedance 4.7

$Z_1=V_{a1}/I_{a1}=Z$=positive sequence impedance 4.8

$Z_2=V_{a2}/I_{a2}=Z$=negative sequence impedance 4.9

From the above results, we can conclude that for a symmetrical static circuit (like that of transformers of fully transposed transmission lines).

i) The positive sequence impedance is the same as the negative sequence impedance.

ii) Zero sequence impedance is much larger than the positive (or negative) sequence impedance. In the absence of the neutral, $Z_n = \infty$

therefore,

$Z_0 = \infty$ and hence $I_{a0} = V_{a0}/Z_0 = V_{a0}/\infty = 0$, as expected.

Example 4.4:

Across a star connected symmetrical impedance load of 10Ω in each phase and a neutral impedance of 3.33Ω, an unbalanced three phase supply with $V_a = 220\angle 0°$, $V_b = 200\angle 110°$ and $V_c = 180\angle -110°$ is applied. Determine the line currents using symmetrical components.

Solution:

since the circuit is symmetrical, we have

$Z_1 = Z_2 = 10\ \Omega$

and, $Z_0 = Z + 3Z_n = 10 + 3(3.33) = 20\ \Omega$

carefully observe the phase voltages shown in fig 4.2

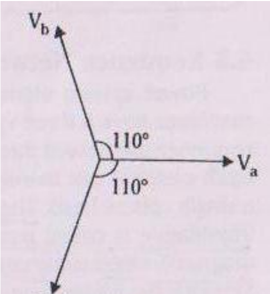


Fig. 4.2

It can be seen that phase sequence is acb. Hence in the equations for the determination of sequence components of phase voltages, the subscripts b and c should be interchanged.

Hence,

$V_{a1} = (1/3)\ (V_a + a.\ V_c + a^2.V_b)$

$= (1/3)\ (220\angle 0° + 180\angle(-110° + 120°) + 200\angle(110° + 240°))$

$= 198.07\angle -0.33°$ volts.

$V_{a2} = (1/3)\ (V_a + a^2.\ V_c + a.V_b)$

$= (1/3)\ (220\angle 0° + 180\angle(-110° + 240°) + 200\angle(110° + 120°))$

$= 9.56\angle -147.7°$ volts.

$V_{a0} = (1/3)\ (V_a + V_c + V_b)$

$= 220\angle 0° + 180\angle -110° + 200\angle 110°$

$= 30.64\angle 11.77°$ volts.

Now,

$I_{a1} = V_{a1}/Z_1$

$=(198.07\angle-0.33°)/10$

$=19.807\angle-0.33°$ A

$I_{a2}=V_{a2}/Z_2$

$=(9.56\angle-147.7°)/10$

$=0.956\angle-147.7°$ A

$I_{a0}=V_{a0}/Z_0$

$=(30.64\angle11.77°)/20$

$=1.532\angle11.77°$ A

The line currents are,

$I_a=I_{a0}+I_{a1}+I_{a2}$

$=1.532\angle11.77°+ 19.807\angle-0.33 +0.956\angle-147.7°$

$=20.49\angle-0.87°$ A

$I_b= I_{a0}+a.I_{a1}+a^2.I_{a2}$ (for phase sequence acb)

$=1.532\angle11.77°+ 19.807\angle(-0.33°+120°) +0.956\angle(-147.7°+240°)$

$=20.27\angle114.4°$ A

$I_c= I_{a0}+a^2.I_{a1}+a.I_{a2}$

$=1.532\angle11.77°+ 19.807\angle(-0.33°+240°) +0.956\angle(-147.7°+120°)$

$=18.85\angle-113.86°$ A

Example 4.2:

Prove that a three phase symmetrically wound alternator generators only positive sequence components of voltages.

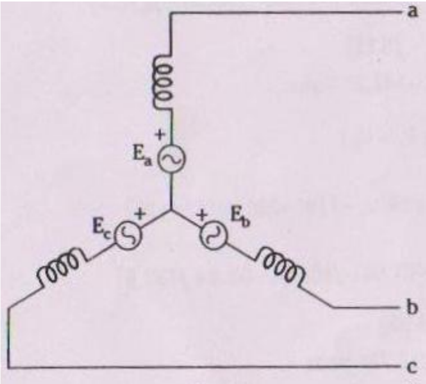


Fig. 4.3

Solution:

Fig. 4.3 depicts an unbalanced synchronous generator. E_a, E_b, E_c are the induced emfs of the three phases. Since the windings are symmetrical, the induced emfs are perfectly balanced.

Let,

$|E_a|=|E_b|=|E_c|=V_p$

Then, if follows that (assuming a abc phase sequence)

$E_a=V_p\angle 0°$

$E_b=V_p\angle -120°$

$E_c=V_p\angle 120°$

Hence the sequence components of voltages are,

$E_{a0}=(1/3)(E_a+E_b+E_c)$

$=(1/3)(V_p\angle 0°+V_p\angle -120°+V_p\angle 120°)$

$=0$

$E_{a1}=(1/3)(E_a+a.E_b+a^2E_c)$

$=(1/3)(V_p\angle 0°+V_p\angle(-120°+120°)+V_p\angle(120°+240°))$

$=V_p$

$=E_a$

$E_{a2}=(1/3)(E_a+a^2.E_b+a.E_c)$

$=(1/3)(V_p\angle 0°+V_p\angle(-120°+240°)+V_p\angle(120°+120°))$

$=0$

From the results obtained above, it can be inferred that a three phase symmetrically wound alternator generators only positive sequence components of voltages.

4.3 Sequence Networks of power system elements:

Power system elements namely transmission lines, transformers and synchronous machines have a three phase symmetry because of which when currents of a particular sequence are passed through these elements, voltage drops of the same sequence appear. Each element can therefore be represented by three decoupled sequence networks on a single phase basis. The impedance or reactance diagram formed using positive sequence impedance is called positive sequence network. Similarly the impedance or reactance diagram formed using negative sequence impedance is called negative sequence network. Similarly the impedance or reactance diagram formed using zero sequence impedance is called zero sequence network.

The sequence networks are very useful in the analysis of unsymmetrical faults in the power system. In unsymmetrical fault analysis of a power system, the positive, negative and zero sequence networks of the system are determined and then they are interconnected suitably to represent the various fault conditions. The sequence currents and voltages during the fault are then calculated from which actual fault currents and voltages can be found. Before proceeding to these fault analysis, we must know the equivalent circuit presented by the power elements to the flow of positive, negative and zero sequence currents respectively.

4.4 Sequence impedances and networks of synchronous generator:

Consider the three phase equivalent circuit of a synchronous generator shown in fig.4.4. The neutral of the generator is grounded through a reactor(impedance Z_n). When the generator is delivering a balanced load or under symmetrical fault conditions, the neutral current is zero. But when the generator is delivering an unbalanced load or during unsymmetrical faults neutral current in flows to neutral from ground via Z_n.

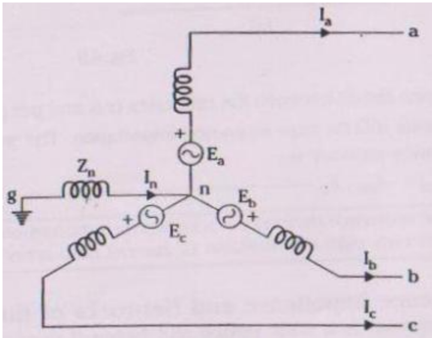


Fig. 4.4

Here, E_a, E_b and E_c are the induced emfs of the three phases. I_a, I_b and I_c are the currents flowing in the lines when a fault(not shown in the figure) takes place at machine terminals. Since a synchronous machine is designed with symmetrical windings, it induces emfs of positive sequence only (This fact is proved in example 4.2).Fig.4.5a shows the three phase positive sequence network model of the synchronous generator Z_n does not appear in the model as I_n=0 for positive sequence currents. E_{a1}, E_{b1} and E_{c1} are the positive sequence generated voltages and Z_1 is the positive sequence impedance. Because of the balanced and symmetrical nature of the system, the three phase system can be replaced by a single phase network as shown in fig 4.5b.

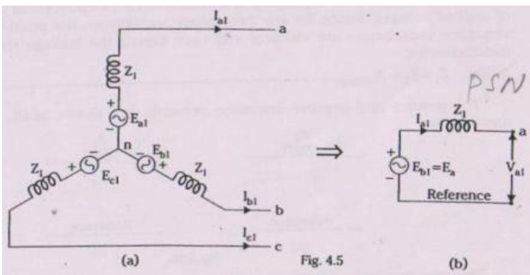


Fig. 4.5

Using the notation E for generated voltage and V for the terminal voltage, the equation that holds good for positive sequence network is,

$V_{a1}=E_{a1}-I_{a1}Z_1=E_a-I_{a1}Z_1$ 4.10

The per phase positive sequence impedance Z_1 in the above case is the subtransient, transient or steady state reactance of the machine depending on whether subtransient, transient or steady state conditions are being studied.

The negative sequence network models of a synchronous generator on a three phase and single-phase basis are shown in fig 4.6a and 4.6b respectively. Negative sequence voltages are not present in the equivalent circuits, as they are not generated in the synchronous machine. However, due to a fault or an unbalanced load, negative sequence currents can flow in the machine. Since negative sequence currents do not flow in the neutral, Z_n does not appear in the model.

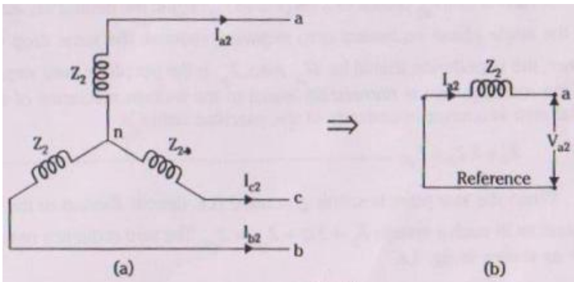


Fig. 4.6

The equation that holds good for the negative sequence network is,

$V_{a2}=-I_{a2}Z_2$ 4.11

It is known that the phase sequence of negative sequence currents is opposite to that of the phase sequence of positive sequence currents. With the flow of negative sequence currents in the stator of rotating field is created which rotates in the opposite direction to that of the positive sequence field and, therefore, at double synchronous speed with respect to the rotor. Currents at double the stator frequency are therefore induced in rotor field and damper winding. In sweeping over the rotor surface, the negative sequence mmf is alternately presented with reluctances of direct and quadrature axis. The negative sequence impedance presented by the machine with this consideration is often defined as,

$Z_2=j((X_g''+X_d'')/2)$; $|Z_2|<|Z_1|$ 4.12

It is the zero sequence currents that flow in the neutral during an unbalance or faulty condition ($I_n=3.I_{a0}$ refer section 3.8). The impedance offered by the synchronous generator to zero sequence currents depend on grounding of the neutral (star point). For a synchronous generator whose star point is grounded through an impedance Z_n, the zero sequence network models are as shown in fig.4.7.

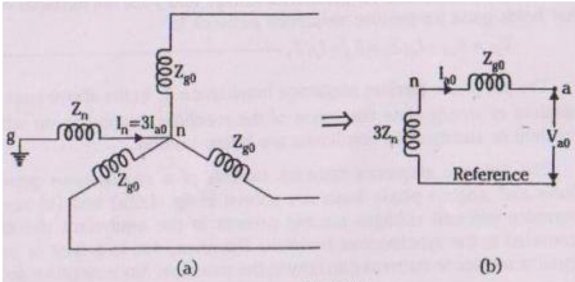


Fig. 4.7

A current of $3.I_{a0}$ produces a drop of $(3\ I_{a0}.Z_n)$ in the neutral impedance Z_n. To show in the single phase equivalent zero sequence network the same drop where current I_{a0} flows, the impedance should be $3Z_n$. Also, Z_{g0} is the per phase zero sequence impedance of the machine and is numerically equal to the leakage reactance of the machine. The total zero sequence impedance of the machine hence is,

$Z_0=3Z_n+Z_{g0}$ 4.13

When the star point is solidly grounded i.e directly shorted to the ground), $Z_n=0$. Therefore in such a system $Z_0=3(0)+Z_{g0}=Z_{g0}$. The zero sequence networks in this case are as shown in fig 4.8.

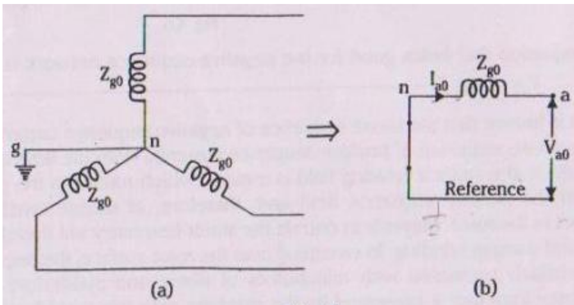


Fig. 4.8

When the star point is grounded, $Z_n=\infty$, Therefore eq. 4.13 becomes $Z_0=\infty+Z_{g0}=\infty$. The zero sequence networks for a ungrounded generator is as shown below.

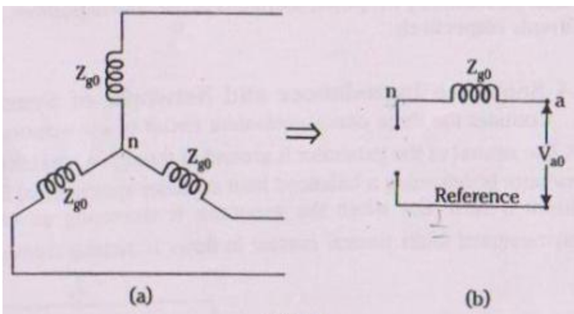


Fig. 4.9

The open circuit between the reference bus and per phase zero sequence impedance Z_{g0} represents infinite zero sequence impedance. The general equation applicable for a zero sequence network is,

$V_{a0}= -I_{a0}.Z_0$ 4.14

Note:

The sequence networks of synchronous motors are same as that of synchronous generators only with the direction of current flow reversed.

4.5 Sequence Impedance and networks of three phase transformer.

A transformer is a static device and hence it does not have anything like a phase sequence by itself, unlike that of a three phase synchronous generator or a synchronous motor. For this reason, the impedance of a transformer is independent of phase sequence of applied voltages. Hence for any three phase transformer, the positive and negative sequence impedances are identical and each equals the leakage reactance of the transformer. i.e

$Z_1=Z_2=X_{leakage}$ 4.15

The positive and negative sequence networks are shown in fig. 4.10a and 4.10b respectively.

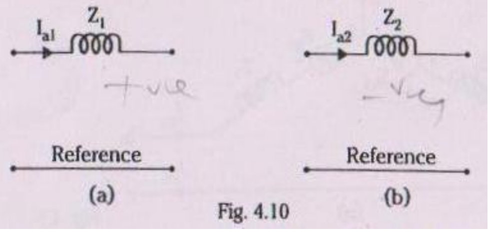


Fig. 4.10

The zero sequence impedance will be equal to positive (or negative) sequence impedances if there is a path for zero sequence current and will be infinity if there is no path for zero sequence currents. The following general observations can be made for zero sequence currents in transformers.

1) If the neutral point (star point) in the star connected winding is not grounded, then there is no path for zero sequence currents in the legs of star connection. i.e, the zero sequence currents flows in the star connected winding and in the lines connected to the winding only when the neutral point is grounded.

2) No zero sequence currents can flow in the lines connected to a delta connection as no return path is available for these currents. Zero sequences currents can however, flow in the legs of a delta-such currents are cause by the presence zero sequence voltages in the delta connection.

Based on the above observations the zero sequence network of three phase transformer can be obtained for any configuration. There is, however, a mechanical way of obtaining the zero sequence networks for any configuration of a three phase transformer (this as well caters the conditions specified above).

The general circuit for any configuration is given in fig4.11.

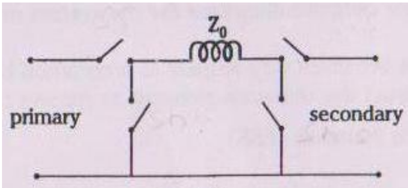


Fig. 4.11

Z_0 is the per phase zero sequence impedance of the winding of the transformer. There are two series and two shunt switches. One series and one shunt switch for both the sides separately. The series switch of a particular side is closed if it is a star connection with its neutral grounded and the shunt switch is closed if that side is delta connected, otherwise they are left open.

Using the above general rule, the zero sequence network for some of the configurations of three phase transformer are presented in table 4.1.

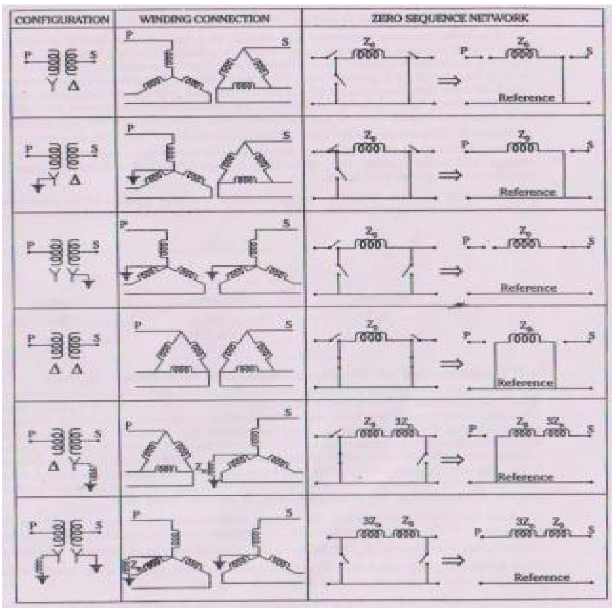


TABLE 4.1

4.6 Sequence impedances and networks of transmission lines.

The series impedance of a transmission line, since it is a static apparatus, is the same for both positive and negative sequence currents. Hence the positive and negative sequence impedances of a transmission line are identical and can be obtained as follows:

$Z_1=Z_2=X_L$ ohms/phase/unit length 4.16

Where,

$X_L=\omega.L$, L=inductance/phase/unit length. This can be computed(for any symmetrically or unsymmetrically spaced transposed single circuit or double circuit transmission line) using the formula.

$L=2(10^{-7}) \log_e(GMD/GMR)$ H/m/phase 4.17

Here GMD is the Geometric Mean Distance of the spacings of the conductors between the phase and GMR is the Geometric Mean Radius of each of the three phases.

However, the zero sequence impedance of a transmission line is of different nature from that of positive and negative sequence impedances. Since the three zero sequence currents of the three phases are in phase, it necessitates a return path either in the ground or in a neutral or in a ground wire. This involves the understanding of current flow and distribution in the ground (earth). Since the ground impedance is heavily dependent on soil conditions, it is essential to make some simplifying assumptions for calculating the zero sequence impedance of a transmission line. Carson's formulas are generally used to compute Z_0 of transmission line. These formulas relate the zero sequence impedance of transmission lines to the physical dimensions of conductors, earth's resistivity, operation voltage and other factors.

Once the positive, negative and zero sequence impedances of the transmission line is known, the corresponding sequence networks can be easily drawn as shown in fig 4.12.

4.7 Construction of sequence networks of a power system.

In the previous sections the sequence networks of important power system elements have been discussed. Using these, complete sequence networks of a power system can be easily constructed. To start with, the positive sequence network is constructed by examination of the one-line diagram of the system. The transition from positive sequence network to negative sequence network is straight forward. Since the positive and negative sequence impedances are identical for static elements (like transformers and transmission lines), the only change necessary in positive sequence network to obtain negative sequence network is in respect of synchronous machines. Each machine is represented by its negative sequence impedance, the negative sequence voltage being zero.

Zero sequence subnetworks for various parts of a system can be easily combined to form complete zero sequence network. No voltage source is present in the zero sequence network. Any impedance (Z_n) included in generator, motor or transformer neutral becomes three times ($3Z_n$) its value in a zero sequence network. Special care needs to be taken of transformers in respect of zero sequence network.

The procedure for drawing sequence networks is illustrated through the following examples.

Example 4.3:
A 250 MVA, 11kV, 3 phase generator is connected to a large system through a transformer and a line as shown in fig below.

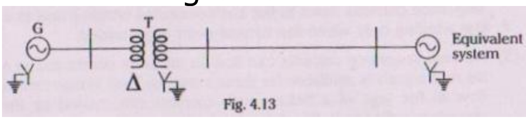


Fig. 4.13

Generator: $X_1 = X_2 = 0.15$ p.u, $X_0 = 0.1$ p.u
Transformer: $X_1 = X_2 = X_0 = 0.12$ p.u
Line: $X_1 = X_2 = 0.25$ p.u, $X_0 = 0.75$ p.u
Equivalent system: $X_1 = X_2 = X_0 = 0.15$ p.u
Draw the sequence network diagrams for the system and indicate all per unit values.
Solution:
all reactances are give with respect to a common base in this problem. Hence we can directly construct the sequence network as follows:

Positive sequence network (PSN):

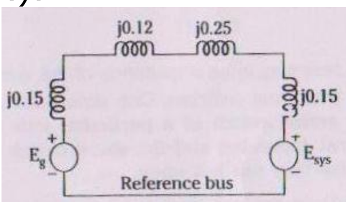


Fig. 4.14

Negative sequence network (NSN):

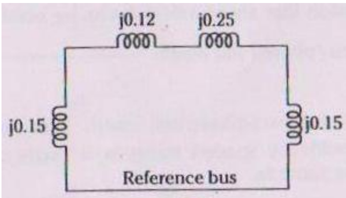


Fig. 4.15

Zero sequence network (ZSN):

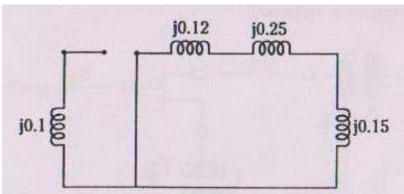


Fig. 4.16

Example 4.4:

Draw the positive, negative and zero sequence networks for the power system shown in fig 4.17.

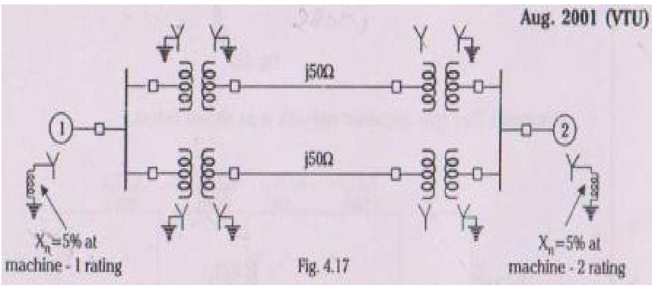


Fig. 4.17

Choose a base of 50MVA, 220kV in the 50Ω transmission lines and mark all reactances in p.u. The ratings of the generators and transformers are:
Generator 1: 25MVA, 11kV, X"=20%.
Generator 2: 25MVA, 11kV, X"=20%.
Three phase transformer (each): 20MVA, 11 Y/220 Y kV, X=15%.
The negative sequence reactance of each synchronous machine is equal to the subtransient reactance. The zero sequence reactance of each machine is 8%. Assume that the zero sequence reactance of lines are 250% of their positive sequence reactances.

Solution:
base values:
We choose a given,
base MVA=50
base kV on 50Ω transmission lines=220
base kV on generator 1=220(11/220)=11
base kV on generator 2=220(11/220)=11

sequence reactances of generators:
Since the ratings of the machines are the same, their reactances are also the same.
Positive sequence reactance=X''_{G1}=subtransient reactance on new base.

$X''_{G1,\ new} = X''_{G1,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$

$= j0.2 \times (50 / 25) \times (11^2 / 11^2)$

$= j\,0.4$ p.u

Negative sequence reactance=X''_{G1}=subtransient reactance on new base.
=j0.4 p.u (as per given data)

Zero sequence reactance=XG0=8% on new base

$= j0.08 \times (50 / 25) \times (11^2 / 11^2)$

=j0.16 p.u

p.u value of generator neutral reactance=XGn=5% on new base.

$= j0.05 \times (50 / 25) \times (11^2 / 11^2)$

=j0.1 p.u.

Sequence reactances of transformers:

Since the ratings of all transformers are identical, their sequence reactances should be one and the same. Also, all the three sequence reactances of transformers are the same.

Hence positive sequence reactance=Negative sequence reactance
=Zero sequence reactance

$X_{p.u,\ new} = X_{p.u,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$
$= j0.15 \times (50 / 20) \times (220^2 / 220^2)$
$= j\ 0.375$ p.u

Sequence reactance of transmission lines:

Since the transmission line is a static apparatus, its positive and negative sequence reactances are one and the same.

Hence, $X_{TL1}=X_{TL2}= X_{TL}(\Omega) \times (MVA)_B / (kV)^2_B= j50 \times 50 / 220^2= j0.052$p.u

However, it is given that the zero sequence reactances of transmission lines are equal to 250% of their positive sequence reactances.

Therefore,

$X_{TL\ .Zero}=2.5(j0.052)=j0.13$p.u

Using these computed values, the sequence networks are drawn as below.

Positive sequence network (PSN):

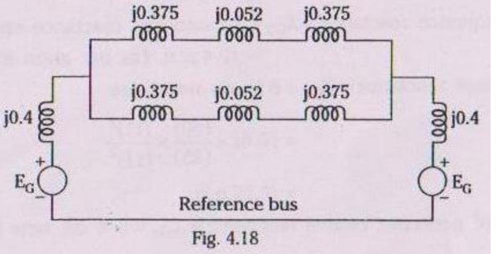


Fig. 4.18

Negative sequence network (NSN):

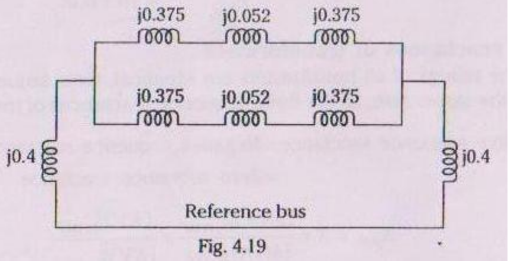


Fig. 4.19

Zero sequence network (ZSN):

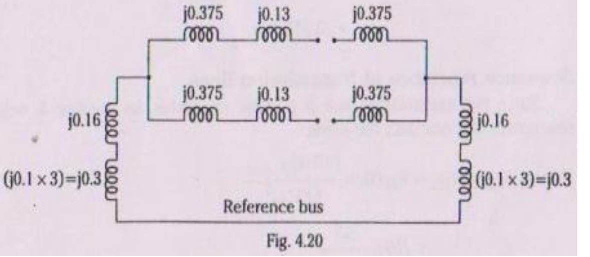


Fig. 4.20

Note:
While drawing zero sequence networks, carefully follow the steps.

Example 4.5:
For the power system whose one line diagram is shown in fig 4.21. Sketch the zero sequence network.

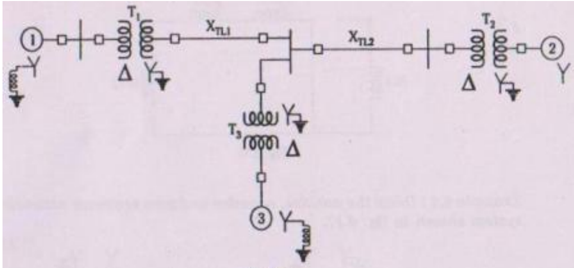


Fig. 4.21

Solution:
The zero sequence network is as shown below.

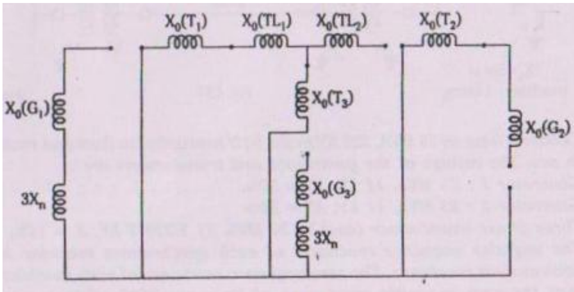
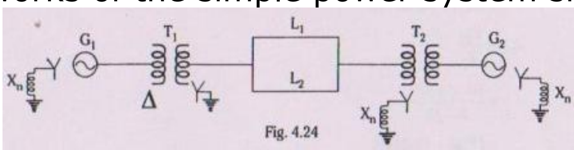


Fig. 4.22

Example 4.8:
Draw the sequence networks of the simple power system shown in fig. 4.24.



Positive sequence network (PSN):

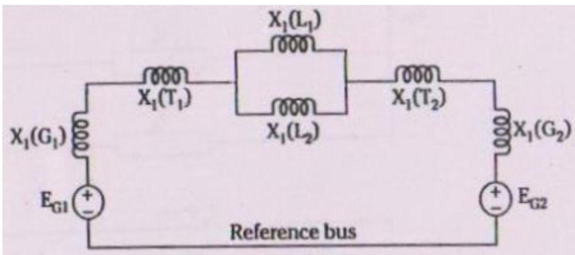


Fig. 4.25

Negative sequence network (NSN):

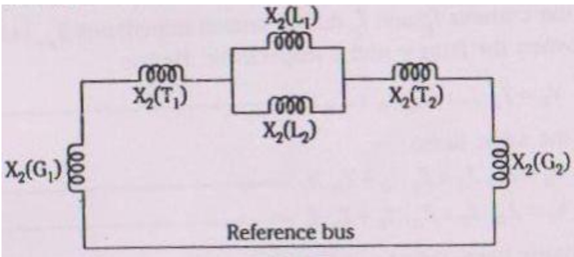


Fig. 4.26

Zero sequence network (ZSN):

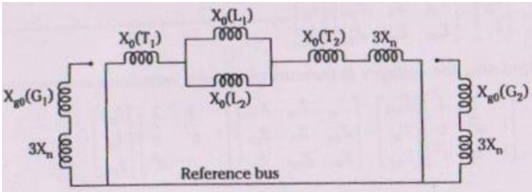


Fig. 4.27

Example 4.9:
The one line diagram of a power system is shown in fig 4.28.
The ratings of the devices are as follows:
G_1 & G_2: 104MVA, 11.8kV, X_1=X_2=0.2p.u; X_0=0.1p.u.
T_1 & T_2: 125MVA, 11Y-220Y kV, X_1=X_2=X_0=0.1p.u.
T_3 & T_4:120MVA, 230Y-6.9Y kV, X_1=X_2=X_0=0.12p.u.
M_1: 175MVA, 6.6kV, X_1=X_2=0.3p.u; X_0=0.15p.u.
M_2: 50MVA, 6.9kV, X_1=X_2=0.3p.u; X_0=0.1p.u.
Transmission line reactances: X_1=X_2=30Ω; X_0=60Ω.

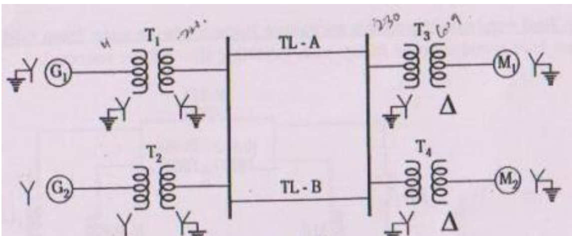


Fig. 4.28

Draw the sequence impedance diagram in p.u on a base of 200MVA, 220kV in the transmission lines. Also, find the equivalent positive sequence impedance as seen from the mid point of line-B.
Solution:
Base values:
We choose from give,
base MVA=200 MVA
base kV on the transmission lines=220

base kV on the generators G_1 & G_2=220(11/220)=11
base kV on the motors M_1 & M_2=220(6.9/230)=6.6
Reactances of G_1 & G_2:
$X_1 = X_2 = X_{p.u,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$
$= j0.2 \times (200 / 104) \times (11.8^2 / 11^2)$
$= j\ 0.44$ p.u
$X_0 = j0.1 \times (200 / 104) \times (11.8^2 / 11^2)$
$= j0.22$ p.u

Reactances of transformers T_1 & T_2:(calculated on primary side of them),
$X_1 = X_2 = X_0 = X_{p.u,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$
$= j0.1 \times (200 / 125) \times (11^2 / 11^2)$
$= j\ 0.16$ p.u

Reactance of transmission lines:
$X_{1TL} = X_{2TL} = X_{TL}(\Omega) \times (MVA)_B / (kV)^2_B = j30 \times 200 / 220^2 = j0.124$ p.u
$X_{0TL} = j60 \times 200 / 220^2 = j0.248$ p.u

Reactances of transformers T_3 & T_4:(calculated on primary side of them),
$X_1 = X_2 = X_0 = X_{p.u,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$
$= j0.12 \times (200 / 120) \times (230^2 / 220^2)$
$= j\ 0.22$ p.u

Reactances of M_1:
$X_1 = X_2 = X_{p.u,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$
$= j0.3 \times (200 / 175) \times (6.6^2 / 6.6^2)$
$= j\ 0.342$ p.u
$X_0 = j0.15 \times (200 / 175) \times (6.6^2 / 6.6^2)$
$= j0.171$ p.u

Reactances of M_2:
$X_1 = X_2 = X_{p.u,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$
$= j0.3 \times (200 / 50) \times (6.9^2 / 6.6^2)$
$= j\ 1.31$ p.u
$X_0 = j0.1 \times (200 / 50) \times (6.9^2 / 6.6^2)$
$= j0.4372$ p.u
Using these values, the sequence networks are drawn as below.
Positive sequence network (PSN):

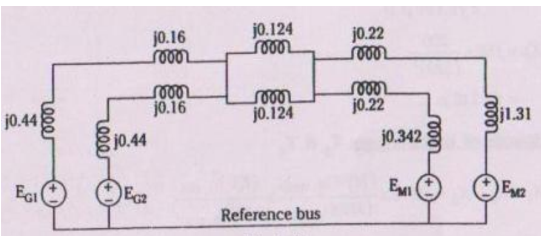


Fig. 4.29

Negative sequence network (NSN):

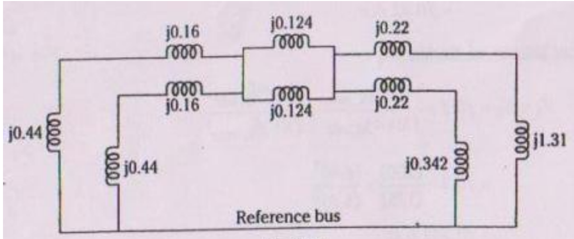


Fig. 4.30

Zero sequence network (ZSN):

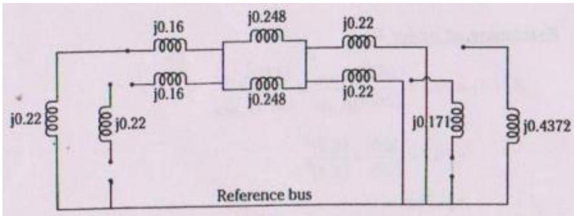


Fig. 4.31

To find equivalent positive sequence impedance as seen from mid point of line-B: The PSN is redrawn as in fig 4.32(shorting the voltage sources)

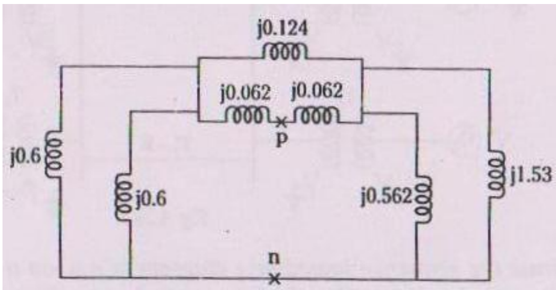


Fig. 4.32

The mid point of line-B is marked as 'P' and point 'n' is a point in the reference. Using elementary circuit theory, the network of fig 4.32 can be simplified as shown in fig 4.33

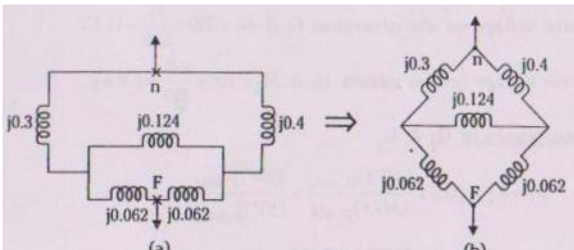


Fig. 4.33

Consider the delta connected network of fig 4.33b, it cab be converted to its equivalent star connection as shown below.

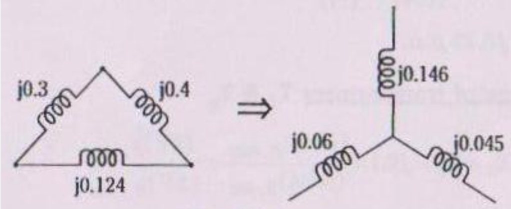


Fig. 4.34

Putting this back into fig 4.33b and simplifying, we get

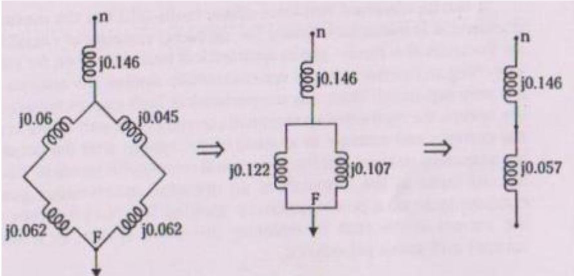


Fig. 4.35

Hence the equivalent positive sequence impedance as seen from the point P is, (j0.146+j0.057)=j0.203 p.u.

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