Module-3

ECONOMIC OPEARATION OF POWER SYSTEM

5.1 INTRODUCTION

One of the earliest applications of on-line centralized control was to provide a central facility, to operate economically, several generating plants supplying the loads of the system. Modern integrated systems have different types of generating plants, such as coal fired thermal plants, hydel plants, nuclear plants, oil and natural gas units etc. The capital investment, operation and maintenance costs are different for different types of plants. The operation economics can again be subdivided into two parts.

i) Problem of *economic dispatch*, which deals with determining the power output of each plant to meet the specified load, such that the overall fuel cost is minimized.

ii) Problem of *optimal power flow*, which deals with minimum – loss delivery, where in the power flow, is optimized to minimize losses in the system. In this chapter we consider the problem of economic dispatch.

During operation of the plant, a generator may be in one of the following states: i) Base supply without regulation: the output is a constant.
ii) Base supply with regulation: output power is regulated based on system load. iii) Automatic non-economic regulation: output level changes around a base setting as area control error changes. iv) Automatic economic regulation: output level is adjusted, with the area load and area control error, while tracking an economic setting.

Regardless of the units operating state, it has a contribution to the economic operation, even though its output is changed for different reasons. The factors influencing the cost of generation are the generator efficiency, fuel cost and transmission losses. The most efficient generator may not give minimum cost, since it may be located in a place where fuel cost is high. Further, if the plant is located far from the load centers, transmission losses may be high and running the plant may become uneconomical. The economic dispatch problem basically determines the generation of different plants to minimize total operating cost.

Modern generating plants like nuclear plants, geo-thermal plants etc, may require capital investment of millions of rupees. The economic dispatch is however determined in terms of fuel cost per unit power generated and does not include capital investment, maintenance, depreciation, start-up and shut down costs etc.

5.2 PERFORMANCE CURVES

INPUT-OUTPUT CURVE

This is the fundamental curve for a thermal plant and is a plot of the input in British thermal units (Btu) per hour versus the power output of the plant in MW as shown in Fig1.

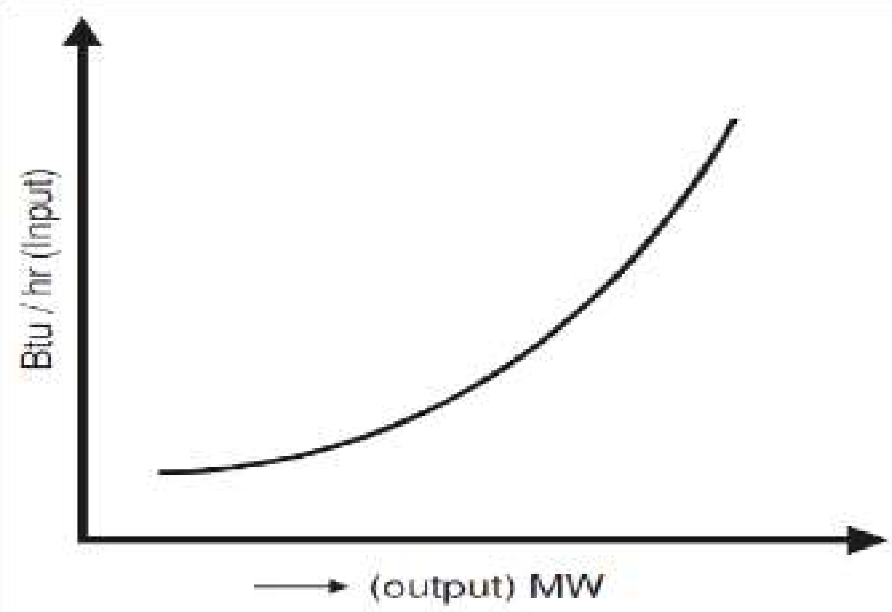


Fig 1: Input – output curve

HEAT RATE CURVE

The heat rate is the ratio of fuel input in Btu to energy output in KWh. It is the slope of the input – output curve at any point. The reciprocal of heat – rate is called fuel –efficiency. The heat rate curve is a plot of heat rate versus output in MW. A typical plot is shown in Fig .2

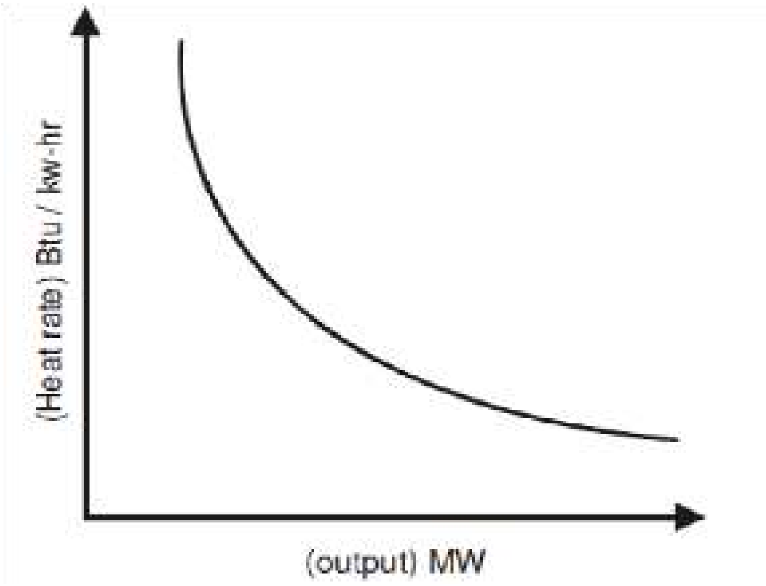


Fig .2 Heat rate curve.

INCREMENTAL FUEL RATE CURVE

The incremental fuel rate is equal to a small change in input divided by the corresponding change in output.

$$\text{Incremental fuel rate} = \frac{\Delta Input}{\Delta Output}$$

The unit is again Btu / KWh. A plot of incremental fuel rate versus the output is shown in Fig 3

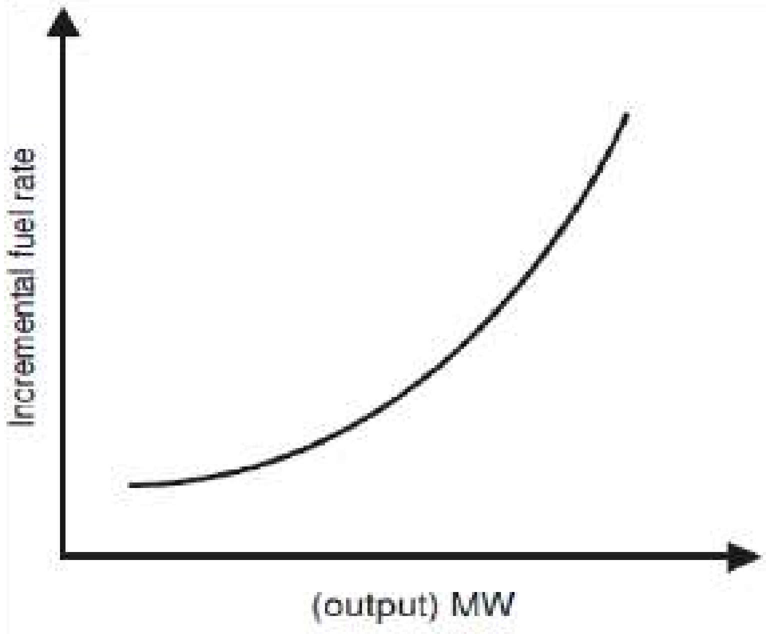


Fig 3: Incremental fuel rate curve

<u>Incremental cost curve</u>

The incremental cost is the product of incremental fuel rate and fuel cost (Rs / Btu or $ / Btu). The curve in shown in Fig. 4. The unit of the incremental fuel cost is Rs / MWh or $ /MWh.

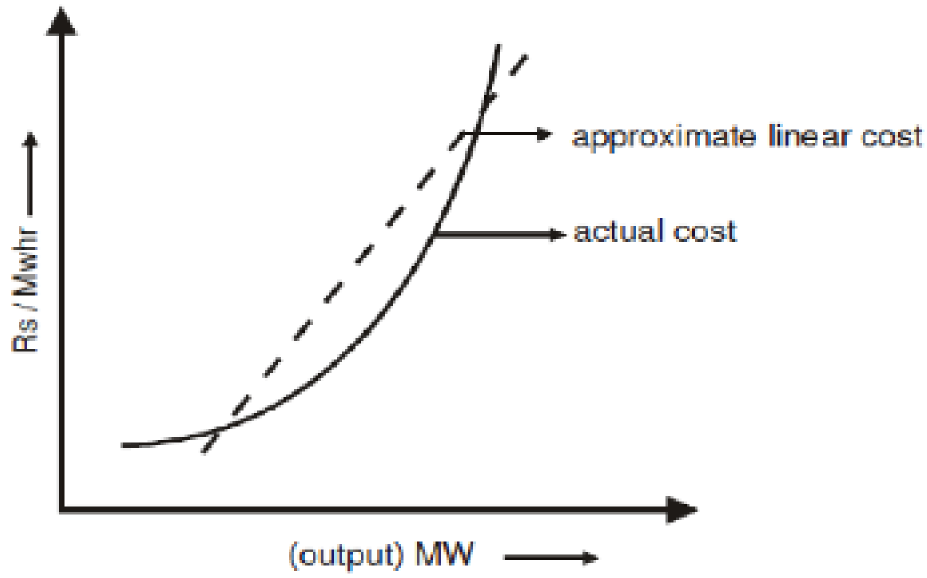


Fig 4: Incremental cost curve

In general, the fuel cost Fi for a plant, is approximated as a quadratic function of the generated output PGi.

$$F_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \quad Rs/h$$

The incremental fuel cost is given by

$$\frac{dF_i}{dP_{Gi}} = b_i + 2c_i P_{Gi} \quad Rs/MWh$$

The incremental fuel cost is a measure of how costly it will be produce an increment of power. The incremental production cost, is made up of incremental fuel cost plus the incremental cost of labour, water, maintenance etc. which can be taken to be some percentage of the incremental fuel cost, instead of resorting to a rigorous mathematical model. The cost curve can be approximated by a linear curve. While there is negligible operating cost for a hydel plant, there is a limitation on the power output possible. In any plant, all units normally operate between PGmin, the minimum loading limit, below which it is technically infeasible to operate a unit and PGmax, which is the maximum output limit.

5.3 ECONOMIC GENERATION SCHEDULING NEGLECTING LOSSES AND GENERATOR LIMITS

The simplest case of economic dispatch is the case when transmission losses are neglected. The model does not consider the system configuration or line impedances. Since losses are neglected, the total generation is equal to the total demand PD. Consider a system with **n**g number of generating plants supplying the total demand PD. If Fi is the cost of plant ***i*** in Rs/h, the mathematical formulation of the problem of economic scheduling can be stated as follows:

Minimize $F_T = \sum_{i=1}^{n_g} F_i$

Such that $\sum_{i=1}^{n_g} P_{Gi} = P_D$

where F_T = total cost.
P_{Gi} = generation of plant i.
P_D = total demand.

This is a constrained optimization problem, which can be solved by Lagrange‟ s method.

LAGRANGE METHOD FOR SOLUTION OF ECONOMIC SCHEDULE

The problem is restated below:

Minimize $F_T = \sum_{i=1}^{n_g} F_i$

Such that $P_D = \sum_{i=1}^{n_g} P_{Gi} = 0$

The augmented cost function is given by

$$£ = F_T + \lambda\left(P_D - \sum_{i=1}^{n_g} P_{Gi} \right)$$

The minimum is obtained when

$$\frac{\partial £}{\partial P_{Gi}} = 0 \text{ and } \frac{\partial £}{\partial \lambda} = 0$$

$$\frac{\partial £}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda = 0$$

$$\frac{\partial £}{\partial \lambda} = P_D - \sum_{i=1}^{n_g} P_{Gi} = 0$$

The second equation is simply the original constraint of the problem. The cost of a plant Fi depends only on its own output PGi, hence

$$\frac{\partial F_T}{\partial P_{Gi}} = \frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}}$$

Using the above,

$$\frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}} = \lambda \ ; \qquad i = 1 \ldots\ldots n_g$$

We can write

$$b_i + 2c_i P_{Gi} = \lambda \qquad i = 1 \ldots\ldots n_g$$

The above equation is called the co-ordination equation. Simply stated, for economic generation scheduling to meet a particular load demand, when transmission losses are neglected and generation limits are not imposed, all plants must operate at equal incremental production costs, subject to the constraint that the total generation be equal to the demand. From we have

$$P_{Gi} = \frac{\lambda - b_i}{2c_i}$$

We know in a loss less system

$$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

Substituting (8.16) we get

$$\sum_{i=1}^{n_g} \frac{\lambda - b_i}{2c_i} = P_D$$

An analytical solution of λ is obtained from (8.17) as

$$\lambda = \frac{P_D + \sum_{i=1}^{n_g} \frac{b_i}{2c_i}}{\sum_{i=1}^{n_g} \frac{1}{2c_i}}$$

It can be seen that l is dependent on the demand and the coefficients of the cost function.

Example 1.

The fuel costs of two units are given by

$F_1 = 1.5 + 20\ P_{G1} + 0.1\ P_{G1}^2$ Rs/h

$F_2 = 1.9 + 30\ P_{G2} + 0.1\ P_{G2}^2$ Rs/h

P_{G1}, P_{G2} are in MW. Find the optimal schedule neglecting losses, when the demand is 200 MW.

Solution:

$$\frac{dF_1}{dP_{G1}} = 20 + 0.2P_{G1} \quad \text{Rs / MWh}$$

$$\frac{dF_2}{dP_{G2}} = 30 + 0.2P_{G2} \quad \text{Rs / MWh}$$

$$P_D = P_{G1} + P_{G2} = 200\ \text{MW}$$

For economic schedule

$$\frac{dF_1}{dP_{G1}} = \frac{dF_2}{dP_{G2}} = \lambda$$

$$20 + 0.2\ P_{G1} = 30 + 0.2\ (200 - P_{G1})$$

Solving we get,

$$P_{G1} = 125\ \text{MW}$$

$$P_{G2} = 75\ \text{MW}$$

$$\lambda = 20 + 0.2\ (125) = 45\ \text{Rs / MWh}$$

Example 2

The fuel cost in \$ / h for two 800 MW plants is given by

$F_1 = 400 + 6.0\ P_{G1} + 0.004\ P_{G1}^2$

$F_2 = 500 + b_2 P_{G2} + c_2 P_{G2}^2$

where P_{G1}, P_{G2} are in MW

(a) The incremental cost of power, λ is \$8 / MWh when total demand is 550MW. Determine optimal generation schedule neglecting losses.

(b) The incremental cost of power is \$10/MWh when total demand is 1300 MW. Determine optimal schedule neglecting losses.

(c) From (a) and (b) find the coefficients b_2 and c_2.

Solution:

a) $$P_{G1} = \frac{\lambda - b_1}{2c_1} = \frac{8.0 - 6.0}{2 \times 0.004} = 250\ \text{MW}$$

$$P_{G2} = P_D - P_{G1} = 550 - 250 = 300\ \text{MW}$$

b) $P_{G1} = \frac{\lambda - b_1}{2C_1} = \frac{10-6}{2 \times 0.004} = 500 \text{ MW}$

$P_{G2} = P_D - P_{G1} = 1300 - 500 = 800 \text{ MW}$

c) $P_{G2} = \frac{\lambda - b_2}{2c_2}$

From (a) $300 = \frac{8.0 - b_2}{2c_2}$

From (b) $800 = \frac{10.0 - b_2}{2c_2}$

Solving we get $b_2 = 6.8$
$c_2 = 0.002$

5.4 ECONOMIC SCHEDULE INCLUDING LIMITS ON GENERATOR(NEGLECTING LOSSES)

The power output of any generator has a maximum value dependent on the rating of the generator. It also has a minimum limit set by stable boiler operation. The economic dispatch problem now is to schedule generation to minimize cost, subject to the equality constraint.

$$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

and the inequality constraint

$$P_{Gi\,(min)} \le P_{Gi} \le P_{Gi\,(max)}\ ;\ i = 1, \ldots\ldots\ldots n_g$$

The procedure followed is same as before i.e. the plants are operated with equal incremental fuel costs, till their limits are not violated. As soon as a plant reaches the limit (maximum or minimum) its output is fixed at that point and is maintained a constant. The other plants are operated at equal incremental costs.

Example 3

Incremental fuel costs in \$ / MWh for two units are given below:

$$\frac{dF_1}{dP_{G1}} = 0.01P_{G1} + 2.0 \text{ \$ / MWh}$$

$$\frac{dF_2}{dP_{G2}} = 0.012P_{G2} + 1.6 \text{ \$ / MWh}$$

The limits on the plants are P_{min} = 20 MW, P_{max} = 125 MW. Obtain the optimal schedule if the load varies from 50 – 250 MW.

Solution:

The incremental fuel costs of the two plants are evaluated at their lower limits and upper limits of generation.

At $P_{G(min)}$ = 20 MW.

$$\lambda_{1(min)} = \frac{dF_1}{dP_{G1}} = 0.01 \times 20 + 2.0 = 2.2 \text{ \$ / MWh}$$

$$\lambda_{2(min)} = \frac{dF_2}{dP_{G2}} = 0.012 \times 20 + 1.6 = 1.84 \text{ \$ / MWh}$$

At $P_{G(Max)}$ =125 Mw

$$\lambda_{1(max)} = 0.01 \times 125 + 2.0 = 3.25 \text{ \$ / MWh}$$

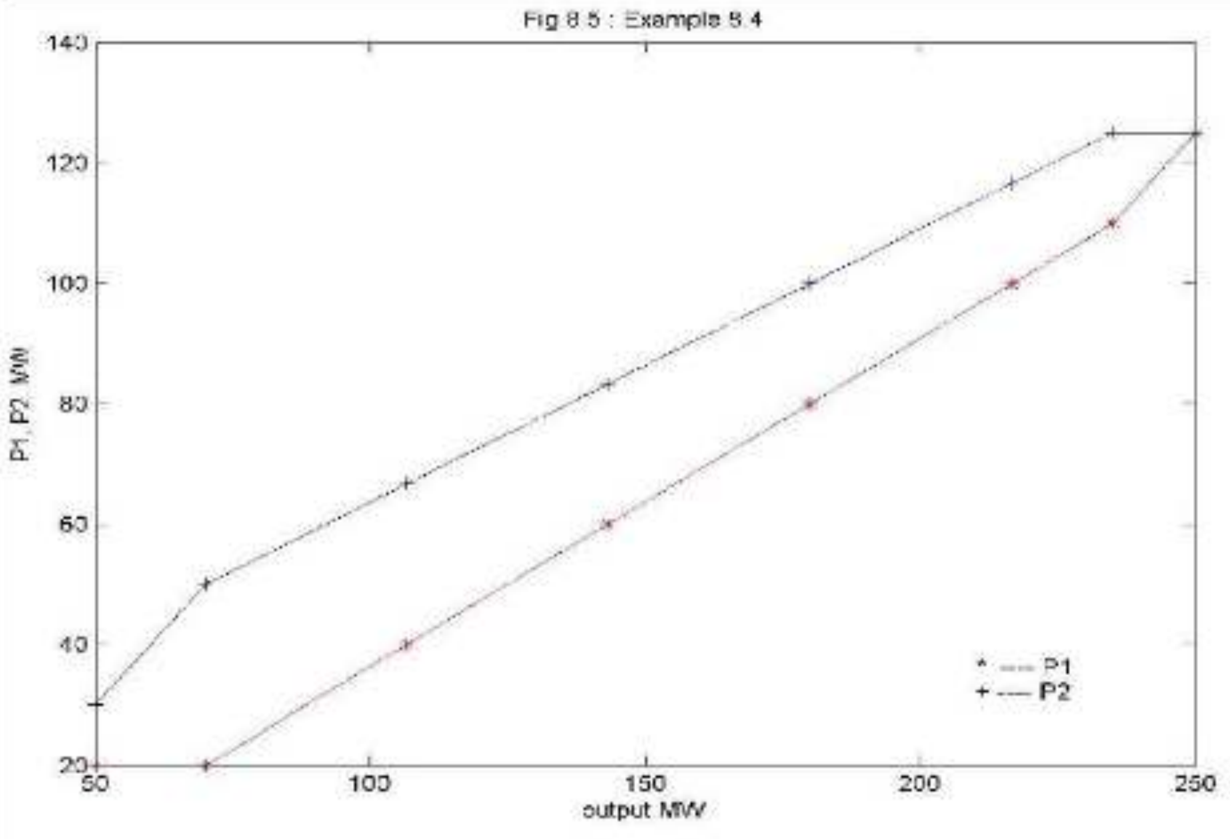
$$\lambda_{2(max)} = 0.012 \times 125 + 1.6 = 3.1 \text{ \$ / MWh}$$

Now at light loads unit 1 has a higher incremental cost and hence will operate at its lower limit of 20 MW. Initially, additional load is taken up by unit 2, till such time its incremental fuel cost becomes equal to 2.2\$ / MWh at PG2 = 50 MW. Beyond this, the two units are operated with equal incremental fuel costs. The contribution of each unit to meet the demand is obtained by assuming different values of l; When l = 3.1 \$ / MWh, unit 2 operates at its upper limit. Further loads are taken up by unit 1. The computations are show in Table

Table Plant output and output of the two units

$\frac{dF_1}{dP_{G1}}$ \$/MWh	$\frac{dF_2}{dP_{G2}}$ \$/MWh	Plant λ \$/MWh	P_{G1} MW	P_{G2} MW	Plant Output MW
2.2	1.96	1.96	20^+	30	50
2.2	2.2	2.2	20^+	50	70
2.4	2.4	2.4	40	66.7	106.7
2.6	2.6	2.6	60	83.3	143.3
2.8	2.8	2.8	80	100	180
3.0	3.0	3.0	100	116.7	216.7
3.1	3.1	3.1	110	125^*	235

For a particular value of l, PG1 and PG2 are calculated using (8.16). Fig 8.5 Shows plot of each unit output versus the total plant output.



For any particular load, the schedule for each unit for economic dispatch can be obtained.

Example 4.

In example 3, what is the saving in fuel cost for the economic schedule compared to the case where the load is shared equally. The load is 180 MW.

Solution:

From Table it is seen that for a load of 180 MW, the economic schedule is PG1 = 80 MW and PG2 = 100 MW. When load is shared equally PG1 = PG2 = 90 MW. Hence, the generation of unit 1 increases from 80 MW to 90 MW and that of unit 2 decreases from 100 MW to 90 MW, when the load is shared equally. There is an increase in cost of unit 1 since PG1 increases and decrease in cost of unit 2 since PG2 decreases.

Increase in cost of unit 1 $= \int_{80}^{90} \left(\frac{dF_1}{dP_{G1}} \right) dP_{G1}$

$$= \int_{80}^{90} (0.01P_{G1} + 2.0) dP_{G1} = 28.5 \text{ \$ / h}$$

Decrease in cost of unit 2 $= \int_{100}^{90} \left(\frac{dF_2}{dP_{G2}} \right) dP_{G2}$

$$= \int_{100}^{90} (0.012P_{G2} + 1.6) dP_{G2} = -27.4 \text{ \$ / h}$$

Total increase in cost if load is shared equally = 28.5 – 27.4 = 1.1 \$ / h

Hence the saving in fuel cost is 1.1 \$ / h if coordinated economic schedule is used.

5.5 ECONOMIC DISPATCH INCLUDING TRANSMISSION LOSSES

When transmission distances are large, the transmission losses are a significant part of the generation and have to be considered in the generation schedule for economic operation.
The mathematical formulation is now stated as

Minimize $F_T = \sum_{i=1}^{n_g} F_i$

Such That $\sum_{i=1}^{n_g} P_{Gi} = P_D + P_L$

where P_L is the total loss.

The Lagrange function is now written as

$$£ = F_T - \lambda\left(\sum_{i=1}^{n_g} P_{Gi} - P_D - P_L\right) = 0$$

The minimum point is obtained when

$$\frac{\partial £}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda\left(1 - \frac{\partial P_L}{\partial P_{Gi}}\right) = 0;\ i = 1\ldots\ldots n_g$$

$$\frac{\partial £}{\partial \lambda} = \sum_{i=1}^{n_g} P_{Gi} - P_D + P_L = 0 \qquad \text{(Same as the constraint)}$$

Since $\frac{\partial F_T}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}}$, (8.27) can be written as

$$\frac{dF_i}{dP_{Gi}} + \lambda\frac{\partial P_L}{\partial P_{Gi}} = \lambda$$

$$\lambda = \frac{dF_i}{dP_{Gi}}\left(\frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}}\right)$$

The term $\frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}}$ is called the penalty factor of plant i, L_i. The coordination equations including losses are given by

$$\lambda = \frac{dF_i}{dP_{Gi}} L_i;\ i = 1\ldots\ldots n_g$$

The minimum operation cost is obtained when the product of the incremental fuel cost and the penalty factor of all units is the same, when losses are considered. A rigorous general expression for the loss PL is given by

$$P_L = \Sigma_m \Sigma_n P_{Gm} B_{mn} P_{Gn} + \Sigma_n P_{Gn} B_{no} + B_{oo}$$

where Bmn, Bno , Boo called loss – coefficients , depend on the load composition. The assumption here is that the load varies linearly between maximum and minimum values. A simpler expression is

$$P_L = \Sigma_m \Sigma_n P_{Gm} B_{mn} P_{Gn}$$

The expression assumes that all load currents vary together as a constant complex fraction of the total load current. Experiences with large systems has shown that the loss of accuracy is not significant if this approximation is used. An average set of loss coefficients may be used over the complete daily cycle in the coordination of incremental production costs and incremental transmission losses. In general, Bmn = Bnm and can be expanded for a two plant system as

$$P_L = B_{11} P_{G1} + 2 B_{12} P_{G1} P_{G2} + B_{22} P_{G2}^2$$

Example 5

A generator is supplying a load. An incremental change in load of 4 MW requires generation to be increased by 6 MW. The incremental cost at the plant bus is Rs 30 /MWh. What is the incremental cost at the receiving end?

Solution:

$$\frac{dF_1}{dP_{G1}} = 30$$

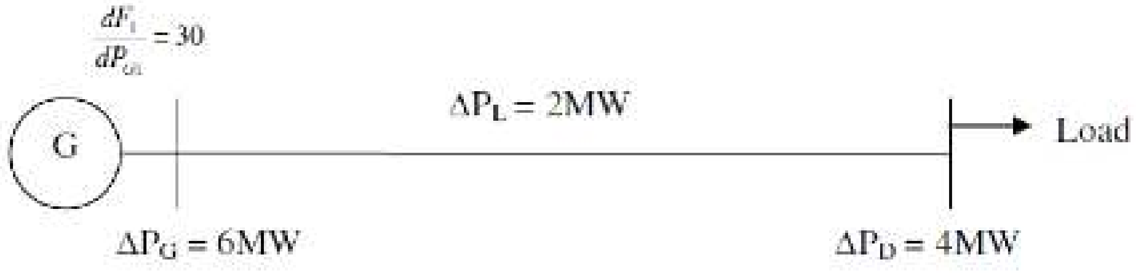


Fig ; One line diagram of example 5

$\Delta P_L = \Delta P_G - \Delta P_D = 2$ MW

λ at receiving end is given by

$$\lambda = \frac{dF_1}{dP_{G1}} \times \frac{\Delta P_G}{\Delta P_D} = 30 \times \frac{6}{4} = 45 \text{ Rs / MWh}$$

$$\text{or } \lambda = \frac{dF_1}{dP_{G1}} \times \frac{1}{1 - \frac{\Delta P_L}{\Delta P_G}} = 30 \times \frac{1}{1 - \frac{2}{6}} = 45 \text{ Rs / MWh}$$

Example 6

In a system with two plants, the incremental fuel costs are given by

$$\frac{dF_1}{dP_{G1}} = 0.01P_{G1} + 20 \text{ Rs / MWh}$$

$$\frac{dF_2}{dP_{G2}} = 0.015P_{G2} + 22.5 \text{ Rs / MWh}$$

The system is running under optimal schedule with $P_{G1} = P_{G2} = 100$ MW.

If $\frac{\partial P_L}{\partial P_{G2}} = 0.2$, find the plant penalty factors and $\frac{\partial P_L}{\partial P_{G1}}$.

Solution:

For economic schedule,

$$\frac{dF_i}{dP_{Gi}} L_i = \lambda ; \qquad L_i = \frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}} ;$$

For plant 2, $P_{G2} = 100$ MW

$$\therefore (0.015 \times 100 + 22.5)\frac{1}{1 - 0.2} = \lambda.$$

Solving, $\lambda = 30 \text{ Rs / MWh}$

$$L_2 = \frac{1}{1 - 0.2} = 1.25$$

$$\frac{dF_1}{dP_{G1}} L_1 = \lambda \Rightarrow (0.01 \times 100 + 20)\ L_1 = 30$$

$$L_1 = 1.428$$

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G1}}}$$

$$1.428 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G1}}} \text{ ; Solving } \frac{\partial P_L}{\partial P_{G1}} = 0.3$$

Example 7

A two bus system is shown in Fig. 8.8 If 100 MW is transmitted from plant 1 to the load, a loss of 10 MW is incurred. System incremental cost is Rs 30 / MWh. Find P_{G1}, P_{G2} and power received by load if

$$\frac{dF_1}{dP_{G1}} = 0.02P_{G1} + 16.0 \text{ Rs / MWh}$$

$$\frac{dF_2}{dP_{G2}} = 0.04P_{G2} + 20.0 \text{ Rs / MWh}$$

5.6 DERIVATION OF TRANSMISSION LOSS FORMULA

An accurate method of obtaining general loss coefficients has been presented by Kron. The method is elaborate and a simpler approach is possible by making the following assumptions:

(i) All load currents have same phase angle with respect to a common reference (ii)
The ratio X / R is the same for all the network branches.

Consider the simple case of two generating plants connected to an arbitrary number of loads through a transmission network as shown in Fig a

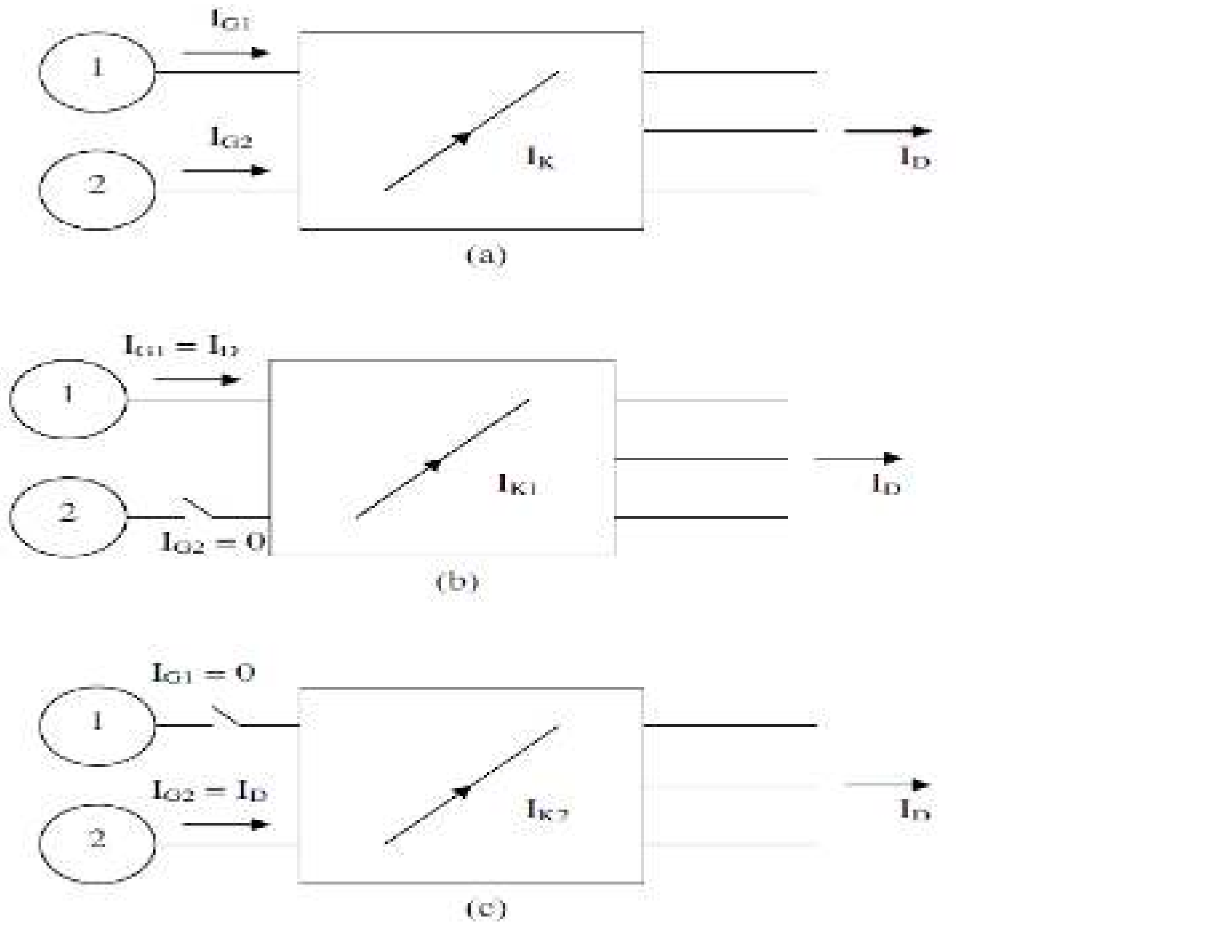


Fig Two plants connected to a number of loads through a transmission network

Let's assume that the total load is supplied by only generator 1 as shown in Fig 8.9b. Let the current through a branch K in the network be I_{K1}. We define

$$N_{K1} = \frac{I_{K1}}{I_D}$$

It is to be noted that $I_{G1} = I_D$ in this case. Similarly with only plant 2 supplying the load current I_D, as shown in Fig 8.9c, we define

$$N_{K2} = \frac{I_{K2}}{I_D}$$

N_{K1} and N_{K2} are called current distribution factors and their values depend on the impedances of the lines and the network connection. They are independent of I_D. When both generators are supplying the load, then by principle of superposition

$$I_K = N_{K1} I_{G1} + N_{K2} I_{G2}$$

where I_{G1}, I_{G2} are the currents supplied by plants 1 and 2 respectively, to meet the demand I_D. Because of the assumptions made, I_{K1} and I_D have same phase angle, as do I_{K2} and I_D. Therefore, the current distribution factors are real rather than complex. Let

$$I_{G1} = I_{G1} \angle \sigma_1 \text{ and } I_{G2} = |I_{G2}| \angle \sigma_2 .$$

where σ_1 and σ_2 are phase angles of I_{G1} and I_{G2} with respect to a common reference. We can write

$$|I_K|^2 = \left(N_{K1}|I_{G1}|\cos\sigma_1 + N_{K2}|I_{G2}|\cos\sigma_2\right)^2 + \left(N_{K1}|I_{G1}|\sin\sigma_1 + N_{K2}|I_{G2}|\sin\sigma_2\right)^2$$

$$= \begin{array}{l} N_{K1}^2 I_{G1}^2\left[\cos^2\sigma_1 + \sin^2\sigma_1\right] + N_{K2}^2|I_{G2}|^2\left[\cos^2\sigma_2 + \sin^2\sigma_2\right] \\ + 2\left[N_{K1}|I_{G1}|\cos\sigma_1 N_{K2}|I_{G2}|\cos\sigma_2 + N_{K1}|I_{G1}|\sin\sigma_1 N_{K2}|I_{G2}|\sin\sigma_2\right] \end{array}$$

$$= N_{K1}^2|I_{G1}|^2 + N_{K2}^2|I_{G2}|^2 + 2N_{K1}N_{K2}|I_{G1}||I_{G2}|\cos(\sigma_1 - \sigma_2)$$

Now $|I_{G1}| = \dfrac{P_{G1}}{\sqrt{3}|V_1|\cos\phi_1}$ and $|I_{G2}| = \dfrac{P_{G2}}{\sqrt{3}|V_2|\cos\phi_2}$

where P_{G1}, P_{G2} are three phase real power outputs of plant1 and plant 2; V_1, V_2 are the line to line bus voltages of the plants and ϕ_1, ϕ_2 are the power factor angles.

The total transmission loss in the system is given by

$$P_L = \sum_K 3|I_K|^2 R_K$$

where the summation is taken over all branches of the network and R_K is the branch resistance. Substituting we get

$$P_L = \frac{P_{G1}^2}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}^2 R_K + \frac{2P_{G1}P_{G2}\cos(\sigma_1 - \sigma_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2}\sum_K N_{K1}N_{K2}R_K + \frac{P_{G2}^2}{|V_2|^2(\cos\phi_2)^2}\sum_K N_{K2}^2 R_K$$

$$P_L = P_{G1}^2 B_{11} + 2P_{G1}P_{G2}B_{12} + P_{G2}^2 B_{22}$$

where $$B_{11} = \frac{1}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}^2 R_K$$

$$B_{12} = \frac{\cos(\sigma_1 - \sigma_2)}{|V_1||V_2|\cos\phi_1 \cos\phi_2} \sum_K N_{K1} N_{K2} R_K$$

$$B_{22} = \frac{1}{|V_2|^2 (\cos\phi_2)^2} \sum_K N_{K2}^2 R_K$$

The loss – coefficients are called the B – coefficients and have unit MW^{-1}.

For a general system with n plants the transmission loss is expressed as

$$P_L = \frac{P_{G1}^2}{|V_1|^2 (\cos\phi_1)^2} \sum_K N_{K1}^2 + \ldots\ldots + \frac{P_{Gn}^2}{|V_n|^2 (\cos\phi_n)^2} \sum_K N_{Kn}^2 R_K$$
$$+ 2 \sum_{\substack{p,q=1 \\ p \neq q}}^{n} \frac{P_{GP} P_{Gq} \cos(\sigma_p - \sigma_q)}{|V_p||V_q| \cos\phi_p \cos\phi_q} \sum_K N_{KP} N_{Kq} R_K$$

In a compact form

$$P_L = \sum_{p=1}^{n} \sum_{q=1}^{n} P_{Gp} B_{Pq} P_{Gq}$$

$$B_{Pq} = \frac{\cos(\sigma_p - \sigma_q)}{|V_p||V_q| \cos\phi_p \cos\phi_q} \sum_K N_{KP} N_{Kq} R_K$$

B – Coefficients can be treated as constants over the load cycle by computing them at average operating conditions, without significant loss of accuracy.

Example 8

Calculate the loss coefficients in pu and MW^{-1} on a base of 50MVA for the network of Fig below. Corresponding data is given below.

$I_a = 1.2 - j\,0.4$ pu	$Z_a = 0.02 + j\,0.08$ pu
$I_b = 0.4 - j\,0.2$ pu	$Z_b = 0.08 + j\,0.32$ pu
$I_c = 0.8 - j\,0.1$ pu	$Z_c = 0.02 + j\,0.08$ pu
$I_d = 0.8 - j\,0.2$ pu	$Z_d = 0.03 + j\,0.12$ pu
$I_e = 1.2 - j\,0.3$ pu	$Z_e = 0.03 + j\,0.12$ pu

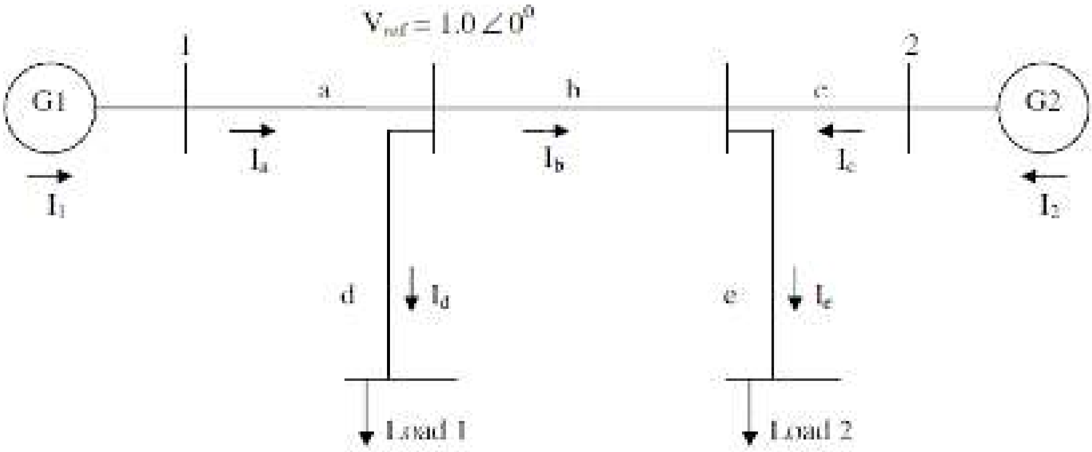


Fig : Example 8

Solution:

Total load current

$I_L = I_d + I_e = 2.0 - j\,0.5 = 2.061\ \angle -14.03^0 A$

$I_{L1} = I_d = 0.8 - j\,0.2 = 0.8246\ \angle -14.03^0\ A$

$\frac{I_{L1}}{I_L} = 0.4; \qquad \frac{I_{L2}}{I_L} = 1.0 - 0.4 = 0.6$

If generator 1, supplies the load then $I_1 = I_L$. The current distribution is shown in Fig a.

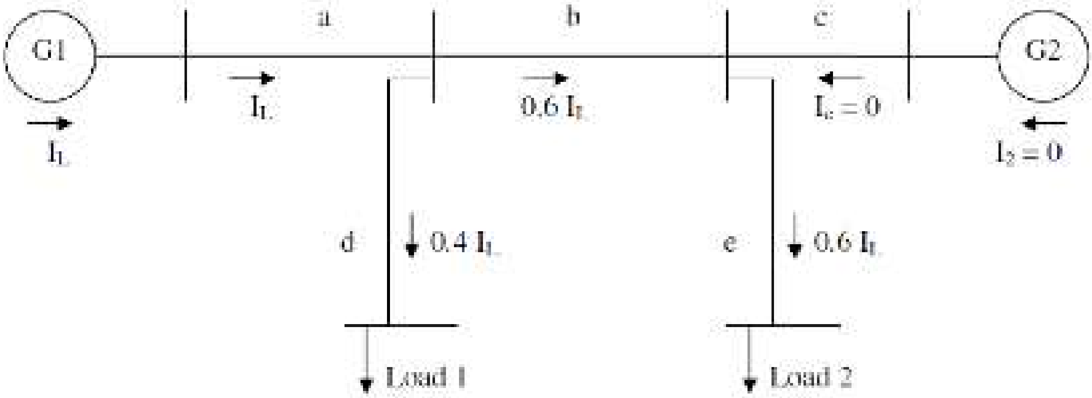


Fig a : Generator 1 supplying the total load

$N_{a1} = \frac{I_a}{I_L} = 1.0; \ N_{b1} = \frac{I_b}{I_L} = 0.6; \ N_{C1} = 0; \ N_{d1} = 0.4; \ N_{e1} = 0.6.$

Similarly the current distribution when only generator 2 supplies the load is shown in Fig b.

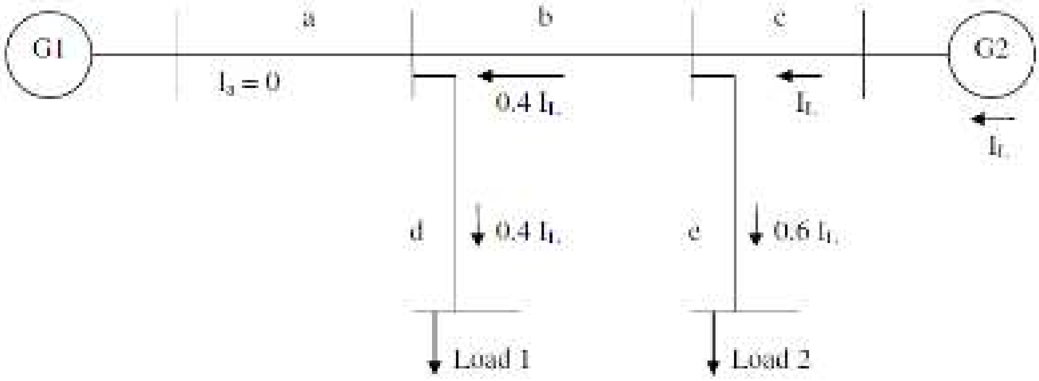


Fig b: Generator 2 supplying the total load

$N_{a2} = 0$; $N_{b2} = -0.4$; $N_{c2} = 1.0$; $N_{d2} = 0.4$; $N_{e2} = 0.6$

From Fig 8.10, $V_1 = V_{ref} + Z_a I_a$

$$= 1\angle 0^0 + (1.2 - j\,0.4)\,(0.02 + j0.08)$$

$$= 1.06\angle 4.78^0 = 1.056 + j\,0.088 \text{ pu.}$$

$$V_2 = V_{ref} - I_b Z_b + I_c Z_c$$

$$= 1.0\ \angle 0^0 - (0.4 - j\,0.2)\,(0.08 + j\,0.32) + (0.8 - j\,0.1)\,(0.02 + j\,0.08)$$

$$= 0.928 - j\,0.05 = 0.93\ \angle -3.10^0 \text{ pu.}$$

Current Phase angles

$$\sigma_1 = \text{angle of } I_1(=I_a) = \tan^{-1}\left(\frac{-0.4}{1.2}\right) = -18.43^0$$

$$\sigma_2 = \text{angle of } I_2(=I_c) = \tan^{-1}\left(\frac{-0.1}{0.8}\right) = -7.13^0$$

$$\cos(\sigma_1 - \sigma_2) = 0.98$$

Power factor angles

$$\phi_1 = 4.78^0 + 18.43 = 23.21^0; \cos\phi_1 = 0.92$$

$$\phi_2 = 7.13^0 - 3.10^0 = 4.03^0; \cos\phi_2 = 0.998$$

$$B_{11} = \frac{\sum_K N_{K1}^{\ 2} R_K}{|V_1|^2 (\cos\phi_1)^2} = \frac{1.0^2 \times 0.02 + 0.6^2 \times 0.08 + 0.4^2 \times 0.03 + 0.6^2 \times 0.03}{(1.06)^2\,(0.920)^2}$$

$= 0.0677$ pu

$= 0.0677 \text{ x } \frac{1}{50} = 0.1354 \times 10^{-2}\ \text{MW}^{-1}$

$$B_{12} = \frac{Cos(\sigma_1 - \sigma_2)}{|V_1||V_2|(\cos\phi_1)(\cos\phi_2)} \sum_K N_{K1} N_{K2} R_K$$

$$= \frac{0.98}{(1.06)(0.93)(0.998)(0.92)}\left[-0.4\times0.6\times0.08+0.4\times0.4\times0.03+0.6\times0.6\times0.03\right]$$

$= -\ 0.00389$ pu

$= -\ 0.0078 \times 10^{-2}\ MW^{-1}$

$$B_{22} = \frac{\sum_{K} N_{K2}^{\ 2} R_K}{|V_2|^2 (\cos\phi_2^{\ 2})}$$

$$= \frac{(-0.4)^2 0.08 + 1.0^2 \times 0.02 + 0.4^2 \times 0.03 + 0.6^2 \times 0.03}{(0.93)^2 (0.998)^2}$$

$= 0.056\text{pu} = 0.112\times10^{-2}\ MW^{-1}$