MODULE-4

ORIFICES AND MOUTHPIECES

Orifice: An opening, in a vessel, through which the liquid flows out is known as orifice. This hole or opening is called an orifice, so long as the level of the liquid on the upstream side is above the top of the orifice.
The typical purpose of an orifice is the measurement of discharge. An orifice may be provided in the vertical side of a vessel or in the base. But the former one is more common.

Types of Orifice

Orifices can be of different types depending upon their size, shape, and nature of discharge. But the following are important from the subject point of view.

- According to size:
  - Small orifice
  - Large orifice

  According to shape:
  - Circular orifice
  - Rectangular orifice
  - Triangular orifice

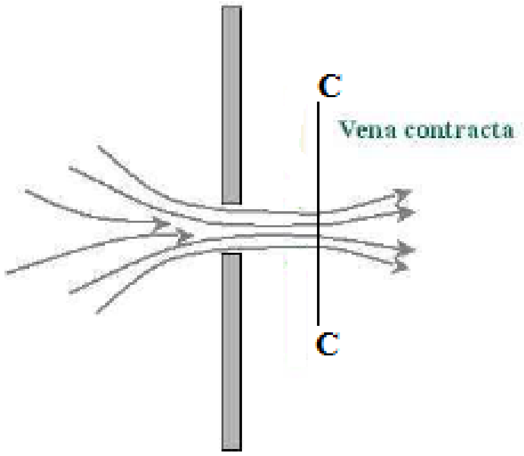
  According to shape of edge:
  - Sharp-edged
  - Bell-mouthed

  According to nature of discharge:
  - Discharging free Orifice
  - Fully submerged Orifice
  - Partially submerged Orifice

Venacontracta

Consider an orifice is fitted with a tank. The liquid particles, in order to flow out through the orifice, move towards the orifice from all directions. A few of the particles first move downward, then take a turn to enter into the orifice and then finally flow through it.
The liquid flowing through the orifice forms a jet of liquid whose area of cross-section is less then that of the orifice. the area of jet of fluid goes on decreasing and at section C-C, the area is minimum.

This section is approximately at a distance of half of diameter of the orifice . At this section, the streamlines are straight and parallel to the each other and perpendicular to the plane of the orifice. This section is called vena-contracta. Beyond this section, the jet diverges and is attracted in the downward direction by the gravity.



Venacontracta

Hydraulic Coefficients

The following four coefficients are known as hydraulic coefficients or orifice coefficients.

1. **Coefficient of Contraction:** The ratio of the area of the jet, at vena-contracta, to the area of the orifice is known as coefficient of contraction.

$$C_c = \frac{area\ of\ the\ jet\ at\ venacontracta}{area\ of\ the\ orifice}$$

The value of Coefficient of contraction varies slightly with the available head of the liquid, size and shape of the orifice. The average value of Cc is 0.64

2. **Coefficient of Velocity:** The ratio of actual velocity of the jet, at Vena-contracta, to the theoretical velocity is known as coefficient of velocity.The theoretical velocity of jet at Vena-contracta is given by the relation, $v = \sqrt{2gh}$

$$C_v = \frac{actual\ velocity\ of\ the\ jet\ at\ Venacontracta}{theoretical\ velocity}$$

The difference between the velocities is due to friction of the orifice. The value of Coefficient of velocity varies slightly with the different shapes of the edges of the orifice.This value is very small for sharp-edged orifices. For a sharp edged orifice, the value of C_v increases with the head of water. theoretical

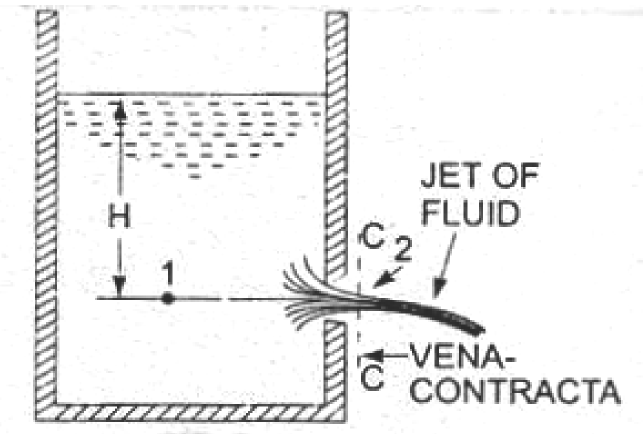
3. **Coefficient of Discharge** The ratio of a actual discharge through an orifice to the theoretical discharge is known as coefficient of discharge. Mathematically coefficient of discharge,

$$C_d = \frac{actual\ discharge}{theoretical\ discharge} = \frac{actual\ velocity \times actual\ area}{theoretical\ velocity \times theoretical\ area} = C_v \times C_c$$

Thus the value of coefficient of discharge varies with the values of C_c and C_v. An average of coefficient of discharge varies from 0.60 to 0.64.

Discharge through the Orifice

Consider a tank fitted with circular orifice in one of its sides as shown in fig. let H be the head of the liquid above the liquid above the center of orifice.



Flow through orifice

Consider two points 1 and 2 as shown in the fig. point 1 is inside the tank ans point 2 at vena-contracta. let the flow be steady and at a constant head H.
Applying Bernoulli's equation between 1 and 2

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

But, z_1=z_2,
Hence,

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}$$

Also

$$\frac{p_1}{\rho g}=H$$

$$\frac{p_2}{\rho g}=0(atmospheric\ pressure)$$

$v_1$is very small in comparision to v_2 as area of the tank is very large compared to area of the jet

$$\therefore H+0=0+\frac{v_2^2}{2g}$$

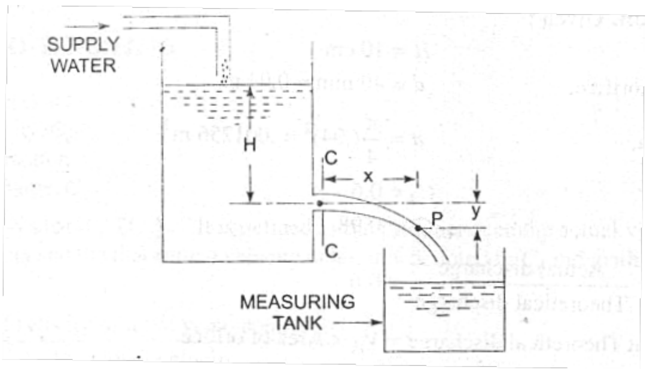
$$\therefore v_2=\sqrt{2gH}$$

This is the expression for the theoretical velocity.

Experimental determination of hydraulic coefficients

Determination of C_d

Water is allowed to flow through a orifice fitted to a tank under a constant head, H as shown in Fig. The water collected in the measuring tank for a known time, t. the height of water in measuring tank



Value of C_v

is noted down. then actual discharge through orifice,

$$Q = \frac{area\ of\ measuring\ tank \times height\ of\ water\ in\ measuring\ tank}{Time(t)}$$

and theoretical discharge = area of the orifice $\times \sqrt{2gH}$

$$C_d = \frac{Q}{a \times \sqrt{2gH}}$$

Determination of C_v

Co-efficient of velocity,

$$C_v = \frac{x}{\sqrt{4yH}}$$

where, x = Horizontal distance traveled by the particle in time 't'
y= vertical distance between p and C-C (refer fig: Value of C_v)

Determination of C_c

$$C_d = C_v \times C_c$$

Mouthpiece

Mouthpiece is a short tube of length not more than two or three times its diameter, provided in a tank or a vessel containing fluid such that it is an extension of the orifice and through which also the fluid may discharge. Both orifice and mouthpiece are usually used for measuring the rate of flow.

Classification of Mouthpiece

1. Based on the position with respect to the tank or vessel to which they are fitted

   (a) External mouthpiece

   (b) Internal mouthpiece

2. Based on the shape:

   (a) Cylindrical mouthpiece

   (b) Convergent mouthpiece

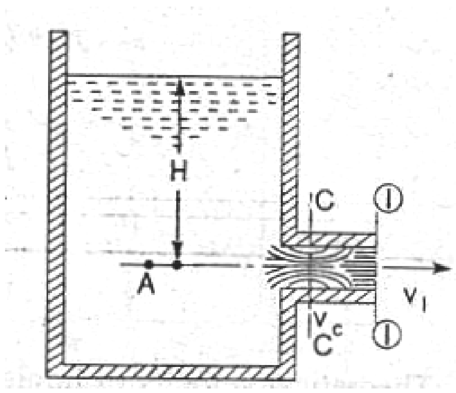
   (c) Convergent-divergent mouthpiece

3. Based on the nature of discharge at outlet of mouthpiece.

   (a) Mouthpieces running full

   (b) Mouthpieces running free

Flow through Mouthpiece



Flow through mouthpiece

Consider a tank having as external cylindrical mouthpiece of C/S area a1, attached to one of its sides as shown in Fig. the jet off liquid enntering the mouthpiece contracts to form a vena-contracta at a section C-C. Beyond this section, the jet again expands and fill the mouthpiece completely.
Let H=Heigh of liquid above the centre of mouthpiece
v_c=Velocity of liquid at C-C section
a_c=Area of flow at vena-contracta
v_1=Velocity of liquid at outlet
a_1=Area of mouthpiece at vena-contracta
C_c=Co-efficient of contraction.

Applying continuity equation at C-C and (1)-(1), we get

$$a_c \times v_c = a_1 \times v_1$$
$$\therefore v_c = \frac{a_1 v_1}{a_1} = \frac{v_1}{a_c/a_1}$$

But

$$\frac{a_c}{a_1} = C_c = Co-efficient\ of\ contraction$$

taking C_d=0.62, we get $\frac{a_c}{a_1} = 0.62$

$$\therefore v_c = \frac{v_1}{0.62}$$

the jet of liquid from section C-C suddenly enlarges at section (1)-(1). Due to sudden enlargement, there wiil be loss of head, h_L^* which is given as

$$h_L^* = \frac{(v_c - v_1)^2}{2g}$$

But,

$$v_c = \frac{v_1}{0.62} = \frac{\left(\frac{v_1}{0.62} - v_1\right)}{2g} = \frac{v_1^2}{2g}\left[\frac{1}{0.62} - 1\right] = \frac{0.375 v_1^2}{2g}$$

Applying Bernoulli's equation to point A (1)-(1)

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_A = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_1 + h_L$$

wherez_A=z_1 ,v_A is negligible,

$$\frac{p_1}{\rho g} = atmospheric\ pressure = 0$$
$$\therefore H + 0 = 0 + \frac{v_1^2}{2g} + 0.375\frac{v_1^2}{2g}$$
$$H = 1.375\frac{v_1^2}{2g}$$
$$v_1 = \sqrt{\frac{2gH}{1.375}} = 0.855\sqrt{2gH}$$

Theoretical velocity of liquid at outlet is v_{th}=$\sqrt{2gH}$

$$\therefore$$

Co-effficinet of velocity for mouthpiece

$$C_v = \frac{Actual\ velocity}{Theoritical\ velocity} = \frac{0.855\sqrt{2gH}}{\sqrt{2gH}} = 0.855$$

$C_c$for mouthpiece =1 as the area of the area of jet of liquid at out let is equal to area of the mouthpiece. Thus,

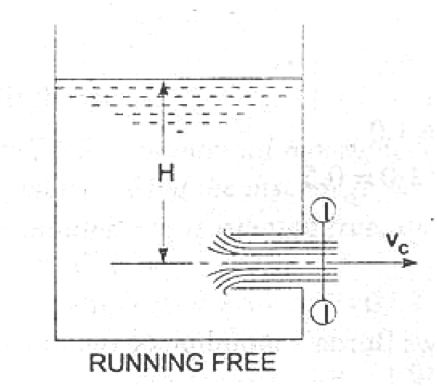
$$C_d = C_v \times C_c = 1.0 \times 0.855 = 0.855$$

Borda's mouthpiece

A short cylindrical tube attached to an orifice in such away that the tube projects inwardly to a tank, is called as Borda's mouthpiece or Re-entrant mouthpiece or internal mouthpiece.

Borda's Mouthpiece running free

If the length of the tube is equal to its diameter, the jet of the liquid comes out from mouthpiece without touching the sides of tube.



Mouthpiece running free

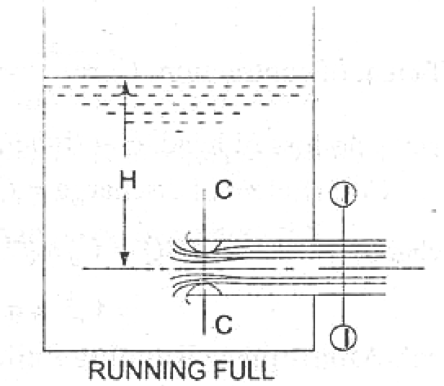
Discharage,

$$Q = 0.5 \times a\sqrt{2gH}$$

where H= height of the fluid above the mouthpiece,
a=area of the mouthpiece

Boarda's Mouthpiece running full

If the length of the tube is about 3 times its diameter, the jet comes out with its diameter equal to diameter of the mouthpiece at outlet.



Mouthpiece running full

Discharage,

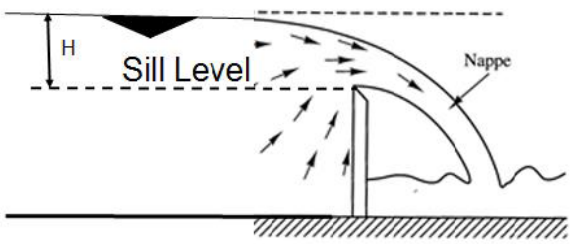
$$Q = 0.707 \times a\sqrt{2gH}$$

where H= height of the fluid above the mouthpiece,
a=area of the mouthpiece

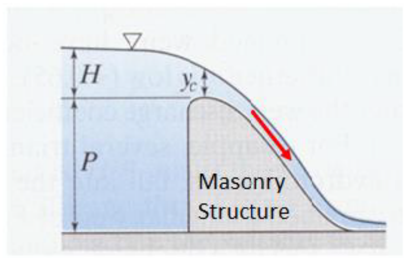
NOTCHES AND WEIRS

A notch is a device used for measuring the rate of flow of a liquid through a small channel or tank. It is an opening in the side of a measuring tank or reservoir such that the water level is always below the top edge of the opening.

A weir is a concrete or a masonry structure , placed in an open channel over which the flow occurs . It is generally in the form of vertical wall, with Bell mouthed edge.



(a)Notch



(b)Weir

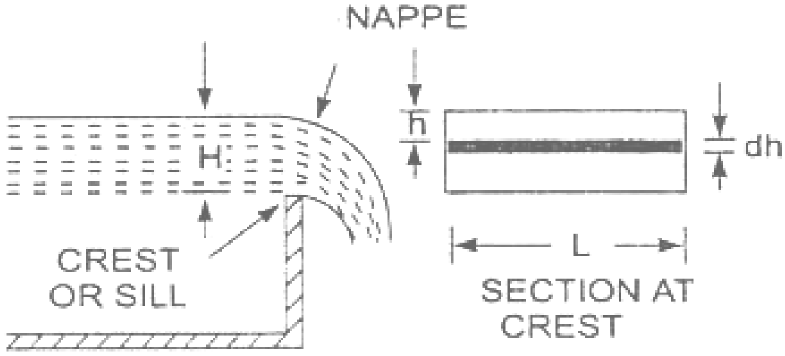
Note: The sheet of water flowing over a notch or a weir is known as Nappe.
The bottom edge of the opening is known as 'Sill' or Crest.

Classification of notches

- According to the shape of opening
  - Rectangular notch
  - Triangular notch
  - Trapezoidal notch
  - Stepped notch
- According to the effect of the sides on the nappe:
  - Notch with end contraction
  - Notch without end contraction or suppressed notch

Discharge over a Rectangular notch

Consider a sharp edge rectangular notch with crest horizontal and normal to direction of flow. Let,
H= Head of water over the crest,
L=length of notch or weir
consider an elementary horizontal strip of water of thickness dh' and length L at a depth 'h' from free surface of water as shown in Fig.
The area of the strip = $L \times dh$



Rectangular Notch

and theoretical velocity of water flowing through strip = $\sqrt{2gh}$
The discharge dQ, through strip is

$$dQ = C_d \times Areaofstrip \times theoriticalvelocity = C_d \times L \times dh \times \sqrt{2gh}$$

where C_d=Co-efficient of discharge.
The total discharge , Q, for the whole notch or weir is determined by integrating the above equation between limits 0 and H.

$$\begin{aligned} Q &= \int_0^H C_d\, L\, \sqrt{2gh}\, dh = C_d \times L \times \sqrt{2g} \int_0^H h^{1/2}\, dh \\ &= C_d \times L \times \sqrt{2g}\, \left[\frac{h^{1/2+1}}{\frac{1}{2}+1}\right]_0^H \\ &= C_d \times L \times \sqrt{2g}\, \left[\frac{h^{3/2}}{\frac{1}{2}}\right]_0^H \\ &= \frac{2}{3} C_d L \sqrt{2g} [H]^{3/2} \end{aligned}$$

Discharge over a Triangular notch

Discharge over a triangular notch or weir is same. Let H= head of the water above the V-notch
θ =angle of notch
consider a horizontal strip of water of thickness 'dh' at a depth of 'h' from free surface as shown in fig. from the fig we have

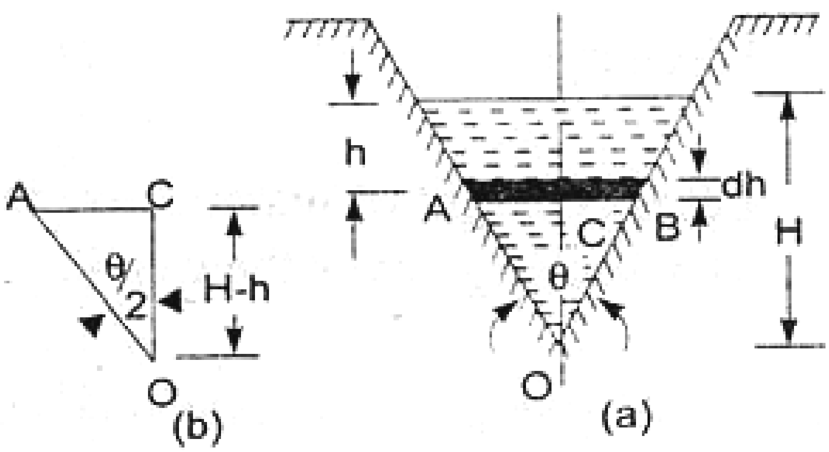
$$\tan\theta/2 = \frac{AC}{OC} = \frac{AC}{H-h}$$

$$\therefore \qquad AC = (H-h)\tan\theta/2$$

$$Width\ of\ strip = AB = 2AC = 2(H-h)\tan\theta/2$$

$$\therefore \qquad area\ of\ the\ strip = 2(H-h)\tan\frac{\theta}{2} \times dh$$

The theroritical velocity of water through strip = $\sqrt{2gh}$



Triangular Notch

$\therefore$ Discharge, dQ, through the strip is

$$
\begin{aligned}
dQ &= C_d \times area\ of strip \times Velocity(theoritical) \\
&= C_d \times 2(H-h)\tan\frac{\theta}{2} \times dh \times \sqrt{2gh}
\end{aligned}
$$

$\therefore$ Total discharge, Q is

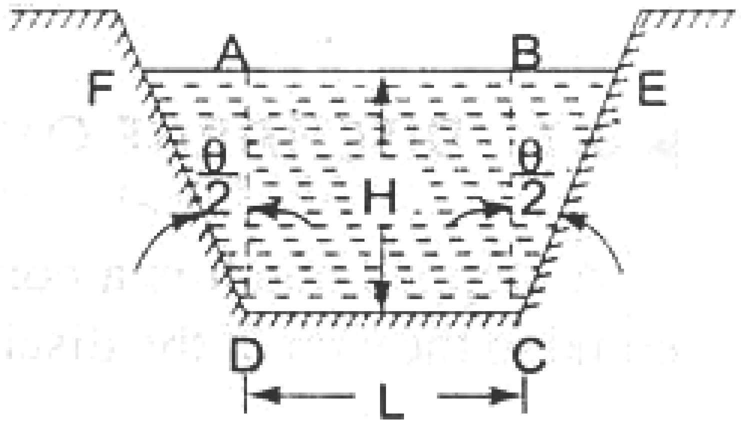
$$
\begin{aligned}
Q &= \int_0^H 2C_d(H-h)\tan\frac{\theta}{2} \times \sqrt{2gh} \times dh \\
&= 2C_d \tan\frac{\theta}{2} \times \sqrt{2g}\int_0^H (Hh^{1/2} - h^{3/2})dh \\
&= 2C_d \tan\frac{\theta}{2}\sqrt{2g}\Big[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\Big]_0^H \\
&= 2C_d \tan\frac{\theta}{2}\sqrt{2g}\Big[\frac{2}{3}H^{5/2} - \frac{2}{5}H^{5/2}\Big] \\
&= \frac{8}{15}C_d \tan\frac{\theta}{2}\sqrt{2g} \times H^{5/2}
\end{aligned}
$$

Advantages of triangular notch/weir or rectangular notch/weir

- Expression for discharge for a right angled V-notch or weir is very simple.
- In case of triangular notch, only one reading, (H) is required for computation of discharge.
- Ventilation of triangular notch is not necessary.
- For measuring low discharge, a triangular notch gives more accurate results than a rectangular notch.

Discharge over a Trapezoidal notch

Trapezoidal notch is combination of rectangular and triangular notch or weir. thus the total discharge will be equal to the sum of discharge through a rectangular notch and discharge through triangular notch as shown in fig below



Trapezoidal Notch

Let H = head of the water above the V-notch
L = Length of the crest of the notch
C_{d_1}= co-efficient of discharge for rectangular portion ABCD
C_{d_2}= co-efficient of discharge for triangular portion [FAD and BCE]
The discharge through rectangular portion ABCD is given by

$$Q_1 = \frac{2}{3} C_{d_1} \times L\sqrt{2g} \times H^{3/2}$$

The discharge through two triangular notches FCA and BCE is equal to discharge single triangular notch of angle θ and it is given by equation as

$$Q_2 = \frac{8}{15} C_{d_2} \tan\frac{\theta}{2}\sqrt{2g} \times H^{5/2}$$

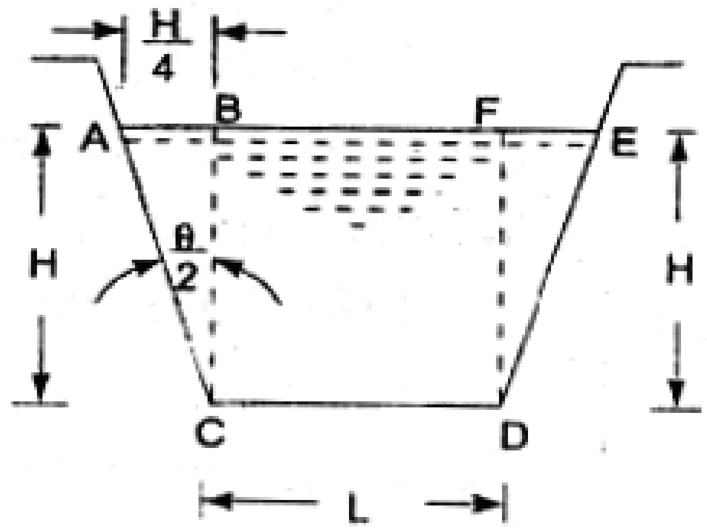
$\therefore$ Discharge through tarpezoidal notch,

$$Q = \frac{2}{3} C_{d_1} \times L\sqrt{2g} \times H^{3/2} + \frac{8}{15} C_{d_2} \tan\frac{\theta}{2}\sqrt{2g} \times H^{5/2}$$

Cipolletti Notch or Weir

Cipolletti weir is trpezoidal weir, which has sides slopes if 1 horizontal to 4 vertical as shown in figure. Thus from fig,

$$\tan\frac{\theta}{2} = \frac{AB}{BC} = \frac{H/2}{H} = \frac{1}{4}$$
$$\therefore \quad \frac{\theta}{2} = \tan^{-1}\frac{1}{4} = 14^o 2'$$



Cipolletti weir

By giving this slopes to the sides an increase in discharge through the triangular portions ABC and DEF of the weir is obtained. if this slope is not provided the weir would be a rectangular one, and due to end contraction, the discharge would decrease. thus in case of Cippolletti weir, the factor of end contraction is not required which is shown below. the discharge through a rectangular weir with two end contraction is

$$Q = \frac{2}{3} \times C_d \times (L - 0.2H) \times \sqrt{2g} \times H^{3/2}$$
$$= \frac{2}{3} \times C_d \times \sqrt{2g} \times H^{3/2} - \frac{2}{15} \times C_d \times \sqrt{2g} \times H^{5/2}$$

Thus due to end contraction, the discharge decreases by $\frac{2}{15} C_d \times \sqrt{2g} \times H^{5/2}$. This decrease discharge can be compensated by giving such slope to the sides that the discharge through two triangular portions is equal to $\frac{2}{15} C_d \times \sqrt{2g} \times H^{5/2}$. Let the slope is given by $\theta/2$. the discharge through a V-notch of angle θ is given by

$$= \frac{8}{15} \times C_d \times \sqrt{2g} \times \tan\frac{\theta}{2} H^{5/2}$$

Thus

$$\frac{8}{15} \times C_d \times \sqrt{2g} \times \tan\frac{\theta}{2} H^{5/2} = \frac{2}{15} \times C_d \times \sqrt{2g} \times \tan\frac{\theta}{2} H^{5/2}$$
$$\therefore \qquad \tan\frac{\theta}{2} = \frac{2}{15} \times \frac{15}{8} = \frac{1}{4}$$

or

$$\theta/2 = 14^o 2^{'}$$

Thus discharge through Cipoletti weir is

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

If the velocity of approach is considered,

$$Q = \frac{2}{3} \times C_d \times (L) \times \sqrt{2g}\left[(H+h_a)^{3/2} - h_a^{3/2}\right]$$

Velocity of Approach

It is defined as the velocity with which the flow approaches/reaches the notch/weir before it flows past it. The velocity of approach for any horizontal element across the notch depends only on its depth below the free surface.

In most of the cases such as flow over a notch/weir in the side of the reservoir, the velocity of approach may be neglected. But, for the notch/weir placed at the end of the narrow channel, the velocity of approach to the weir will be substantial and the head producing the flow will be increased by the kinetic energy of the approaching liquid.

Thus, if $v_a$is the velocity of approach, then the additional head ha due to velocity of approach, acts on the water flowing over the notch or weir. So, the initial and final height of water over the notch/weir will be $(H+h_a)$ and ha respectively. It may be determined by finding the discharge over the notch/weir neglecting the velocity of approach i.e.

$$v_a = \frac{Q}{A}$$

where 'Q' is the discharge over the notch/weir and 'A' is the cross-sectional area of channel on the upstream side of the weir/notch. Additional head corresponding to the velocity of approach will be,

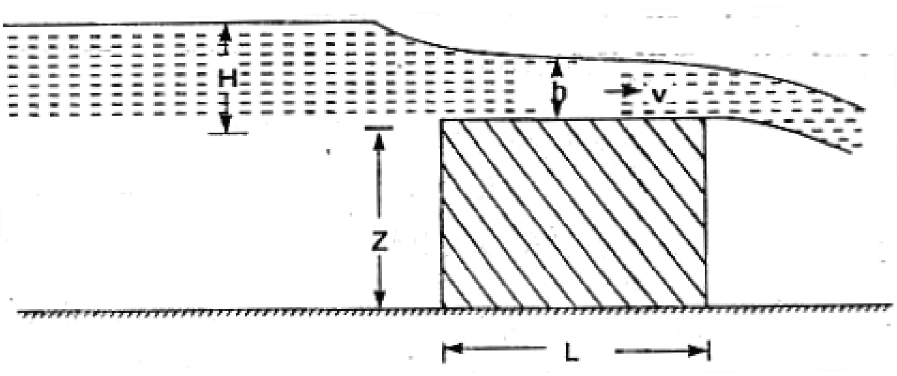
$$H_a = \frac{v_a^2}{2g}$$

Example:- The discharge over a rectangular notch/weir of width L,

$$Q = \frac{2}{3} \times C_d \times (L) \times \sqrt{2g}\left[(H+h_a)^{3/2} - h_a^{3/2}\right]$$

Broad crested weir

A weir having a wide crest is known as broad crested weir. Broad-crested weirs differ from thin-plate and narrow-crested weirs by the fact that different flow pattern is developed.



Broad crested weir

Condition for a weir to be broad or narrow.

Let H= height of water, above the crest,
L=length of crest.

- If 2L $>$ H, the weir is called broad crested weir.
- If 2L $<$ H, the weir is called narrow crested weir.

(Refer above fig.)
Let h= head of water at the middle o weir which is constant
v=velocity of flow over the weir applying bernoullies equation to the still water surface on U/S side and running water at the end of the weir,

$$0+0+H=0+\frac{v^2}{2g}+h$$
$$\frac{v^2}{2g}=H-h$$
$$v=\sqrt{2g(H-h)}$$

The discharge over weir

$$Q=C_d\times Area\ of\ flow\times velocity$$
$$=C_d\times L\times h\times\sqrt{2g(H-h)}$$
$$=C_d\times L\times\sqrt{2g(Hh^2-h^3)}$$

discharge is maximum if (Hh^2-h^3) is maximum or

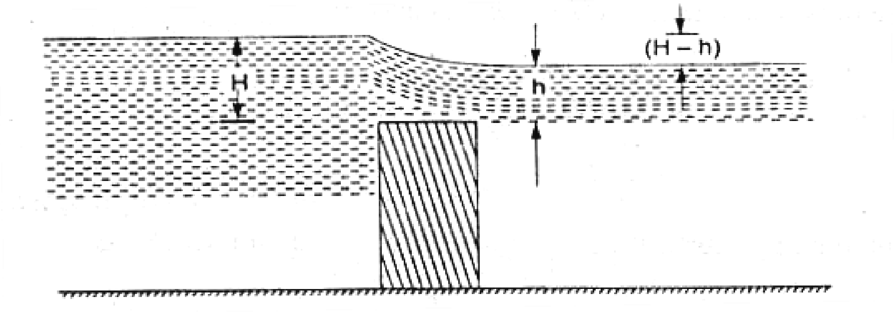
$$\frac{d}{dh}(Hh^2-h^3)=0\ or\ h=\frac{2}{3}H\therefore$$

Q$_{max}$ will be otained by substituting this valve of h in the above discharge equation

$$Q_max=C_d\times L\times\sqrt{2g\left[H\times\left(\frac{2}{3}H\right)^2-\left(\frac{2}{3}H\right)^3\right]}$$
$$=C_d\times L\times\sqrt{2g}\sqrt{\frac{4}{9}H^3-\frac{8}{27}H^3}$$
$$=C_d\times L\times\sqrt{2g}\times0.3849\times H^(3/2)$$
$$=1.705\times C_d\times L\times H^(3/2)$$

Submerged weir

When the water level on the down stream side of a weir is above the crest of the weir, then weir is said to be submerged weir. Below fig shows a submerged weir.



Submerged weir

The total discharge is obtained by dividing the weir into two parts. The portion between U/S and D/S water surface may be treated as free weir and the portion between D/S water surface and crest of weir as a drowned weir. Total disharge is given by

$$Q = \frac{2}{3}C_{d1} \times L \times \sqrt{2g}[H-h]^{3/2} + C_{d2} \times L \times h \times \sqrt{2g(H-h)}$$