# MODULE-1

## Introduction:

A system is an arrangement of or a combination of different physical components connected or related in such a manner so as to form an entire unit to attain a certain objective.
Control system is an arrangement of different physical elements connected in such a manner so as to regulate, director command itself to achieve a certain objective

Any control system consists of three essential components namely input, system and output. The input is the stimulus or excitation applied to a system from an external energy source. A system is the arrangement of physical components and output is the actual response obtained from the system. The control system may be one of the following type.

1) man made
2) natural and / or biological and
3) hybrid consisting of man made and natural or biological.

Requirements of good control system are accuracy, sensitivity, noise, stability, bandwidth, speed, oscillations

Types of control systems

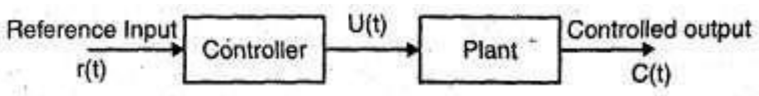
Control systems are classified into two general categories based upon the control action which is responsible to activate the system to produce the output viz.

1) Open loop control system in which the control action is independent of the out put.

2) Closed loop control system in which the control action is some how dependent upon the output and are generally called as feedback control systems.

## Open Loop System

It is a system in which control action is independent of output. To each reference input there is a corresponding output which depends upon the system and its operating conditions. The accuracy of the system depends on the calibration of the system. In the presence of noise or disturbances open loop control will not perform satisfactorily.




Example: Automatic hand driver, automatic washing machine, bread toaster, electric lift, traffic signals, coffee server, theatre lamp etc.

**Advantages of open loop system:**

1. They are simple in construction and design.
2. They are economic.
3. Easy for maintenance.
4. Not much problem of stability.
5. Convenient to use when output is difficult to measure.

**Disadvantages of open loop system**

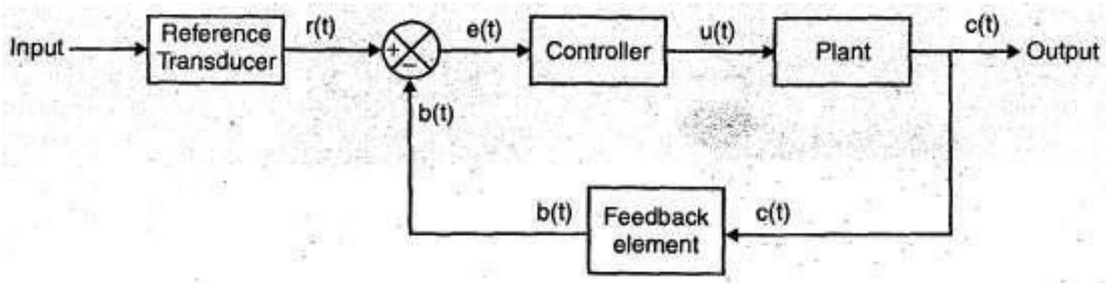
1. Inaccurate and unreliable because accuracy is dependent on accuracy of calibration.
2. Inaccurate results are obtained with parameter variations, internal disturbances.

3. To maintain quality and accuracy, recalibration of controller is necessary from time to time.

# A closed loop control system:

Is a system in which the control action depends on the output. In closed loop control system the actuating error signal, which is the difference between the input signal and the feed back signal (out put signal or its function) is fed to the controller.

The elements of closed loop system are command, reference input, error detector, control element controlled system and feedback element.




**Elements of closed loop system are:**

1. **Command :** The command is the externally produced input and independent of the feedback control system.
2. **Reference Input Element:** It is used to produce the standard signals proportional to the command.
3. **Error Detector :** The error detector receives the measured signal and compare it with reference input. The difference of two signals produces error signal.
4. **Control Element :** This regulates the output according to the signal obtained from error detector.
5. **Controlled System :** This represents what we are controlling by feedback loop.
6. **Feedback Element :** This element feedback the output to the error detector for comparison with the reference input.

**Example:** Automatic electric iron, servo voltage stabilizer, sun-seeker solar system, water level controller, human perspiration system.

**Advantages of closed loop system:**

1. Accuracy is very high as any error arising is corrected.
2. It senses changes -in output due to environmental or parametric change, internal disturbance etc. and corrects the same.
3. Reduce effect of non-linearities.
4. High bandwidth.
5. Facilitates automation.

**Disadvantages**

1. Complicated in design and maintenance costlier.
2. System may become unstable.

## Concepts of feedback:

Feedback system is that in which part of output is feeded back to input. In feedback system corrective action starts only after the output has been affected.

## Requirements of good control system :

Requirements of good control system are,

1. Accuracy
2. Sensitivity
3. Noise
4. Stability
5. Bandwidth
6. Speed
7. Oscillations

## Types of controllers:

An automatic controller compares the actual value of the system output with the reference input (desired value), determines the deviation, and produces a control signal that will reduce the deviation to zero or a small value. The manner in which the automatic controller produces the control signal is called the control action. The controllers may be classified according to their control actions as,

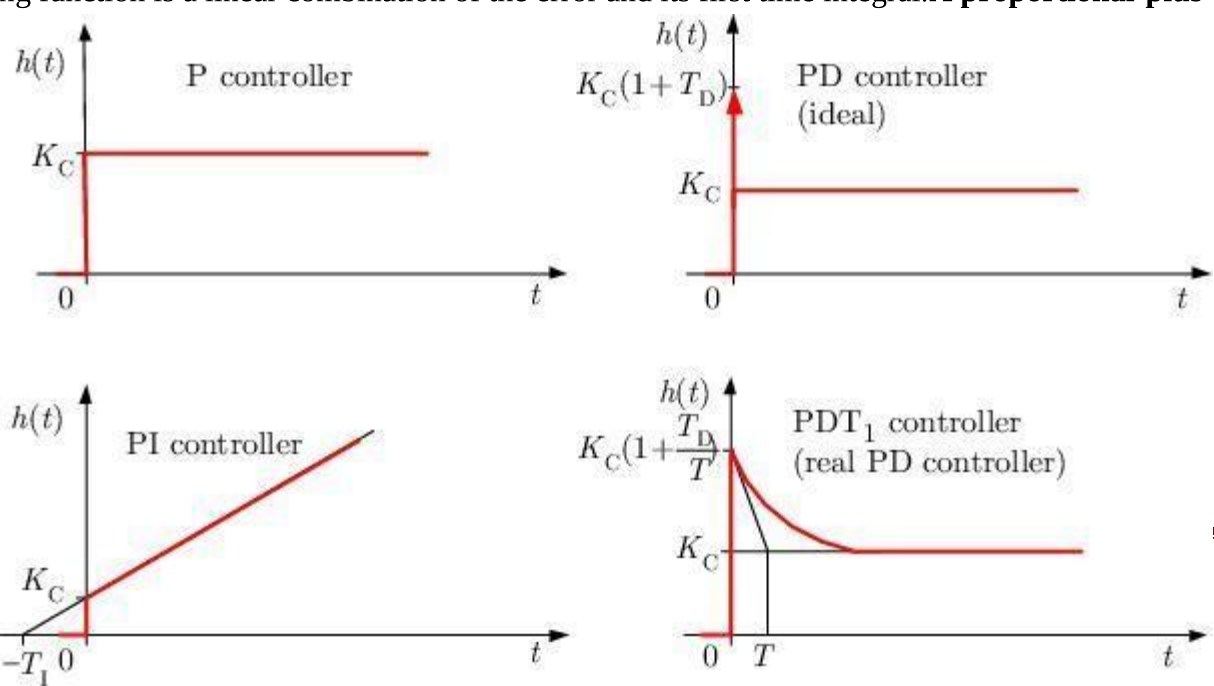
1. Proportional controllers.
2. Integral controllers.
3. Proportional-plus- integral controllers.
4. Proportional-plus-derivative controllers.
5. Proportional-plus- integral-plus-derivative controllers

**A proportional control system** is a feedback control system in which the output forcing function is directly proportional to error.

**A integral control system** is a feedback control system in which the output forcing function is directly proportional to the first time integral of error.

**A proportional-plus-derivative control system** is a feedback control system in which the output forcing function is a linear combination of the error and its first time derivative.

**A proportional-plus- integral control system** is a feedback control system in which the output forcing function is a linear combination of the error and its first time integral.**A proportional-plus-**



**derivative-plus- integral control system** is a feedback control system in which the output forcing function is a linear combination of the error, its first time derivative and its first time integral.

## OUTCOMES:

At the end of the unit, the students are able to:

- Different types of control system.
- Different types of controllers.
- Ideal requirements of a good control system.

## SELF-TEST QUESTIONS:

1. Name three applications of control systems.
2. Give three examples of open- loop systems.
3. Explain open loop control system.
4. Explain Closed loop control system.
5. Describe the requirements of ideal control system.
6. Explain controllers in a control system.
7. Explain Proportional controllers.
8. Explain Integral Proportional Integral controllers.
9. Explain Proportional Integral Differential controllers.

# MATHEMATICAL MODELS

**LESSON STRUCTURE:**

**Modeling of Control Systems**
**Modeling of Mechanical Systems**
**Mathematical Modeling of Electrical System**
**Force Voltage Analogy**
**Force Current Analogy**

**OBJECTIVES:**

To develop mathematical model for the mechanical, electrical, servo mechanism and hydraulic systems.

## Modeling of Control Systems:

The first step in the design and the analysis of control system is to build physical and mathematical models. A control system being a collection of several physical systems (sub systems) which may be of mechanical, electrical electronic, thermal, hydraulic or pneumatic type. No physical system can be represented in its full intricacies. Idealizing assumptions are always made for the purpose of analysis and synthesis. An idealized representation of physical system is called a Physical Model.

Control systems being dynamic systems in nature require a quantitative mathematical description of the system for analysis. ***This process of obtaining the desired mathematical description of the system is called Mathematical Modeling***.

In Unit 1, we have learnt the basic concepts of control systems such as open loop and feedback control systems, different types of Control systems like regulator systems, follow-up systems and servo mechanisms. We have also discussed a few simple applications. In this chapter we learn the concepts of modeling.

The requirements demanded by every control system are many and depend on the system under consideration. Major requirements are 1) Stability 2) Accuracy and 3) Speed of Response. An ideal control system would be stable, would provide absolute accuracy (maintain zero error despite disturbances) and would respond instantaneously to a change in the reference variable. Such a system cannot, of course, be produced. However, study of automatic control system theory would provide the insight necessary to make the most effective compromises so that the engineer can design the best possible system. One of the important steps in the study of control systems is modeling. Following section presents modeling aspects of various systems that find application in control engineering.

The basic models of dynamic physical systems are differential equations obtained by the application of appropriate laws of nature. Having obtained the differential equations and where possible the numerical values of parameters, one can proceed with the analysis. When the mathematical model of a physical system is solved for various input conditions, the results represent the dynamic response of the system. The mathematical model of a system is linear, if it obeys the principle of ***superposition and homogeneity***.

A mathematical model is ***linear***, if the differential equation describing it has coefficients, which are either functions of the independent variable or are constants. If the coefficients of the describing differential equations are functions of time (the independent variable), then the mathematical model is ***linear time-varying***. On the other hand, if the coefficients of the describing differential equations are constants, the model is ***linear time-invariant***. Powerful mathematical tools like the Fourier and Laplace transformations are available for use in linear systems. Unfortunately no physical system in nature is perfectly linear. Therefore certain assumptions must always be made to get a linear model.

Usually control systems are complex. As a first approximation a simplified model is built to get a general feeling for the solution. However, improved model which can give better accuracy can then be obtained for a complete analysis. Compromise has to be made between simplicity of the model and accuracy. It is difficult to consider all the details for mathematical analysis. Only most important features are considered to predict behavior of the system under specified conditions. A more complete model may be then built for complete analysis.

## Modeling of Mechanical Systems:

Mechanical systems can be idealized as spring- mass-damper systems and the governing differential equations can be obtained on the basis of Newton's second law of motion, which states that

F = ma: for rectilinear motion

where F: Force, m: mass and a: acceleration (with consistent units)

T = I α: or Jα for rotary motion

where T: Torque, I or J: moment of inertia and α: angular acceleration (with consistent units)

Mass / inertia and the springs are the energy storage elements where in energy can be stored and retrieved without loss and hence referred as conservative elements. Damper represents the energy loss (energy absorption) in the system and hence is referred as dissipative element. Depending upon the choice of variables and the coordinate system, a given physical model may lead to different mathematical models. The minimum number of independent coordinates required to determine completely the positions of all parts of a system at any instant of time defines the degrees of freedom (DOF) of the system. A large number of practical systems can be described using a finite number of degrees of freedom and are referred as discrete or lumped parameter systems. Some systems, especially those involving continuous elastic members, have an infinite number of degrees of freedom and are referred as continuous or distributed systems. Most of the time, continuous systems are approximated as discrete systems, and solutions are obtained in a simpler manner. Although treatment of a system as continuous gives exact results, the analysis methods available for dealing with continuous systems are limited to a narrow selection of problems. Hence most of the practical systems are studied by treating them as finite lumped masses, springs and dampers. In general, more accurate results are obtained by increasing the number of masses, springs and dampers-that is, by increasing the number of degrees of freedom.

Mechanical systems can be of two types:

1) Translation Systems
2) Rotational Systems.

The variables that described the motion are displacement, velocity and acceleration.

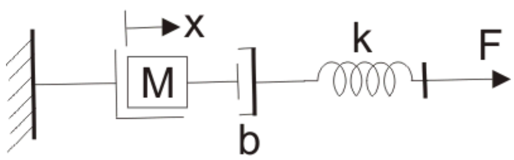
And also we have three parameters-

- Mass which is represented by 'M'.
- Coefficient of viscous friction which is represented by 'B'.
- Spring constant which is represented by 'K'.

In rotational mechanical type of systems we have three variables-

- Torque which is represented by 'T'.
- Angular velocity which is represented by 'ω'
- Angular displacement represented by 'θ'

Now let us consider the linear displacement mechanical system which is shown below-




spring mass mechanical system

We have already marked various variables in the diagram itself. We have x is the displacement as shown in the diagram. From the Newton's second law of motion, we can write force as

$$F_1 = M\frac{dx^2}{d^2y} + B\frac{dt}{dy} + Kx$$
$$F_2 = B\frac{dt}{dx}$$
$$F_3 = Kx$$

From the diagram we can see that the,

$$F = F_1+F_2+F_3$$

On substituting the values of $F_1$, $F_2$ and $F_3$ in the above equation and taking the Laplace transform we have the transfer function as,

$$T = \frac{1}{Ms^2 + Bs + K}$$

# Mathematical Modeling of Electrical System:

In electrical type of systems we have three variables -

- Voltage which is represented by 'V'.
- Current which is represented by 'I'.
- Charge which is represented by 'Q'.

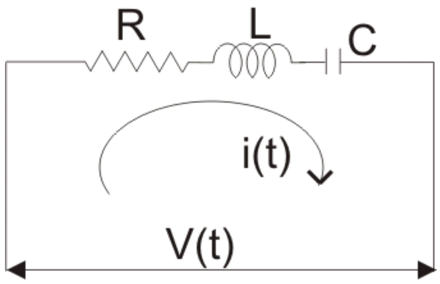
And also we have three parameters which are active and passive elements –

- Resistance which is represented by 'R'.
- Capacitance which is represented by 'C'.
- Inductance which is represented by 'L'.

Now we are in condition to derive analogy between electrical and mechanical types of systems. There are two types of analogies and they are written below:

## Force Voltage Analogy :

In order to understand this type of analogy, let us consider a circuit which consists of series combination of resistor, inductor and capacitor.




A voltage V is connected in series with these elements as shown in the circuit diagram. Now from the circuit diagram and with the help of KVL equation we write the expression for voltage in terms of charge, resistance, capacitor and inductor as,

$$V = L\frac{dt^2}{d^2q} + R\frac{dt}{dq} + \frac{q}{C}$$

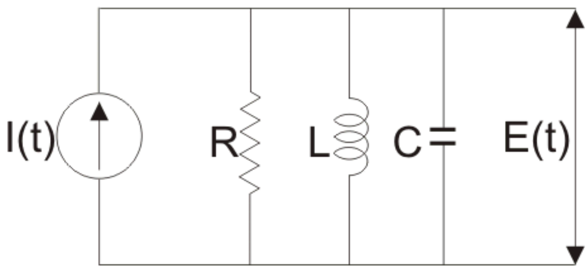
Now comparing the above with that we have derived for the mechanical system we find that-

1. Mass (M) is analogous to inductance (L).
2. Force is analogous to voltage V.
3. Displacement (x) is analogous to charge (Q).
4. Coefficient of friction (B) is analogous to resistance R and
5. Spring constant is analogous to inverse of the capacitor (C).

This analogy is known as force voltage analogy.

## Force Current Analogy :

In order to understand this type of analogy, let us consider a circuit which consists of parallel combination of resistor, inductor and capacitor.




A voltage E is connected in parallel with these elements as shown in the circuit diagram. Now from the circuit diagram and with the help of KCL equation we write the expression for current in terms of flux, resistance, capacitor and inductor as,

$$I = C\frac{dt^2}{d^2\psi} + \frac{1}{R}\frac{dt}{d\psi} + \frac{\psi}{L}$$

Now comparing the above with that we have derived for the mechanical system we find that,

1. Mass (M) is analogous to Capacitor (C).
2. Force is analogous to current I.
3. Displacement (x) is analogous to flux (ψ).
4. Coefficient of friction (B) is analogous to resistance 1/ R and
5. Spring constant K is analogous to inverse of the inductor (L).

This analogy is known as force current analogy.

**OUTCOMES:**

At the end of the unit, the students are able to:

- Mathematical modeling of mechanical, electrical, servo mechanism and hydraulic systems.
- To find Transfer function of a system.

**SELF-TEST QUESTIONS:**

1. What mathematical model permits easy interconnection of physical systems?
2. Define the transfer function.
3. What are the component parts of the mechanical constants of a motor‘s transfer function?
4. Derive the transfer function of a Spring - Mass-Damper –system.
5. Differentiate between FI and FV analogy.
6. Obtain Transfer function of Armature controlled DC motor.
7. Derive transfer function for the Electrical system shown in Figure below.