

# MODULE-2

## Moment Distribution Method

- (\*) Procedure: 1) FEM  
 2) Distribution factor (DF)  
 3) Moment Distribution Table  
 4) Diagrams (SFD, BMD & EC)

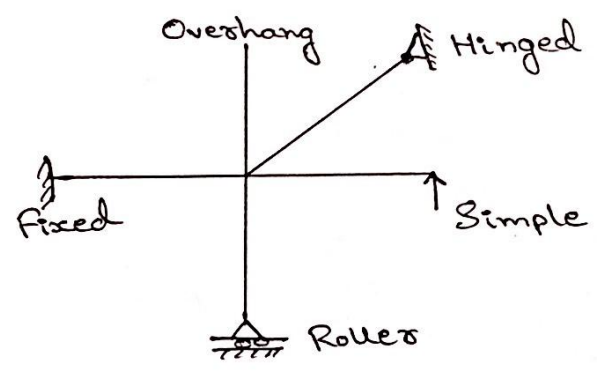
1) FEM:  
 Refer Unit-2 - Slope Deflection method.

2) Distribution factor (DF):  
 (For intermediate support joints)

⇒

Joint	Member	Relative Stiffness (k)	$\Sigma k$	$DF = \frac{k}{\Sigma k}$

⇒ Relative Stiffness (k)



- a) For Fixed (or) Continuous support →  $(k = \frac{I}{l})$
- b) For Simple, Roller (or) Hinge →  $(k = \frac{3}{4} \times \frac{I}{l})$
- c) For Overhang →  $(k = 0)$

### 3) Moment Distribution Table:

- ⇒ (a) If the far end is fixed (or) Continuous  
Every 50% of moment with same sign.
- (b) If the far end is "not Continuous", then  
there is no transfer of moments.

### ⇒ M.D. Table:

Joint	A	B	C	D		
Member	AB	BA	BC	CB	CD	DC
DF						
FEM						
Bal C.O						
Bal C.O						
Bal C.O						
Bal C.O						
Final Moments						

### 4) Diagrams (SFD, BMD & EC):

Refer Unit-2 — S.D. Notes

### (\*) Sinking and Rotation of Support

#### ① FEM:

(a) Additional Moment due to rotation

$$\text{@ near end} = \frac{4EI\theta}{l}$$

$$\text{@ Far end} = \frac{2EI\theta}{l}$$

(b) Additional moment due to sinking.

$$\text{" } -\frac{6EIS}{l^2} \text{"}$$

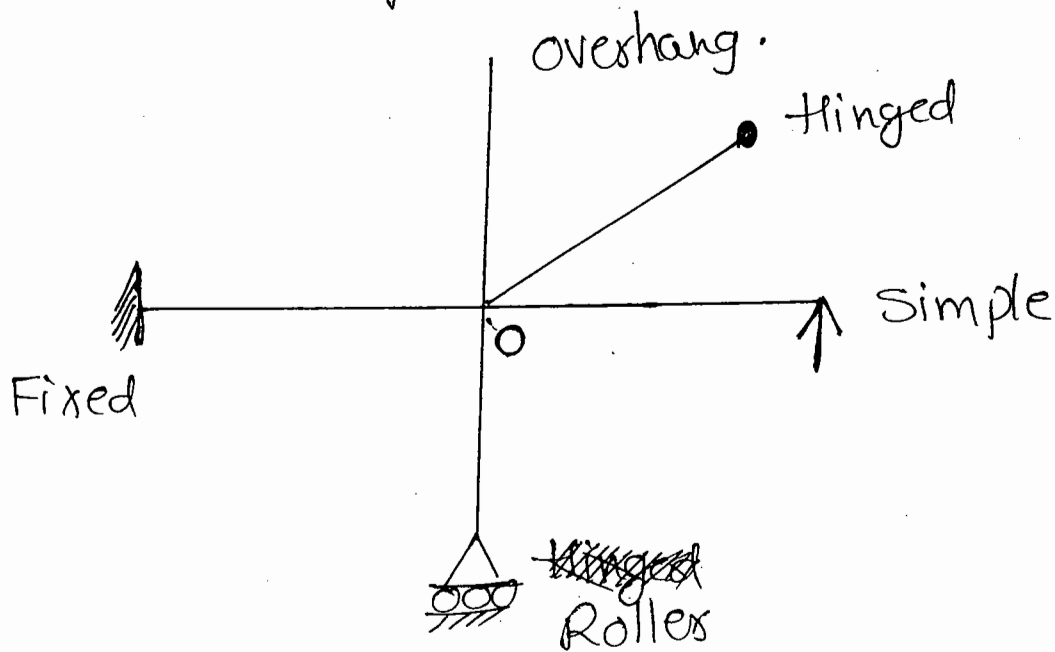
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# (III) Moment Distribution Method

(35)

Relative stiffness =  $k = \frac{I}{l}$

"The ratio of  $M, I$  to the span of beam is called relative stiffness."



(a) For "Fixed end" or "Continuous" support

$$k = \frac{I}{l}$$

(b) For "Simple" or "Hinge" or "Roller" support

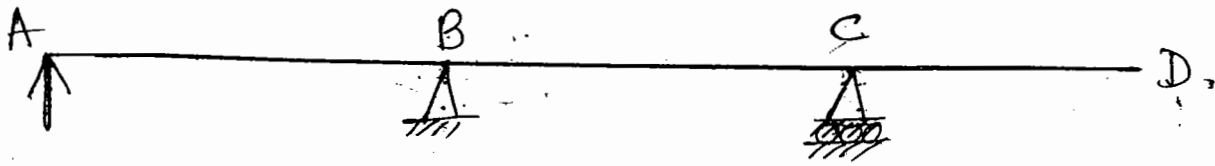
$$k = \frac{3}{4} \frac{I}{l}$$

(c) For "Overhang"

$$k = 0$$

## Continuous support:

(36)



(i) w.r.t to "B"  $\rightarrow$  A is Not continuous  $k = \frac{3}{4} \frac{I}{l}$

C is Not continuous  $k = \frac{3}{4} \frac{I}{l}$

(ii) w.r.t to "C"  $\rightarrow$  B is Continuous  $k = \frac{I}{l}$

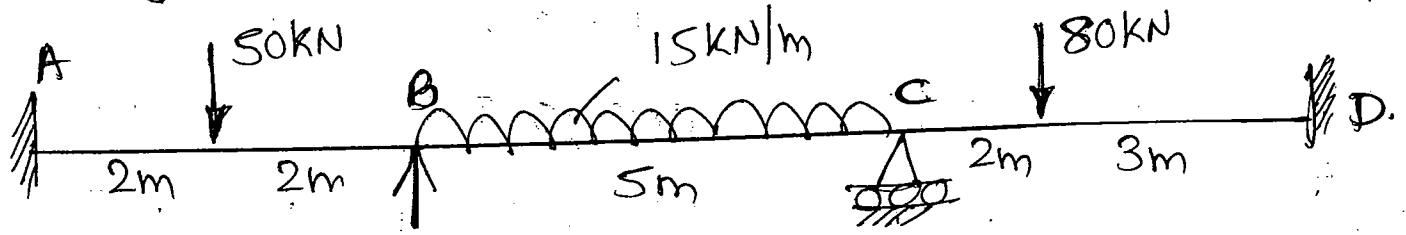
D is Overhang  $k = 0$

## Carry over of Moments:-

(i) If the far end is "fixed" or "Continuous"  
take or carry 50% of moment with  
same sign,

(ii) If far end is Not continuous, then  
there is no Transfer of moment.

Eg:-1] Analyse the continuous beam shown (3+)  
 by MID method. Draw SFD, BMD & EC.



Sol<sup>n</sup>

(a) FEM

$$M_{FAB} = -\frac{Wl}{8} = -25 \text{ kN-m}, \quad M_{FBA} = +25 \text{ kN-m}$$

$$M_{FBC} = -\frac{wl^2}{12} = -31.25, \quad M_{FCB} = +\frac{wl^2}{12} = 31.25$$

$$M_{FCD} = -\frac{Wab^2}{l^2} = -57.6, \quad M_{FC\cancel{D}} = +\frac{Wab^2}{l^2} = 38.4$$

(b) Distribution Factor (For Intermediate support)

Joint	Member	Relative stiffness = K	Sum $\Sigma K$	$DF = \frac{K}{\Sigma K}$
B	BA	$\left(\frac{I}{l}\right) = \frac{I}{4} = 0.25I$	$0.45I$	$\frac{0.25}{0.45} = 0.56$
	BC	$\left(\frac{I}{l}\right) = \frac{I}{5} = 0.20I$		$\frac{0.2}{0.45} = 0.44$
C	CB	$\left(\frac{I}{l}\right) = \frac{I}{5} = 0.20I$	$0.4I$	0.5
	CD	$\left(\frac{I}{l}\right) = \frac{I}{5} = 0.2I$		0.5

(c) Moment Distribution Table

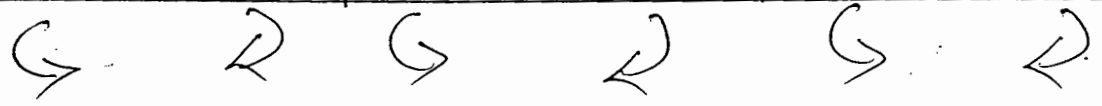
A (Fixed)

B ✓

C ✓

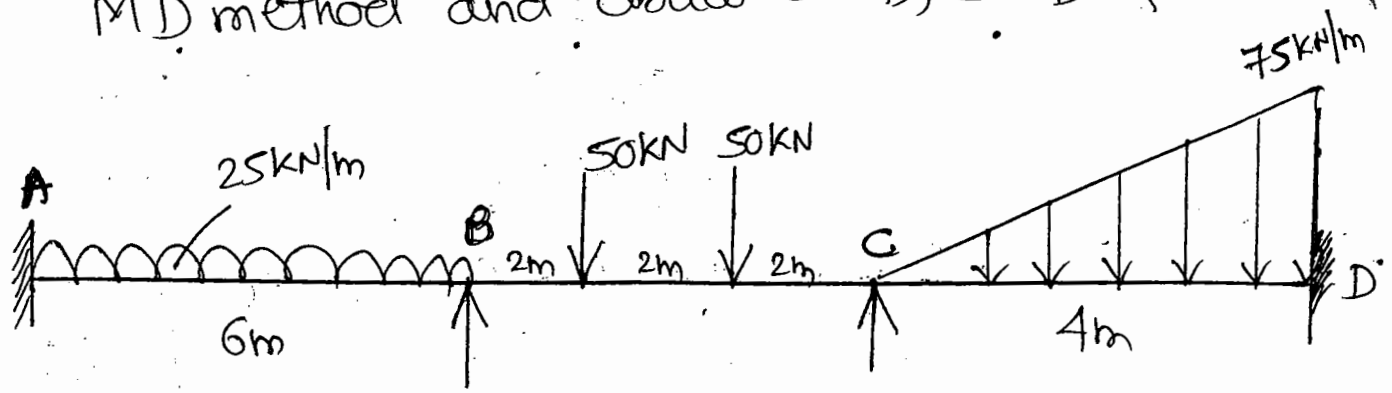
D (Fixed)

AB	BA	BC	CB	CD	DC	Member
-	0.56	0.44	0.5	0.5	-	DF
-25	25	-31.25	31.25	-57.6	38.4	FEM
1.75 ← 3.5	2.75	13.18	13.18	6.59 →	-	Balance
1.75	6.59	1.37	1.37	6.59	-	Carry over
-1.84 ← -3.69	-2.90	-0.68	-0.68	-0.34 →	-	Bal
-1.84	-0.34	-1.45	-1.45	-0.34	-	C.O
0.09 ← 0.19	0.15	0.73	0.73	0.36 →	-	Bal
0.09	0.36	0.075	0.075	0.36	-	C.O
-0.10 ← -0.20	-0.16	-0.037	-0.037	-0.018 →	-	Bal
-0.10	-0.018	-0.08	-0.08	-0.018	-	C.O
0.01	0.008	0.04	0.04	-	-	Bal
25.10	<del>24.81</del> 24.81	-24.81	44.40	-44.40	44.992	Final Moments



Draw SFD, BMD and EC.

Eg:- 2] Analyse the beam shown by MD method and draw BMD, SFD & EC. (30)



50/2

(a) FEM

$$M_{FAB} = -\frac{wL^2}{12} = -75, \quad M_{FBA} = +75$$

$$M_{FBC} = -\frac{Wab^2}{12} = -\left[ \frac{50 \times 2 \times 4^2}{6^2} + \frac{50 \times 4 \times 2^2}{6^2} \right] = -66.67$$

$$M_{FCB} = +\frac{Wab^2}{12} = +66.67$$

$$M_{FCD} = -\frac{wL^2}{30} = \frac{-75 \times 4^2}{30} = -40$$

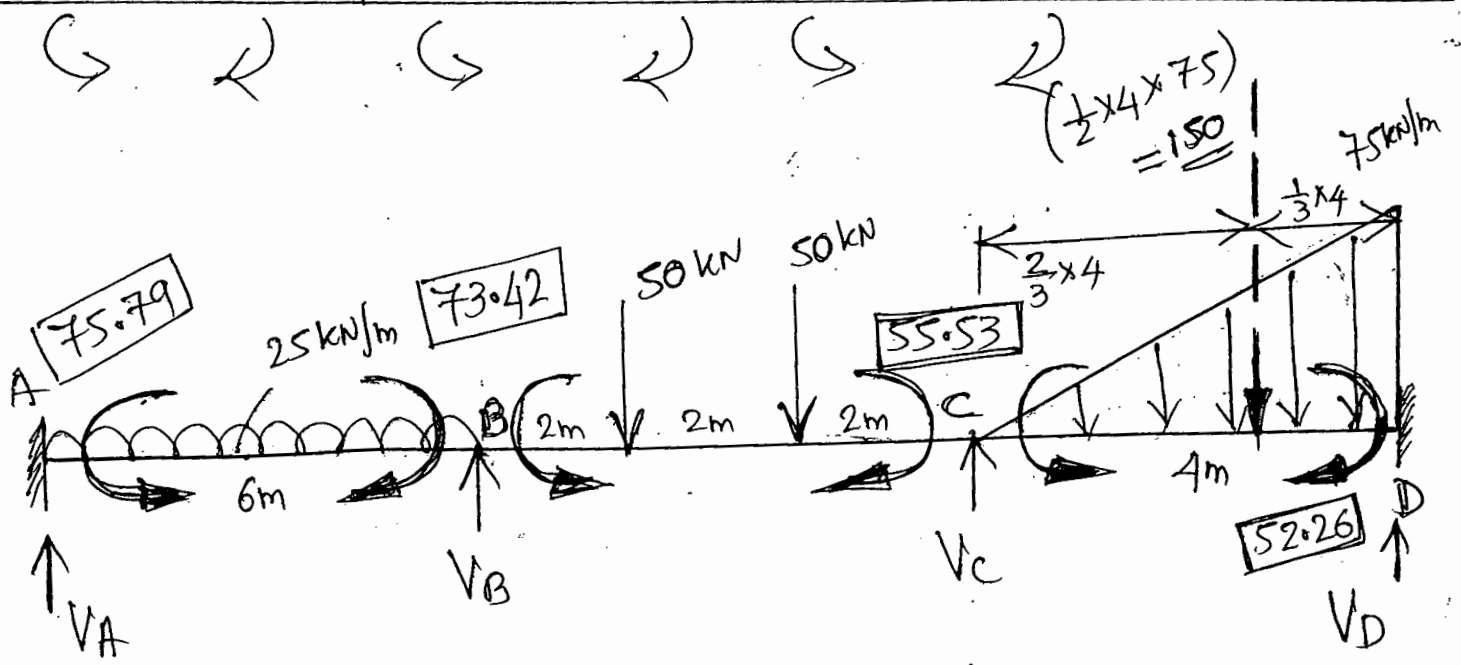
$$M_{FDC} = \frac{+wL^2}{20} = +60$$

(b) D.F: (For Intermediate Support)

	Members	Relative stiffness = k	$\Sigma k$	$DF = \frac{k}{\Sigma k}$
B	BA	$I/l = I/6 = 0.167I$	0.333I	0.5
	BC	$I/l = I/6 = 0.167I$		0.5
C	CB	$I/l = I/6 = 0.167I$	0.417I	0.4
	CD	$I/l = I/4 = 0.25I$		0.6

(c) M.D. Table

A (Fixed)	B	C	D (Fixed)	Member		
AB	BA	BC	CB	CD	DC	Member
	0.5	0.5	0.40	0.60		DF
-75	75	-66.67	+66.67	-40	60	FEM
-2.09	-4.17	-4.17	-10.67	-16.0	-8	Balance C.O
1.34	2.67	2.67	0.84	1.25	0.63	Bal C.O
-0.11	-0.21	-0.21	-0.54	-0.80	-0.14	Bal C.O
0.07	0.14	0.14	+0.105	+0.106	0.03	Bal C.O
	-0.101	-0.101	-0.103	-0.104		Bal
-75.79	73.42	-73.42	55.53	-55.53	52.26	Final Values





# Reactions

$$\sum V = 0, V_A + V_B + V_C + V_D = 25 \times 6 + 2 \times 50 + 150 = 400 \text{ --- (i)}$$

$$\sum M_B = 0 \text{ (LHS)}$$

$$V_A \times 6 - 25 \times 6 \times 6/2 - 75.79 + 73.42 = 0 \quad \boxed{V_A = 75.4}$$

$$\sum M_C = 0 \text{ (RHS)}$$

$$-V_D \times 4 + (150 \times \frac{2}{3} \times 4) - 55.53 + 52.26 = 0$$

$$\boxed{V_D = 99.18}$$

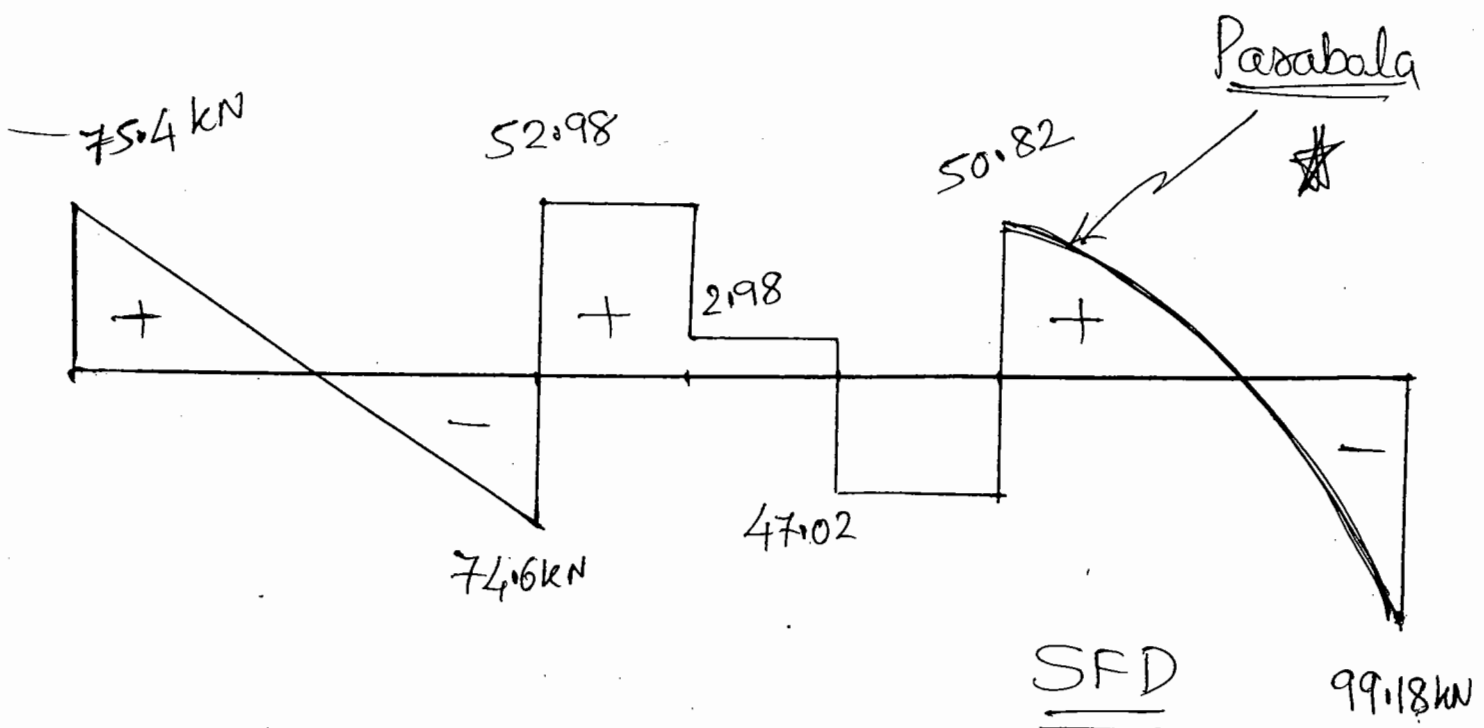
$$\sum M_C = 0 \text{ (LHS)}$$

$$75.4 \times 12 + V_B \times 6 - 25 \times 6 \times 9 - 50 \times 2 - 50 \times 4$$

$$- 75.79 + 73.42 - 73.42 + 55.53 = 0$$

$$\boxed{V_B = 127.58}$$

$$\text{From (i)} \quad \boxed{V_C = 97.84}$$

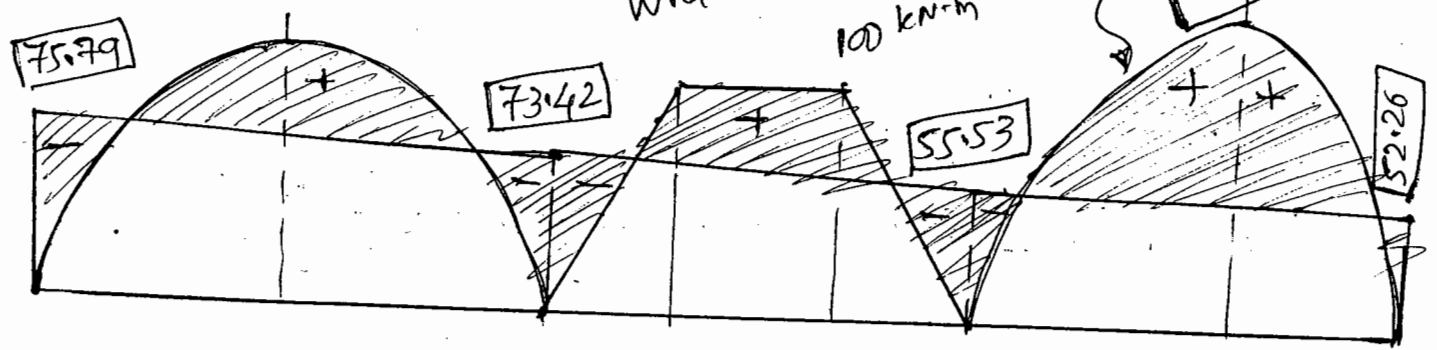


$$\frac{wL^2}{8} = 112.5$$

$$W \cdot a = 100 \text{ kN}\cdot\text{m}$$

$$100 \text{ kN}\cdot\text{m}$$

Cubic Parabola



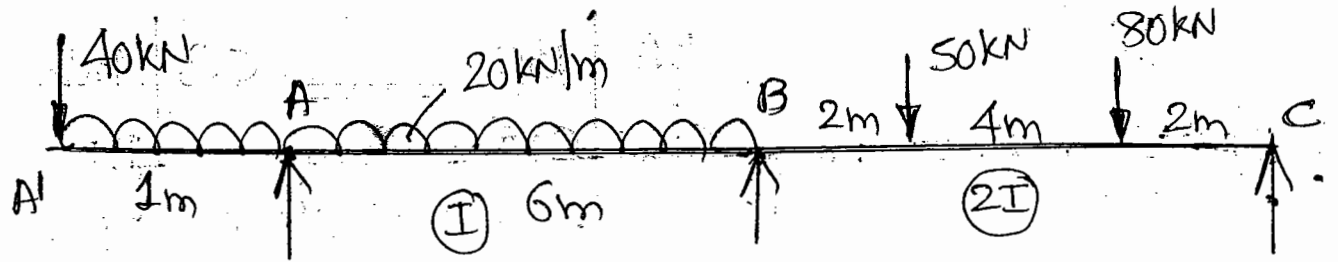
$$M = 0.06415 w L^2 \star$$

$$= 0.06415 \times 75 \times (4)^2 = 77 \text{ kN}\cdot\text{m}$$

$$= 0.577 \times 4 = 2.308 \text{ m}$$

— x =

Eg:- 3] Analyse the beam shown by M.D. method. Draw SFD, BMD & EC.

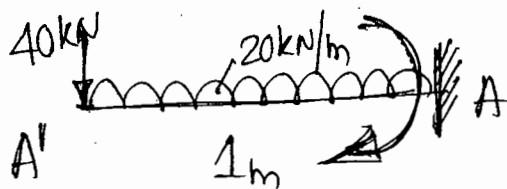


(a) FEM

$$M_{FAB} = -\frac{wL^2}{12} = -60 \text{ kN-m}, \quad M_{FBA} = +60 \text{ kN-m}$$

$$M_{FBC} = -\frac{wab^2}{J^2} = -\left[ \frac{50 \times 2 \times 6^2}{8^2} + \frac{80 \times 6 \times 2^2}{8^2} \right] = -86.25 \text{ kN-m}$$

$$M_{FCB} = +\frac{Wa^2b}{J^2} = +\left[ \frac{50 \times 2^2 \times 6}{8^2} + \frac{80 \times 6^2 \times 2}{8^2} \right] = +108.75 \text{ kN-m}$$



$$M_{AA'} = +40 \times 1 + 20 \times 1 \times \frac{1}{2} = +50 \text{ kN-m}$$

(b) D.F. (For Intermediate)

	Member	K	$\Sigma K$	$DF = \frac{K}{\Sigma K}$
A	AA'	0 ( $\because$ Overhang)		0
	AB	$\left(\frac{I}{L}\right) = \frac{I}{6} = 0.167 I$	0.167 I	1
B	BA	$\left(\frac{3}{4}\left(\frac{I}{L}\right)\right) = \frac{3}{4}\left(\frac{I}{6}\right) = 0.125 I$		0.40
	BC	$\left(\frac{3}{4}\left(\frac{I}{L}\right)\right) = \frac{3}{4}\left(\frac{2I}{8}\right) = 0.1875 I$	0.3125 I	0.60

M.D. Table

★ Simple Hinge  
C Roller

4

A ✓

B ✓

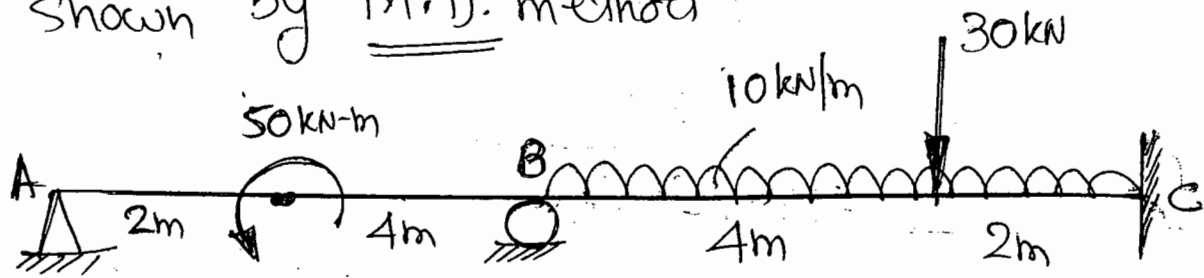
AA'	AB	BA	BC	CB	Members
0	1	0.4	0.6		DF
50	-60	60	-86.25	108.75	FEM
			-54.37	-108.75	Release C.O
50	-60	60	-140.62	0	Initial Values
0	10	32.25	48.37	0	Bal
	0	5		0	C.O
		-2	-3	0	Bal
	0			0	C.O
50	-50	95.25	-95.25	0	Final Values



Refer S.D. Notes for SFD, BMD.

Eg:- 4] Analyse the continuous beam

shown by M.D. method.



Sol<sup>n</sup>

(a) FEM

$$M_{FAB} = -\frac{M_b(2a-b)}{l^2} = \frac{-50 \times 4(2 \times 2 - 4)}{6^2} = 0$$

$$M_{FBA} = -\frac{M_a(2b-a)}{l^2} = \frac{-50 \times 2(2 \times 4 - 2)}{6^2} = -16.67 \text{ kN-m}$$

$$M_{FBC} = -\frac{wl^2}{12} - \frac{wab^2}{l^2} = -43.33 \text{ kN-m}$$

$$M_{FCB} = +\frac{wl^2}{12} + \frac{wa^2b}{l^2} = 56.67 \text{ kN-m}$$

(b) D.F. (For Intermediate support)




		k	$\Sigma k$	$DF = \frac{k}{\Sigma k}$
B	BA	$\frac{3(I)}{4(l)} = \frac{3(I)}{4(6)} = 0.125 I$	0.292 I	0.43
	BC	$\left(\frac{I}{l}\right) = \frac{I}{6} = 0.167 I$		0.57

M.D. Table

A (Hinge)

✓  
B

C (Fixed)

AB	BA	BC	CB	Members
-	0.43	0.57		DF
0	-16.67	-43.33	56.67	FEM
0 	0			Release (A) C.O.
0	-16.67	-43.33	56.67	Initial Values
0 	25.80	34.20 	17.10	Bal C.O.
0	9.13	-9.13	73.77	Final Values



Draw BMD, SFD.

# Sinking and Rotation of Support (47)

## Additional Moment due to Rotation

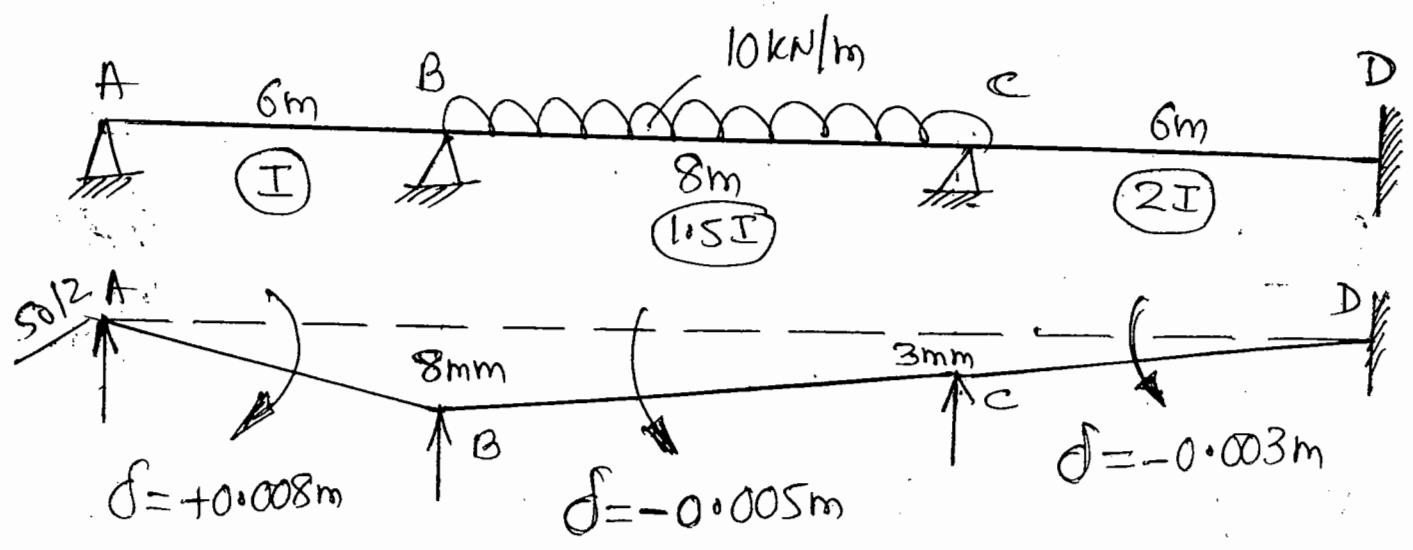
$$(i) \text{ Additional Moment } \left. \begin{array}{l} \text{at } \underline{\text{Near end}} \end{array} \right\} = \frac{4EI\theta}{l}$$

$$(ii) \text{ Additional moment } \left. \begin{array}{l} \text{at } \underline{\text{Far end}} \end{array} \right\} = \frac{2EI\theta}{l}$$

$$(iii) \text{ Additional moment } \left. \begin{array}{l} \text{due to } \underline{\text{Sinking}} \end{array} \right\} = \frac{-6EI\delta}{l^2}$$

★ ∴ The above additional moments are added to F.E.M ★

Eg:- 5] Analyse the continuous beam shown by M.D. method and draw SFD, BMD. Support B and C settle by 8mm and 3mm respt.  $EI = 2 \times 10^4 \text{ kN/m}^2$



(a) FEM

$$M_{FAB} = 0 - \frac{6EI\delta}{l^2} = 0 - \frac{6(1 \times 2 \times 10^4)(0.008)}{6^2} = -26.67$$

$$M_{FBA} = 0 - \frac{6EI\delta}{l^2} = 0 - \frac{6(1 \times 2 \times 10^4)(0.008)}{6^2} = -26.67$$

$$M_{FBC} = -\frac{wl^2}{12} - \frac{6EI\delta}{l^2} = \frac{-10 \times 8^2}{12} - \frac{6(1.5 \times 2 \times 10^4)(-0.005)}{8^2} = -39.27 \text{ kN-m}$$

$$M_{FCB} = +\frac{wl^2}{12} - \frac{6EI\delta}{l^2} = \frac{10 \times 8^2}{12} - \frac{6(1.5 \times 2 \times 10^4)(-0.005)}{8^2} = +67.40 \text{ kN-m}$$



$$M_{FCD} = 0 - \frac{6EI\delta}{l^2} = 0 - \frac{6(2 \times 10^7)(-0.003)}{6^2} = +20 \text{ kN}\cdot\text{m}$$

$$M_{FDC} = 0 - \frac{6EI\delta}{l^2} = +20 \text{ kN}\cdot\text{m}$$

(b) D.F

		k	$\Sigma k$	$DF = \frac{k}{\Sigma k}$
B	BA	$\frac{3}{4}(\frac{I}{l}) = \frac{3}{4} \times \frac{I}{6} = 0.125I$	0.3125I	0.4
	BC	$\frac{I}{l} = \frac{1.5I}{8} = 0.1875I$		0.6
C	CB	$I/l = \frac{1.5I}{8} = 0.1875I$	0.5200I	0.36
	CD	$I/l = \frac{2I}{6} = 0.333I$		0.64

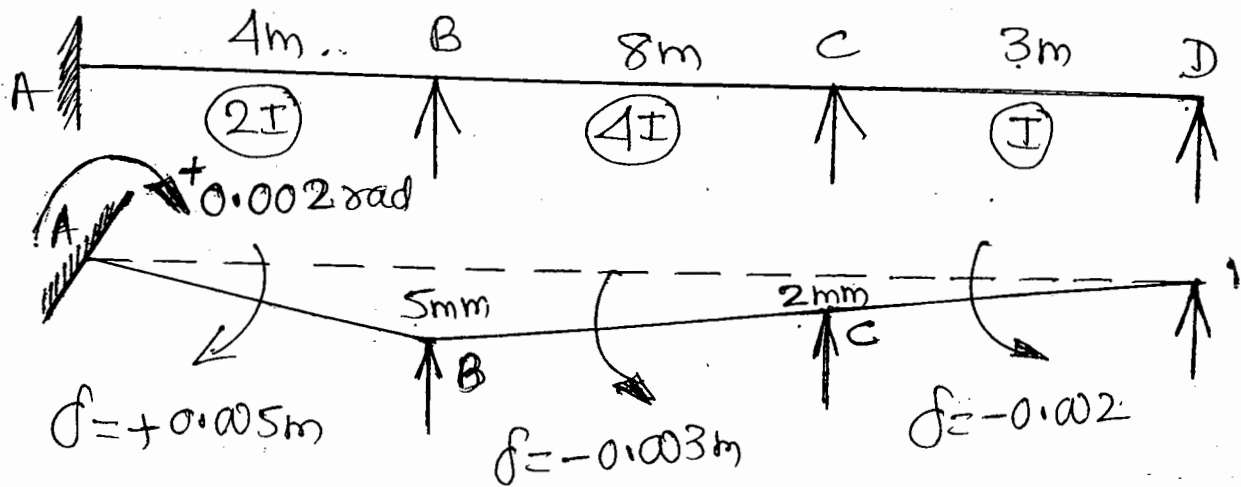
(c) M.D Table

AB	BA	BC	CB	CD	DC	Member
	0.4	0.6	0.36	0.64		DF
-26.67	-26.67	-39.27	67.40	20	20	FEM
+26.67	→ 13.33					Release (A) C.O
0	-13.33	-39.27	67.40	20	20	Initial
0	← 21.04	31.56 -15.88	↔ 15.76	-31.46 -55.94	→ -27.97	Bal C.O
0	← 6.35	9.53 -2.83	↔ 4.76	-5.67 -10.08	→ -5.04	Bal C.O
0	← 1.13	1.70 -0.85	↔ 0.85	-1.71 -3.04	→ -1.52	Bal C.O
0	← 0.34	0.51 -0.15	↔ 0.25	-0.30 -0.54	→ -0.27	Bal C.O
0	+0.06	0.09	-0.09	-0.16		Bal
	15.59	-15.59	49.79	-49.79	-14.80	Final

Eg:- 6] fig shows a continuous beam ABCD. (50)

Analyse the beam by M.D method. If the End "A" rotates by  $0.002$  radians in the clockwise order & support 'B' sinks by  $5\text{mm}$  & 'C' by  $2\text{mm}$ . Take

$$EI = 18000 \text{ kN-m}^2$$



(a) FEM

$$M_{FAB} = 0 + \frac{4EI\theta}{l} - \frac{6EI\delta}{l^2}$$

$$= 0 + \frac{4(2 \times 18000)(0.002)}{4} - \frac{6(2 \times 18000)(0.005)}{4^2} = \underline{\underline{4.5}}$$

FEM

$$M_{FBA} = 0 + \frac{2EI\theta}{l} - \frac{6EI\delta}{l^2}$$

$$= 0 + \frac{2(2 \times 18000)(0.002)}{4} - \frac{6(2 \times 18000)(0.005)}{4^2} = \underline{\underline{-31.5}}$$

$$M_{FBC} = 0 - \frac{6EI\delta}{l^2} = 0 - \frac{6(4 \times 18000)(-0.003)}{8^2} = \underline{\underline{20.25}}$$

$$M_{FCB} = 0 - \frac{6EI\delta}{l^2} = \underline{\underline{20.25}}$$

$$M_{FCD} = 0 - \frac{6EI\theta}{L^2} = - \frac{6(1 \times 18000)(-0.002)}{3^2} \quad (51)$$
$$= \underline{\underline{24 \text{ kN}\cdot\text{m}}}$$

$$M_{FDC} = 0 - \frac{6EI\theta}{L^2} = \underline{\underline{24 \text{ kN}\cdot\text{m}}}$$

(b)

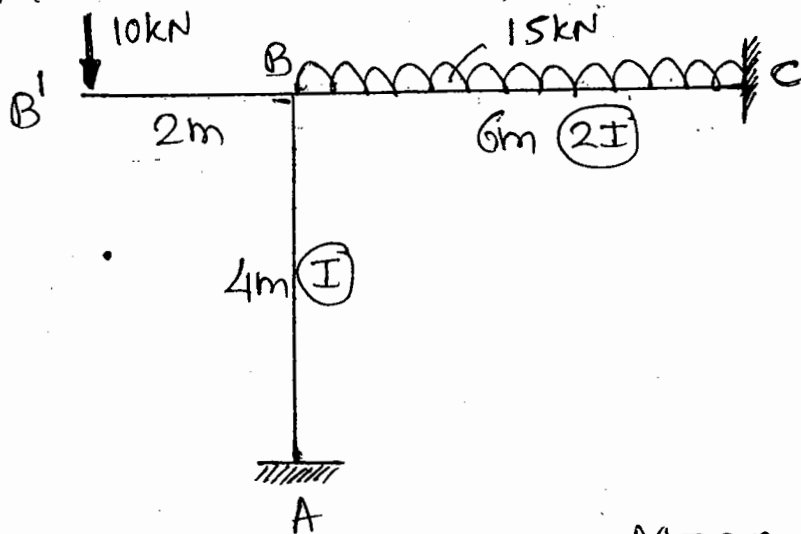


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Non-Sway Frames :-

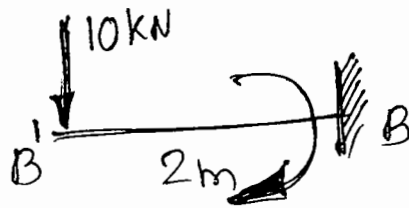
(53)

Eg:- 1] Analyse the rigid frame by M.D. method. Draw SFD, BMD & EC.



(a) FEM:  $M_{FAB} = M_{FBA} = 0$

$M_{FBC} = -\frac{wl^2}{12} = -45$ ,  $M_{FCB} = +45 \text{ kN-m}$



$M_{BB'} = +10 \times 2 = +20 \text{ kN-m}$   
(clockwise resisting moment)

(b) D.F (For Intermediate)

		K	$\Sigma K$	$DF = \frac{K}{\Sigma K}$
	BA	$I/l = I/4 = 0.25I$		0.43
B	BC	$I/l = \frac{2I}{6} = 0.33I$	0.58I	0.57
	BB'	0		0

(C) M.D. Table

BB'	AB	BA	BC	CB	Members
0		0.43	0.57		DF
20	0	0	-45	45	FEM
0		10.75	14.25		Bal
—	5.37	—	—	7.13	C:0
20	5.37	10.75	-30.75	52.13	Final Values.

$\curvearrowright$     $\curvearrowright$     $\curvearrowleft$     $\curvearrowright$     $\curvearrowleft$

At B"  $M_{BA} + M_{BC} + M_{BB'} = 0$

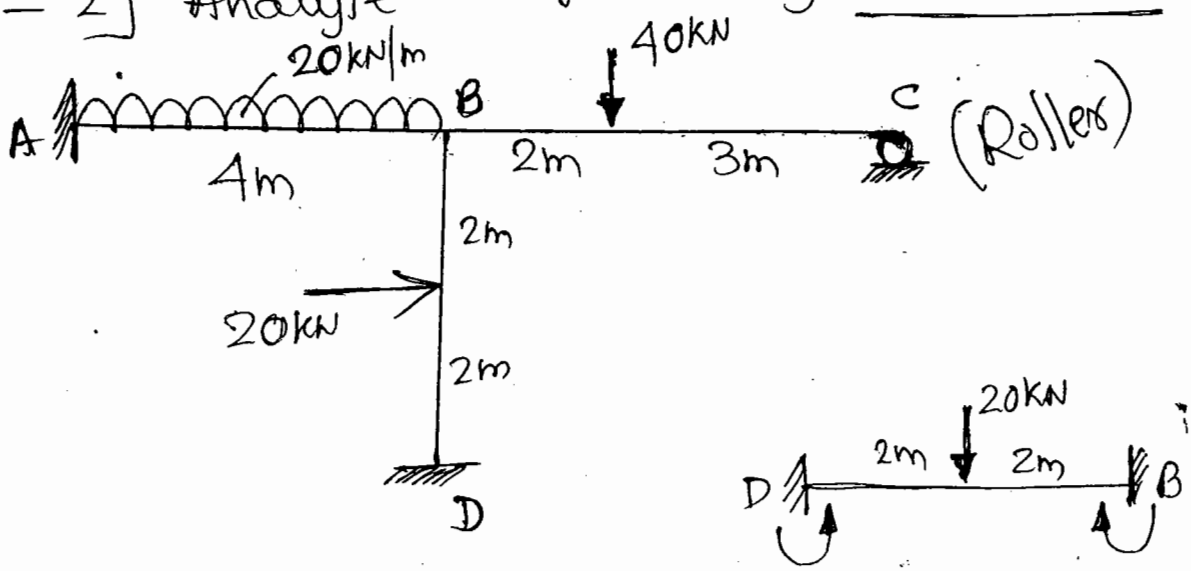
$0 - 45 + 20 = -25$

Refer S.D. method for Reaction,

SFD & BMD.

== x ==

Eg:- 2] Analyse the frame by M.D. method



(a) FEM :  
 $M_{FAB} = -\frac{wL^2}{12} = -26.67, M_{FBA} = +26.67$   
 $M_{FBC} = -\frac{wab^2}{12} = -28.8, M_{FCB} = +\frac{Wa^2b}{12} = +19.2$   
 $M_{FDB} = -\frac{wL}{8} = -10, M_{FBD} = +10$

(b) D.F. : (For Intermediate)

		K	$\Sigma K$	$DF = \frac{K}{\Sigma K}$
B	BA	$I/l = I/4 = 0.25I$	0.65I	0.38
	BC	$\frac{3}{4}(I/l) = \frac{3}{4}(I/5) = 0.15I$		0.24
	BD	$I/l = I/4 = 0.25I$		0.38

(c)

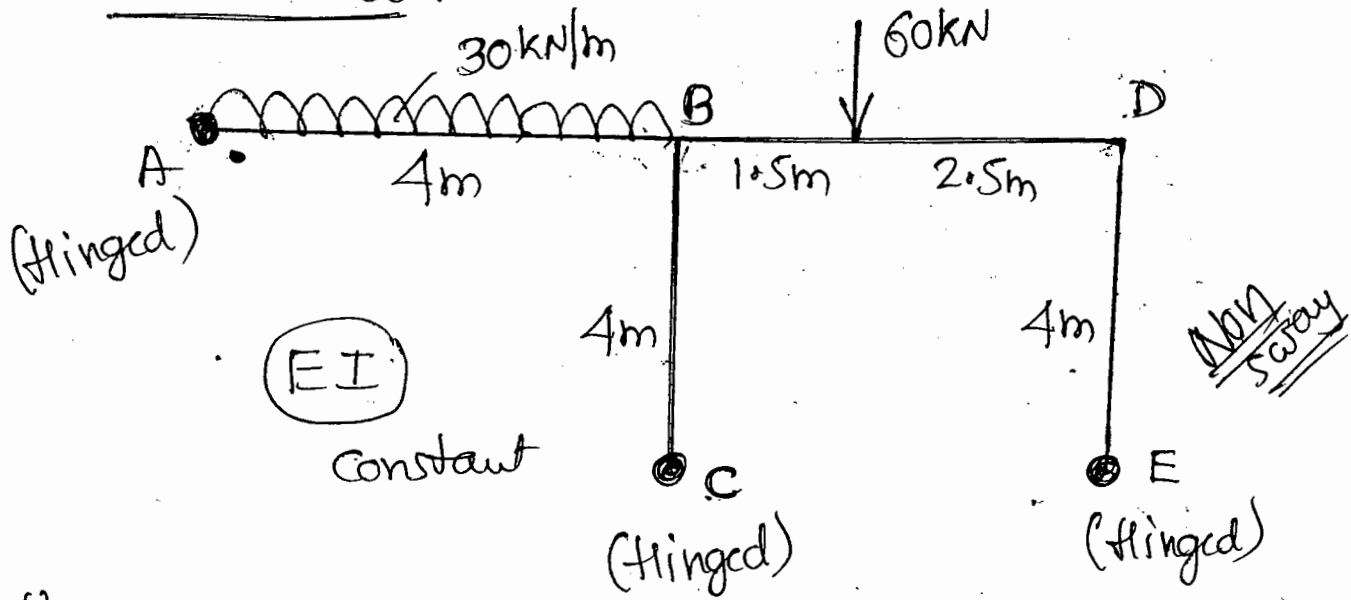
\* "e" Roller

AB	BA	BD	<del>DB</del>	BC	CB	Member
	0.38	0.38		0.24		DF
-26.67	26.67	+10	-10	-28.8	19.2	FEM
				-9.60 ←	19.2	Release C.O.
-26.67	26.67	10	-10	-38.40	0	Initial Values
0.33 ←	0.66	0.66 →	0.33	0.41 →	0	Bal C.O.
-26.33	27.33	10.66	-9.67	-37.99	0	Final Values
↻	↻	↻	↻	↻	○	

Refer S.D. Notes For BMD.

At "B"  $M_{BA} + M_{BC} + M_{BD} = 0$

Eg:- 3] Analyse the frame shown by MD method.



Sol<sup>n</sup>

(a) FEM

$$M_{FAB} = -\frac{wL^2}{12} = -40, \quad M_{FBA} = +40$$

$$M_{FBD} = -35.16, \quad M_{FDB} = +21.10 \text{ kN-m}$$

(b) D.F (For Intermediate)

		$K$	$\Sigma K$	$DF = \frac{K}{\Sigma K}$
B	BA	$\frac{3}{4} \left( \frac{I}{4} \right) = 0.187I$	0.625I	0.30
	BC	$\frac{3}{4} \left( \frac{I}{4} \right) = 0.187I$		0.30
	BD	$\frac{I}{4} = 0.25I$		0.40
D	DB	$\frac{I}{4} = 0.25I$	0.437I	0.57
	DE	$\frac{3}{4} \left( \frac{I}{4} \right) = 0.187I$		0.43

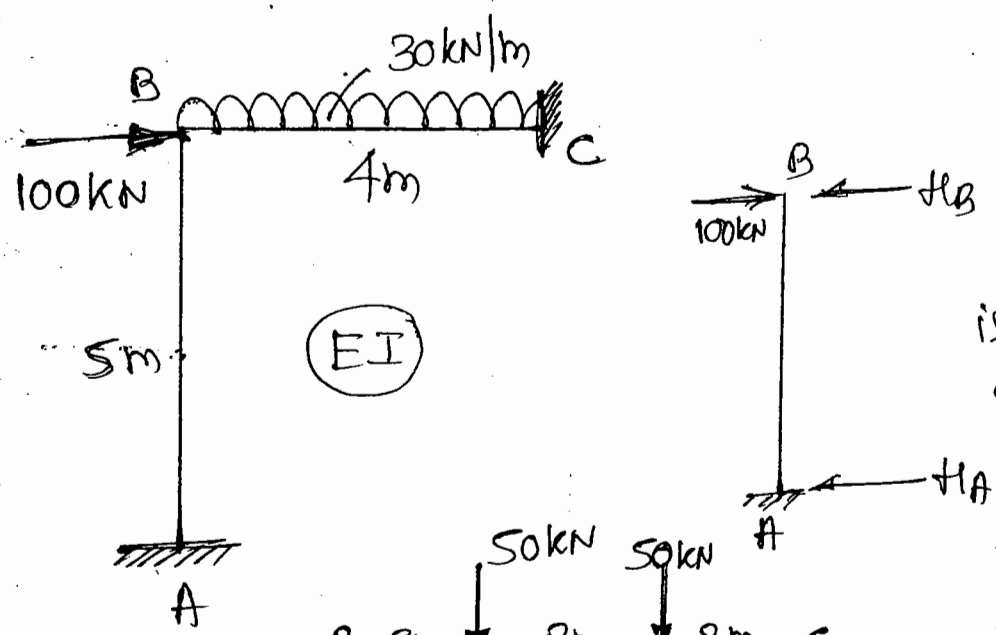
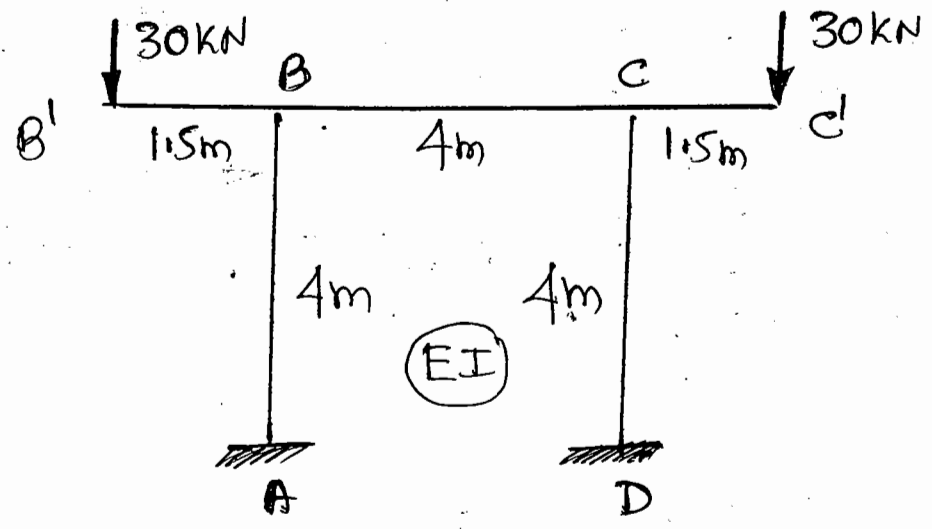
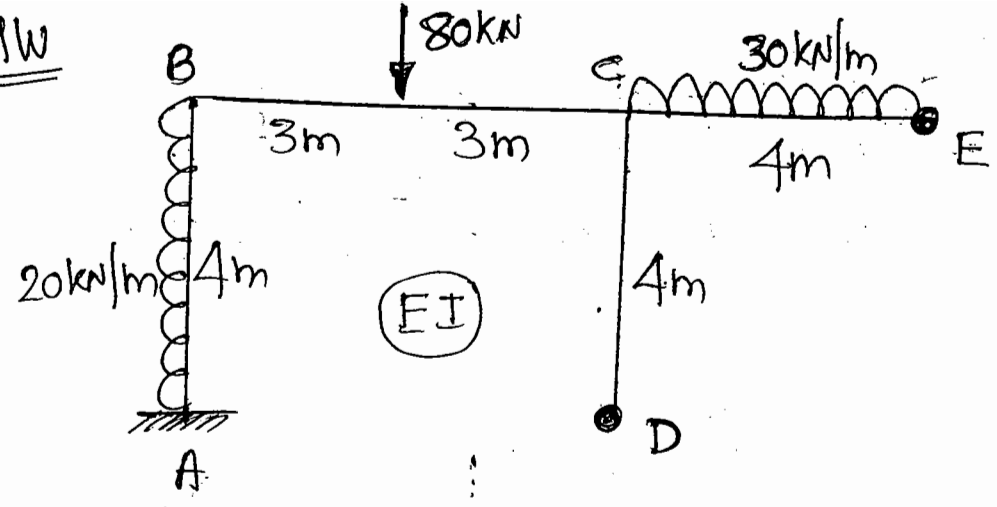


M.D. Table:

AB	BA	BC	CB	BD	DB	DE	ED	Member
-40	0.30	0.30	0	0.40	0.57	0.43		DF
+40	40	0	0	-35.16	21.10	0	0	FEM
0	20							Release C.O
0	60	0	0	-35.16	21.10	0	0	Initial
0	-7.45	-7.45	0	-9.94	-12.03	-9.07	0	Bal
0	1.80	1.80	0	-6.01	-4.97		0	C.O
0	1.80	1.80	0	2.40	2.83	2.14	0	Bal
0	-0.43	-0.43	0	1.42	1.20		0	C.O
0	0.10	0.10	0	-0.57	-0.68	-0.52	0	Bal.
0	-0.102	-0.102	0	-0.34	-0.28		0	C.O
0	54	-6	0	0.14	0.16	0.12	0	Bal
			0	0.08	0.07		0	C.O
			0	-0.04	-0.04	-0.03		Bal
			0	-48.02	7.36	-7.36	0	Final

$M_{BA} + M_{BC} + M_{BD} = 0$  | At "D"  $M_{DB} + M_{DE} = 0$   
 $M_{AB} + M_{BC} + M_{BD} = 0$  | At "B"  $M_{DB} + M_{DE} = 0$

HW



100 kN load  
is used only  
at the time of  
SPD.

