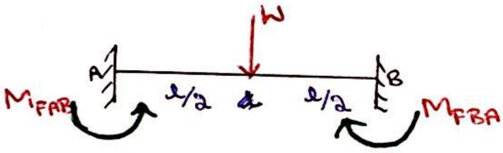
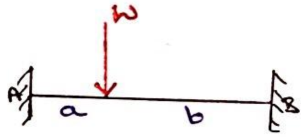
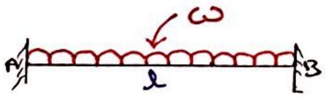
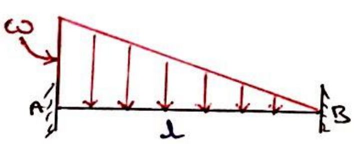
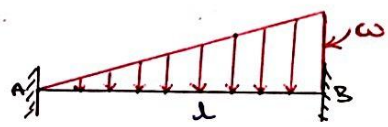
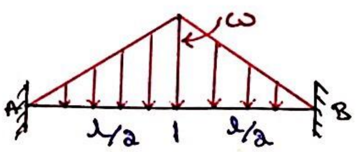
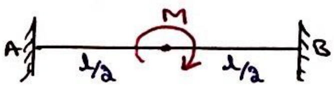
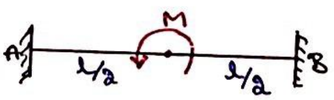
Sunil.H.G

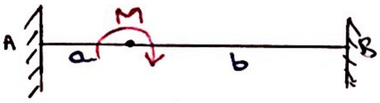
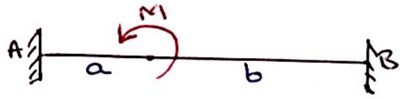
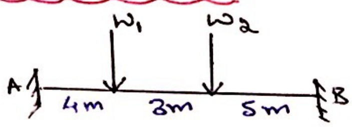
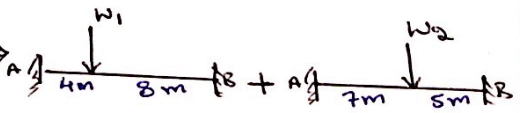
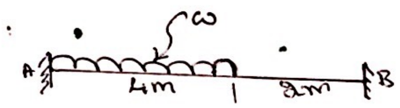
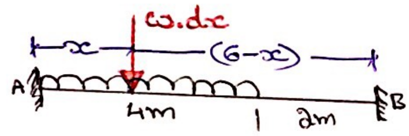
MODULE-1

SLOPE - DEFLECTION METHOD

(*) Procedure:
1) FEM
2) S.D. equations
3) Equilibrium Conditions
4) Final Moments
5) Diagrams (SFD, BMD & EC)

1) FEM (Fixed End Moments)

SL No.	Load pattern	M_{FAB}	M_{FBA}
1.		$-\frac{Wl}{8}$	$+\frac{Wl}{8}$
2.		$-\frac{Wab^2}{l^2}$	$+\frac{Wa^2b}{l^2}$
3.		$-\frac{\omega l^2}{12}$	$+\frac{\omega l^2}{12}$
4.		$-\frac{\omega l^2}{20}$	$+\frac{\omega l^2}{30}$
5.		$-\frac{\omega l^2}{30}$	$+\frac{\omega l^2}{20}$
6.		$-\frac{5\omega l^2}{96}$	$+\frac{5\omega l^2}{96}$
7.		$+\frac{M}{4}$	$+\frac{M}{4}$
8.		$-\frac{M}{4}$	$-\frac{M}{4}$

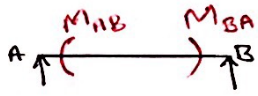
Sl. No.	Load pattern	M_{FAB}	M_{FBA}
9.		$+\frac{Mb}{l^2}(2a-b)$	$+\frac{Ma}{l^2}(2b-a)$
10.		$-\frac{Mb}{l^2}(2a-b)$	$-\frac{Ma}{l^2}(2b-a)$
	Other Cases:		
11.			
	⇒ 	$-\left[\frac{W_1ab^2}{l^2}+\frac{W_2ab^2}{l^2}\right]$	$+\left[\frac{W_1a^2b}{l^2}+\frac{W_2a^2b}{l^2}\right]$
12.			
	⇒ 	$-\frac{wab^2}{l^2}$ $=-\int_0^4\frac{(w.dx)(x)(6-x)^2}{l^2}$	$+\frac{wa^2b}{l^2}$ $=+\int_0^4\frac{(w.dx)(x)^2(6-x)}{l^2}$

(*) Overhang Portion ⟶ No FEM

2) Slope-Deflection equation

$$M_{AB}=\frac{2EI}{l}\left[2\theta_A+\theta_B-\frac{3\delta}{l}\right]+M_{FAB}$$

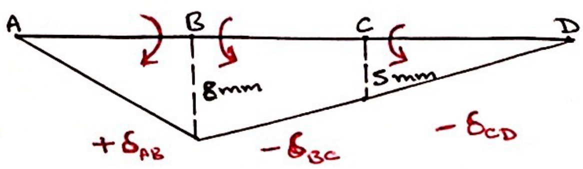
$$M_{BA}=\frac{2EI}{l}\left[2\theta_B+\theta_A-\frac{3\delta}{l}\right]+M_{FBA}$$



(*) @ Fixed Support ⟶ $\theta=0$

No Sinking (or) Non-sway ⟶ $\delta=0$

(*) Write rotation symbol (↻) @ the max. deflection point. Always the Arrow head should be below the line.



Sign: (↻+) (↺−)

δ-Value: Difference b/w Deflection @ the ends.

(*) Overhang portion ⟶ No S-D equation.

3> Equilibrium Conditions:

→ @ intermediate Support joint : $\Sigma M=0$

→ @ Last joint : $M=0$
(Simple, Roller (or) Hinged Support)

4> Final Moments :

→ Substitute 'θ' values in S-D equation and get the Final Moments.

(*) Overhang portion ⟶ Calculate Final Moment directly.

5> Diagrams (SFD, BMD & EC) :

(a) SFD:

→ Draw FBD –
- Write given beam line, points & distances.
- Write given loads
- Put vertical reaction at ~~&~~ all the Supports.
- Write the final moments.

→ Calculate Support Reactions –

~~ΣH=0~~ $\Sigma H=0$
$\Sigma V=0$
& $\Sigma M=0$

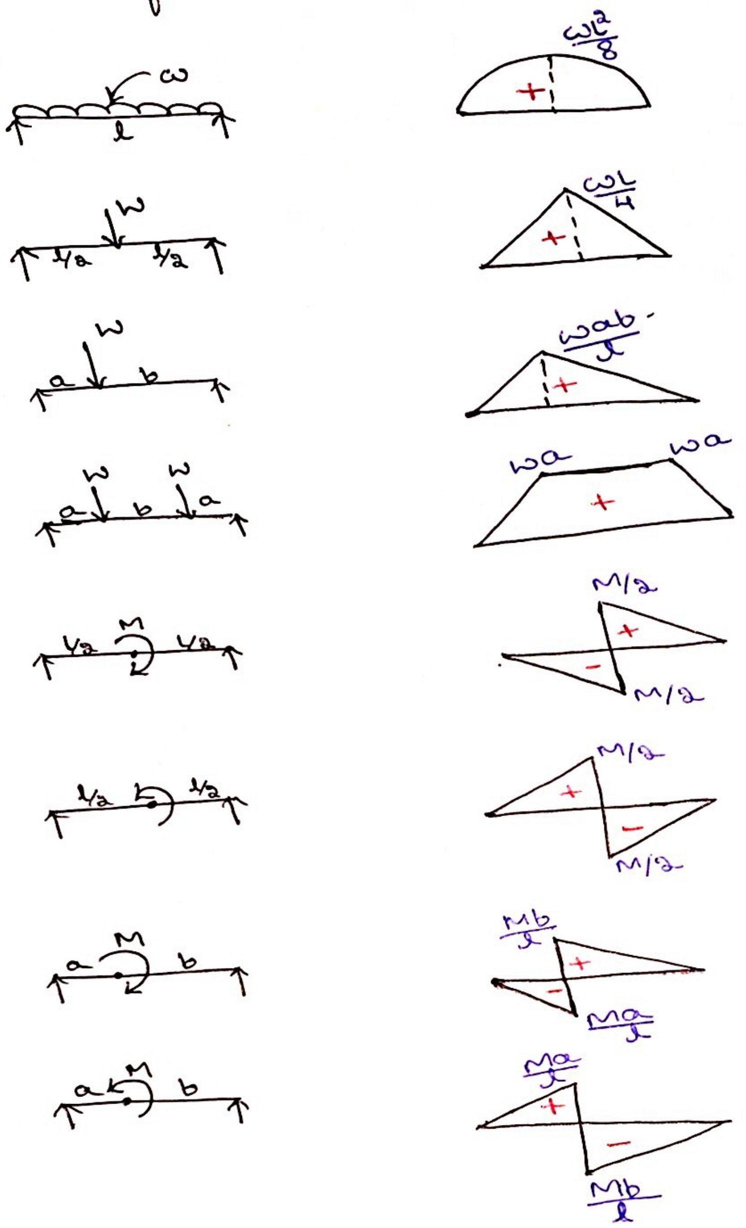
→ Write SFD.

(b) BMD :

BMD ⟶ (Free BMD + Final BMD)

NOTE:
(*) Simply Supported beams – @ ends : $M=0$
(*) For Special load pattern, Consider the Beam portion Seperately & Calculate moments @ intermediate points & draw BMD.
(*) Final BMD – Draw a line at the tail side of the rotation arrow & get Final BMD.

(*) BMD for standard load cases.

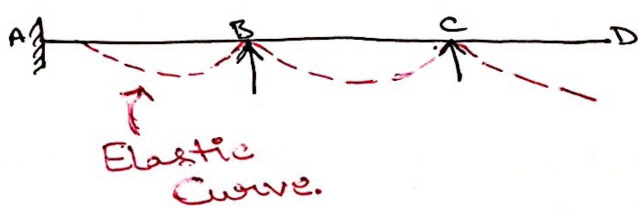


(c) Elastic Curve:

(*) Fixed end → Dont join the line directly

other Supports → join the line directly.

Overhang → Dont close the curve.



Date 24/08/08

Structural Analysis - II

(a) Sign Convention

(1) Reaction

$\Sigma V = 0$, ↑ +ve ↓ −ve

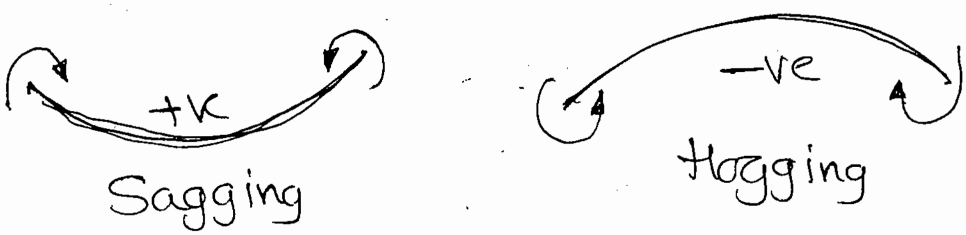
$\Sigma H = 0$, ⟶ +ve ⟵ −ve

$\Sigma M = 0$, ↻ +ve ↺ −ve

(2) Shear Force

From "Left" to "Right" ↑ +ve ↓ −ve

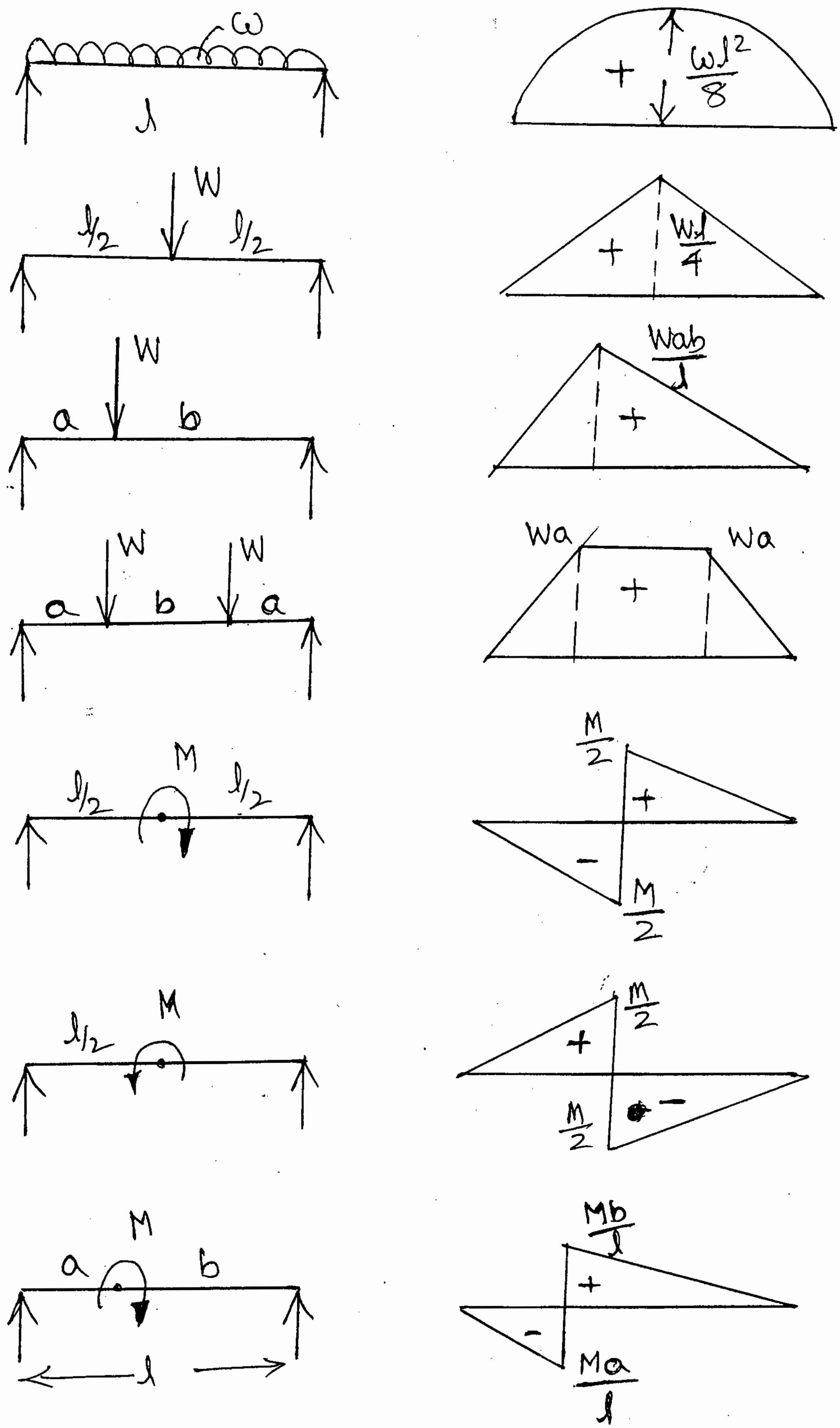
(3) Bending Moment



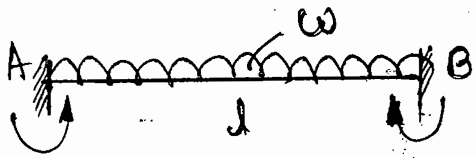
Or Clockwise Moment ⟶ +ve

Anti-clockwise " ⟶ −ve

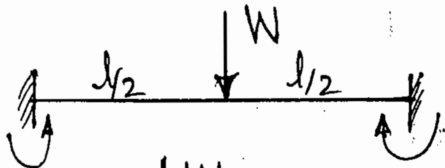
(b) BMD: (Free BMD)



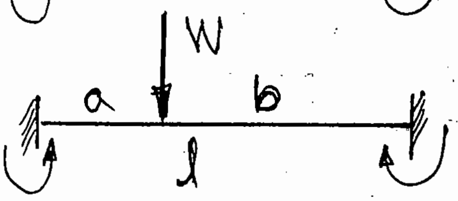
(c) Fixed End Moments :-



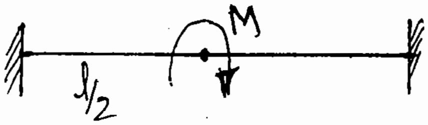
$$M_{FAB} = -\frac{\omega l^2}{12}, \quad M_{FBA} = +\frac{\omega l^2}{12}$$



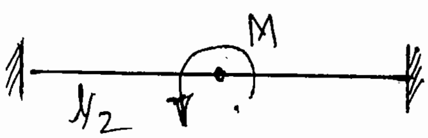
$$M_{FAB} = -\frac{Wl}{8}, \quad M_{FBA} = +\frac{Wl}{8}$$



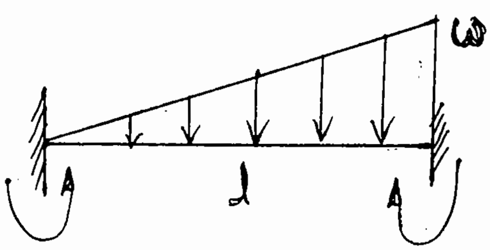
$$M_{FAB} = -\frac{Wab^2}{l^2}, \quad M_{FBA} = +\frac{Wa^2b}{l^2}$$



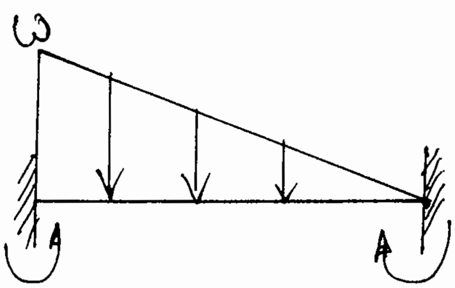
$$M_{FAB} = M_{FBA} = +\frac{M}{4}$$



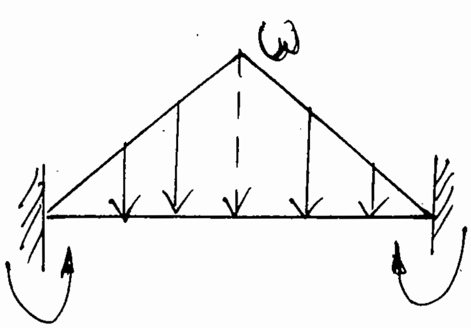
$$M_{FAB} = M_{FBA} = -\frac{M}{4}.$$



$$M_{FAB} = -\frac{\omega l^2}{30}, \quad M_{FBA} = +\frac{\omega l^2}{20}$$

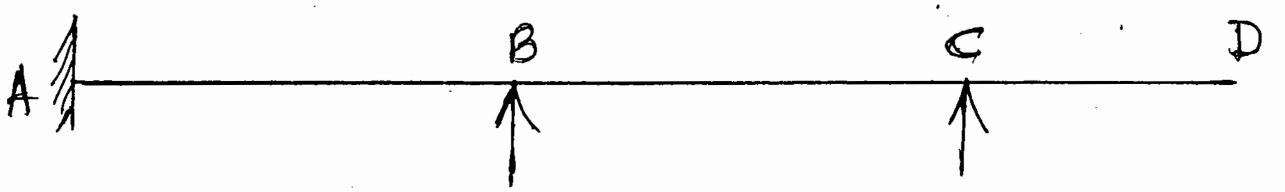


$$M_{FAB} = -\frac{\omega l^2}{20}, \quad M_{FBA} = +\frac{\omega l^2}{30}$$



$$M_{FAB} = \frac{5\omega l^2}{96} \qquad M_{FBA} = +\frac{5\omega l^2}{96}$$

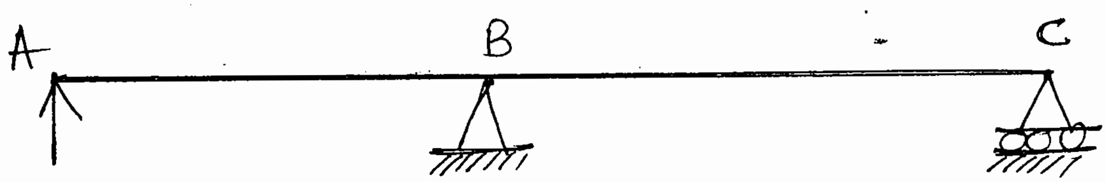
(d) (1) Intermediate Support :



If there is a span on both sides of supports then it is called "intermediate" support

∴ 'B' & 'C' → Intermediate

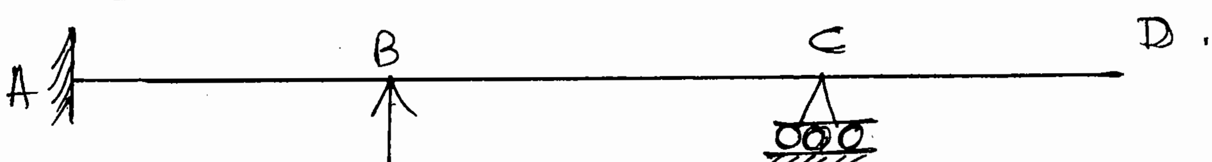
(2) Last Support :



If span is on one side only, then it is called last simple or hinge or Roller support.

∴ At last support $\boxed{\text{Moment} = 0}$

(3) Equillibrium Condition :-

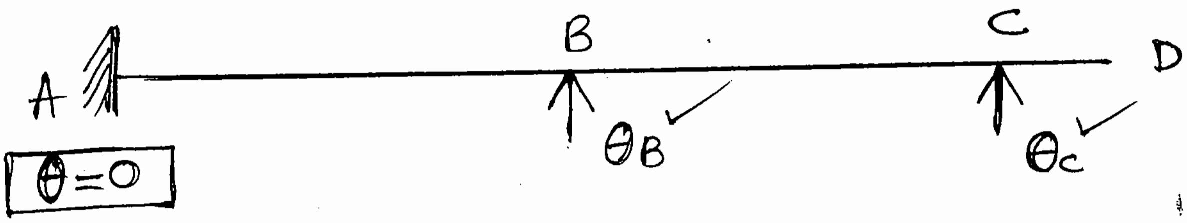


At intermediate support, $\boxed{\text{Sum of Moment} = 0}$

At "B" $\boxed{M_{BA} + M_{BC} = 0}$

At "C" $\boxed{M_{CB} + M_{CD} = 0}$

(I) Slope Deflection Method :



Basic Equation

$$M_{AB} = \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right] + M_{FAB}$$

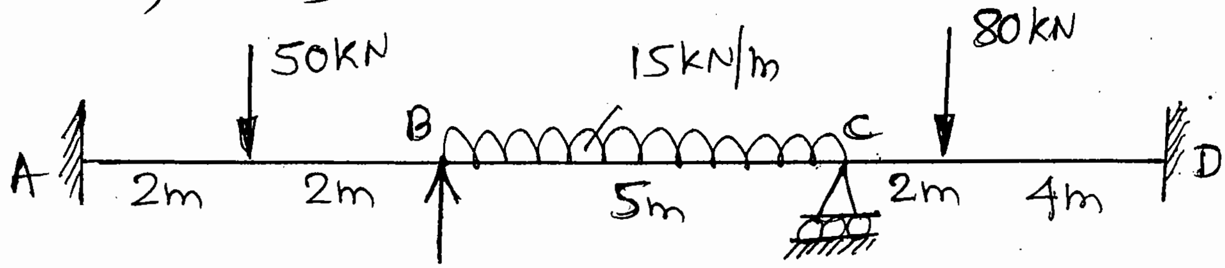
$$M_{BA} = \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right] + M_{FBA}$$

$\theta \rightarrow$ Slope or Rotation

$\delta \rightarrow$ Sinking or Settlement

$EI \rightarrow$ Flexural Rigidity.

Eg:- 1] Analyse the continuous beam shown by S.D. method and draw BMD, SFD and EC.



EI → Constant

Sol^n

(a) FEM

$$M_{FAB} = -\frac{Wl}{8} = \frac{-50\times4}{8} = -25\,kN\text{-}m$$

$$M_{FBA} = +\frac{Wl}{8} = +25\,kN\text{-}m$$

$$M_{FBC} = -\frac{wl^2}{12} = -\frac{15\times5^2}{12} = -31.25$$

$$M_{FCB} = +\frac{wl^2}{12} = +31.25$$

$$M_{FCD} = -\frac{Wab^2}{l^2} = -\frac{80\times2\times4^2}{6^2} = -71.11\,kN\text{-}m$$

$$M_{FDC} = +\frac{Wa^2b}{l^2} = \frac{80\times2^2\times4}{6^2} = +35.56\,kN\text{-}m$$

(b) S.D. Equation:

$\theta_A = \theta_D = 0$ ($\because$ Fixed Support)

$\delta = 0$ ($\because$ No sinking)

$$\boxed{M_{AB} = \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right] + M_{FAB}}$$

$$M_{AB} = \frac{2EI}{4}\left[0 + \theta_B - 0\right] - 25 = 0.5EI\theta_B - 25 \quad \text{—(i)}$$

$$M_{BA} = \frac{2EI}{4}\left[2\theta_B + 0 - 0\right] + 25 = EI\theta_B + 25 \quad \text{—(ii)}$$

$$M_{BC} = \frac{2EI}{5}\left[2\theta_B + \theta_C - 0\right] - 31.25$$

$$= 0.8EI\theta_B + 0.4EI\theta_C - 31.25 \quad \text{—(iii)}$$

$$M_{CB} = \frac{2EI}{5}\left[2\theta_C + \theta_B - 0\right] + 31.25$$

$$= 0.8EI\theta_C + 0.4EI\theta_B + 31.25 \quad \text{—(iv)}$$

$$M_{CD} = \frac{2EI}{6}\left[2\theta_C + 0 - 0\right] - 71.11 = 0.667EI\theta_C - 71.11 \quad \text{—(v)}$$

$$M_{DC} = \frac{2EI}{6}\left[0 + \theta_C - 0\right] + 35.56 = 0.333EI\theta_C + 35.56 \quad \text{—(vi)}$$

(c) <u>Equillibrium Condition</u> :–

(1) At "B" $\boxed{M_{BA} + M_{BC} = 0}$ → Intermediate support

$$\left[EI\theta_B + 25\right] + \left[0.8EI\theta_B + 0.4EI\theta_C - 31.25\right] = 0$$

$$1.8EI\theta_B + 0.4EI\theta_C = 6.25 \rightarrow \text{(I)}$$

(2) At "C" $\boxed{M_{CB} + M_{CD} = 0}$

$$\left[0.8EI\theta_C + 0.4EI\theta_B + 31.25\right] + \left[0.667EI\theta_C - 71.11\right] = 0$$

$$0.4EI\theta_B + 1.467EI\theta_C = 39.86 \rightarrow \text{(II)}$$

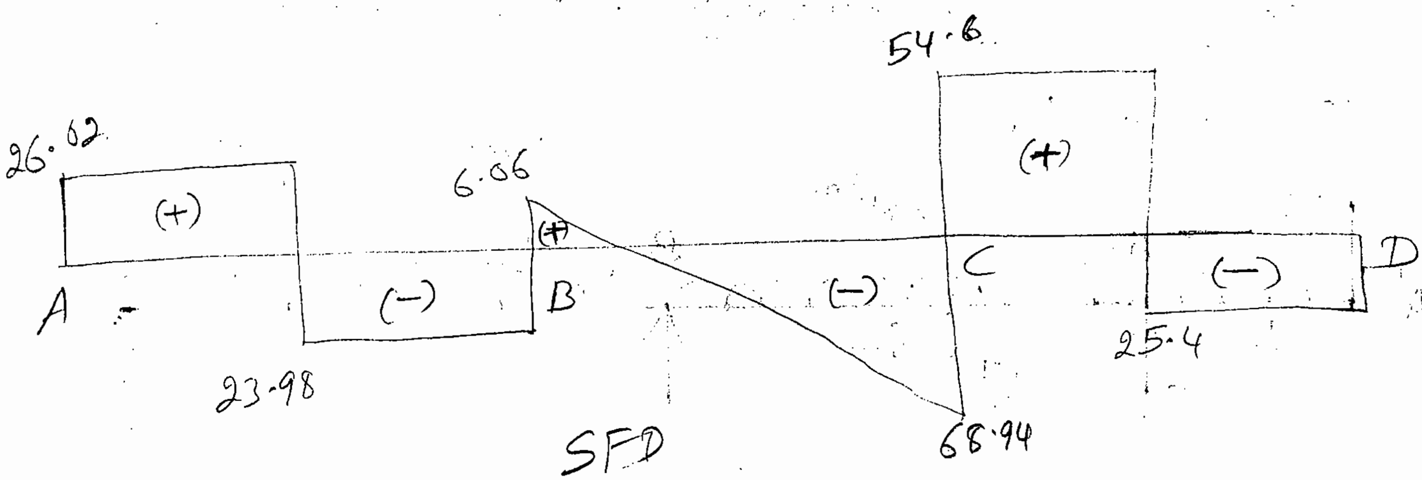
Solving
$$\theta_B = -2.73/EI$$
$$\theta_C = +27.91/EI$$

(d) Final Moments [Substitute 'θ' values in eq^n (i) to (vi)]

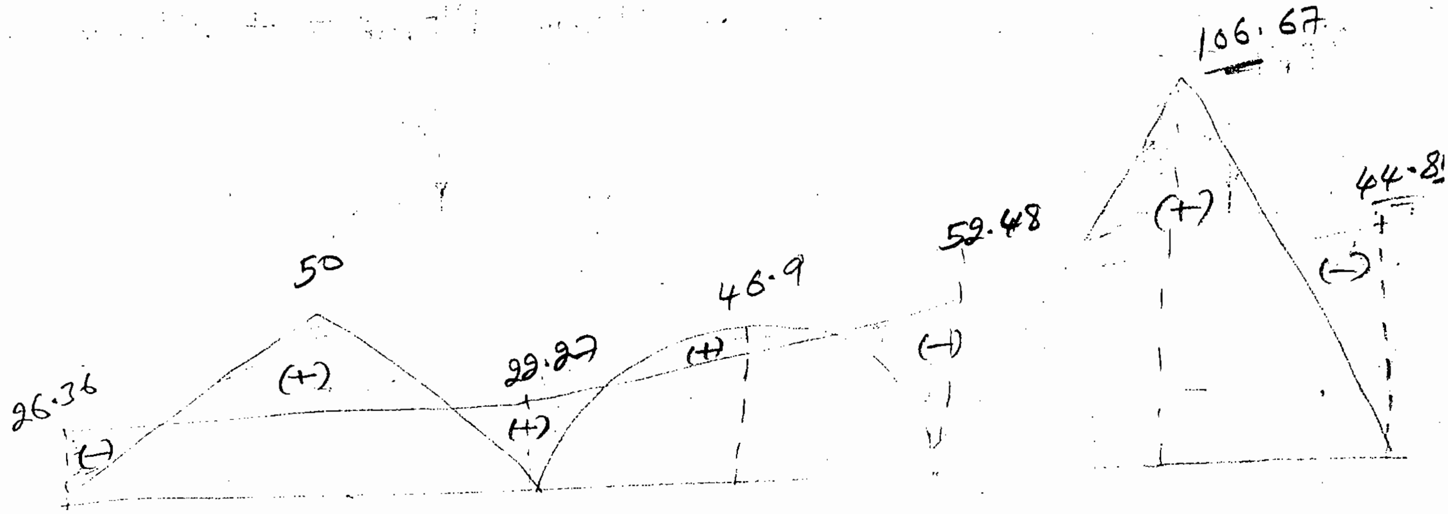
$M_{AB} = -26.36$ kN-m $M_{CB} = 52.48$ kN-m

$M_{BA} = 22.27$ kN-m $M_{CD} = -52.49$ kN-m

$M_{BC} = -22.27$ kN-m $M_{DC} = 44.85$ kN-m

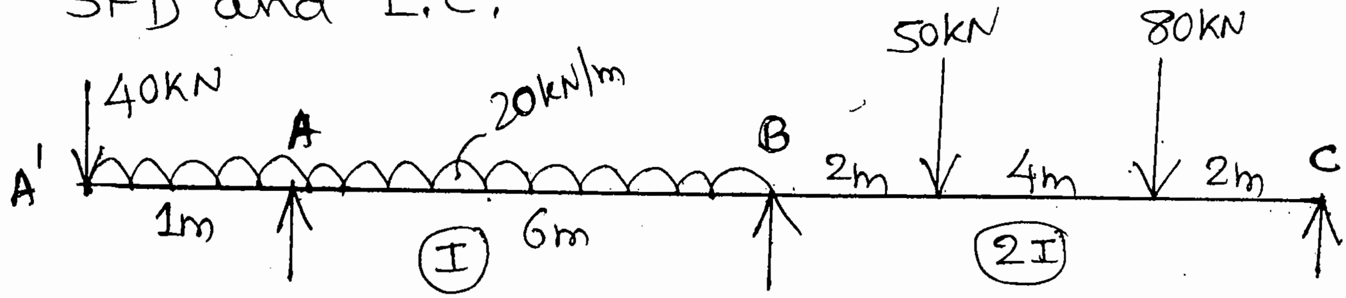

54.6
26.02
6.06
(+)
(+)
(+)
A
(−)
B
(−)
C
(−)
D
23.98
25.4
68.94

SFD


106.67
44.81
52.48
50
46.9
(+)
(−)
22.27
26.36
(+)
(+)
(−)
(−)
(+)

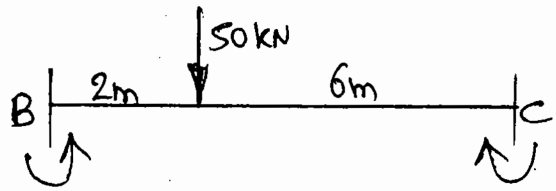
BMD

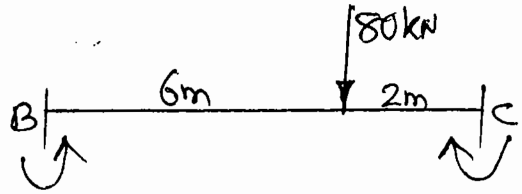
Eg:-2] Analyse the continuous beam shown by S.D. method. And draw BMD, SFD and E.C.



Sol^n (a) FEM

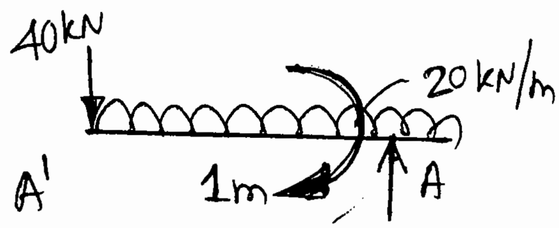
$$M_{FAB} = -\frac{wl^2}{12} = -\frac{20\times6^2}{12} = -60\text{ kN-m},\quad M_{FBA} = +60\text{ kN-m}$$





$$M_{FBC} = -\frac{Wab^2}{l^2} = -\left[\frac{50\times2\times6^2}{8^2} + \frac{80\times6\times2^2}{8^2}\right] = -86.25\text{ kN-m}$$

$$M_{FCB} = +\frac{Wa^2b}{l^2} = +\left[\frac{50\times2^2\times6}{8^2} + \frac{80\times6^2\times2}{8^2}\right] = +108.75\text{ kN-m}$$



$$M_{AA'} = +\left[40\times1 + 20\times1\times\tfrac{1}{2}\right]$$

$$= +50\text{ kN-m}$$

+ve sign for clockwise resisting moment.

(b) S.D. Equation

$\delta = 0$ (No sinking)

$$M_{AB} = \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right] + M_{FAB}$$

$$M_{AB} = \frac{2(1 \times EI)}{6}\left[2\theta_A + \theta_B - 0\right] - 60$$

$$= (0.667EI)\theta_A + (0.333EI)\theta_B - 60 \quad \text{—(i)}$$

$$M_{BA} = \frac{2(1 \times EI)}{6}\left[2\theta_B + \theta_A - 0\right] + 60$$

$$= (0.667EI)\theta_B + (0.333EI)\theta_A + 60 \quad \text{—(ii)}$$

$$M_{BC} = \frac{2(2EI)}{8}\left[2\theta_B + \theta_C - 0\right] - 86.25$$

$$= EI\theta_B + 0.5EI\theta_C - 86.25 \quad \text{—(iii)}$$

$$M_{CB} = \frac{2(2EI)}{8}\left[2\theta_C + \theta_B - 0\right] + 108.75$$

$$= EI\theta_C + 0.5EI\theta_B + 108.75 \quad \text{—(iv)}$$

★ Note:- There is NO S.D. Equation for "overhang"

(c) Equillibrium Condition :-

(i) At "A" $\boxed{M_{AA'} + M_{AB} = 0}$

$$[50] + [0.667EI\theta_A + 0.333EI\theta_B - 60] = 0$$

$$(0.667EI)\theta_A + (0.333EI)\theta_B = 10 \rightarrow \text{(I)}$$

(ii) At "B" $\boxed{M_{BA} + M_{BC} = 0}$

$$(0.333\,EI)\theta_A + (1.667\,EI)\theta_B + (0.5\,EI)\theta_C = 26.25 \rightarrow \text{(II)}$$

(iii) At "C" $\boxed{M_{CB} = 0}$ (∵ last Simple or Hinge or Roller support)

$$(0.5\,EI)\theta_B + (1\,EI)\theta_C = -108.75 \longrightarrow \text{(III)}$$

Solving

$$\theta_A = \frac{-15.197}{EI} \qquad \theta_C = \frac{-138.98}{EI}$$

$$\theta_B = \frac{60.47}{EI}$$

(d) Final Moment:

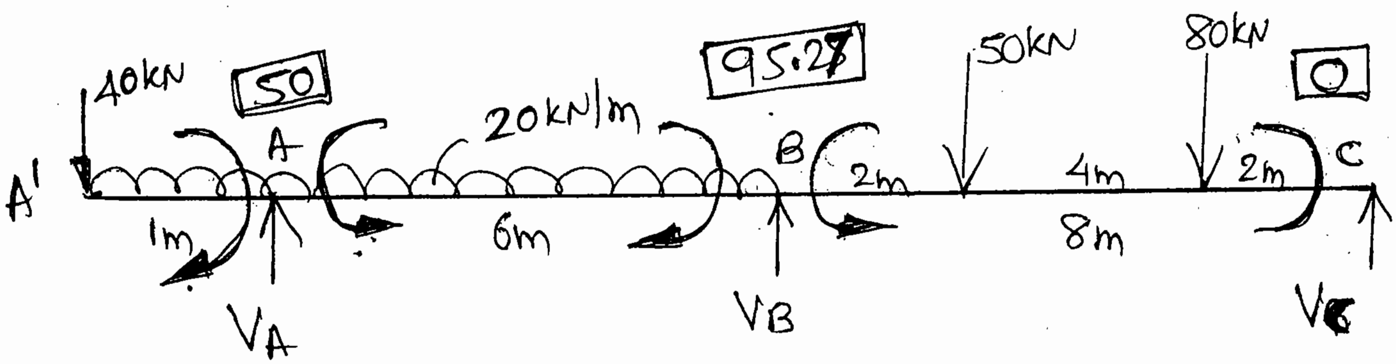
$M_{AB} = -50$ kN-m

$M_{BA} = 95.27$ kN-m

$M_{BC} = -95.27$ "

$M_{CB} = 0$

$M_{AA'} = +50$ kN-m



$$\Sigma V = 0, \quad V_A + V_B + V_C = 40 + 50 + 80 + 20 \times 7 = 310 \quad \text{(i)}$$

$$\Sigma M_B = 0 \text{ (RHS)} \quad -V_C \times 8 + 50 \times 2 + 80 \times 6 - 95.27 = 0$$

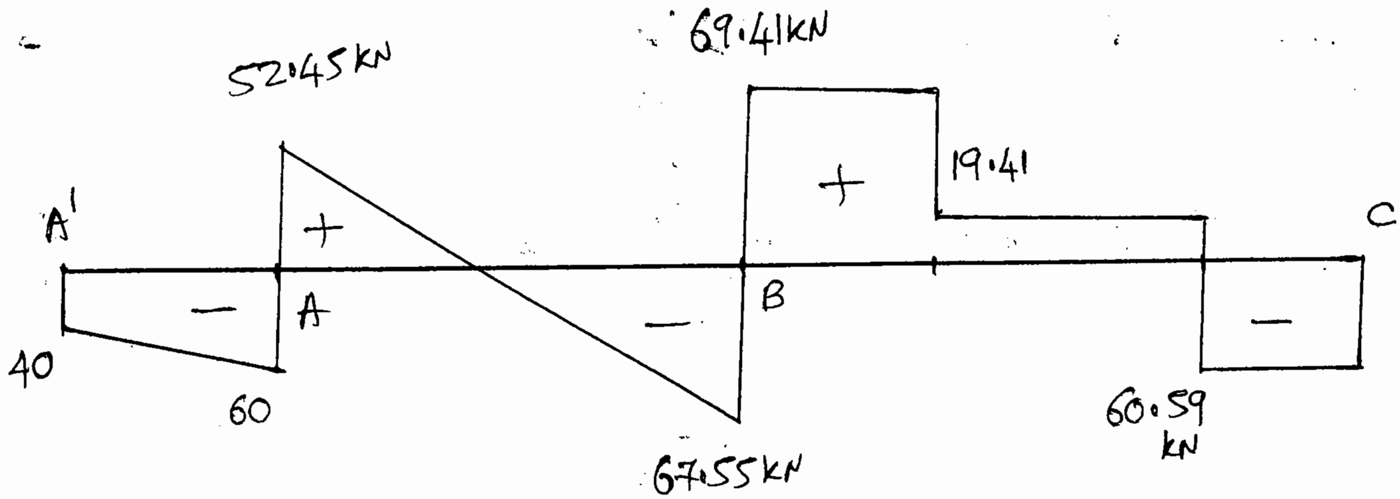
$$V_C = 60.59 \text{ kN}$$

$\Sigma M_B = 0$ (LHS)

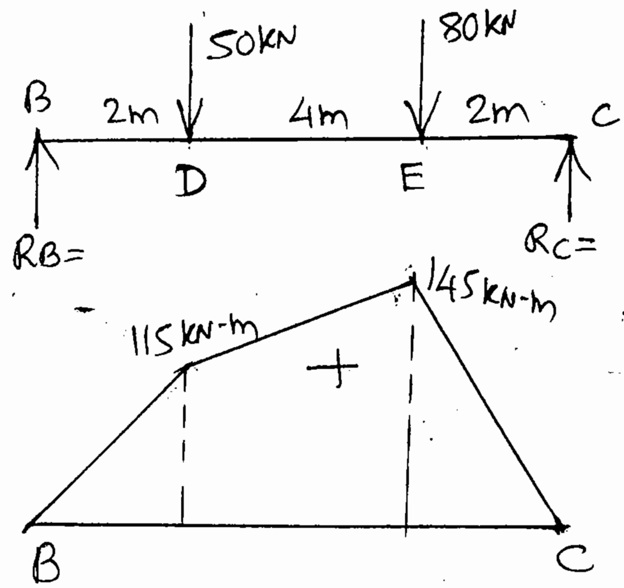
$$V_A \times 6 - 40 \times 7 - 20 \times 7 \times 7/2 + 50 - 50 + 95.27 = 0$$

$$V_A = 112.45 \text{ kN}$$

From(i) $112.45 + V_B + 60.59 = 310$ $\therefore V_B = 136.96$



Free BMD for "BC"



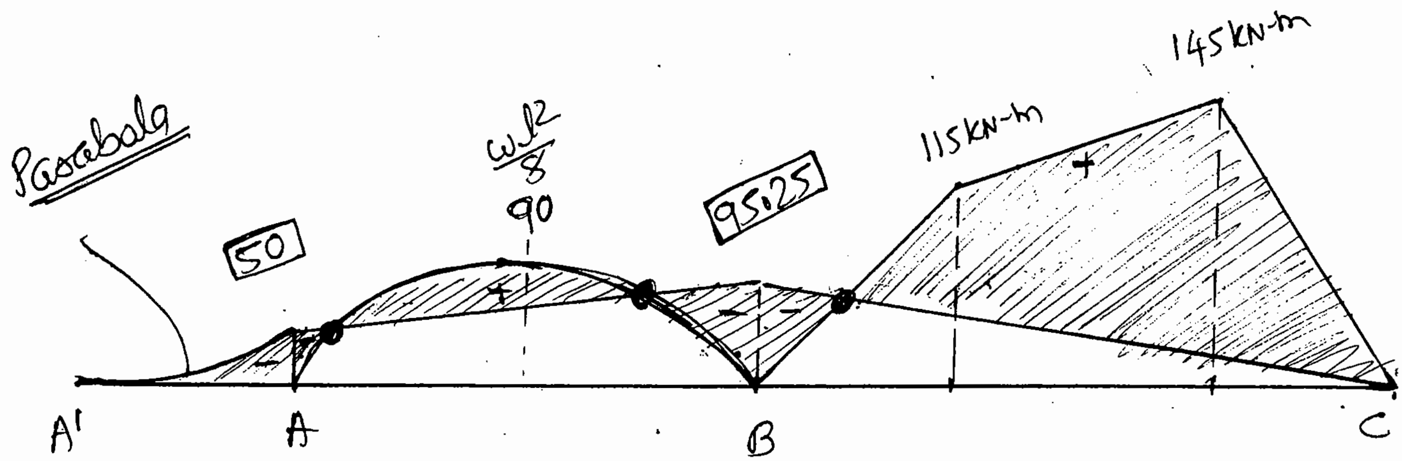
Reaction

$R_C = 72.5$ kN

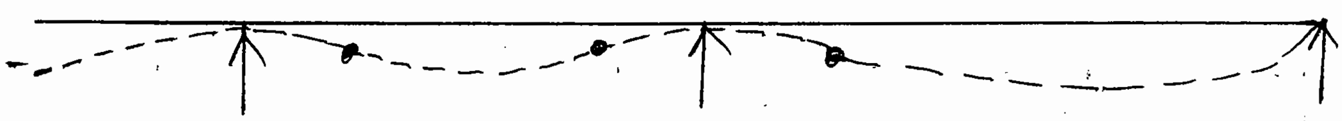
$R_B = 57.5$ kN

$\therefore M_D = R_B \times 2 = 115$ kN-m

$M_E = R_C \times 2 = 145$ kN-m



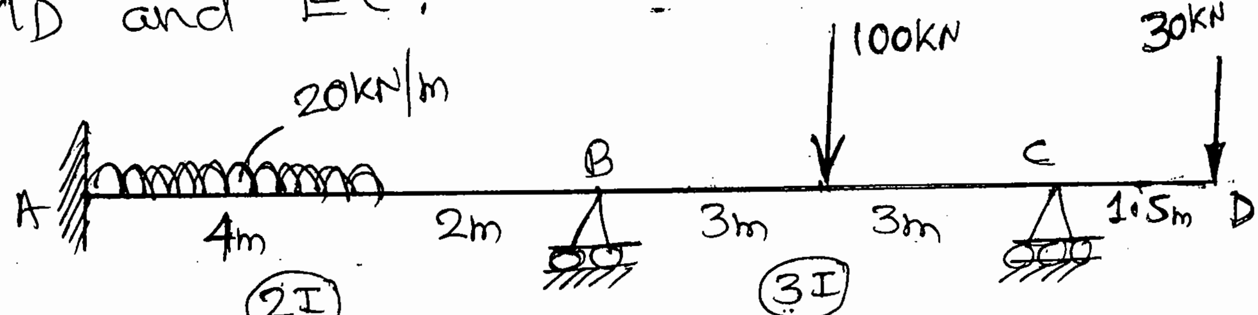
EC



EC

=== X ===

Eg:- 3 Analyse the continuous beam shown by S.D. method Draw SFD, BMD and EC.

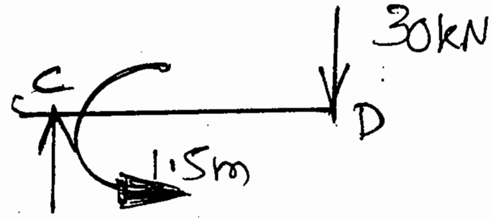


Sol:

(a) FEM

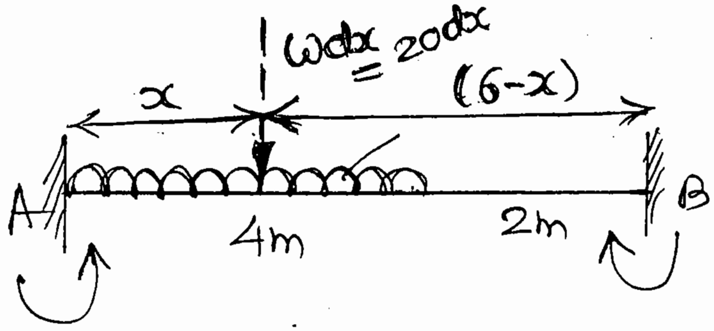
$$M_{FBC} = -\frac{Wl}{8} = -\frac{100 \times 6}{8} = -75 \text{ kN-m}$$

$$M_{FCB} = +\frac{Wl}{8} = +75 \text{ kN-m}$$



$M_{CD} = -30 \times 1.5 = -45$ kN-m ★

-ve sign for "Anticlockwise" Resisting Moment



$W = \omega \cdot dx = 20dx$

$a = x$

$b = (6-x)$

$l = 6m$

$$M_{FAB} = -\frac{Wab^2}{l^2} = -\int_0^4 \frac{(20dx)(x)(6-x)^2}{6^2} = -53.33 \text{ kN-m}$$

$$M_{FBA} = +\frac{Wa^2b}{l^2} = +\int_0^4 \frac{(20dx)(x)^2(6-x)}{6^2} = +35.56 \text{ kN-m}$$

(b) S.D. Equation :

$\theta_A = 0$ ($\because$ fixed)

$\delta = 0$ ($\because$ No sinking)

For overhang there is NO SD equation

$$M_{AB} = \frac{2(2EI)}{6}\left[0 + \theta_B - 0\right] - 53.33$$

$$= 0.667 EI\theta_B - 53.33 \quad \text{—(i)}$$

$$M_{BA} = \frac{2(2EI)}{6}\left[2\theta_B + 0 - 0\right] + 35.56$$

$$= 1.333 EI\theta_B + 35.56 \quad \text{—(ii)}$$

$$M_{BC} = \frac{2(3EI)}{6}\left[2\theta_B + \theta_C - 0\right] - 75$$

$$= 2EI\theta_B + EI\theta_C - 75 \quad \text{— (III)}$$

$$M_{CB} = \frac{2(3EI)}{6}\left[2\theta_C + \theta_B - 0\right] + 75$$

$$= 2EI\theta_C + EI\theta_B + 75 \longrightarrow \text{(IV)}$$

(c) Equillibrium Condition

(i) At "B" $\boxed{M_{BA} + M_{BC} = 0}$

$$3.333EI\theta_B + EI\theta_C = 39.44 \longrightarrow \text{(I)}$$

(ii) At 'C' $\boxed{M_{CB} + M_{CD} = 0}$

$$EI\theta_B + 2EI\theta_C = -30 \longrightarrow \text{(II)}$$

Solving

$$\theta_B = 19.21/EI$$

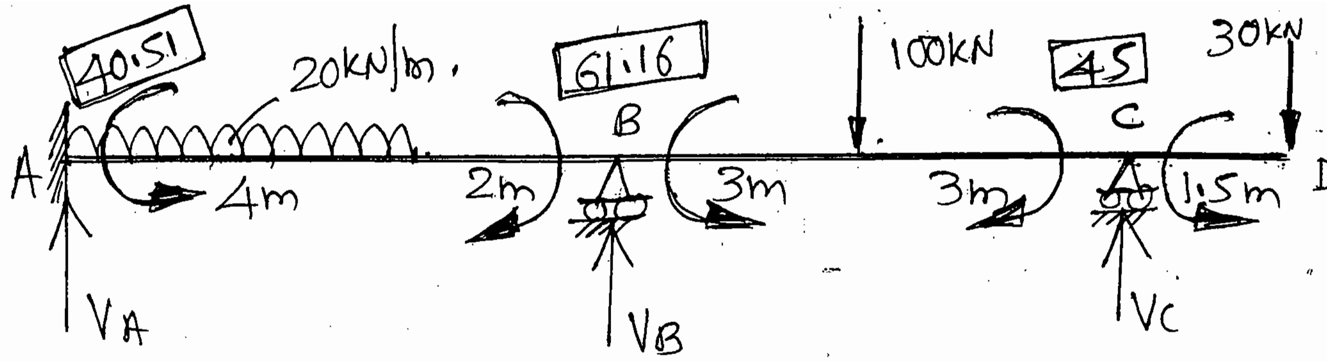
$$\theta_C = -24.61/EI$$

(d) Final Moment

$M_{AB} = -40.51$ kN-m ↻ $M_{BC} = -61.18$ kN-m ↻

$M_{BA} = 61.16$ kN-m ↺ $M_{CB} = 45$ kN-m ↺

$M_{CD} = -45$ kN-m ↻



$$\Sigma V = 0, \quad 20 \times 4 + 100 + 30 = V_A + V_B + V_C$$

$$\therefore \quad V_A + V_B + V_C = 210 \longrightarrow (i)$$

$\Sigma M_B = 0$, (LHS)

$$V_A \times 6 - 20 \times 4 \times \left(\tfrac{4}{2} + 2\right) - 40.51 + 61.16 = 0$$

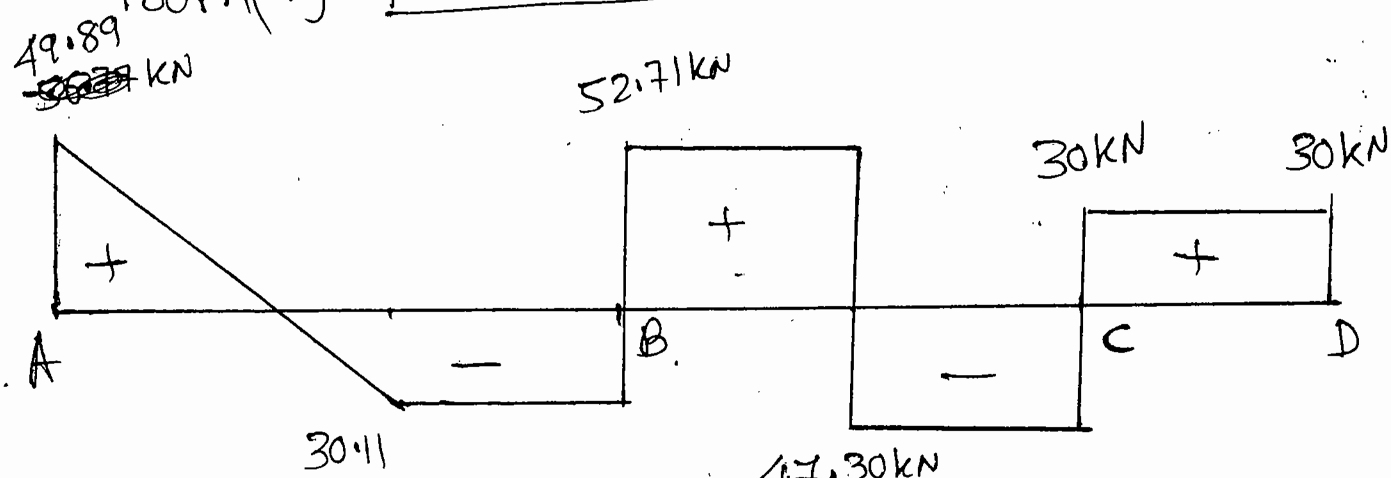
$$\boxed{V_A = \cancel{56.77}\ 49.89}\ \text{kN}$$

$\Sigma M_B = 0$ (RHS)

$$-V_C \times 6 + 100 \times 3 + 30 \times 7.5 - 61.16 + 45 - 45 = 0$$

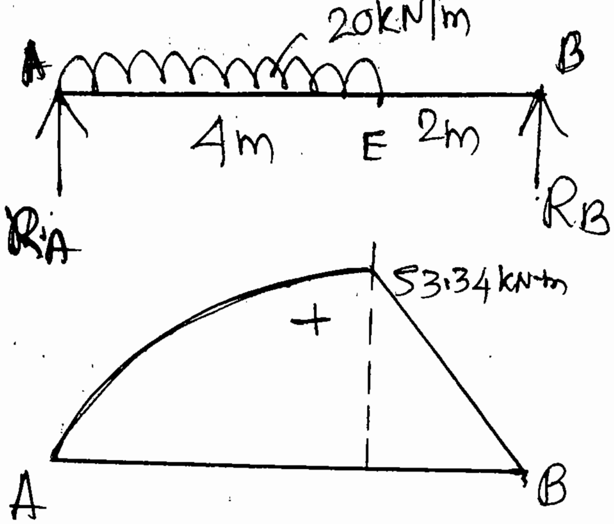
$$\boxed{V_C = 77.30\ \text{kN}}$$

From (i) $\boxed{V_B = \cancel{78.93}}\ 82.81\ \text{kN}$



SFD

Free BMD for AB

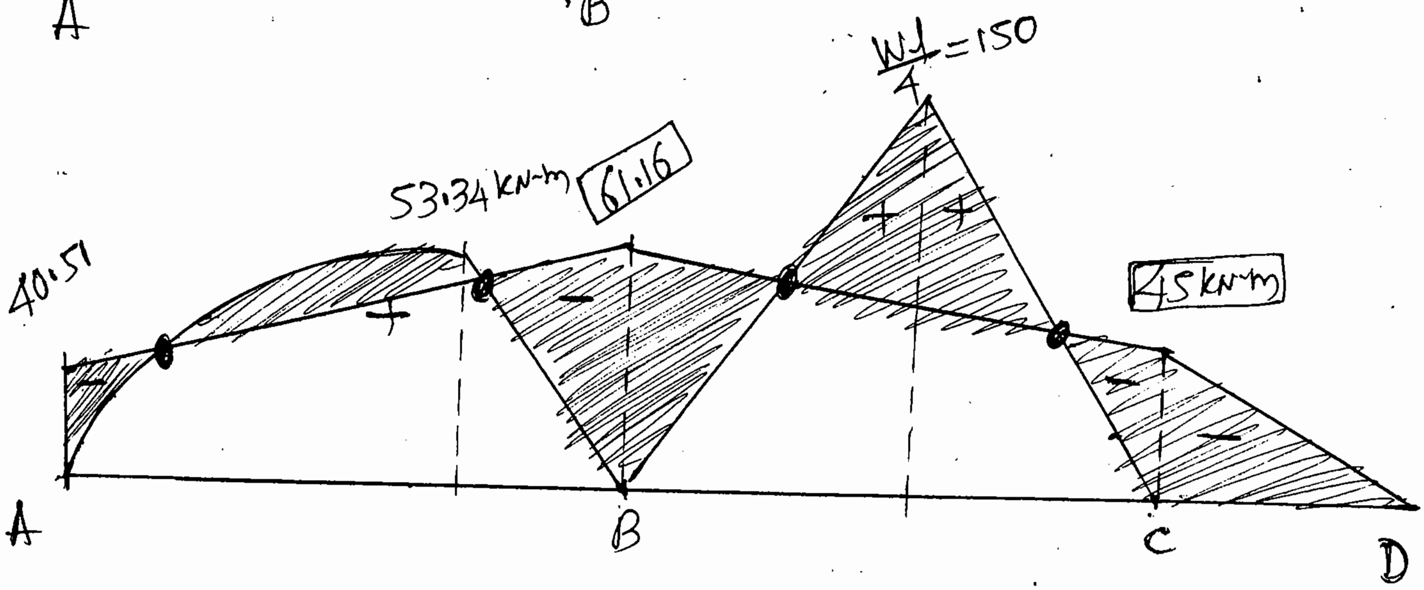


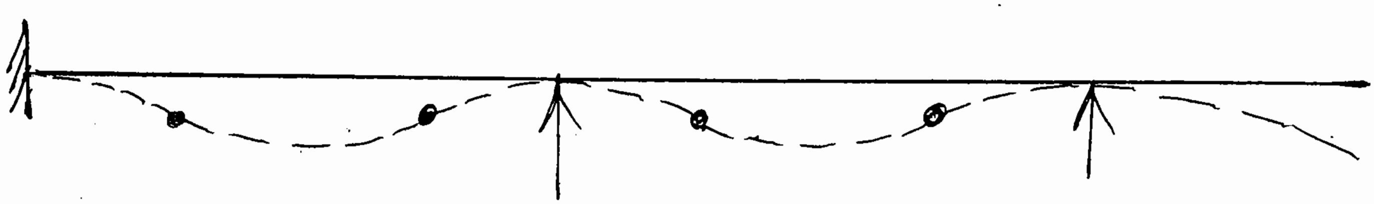
$$R_A = 53.33\,kN$$

$$R_B = 26.67\,kN$$

$$\therefore M_E = R_B \times 2$$

$$= 53.34\,kN\text{-}m$$

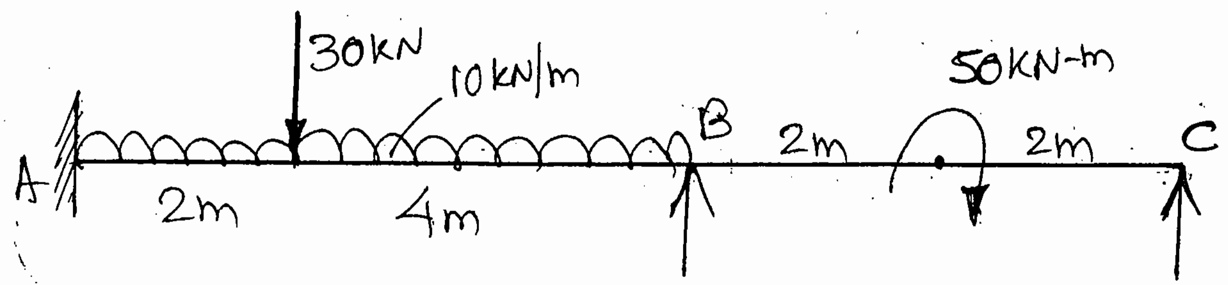




— × —

Sinking of Support

Eg:- 4] Analyse the beam shown by SD method and draw BMD, SFD & EC. The support 'B' sinks by 5mm. Take $E = 210 GPa$, $I = 0.1 G mm^4$



Sol^n

$$E = 210 GPa = 210 \times 10^9 \times 10^{-6} = 210 \times 10^3 \, N/mm^2$$

$$I = 0.1 G mm^4 = 0.1 \times 10^9 mm^4$$

$$EI = \left(\underset{N/mm^2}{210 \times 10^3}\right)\left(\underset{mm^4}{0.1 \times 10^9}\right) = 2.1 \times 10^{13} \, N\text{-}mm^2$$

$$EI = \frac{2.1 \times 10^{13}}{(1000)(1000)^2} = 2.1 \times 10^4 \, kN\text{-}m^2$$

(a) FEM

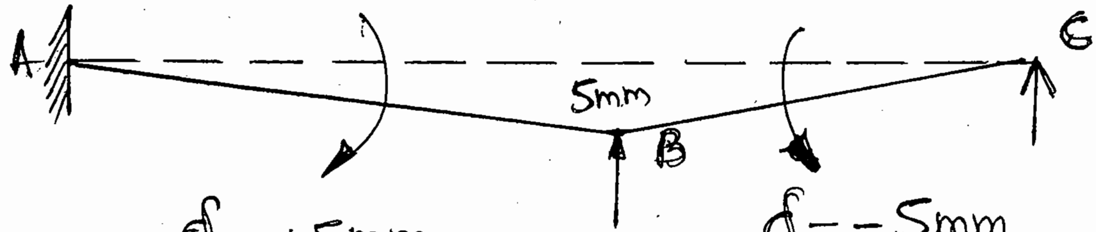
$$M_{FAB} = -\frac{\omega l^2}{12} - \frac{Wab^2}{l^2} = \frac{-10 \times 6^2}{12} - \frac{30 \times 2 \times 4^2}{6^2} = -56.67$$

$$M_{FBA} = +\frac{\omega l^2}{12} + \frac{Wa^2 b}{l^2} = +\frac{10 \times 6^2}{12} + \frac{30 \times 2^2 \times 4}{6^2} = +43.33$$

$$M_{FBC} = M_{FCB} = +\frac{M}{4} = +12.5 \, kN\text{-}m$$

(b) SD Equation :

$\theta_A = 0$,



$\delta = +5mm$
$= +0.005m$

$\delta = -5mm$
$= -0.005m$

$$M_{AB} = \frac{2EI}{6}\left[0 + \theta_B - \frac{3 \times 0.005}{6}\right] - 56.67$$

$$= \frac{2(2.1 \times 10^4)}{6}\left[\theta_B - 2.5 \times 10^{-3}\right] - 56.67$$

$$= 7000\,\theta_B - 74.17 \longrightarrow (i)$$

$$M_{BA} = \frac{2(2.1 \times 10^4)}{6}\left[2\theta_B + 0 - 2.5 \times 10^{-3}\right] + 43.33$$

$$= 14000\,\theta_B + 25.83 \longrightarrow (ii)$$

$$M_{BC} = \frac{2(2.1 \times 10^4)}{4}\left[2\theta_B + \theta_C - \frac{3(-0.005)}{4}\right] + 12.5$$

$$= 21000\,\theta_B + 10500\,\theta_C + 51.87 \longrightarrow (iii)$$

$$M_{CB} = \frac{2(2.1 \times 10^4)}{4}\left[2\theta_C + \theta_B - \frac{3(-0.005)}{4}\right] + 12.5$$

$$= 21000\,\theta_C + 10500\,\theta_B + 51.87 \longrightarrow (iv)$$

(c) Equillibrium Condition

① At "B" $\boxed{M_{BA} + M_{BC} = 0}$

$$35000\,\theta_B + 10500\,\theta_C = -77.7 \rightarrow \text{(I)}$$

② At 'C' $\boxed{M_{CB} = 0}$

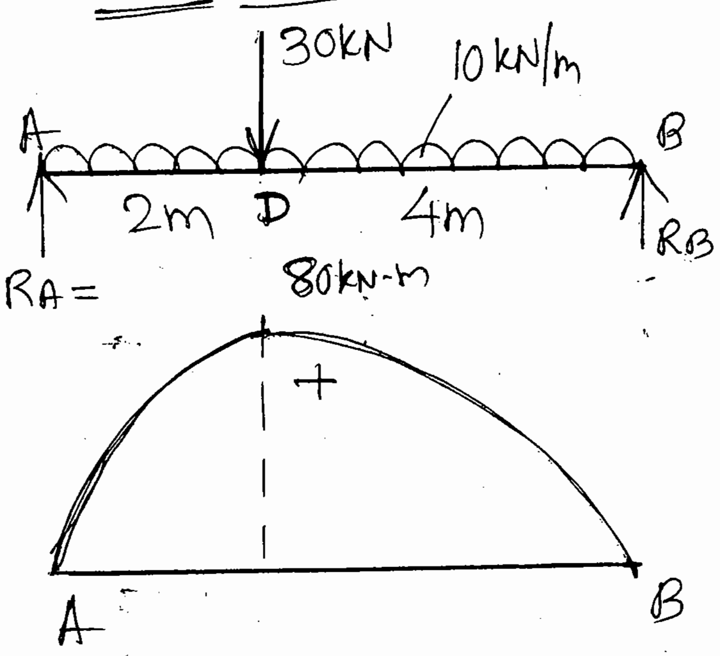
$$10500\,\theta_B + 21000\,\theta_C = -51.87 \rightarrow \text{(II)}$$

Solving
$$\theta_B = -1.74 \times 10^{-3}$$
$$\theta_C = -1.6 \times 10^{-3}$$

(d) Final Moment :

$M_{AB} = -86.35$ kN-m ↻
$M_{BA} = 1.47$ kN-m ↺
$M_{BC} = -1.47$ kN-m ↻
$M_{CB} = 0$

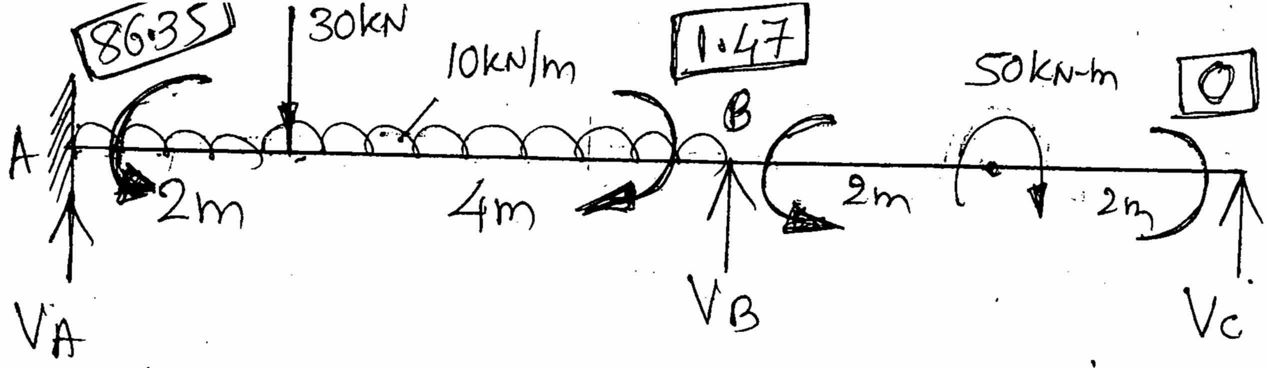
Free BMD for "AB"



$R_A = 50$ kN

$R_B = 40$ kN

$$\therefore M_D = R_B \times 4 - 10 \times \frac{4 \times 4}{2}$$
$$= 80 \text{ kN-m}$$



$$V_A + V_B + V_C = 30 + 10 \times 6 = 90 \rightarrow (i)$$

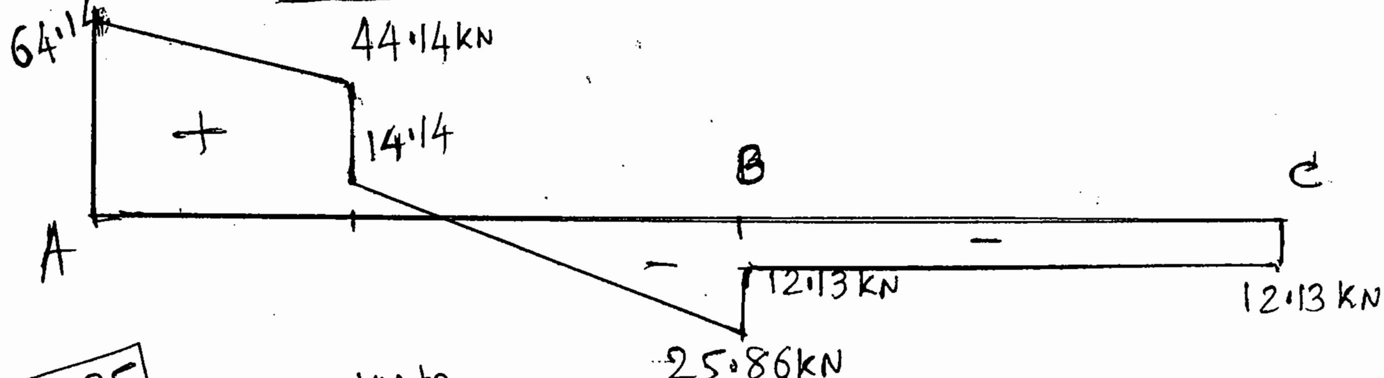
$\Sigma M_B = 0$, (LHS)

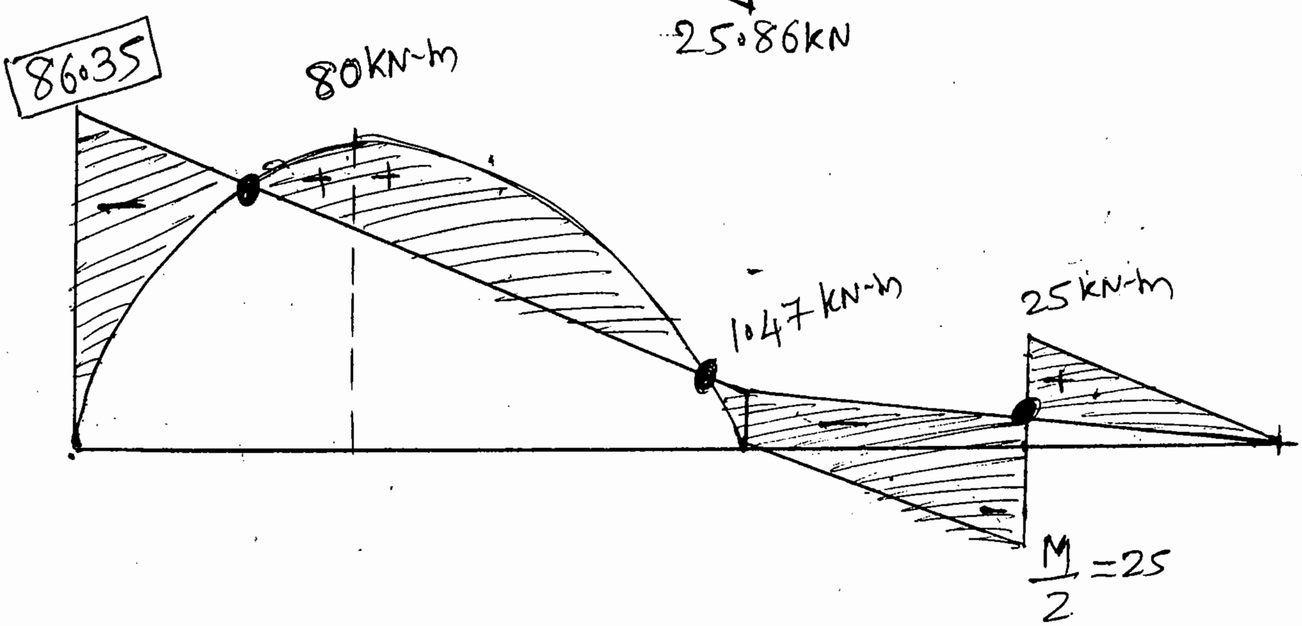
$$V_A \times 6 - 30 \times 4 - 10 \times 6 \times 6/2 - 86.35 + 1.47 = 0$$

$$\boxed{V_A = 64.14}$$

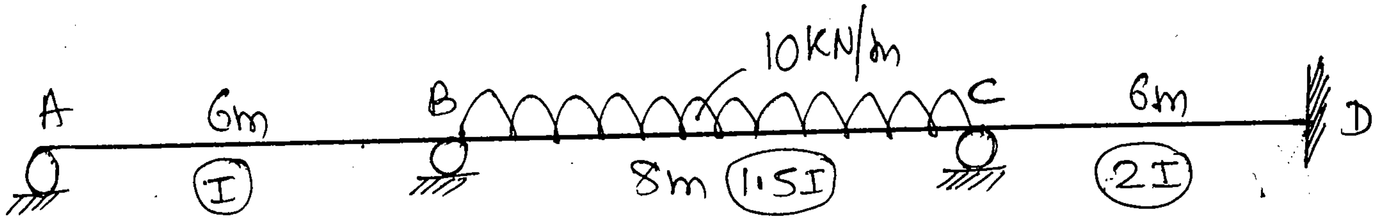
$\Sigma M_B = 0$ (RHS) $-V_C \times 4 + 50 - 1.47 = 0$ $\boxed{V_C = 12.13}$

From (i) $\boxed{V_B = 13.72}$



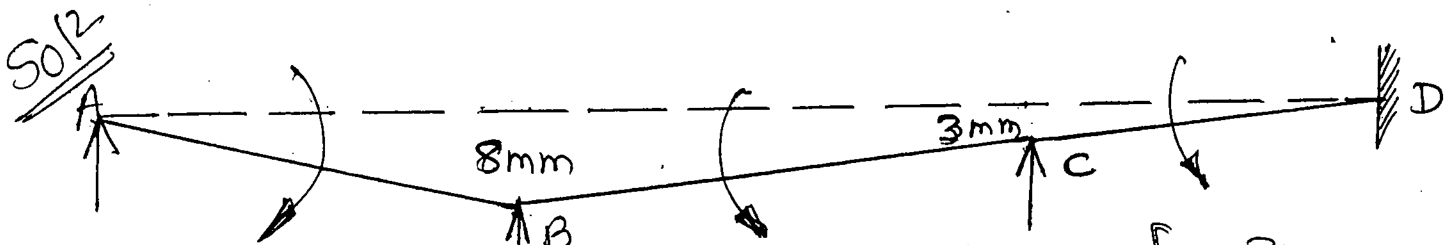


Eg:- 5] Analysis the beam shown by SD method. Draw BMD, SFD & EC.



Support B & C settles by 8mm and 3mm respt. Take $EI = 2\times10^4$ KN-m^2.

Sol:



$\delta = +8mm$
$0.008m.$

$\delta = -5mm$
$= -0.005m.$

$\delta = -3mm$
$= -0.003m$

(a) FEM

$$M_{FBC} = \frac{-\omega l^2}{12} = -53.33, \quad M_{FCB} = +53.33$$

(b) SD Equation :

$$\theta_D = 0$$

$$M_{AB} = \frac{2(2\times10^4)}{6}\left[2\theta_A + \theta_B - \frac{3(0.008)}{6}\right] + 0$$

$$= 13333.33\,\theta_A + 6666.67\,\theta_B - 26.67 \rightarrow (i)$$

$$M_{BA} = \frac{2(2\times10^4)}{6}\left[2\theta_B + \theta_A - \frac{3(0.008)}{6}\right] + 0$$

$$= 13333.33\,\theta_B + 6666.67\,\theta_A - 26.67 \quad \text{— (II)}$$

$$M_{BC} = \frac{2(1.5\times2\times10^4)}{8}\left[2\theta_B + \theta_C - \frac{3(-0.005)}{8}\right] - 53.33$$

$$= 15000\,\theta_B + 7500\,\theta_C - 39.26 \quad \text{— (III)}$$

$$M_{CB} = 7500\left[2\theta_C + \theta_B - \frac{3(-0.005)}{8}\right] + 53.33$$

$$= 15000\,\theta_C + 7500\,\theta_B + 67.39 \quad \text{— (IV)}$$

$$M_{CD} = \frac{2(2\times2\times10^4)}{6}\left[2\theta_C + 0 - \frac{3(-0.003)}{6}\right]$$

$$= 26666.67\,\theta_C + 20 \longrightarrow \text{(V)}$$

$$M_{DC} = 13333.33\left(0 + \theta_C - \frac{3(-0.003)}{6}\right)$$

$$= 13333.33\,\theta_C + 20 \longrightarrow \text{(VI)}$$

(c) Equillibrium Condition

(i) $\boxed{M_{AB} = 0}$

$$13333.33\,\theta_A + 6666.67\,\theta_B = 26.67 \longrightarrow \text{(I)}$$

(ii) $\boxed{M_{BA} + M_{BC} = 0}$

$$28333.33\,\theta_B + 6666.67\,\theta_A + 7500\,\theta_C = 65.93 \rightarrow \text{(II)}$$

(iii) $\boxed{M_{CB} + M_{CD} = 0}$

$$7500\,\theta_B + 41666.67\,\theta_C = -87.39 \rightarrow \text{(III)}$$

Solving, $\theta_A = +5.56\times10^{-4}$ $\quad\theta_C = -2.617\times10^{-3}$

$\theta_B = 2.889\times10^{-3}$

(d) Final Values

$M_{AB} \approx 0$

$M_{BA} = 15.55$ kN-m

$M_{BC} = -15.55$ kN-m

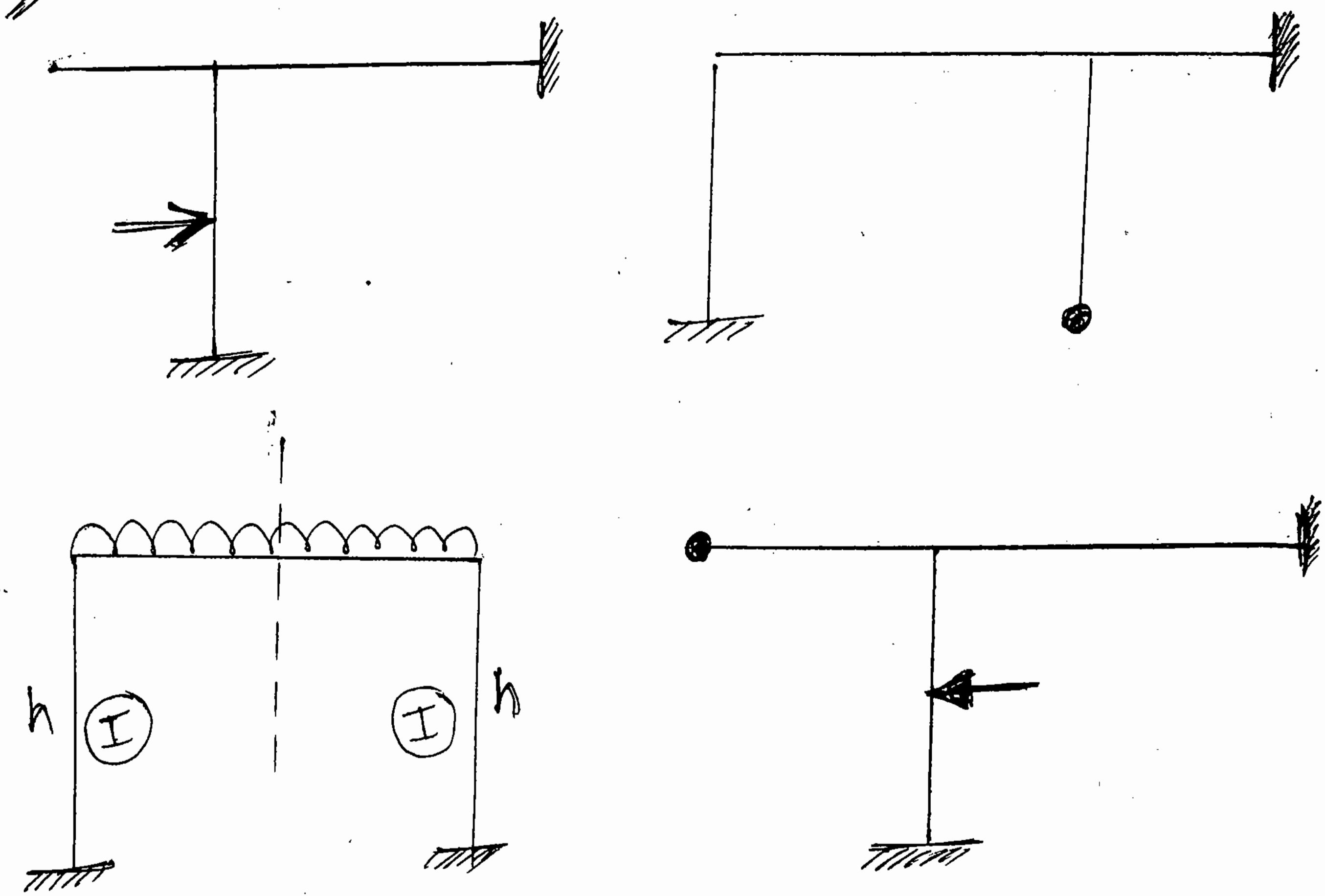
$M_{CB} = 49.80$ kN-m

$M_{CD} = -49.80$ kN-m

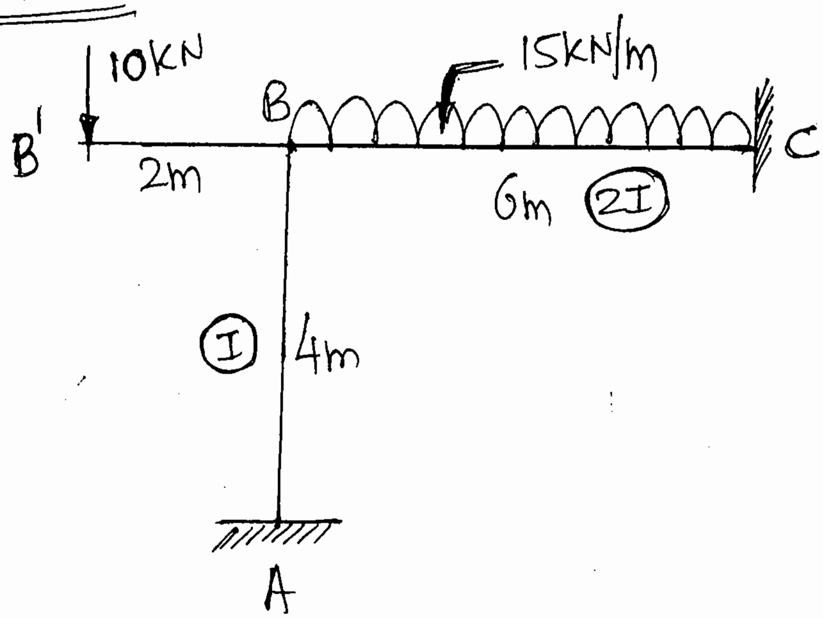
$M_{DC} = -14.89$ kN-m

Date 14/9/18

Non Sway Frame :



Eg:-1] Analyse the frame shown by S.D. method and draw BMD, SFD & EC.

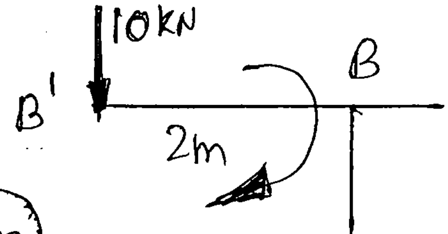


(a) FEM ;

$M_{FAB} = M_{FBA} = 0$

$M_{FBC} = -\frac{wl^2}{12} = -\frac{15(6)^2}{12} = -45\,kN\text{-}m$

$M_{FCB} = +45.$



$M_{BB'} = +10 \times 2 = +20\,kN\text{-}m$

+ve sign for clockwise resisting moment.

(b) S.D. equation :-

$\theta_A = 0, \quad \theta_C = 0$ ($\because$ Fixed)

$\delta = 0$ ($\because$ Non-sway)

There is no equation for overhang BB'

$M_{AB} = \frac{2(EI)}{4}\left[\theta_B\right] = 0.5EI(\theta_B)$ —(i)

$M_{BA} = \frac{2EI}{4}\left[2\theta_B\right] = EI(\theta_B)$ —(ii)

$M_{BC} = \frac{2(2EI)}{6}\left[2\theta_B\right] - 45 = 1.33EI(\theta_B) - 45$ —(iii)

$M_{CB} = \frac{2(2EI)}{6}\left[\theta_B\right] + 45 = 0.666EI(\theta_B) + 45$ —(iv)

(c) Equillibrium Condition

At Intermediate joint "B"

$$\boxed{M_{BA} + M_{BC} + M_{BB'} = 0}$$

$$[EI\theta_B] + [1.33EI\theta_B - 45] + [20] = 0$$

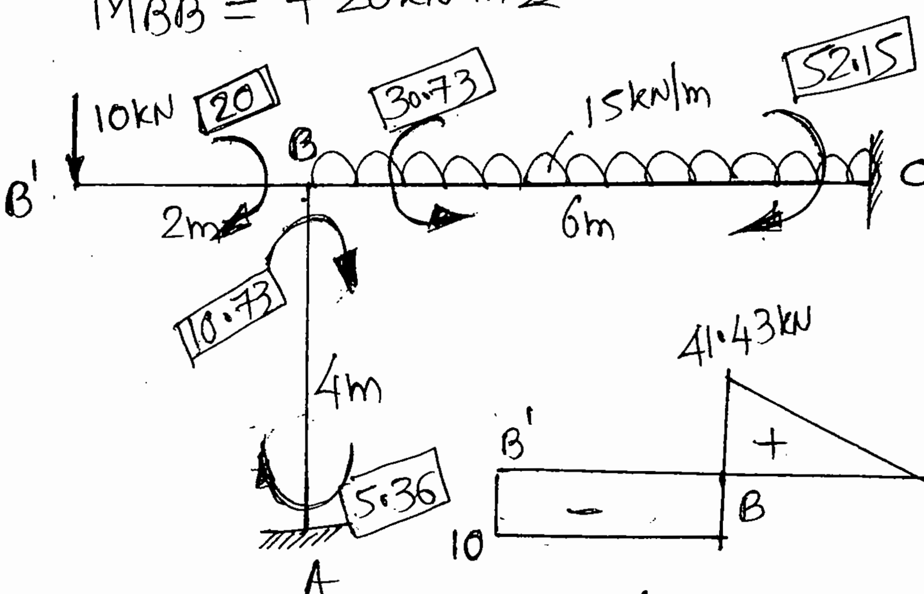
$$\therefore \theta_B = \frac{10.73}{EI}$$

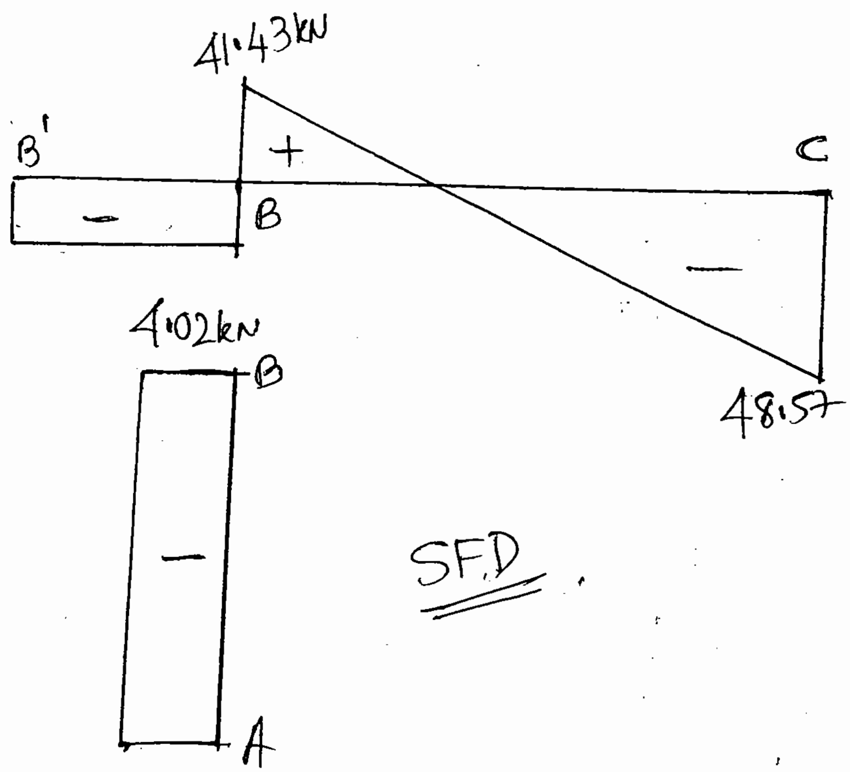
(d) Final Moment :- Substitute in eqⁿ (i) to (iv)

$M_{AB} = 5.36$ kN-m $M_{BC} = -30.73$ kN-m

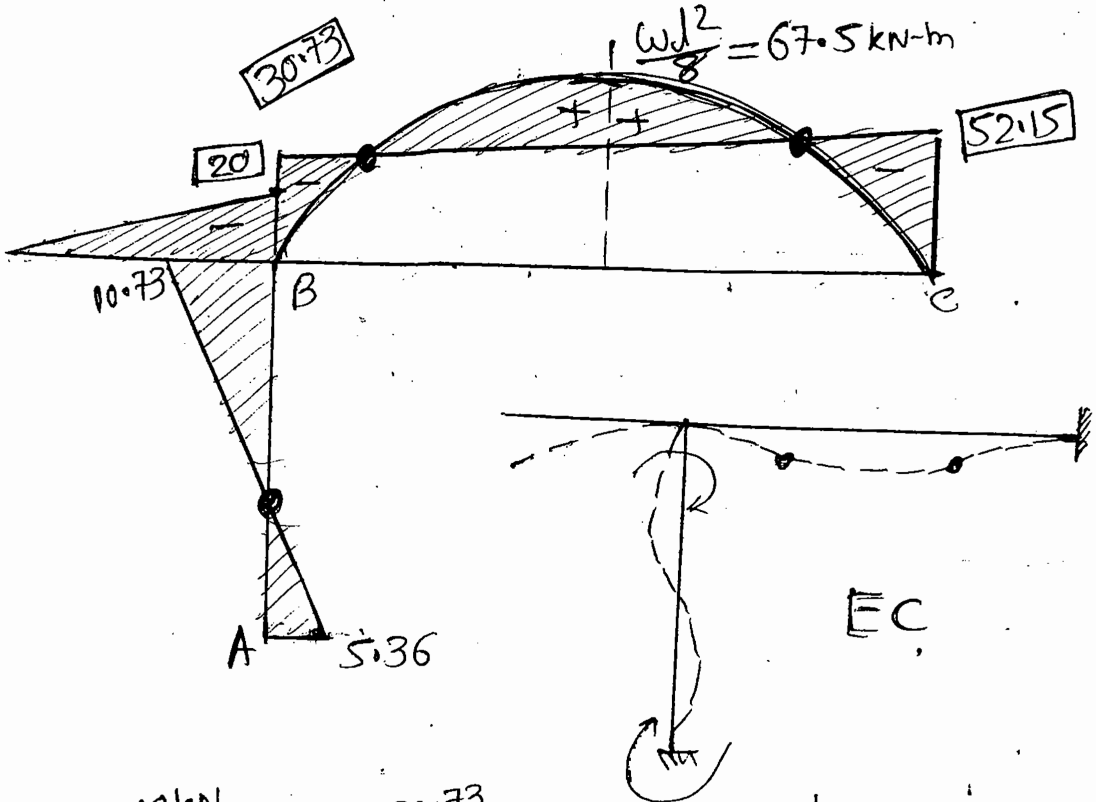
$M_{BA} = 10.73$ kN-m $M_{CB} = 52.15$ kN-m.

$M_{BB'} = +20$ kN-m

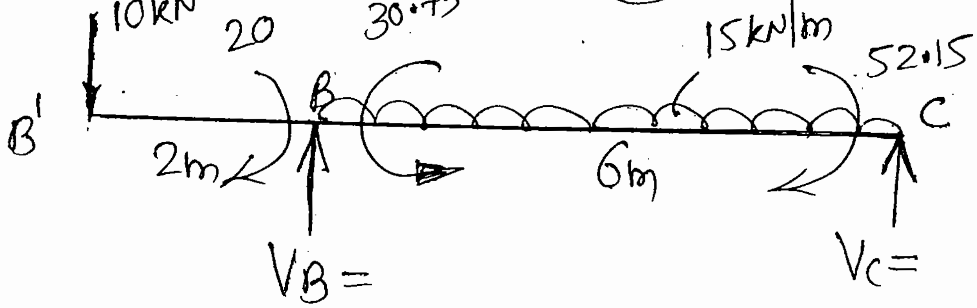




SFD



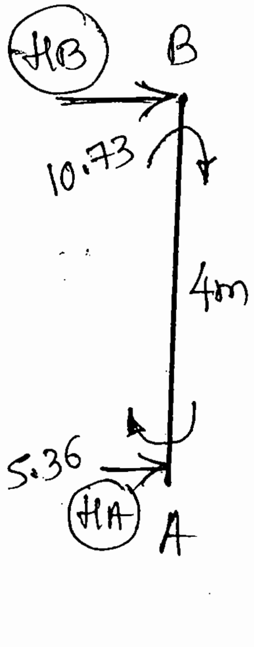
SFD



$V_A + V_B = 10 + 15 \times 6 = 100$ — (i)

$\Sigma M_C = 0, \quad -10 \times 8 + V_B \times 6 - 15 \times 6 \times 6/2 + 20 - 30.73 + 52.15 = 0$

$\boxed{V_B = 51.43}$ & $\boxed{V_C = 48.57}$



$\Sigma H = 0, \quad H_A + H_B = 0$

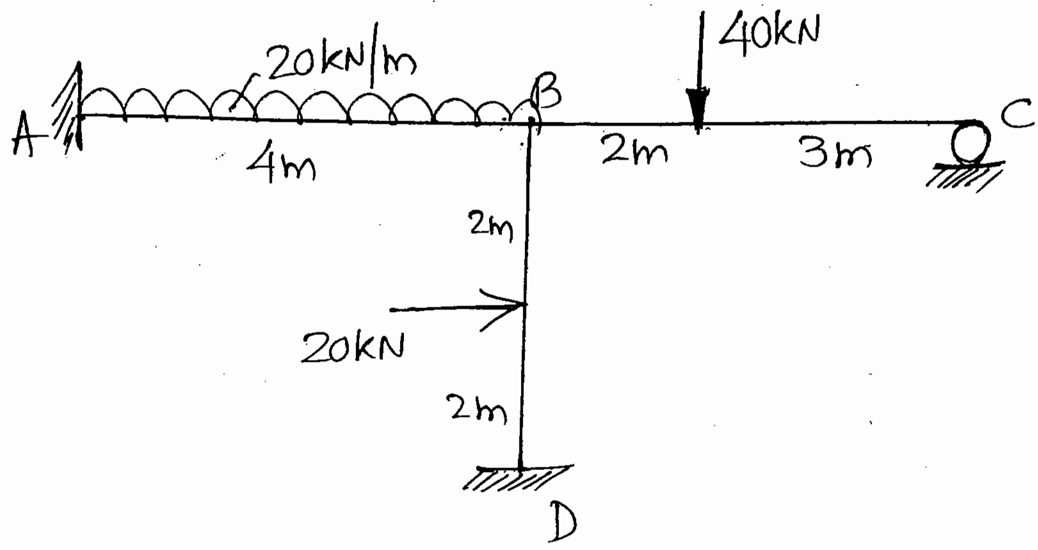
$H_B \times 4 + 10.73 + 5.36 = 0$

$\boxed{H_B = -4.02}$

& $\boxed{H_A = +4.02}$

=x=

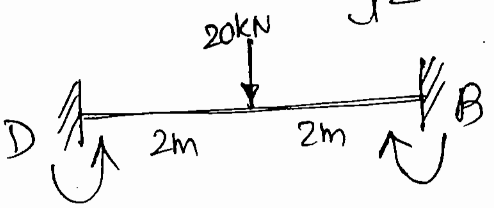
Eg:- 2] Analyse the frame shown by S.D. method. Draw BMD, SFD, EC



Sol: (a) FEM

$M_{FAB} = -\frac{wl^2}{12} = -26.67$, $M_{FBA} = +26.67$

$M_{FBC} = -\frac{Wab^2}{l^2} = -28.8$, $M_{FCB} = +\frac{Wa^2b}{l^2} = +19.2$



$M_{FDB} = -\frac{Wl}{8} = -10$

$M_{FBD} = +10$

(b) S.D. Equation :

$\theta_A = \theta_D = 0$ (Fixed)

$\delta = 0$ (Non-Sway)

$M_{AB} = \frac{2EI}{4}\left[\theta_B\right] - 26.67 = 0.5EI\theta_B - 26.67$ —(i)

$M_{BA} = \frac{2EI}{4}\left[2\theta_B\right] + 26.67 = EI\theta_B + 26.67$ —(ii)

$$M_{BC} = \frac{2EI}{5}\left[2\theta_B + \theta_C\right] - 28.8$$

$$= 0.8EI(\theta_B) + 0.4EI\theta_C - 28.8 \text{ —(III)}$$

$$M_{CB} = \frac{2EI}{5}\left[2\theta_C + \theta_B\right] + 19.2$$

$$= 0.8EI\theta_C + 0.4EI\theta_B + 19.2 \text{ —(IV)}$$

$$M_{BD} = \frac{2EI}{4}\left[2\theta_B\right] + 10 = EI(\theta_B) + 10 \text{ —(V)}$$

$$M_{DB} = \frac{2EI}{4}\left[\theta_B\right] - 10 = 0.5EI(\theta_B) - 10 \text{ —(VI)}$$

(c) <u>Equillibrium Condition</u> :—

at "B" $\boxed{M_{BA} + M_{BC} + M_{BD} = 0}$

$$2.8EI(\theta_B) + 0.4EI(\theta_C) = -7.87 \longrightarrow (I)$$

At "C" $\boxed{M_{CB} = 0}$

$$0.4EI(\theta_B) + 0.8EI(\theta_C) = -19.2 \longrightarrow (II)$$

solving

$$\theta_B = \frac{0.67}{EI}$$

$$\theta_C = -\frac{24.33}{EI}$$

(d) Final Moment

$M_{AB} = -26.33 \text{ kN-m}$ ↻ $\qquad M_{CB} = 0$

$M_{BA} = 27.34 \text{ kN-m}$ ↺ $\qquad M_{BD} = 10.67 \text{ kN-m}$ ↻

$M_{BC} = -38.00 \text{ kN-m}$ ↻ $\qquad M_{DB} = -9.66 \text{ kN-m}$ ↻