Module 1

Introduction to Limit State Design & Serviceability

Contents

1.1 Introduction

A structure refers to a system of connected parts used to support forces (loads). Buildings, bridges and towers are examples for structures in civil engineering. In buildings, structure consists of walls floors, roofs and foundation. In bridges, the structure consists of deck, supporting systems and foundations. In towers the structure consists of vertical, horizontal and diagonal members along with foundation.

A structure can be broadly classified as (i) sub structure and (ii) super structure. The portion of building below ground level is known as sub-structure and portion above the ground is called as super structure. Foundation is sub structure and plinth, walls, columns, floor slabs with or without beams, stairs, roof slabs with or without beams etc are super-structure.

Many naturally occurring substances, such as clay, sand, wood, rocks natural fibers are used to construct buildings. Apart from this many manmade products are in use for building construction. Bricks, tiles, cement concrete, concrete blocks, plastic, steel & glass etc are manmade building materials.

1.2 Objectives

1. To understand various design philosophies.
2. To understand the necessity of reinforcement in RC structure.
3. To understand the stress block parameter of RC beam section.
4. To understand the necessity of partial safety in design of RC member.

1.3 Advantages Disadvantages of RC members

Advantages

- It has high tensile and compressive strength
- It is more durable and may long up to 100 years
- It imparts ductility
- Raw materials used for construction of RC buildings are easily available and can be transported.
- Overall cost for constructing a building using RC proves to be economical compared to steel and pre-stressed structures.
- RC components can be moulded to any desired shape , if formwork is designed properly.
- If RC structures are properly designed then it can resist the earthquake forces.

Disadvantage

- Tensile strength of RC member is about 1/10th of its compressive strength

1.4 Materials required for RC member

a. Concrete

Concrete is a product obtained artificially by hardening of the mixture of cement, sand, gravel and water in predetermined proportions. Depending on the quality and proportions of the ingredients used in the mix the properties of concrete vary almost as widely as different kinds of stones. Concrete has enough strength in compression, but has little strength in tension. Due to this, concrete is weak in bending, shear and torsion. Hence the use of plain concrete is limited applications where great compressive strength and weight are the principal requirements and where tensile stresses are either totally absent or are extremely low.

Properties of Concrete

1. **Grade of concrete**

Mild	M20
Moderate	M25
Severe	M30
Very Severe	M35
Extreme	M40

2. Tensile strength

Fcr = 0.7* √fck

3. Modulus of elasticity

Ec = 5000*√fck

4. Shrinkage of concrete: Depends on

- Constituents of concrete
- Size of the member
- Environmental conditions

5. Creep of concrete: Depends on

- Strength of the concrete
- Stress in concrete
- Duration of loading

6. Durability: Mainly depends on

- Type of Environment
- Cement content
- Water cement ratio
- Workmanship
- Cover to the reinforcement

7. Cover to the reinforcement

Nominal cover is essential

- Resist corrosion
- Bonding between steel and concrete

b) Reinforcements

- Bamboo, natural fibers (jute, coir etc) and steel are some of the types of reinforcements

Roles of reinforcement in RCC

- To resist Bending moment in case of flexural members
- To reduce the shrinkage of concrete
- To improve the load carrying capacity of the compression member
- To resist the effect of secondary stresses like temperature etc.
- To prevent the development of wider cracks formed due to tensile stress

Advantages of Steel Reinforcement

- It has high tensile and compressive stress
- It is ductile in nature
- It has longer life
- It allows easy fabrication (easy to cut, bend or weld)
- It is easily available
- It has low co-efficient of thermal expansion same as that of concrete

Disadvantages of Steel Reinforcement

- More prone to corrosion
- Loses its strength when exposed to high temperature

Classification of Steel bars

1. Mild Steel plain bars

- Cold worked steel bars
- Hot rolled mild steel bars

Eg: Fe250

2. High Yield Strength Deformed (HYSD) Bars

Eg: Fe415 & Fe500

3. Steel wire Fabric

4. Structural Steel

5. CRS and TMT

1.5 Introduction to RCC Design

Objective:-

1. Structure should perform satisfactorily during its life span
2. Structure should take up the forces which are likely and deform within the limit
3. The structure should resist misuse or fire.

Design of RC member involves

1. Deciding the size or dimension of the structural element and amount of reinforcement required.
2. To check whether the adopted size perform satisfactorily during its life span.

1.6 Methods of Design or Design philosophy

1. Working stress method
2. Ultimate or load factor method
3. Limit state method

Working Stress Method – Based on Elastic theory

Assumptions**:-**

- Plane section remains plane before and after deformation takes place
- Stress –strain relation under working load, is linear for both steel and concrete
- Tensile stress is taken care by reinforcement and none of them by concrete
- Modular Ratio between steel and concrete remains constant.

Modular ratio

$$m = \frac{Es}{Ec} = \frac{280}{\sigma_{cbc}}$$

Where σ_{cbc} = is permissible stress

Advantages:-

1. Method is simple
2. Method is reliable
3. Stress is very low under working condition , therefore serviceability is automatically satisfied

Limitations:-

1. Stress strain relation for concrete is not linear for concrete
2. It gives an idea that failure load = working load * factor of safety, but it is not true
3. This method gives uneconomical section

. Ultimate load method or Load factor method

- This method uses design load = ultimate load * load factor
- Load factor = $\frac{Collapse\ Load}{Working\ Load}$
- This method gives slender and thin section which results in excessive deflection and cracks
- This method does not take care of shrinkage of concrete
- This method does not take of serviceability

Limit State Method

Limit state is an acceptable limit for both safety and serviceability before which failure occurs

1. Limit state of collapse
2. Limit state of serviceability
3. Other limit state

Limit state of Collapse

The structure may get collapse because of

- ➢ Rupture at one or more cross-sections
- ➢ Buckling
- ➢ Overturning

While designing the structure following ultimate stresses should be considered

1. Flexure
2. Shear
3. Torsion
4. Tension
5. Compression

Limit state of Serviceability

a) Limit state of deflection

- ➢ Lack of safety
- ➢ Appearance
- ➢ Ponding of water
- ➢ Misalignment in machines
- ➢ Door, window frames, flooring materials undergoes crack

Methods for controlling deflecting

- Empherical formula – span/depth
- Theoretical - dimension

b) Limit state of cracking

- Appearance
- Lack of safety
- Leakage
- Creation of maintenance problem
- Reduction in stiffness with increase in deflection
- corrosion

Other Limit states

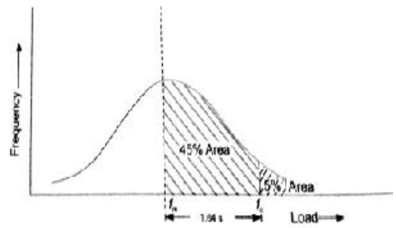
a) Vibration
b) Fire resistance
c) Chemical and environmental actions
d) Accidental loads

1.7 Types of Loads on RCC Structures

1. Dead Load IS 875 (Part 1)1987
2. Live Load IS 875 (Part 2)1987
3. Wind Load IS 875 (Part 3)1987
4. Snow Load IS 875 (Part 4)1987
5. Earthquake Load IS 1893 2002
    - Low intensity Zone (IV or less) – Zone II
    - Moderate intensity Zone (VII) – Zone III
    - Severe intensity Zone (VIII) – Zone IV
    - Very Severe intensity Zone (IX and above) – Zone V

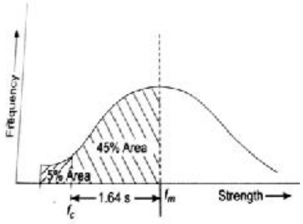
1.8 Characteristic load

Characteristic load = Mean Load+1.64S



Characteristic Strength

Characteristic Strength = Mean Strength -1.64S



1.9 Partial safety factor

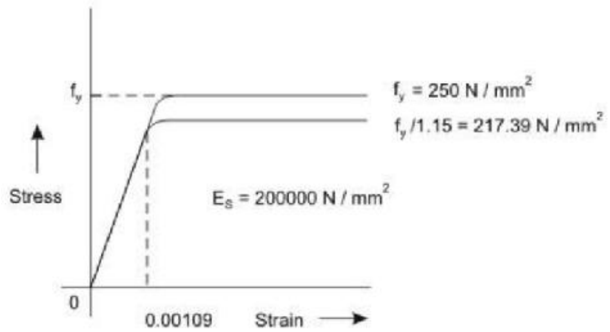
2. For material

$$f_d = \frac{f}{\gamma_m}$$

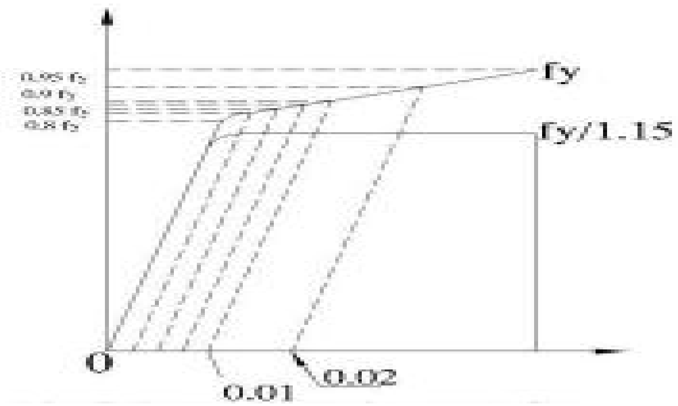
3. For load

$F_d = F * \gamma_f$

1.10 Stress-strain curves for reinforcement



Stress-strain curve for Mild steel (idealized) (Fe 250) with definite yield point



Stress-strain curve for cold worked deform bar

Figures show the representative stress-strain curves for steel having definite yield point and not having definite yield point, respectively. The characteristic yield strength *fy* of steel is assumed as the minimum yield stress or 0.2 per cent of proof stress for steel having no definite yield point. The modulus of elasticity of steel is taken to be 200000 N/mm^2

For mild steel, the stress is proportional to the strain up to the yield point. Thereafter, post yield strain increases faster while the stress is assumed to remain at constant value of *fy*.

1.11 Limit state of collapse in flexure

The behaviour of reinforced concrete beam sections at ultimate loads has been explained in detail in previous section. The basic assumptions involved in the analysis at the ultimate limit state of flexure (Cl. 38.1 of the Code) are listed here.

a) Plane sections normal to the beam axis remain plane after bending, i.e., in an initially straight beam, strain varies linearly over the depth of the section.

b) The maximum compressive strain in concrete (at the outermost fibre) *cu* shall be taken as 0.0035 in bending.

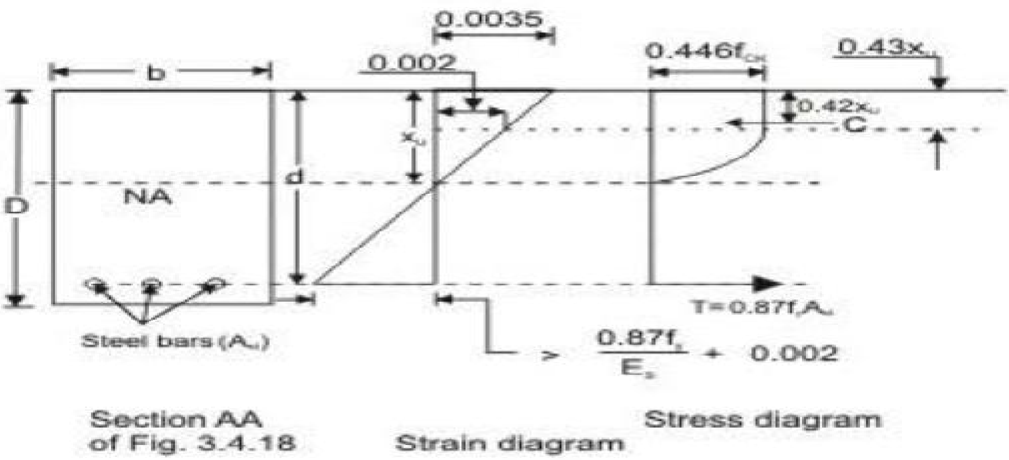
c) The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test. An acceptable stress-strain curve is given below in figure 1.6. For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor y, = 1.5 shall be applied in addition to this.

d) The tensile strength of the concrete is ignored.

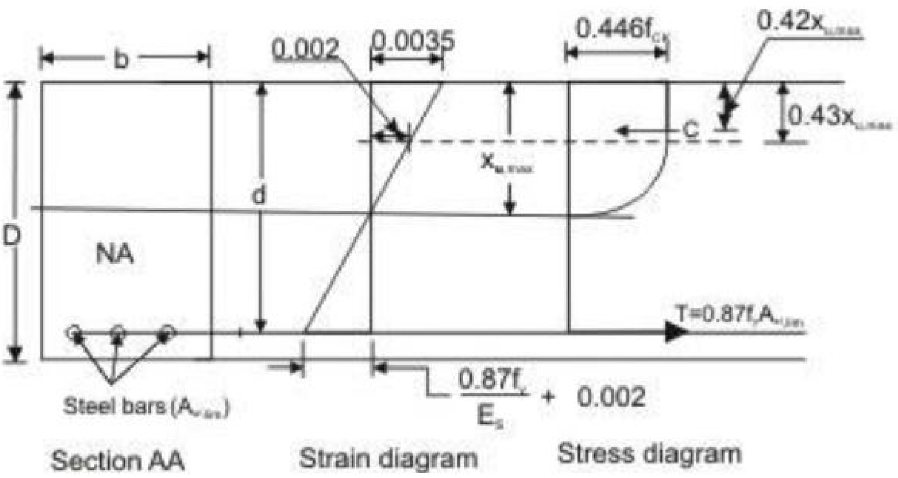
e) The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. Typical curves are given in figure 1.3. For design purposes the partial safety factor equal to 1.15 shall be applied.

f) The maximum strain in the tension reinforcement in the section at failure shall not be less than $\frac{f_y}{1.15E_s}+0.002$

1.12 Limiting Depth of Neutral Axis



Rectangular beam under flexure *xu* < *xu*max



Rectangular beam under flexure *xu* = *xu*,max

Based on the assumption given above, an expression for the depth of the neutral axis at the ultimate limit state, *xu* , can be easily obtained from the strain diagram in Fig

Considering similar triangles,

$$\frac{x_u}{d} = \frac{0.0035}{0.0035 + \frac{0.87 f_y}{E_s} + 0.002} \tag{1}$$

According to IS 456:2000 cl no 38.1 (f) ,when the maximum strain in tension reinforcement is equal to $\frac{0.87 f_y}{E_s} + 0.002$, then the value of neutral axis will be $x_{u,\max}$.

Therefore, $$\frac{x_{u,\max}}{d} = \frac{0.0035}{0.0035 + \frac{0.87 f_y}{E_s} + 0.002} \tag{2}$$

The values of *xu*,max for different grades of steel, obtained by applying Eq. (2), are listed in table.

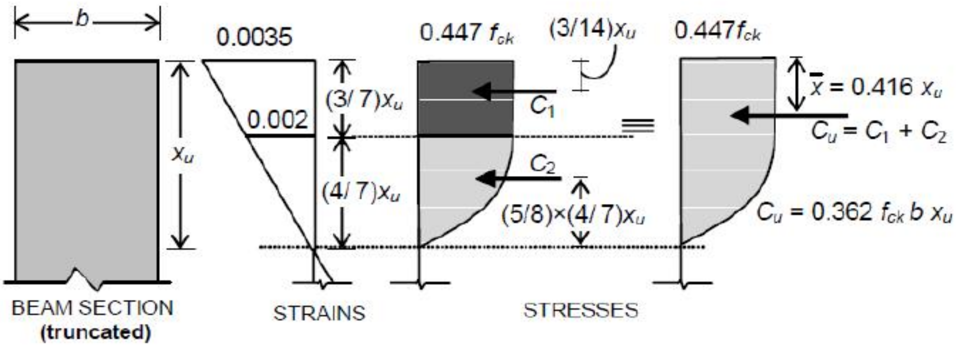
Limiting depth of neutral axis for different grades of steel

Steel Grade	Fe 250	Fe 415	Fe 500
$x_{u,\max}/d$	0.5313	0.4791	0.4791

The limiting depth of neutral axis *xu*,max corresponds to the so-called balanced section, i.e., a section that is expected to result in a balanced‘ failure at the ultimate limit state in flexure. If the neutral axis depth *xu* is less than *xu*,max , then the section is under-reinforced (resulting in a tension‘ failure); whereas if *xu* exceeds *xu*,max , it is over-reinforced (resulting in a compression‘ failure).

1.13 Analysis of Singly Reinforced Rectangular Sections

Analysis of a given reinforced concrete section at the ultimate limit state of flexure implies the determination of the *ultimate moment MR of resistance* of the section. This is easily obtained from the couple resulting from the flexural stresses



Concrete stress-block parameters in compression

$M_R = C * z = T * z$

where *C* and *T* are the resultant (ultimate) forces in compression and tension respectively and *z* is the lever arm.

$T = 0.87\, f_y A_{st}$

Concrete Stress Block in Compression

In order to determine the magnitude of Cu and its line of action, it is necessary to analyze the concrete stress block in compression. As ultimate failure of a reinforced concrete beam in flexure occurs by the crushing of concrete, for both under- and over-reinforced beams, the shape of the compressive stress distribution (‗stress block‘) at failure will be, in both cases, as shown in Fig. The value of Cu can be computed knowing that the compressive stress in concrete is uniform at 0.447 fck for a depth of 3xu/ 7, and below this it varies parabolically over a depth of 4xu/ 7 to zero at the neutral axis.

For a rectangular section of width *b*,

$$C_u = 0.447 f_{ck}\, b\left[\frac{3x_u}{7} + \left(\frac{2}{3} x \frac{4x_u}{7}\right)\right]$$

Therefore, Cu = 0.36 * fck * b * xu

Also, the line of action of *Cu* is determined by the centroid of the stress block, located at a distance *x* from the concrete fibres subjected to the maximum compressive strain.

Accordingly, considering moments of compressive forces *C u*, *C*1 and *C*2 about the maximum compressive strain location,

$$(0.362 f_{ck} b x_u)\,\mathrm{x}\,\bar{x} - (0.447 f_{ck} b x_u)\left[\left(\frac{3}{7}\right)\left(\frac{1.5x_u}{7}\right)+\left(\frac{2}{3}\,\mathrm{x}\,\frac{4}{7}\right)\left(x_u-\frac{5}{8}\,\mathrm{x}\,\frac{4x_u}{7}\right)\right]$$

Solving $\bar{x} = 0.416x_u$

Depth of Neutral Axis

For any given section, the depth of the neutral axis should be such that *Cu* = *T*, satisfying equilibrium of forces.

Equating *C* = *T*,

$$x_u - \frac{0.87 f_y A_{st}}{0.361 f_{ck} b}$$, valid only if resulting $x_u \le x_{u,\max}$

1.14 Ultimate Moment of Resistance

The ultimate moment of resistance MR of a given beam section is

Accordingly, in terms of the concrete compressive strength,

$M_{uR} = 0.361 f_{ck} b x_u (\mathrm{d} - 0.416\mathrm{x}_u)$ *for all* x_u

Alternatively, in terms of the steel tensile stress,

$M_{uR} = f_{st}\,\mathrm{A}_{st}(\mathrm{d} - 0.416\mathrm{x}_u)$ *for all* x_u

With $f_{st}=0.87f_y$ for $x_u \le x_{u,\max}$

Limiting Moment of Resistance

The *limiting moment of resistance* of a given (singly reinforced, rectangular) section, according to the Code (Cl. G−1.1), corresponds to the condition, defined by Eq. (2). From Eq. (9), it follows that:

$$M_{u,\lim} = 0.361 f_{ck} b x_{u,\max}(\mathrm{d} - 0.416\mathrm{x}_{u,\max}) \quad (11)$$

$$M_{u,\lim} = 0.361 f_{ck}\left(\frac{x_{u,\max}}{d}\right)\left(1-\frac{0.416\mathrm{x}_{u,\max}}{d}\right)bd^2 \quad (11a)$$

1.15 Modes of failure: Types of section

A reinforced concrete member is considered to have failed when the strain of concrete in extreme compression fibre reaches its ultimate value of 0.0035. At this stage, the actual strain in steel can have the following values:

(a) Equal to failure strain of steel

(b) More than failure strain, corresponding to under reinforced section.

(c) Less than failure strain corresponding to over reinforced section.

Thus for a given section, the actual value of *xu / d* can be determined from Eq. (7). Three cases arise.

Case-1: xu /*d* equal to the limiting value *xu*,max */d* : Balanced section.

Case-2: *xu /d* less than limiting value: under-reinforced section.

Case-3: *xu /d* more than limiting value: over-reinforced section.

Balanced Section

In balanced section, the strain in steel and strain in concrete reach their maximum values simultaneously. The percentage of steel in this section is known as critical or limiting steel percentage. The depth of neutral axis (NA) is $x_u = x_{u,max}$.

Under-reinforced section

An under-reinforced section is the one in which steel percentage (pt) is less than critical or limiting percentage (*pt*,lim). Due to this the actual NA is above the balanced NA and
$x_u < x_{u,max}$.

Over-reinforced section

In the over reinforced section the steel percentage is more than limiting percentage due to which NA falls below the balanced NA and $x_u > x_{u,max}$. Because of higher percentage of steel, yield does not take place in steel and failure occurs when the strain in extreme fibres in concrete reaches its ultimate value.

1.16 General Aspects of Serviceability:

The members are designed to withstand safely all loads liable to act on it throughout its life using the limit state of collapse. These members designed should also satisfy the serviceability limit states. To satisfy the serviceability requirements the deflections and cracking in the member should not be excessive and shall be less than the permissible values.

Apart from this the other limit states are that of the durability and vibrations. Excessive values beyond this limit state spoil the appearance of the structure and affect the partition walls, flooring etc. This will cause the user discomfort and the structure is said to be unfit for use.

The different load combinations and the corresponding partial safety factors to be used for the limit state of serviceability are given in Table 18 of IS 456:2000.

Load combination	Limit State of Collapse			Limit state of serviceability		
	DL	IL	WL	DL	IL	WL
DL + IL	1.5		1.0	1.0	1.0	-
DL + WL	1.5 or 0.9	-	1.5	1.0	-	1.0
DL + IL + WL	1.2			1.0	0.8	0.8

Limit state of serviceability for flexural members:

Deflection

The check for deflection is done through the following two methods specified by IS 456:2000 (Refer clause 42.1)

1 Empirical Method

In this method, the deflection criteria of the member is said to be satisfied when the actual value of span to depth ratio of the member is less than the permissible values. The IS code procedure for calculating the permissible values are as given below

a. Choosing the basic values of span to effective depth ratios (l/d) from the following, depending on the type of beam

1. Cantilever = 8
2. Simply supported = 20
3. Continuous = 26

b. Modify the value of basic span to depth ratio to get the allowable span to depth ratio.

Allowable l/d = Basic l/d x M_t x M_c x M_f

Where, M_t = Modification factor obtained from fig 4 IS 456:2000. It depends on the area of tension reinforcement provided and the type of steel.

M_c = Modification factor obtained from fig 5 IS 456:2000. This depends on the area of compression steel used.

M_f = Reduction factor got from fig 6 of IS 456:2000

Note: The basic values of l/d mentioned above is valid upto spans of 10m. The basic values are multiplied by 10 / span in meters except for cantilever. For cantilevers whose span exceeds 10 m the theoretical method shall be used.

2 Theoretical method of checking deflection

The actual deflections of the members are calculated as per procedure given in annexure 'C' of IS 456:2000. This deflection value shall be limited to the following

i. The final deflection due to all loads including the effects of temperature, creep and shrinkage shall not exceed span / 250.

ii. The deflection including the effects of temperature, creep and shrinkage occurring after erection of partitions and the application of finishes shall not exceed span/350 or 20 mm whichever is less.

Cracking in structural members

Cracking of concrete occurs whenever the tensile stress developed is greater than the tensile strength of concrete. This happens due to large values of the following:

1. Flexural tensile stress because of excessive bending under the applied load
2. Diagonal tension due to shear and torsion
3. Direct tensile stress under applied loads (for example hoop tension in a circular tank)
4. Lateral tensile strains accompanying high axis compressive strains due to Poisson's effect (as in a compression test)
5. Settlement of supports

In addition to the above reasons, cracking also occurs because of

1. Restraint against volume changes due to shrinkage, temperature creep and chemical effects.
2. Bond and anchorage failures

Cracking spoils the aesthetics of the structure and also adversely affect the durability of the structure. Presence of wide cracks exposes the reinforcement to the atmosphere due to which the reinforcements get corroded causing the deterioration of concrete. In some cases, such as liquid retaining structures and pressure vessels cracks affects the basic functional requirement itself (such as water tightness in water tank).

Permissible crack width

The permissible crack width in structural concrete members depends on the type of structure and the exposure conditions. The permissible values are prescribed in clause 35.3.2 IS 456:2000 and are shown in table below

Table: Permissible values of crack width as per IS 456:2000

No.	Types of Exposure	Permissible widths of crack at surface (mm)
1	Protected and not exposed to aggressive environmental conditions	0.3
2	Moderate environmental conditions	0.2

Control of cracking

The check for cracking in beams are done through the following 2 methods specified in IS 456:2000 clause 43.1

1. By empirical method:

In this method, the cracking is said to be in control if proper detailing (i.e. spacing) of reinforcements as specified in clause 26.3.2 of IS 456:2000 is followed. These specifications regarding the spacing have been already discussed under heading general specifications. In addition, the following specifications shall also be considered

i. In the beams where the depth of the web exceeds 750 mm, side face reinforcement shall be provided along the two faces. The total area of such reinforcement shall not be less than 0.1% of the web area and shall be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness whichever is less. (Refer clause 25.5.1.3 IS456:2000)

ii. The minimum tension reinforcement in beams to prevent failure in the tension zone by cracking of concrete is given by the following

As = 0.85 fy / 0.87 fy (Refer clause 26.5.1.1 IS 456:2000)

iii. Provide large number of smaller diameter bars rather than large diameter bars of the same area. This will make the bars well distributed in the tension zone and will reduce the width of the cracks.

2. By crack width computations In the case of special structures and in aggressive environmental conditions, it is preferred to compute the width of cracks and compare them with the permissible crack width to ensure the safety of the structure at the limit state of serviceability. The

IS 456-2000 has specified an analytical method for the estimation of surface crack width in Annexure-F which is based on the British Code (BS : 8110) specifications where the surface crack width is less than the permissible width, the crack control is said to be satisfied.

Problems:

1. Given the following data of a simply supported T beam, check the deflection criteria by empirical method
   Width of the beam (b) = 230 mm
   Effective depth (d) = 425 mm
   Effective span = 8.0 m
   Area of tension steel required = 977.5 mm^2
   Area of tension steel provided = 1256 mm^2
   Area of compression steel provided = 628 mm^2
   Type of steel = Fe 415
   Width of flange (b_f) = 0.9 m
   Width of web (b_w) = 0.3 m

Solution:

Basic $\frac{l}{d} = 20$ for simply supported beam from clause 23.2.1

Allowable $\frac{l}{d}$ = Basic $\frac{l}{d}$ x M_t x M_c x M_f

$$P_t = \frac{1265\ x\ 100}{230\ x\ 425} = 1.30\ \%$$

$$f_s = \frac{0.58\ f_y\ \times\ Area\ of\ steel\ required}{Area\ of\ steel\ provided}$$

$$f_s = \frac{0.58\ \times\ 415\ \times\ 977.5}{1256} = 187.3$$

From fig 4, for $P_t = 1.3\%$, $f_s = 187.5\ N/mm^2$

M_t

$$P_c = \frac{628\ \times\ 100}{230\ \times\ 425} = 0.65\%$$

From fig 5, for $P_c = 0.65\%$, M_c

From fig 6, for $\frac{b_w}{b_f} = \frac{0.30}{0.90} = 0.33$, M_f

Substituting a, b and c in equation (1)

We get allowable $\frac{l}{d} = 20 \times 1.1 \times 1.15 \times 0.80 = 20.2$

Actual $\frac{l}{d} = \frac{8}{0.425} = 18.82 <$ allowable $\frac{l}{d}$

Hence OK

2. A rectangular beam continuous over several supports has a width of 300 mm and overall depth of 600 mm. The effective length of each of the spans of the beam is 12.0 m. The effective cover is 25 mm. Area of compression steel provided is 942 mm^2 and area of tension steel provided is 1560 mm^2. Adopting Fe 500 steel estimate the safety of the beam for deflection control using the empirical method

Solution:

Allowable $\frac{l}{d}$ = Basic $\frac{l}{d} \times M_t \times M_c \times M_f$

Basic $\frac{l}{d} = 26$ as the beam is continuous

$$f_s = \frac{0.58\ f_y\ \times\ Area\ of\ steel\ required}{Area\ of\ steel\ provided}$$

$$f_s = \frac{0.58\ \times\ 500\ \times\ 1560}{1560} = 290$$

From fig 4, for $f_s = 290$, $P_t = 0.90$, $M_t = 0.9$

From fig 5, for $P_c = 0.54\%$, M_c .(b)

From fig 6, for $\frac{b_w}{b_f} = 1.0$, M_f (c)

The equation (1) shall be multiplied by $\frac{10}{span}$ $i.e$ $\frac{10}{12}$ as the span of the beam is greater than 10.0 m

Allowable $\frac{l}{d} = \frac{10}{12}$ x 26 x 0.9 x 1.15 x 1 = 22.4

Actual $\frac{l}{d} = \frac{12}{0.575} = 20.86 <$ allowable $\frac{l}{d}$

Hence deflection control is satisfied.

3. Find the effective depth based on the deflection criteria of a cantilever beam of 6m span. Take f_y = 415 N/mm^2, Pt = 1%, Pc = 1%.

Solution:

Allowable $\frac{l}{d}$ = Basic $\frac{l}{d}$ x M_t x M_c x M_f

Basic $\frac{l}{d}$ = 7 for cantilever beam

Assume $\frac{A_{st}\ required}{A_{st}\ provided} = 1.0$

f_s = 0.58 x 415 x 1= 240.7

From fig 4, for f_s = 240, P_t = 1%, M_t = 1.0

From fig 5, for P_c = 1%, M_c = 1.25

From fig 6, for $\frac{b_w}{b_f}$ = 1.0, M_f = 1

Allowable $\frac{l}{d}$ = 7 x 1.0 x 1.25 x 1.0 = 8.75

$d = \frac{l}{8.75} = \frac{6000}{8.75}$ = 685 mm

4. A simply supported beam of rectangular cross section 250mm wide and 450mm overall depth is used over an effective span of 4.0m. The beam is reinforced with 3 bars of 20mm diameter Fe 415 HYSD bars at an effective depth of 400mm. Two anchor bars of 10mm diameter are provided. The self weight of the beam together with the dead load on the beam is 4 kN/m. Service load acting on the beam is 10 kN/m. Using M20 grade concrete, compute
   a. Short term deflection
   b. Long term deflection

Solution:

Data b = 250 mm, D = 450 mm, d = 400 mm, f_y = 415 N/mm^2

A_{st} = 3 x $\frac{\pi}{4}$ x 20^2 = 942 mm^2, l = 4.0 m, D.L = 4 kN/m, Service load = 10 kN/m,

Total load = 14 kN/m, f_{ck} = 20, A_{sc} = 2 x $\frac{\pi}{4}$ x 10^2 = 158 mm^2

E_s = 2.1 x 10^5 , Ec = 5000 $\sqrt{f_{ck}}$ = 22360 N/mm^2

$$m = \frac{280}{3\,\sigma_{cbc}} = \frac{280}{3\,x\,7} = 13.3$$

$f_{cr} = 0.7\,\sqrt{f_{ck}} = 0.7\,\sqrt{20} = 3.13$ N/mm^2

a. Short term deflection

To determine the depth of N.A

Equating the moment of compression area to that of the tension area, we get

$$b * x * \frac{x}{2} = m * A_{st} * (d-x)$$

t the steel into equivalent concrete area

$$250 * \frac{x^2}{2} = 13 * 942 * (400-x)$$

Solving, x = 155 mm from the top

Cracked MOI $I_r = \frac{250 \times 155^3}{12} + (250 \times 155) \text{ x } (155/2)^2 + 13 \text{ x } 942\ (400 \quad 155)$

$= 10.45 \text{ x } 10^8 \text{ mm}^4$

(2) I_{gr} = Gross MOI = $\frac{250\,x\,450^3}{12} = 18.98 \text{ x } 10^8 \text{ mm}^4$

(3) M = Maximum BM under service load

$$M = \frac{w\ l^2}{8} = \frac{14 \times 4^2}{8} = 28 \text{ kN} = 28 \text{ x } 10^6 \text{ N-mm}$$

(4) Cracked moment of inertia

$$M_r = \frac{f_{cr}\,I_{gr}}{y_t} = \frac{3.13 \times 18.98 \times 10^8}{0.5 \times 450} = 26 \text{ x } 10^6 \text{ N-mm}$$

Lever arm = z = $\left(d - \frac{x}{3}\right)$

$= \left(400 - \frac{155}{3}\right) = 348.34$ mm

(5) $I_{eff} = \left[\frac{I_r}{1.2 - \left(\frac{m_r}{m}\right)\left(\frac{z}{d}\right)\left(1 - \frac{x}{d}\right)\left(\frac{b_w}{b}\right)}\right]$

$$\left[\frac{10.45 \times 10^8}{1.2 - \left(\frac{26 \times 10^6}{28 \times 10^6}\right)\left(\frac{348.34}{400}\right)\left(1 - \frac{155}{400}\right)(1)}\right]$$

$I_{eff} = 14.93 \times 10^8 \text{ mm}^4$

Further $I_r < I_{eff} < I_{gr}$

(6) Maximum short term deflection

$$a_{i(perm)} = \frac{K_w\, w\, l^4}{E_c\, I_{eff}} = \frac{5}{384}\ \frac{14 \times (4000)^2}{22360 \times 14.93 \times 10^8} = 1.39 \text{ mm}$$

$$K_w = \frac{5}{384} \text{ for SSB with UDL}$$

b. Long term deflection

(1) Shrinkage deflection (a_{cs}):

$a_{cs} = K_3 \quad _{cs} L^2$

$K_3 = 0.125$ for simply supported beam from Annexure C-3.1

$_{cs}$ = Shrinkage curvature = $K_4 \left(\frac{\epsilon_{cs}}{D}\right)$

ϵ_{cs} = Ultimate shrinkage strain of concrete (refer 6.2.4) = 0.0003

$$P_t = \frac{100 \times 942}{250 \times 400} = 0.942$$

$$P_c = \frac{100 \times 158}{250 \times 400} = 0.158$$

$P_t - P_c = (0.942 \quad 0.158) = 0.784$

$P_t - P_c = (0.942 \quad 0.158) = 0.784$

This is greater than 0.25 and less than 1.0 Hence ok.

Therefore $K_4 = 0.72 \times \frac{P_t - P_c}{\sqrt{P_t}} = 0.72 \times \frac{0.942 - 0.158}{\sqrt{0.942}}$

$K_4 = 0.58$

$$_{cs} = \frac{0.58 \times 0.0003}{450} = 3.866 \times 10^{-7}$$

$a_{cs} = K_3 \quad _{cs} L^2$

$= 0.125 \times 3.866 \times 10^{-7} \times (4000)^2$

$= 0.773$ mm

(2) Creep deflection [$a_{cc(perm)}$]

Creep deflection $a_{cc(perm)} = a_{icc(perm)} \quad a_{i(perm)}$

Where, $a_{cc(perm)}$ = creep deflection due to permanent loads

$a_{icc(perm)}$ = short term deflection + creep deflection

$a_{i(perm)}$ = short term deflection

$$a_{icc\ (perm)} = K_w \left(\frac{w\, l^3}{E_{ce}\, I_{eff}} \right)$$

$$E_{ce} = \frac{E_c}{(1+\theta)} = \frac{E_c}{(1+1.6)}$$

θ = Creep coefficient = 1.6 for 28 days loading

$a_{icc(perm)}$ = 2.6 x short term deflection

= 2.6 x $a_{i(perm)}$

= 2.6 x 1.39 = 3.614 mm

Creep deflection $a_{cc(perm)}$ = 3.614 1.39 = 2.224 mm

Total long term deflection = shrinkage deflection + Creep deflection

= 0.773 + 2.224 = 3.013 mm

Total deflection = Short term deflection + Long term deflection

= 1.39 + 3.013 = 4.402 mm

1.17 Outcome

1. Able to know various design philosophies.
2. Able to know the necessity of reinforcement in RC structure.
3. Able to know the stress block parameter of RC beam section.
4. Able to know the necessity of partial safety in design of RC member.

1.18 Assignment questions

1. What are the modes of failure of singly reinforced beam?
2. What are the methods of design philosophies?
3. What is moment of resistance?
4. What are the loads that are likely to act on the structure?
5. What is singly reinforced beam?

1.19 Future Study

https://nptel.ac.in/courses/105105105/