# Module-3
# LATERAL EARTH PRESSURE

**Structure**



## 3.0 Objectives

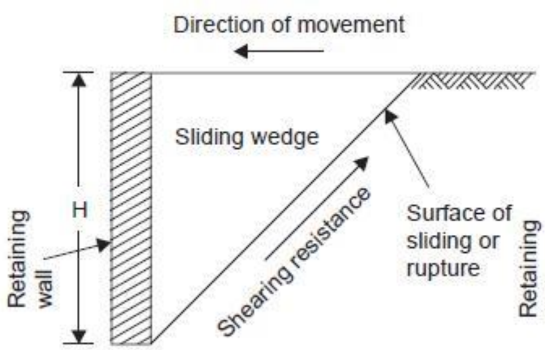
- To understand earth pressure and its importance
- To determine magnitude of LEP by applying Rankines and Coulomb’s theory and graphical methods
- To understand stability slopes, its causes and importance
- To estimate factor of safety of slopes by analytical and graphical method

## 3.1 Introduction

Soil is neither a solid nor a liquid, but it exhibits some of the characteristics of both. One of the characteristics similar to that of a liquid is its tendency to exert a lateral pressure against any object in contact. This important property influences the design of retaining walls, abutments, bulkheads, sheet pile walls, basement walls and underground conduits which retain or support soil, and, as such, is of very great significance. Retaining walls are constructed in various fields of civil engineering, such as hydraulics and irrigation structures, highways, railways, tunnels, mining and military engineering.

## 3.2 Lateral earth pressures

Lateral earth pressure is the force exerted by the soil mass upon an earth- retaining structure, such as a retaining wall. There are two distinct kinds of lateral earth pressure; the nature of each is to be clearly understood. First, let us consider a retaining wall which holds back a mass of soil. The soil exerts a push against the wall by virtue of its tendency to slip laterally and seek its natural slope or angle of repose, thus making the wall to move slightly away from the backfilled soil mass. This kind of pressure is known as the ‗active' earth pressure of the soil. The soil, being the actuating element, is considered to be active and hence the name active earth pressure. Next, let us imagine that in some manner the retaining wall is caused to move toward the soil. In such a case the retaining wall or the earth-retaining structure is the actuating element and the soil provides the resistance which soil develops in response to movement of the structure toward it is called the ‗passive earth pressure', or more appropriately ‗passive earth resistance' which may be very much greater than the active earth pressure. The surface over which the sheared-off soil wedge tends to slide is referred to as the surface of ‗sliding' or ‗rupture'. The limiting values of both the active earth pressure and passive earth resistance for a given soil depend upon the amount of movement of the structure. In the case of active pressure, the structure tends to move away from the soil, causing strains in the soil mass, which in turn, mobilise shearing stresses; these stresses help to support the soil mass and thus tend to reduce the pressure exerted by the soil against the structure. This is indicated in Fig.



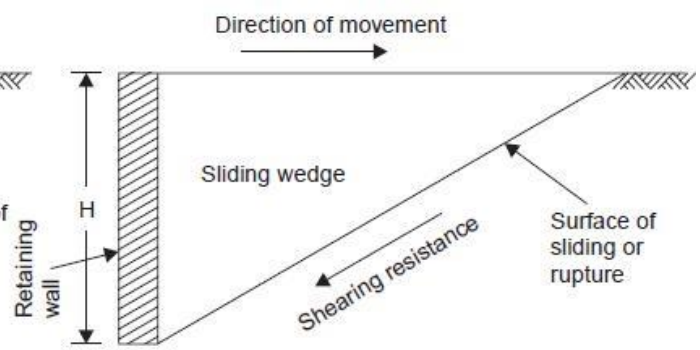




Fig.Conditions in the case of pressure passive earth resistance Fig.Conditions in the case of active

In the case of passive earth resistance also, internal shearing stresses develop, but act in the opposite direction to those in the active case and must be overcome by the movement of the structure. This difference in direction of internal stresses accounts for the difference in magnitude between the active earth pressure and the passive earth resistance. The conditions obtaining in the passive case are indicated in Fig.

**3.3 Earth pressure at rest**

Active pressures are accompanied by movements directed away from the soil, and passive resistances are accompanied by movements towards the soil. Logically, therefore, there must be a situation intermediate between the two when the retaining structure is perfectly stationary and does not move in either direction. The pressure which develops in this condition is called _earth pressure at rest'. Its value is a little larger than the limiting value of active pressure, but is considerably less than the maximum passive resistance. This is indicated in Fig.

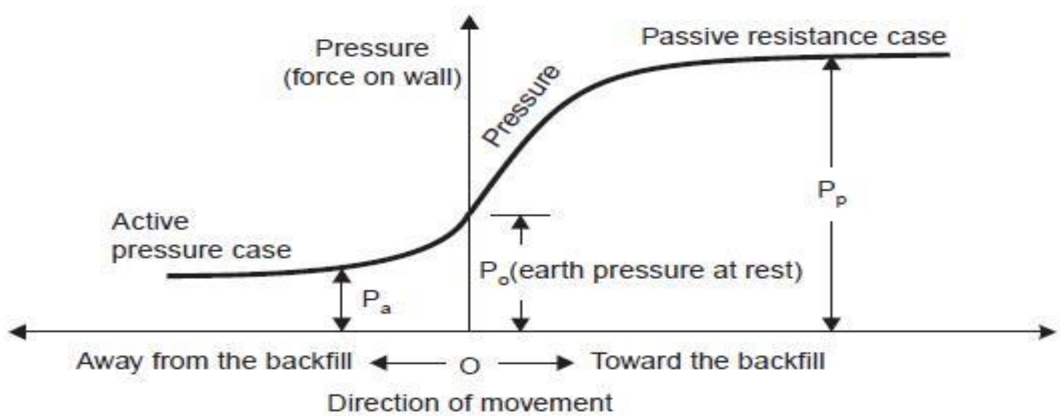



**Fig.** Relation between lateral earth pressure and movement of wall

Earth pressure at rest may be obtained theoretically from the theory of elasticity applied to an element of soil, remembering that the lateral strain of the element is zero. Referring to Fig. (*a*), the principal stresses acting on an element of soil situated at a depth *z* from the surface in semi-infinite, elastic, homogeneous and isotropic soil mass are $\sigma_v$, $\sigma_h$ and $\sigma_h$ as shown. $\sigma_v$ and $\sigma_h$ denoting the stresses in the vertical and horizontal directions respectively.

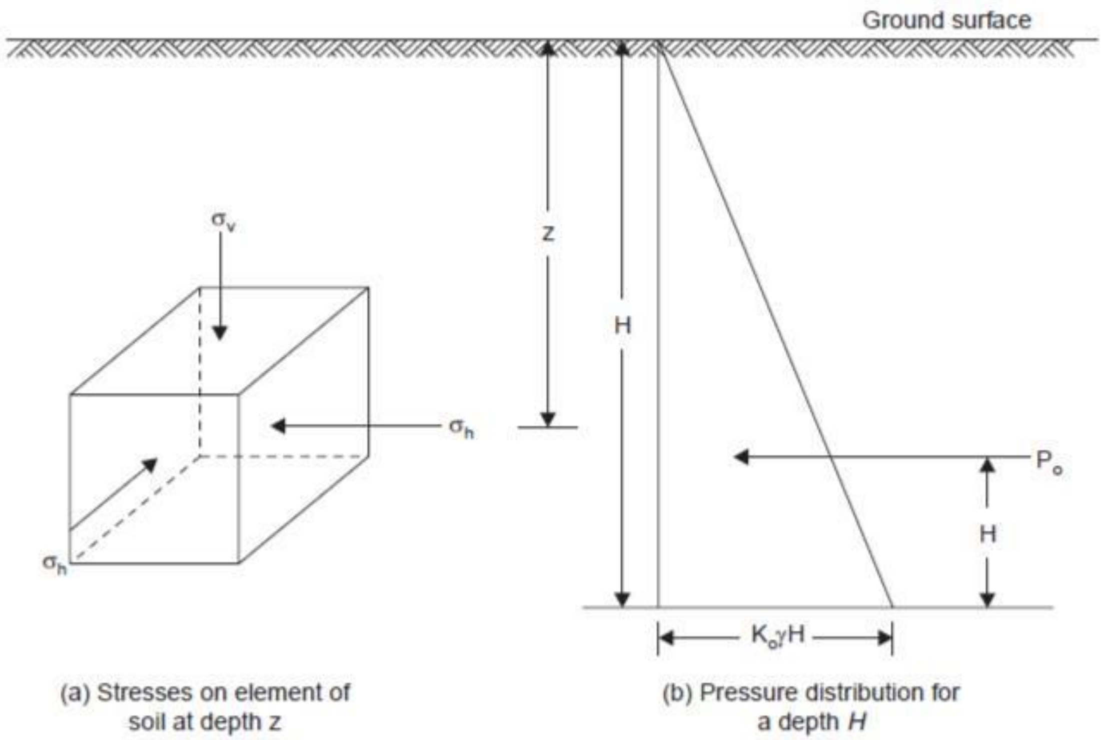



**Fig.** Stress conditions relating to earth pressure at rest

The soil deforms vertically under its self-weight but is prevented from deforming laterally because of an infinite extent in all lateral directions. Let *Es* and ν be the modulus of elasticity and Poisson‘s ratio of the soil respectively.

$$\text{Lateral strain,} \qquad \varepsilon_h = \frac{\sigma_h}{E_s} - \upsilon\left(\frac{\sigma_v}{E_s} + \frac{\sigma_h}{E_s}\right) = 0$$

$$\therefore \qquad \frac{\sigma_h}{\sigma_v} = \frac{\upsilon}{(1-\upsilon)}$$

But $\sigma v = \gamma.\ z$, where γ is the appropriate unit weight of the soil depending upon its condition.

$$\sigma_h = \left(\frac{\upsilon}{1-\upsilon}\right).\ \gamma.\ z$$

$$K_0 = \left(\frac{\upsilon}{1-\upsilon}\right)$$

$$\sigma_h = K_0.\,\gamma.z$$

The distribution of the earth pressure at rest with depth is obviously linear (or of hydrostatic nature) for constant soil properties such as *E,* υ, and γ, as shown in Fig. If a structure such as a retaining wall of height *H* is interposed from the surface and imagined to be held without yield, the total thrust on the wall unit length *P*0, is given by

$$P_0 = \int_0^H \sigma_h.dz = \int_0^H K_0.\gamma z.dz = \frac{1}{2}K_0.\gamma.H^2$$

This is considered to act at (1/3) *H* above the base of wall. As has been indicated in the previous chapter, choosing an appropriate value for the Poisson's ratio, ν, is by no means easy; this is the limitation in arriving at $K_0$ from equation. Various researchers proposed empirical relationships for $K_0$, some of which are given below.

$$K_0 = (1 - \sin\phi') \text{ (Jaky, 1944)}$$
$$K_0 = 0.9\,(1 - \sin\phi') \text{ (Fraser, 1957)}$$
$$K_0 = 0.19 + 0.233 \log I_p \text{ (Kenney, 1959)}$$
$$K_0 = [1 + (2/3)\sin\phi'] \left(\frac{1-\sin\phi}{1+\sin\phi}\right) \text{ (Kezdi, 1962)}$$
$$K_0 = (0.95 - \sin\phi') \text{ (Brooker and Ireland, 1965)}$$

φ′ in these equations represents the effective angle of friction of the soil and *Ip*, the plasticity index. Brooker and Ireland (1965) recommend Jaky's equation for cohesioness soils and their own equation, given above, for cohesive soils. However, Alpan (1967) recommends Jaky's equation for cohesionless soils and Kenney equation for cohesive soils as does Kenney (1959).

Certain values of the coefficient of earth pressure at rest are suggested for different soils, based on field data, experimental evidence and experience. These are given in Table

| S.No. | Soil | | | $K_0$ |
|---|---|---|---|---|
| 1 | Loose Sand ($e$ = 0.8) | | | |
| | | dry | ... | 0.64 |
| | | Saturated | ... | 0.46 |
| 2 | Dense sand ($e$ = 0.6) | | | |
| | | dry | ... | 0.49 |
| | | saturated | ... | 0.36 |
| 3 | Sand (compacted in layers) | | ... | 0.80 |
| 4 | Soft clay ($I_p$ = 30) | | ... | 0.60 |
| 5 | Hard clay ($I_p$ = 9) | | ... | 0.42 |
| 6 | Undisturbed Silty clay ($I_p$ = 45) | | ... | 0.57 |

**3.4 Earth pressure theories**

The magnitude of the lateral earth pressure is evaluated by the application of one or the other of the so-called _lateral earth pressure theories‘ or simply _earth pressure theories‘. The problem of determining the lateral pressure against retaining walls is one of the oldest in the field of engineering. A French military engineer, Vauban, set forth certain rules for the design of revetments in 1687. Since then, several investigators have proposed many theories of earth pressure after a lot of experimental and theoretical work. Of all these theories, those given by Coulomb and Rankine stood the test of time and are usually referred to as the ―Classical earth pressure theories‖. These theories are considered reliable in spite of some limitations and are considered basic to the problem. These theories have been developed originally to apply to cohesionless soil backfill, since this situation is considered to be more frequent in practice and since the designer will be on the safe side by neglecting cohesion. Later researchers gave necessary modifications to take into account cohesion, surcharge, submergence, and so on. Some have evolved graphical procedures to evaluate the total thrust on the retaining structure. Although Coulomb presented his theory nearly a century earlier to Rankine‘s theory, Rankine‘s theory will be presented first due to its relative simplicity.

### 3.4.1 Rankine's theory

Rankine (1857) developed his theory of lateral earth pressure when the backfill consists of dry, cohesionless soil. The theory was later extended by Resal (1910) and Bell (1915) to be applicable to cohesive soils.

**The following are the important assumptions in Rankine's theory:**

(*i*) The soil mass is semi infinite, homogeneous, dry and cohesionless.

(*ii*) The ground surface is a plane which may be horizontal or inclined.

(*iii*) The face of the wall in contact with the backfill is vertical and smooth. In other words, the friction between the wall and the backfill is neglected (This amounts to ignoring the presence of the wall).

(*iv*) The wall yields about the base sufficiently for the active pressure conditions to develop; if it is the passive case that is under consideration, the wall is taken to be pushed sufficiently towards the fill for the passive resistance to be fully mobilised. (Alternatively, it is taken that the soil mass is stretched or gets compressed adequately for attaining these states, respectively. Friction between the wall and fill is supposed to reduce the active earth pressure on the wall and increase the passive resistance of the soil. Similar is the effect of cohesion of the fill soil). Thus it is seen that, by neglecting wall friction as also cohesion of the backfill, the geotechnical engineer errs on the safe side in the computation of both the active pressure and passive resistance. Also, the fill is usually of cohesionless soil, wherever possible, from the point of view of providing proper drainage.

### 3.4.2 Coulomb's wedge theory

**The primary assumptions in Coulomb's wedge theory are as follows:**

1. The backfill soil is considered to be dry, homogeneous and isotropic; it is elastically underformable but breakable, granular material, possessing internal friction but no cohesion.

2. The rupture surface is assumed to be a plane for the sake of convenience in analysis. It passes through the heel of the wall. It is not actually a plane, but is curved and this is known to Coulomb.

3. The sliding wedge acts as a rigid body and the value of the earth thrust is obtained by considering its equilibrium.

4. The position and direction of the earth thrust are assumed to be known. The thrust acts on the back of the wall at a point one-third of the height of the wall above the base of the wall and makes an angle δ, with the normal to the back face of the wall. This is an angle of friction between the wall and backfill soil and is usually called ‗wall friction'.

5. The problem of determining the earth thrust is solved, on the basis of two-dimensional case of ‗plane strain'. This is to say that, the retaining wall is assumed to be of great length and all conditions of the wall and fill remain constant along the length of the wall. Thus, a unit length of the wall perpendicular to the plane of the paper is considered.

6. When the soil wedge is at incipient failure or the sliding of the wedge is impending, the theory gives two limiting values of earth pressure, the least and the greatest (active and passive), compatible with equilibrium. The additional inherent assumptions relevant to the theory are as follows:

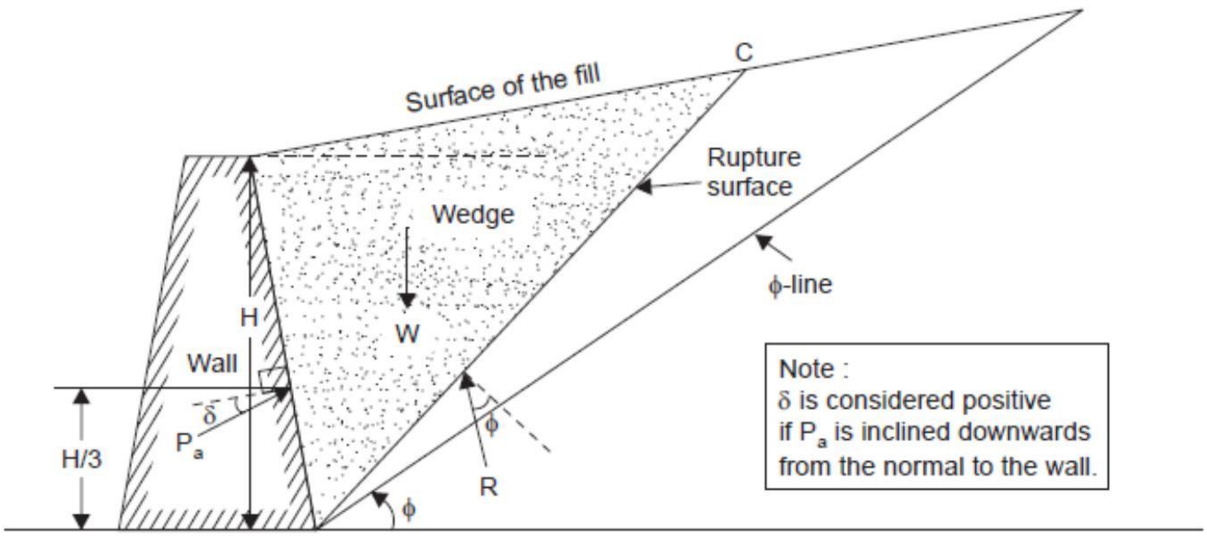
7. The soil forms a natural slope angle, φ, with the horizontal, without rupture and sliding. This is called the angle of repose and in the case of dry cohesionless soil, it is nothing but the angle of internal friction. The concept of friction was understood by Coulomb.

8. If the wall yields and the rupture of the backfill soil takes place, a soil wedge is torn off from the rest of the soil mass. In the active case, the soil wedge slides sideways and downward over the rupture surface, thus exerting a lateral pressure on the wall. In the case of passive earth resistance, the soil wedge slides sideways and upward on the rupture surface due to the forcing of the wall against the fill. These are illustrated in Fig.

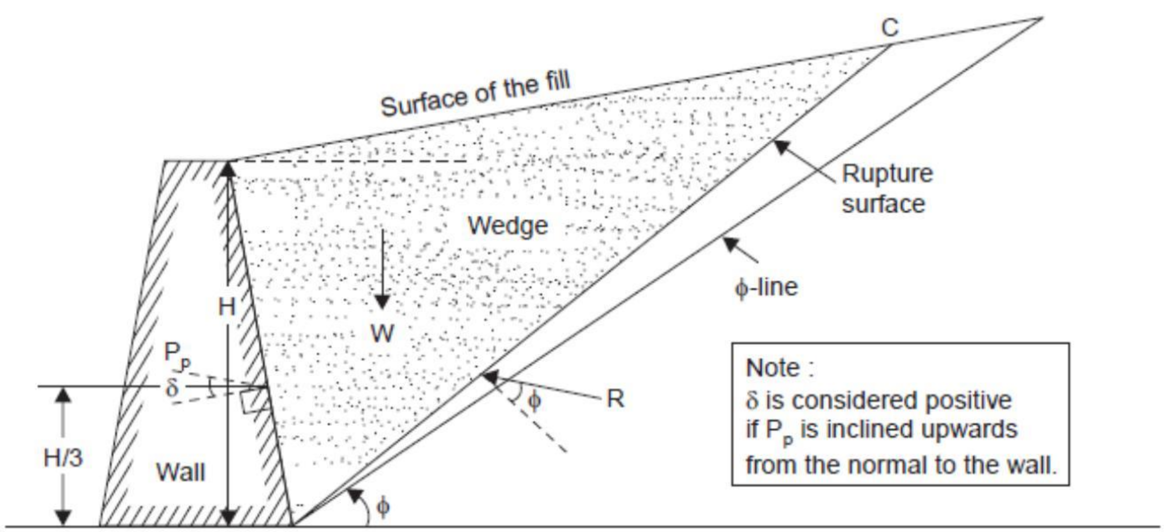
9. For a rupture plane within the soil mass, as well as between the back of the wall and the soil, Newton's law of friction is valid (that is to say, the shear force developed due to friction is the coefficient of friction times the normal force acting on the plane). This angle of friction, whose tangent is the coefficient of friction, is dependent upon the physical properties of the materials involved.

10. The friction is distributed uniformly on the rupture surface.

11. The back face of the wall is a plane.




(a) Active earth pressure




(b) Passive earth resistance

**Limitations of coulomb's theory**

Also note that Coulomb's theory treats the soil mass in the sliding wedge in its entirety. The assumptions permit one to treat the problem as a statically determinate one. Coulomb's theory is applicable to inclined wall faces, to a wall with a broken face, to a sloping backfill curved backfill surface, broken backfill surface and to concentrated or distributed surcharge loads. One of the main deficiencies in Coulomb's theory is that, in general, it does not satisfy the static equilibrium condition occurring in nature. The three forces (weight of the sliding wedge, earth pressure and soil reaction on the rupture surface) acting on the sliding wedge do not meet at a common point, when the sliding surface is assumed to be planar. Even the wall friction was not originally considered but was introduced only some time later. Regardless of this deficiency and other assumptions, the theory gives useful results in practice; however, the soil constants should be determined as accurately as possible.

## 3.5 Lateral Earth Pressure of Cohesive Soil

### 3.5.1 Active Earth Pressure of Cohesive Soil

The lateral earth pressure of cohesive soil may be obtained from the Coulomb's wedge theory; however, one should take cognisance of the tension zone near the surface of the cohesive backfill and consequent loss of contact and loss of adhesion and friction at the back of the wall and along the plane of rupture, so as to avoid getting erroneous results. The trial wedge method may be applied to this case as illustrated in Fig. The following five forces act on a trial wedge:

1. Weight of the wedge including the tension zone, *W*.
2. Cohesion along the wall face or adhesion between the wall and the fill, *Ca*.
3. Cohesion along the rupture plane, *C*.
4. Reaction on the plane of failure, *R*, acting at φ to the normal to the plane of failure.
5. Active thrust, *Pa*, acting at δ to the normal to the face of the wall.
6. The total adhesion force, *Ca*, is given by

$$C_a = c_a \cdot \overrightarrow{BF}$$

where $c_a$ is the unit adhesion between the wall and the fill, which cannot be greater than the unit cohesion, $c$, of the soil. $c_a$ may be obtained from tests; however, in the absence of data, $c_a$ may be taken as equal to $c$ for soils with $c$ up to 50 kN/m$^2$, $ca$ may be limited to 50 kN/m$^2$ for soils with $c$ greater than this value. (Smith, 1974).

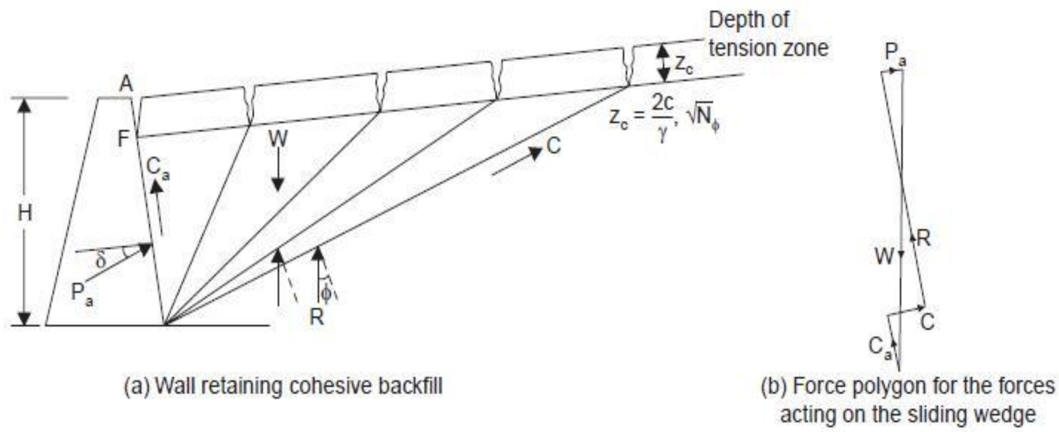



**Fig.** Active earth pressure of cohesive soil—trial wedge method—Coulomb‘s theory

The total cohesion force, *C*, is given by

$$C = c.\overrightarrow{BC}$$

c being the unit cohesion of the fill soil and BC→ is the length of the rupture plane. The three forces W, Ca, and C are fully known and the directions of the other two unknown forces R and Pa are known; the vector polygon may therefore be completed as shown in Fig. (b), and the value of Pa may be scaled-off. A number of such trial wedges may be analyzed and the maximum of all Pa values chosen as the active thrust. The rupture plane may also be located. The final value of the thrust on the wall is the resultant of Pa and Ca. Culmann‘s method may also be adapted to suit this case, as illustrated in fig.

### 3.5.2 Passive Earth Pressure of Cohesive Soil

The procedure adopted to determine the active earth pressure of cohesive soil from Coulomb‘s theory may also be used to determine the passive earth resistance of cohesive soil. The points of difference are that the signs of friction angles, φ and δ, will be reversed and the directions of $C_a$ and $C$ also get reversed. Either the trial wedge approach or Culmann‘s approach may be used but one has also to consider the effect of the tensile zone in reducing $C_a$ and $C$. However, it must be noted that the Coulomb theory with plane rupture surfaces is not applicable to the case of passive resistance. Analysis must be carried out, strictly speaking, using curved rupture surfaces such as logarithmic spirals (Terzaghi, 1943), so as to avoid overestimation of passive resistance.

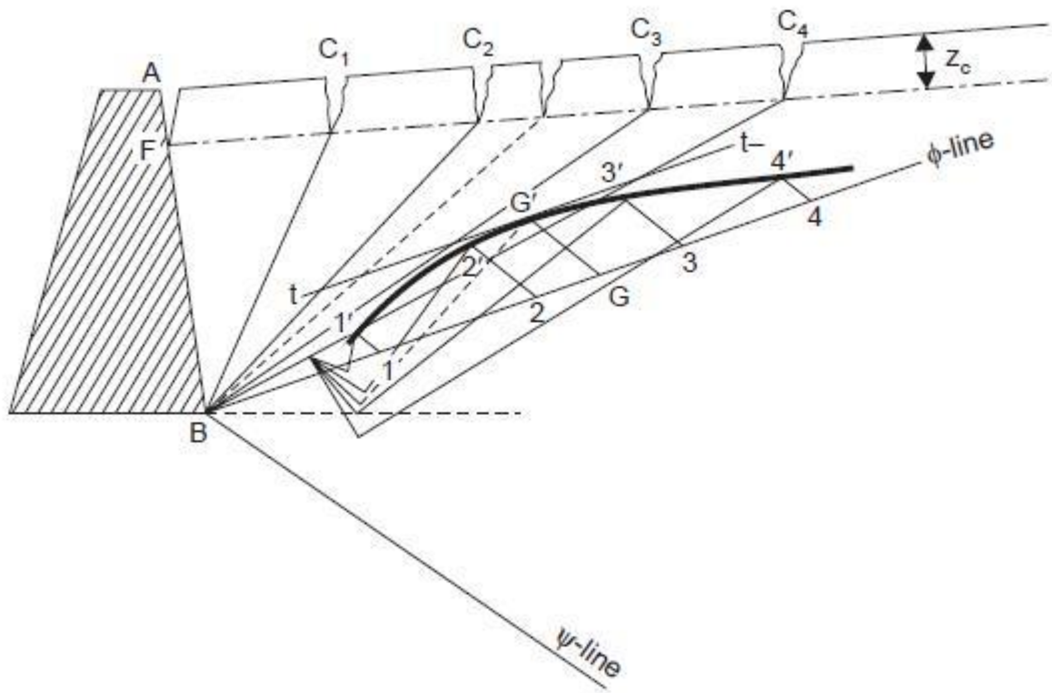



**Fig.** Culmann‘s method adapted to allow for cohesion

### 3.6 Lateral earth pressure for cohesionless soil

### 3.6.1 Active Earth Pressure of Cohesionless Soil

A simple case of active earth pressure on an inclined wall face with a uniformly sloping backfill may be considered first. The backfill consists of homogeneous, elastic and isotropic cohesionelss soil. A unit length of the wall perpendicular to the plane of the paper is considered. The forces acting on the sliding wedge are (*i*) *W*, weight of the soil contained in the sliding wedge, (*ii*) *R*, the soil reaction across the plane of sliding, and (*iii*) the active thrust $P_a$ against the wall, in this case, the reaction from the wall on to the sliding wedge, as shown in Fig.

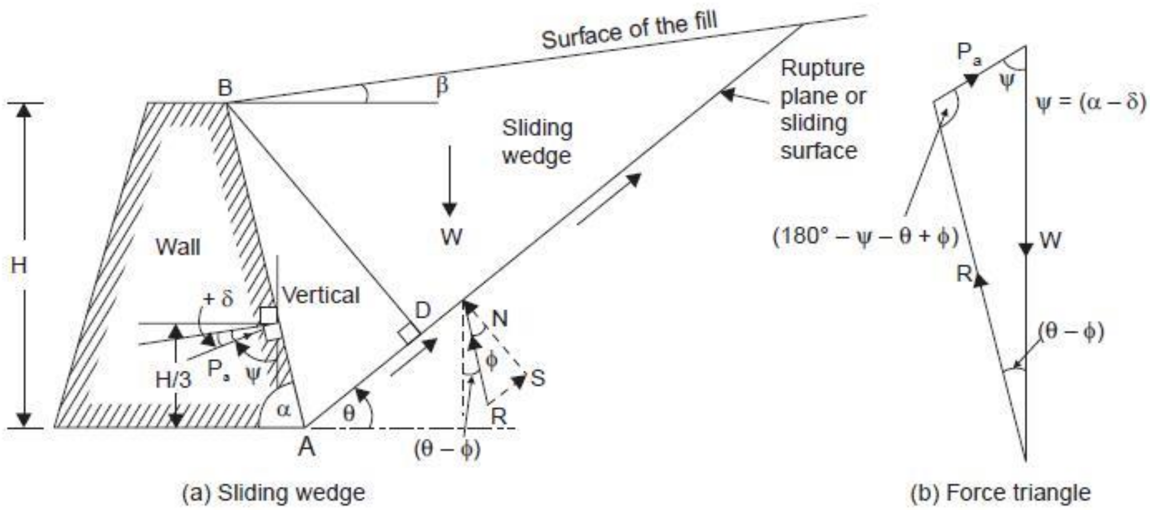



**Fig.** Active earth pressure of cohesionless soil—Coulomb's theory

The triangle of forces is shown in Fig. (*b*). With the nomenclature of Fig. one may proceed as follows for the determination of the active thrust, *Pa*: $W = \gamma$ (area of wedge *ABC*)

$$\Delta ABC = \frac{1}{2}AC.\ BD,\ BD \text{ being the altitude on to } AC.$$

$$AC = AB.\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)}$$

$$BD = AB.\ \sin(\alpha+\theta)$$

$$AB = \frac{H}{\sin\alpha}$$

Substituting and simplifying,

$$W = \gamma\frac{H^2}{2\sin^2\alpha}.\sin(\theta+\alpha).\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)}$$

From the triangles of forces,

$$\frac{P_a}{\sin(\theta-\phi)} = \frac{W}{\sin(180^\circ-\psi-\theta+\phi)}$$

$$P_a = W.\frac{\sin(\theta-\phi)}{\sin(180^\circ-\psi-\theta+\phi)}$$

Substituting for *W*,

$$P_a = \frac{1}{2}\frac{\gamma H^2}{\sin^2\alpha}\cdot\frac{\sin(\theta-\phi)}{\sin(180^\circ-\psi-\theta+\phi)}\cdot\frac{\sin(\theta+\alpha).\sin(\alpha+\beta)}{\sin(\theta-\beta)}$$

The maximum value of *Pa* is obtained by equating the first derivative of *Pa* with respect to θ to zero;

$$\text{or } \frac{\partial P_a}{\partial\theta} = 0, \text{ and substituting the corresponding value of } \theta.$$

The value of *Pa* so obtained is written as

$$P_a = \frac{1}{2}\gamma.H^2.\frac{\sin^2(\alpha+\phi)}{\sin^2\alpha\sin(\alpha-\delta)\left[1+\sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi-\beta)}{\sin(\alpha-\delta)\sin(\alpha+\beta)}}\right]^2}$$

This is usually written as

$$P_a = \frac{1}{2}\gamma H^2.K_a,$$

$$K_a = \frac{\sin^2(\alpha+\phi)}{\sin^2\alpha.\sin(\alpha-\delta)\left[1+\sqrt{\dfrac{\sin(\phi+\delta).\sin(\phi-\beta)}{\sin(\alpha-\delta)\sin(\alpha+\beta)}}\right]^2}$$

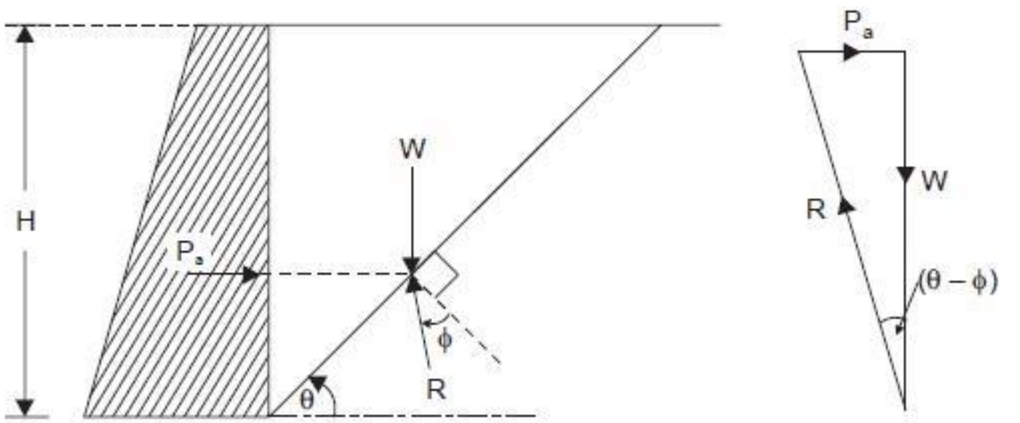
$K_a$ being the coefficient of active earth pressure. For a vertical wall retaining a horizontal backfill for which the angle of wall friction is equal to φ, *Ka* reduces to

$$K_a = \frac{\cos\phi}{(1+\sqrt{2}\sin\phi)^2}$$

by substituting α = 90°, β = 0°, and δ = φ. For a smooth vertical wall retaining a backfill with horizontal surface, α = 90°, δ = 0, and β = 0;

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \tan^2(45° - \phi/2) = 1/N_\phi,$$

which is the same as the Rankine value. In fact, for this simple case, one may proceed from fundamentals as follows:




(a) Sliding wedge (b) Triangle of forces

**Fig.** Active earth pressure of cohesionless soil special case: α = 90°, δ = β = 0°

$$P_a = W\tan(\theta - \phi),$$
$$W = \frac{1}{2}\gamma H^2 . \cot\phi$$
$$P_a = \frac{1}{2}\gamma H^2 \cot\theta \tan(\theta - \phi)$$

For maximum value of $P_a$, $\frac{\partial P_a}{\partial \theta} = 0$

$$\therefore \qquad \frac{\partial P_a}{\partial \theta} = \frac{1}{2}\gamma H^2\left[-\frac{\tan(\theta-\phi)}{\sin^2\theta} + \frac{\cot\theta}{\cos^2(\theta-\phi)}\right] = 0$$

or $$\frac{-\sin(\theta-\phi)\cos(\theta-\phi) + \sin\theta\cos\theta}{\sin^2\theta\cos^2(\theta-\phi)} = 0$$

or $\sin\phi\cos(2\theta - \phi) = 0$, on simplification.

$\therefore \qquad \cos(2\theta - \phi) = 0 \qquad$ or $\qquad \theta = 45° + \phi/2$

$$P_a = \frac{1}{2}\gamma H^2 \tan^2(45° - \phi/2)$$

as obtained by substitution in the general equation. Ironically, this approach is sometimes known as ‗Rankine‘s method of Trial Wedges‘. A few representative values of *Ka* from Eq. 13.34 for certain values of φ, δ, α and β are

| δ↓φ→ | 20° | 30° | 40° |
|---|---|---|---|
| | | α = 90°, β = 0° | |
| 0° | 0.49 | 0.33 | 0.22 |
| 10° | 0.45 | 0.32 | 0.21 |
| 20° | 0.43 | 0.31 | 0.20 |
| 30° | ... | 0.30 | 0.20 |
| | | α = 90°, β = 10° | |
| 0° | 0.51 | 0.37 | 024 |
| 10° | 0.52 | 0.35 | 0.23 |
| 20° | 0.52 | 0.34 | 0.22 |
| 30° | ... | 0.33 | 0.22 |
| | | α = 90°, β = 20° | |
| 0° | 0.88 | 0.44 | 0.27 |
| 10° | 0.90 | 0.43 | 0.26 |
| 20° | 0.94 | 0.42 | 0.25 |
| 30° | ... | 0.42 | 0.25 |

It may be observed that the theoretical solution is thus rather complicated even for relatively simple cases. This fact has led to the development of graphical procedures for arriving at the total thrust on the wall. Poncelet (1840), Culmann (1866), Rebhann (1871), and Engessor (1880) have given efficient graphical solutions, some of which will be dealt with in the subsequent subsections. An obvious grpahical approach that suggests itself if the ―Trial-Wedge method‖. In this method, a few trial rupture surfaces are assumed at varying inclinations, θ, with the horizontal and passing through the heel of the wall; for each trial surface the triangle of forces is completed and the value of *Pa* found. A θ – *Pa* plot is made which should appear somewhat as shown in Fig. If an adequate number of intelligently planned trial rupture surface are analysed. The maximum value of *Pa* from this plot gives the anticipated total active thrust on the wall per lineal unit and the corresponding value of θ, the inclination of the most probable rupture surface. Wall friction At this juncture, a few comments on wall friction may be appropriate. In the active case, the outward stretching leads to a downward motion of the backfill soil relative to the wall. Such a downward shear force upon the wall is called ‗positive‘ wall friction for the active case.

This leads to the upward inclination of the active thrust exerted on the sliding wedge as shown in Fig. (*a*). This means that the active thrust exerted on the wall will be directed with a downward inclination. In the passive case, the horizontal compression must be accompanied by an upward bulging of the soil and hence there tends to occur an upward shear on the wall. Such an upward shear on the wall is said to be ‗positive' wall friction for the passive case. This leads to the downward inclination of the passive thrust exerted on the sliding wedge as shown in fig. 13.24 (*a*); this means that the passive resistance exerted on the wall will be directed with upward inclination. In the active case wall friction is almost always positive. Sometimes, under special conditions, such as when part of the backfill soil immediately behind the wall is excavated for repair purposes and the wall is braced against the remaining earth mass of the backfill, negative wall friction might develop. Either positive or negative wall friction may develop in the passive case. This sign of wall friction must be determined from a study of motions expected for each field situation. Once wall friction is present, the shape of the rupture surface is curved and not plane. The nature of the surface for positive and negative values of wall friction is shown in Figs. (*a*) and (*b*), respectively.

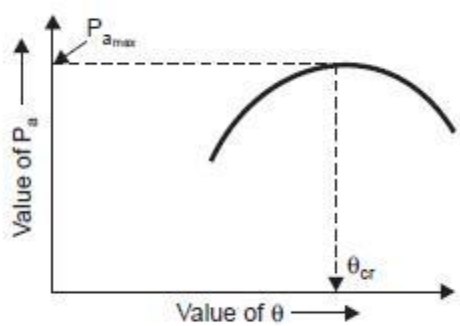



Fig. Angle of inclination of trial rupture plane versus active thrust

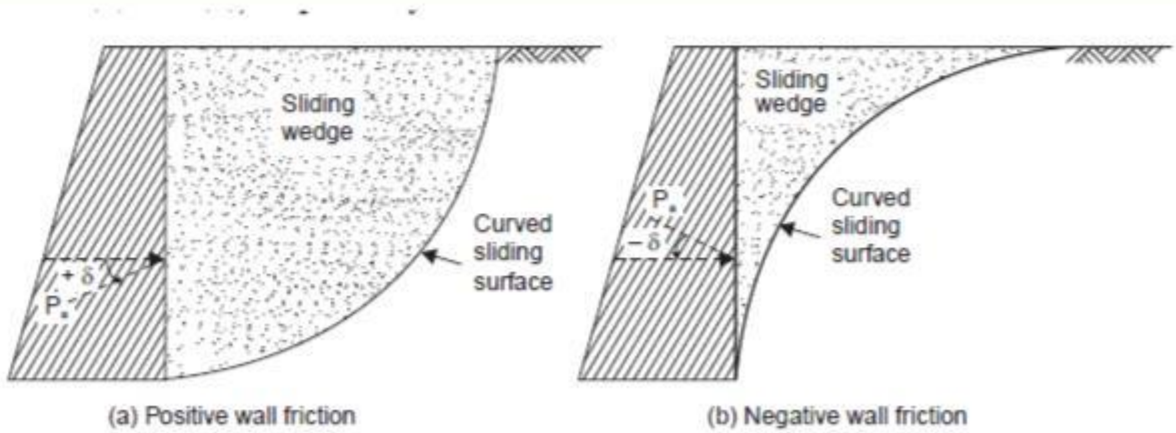



**Fig.** Positive and negative wall friction for active case along with probable shape of sliding surface

The angle of wall friction, δ, will not be greater than φ; at the maximum it can equal φ, for a rough wall with a loose fill. For a wall with dense fill, δ will be less than φ. It may range from 12 φ to 34 φ in most cases; it is usually assumed as (2/3) φ in the absence of precise data. The possibility of δ shifting from +φ to –φ in the worst case should be considered in the design of a retaining wall. The value of Ka for the case of a vertical wall retaining a fill with a level surface, in which φ ranges from 20° to 40° and δ ranges from 0° to φ, may be obtained from the chart given in Fig.

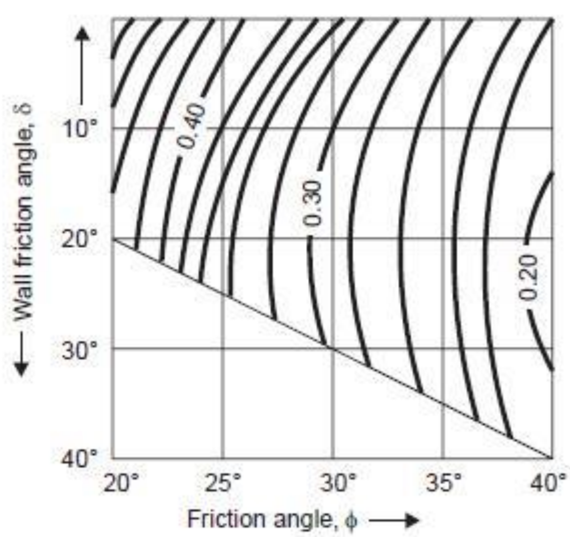



**Fig.** Coefficient of active pressure as a function of wall friction

The influence of wall friction on *Ka* may be understood from this chart to some extent. The assumption of plane failure in the active case of the Coulomb theory is in error by only a relatively small amount. It has been shown by Fellenius that the assumption of circular arcs for failure surfaces leads to active thrusts that generally do not exceed the corresponding values from the Coulomb theory by more than 5 per cent.

### 3.6.2 Passive Earth Pressure of Cohesionless Soil

The passive case differs from the active case in that the obliquity angles at the wall and on the failure plane are of opposite sign. Plane failure surface is assumed for the passive case also in the Coulomb theory but the critical plane is that for which the passive thrust is minimum. The failure plane is at a much smaller angle to the horizontal than in the active case, as shown in Fig. The triangle of forces with the usual nomenclature, the passive resistance *PP* may be determined as follows:

$$W = \frac{1}{2}\frac{\gamma H^2}{\sin^2\alpha}.\sin(\theta+\alpha).\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)}, \text{ as in the active case.}$$

From the triangle of forces

$$\frac{P_p}{\sin(\theta+\phi)} = \frac{W}{\sin(180^\circ-\psi-\theta-\phi)}$$

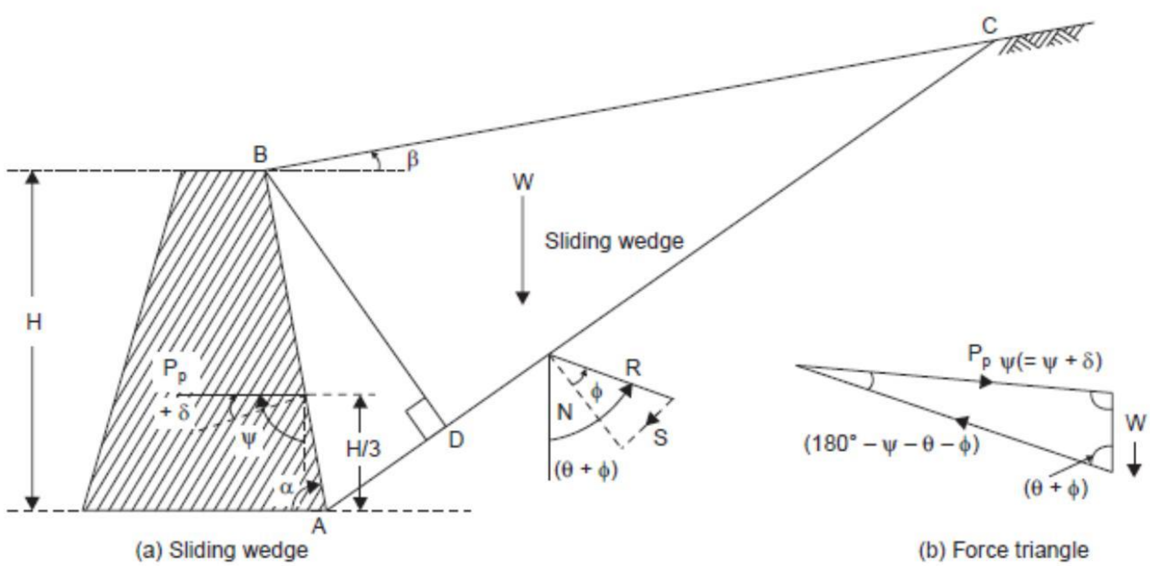



**Fig.** Passive earth pressure of cohesionless soil—Coulomb‘s theory

$$P_p = W.\frac{\sin(\theta+\phi)}{\sin(180^\circ-\psi-\theta-\phi)}$$

Substituting for *W,*

$$P_p = \frac{1}{2}.\frac{\gamma H^2}{\sin^2\alpha}.\sin(\theta+\alpha).\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)}.\frac{\sin(\theta+\alpha)}{\sin(180^\circ-\psi-\theta-\phi)}$$

The minimum value of *Pp* is obtained by differentiating with respect to θ

$$P_p = \frac{1}{2}\gamma H^2.K_p$$

$$K_p = \frac{\sin^2(\alpha-\phi)}{\sin^2\alpha.\sin(\alpha+\delta)\left[1-\sqrt{\frac{\sin(\theta+\delta).\sin(\phi+\beta)}{\sin(\alpha+\delta).\sin(\alpha+\beta)}}\right]^2}$$

*KP* being the coefficient of passive earth resistance. For a vertical wall retaining a horizontal backfill and for which the friction is equal to φ, α = 90°, β = 0°, and δ = φ, and *Kp* reduces to

$$K_p = \frac{\cos^2\phi}{\cos\phi\left[1-\sqrt{\frac{2\sin\phi\cos\phi.\sin\phi}{\cos\phi}}\right]^2}$$

$$K_p = \frac{\cos\phi}{(1-\sqrt{2}\sin\phi)^2}$$

For a smooth vertical wall retaining a horizontal backfill, α = 90°, β = 0° and δ = 0°;

$$K_P = \frac{\cos^2\phi}{(1-\sin\phi)^2} = \frac{1-\sin^2\phi}{(1-\sin\phi)^2} = \frac{(1+\sin\phi)}{(1-\sin\phi)} = \tan^2(45^\circ+\phi/2) = N_\phi,$$

which is the same as the Rankine value. For this simple case, it is possible to proceed from fundamentals, as has been shown for the active case. [(θ + φ) takes the place of (θ – φ) and (45° + φ/2) that of (45° – φ/2) in the work relating to the active case.] Coulomb‘s theory with plane surface of failure is valid only if the wall friction is zero in respect of passive resistance. The passive resistance obtained by plane failure surfaces is very much more than that obtained by assuming curved failure surfaces, which are nearer truth especially when wall friction is present. The error increases with increasing wall friction. This leads to errors on the unsafe side.

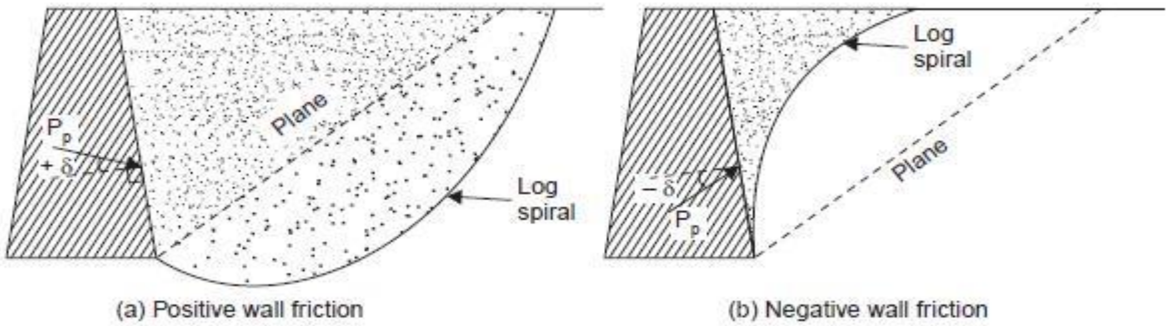



**Fig.** Curved failure surface for estimating passive resistance

Terzaghi (1943) has presented a more rigorous type of analysis assuming curved failure surface (logarithmic spiral form) which resembles those shown in Fig.. Terzaghi states that when δ is less than (1/3) φ, the error introduced by assuming plane rupture surfaces instead of curved ones in estimating the passive resistance is not significant; when δ is greater than (1/3) φ, the error is significant and hence cannot be ignored. This situation calls for the use of analysis based on curved rupture surfaces as given by Terzaghi; alternatively, charts and tables prepared by Caquot and Kerisel (1949) may be used. Extracts of such results are presented in Table 13.3 and Fig.

| δ ↓ φ → | *10°* | *20°* | *30°* | *40°* |
|---|---|---|---|---|
| 0° | 1.42 | 2.04 | 3.00 | 4.60 |
| φ/2 | 1.56 | 2.60 | 4.80 | 10.40 |
| φ | 1.65 | 3.00 | 6.40 | 17.50 |
| −φ | 0.72 | 0.58 | 0.54 | 0.52 |

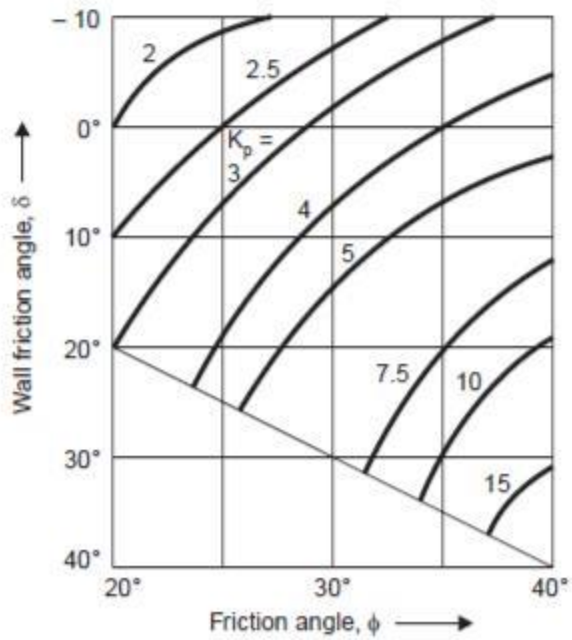
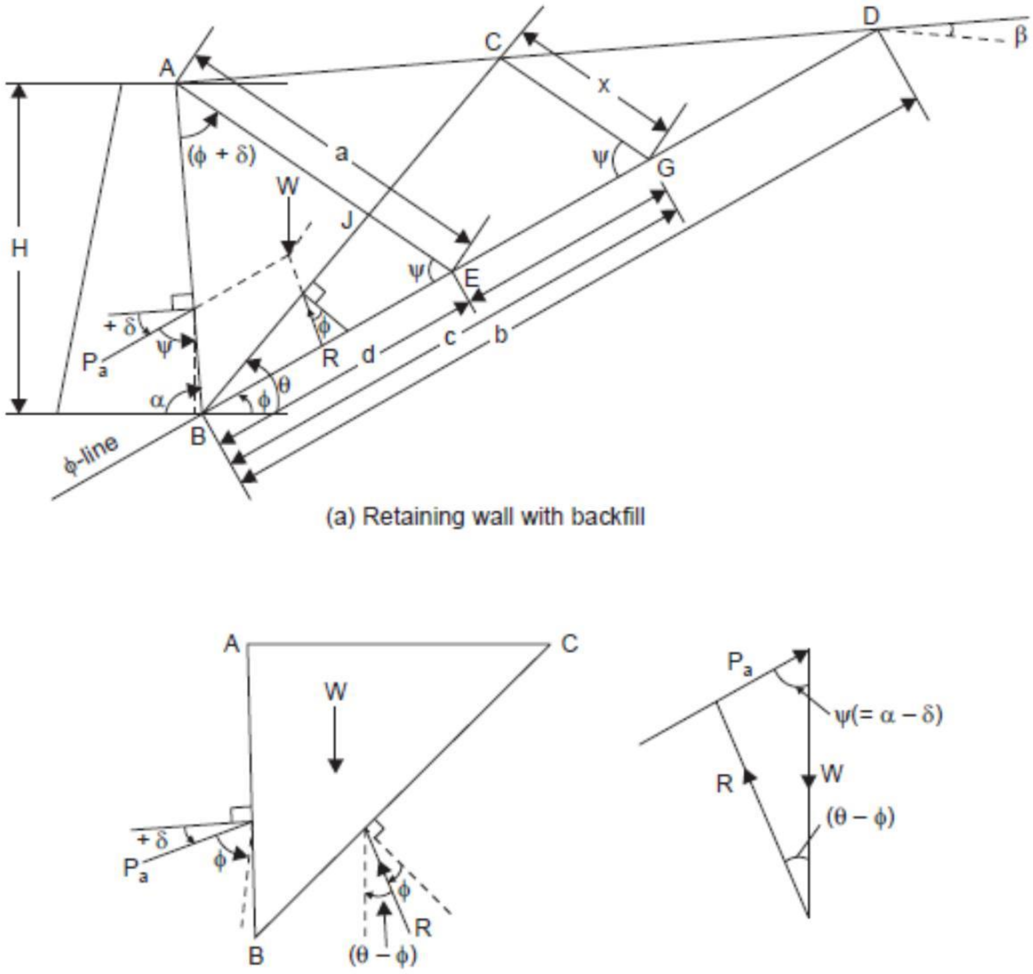



**Fig.** Chart for passive pressure coefficient(After Caquot and Kerisel, 1949)

Alternatively, Sokolovski's (1965) method may be used. This also gives essentially the same results. The theoretical predictions regarding passive resistance with wall friction are not well confirmed by experimental evidence as those regarding active thrust and hence cannot be used with as much confidence. Tschebotarioff (1951) gives the results of a few large-scale laboratory tests in this regard.

### 3.7 Rebhann's Condition and Graphical Method

Rebhann (1871) is credited with having presented the criterion for the direct location of the failure plane assumed in the Coulomb's theory. His presentation is somewhat as follows: Figure (*a*) represent a retaining wall retaining a cohesionless backfill inclined at +β to the horizontal. Let *BC* be the failure plane, the position of which is to be determined.




(a) Retaining wall with backfill

(b) Forces on the sliding wedge

(c) Force triangle

**Fig.** Rebhann‘s condition for Coulomb‘s wedge theory— Location of failure plane for the active case.

Figure (*b*) represents the forces on the sliding wedge and Fig. (*c*) represents the force triangle. Let *BD* be a line inclined at φ to the horizontal through *B*, the heel of the wall, *D* being the intersection of this φ-line with the surface of the backfill. The value of *Pa* depends upon the angle θ relating to the location of the failure plane. *Pa* will be zero when θ = φ, and increases with an increase in θ up to a limit, beyond which it decreases and reaches zero again when θ = 180° – α. The situations when *Pa* is zero are both ridiculous, since in the first case, no wall is required to retain a soil mass at an angle φ and in the second, the failure wedge has no mass. Thus, the failure plane will lie between the φ-line and the back of the wall. Let *AE* be drawn at an angle (φ + δ) to the wall face *AB* to meet the φ-line in *E*. Let *CG* be drawn parallel to *AE* to meet the φ-line in *G*. Let the distances be denoted as follows: *AE* = *a BG* = *c CG* = *x BD* = *b BE* = *d* It is required to determine the criterion for which *Pa* is the maximum, which is supposed to give the correct location of the failure surface. Weight of the soil in the sliding wedge

$W = \gamma.\ ( \quad ABC)$

$$= \gamma.\ (\Delta ABD - \ BCD)$$

$$= \gamma.\ (b/2).\ (\sin \psi)\ (a - x)$$

Value of thrust on the wedge (the same as the thrust on the wall).

$$P_a = \frac{W.x}{c}, \text{ since } \Delta BCG \text{ is similar to the triangle of forces.}$$

$$\therefore \quad P_a = \frac{\gamma bx}{2c}(a - x).\sin\psi \quad \ldots$$

If $\dfrac{DG}{CG} = k,\ c = b - kx$

$$\therefore \quad P_a = \frac{\gamma bx}{2(b - kx)}.(a - x)\sin\psi$$

For the value of $P_a$ to be a maximum, $\dfrac{\partial P_a}{\partial x} = 0$,

since *x* is the only value which varies with the orientation of the failure plane.

$$\therefore \quad \frac{\partial P_a}{\partial x} = (b - kx)(a - 2x) + kx(a - x) = 0$$

$$(a - x)\ (b - kx + kx) - x(b - kx) = 0$$

$$b(a - x) = cx$$

Multiplying throughout by $\frac{1}{2}\sin\psi$,

$$\tfrac{1}{2}\,ba \sin\psi - \tfrac{1}{2}\,bx \sin\psi = \tfrac{1}{2}\,cx \sin\psi$$

$$\Delta ABD - \Delta BCD = \Delta BCG$$

$$\Delta ABC = \Delta BCG$$

This equation signifies that for *EC* to be the failure plane the requirement is that the area of the failure wedge *ABC* be equal to the area of the triangle *BCG*. This is known as ―Rebhann‘s condition‖, since it was demonstrated first by Rebhann is 1871. The triangles *ABC* and *BCG* which are equal have a common base *BC*; hence their altitudes on to *BC* should be equal; or *AJ*. sin ∠*AJB* = *CG*. sin ∠*BCG* But ∠*AJB* = ∠*BCG* as *CG* is parallel to *AJ*. This leads to *CG* = *AJ* = *x*; and *JE* = *a* – *x* Triangles DAE and DCG are similar.

$$\text{Hence } \frac{(b-d)}{(b-c)}.x = a$$

Also, triangles *BCG* and *BJE* are similar. Consequently,*d/c .x* = *a* – *x* Subtracting one from the another,

$$x\left(\frac{b-d}{b-c}-\frac{d}{c}\right) = x$$

Simplifying,

$$c^2 = bd$$

$$c = \sqrt{bd}$$

Thus if *c* is known, the position of *G* and hence that of the most dangerous rupture *p* surface, *BC*, can be determined and the weight of the sliding wedge, *W*, and the active thrust, *Pa*, can be calculated. The relationship expressed by Eq. 13.44 is called the ―Poncelet Rule‖ after Poncelet (1840). It is obvious that Rebhann‘s condition leads one to Poncelet‘s rule and the satisfaction of one of these two implies that of the other automatically.

$$cx = b(a-x)$$

$$x = \frac{ab}{b+c}$$

$$P_a = \frac{1}{2}\gamma x^2.\sin\psi$$

$$\psi = \alpha - \delta$$

$$c = \sqrt{bd}$$

$$x = \frac{ab}{b+c}$$

$$P_a = \frac{1}{2}\gamma x^2.\sin\psi$$

which gives an analytical procedure for the computation of the active thrust by Coulomb‘s wedge theory. However, elegant graphical methods have been devised and are preferred to the analytical approach, in view of their versatility, coupled with simplicity. The graphical method to follow is given by Poncelet and it is also sometimes known as the Rebhann‘s graphical method, since it is based on Reghann‘s condition. The steps involved in the graphical method are as follows, with reference to Fig.

(*i*) Let *AB* represent the backface of the wall and *AD* the backfill surface.

(*ii*) Draw *BD* inclined at φ with the horizontal from the heel *B* of the wall to meet the backfill surface in *D*.

(*iii*) Draw *BK* inclined at ψ(= α – δ) with *BD*, which is the ψ-line.

(*iv*) Through *A*, draw *AE* parallel to the ψ-line to meet *BD* in *E*. (Alternatively, draw *AE* at (φ + δ) with *AB* to meet *BD* in *E*).

(*v*) Describe a semi-circle on *BD* as diameter.

(*vi*) Erect a perpendicular to *BD* at *E* to meet the semi-circle in *F*.

(*vii*) With *B* as centre and *BF* as radius draw an arc to meet *BD* in *G*.

(*viii*) Through *G*, draw a parallel to the ψ-line to meet *AD* in *C*.

(*ix*) With *G* as centre and *GC* as radius draw an arc to cut *BD* in *L*; join *CL* and also draw a perpendicular *CM* from *C* on to *LG*.

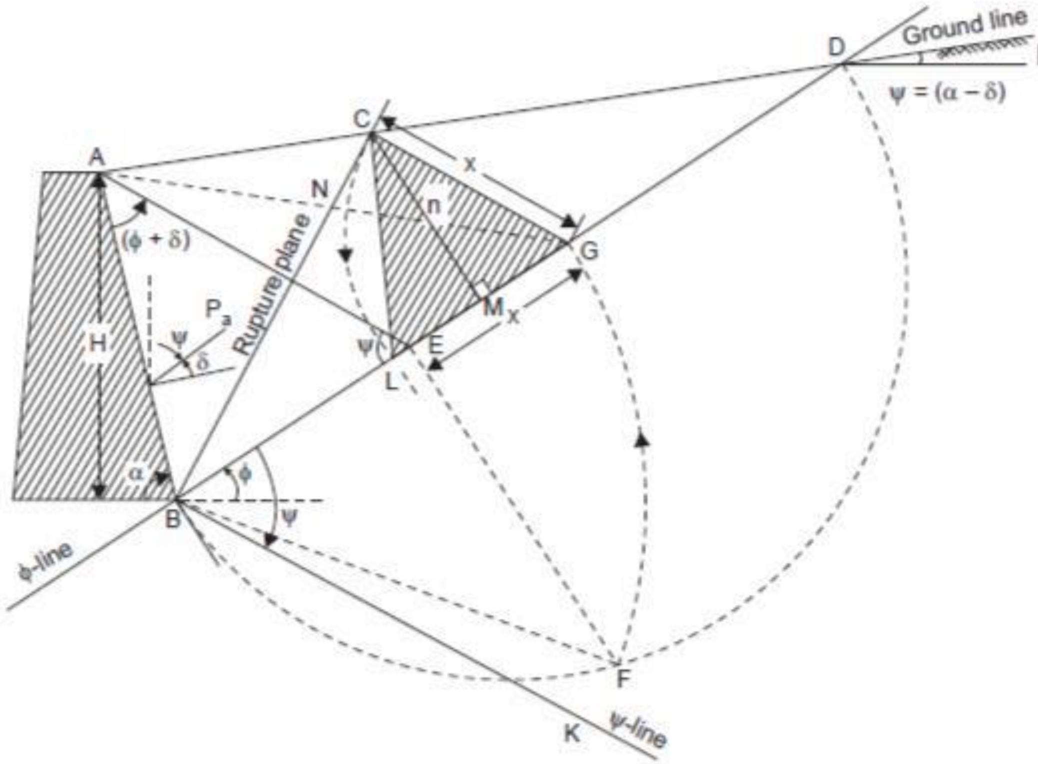



**Fig.** Poncelet graphical construction for active thrust

*BC* is the required rupture surface. The criterion may be checked as follows: Since *ABC* = *BCG*, and *BC* is their common base, their altitudes on *BC* must be equal; or *AN* sin ∠*ANB* = *NG* sin ∠*GNC* that is to say *AN* = *NG*, since ∠*ANB* = ∠*GNC*. (*N* is intersection of *AG* and *BC*). Thus, if *AN* and *NG* are measured and found to be equal, the construction is correct. The active thrust

*Pa* is given by

$$P_a = \frac{1}{2}\gamma x^2 . \sin\psi, \text{ where } CG = LG = x$$
$$= \gamma.(\Delta CGL)$$
$$= \frac{1}{2}\gamma.x.n, \text{ where } n = CM, \text{ the altitude on the } LG.$$

(Incidentally, with the notation of Fig. 13.27 (*a*), it may be easily understood that *W* =1/2 γ.*x*.*n*.

### 3.8 Culmann's Graphical Method

Karl Culmann (1866) gave his own graphical method to evaluate the earth pressure from Coulomb the earth pressure and to locate the most dangerous rupture surface according to Coulomb's wedge theory. This method has more general application than Poncelet's and is, in fact, a simplified version of the more general trial wedge method. It may be conveniently used for ground surface of any shape, for different types of surcharge loads, and for layered backfill with different unit weights for different layers. With reference to Fig. 13.33 (*b*), the force triangle may be imagined to the rotated clockwise through an angle (90° – φ), so as to bring the vector *W* parallel to the φ-line; in that case, the reaction, *R* will be parallel to the rupture surface, and the active thrust, *Pa*, parallel to the ψ-line.'s theory. Culmann's method permits one to determine graphically the magnitude of

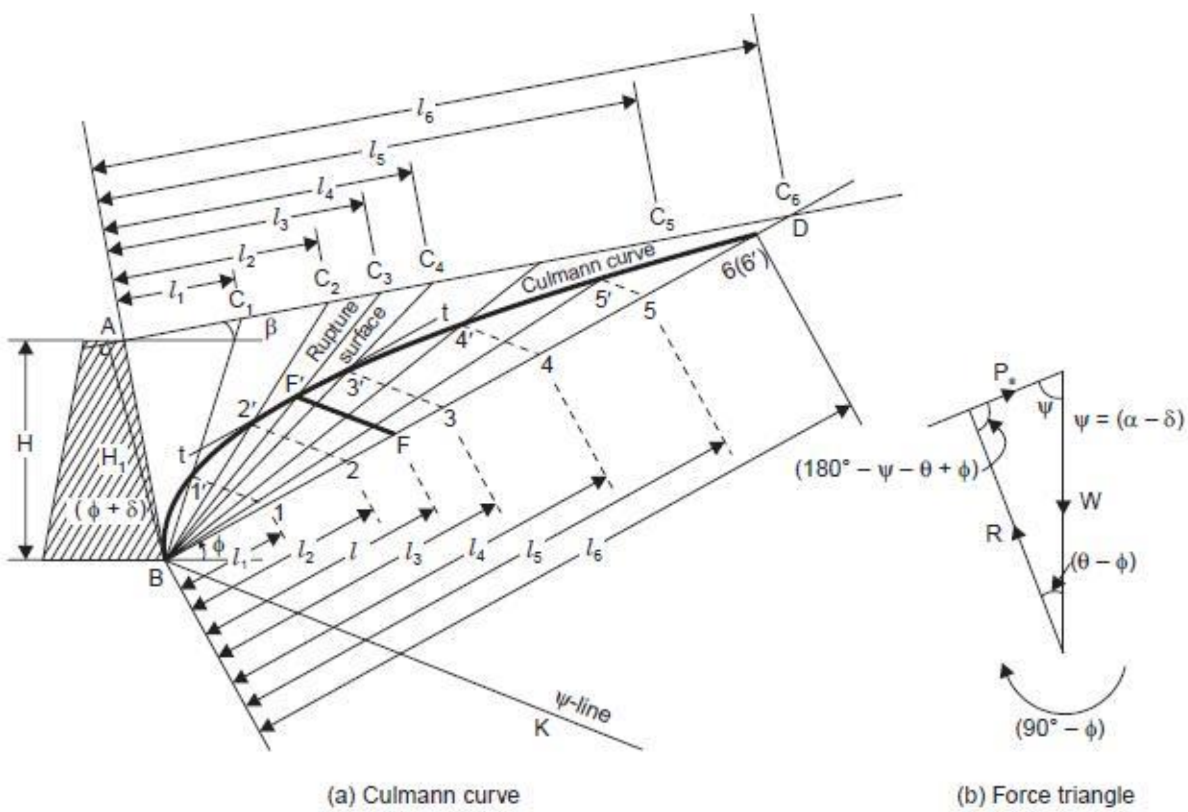



**Fig.** Culmann's graphical method for active thrust

Hence, if weights of the various sliding wedges arising out of arbitrarily assumed sliding surface are set off to a convenient force scale on the φ-line from the heel of the wall and if lines parallel to the ψ-line are drawn from the ends of these weight vectors to meet the respective assumed rupture lines, the force triangle for each of these sliding wedges will be complete. The end points of the active thrust vectors, when joined in a sequence, form what is known as the ―Culmann-curve‖. The maximum value of the active thrust may be obtained from this curved by drawing a tangent parallel to the φ-line, which represents the desired active thrust, *Pa*. The corresponding rupture surface, which represents the most dangerous rupture surface, may be obtained by the line joining the heel of the wall to the end of the maximum pressure vector.

The steps in the construction may be set out as follows:

(*i*) Draw the ground line, φ-line, and ψ-line, and the wall face *AB*.

(*ii*) Choose an arbitrary failure plane *BC*1. Calculate weight of the wedge *ABC* and plot it as *B*-1 to a convenient scale on the φ-line.

(*iii*) Draw 1 – 1′ parallel to the ψ-line through 1 to meet *BC*1 in 1′. 1′ is a point on the Culmann line.

(*iv*) Similarly, take some more failure planes *BC*2, *BC*3, ..., and repeat the steps (*ii*) and (*iii*) to establish points 2′, 3′, ...

(*v*) Join *B*, 1′, 2′, 3′, etc., smoothly to obtain the Culmann curve.

(*vi*) Draw a tangent *t-t*, to the Culmann line parallel to the φ-line. Let the point of the tangency be *F′*

(*vii*) Draw *F′F* parallel to the ψ-line to meet the φ-line in *F*.

(*viii*) Join *BF′* and produce it to meet the ground line in *C*.

(*ix*) *BF′C* represents the failure surface and *F F ′* represents *Pa* to the same scale as that chosen to represent the weights of wedges. If the upper surface of the backfill is a plane, as shown in Fig., the weights of wedges will be proportional to the distances $l_1$, $l_2$ ... (bases), since they have a common-height, *H*1. Thus *B*-1, *B*-2, etc ..., may be made equal or proportional to $l_1$, $l_2$, etc. The sector scale may be easily obtained by comparing *BF* with the weight of wedge *ABC*.

Thus $P_a = \frac{\overrightarrow{F'F}}{\overrightarrow{BF}} \frac{1}{2} \gamma(H_1)\, 1. \frac{\overrightarrow{F'F}}{l} = \frac{1}{2} . \gamma H_1 (\overrightarrow{EF})$, if the bases themselves are used to represent the weight vector.

**Comparison of Coulomb's Theory with Rankine's Theory**

The following are the important points of comparison:

(*i*) Coulomb considers a retaining wall and the backfill as a system; he takes into account the friction between the wall and the backfill, while Rankine does not.

(*ii*) The backfill surface may be plane or curved in Coulomb‘s theory, but Rankine‘s allows only for a plane surface.

(*iii*) In Coulomb‘s theory, the total earth thrust is first obtained and its position and direction of the earth pressure are assumed to be known; linear variation of pressure with depth is tacitly assumed and the direction is automatically obtained fromthe concept of wall friction. In Rankine‘s theory, plastic equilibrium inside a semiinfinite soil mass is considered, pressures evaluated, a retaining wall is imagined to be interposed later, and the location and magnitude of the total earth thrust are established mathematically.

(*iv*) Coulomb‘s theory is more versatile than Rankine‘s in that it can take into account any shape of the backfill surface, break in the wall face or in the surface of the fill, effect of stratification of the backfill, effect of various kinds of surcharge on earth pressure, and the effects of cohesion, adhesion and wall friction. it lends itself to elegant graphical solutions and gives more reliable results, especially in the determination of the passive earth resistance; this is inspite of the fact that static equilibrium condition does not appear to be satisfied in the analysis.

(*v*) Rankine‘s theory is relatively simple and hence is more commonly used, while Coulomb‘s

theory is more rational and versatile although cumbersome at times; therefore, the use of the latter is called for in important situations or problems.

**Problems**

1. A retaining wall, 6 m high, retains dry sand with an angle of friction of 30° and unit weight of 16.2 kN/$m^3$. Determine the earth pressure at rest. If the water table rises to the top of the wall, determine the increase in the thrust on the wall. Assume the submerged unit weight of sand as 10 kN/$m^3$.

(*a*) Dry backfill:

$\varphi = 30°$ $H = 6$ m

$K0 = 1 - \sin 30° = 0.5$

(Also $K_0 = 0.5$ for medium dense sand)

$\sigma_0 = K_0 \gamma . H$

$$= \frac{0.5 \times 16.2 \times 600}{1000} \mathrm{N / cm^2}$$
$$= 48.6\ \mathrm{kN/m^2}$$

Thrust per metre length of the wall = 48.6 ×1/2× 6 = **145.8 kN**

(*b*) Water level at the top of the wall

The total lateral thrust will be the sum of effective and neutral lateral thrusts.

Effective lateral earth thrust, $P_0 = 1/2\ K_0\ \gamma . H^2$

$= 1/2 \times 0.5 \times 10 \times 6 \times 6$ kN / m.run

= 90 kN/m. run

Neutral lateral pressure $Pw = 1/2 \gamma_w H^2$

$\approx 1/2 \times 10 \times 6 \times 6$ kN/ m. run

≈ 180 kN/m. run

Total lateral thrust = **270 kN/m. run**

Increase in thrust = **124.2 kN/m. run**

This represents an increase of about **85.2%** over that of dry fill.

2. What are the limiting values of the lateral earth pressure at a depth of 3 metres in a uniform sand fill with a unit weight of 20 kN/m3 and a friction angle of 35°? The ground surface is level. If a retaining wall with a vertical back face is interposed, determine the total active thrust and the total passive resistance which will act on the wall.

Depth, $H$ = 3 m

$\gamma$ =20 kN/ m $^3$

$\Phi$=35$^0$

for sand fill with level surface.

Limiting values of lateral earth pressure:

$$\text{Active pressure} \quad = K_a.\gamma H = \frac{1-\sin 35^\circ}{1+\sin 35^\circ} \times 20 \times 3$$

= 0.271 × 60

= **16.26 kN/m$^2$**

$$\text{Passive pressure} \quad = K_p.\ \gamma H = \frac{1+\sin 35^\circ}{1-\sin 35^\circ} \times 20 \times 3$$

= 3.690 × 60

= **221.4 kN/m$^2$**

Total active thrust per metre run of the wall

$$P_a = \frac{1}{2}\gamma H^2 K_a = 16.26 \times \frac{1}{2} \times 3 = \mathbf{24.39\ kN}$$

Total passive resistance per metre run of the wall

$$P_p = \frac{1}{2}\gamma H^2 . K_p = 221.4 \times \frac{1}{2} \times 3 = \mathbf{332.1\ kN}$$