15 CV 33 FLUID MECHANICS NOTES

MODULE-1

- **Fluids & Their Properties**
- **Fluid Pressure and Its Measurements**

by

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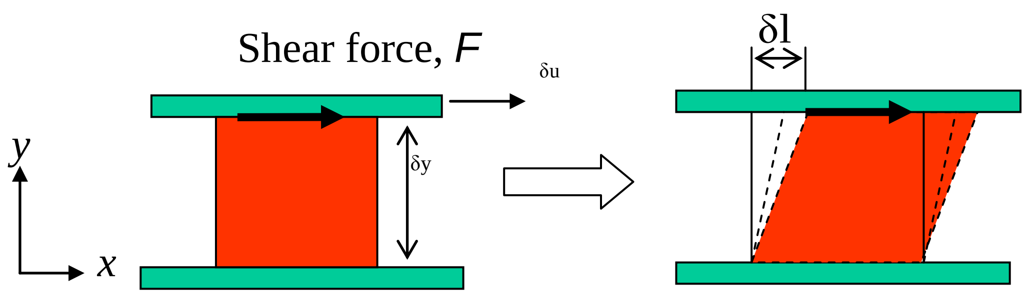
Module -1: Fluids & Their Properties:
Concept of fluid, Systems of units. Properties of fluid; Mass density, Specific weight, Specific gravity, Specific volume, Viscosity, Cohesion, Adhesion, Surface tension& Capillarity. Fluid as a continuum, Newton's law of viscosity (theory &problems).Capillary rise in a vertical tube and between two plane surfaces (theory & problems). Vapor pressure of liquid, compressibility and bulk modulus, capillarity, surface tension, pressure inside a water droplet, pressure inside a soap bubble and liquid jet. Numerical problems

1.0 INTRODUCTION: In general matter can be distinguished by the physical forms known as solid, liquid, and gas. The liquid and gaseous phases are usually combined and given a common name of fluid. Solids differ from fluids on account of their molecular structure (spacing of molecules and ease with which they can move). The intermolecular forces are large in a solid, smaller in a liquid and extremely small in gas.

Fluid mechanics is the study of fluids at rest or in motion. It has traditionally been applied in such area as the design of pumps, compressor, design of dam and canal, design of piping and ducting in chemical plants, the aerodynamics of airplanes and automobiles. In recent years fluid mechanics is truly a 'high-tech' discipline and many exciting areas have been developed like the aerodynamics of multistory buildings, fluid mechanics of atmosphere, sports, and micro fluids.

1.1 DEFINITION OF FLUID: A *fluid* is a substance which deforms continuously under the action of shearing forces, however small they may be. Conversely, it follows that: If a fluid is at rest, there can be no shearing forces acting and, therefore, all forces in the fluid must be perpendicular to the planes upon which they act.



Fluid deforms continuously under the action of a shear force

$$\tau_{yx} = \frac{dF_x}{dA_y} = f(\text{Deformation Rate})$$

Shear stress in a moving fluid:

Although there can be no shear stress in a fluid at rest, shear stresses are developed when the fluid is in motion, if the particles of the fluid move relative to each other so that they have different velocities, causing the original shape of the fluid to become distorted. If, on the other hand, the velocity of the fluid is same at every point, no shear stresses will be produced, since the fluid particles are at rest relative to each other.

Differences between solids and fluids: The differences between the behaviour of solids and fluids under an applied force are as follows:

i. For a solid, the strain is a function of the applied stress, providing that the elastic limit is not exceeded. For a fluid, the rate of strain is proportional to the applied stress.

ii. The strain in a solid is independent of the time over which the force is applied and, if the elastic limit is not exceeded, the deformation disappears when the force is removed. A fluid continues to flow as long as the force is applied and will not recover its original form when the force is removed.

Differences between liquids and gases:

Although liquids and gases both share the common characteristics of fluids, they have many distinctive characteristics of their own. A liquid is difficult to compress and, for many purposes, may be regarded as incompressible. A given mass of liquid occupies a fixed volume, irrespective of the size or shape of its container, and a free surface is formed if the volume of the container is greater than that of the liquid.

A gas is comparatively easy to compress (Fig.1). Changes of volume with pressure are large, cannot normally be neglected and are related to changes of temperature. A given mass of gas has no fixed volume and will expand continuously unless restrained by a containing vessel. It will completely fill any vessel in which it is placed and, therefore, does not form a free surface.

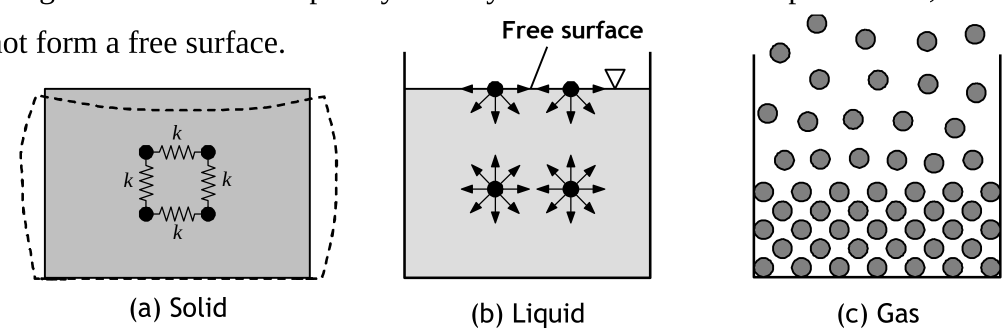


Fig.1 Comparison of Solid, Liquid and Gas

1.2 Systems of Units:

The official international system of units (System International Units). Strong efforts are underway for its universal adoption as the exclusive system for all engineering and science, but older systems, particularly the CGS and FPS engineering gravitational systems are still in use and probably will be around for some time. The chemical engineer finds many physiochemical data given in CGS units; that many calculations are most conveniently made in fps units; and that SI units are increasingly encountered in science and engineering. Thus it becomes necessary to be expert in the use of all three systems.

SI system:

Primary quantities:

Quantity	*Unit*
Mass in Kilogram	kg
Length in Meter	m
Time in Second	s or as sec
Temperature in Kelvin	K
Mole	mol

Derived quantities:

Quantity	*Unit*
Force in Newton (1 N = 1 kg.m/s^2)	N
Pressure in Pascal (1 Pa = 1 N/m^2)	N/m^2
Work, energy in Joule (1 J = 1 N.m)	J
Power in Watt (1 W = 1 J/s)	W

CGS Units:
The older centimeter-gram-second (cgs) system has the following units for derived quantities:

Quantity	*Unit*
Force in dyne (1 dyn = 1 g.cm/s^2)	dyn
Work, energy in erg (1 erg = 1 dyn.cm = 1 x 10^{-7} J)	erg
Heat Energy in calorie (1 cal = 4.184 J)	cal

Dimensions: Dimensions of the primary quantities:

Fundamental dimension	*Symbol*
Length	L
Mass	M
Time	t
Temperature	T

Dimensions of derived quantities can be expressed in terms of the fundamental dimensions.

Quantity	*Representative symbol*	*Dimensions*
Angular velocity	ω	t^{-1}
Area	A	L^2
Density	ρ	M/L^3
Force	F	ML/t^2
Kinematic viscosity	ν	L^2/t
Linear velocity	v	L/t

1.3 Properties of fluids:

1.3.1 Mass density or Specific mass (ρ):

Mass density or specific mass is the mass per unit volume of the fluid.

$$\therefore \quad \rho = \frac{Mass}{Volume}$$

$$\rho = \frac{\mathrm{M}}{\mathrm{V}} \text{ or } \frac{\mathrm{dM}}{\mathrm{dV}}$$

Unit: kg/m^3

With the increase in temperature volume of fluid increases and hence mass density decreases in case of fluids as the pressure increases volume decreases and hence mass density increases.

1.3.2 Weight density or Specific weight (γ):

Weight density or Specific weight of a fluid is the weight per unit volume.

$$\therefore \quad \gamma = \frac{Weight}{Volume} = \frac{\mathrm{W}}{\mathrm{V}} \text{ or } \frac{\mathrm{dW}}{\mathrm{dV}}$$

Unit: N/m^3 or Nm^{-3}.

With increase in temperature volume increases and hence specific weight decreases.

With increases in pressure volume decreases and hence specific weight increases.

Note: Relationship between mass density and weight density:

We have $\gamma = \dfrac{Weight}{Volume}$

$$\gamma = \frac{mass\ x\ g}{Volume}$$

$$\gamma = \rho \text{ x } g$$

1.3.3 Specific gravity or Relative density (S):

It is the ratio of density of the fluid to the density of a standard fluid.

$$S = \frac{\rho_{\text{fluid}}}{\rho_{\text{standard fluid}}}$$

Unit: It is a dimensionless quantity and has no unit.

In case of liquids water at 4^oC is considered as standard liquid. $\rho_{water} = 1000$ kg/m^3

1.3.4 Specific volume ($\forall$): It is the volume per unit mass of the fluid.

$$\therefore \quad \forall = \frac{Volume}{mass} = \frac{V}{M} \ or \ \frac{dV}{dM}$$

Unit: m^3/kg

As the temperature increases volume increases and hence specific volume increases. As the pressure increases volume decreases and hence specific volume decreases.

Solved Problems:

Ex.1 Calculate specific weight, mass density, specific volume and specific gravity of a liquid having a volume of $4 m^3$ and weighing 29.43 kN. Assume missing data suitably.

$$\gamma = ?$$
$$\rho = ?$$
$$\forall = ?$$
$$S = ?$$
$$V = 4\,m^3$$
$$W = 29.43\ kN$$
$$= 29.43 \times 10^3\,N$$

$$\gamma = \frac{W}{V}$$
$$= \frac{29.43X10^3}{4}$$
$$\gamma = 7357.58\ N/m^3$$

To find ρ - Method 1:

$$W = mg$$

$$29.43 \text{ x } 10^3 = m \text{ x } 9.81$$

$$m = 3000\ kg$$

$$\therefore\ \rho = \frac{m}{v} = \frac{3000}{4}$$

$$\rho = 750\ kg/m^3$$

Method 2:

$$\gamma = \rho\ g$$

$$7357.5 = \rho\ 9.81$$

$$\rho = 750\ kg/m^3$$

$$\rho = \frac{M}{V}$$

i) $\forall = \frac{V}{M}$

$$= \frac{4}{3000}$$

$$\forall = 1.33 \text{ x } 10^{-3} m^3/kg$$

$$\forall = \frac{V}{M}$$

$$\forall = \frac{1}{\rho} = \frac{1}{750}$$

$$\forall = 1.33X10^{-3} m^3/kg$$

$$S = \frac{\gamma}{\gamma_{Standard}} \qquad S = \frac{\rho}{\rho_{Standaard}}$$

$$= \frac{7357.5}{9810} \qquad \text{or} \qquad S = \frac{750}{1000}$$

$$S = 0.75 \qquad S = 0.75$$

Ex.2 Calculate specific weight, density, specific volume and specific gravity and if one liter of Petrol weighs 6.867N.

$$\gamma = \frac{W}{V}$$

$$V = 1\,\text{Litre}$$

$$= \frac{6.867}{10^{-3}}$$

$$V = 10^{-3}\,\text{m}^3$$

$$W = 6.867\text{N}$$

$$\gamma = 6867\,\text{N}/\text{m}^3$$

$$S = \frac{\gamma}{\gamma_{S\tan dard}}$$

$$\rho = s\ g$$

$$= \frac{6867}{9810}$$

$$6867 = \rho\ \text{x}\ 9.81$$

$$\rho = 700\text{kg}/\text{m}^3$$

$$S = 0.7$$

$$\forall = \frac{V}{M}$$

$$M = 6.867 \div 9.81$$

$$= \frac{10^{-3}}{0.7}$$

$$M = 0.7\ \text{kg}$$

$$\forall = 1.4x10^{-3} m^3 / kg$$

Ex.3 Specific gravity of a liquid is 0.7 Find i) Mass density ii) specific weight. Also find the mass and weight of 10 Liters of liquid.

$$S = \frac{\gamma}{\gamma_{Standard}}$$

$$0.7 = \frac{\gamma}{9810}$$

$$\gamma = 6867 N/m^3$$

$$\gamma = \rho g$$

$$6867 = \rho \text{ x } 9.81$$

$$\rho = 700 \text{ kg}/\text{m}^3$$

$$S = 0.7$$
$$V = ?$$
$$\rho = ?$$
$$M = ?$$
$$W = ?$$
$$V = 10 \text{ litre}$$
$$= 10\text{x}10^{-3}\text{m}^3$$

$$S = \frac{\rho}{\rho_{Standard}}$$

$$0.7 = \frac{\rho}{1000}$$

$$\rho = 700\text{kg}/\text{m}^3$$

$$\rho = \frac{M}{V}$$

$$700 = \frac{M}{10\text{x}10^{-3}}$$

$$M = 7\text{kg}$$

1.3.5 Viscosity: Viscosity is the property by virtue of which fluid offers resistance against the flow or shear deformation. In other words, it is the reluctance of the fluid to flow. Viscous force is that force of resistance offered by a layer of fluid for the motion of another layer over it.

In case of liquids, viscosity is due to cohesive force between the molecules of adjacent layers of liquid. In case of gases, molecular activity between adjacent layers is the cause of viscosity.

- **Newton's law of viscosity:**

Let us consider a liquid between the fixed plate and the movable plate at a distance 'Y' apart, 'A' is the contact area (Wetted area) of the movable plate, 'F' is the force required to move the plate with a velocity 'U' According to Newton's law shear stress is proportional to shear strain. (Fig.2)

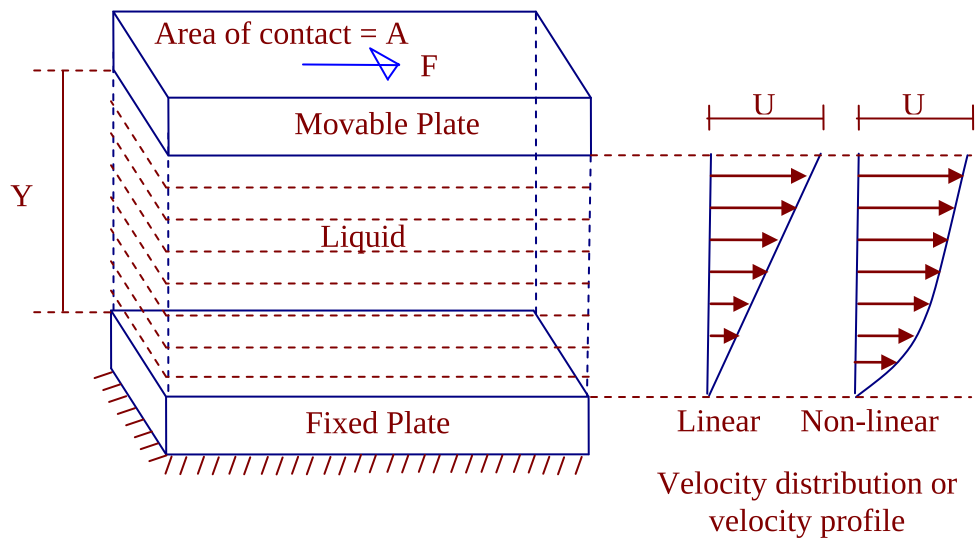


Fig.2 Definition diagram of Liquid viscosity

- F α A
- F α $\frac{1}{Y}$
- F α U

$$\therefore F \alpha \frac{AU}{Y}$$

$$F = \mu . \frac{AU}{Y}$$

'μ' is the constant of proportionality called Dynamic Viscosity or Absolute Viscosity or Coefficient of Viscosity or Viscosity of the fluid.

$$\frac{F}{A} = \mu . \frac{U}{Y} \longrightarrow \therefore \tau = \mu \frac{U}{Y}$$

‘τ’ is the force required; Per Unit area called ‘Shear Stress’. The above equation is called Newton’s law of viscosity.

Velocity gradient or rate of shear strain:

It is the difference in velocity per unit distance between any two layers.

If the velocity profile is linear then velocity gradient is given by $\frac{U}{Y}$. If the velocity profile is non – linear then it is given by $\frac{du}{dy}$.

- Unit of force (F): N.
- Unit of distance between the twp plates (Y): m
- Unit of velocity (U): m/s
- Unit of velocity gradient : $\frac{U}{Y} = \frac{m/s}{m} = /s = s^{-1}$
- Unit of dynamic viscosity (τ): $\tau = \mu \,.\, \frac{u}{y}$

$$\mu = \frac{\tau y}{U}$$

$$\Rightarrow \frac{N/m^2 \,.\, m}{m/s}$$

$$\mu \Rightarrow \frac{N\text{-}sec}{m^2} \quad \text{or} \quad \mu \Rightarrow P_a\text{-}S$$

NOTE: In CGS system unit of dynamic viscosity is $\frac{dyne \,.\, S}{Cm^2}$ and is called poise (P).

If the value of μ is given in poise, multiply it by 0.1 to get it in $\frac{NS}{m^2}$.

1 Centipoises = 10^{-2} Poise.

- **Effect of Pressure on Viscosity of fluids:**

  Pressure has very little or no effect on the viscosity of fluids.

- **Effect of Temperature on Viscosity of fluids:**

1. *Effect of temperature on viscosity of liquids:* Viscosity of liquids is due to cohesive force between the molecules of adjacent layers. As the temperature increases cohesive force decreases and hence viscosity decreases.

2. *Effect of temperature on viscosity of gases:* Viscosity of gases is due to molecular activity between adjacent layers. As the temperature increases molecular activity increases and hence viscosity increases.

- **Kinematics Viscosity:** It is the ratio of dynamic viscosity of the fluid to its mass density.

$$\therefore \text{Kinematic Viscosity} = \frac{\mu}{\rho}$$

Unit of KV:

$$KV \Rightarrow \frac{\mu}{\rho}$$

$$\Rightarrow \frac{NS/m^2}{kg/m^3}$$

$$= \frac{NS}{m^2} x \frac{m^3}{kg}$$

$$= \left(\frac{kg\,m}{s^2}\right) x \frac{s}{m^2} x \frac{m^3}{kg} = m^2/s$$

$$F = ma$$

$$N = Kg.m/s^2$$

$$\therefore \text{Kinematic Viscosity} = m^2/s$$

NOTE: Unit of kinematics Viscosity in CGS system is cm^2/s and is called stoke (S)

If the value of KV is given in stoke, multiply it by 10^{-4} to convert it into m^2/s.

The Fig. 3 illustrates how μ changes for different fluids.

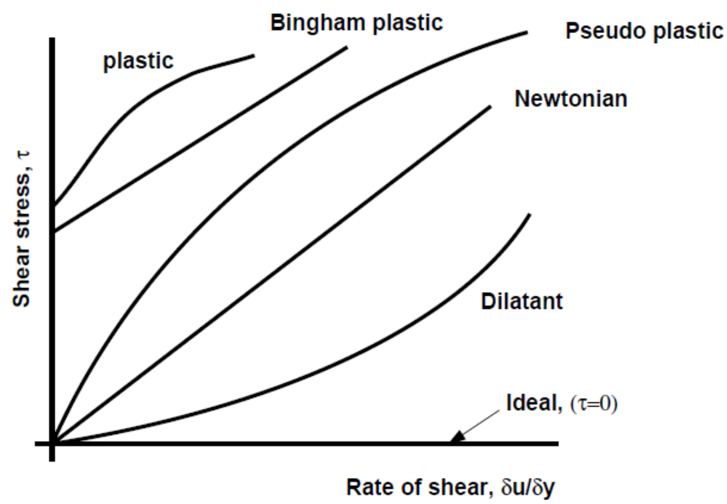
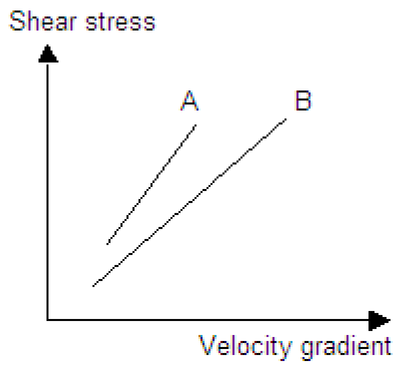


Fig.3 Variation of Viscosity based on Behaviour of Liquids

- Plastic: Shear stress must reach a certain minimum before flow commences.
- Bingham plastic: As with the plastic above a minimum shear stress must be achieved. With this classification n = 1. An example is sewage sludge.
- Pseudo-plastic: No minimum shear stress necessary and the viscosity decreases with rate of shear, e.g. colloidal substances like clay, milk and cement.
- Dilatant substances; Viscosity increases with rate of shear e.g. quicksand.
- Thixotropic substances: Viscosity decreases with length of time shear force is applied e.g. thixotropic jelly paints.
- Rheopectic substances: Viscosity increases with length of time shear force is applied
- Viscoelastic materials: Similar to Newtonian but if there is a sudden large change in shear they behave like plastic

The figure shows the relationship between shear stress and velocity gradient for two fluids, A and B. Comment on the Liquid ‘A’ and Liquid ‘B’ ?



Comments: (i) The dynamic viscosity of liquid A > the dynamic viscosity of liquid B
(ii) Both liquids follow Newton's Law of Viscosity

Solved Problems:

1. Viscosity of water is 0.01 poise. Find its kinematics viscosity if specific gravity is 0.998.

Kinematics viscosity = ?

$\mu = 0.01P$

$= 0.01x0.1$

$S = 0.998$

$$S = \frac{\rho}{\rho_{s\tan drad}}$$

$$\mu = 0.001 \frac{NS}{m^2}$$

$$\therefore \text{Kinmetic Vis cosity} = \frac{\mu}{\rho}$$

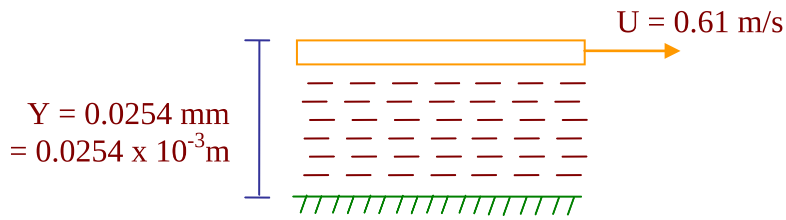
$$0.998 = \frac{\rho}{1000}$$

$$= \frac{0.001}{998}$$

$$KV = 1 \times 10^{\ 6} m^2/s$$

$$\rho = 998\ kg/m^3$$

2. A Plate at a distance 0.0254mm from a fixed plate moves at 0.61m/s and requires a force of 1.962N/m^2 area of plate. Determine dynamic viscosity of liquid between the plates.



$$\tau = 1.962 \text{ N/m}^2$$

$$\mu = ?$$

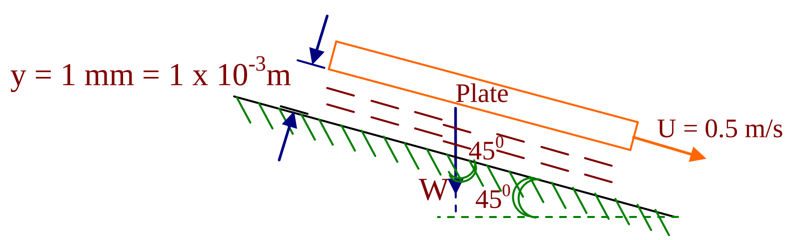
Assuming linear velocity distribution

$$\tau = \mu \frac{U}{Y}$$

$$1.962 = \mu \text{ x } \frac{0.61}{0.0254 \text{ x } 10^{-3}}$$

$$\mu = 8.17 \text{ x } 10^{-5} \frac{NS}{m^2}$$

3. A plate having an area of 1m^2 is dragged down an inclined plane at 45^0 to horizontal with a velocity of 0.5m/s due to its own weight. Three is a cushion of liquid 1mm thick between the inclined plane and the plate. If viscosity of oil is 0.1 PaS find the weight of the plate.



A =1m^2

U = 0.5m/s

Y = 1x10^{-3}m

μ = 0.1NS/m^2

$W = ?$

$F = W \text{ x } \cos 45^0$

$= W \text{ x } 0.707$

$F = 0.707W$

$$\tau = \frac{F}{A}$$

$$\tau = \frac{0.707W}{1}$$

$$\tau = 0.707W N/m^2$$

Assuming linear velocity distribution,

$$\tau = \mu.\frac{U}{Y}$$

$$0.707W = 0.1 \text{ x } \frac{0.5}{1 \text{ x } 10^{-3}}$$

$$W = 70.72 \text{ N}$$

4. A flat plate is sliding at a constant velocity of 5 m/s on a large horizontal table. A thin layer of oil (of absolute viscosity = 0.40 N-s/m^2) separates the plate from the table. Calculate the thickness of the oil film (mm) to limit the shear stress in the oil layer to 1 kPa,

   Given : τ = 1 kPa = 1000 N/m2; U = 5m/s; μ = 0.4 N-s/m^2

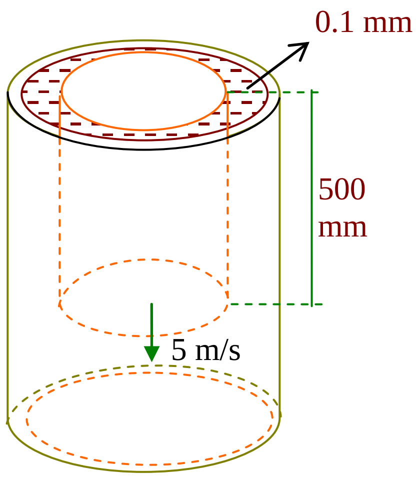
   Applying Newton's Viscosity law for the oil film -

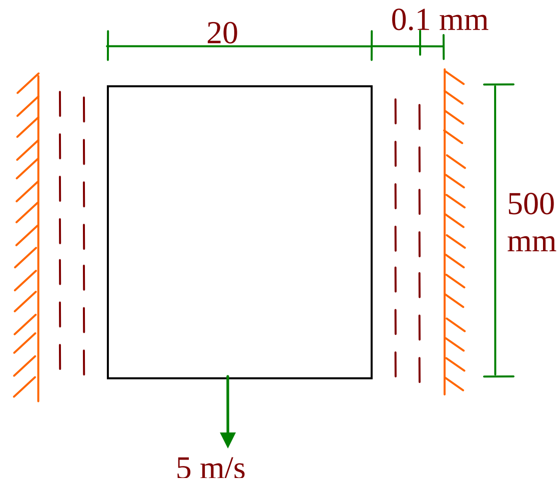
$$\tau = \mu \frac{du}{dy} = \mu \frac{U}{y}$$

$$1000 = 0.4 \frac{5}{y}$$

$$y = 2 \text{ x } 10^{-3} = 2 \text{ mm}$$

5. A shaft of ϕ 20mm and mass 15kg slides vertically in a sleeve with a velocity of 5 m/s. The gap between the shaft and the sleeve is 0.1mm and is filled with oil. Calculate the viscosity of oil if the length of the shaft is 500mm.





$D = 20mm = 20x10^{-3}m$

$M = 15$ kg

$W = 15x\ 9.81$

$\underline{W = 147.15N}$

$y = 0.1mm$

$\underline{y = 0.1 \times 10^{-3} mm}$

$U = 5m/s$

$F = W$

$\underline{F = 147.15N}$

$\mu = ?$

$A = \Pi\ D\ L$

$A = \Pi\ \times 20 \times 10^{-3} \times 0.5$

$\underline{A = 0.031\ m^2}$

$$\tau = \mu . \frac{U}{Y}$$

$$4746.7 = \mu \, x \frac{5}{0.1x10^{-3}}$$

$$\mu = 0.095 \ \frac{NS}{m^2}$$

$$\tau = \frac{F}{A}$$

$$= \frac{147.15}{0.031}$$

$$\tau = 4746.7 N / m^2$$

6. If the equation of velocity profile over 2 plate is V= $2y^{2/3}$. in which 'V' is the velocity in m/s and 'y' is the distance in 'm' . Determine shear stress at (i) y = 0 (ii) y = 75mm. Take μ = 8.35P.
   a. at y = 0
   b. at y = 75mm

      = 75 x 10^{-3}m

      τ = 8.35 P

      $= 8.35 \text{ x } 0.1 \ \frac{NS}{m^2}$

      $= 0.835 \ \frac{NS}{m^2}$

$$V = 2y^{2/3}$$

$$\frac{dv}{dy} = 2x\,\frac{2}{3}\,y^{2/3-1}$$

$$= \frac{4}{3}\,y^{-1/3}$$

$$at,\ y = 0, \frac{dv}{dy} = 3\frac{4}{\sqrt[3]{0}} = \infty$$

$$at,\ y = 75x10^{-3}m, \frac{dv}{dy} = 3\frac{4}{\sqrt[3]{75x10^{-3}}}$$

$$\frac{dv}{dy} = 3.16/s$$

$$\tau = \mu.\frac{dv}{dy}$$

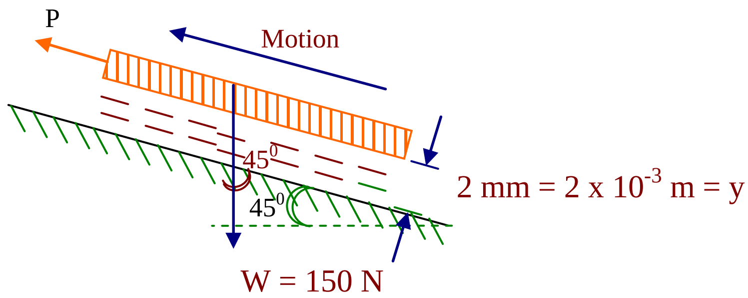
$$at, y = 0,\ \ \tau = 0.835x\infty$$

$$\tau = \infty$$

$$at, y = 75\,x10^{-3}\,m, \tau = 0.835\ x\ 3.16$$

$$\tau = 2.64\ N/m^2$$

7. A circular disc of 0.3m dia and weight 50 N is kept on an inclined surface with a slope of 45^0. The space between the disc and the surface is 2 mm and is filled with oil of dynamics viscosity $\frac{1NS}{m^2}$. What force will be required to pull the disk up the inclined plane with a velocity of 0.5m/s.



D = 0.3m

$$A = \frac{\prod x\ 0.3m^2}{4}$$

$\underline{A = 0.07m^2}$

W = 50N

$$\mu = 1\ \frac{NS}{m^2}$$

$$\underline{y = 2x10^{-3}m}$$

$$U = 0.5m/s$$

$$\tau = \mu.\frac{U}{Y}$$

$$\left(\frac{P-35,35}{0.07}\right) = 1x\ \frac{0.5}{2x10^{-3}}$$

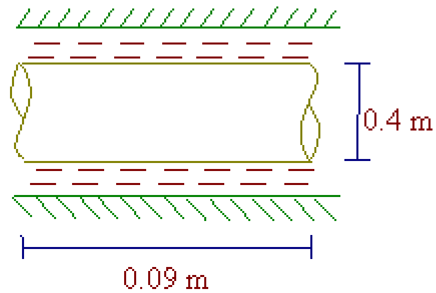
$$P = 52.85N$$

$$F = P - 50\cos 45$$

$$F = (P - 35,35)$$

$$\upsilon = \frac{(P-35.35)}{0.07}N/m^2$$

8. Dynamic viscosity of oil used for lubrication between a shaft and a sleeve is 6 P. The shaft is of diameter 0.4 m and rotates at 190 rpm. Calculate the power lost in the bearing for a sleeve length of 0.09 m .Thickness of oil is 1.5 mm.



$\mu = 6 = 0.6 \frac{NS}{m^2}$

N = 190 rpm

Power lost = ?

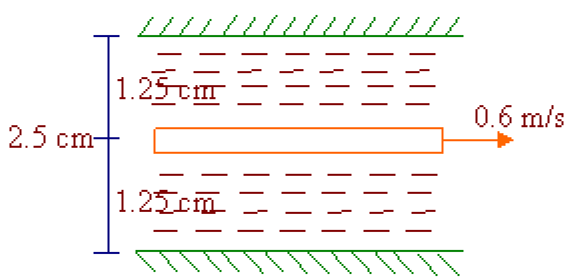
A = Π D L

= Π x 0.4 x0.09 A = $0.11 m^2$

Y = 1.5×10^{-3} m

9. Two large surfaces are 2.5 cm apart. This space is filled with glycerin of absolute viscosity 0.82 NS/m^2. Find what force is required to drag a plate of area $0.5 m^2$ between the two surfaces at a speed of 0.6m/s. (i) When the plate is equidistant from the surfaces, (ii) when the plate is at 1cm from one of the surfaces.

Case (i) When the plate is equidistant from the surfaces,



$$U = \frac{\Pi DN}{60}$$

$$= \frac{\Pi \text{ x } 0.4 \text{ x } 190}{60}$$

$$U = 3.979 \text{ m/s}$$

$$\tau = \mu.\frac{U}{Y}$$

$$= 0.6 \text{ x } \frac{3.979}{1.5 \text{ x } 10^{-3}}$$

$$\tau = 1.592 \text{ x } 10^{3} \text{ N/m}^{2}$$

$$\frac{F}{A} = 1.59 \text{ x } 10^{3}$$

$$F = 1.591 \text{x } 10^{3} \text{ x } 0.11$$

$$F = 175.01 \text{ N}$$

$$\boldsymbol{T} = \boldsymbol{F} \ \boldsymbol{x} \ \boldsymbol{R}$$

$$= 175.01\boldsymbol{x}0.2$$

$$\boldsymbol{T} = 35\boldsymbol{Nm}$$

$$\boldsymbol{P} = \frac{2\Pi\boldsymbol{NT}}{60{,}000}$$

$$\boldsymbol{P} = 0.6964\boldsymbol{KW}$$

$$\boldsymbol{P} = 696.4\boldsymbol{W}$$

Let F_1 be the force required to overcome viscosity resistance of liquid above the plate and F_2 be the force required to overcome viscous resistance of liquid below the plate. In this case $F_1 = F_2$. Since the liquid is same on either side or the plate is equidistant from the surfaces.

$$\tau_1 = \mu_1 \frac{U}{Y}$$

$$\tau_1 = 0.82x\frac{0.6}{0.0125}$$

$$\tau_1 = 39.36 N/m^2$$

$$\frac{F_1}{A} = 39.36$$

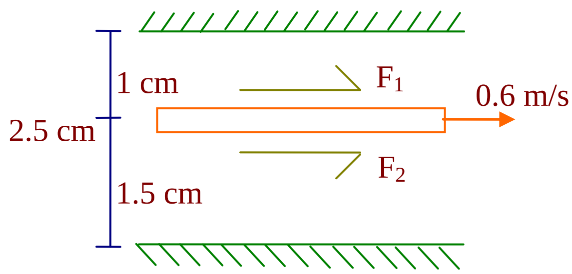
$$F_1 = 19.68N$$

$\therefore$ Tatal force required to drag the plate $=F_1 + F_2 = 19.68+19.68$

$F= 39.36N$

Case (ii) when the plate is at 1cm from one of the surfaces.

Here $F_1 \neq F_2$



$$\frac{F_1}{A} = 49.2$$

$$F_1 = 49.2x0.5$$

$$F_1 = 24.6N$$

$$\frac{F_2}{A} = 32.8$$

$$F_2 = 32.8x0.5$$

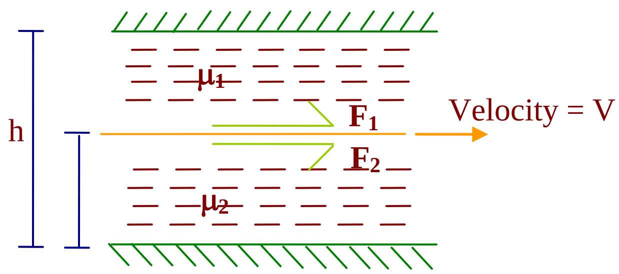
$$F_2 = 16.4N$$

Total Force $F = F_1 + F_2 = 24.6 + 16.4$

$F = 41N$

10. Through a very narrow gap of ht a thin plate of large extent is pulled at a velocity `V'. On one side of the plate is oil of viscosity μ_1 and on the other side there is oil of viscosity μ_2. Determine the position of the plate for the following conditions.
    i. Shear stress on the two sides of the plate is equal.
    ii. The pull required, to drag the plate is minimum.

Condition 1: Shear stress on the two sides of the plate is equal $F_1 = F_2$



$$y = ? \text{ for } F_1 = F_2$$

$$\tau = \mu . \frac{U}{Y}$$

$$\frac{F}{A} = \mu . \frac{U}{Y}$$

$$F = A\mu . \frac{U}{Y}$$

$$F_1 = \frac{A \mu_1 V}{(h - y)}$$

$$F_2 = \frac{A \mu_2 V}{y}$$

$$F_1 = F_2$$

$$\frac{A\mu_1 V}{h-y} = \frac{A\mu_2 V}{y}$$

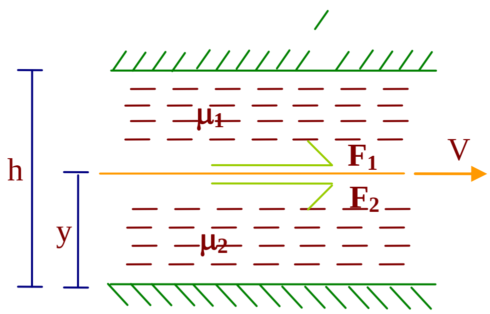
$$\mu_1 y = \mu_2 (h-y)$$

$$\mu_1 y + \mu_2 y = \mu_2 h$$

$$y = \frac{\mu_2 h}{\mu_1 + \mu_2} \text{ or } y = \frac{h}{\frac{\mu_1}{\mu_2} + 1}$$

Condition 2: The pull required, to drag the plate is minimum (i.e.$[\frac{dF}{dy}]_{minimum}$)

$\therefore$ Total drag forced required



$y = ?$ *if*, $F_1 + F_2$ *is to be* min*imum*

$$F_1 = \frac{A\mu_1 V}{h\ \ y}$$

$$F_2 = \frac{A\mu_2 V}{y}$$

$$F = F_1 + F_2$$

$$F = \frac{A\mu_1 V}{h \quad y} + \frac{A\mu_2 V}{y}$$

$$\text{For } F \text{ to be } \min . \ \frac{dF}{dy} = 0$$

$$\frac{dF}{dy} = 0 = +A\mu_1 V \equiv (h \quad y)^{\ 2} \quad A\mu_2 V y^{\ 2}$$

$$= \frac{V\mu_1 A}{(h \quad y)^{\ 2}} \quad \frac{V\mu_2 A}{y^2}$$

$$\frac{(h \quad y)^2}{y^2} = \frac{\mu_1}{\mu_2}$$

$$\frac{h \quad y}{y} = \sqrt{\frac{\mu_1}{\mu_2}}$$

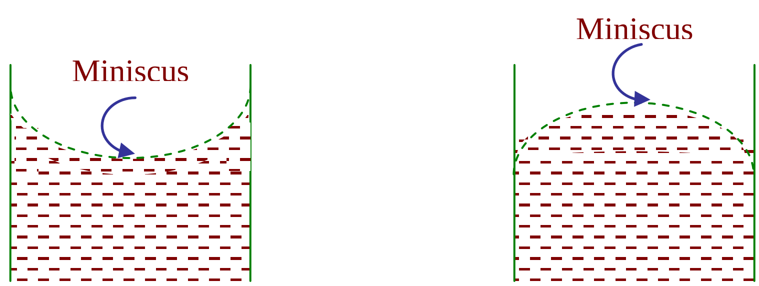
$$(h \quad y) = y\sqrt{\frac{\mu_1}{\mu_2}}$$

$$h = y\sqrt{\frac{\mu_1}{\mu_2}} + y$$

$$h = y \quad 1 + \sqrt{\frac{\mu_1}{\mu_2}}$$

$$\therefore y = \frac{h}{1 + \sqrt{\frac{\mu_1}{\mu_2}}}$$

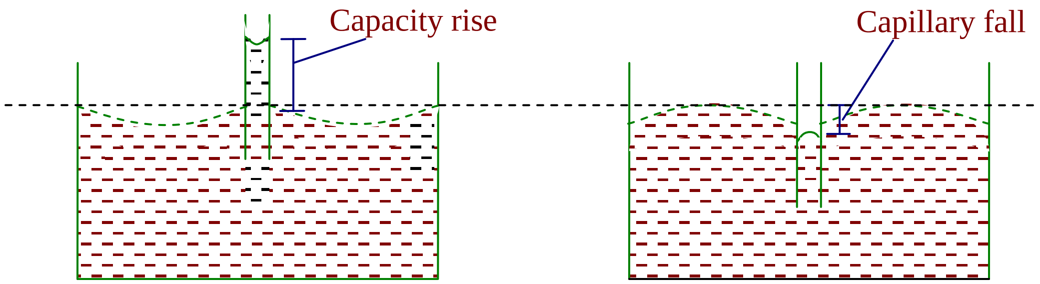
1..3.6 Capillarity :



Cohesion < Adhesion
Eg: Water

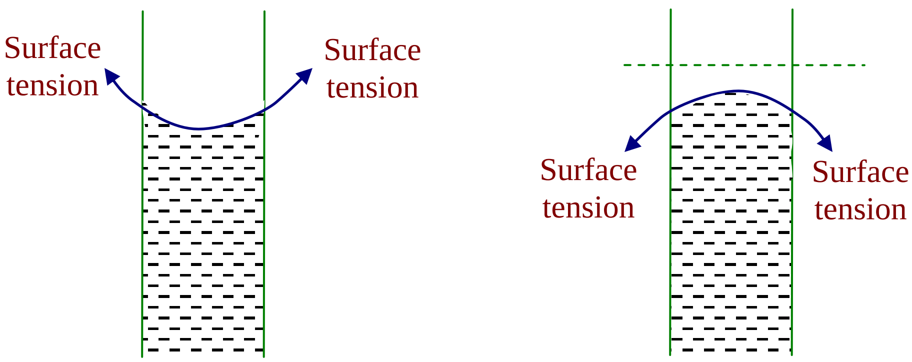
Cohesion > Adhesion
Eg: Mercury

Any liquid between contact surfaces attains curved shaped surface as shown in figure. The curved surface of the liquid is called Meniscus. If adhesion is more than cohesion then the meniscus will be concave. If cohesion is greater than adhesion meniscus will be convex.



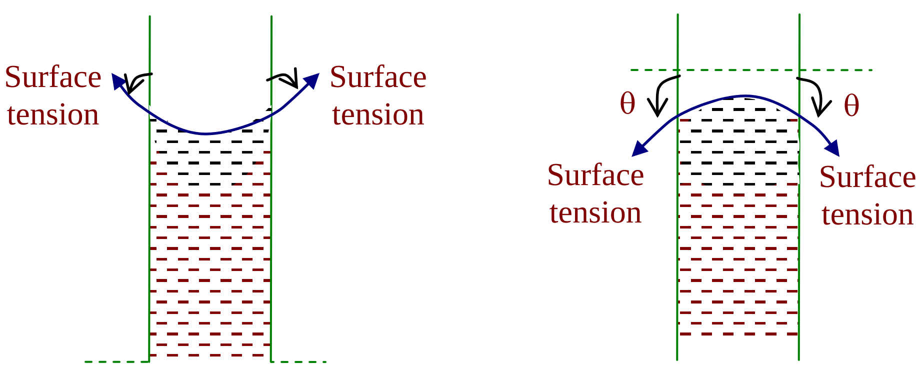
Cohesion < Adhesion
Eg: Water

Cohesion > Adhesion
Eg: Mercury



Capillarity is the phenomena by which liquids will rise or fall in a tube of small diameter dipped in them. Capillarity is due to cohesion adhesion and surface tension of liquids. If adhesion is more than cohesion then there will be capillary rise. If cohesion is greater than adhesion then will be capillary fall or depression. The surface tensile force supports capillary rise or depression.

Angle of contact:

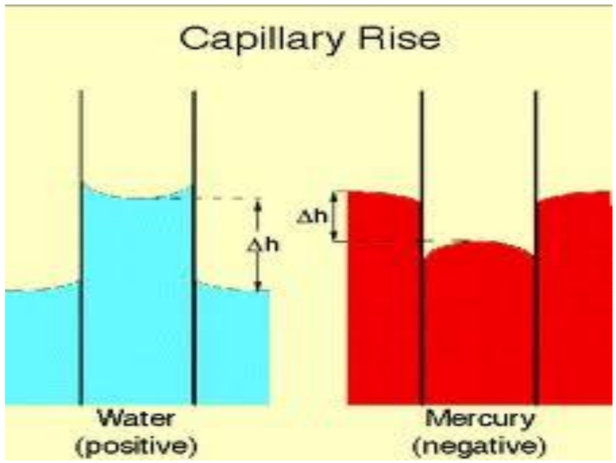


θ → Angle of contact
→ Acute

θ → Angle of contact
→ Obtuse

Note:

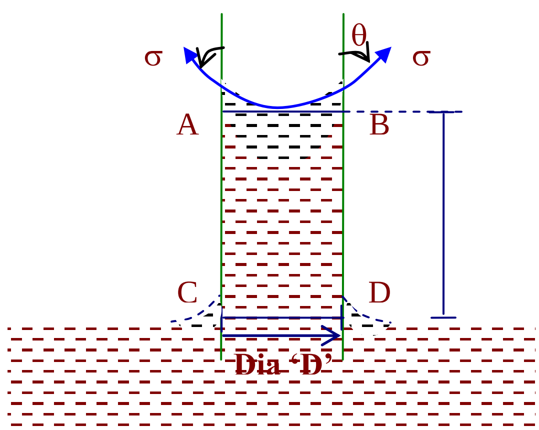
The angle between surface tensile force and the vertical is called angle of contact. If adhesion is more than cohesion then angle of contact is obtuse.



- **To derive an expression for the capillary rise of a liquid in small tube dipped in it:**

Let us consider a small tube of diameter 'D' dipped in a liquid of specific weight γ. 'h' is the capillary rise. For the equilibrium,

Vertical force due to surface tension = Weight of column of liquid ABCD



$$[\sigma(\Pi D)]\cos\theta = \gamma \text{ x volume}$$

$$[\sigma(\Pi D)]\cos\theta = \gamma \text{ x } \frac{\Pi D^2}{4} \text{ x h}$$

$$h = \frac{4\sigma\cos\theta}{\gamma D}$$

It can be observed that the capillary rise is inversely proportional to the diameter of the tube.

Note:

The same equation can be used to calculate capillary depression. In such cases ‘ θ ’ will be obtuse ‘h’ works out to be –ve.

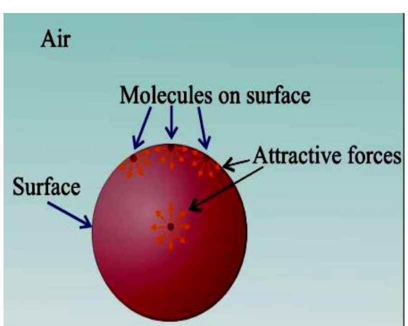
Excess Pressure inside a Water Droplet:

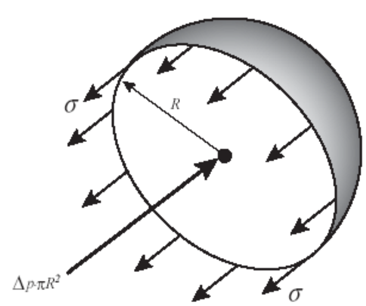
Pressure inside a Liquid droplet: Liquid droplets tend to assume a spherical shape since a sphere has the smallest surface area per unit volume.

The pressure inside a drop of fluid can be calculated using a free-body diagram of a spherical shape of radius R cut in half, as shown in Figure below and the force developed around the edge of the cut sphere is 2πRσ. This force must be balance with the difference between the internal pressure pi and the external pressure Δp acting on the circular area of the cut. Thus,

$$2\pi R\sigma = \Delta p \pi R^2$$

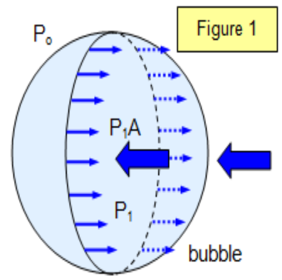
$$\Delta p = (p_{internal} - p_{external}) = \frac{2\times\sigma}{R} = \frac{4\times\sigma}{D}$$





The excess pressure within a Soap bubble:

The fact that air has to be blown into a drop of soap solution to make a bubble should suggest that the pressure within the bubble is greater than that outside. This is in fact the case: this excess pressure creates a force that is just balanced by the inward pull of the soap film of the bubble due to its surface tension.



Consider a soap bubble of radius r as shown in Figure 1. Let the external pressure be P_o and the internal pressure P_1. The excess pressure ΔP within the bubble is therefore given by: Excess pressure $\Delta p = (P_1 - P_0)$

Consider the left-hand half of the bubble. The force acting from right to left due to the internal excess pressure can be shown to be PA, where A is the area of a section through the centre of the bubble. If the bubble is in equilibrium this force is balanced by a force due to surface tension acting from left to right. This force is $2x2\pi r\sigma$ (the factor of 2 is necessary because the soap film has two sides) where 'σ' is the coefficient of surface tension of the soap film. Therefore

$2x2\pi r\sigma = \Delta pA = \Delta p\pi r^2$ giving:

Excess pressure in a soap bubble (P) = $4\sigma/r$

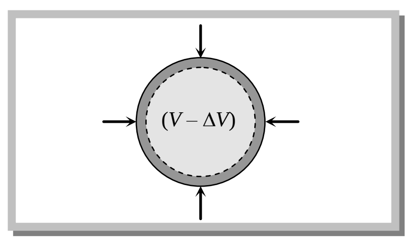
Bulk Modulus (K):

When a solid or fluid (liquid or gas) is subjected to a uniform pressure all over the surface, such that the shape remains the same, then there is a change in volume.

Then the ratio of normal stress to the volumetric strain within the elastic limits is called as Bulk modulus. This is denoted by K.

$$K = \frac{\text{Normal stress}}{\text{volumetric strain}}$$

$$K = \frac{F/A}{-\Delta V/V} = \frac{-pV}{\Delta V}$$



where p = increase in pressure; V = original volume; ΔV = change in volume

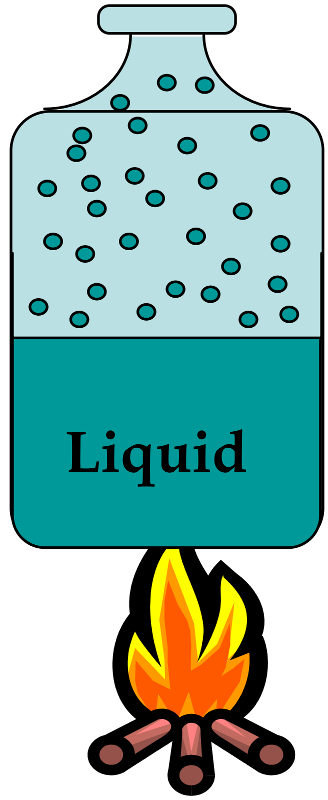
The negative sign shows that with increase in pressure p, the volume decreases by ΔV i.e. if p is positive, ΔV is negative. The reciprocal of bulk modulus is called compressibility.

$$\boldsymbol{C = Compressibility = \frac{1}{K} = \frac{\Delta V}{pV}}$$

S.I. unit of compressibility is N–1m2 and C.G.S. unit is dyne–1 cm2.

Vpour Pressure:

Vapor pressure is defined as the pressure at which a liquid will boil (vaporize) and is in equilibrium with its own vapor (fig). Vapor pressure rises as temperature rises. For example, suppose you are camping on a high mountain (say 3,000m in altitude); the atmospheric pressure at this elevation is about 70 kPa and the boiling temperature is around 90°C. This has consequences for cooking. For example, eggs have to be cooked longer at elevation to become hard-boiled since they cook at a lower temperature.



A pressure cooker has the opposite effect. Namely, the tight lid on a pressure cooker causes the pressure to increase above the normal atmospheric value. This causes water to boil at a temperature even greater than 100°C; eggs can be cooked a lot faster in a pressure cooker!

Vapor pressure is important to fluid flows because, in general, pressure in a flow decreases as velocity increases. This can lead to ***cavitation***, which is generally destructive and undesirable. In particular, at high speeds the local pressure of a liquid sometimes drops below the vapor pressure of the liquid. In such a case, ***cavitation*** occurs. In other words, a "cavity" or bubble of vapor appears because the liquid vaporizes or boils at the location where the pressure dips below the local vapor pressure.

Cavitation is not desirable for several reasons. First, it causes noise (as the cavitation bubbles collapse when they migrate into regions of higher pressure). Second, it can lead to inefficiencies and reduction of heat transfer in pumps and turbines (turbo machines). Finally, the collapse of these cavitation bubbles causes pitting and corrosion of blades and other surfaces nearby. The left figure below shows a cavitating propeller in a water tunnel, and the right figure shows cavitation damage on a blade.

Problems:

1. Capillary tube having an inside diameter 5mm is dipped in water at 20^0. Determine the heat of water which will rise in tube. Take σ =0.0736N/m at 20^0 C.

$$h = \frac{4\sigma\cos\theta}{\gamma D}$$

$$= \frac{4 \text{ x } 0.0736 \text{ x } \cos\theta}{9810 \text{ x } 5 \text{ x } 10^{-3}}$$

$$h = 6 \text{ x } 10^{-3} \text{m}$$

$\theta = 0^0$ (assumed)

$\gamma = 9810 \text{N/m}^3$

2. Calculate capillary rise in a glass tube when immersed in Hg at 20^0c. Assume σ for Hg at 20^0c as 0.51N/m. The diameter of the tube is 5mm. θ = 130^0c.

$$h = \frac{4\sigma\cos\theta}{\gamma D}$$

$$h = -1.965 \text{ x} 10^{-3} \text{m}$$

$$S = \frac{\gamma}{\gamma_{Standard}}$$

$$13.6 = \frac{\gamma}{9810}$$

$$\gamma = 133.416 \text{ x } 10^3 \text{N/m}^3$$

-ve sign indicates capillary depression.

3. Determine the minimum size of the glass tubing that can be used to measure water level if capillary rise is not to exceed 2.5mm. Take σ = 0.0736 N/m.

$$h = \frac{4\sigma\cos\theta}{\gamma D}$$

$$D = \frac{4 \text{ x } 0.0736 \text{ x} \cos 0}{9810 \text{ x } 2.5 \text{x } 10^{-3}}$$

$$D = 0.012 \text{m}$$

$$D = 12 \text{mm}$$

$D = ?$

$h = 2.5 \text{x} 10^{-3} \text{m}$

$\sigma = 0.0736 \text{ N/m}$

4. A glass tube 0.25mm in diameter contains Hg column with air above it. If σ = 0.51N/m, what will be the capillary depression? Take θ = - 40^0 or 140^0.

$$h = \frac{4\sigma\cos\theta}{\gamma D}$$

$$D = 0.25x10^{-3} m$$

$$\sigma = 0.51 N/m$$

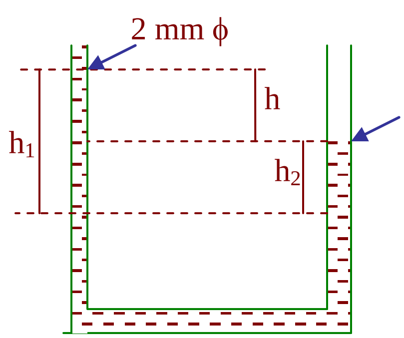
$$= \frac{4x0.51x\cos140}{133.146x10^{-3} x0.25x10^{-3}}$$

$$\theta = 140$$

$$h = -46.851x10^{-3} m$$

$$\gamma = 133.416 \text{ x } 10^3 \text{ N/m}^2$$

5. If a tube is made so that one limb is 20mm in ϕ and the other 2mm in ϕ and water is poured in the tube, what is the difference in the level of surface of liquid in the two limbs. σ = 0.073 N/m for water.



$$h_1 = h = \frac{4\sigma\cos\theta}{\gamma D}$$

$$= \frac{4 \text{ x } 0.073 \text{ x } \cos o}{9810 \text{ x } (20 \text{x} 10^{-3})}$$

$$= 0.01488\text{m}$$

$$h_2 = \frac{4 \text{ x } 0.073\text{x} \cos o}{9810 \text{ x } (20 \text{x} 10^{-3})}$$

$$= 1.488 \text{ x } 10^{-3}\text{m}$$

$$h = h_1 - h_2$$

$$= 0.01339\text{m}$$

$$h = 13.39\text{mm}$$

6. A clean glass tube is to be selected in the design of a manometer to measure the pressure of kerosene. Specific gravity of kerosene = 0.82 and surface tension of kerosene = 0.025 N/m. If the capillary rise is to be limited to 1 mm, calculate the smallest diameter (cm) of the glass tube

Soln. Given For kerosene σ = 0.025 N/m ; Sp.Gr. = 0.82; h_{max} = 1mm

Assuming contact angle θ = 0°, $\gamma_{kerosene}$ =0 .82 x 9810 = 8044.2 N/m^3

Let 'd' be the smallest diameter of the glass tube in Cm

Then using formula for capillary rise in (h)

$$h = \frac{4\ \sigma\ \text{Cos}\theta}{\gamma_{kerosene}(\frac{d_{cm}}{100})} = \frac{4\text{x}0.025\ \text{Cos}0^{\circ}}{8044.2\text{x}(\frac{d_{cm}}{100})} = \frac{1}{1000}$$

$$d_{cm} = 1.24\ \text{Cm}$$

7. The surface tension of water in contact with air at 20°C is 0.0725 N/m. The pressure inside a droplet of water is to be 0.02 N/cm^2 greater than the outside pressure. Calculate the diameter of the droplet of water.

   Given: Surface Tension of Water σ =0.0725 N/m, Δp= 0.02 N/cm^2 = 0.02×10^{-4}N/m^2

   Let 'D' be the diameter of jet

$$\Delta \boldsymbol{p} = \frac{4\sigma}{\boldsymbol{D}}$$

$$0.02 \times 10^{-4} = \frac{4 \times 0.0725}{\boldsymbol{D}}$$

$$D = 0.00145\text{m} = 1.45\text{mm}$$

8. Find the surface tension in a soap bubble of 40mm diameter when inside pressure is 2.5 N/m^2 above the atmosphere.

   Given: D = 40mm = 0.04 m, Δp = 2.5 N/m^2

   Let 'σ' be the surface tension of soap bubble

$$\Delta \boldsymbol{p} = \frac{8\sigma}{\boldsymbol{D}}$$

$$2.5 = \frac{4\sigma}{0.04}$$

$$\sigma = 0.0125 \text{ N/m}$$

9. Determine the bulk modulus of elasticity of a liquid, if the pressure of the liquid is Increased from 70 N/cm^2 to 130 N/cm^2 The volume of the liquid decreases by 0.15per cent

   Given: Initial Pressure = 70 N/cm^2, Final Pressure = 130 N/cm^2
   Decrease in Volume = 0.15%

   $\therefore$ Δp = Increase in Pressure = (130-70) = 60 N/cm^2

$$\boldsymbol{K} = \frac{\Delta \boldsymbol{p}}{\left(-\frac{\Delta\forall}{\forall}\right)} = \frac{60}{\left(\frac{0.15}{100}\right)} = 4\times10^4 \text{ N/cm}^2$$

Module -1: 2.Fluid Pressure and Its Measurements:

Definition of pressure, Pressure at a point, Pascal's law, Variation of pressure with depth. Types of pressure. Measurement of pressure using simple, differential & inclined manometers (theory & problems). Introduction to Mechanical and electronic pressure measuring devices.

2.0 INTRODUCTION: Fluid is a state of matter which exhibits the property of flow. When a certain mass of fluids is held in static equilibrium by confining it within solid boundaries (Fig.1), it exerts force along direction perpendicular to the boundary in contact. This force is called fluid pressure (compression).

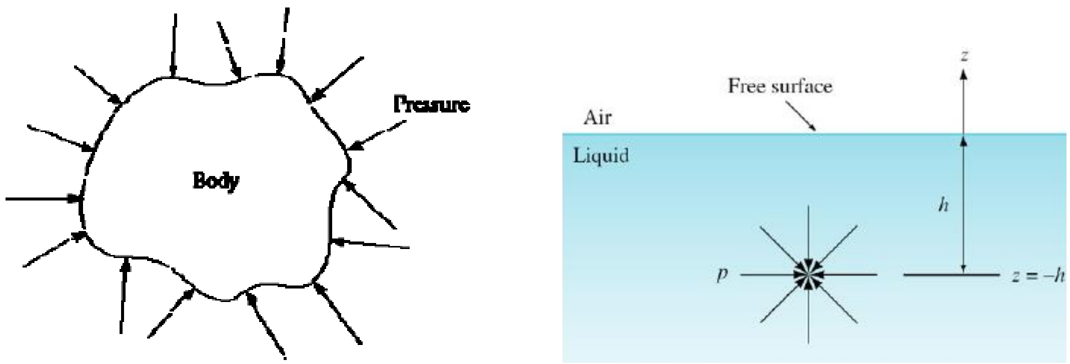


Fig.1 Definition of Pressure

In fluids, gases and liquids, we speak of pressure; in solids this is normal stress. For a fluid at rest, the pressure at a given point is the same in all directions. Differences or gradients in pressure drive a fluid flow, especially in ducts and pipes.

2.1 Definition of Pressure: Pressure is one of the basic properties of all fluids. Pressure (p) is the force (F) exerted on or by the fluid on a unit of surface area (A). Mathematically expressed:

$$\boldsymbol{p} = \frac{\boldsymbol{F}}{\boldsymbol{A}} \quad \left(\frac{\boldsymbol{N}}{\boldsymbol{m}^2}\right)$$

The basic unit of pressure is Pascal (Pa). When a fluid exerts a force of 1 N over an area of 1m^2, the pressure equals one Pascal, i.e., 1 Pa = 1 N/m^2.Pascal is a very small unit, so that for typical power plant application, we use larger units:

Units: 1 kilopascal (kPa) = 10^3 Pa, and

1 megapascal (MPa) = 10^6 Pa = 10^3 kPa.

2.2 Pressure at a Point and Pascal's Law:

Pascal's Principle: Pressure extends uniformly in all directions in a fluid.

By considering the equilibrium of a small triangular wedge of fluid extracted from a static fluid body, one can show (Fig.2) that for *any* wedge angle θ, the pressures on the three faces of the wedge are equal in magnitude:

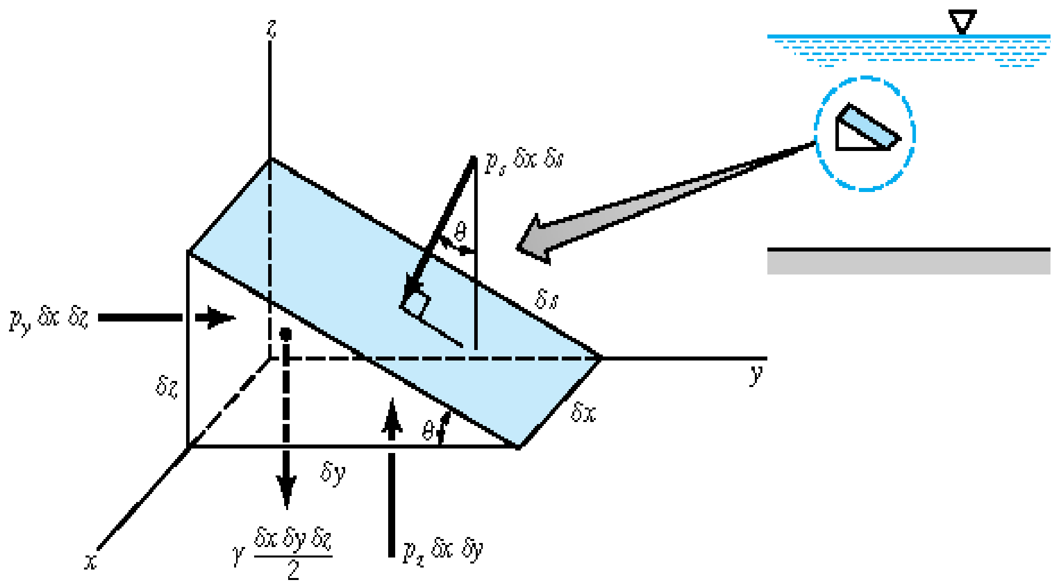


Fig.2 Pascal's Law

Independent of $p_x = p_y = p_z$ independent of 'θ'

Pressure at a point has the same magnitude in all directions, and is called **isotropic.**

This result is known as **Pascal's law**.

2.3 Pascal's Law: In any closed, static fluid system, a pressure change at any one point is transmitted undiminished throughout the system.

2.3.1 Application of Pascal's Law:

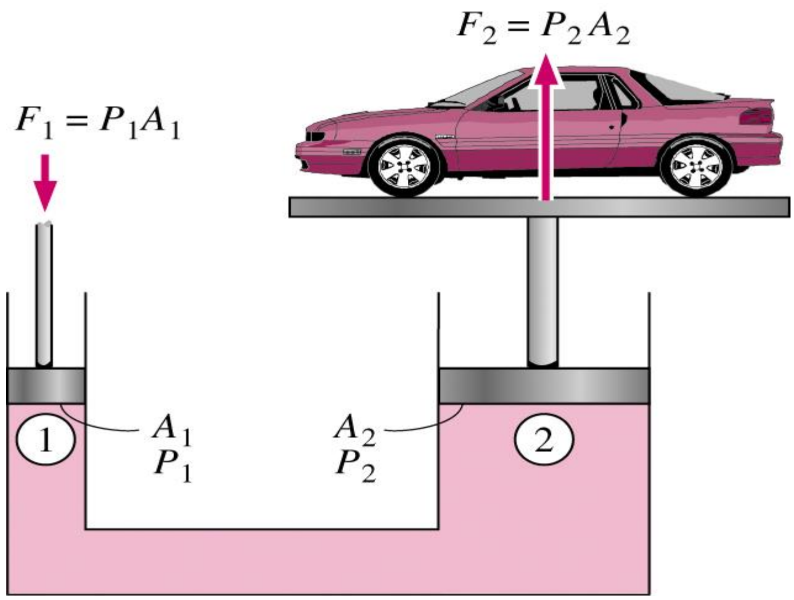


Fig.3 Application of Pascal's Law

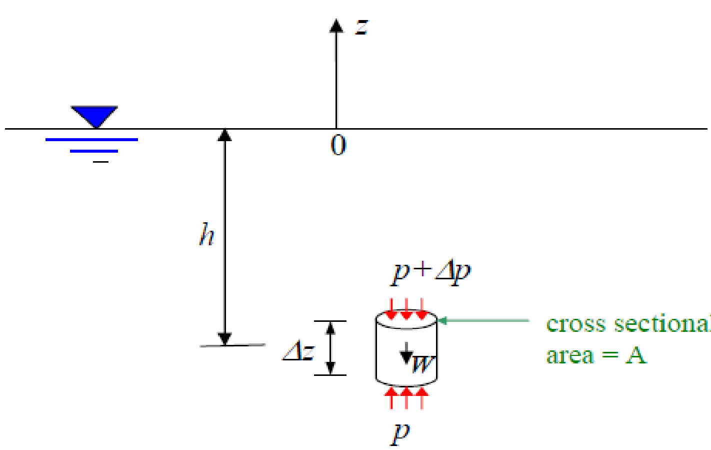
- Pressure applied to a confined fluid increases the pressure throughout by the same amount.
- In picture, pistons are at same height:

$$P_1 = P_2 \rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2} \rightarrow \frac{F_2}{F_1} = \frac{A_2}{A_1}$$

- Ratio A_2/A_1 is called ideal mechanical advantage

2.4 Pressure Variation with Depth:

Consider a small vertical cylinder of fluid in equilibrium, where *positive z is pointing vertically upward*. Suppose the origin z = 0 is set at the free surface of the fluid. Then the pressure variation at a depth *z* = -*h* below the free surface is governed by



$$(p+\Delta p)A+W = pA$$

$$\Rightarrow \Delta pA+\rho gA\Delta z=0$$

$$\Rightarrow \Delta p = -\rho g\Delta z$$

$$\Rightarrow \frac{dp}{dz} = -\rho g \quad \text{or} \quad \frac{dp}{dz} = -\gamma \quad Eq.(1) \quad (\text{as } \Delta z \to 0)$$

Therefore, the hydrostatic pressure increases linearly with depth at the rate of the specific weight $\gamma = \rho g$ of the fluid.

Homogeneous fluid: ρ **is constant**

By simply integrating the above equation-1:

$$\int dp = -\int \rho g \, dz \quad \Rightarrow \quad p = -\rho gz + C$$

Where *C* is constant of integration

When $z = 0$ (on the free surface), $p = C = p_0$ = (the atmospheric pressure).

Hence, $$p = -\rho gz + p_0$$

Pressure given by this equation is called ABSOLUTE PRESSURE, i.e., measured above perfect vacuum.

However, for engineering purposes, it is more convenient to measure the pressure above a datum pressure at atmospheric pressure. By setting $p_0 = 0$,

$$p = -\rho gz + 0 = -\rho gz = \rho gh$$

$$p = \gamma h$$

The equation derived above shows that when the density is constant, **the pressure in a liquid at rest increases linearly with depth from the free surface.**

For a given pressure intensity 'h' will be different for different liquids since, 'γ' will be different for different liquids.

$$\therefore h = \frac{P}{\gamma}$$

Hint-1: To convert head of 1 liquid to head of another liquid.

$$\mathbf{S} = \frac{\gamma}{\gamma_{\mathbf{Staandard}}}$$

$$\mathbf{S}_1 = \frac{\gamma_1}{\gamma_{\mathbf{Staandard}}}$$

$$p = \gamma_1 h_1$$

$$\therefore \gamma_1 = \mathbf{S}_1 \gamma_{\mathbf{Staandard}}$$

$$p = \gamma_2 h_2$$

$$\gamma_{21} = \mathbf{S}_2 \gamma_{\mathbf{Staandard}}$$

$$\boxed{\gamma_1 h_1 = \gamma_2 h_2}$$

$$\therefore \mathbf{S}_1\ \gamma_{\mathbf{Standard}}\ \mathbf{h}_1 = \mathbf{S}_2\ \gamma_{\mathbf{Standard}}\ \mathbf{h}_2$$

$$\boxed{\mathbf{S}_1\mathbf{h}_1 = \mathbf{S}_2\mathbf{h}_2}$$

Hint: 2 S_{water} x h_{water} = S_{liquid} x h_{liquid}

1x h_{water} = S_{liquid} x h_{liquid}

$$\boxed{h_{water} = S_{liquid} \times h_{liquid}}$$

Pressure head in meters of water is given by the product of pressure head in meters of liquid and specific gravity of the liquid.

Eg: 10meters of oil of specific gravity 0.8 is equal to 10x0.8 = 8 meters of water.

Eg: Atm pressure is 760mm of Mercury.

NOTE: $P \quad = \quad \gamma \quad h$

$\downarrow \quad \downarrow \quad \downarrow$

kPa $\quad \frac{kN}{m^3} \quad$ m

Solved Examples:

Ex. 1. Calculate intensity of pressure due to a column of 0.3m of (a) water (b) Mercury (c) Oil of specific gravity-0.8.

Soln: (a) Given: h = 0.3m of water

$$\gamma_{water} = 9.81 \frac{kN}{m^3}$$

$$p = ?$$

$$p_{water} = \gamma_{water} h_{water}$$

$$p_{water} = 2.943\ kPa$$

(b) Given: h = 0.3m of Hg

$\gamma_{mercury}$ = Sp.Gr. of Mercury X γ_{water} = 13.6 x 9.81

$\gamma_{mercury}$ = 133.416 kN/m^3

$p_{mercury} = \gamma_{mercury}\ h_{mercury}$

= 133.416 x 0.3

p = 40.025 kPa or 40.025 kN/m^2

(c) Given: h = 0.3 of Oil Sp.Gr. = 0.8

γ_{oil} = Sp.Gr. of Oil X γ_{water} = 0.8 x 9.8

γ_{oil} = 7.848 kN/m^3

$p_{oil} = \gamma_{oil}\ h_{oil}$

= 7.848 x 0.3

p_{oil} = 2.3544 kPa or 2.3544 kN/m^2

Ex.2. Intensity of pressure required at a point is 40kPa. Find corresponding head in (a) water (b) Mercury (c) oil of specific gravity-0.9.

Solution: Given Intensity of pressure at a point 40 kPa i.e. p = 40 kN/m^2

(a) Head of water h_{water} =?

$$\mathbf{h}_{water} = \frac{\mathbf{p}}{\gamma_{water}} = \frac{40}{9.81}$$

$$\mathbf{h}_{water} = \mathbf{4.077\,m\ of\ water}$$

(b) Head of mercury 'h$_{mercury}$ =?

$\gamma_{mercury}$ = Sp.Gr. of Mercury X γ_{water} = 13.6 x 9.81

$\gamma_{mercury}$ = 133.416 kN/m^3

$$h_{mercury} = \frac{p}{\gamma_{mercury}} = \frac{40}{133.416}$$

$$h_{water} = 0.3\,\mathbf{m\ of}\ \boldsymbol{mercury}$$

(c) Head of oil 'h$_{oil}$ =?

γ_{oil} = Sp.Gr. of Oil X γ_{water} = 0.9 x 9.81

γ_{oil} = 8.829 kN/m^3

$$h_{oil} = \frac{p}{\gamma_{oil}} = \frac{40}{8.829}$$

$$h_{oil} = 4.53\,\mathbf{m\ of}\ \boldsymbol{oil}$$

Ex.3 Standard atmospheric pressure is 101.3 kPa Find the pressure head in (i) Meters of water (ii) mm of mercury (iii) m of oil of specific gravity 0.6.

(i) Meters of water h_{water}

$p = \gamma_{water}\, h_{water}$

$101.3 = 9.81 \text{ x } h_{water}$

h_{water} = 10.3 m of water

(ii) Meters of water h_{water}

$p = \gamma_{mercury} \text{ x } h_{mercury}$

$101.3 = (13.6 \text{x} 9.81) \text{ x } h_{mercury}$

h = 0.76 m of mercury

(iii) $p = \gamma_{oil}\, h_{oil}$

101.3 = (0.6 x 9.81) x h

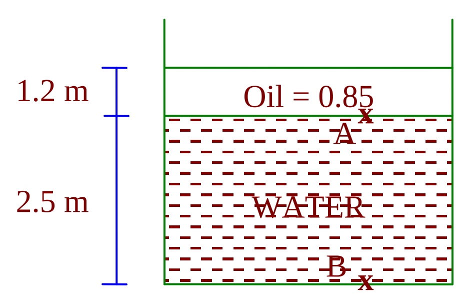
h = 17.21m of oil of S = 0.6

Ex.4 An open container has water to a depth of 2.5m and above this an oil of S = 0.85 for a depth of 1.2m. Find the intensity of pressure at the interface of two liquids and at the bottom of the tank.

(i) At the Oil - water interface

$$p_A = \gamma_{0il}\ h_{oil} = (0.85 \text{ x } 9.81) \text{ x } 1.2$$

$$p_A = 10\ \mathbf{kPa}$$



(ii) At the bottom of container

$$p_B = \gamma_{oil}\ x h_{oil} + \gamma_{water} + h_{water}$$

$$p_B = p_A + \gamma_{water} h_{water}$$

$$p_B = 10\,\mathbf{kPa} + 9.81 \text{ x } 2.5$$

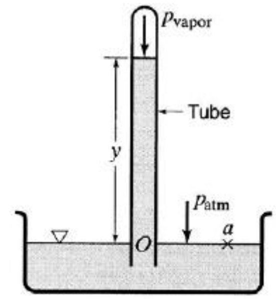
$$p_B = 34.525\,\mathbf{kPa}$$

2.5 Types of Pressure: Air above the surface of liquids exerts pressure on the exposed surface of the liquid and normal to the surface.

- ## Atmospheric pressure

The pressure exerted by the atmosphere is called atmospheric pressure. Atmospheric pressure at a place depends on the elevation of the place and the temperature.

Atmospheric pressure is measured using an instrument called 'Barometer' and hence atmospheric pressure is also called Barometric pressure. *However, for engineering purposes, it is more convenient to measure the pressure above a datum pressure at atmospheric pressure.* By setting $p_{atmophere} = 0$,



Mercury barometer

$$\boxed{p = -\rho gz = \rho gh}$$

Unit: kPa . 'bar' is also a unit of atmospheric pressure 1-bar = 100 kPa.= 1 kg/cm^2

- **Absolute pressure:** **Absolute pressure** at a point is the intensity of pressure at that point measured with reference to absolute vacuum or absolute zero pressure. Absolute pressure at a point is the intensity of pressure at that point measured with reference to absolute vacuum or absolute zero pressure (Fig.4) .

  Absolute pressure at a point can never be negative since there can be no pressure less than absolute zero pressure.

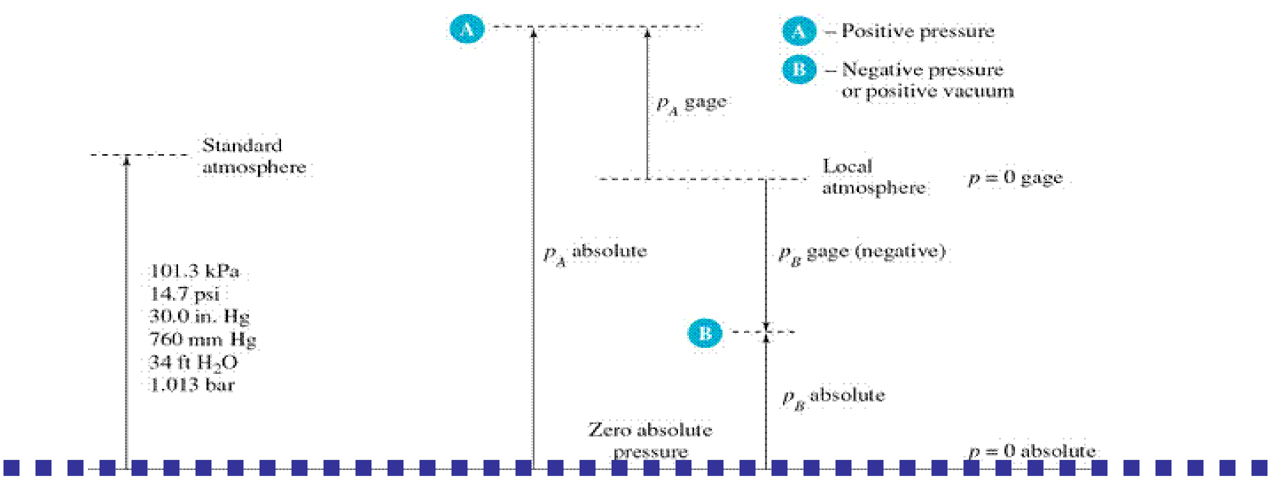


Fig.4 Definition of Absolute Pressure, Gauge Pressure and Vacuum Pressure

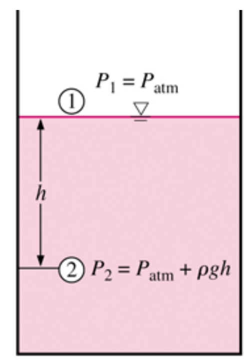
Gauge Pressure: If the intensity of pressure at a point is measurement with reference to atmosphere pressure, then it is called gauge pressure at that point.

Gauge pressure at a point may be more than the atmospheric pressure or less than the atmospheric pressure. Accordingly gauge pressure at the point may be positive or negative (Fig.4)

Negative gauge pressure: It is also called vacuum pressure. From the figure, It is the pressure measured below the gauge pressure (Fig.4).

Absolute pressure at a point = Atmospheric pressure ± Gauge pressure

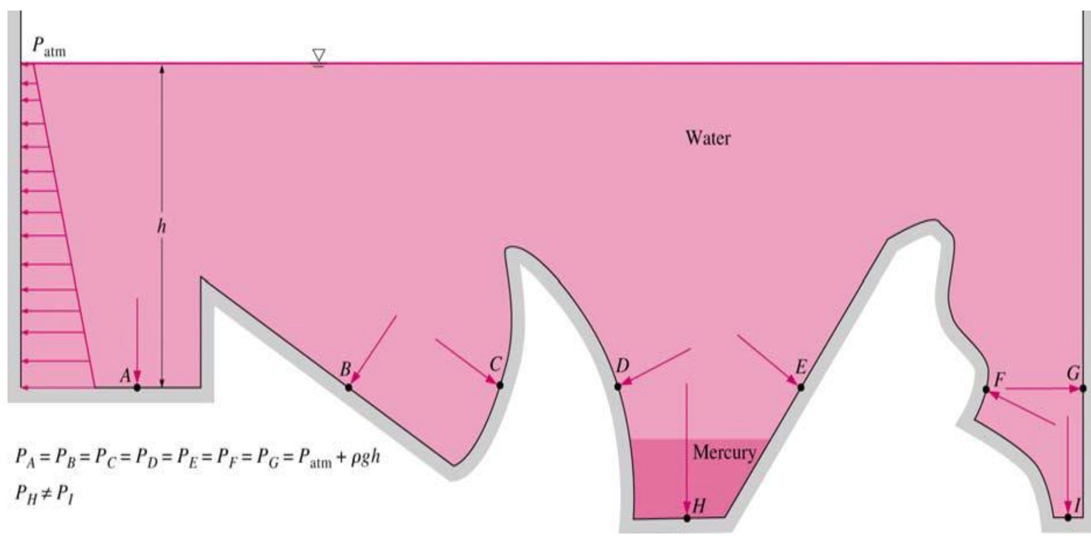
NOTE: If we measure absolute pressure at a Point below the free surface of the liquid, then, $\boxed{p_2 \text{ (absolute)} = \gamma .\ h + p_{atm}}$ $p_1 = p_{atm}$



If gauge pressure at a point is required, then atmospheric pressure is taken as zero, then,

$$\boxed{p_2 \text{ (gauge)} = \gamma .\ h = \rho gh}$$

Also, the pressure is the same at all points with the same depth from the free surface regardless of geometry, provided that the points are interconnected by the same fluid. However, the thrust due to pressure is perpendicular to the surface on which the pressure acts, and hence its direction depends on the geometry.



Solved Example: Convert the following absolute pressure to gauge pressure:

(a) 120kPa (b) 3kPa (c) 15m of H_2O (d) 800mm of Hg.

Solution:

(a) $p_{abs} = p_{atm} + p_{gauge}$

$\therefore\ p_{gauge} = p_{abs} - p_{atm} = 120 - 101.3 = 18.7$ kPa

(b) $p_{gauge} = 3\text{-}101.3 = -98.3$ kPa

$p_{gauge} = 98.3$ kPa (vacuum)

(c) $h_{abs} = h_{atm} + h_{gauge}$

$15 = 10.3 + h_{gauge}$

$h_{gauge} = 4.7$m of water

(d) $h_{abs} = h_{atm} + h_{gauge}$

$800 = 760 + h_{gauge}$

$h_{gauge} = 40$ mm of mercury

2.6 Vpour Pressure:

Vapor pressure is defined as the pressure at which a liquid will boil (vaporize) and is in equilibrium with its own vapor. Vapor pressure rises as temperature rises. For example, suppose you are camping on a high mountain (say 3,000 m in altitude); the atmospheric pressure at this elevation is about 70 kPa and the boiling temperature is around 90°C. This has consequences for cooking. For example, eggs have to be cooked longer at elevation to become hard-boiled since they cook at a lower temperature.

A pressure cooker has the opposite effect. Namely, the tight lid on a pressure cooker causes the pressure to increase above the normal atmospheric value. This causes water to boil at a temperature even greater than 100°C; eggs can be cooked a lot faster in a pressure cooker!

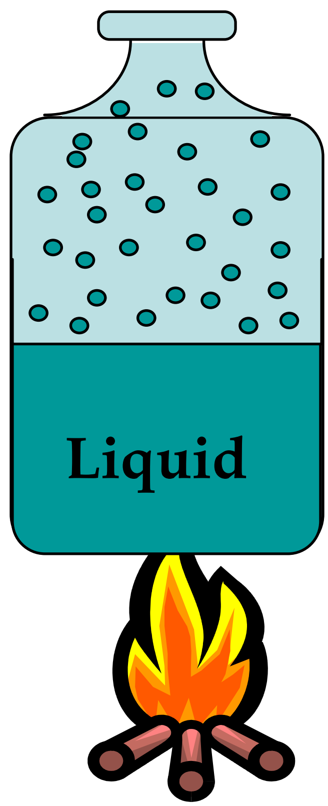


Fig.5

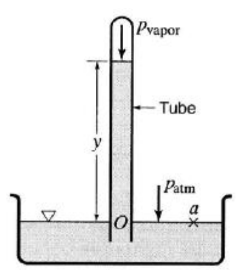
Vapor pressure is important to fluid flows because, in general, pressure in a flow decreases as velocity increases. This can lead to ***cavitation***, which is generally destructive and undesirable. In particular, at high speeds the local pressure of a liquid sometimes drops below the vapor pressure of the liquid. In such a case, ***cavitation*** occurs. In other words, a "cavity" or bubble of vapor appears because the liquid vaporizes or boils at the location where the pressure dips below the local vapor pressure.

Cavitation is not desirable for several reasons. First, it causes noise (as the cavitation bubbles collapse when they migrate into regions of higher pressure). Second, it can lead to inefficiencies and reduction of heat transfer in pumps and turbines (turbo machines). Finally, the collapse of these cavitation bubbles causes pitting and corrosion of blades and other surfaces nearby. The left figure below shows a cavitating propeller in a water tunnel, and the right figure shows cavitation damage on a blade.

<u>2.7 Measurement of Pressure:</u> Measurement of pressure

- Barometer
- Simple manometer
- Piezometer column
- Bourdon gage
- Pressure transducer

2.7.1 Barometer: A *barometer* is a device for measuring atmospheric pressure. A simple barometer consists of a tube more than 760 mm long inserted in an open container of mercury with a closed and evacuated end at the top and open tube end at the bottom and with mercury extending from the container up into the tube.



Mercury barometer

Strictly, the space above the liquid cannot be a true vacuum. It contains mercury vapor at its saturated vapor pressure, but this is extremely small at room temperatures (e.g. 0.173 Pa at 20^{o}C). The atmospheric pressure is calculated from the relation P_{atm} = ρgh where ρ is the density of fluid in the barometer.

$$p_{at'o'} = \gamma_{\text{mercury}} \text{ x } y + p_{vapor} = p_{atm}$$

With negligible $p_{vapor} = 0$

$$\boxed{p_{atm} = \gamma_{\text{mercury}} \times y}$$

2.7.2 Simple Manometer: Simple monometers are used to measure intensity of pressure at a point. They are connected to the point at which the intensity of pressure is required. Such a point is called gauge point

♦ Types of Simple Manometers

Common types of simple manometers are

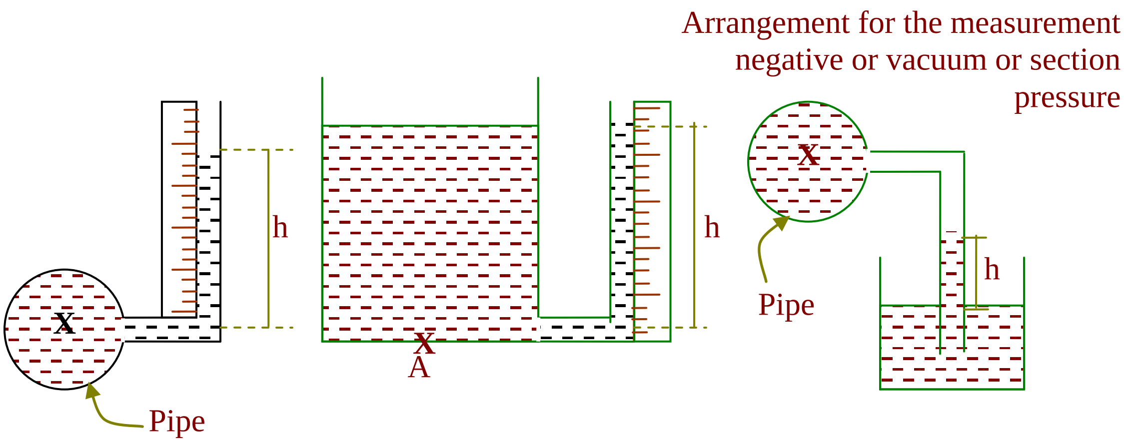
a) Piezometers

b) U-tube manometers

c) Single tube manometers

d) Inclined tube manometers

a) Piezometers



Piezometer consists of a glass tube inserted in the wall of the vessel or pipe at the level of point at which the intensity of pressure is to be measured. The other end of the piezometer is exposed to air. The height of the liquid in the piezometer gives the pressure head from which the intensity of pressure can be calculated.

To minimize capillary rise effects the diameters of the tube is kept more than 12mm.

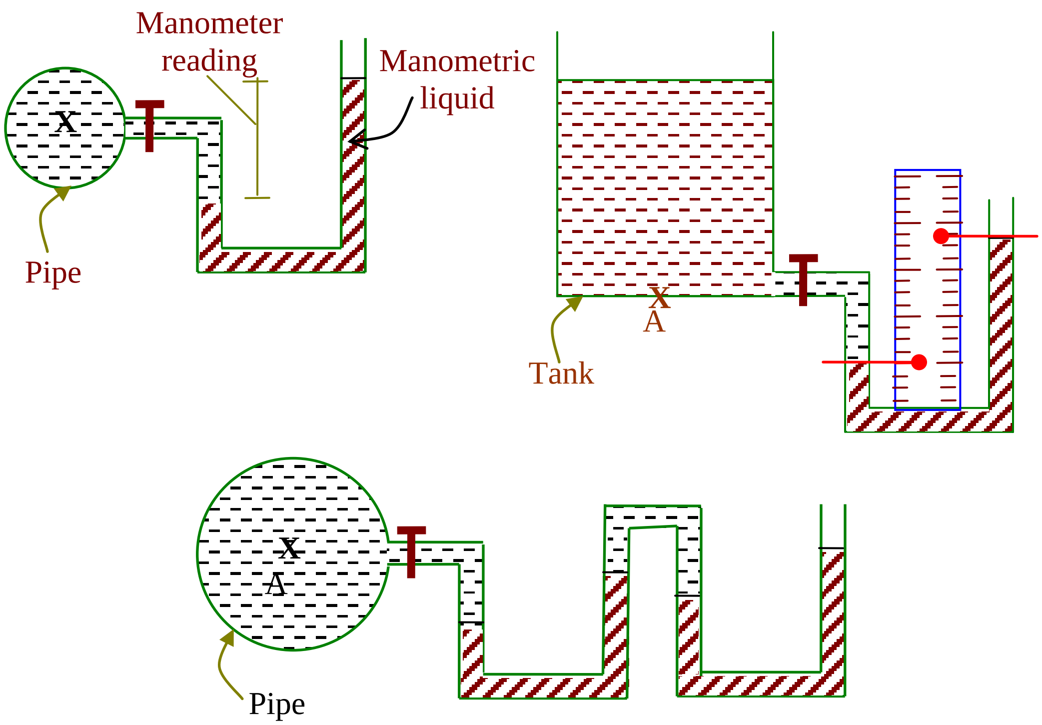
Merits

- Simple in construction
- Economical

Demerits

- Not suitable for high pressure intensity.
- Pressure of gases cannot be measured.

(b) U-tube Manometers:



A U-tube manometers consists of a glass tube bent in U-Shape, one end of which is connected to gauge point and the other end is exposed to atmosphere. U-tube consists of a liquid of specific of gravity other than that of fluid whose pressure intensity is to be measured and is called monometric liquid.

- **Manometric liquids**
    - Manometric liquids should neither mix nor have any chemical reaction with the fluid whose pressure intensity is to be measured.
    - It should not undergo any thermal variation.
    - Manometric liquid should have very low vapour pressure.
    - Manometric liquid should have pressure sensitivity depending upon the magnitude. Of pressure to be measured and accuracy requirement.

Gauge equations are written for the system to solve for unknown quantities.

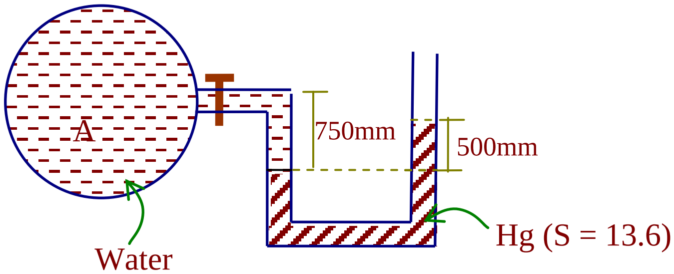
- **To write the gauge equation for manometers**

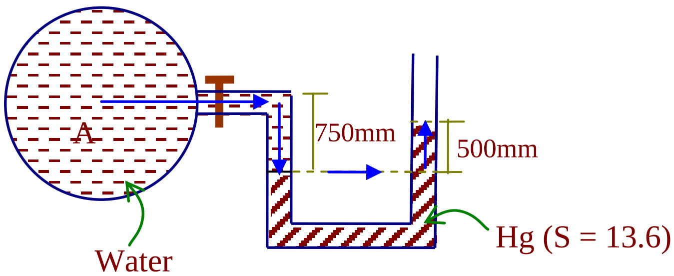
Steps:

1. Convert all given pressure to meters of water and assume unknown pressure in meters of waters.
2. Starting from one end move towards the other keeping the following points in mind.
    - Any horizontal movement inside the same liquid will not cause change in pressure.
    - Vertically downward movement causes increase in pressure and upward motion cause decrease in pressure.
    - Convert all vertical columns of liquids to meters of water by multiplying them by corresponding specify gravity.
    - Take atmospheric pressure as zero (gauge pressure computation).
3. Solve for the unknown quantity and convert it into the required unit.

Solved Problem:

1. Determine the pressure at A for the U- tube manometer shown in fig. Also calculate the absolute pressure at A in kPa.





Let ‘h_A’ be the pressure head at ‘A’ in ‘meters of water’.

$$h_A + 0.75 - 0.5\,x13.6 = 0$$

$$h_A = 6.05\,m\ of\ \ water$$

$$p = \gamma\, h$$

$$= 9.81\,x\,6.05$$

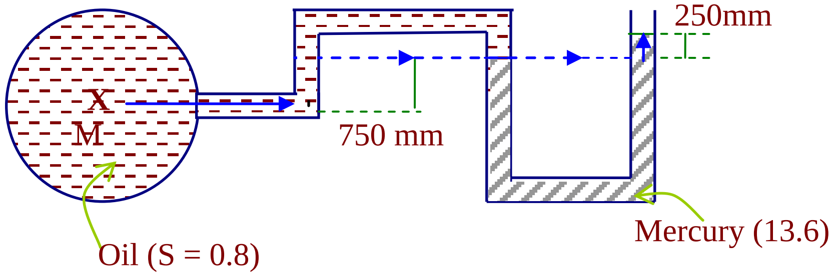
$$p = 59.35\ kPa(gauge\ \ pressure)$$

$$p_{abs} = p_{atm} + p_{gauge}$$

$$= 101.3 + 59.35$$

$$p_{abc} = 160.65\ kPa$$

2. For the arrangement shown in figure, determine gauge and absolute pressure at the point M.



Let 'h_M' be the pressure head at the point 'M' in m of water,

$h_M - 0.75 \times 0.8 - 0.25 \times 13.6 = 0$

$h_M = 4$ m of water

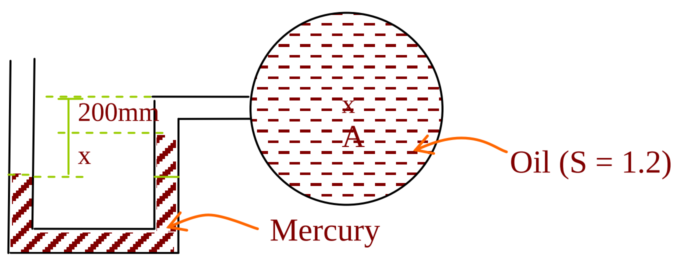
$p = \gamma h$

$p = 39.24 \text{kPa}$

$p_{abs} = 101.3 + 39.24$

$p_{abs} 140.54 \text{kPa}$

3. If the pressure at 'At' is 10 kPa (Vacuum) what is the value of 'x'?



p_A = 10 kPa (Vacuum)

p_A = - 10 kPa

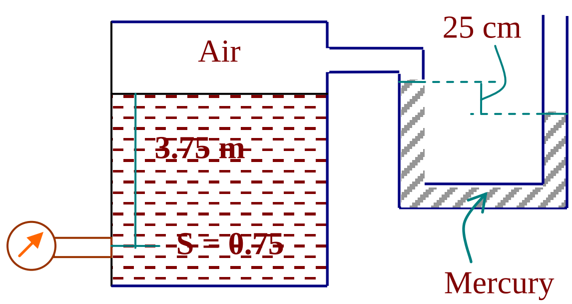
$$\frac{p_A}{\gamma} = \frac{-10}{9.81} = -1.019 \text{ m of water}$$

$$h_A = -1.019 \text{ m of water}$$

$$-1.019 + 0.2 \times 1.2 + x(13.6) = 0$$

$$x = 0.0572 \text{ m}$$

4. The tank in the accompanying figure consists of oil of S = 0.75. Determine the pressure gauge reading in $\frac{kN}{m^2}$.



Let the pressure gauge reading be ‘h’ m of water

h – 3.75 x 0.75 + 0.25 x 13.6 = 0

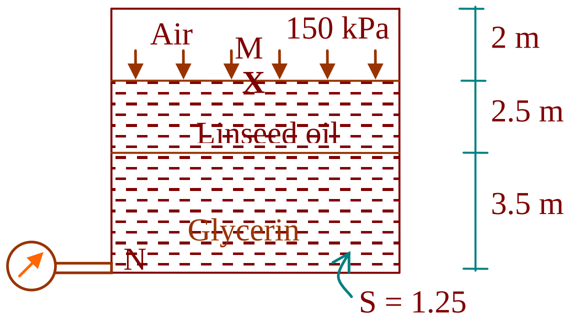
h = - 0.5875 m of water

p = γ h

p = -5.763 kPa

p = 5.763 kPa (Vacuum)

5. A closed tank is 8m high. It is filled with Glycerine up to a depth of 3.5m and linseed oil to another 2.5m. The remaining space is filled with air under a pressure of 150 kPa. If a pressure gauge is fixed at the bottom of the tank what will be its reading. Also calculate absolute pressure. Take relative density of Glycerine and Linseed oil as 1.25 and 0.93 respectively.



$$P_H = 150\,\text{kPa}$$

$$h_M = \frac{150}{9.81}$$

$$h_M = 15.29\,\text{m of water}$$

Let 'h_N' be the pressure gauge reading in m of water.

h_N -3.5 x 1.25 -2.5 x 0.93 =15.29

h_N = 21.99 m of water

p = 9.81 x 21.99

p = 215.72 kPa (gauge)

p_{abs} = 317.02 kPa

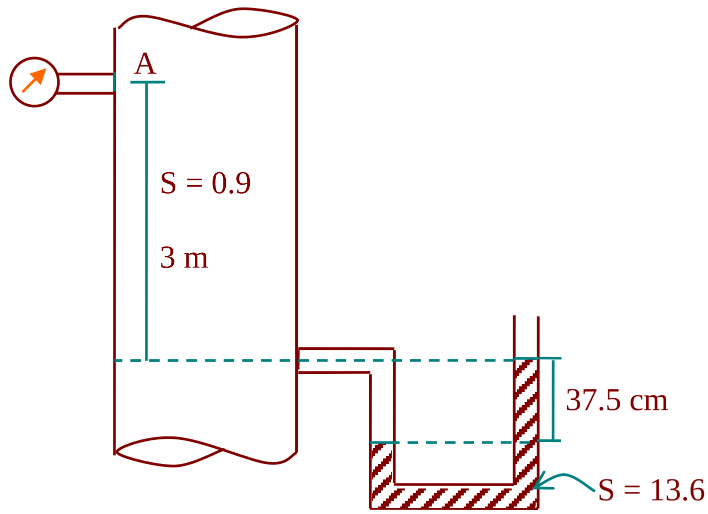
6. A vertical pipe line attached with a gauge and a manometer contains oil and Mercury as shown in figure. The manometer is opened to atmosphere. What is the gauge reading at 'A'? Assume no flow in the pipe.

h_A-3 x 0.9 + 0.375 x 0.9 - 0.375 x 13.6 = 0

h_A = 2.0625 m of water

p = γ x h

= 9.81 x 21.99



p = 20.23 kPa (gauge)

$p_{abs} = 101.3 + 20.23$

$p_{abs} = 121.53$ kPa

- **DIFFERENTIAL MANOMETERS**

Differential manometers are used to measure pressure difference between any two points. Common varieties of differential manometers are:

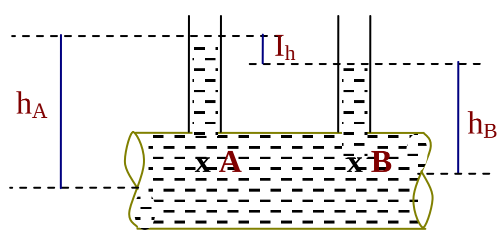
(a) Two piezometers.

(b) Inverted U-tube manometer.

(c) U-tube differential manometers.

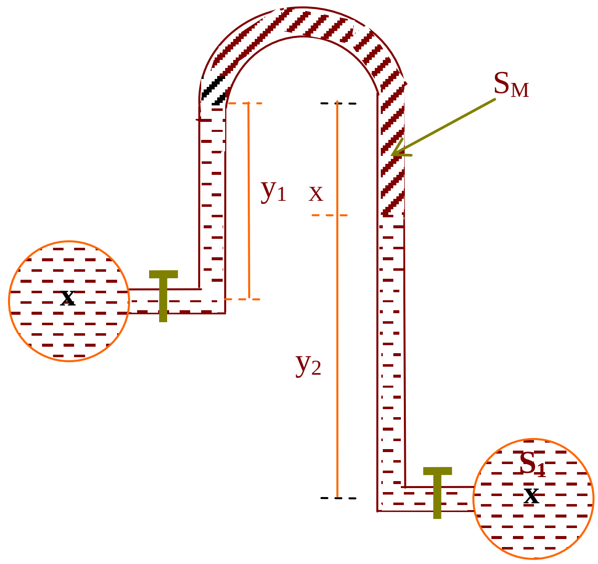
(d) Micro manometers.

(a) Two Pizometers



The arrangement consists of two pizometers at the two points between which the pressure difference is required. The liquid will rise in both the piezometers. The difference in elevation of liquid levels can be recorded and the pressure difference can be calculated. It has all the merits and demerits of piezometer.

(b) Inverted U-tube manometers:



Inverted U-tube manometer is used to measure small difference in pressure between any two points. It consists of an inverted U-tube connecting the two points between which the pressure difference is required. In between there will be a lighter sensitive manometric liquid. Pressure difference between the two points can be calculated by writing the gauge equations for the system.

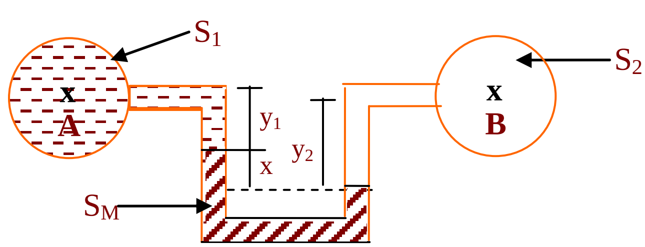
Let 'h_A' and 'h_B' be the pr head at 'A' and 'B' in meters of water

$h_A – (Y_1 S_1) + (x S_M) + (y_2 S_2) = h_B.$

$h_A – h_B = S_1 y_1 – S_M x – S_2 y_2,$

$p_A – p_B = \gamma (h_A – h_B)$

(c) U-tube Differential manometers



A differential U-tube manometer is used to measure pressure difference between any two points. It consists of a U-tube containing heavier manometric liquid, the two limbs of which are connected to the gauge points between which the pressure difference is required. U-tube differential manometers can also be used for gases. By writing the gauge equation for the system pressure difference can be determined.

Let 'h_A' and 'h_B' be the pressure head of 'A' and 'B' in meters of water

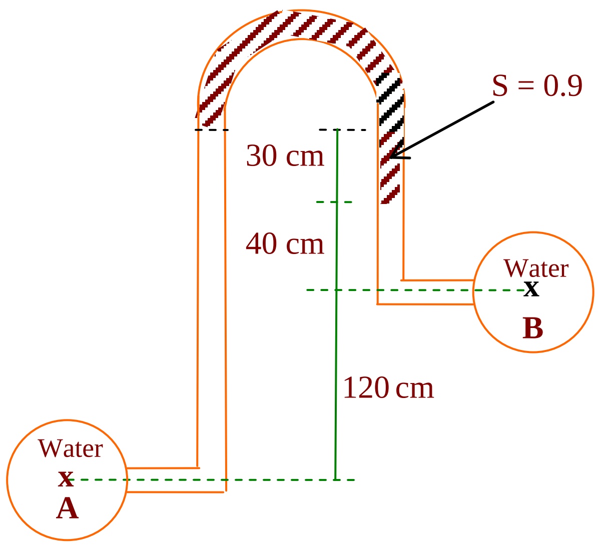
$h_A + S_1 Y_1 + x S_M - Y_2 S_2 = h_B$

$h_A - h_B = Y_2 S_2 - Y_1 S_1 - x S_M$

Solved Problems:

(1) An inverted U-tube manometer is shown in figure. Determine the pressure difference between A and B in N/M^2.

Let h_A and h_B be the pressure heads at A and B in meters of water.



$h_A - (190 \times 10^{-2}) + (0.3 \times 0.9) + (0.4)\ 0.9 = h_B$

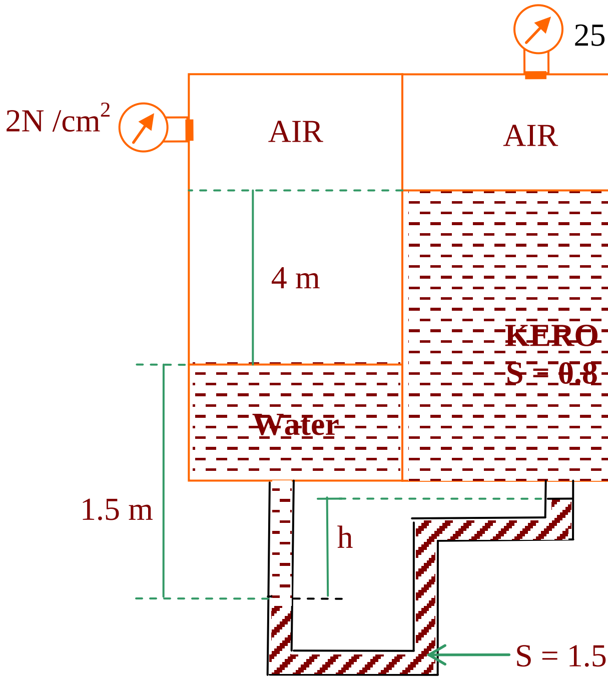
$h_A - h_B = 1.23$ meters of water

$p_A - p_B = \gamma\ (h_A - h_B) = 9.81 \times 1.23$

$p_A - p_B = 12.06$ kPa

$p_A - p_B = 12.06 \times 10^3\ N/m^2$

2. In the arrangements shown in figure. Determine the 'h'.



= - 0.25 x 13.6

= - 3.4 m of water

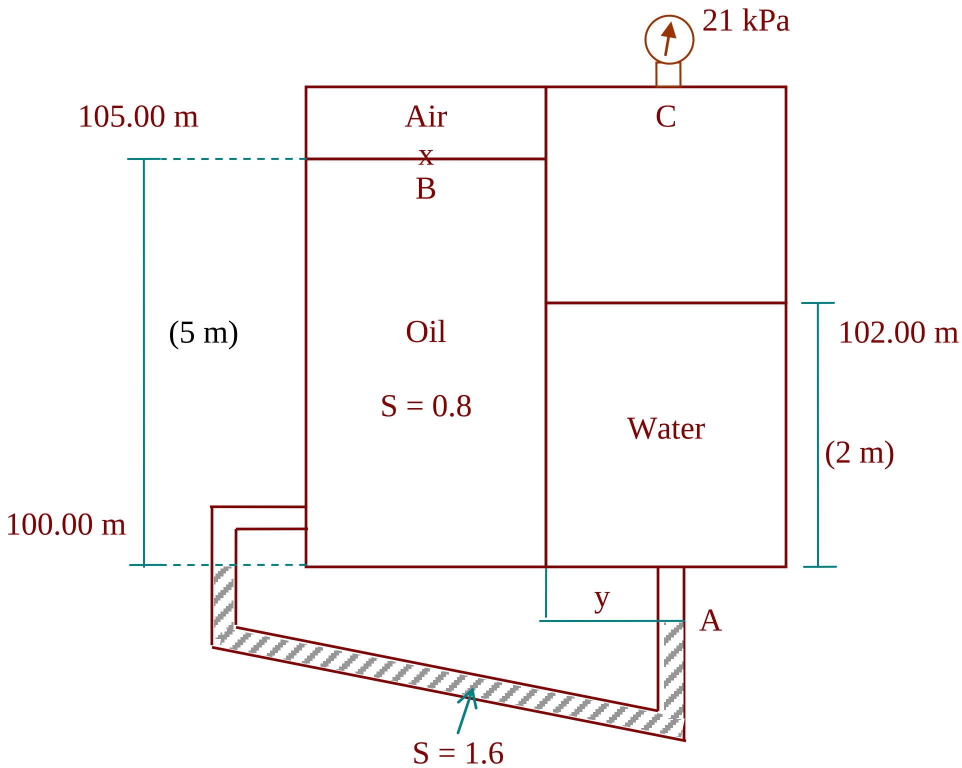
$2N/cm^2 = 2 \times 100^2 \; N/m^2 =$ 20 kPa

h = 20 9.81

h = 2.038 meters of water

2.038 + 1.5 – (4 + 1.5 – h) 0.8 = – 3.4, h = 3.6 m

3. In figure given, the air pressure in the left tank is 230 mm of Mercury (Vacuum). Determine the elevation of gauge liquid in the right limb at A. If liquid in the right tank is water.



$$h_c = \frac{Pc}{\gamma}$$

h_B = 230mm of Hg

$$\frac{21}{9.81}$$

$$h_c = 2.14m\,of\ water$$

= 0.23 x 13.6

h_B = - 3.128 m of water

– 3.128 + 5 x 0.8 + y x 1.6 – (y + 2) = 2.14

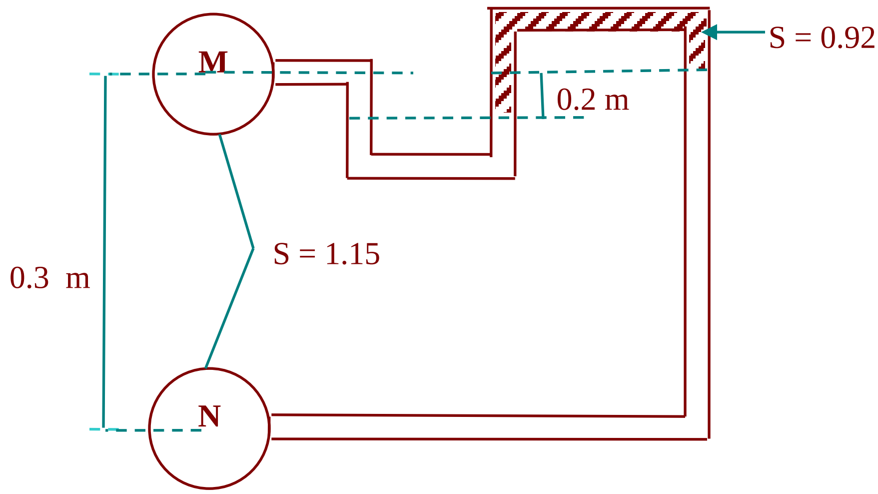
– 3.128 + 5 x 0.8 + y x 1.6 – y – 2 = 2.14

y = 5.446 m

∴ Elevation of A = 100 – 5.446

Elevation of A = 94.553m

4. Compute the pressure different between 'M' and 'N' for the system shown in figure.



Let 'h_M' and ' h_N' be the pressure heads at M and N in m of water.

hm + y x 1.15 – 0.2 x 0.92 + (0.3 – y + 0.2) 1.15 = hn

hm + 1.15 y – 0.184 + 0.3 x 1.15 – 1.15 y + 0.2 x 1.15 = hn

hm + 0.391 = hn

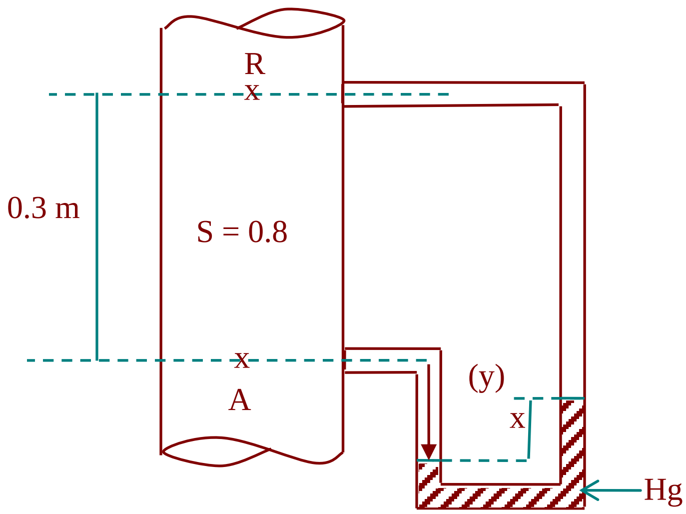
hn – hm = 0.391meters of water

$p_n - p_m = \gamma\,(h_N - h_m)$

$= 9.81 \times 0.391$

$p_n - p_m = 3.835$ kPa

5. Petrol of specify gravity 0.8 flows up through a vertical pipe. A and B are the two points in the pipe, B being 0.3 m higher than A. Connection are led from A and B to a U–tube containing Mercury. If the pressure difference between A and B is 18 kPa, find the reading of manometer.



$p_A - p_B = 18kPa$

$$\frac{P_A - P_B}{\gamma}$$

$$h_A - h_B = \frac{18}{9.81}$$

$$h_A - h_B = 1.835m\ of\ water$$

$h_A + y \text{ x } 0.8 - x\ 13.6 - (0.3 + y - x)\ 0.8 = h_B$

$h_A - h_B = -0.8y + 13.66\ x + 0.24 + 0.8\ y - 0.8\ x$

$$h_A - h_B = 12.8x + 0.24$$

$1.835 = 12.8x + 0.24$

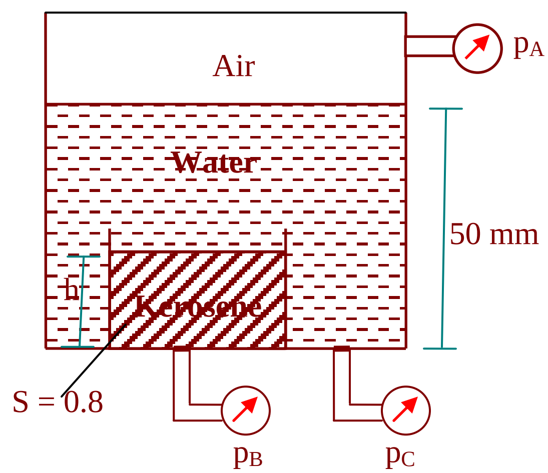
$x = 0.1246$ m

6. A cylindrical tank contains water to a height of 50mm. Inside is a small open cylindrical tank containing kerosene at a height specify gravity 0.8. The following pressures are known from indicated gauges.

p_B = 13.8 kPa (gauge)

p_C = 13.82 kPa (gauge)

Determine the gauge pressure p_A and height h. Assume that kerosene is prevented from moving to the top of the tank.



p_C = 13.82 kPa

h_C = 1.409 m of water

p_B = 13.8 kPa

h_B = 1.407 meters of water

$1.409 - 0.05 = h_A \;\; \therefore \; h_A = 1.359$ meters of water

$\therefore \; p_A = 1.359 \times 9.81$

$\therefore \; p_A = 13.33$ kPa

$h_B - h \times 0.8 - (0.05 - h) = h_A$

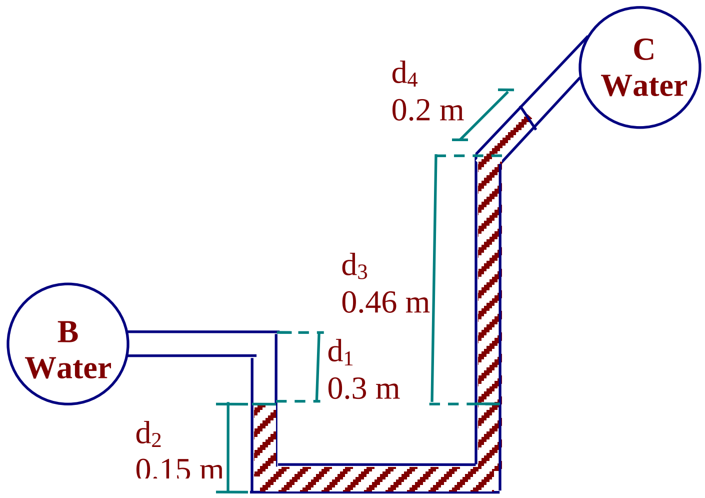
$1.407 - 0.8\,h - 0.05 + h = 1.359$

$0.2\,h = 1.359 - 1.407 + 0.05$

$0.2\,h = 0.002$

$h = 0.02$ m

7. Find the pressure different between A and B if d_1 = 300mm, d_2 = 150mm,d_3 = 460mm, d_4 = 200mm and 13.6.



Let h_A and h_B be the pressure head at A and B in m of water.

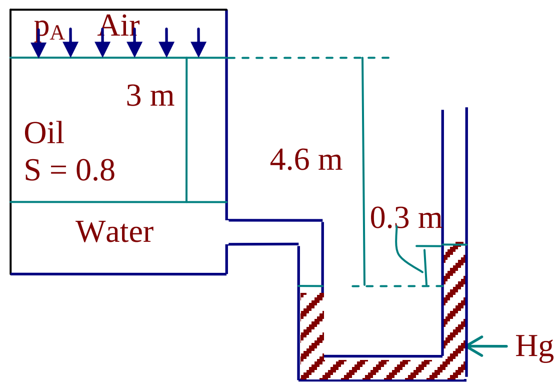
$h_A + 0.3 - (0.46 + 0.2 \text{ Sin } 45)\ 13.6 = h_B$

$h_A - h_B = 7.88$m of water

$p_A - p_B = (7.88)\ (9.81)$

$p_A - p_B = 77.29$ kPa

8. What is the pressure p_A in the fig given below? Take specific gravity of oil as 0.8.



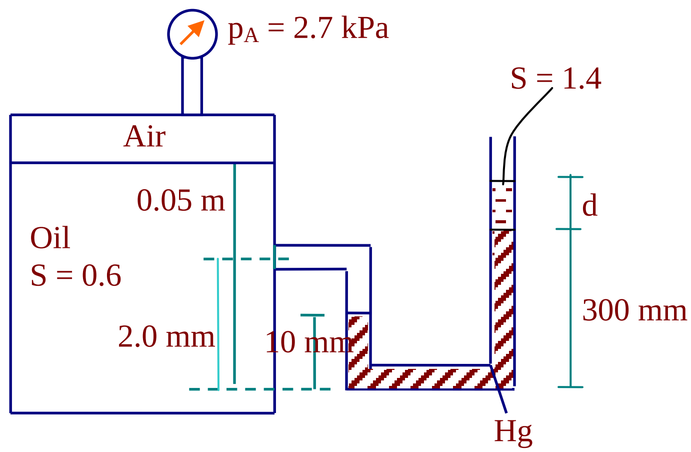
$h_A + (3 \times 0.8) + (4.6 - 0.3)\ (13.6) = 0$

$h_A = 2.24$ m of oil

$p_A = 9.81 \times 2.24$

$p_A = 21.97$ kPa

9. Find 'd' in the system shown in fig. If $p_A = 2.7$ kPa



$$h_A = \frac{p_A}{\gamma} = \frac{2.7}{9.81}$$

$$h_A = 0.2752\,m\ of\ \ water$$

$$h_A + (0.05\,x\,0.6) + (0.05 + 0.02 - 0.01)0.6$$

$$+ (0.01x13.6) - (0.03x13.6) - d\,x1.4) = 0$$

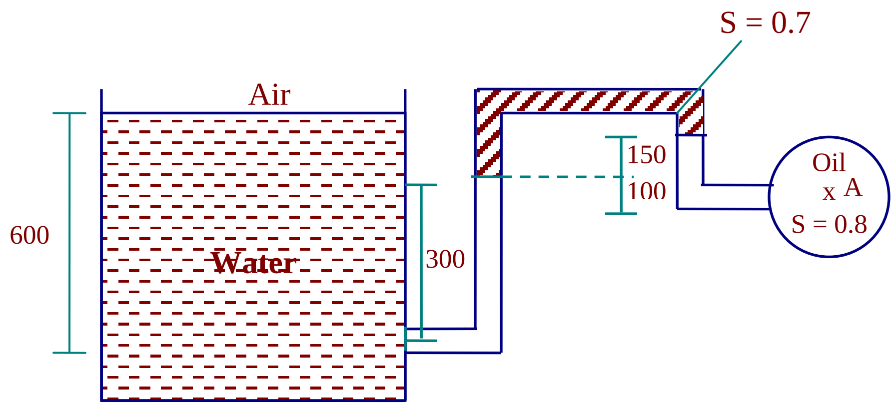
$$0.0692 - 1.4d = 0$$

$$d = 0.0494\,m$$

or

$$d = 49.4\,mm$$

10. Determine the absolute pressure at ‘A’ for the system shown in fig.



h_A - (0.25 x 0.8) + (0.15 x 0.7) + (0.3 x 0.8)-(0.6) = 0

h_A = 0.455 m of water

p_A = h_A x 9.81

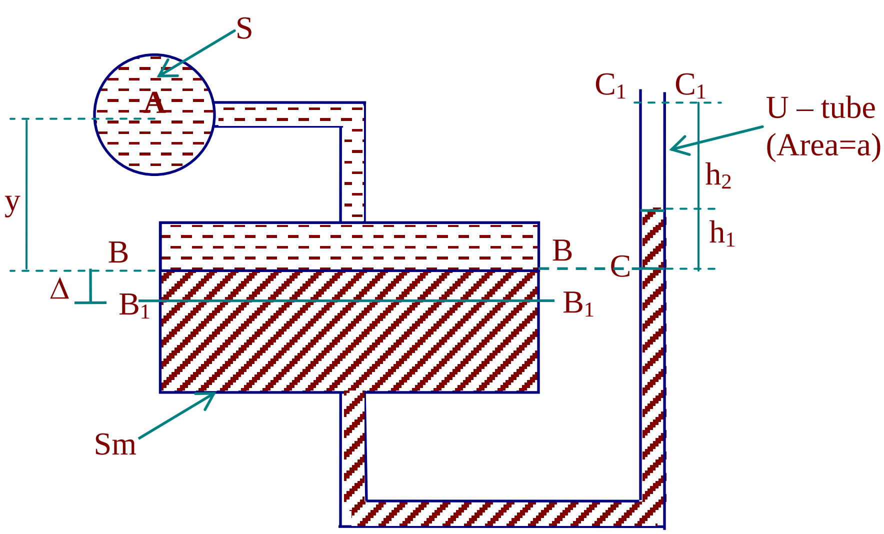
p_A = 4.464 kPa

p_{abs} = 101.3 + 4.464

p_{abs} = 105.764 kPa

SINGLE COLUMN MANOMETER:

Single column manometer is used to measure small pressure intensities.



A single column manometer consists of a shallow reservoir having large cross sectional area when compared to cross sectional area of U – tube connected to it. For any change in pressure, change in the level of manometeric liquid in the reservoir is small (Δ) and change in level of manometric liquid in the U- tube is large.

To derive expression for pressure head at A:

BB and CC are the levels of manometric liquid in the reservoir and U-tube before connecting the point A to the manometer, writing gauge equation for the system we have,

$+ y \times S - h_1 \times S_m = 0$

$\therefore Sy = S_m h_1$

Let the point A be connected to the manometer. B_1B_1 and C_1 C_1 are the levels of manometeric liquid. Volume of liquid between BBB_1B_1 = Volume of liquid between CCC_1C_1

$$A\Delta = a\, h_2$$

$$\Delta = \frac{ah_2}{A}$$

Let 'h_A' be the pressure head at A in m of water.

$h_A + (y + \Delta)\, S - (\Delta + h_1 + h_2)\, Sm = 0$

$h_A = (\Delta + h_1 + h_2)\, Sm - (y + \Delta)\, S$

$= \Delta\, Sm + \underline{h_1\, Sm} + h_2\, Sm - \underline{yS} - \Delta S$

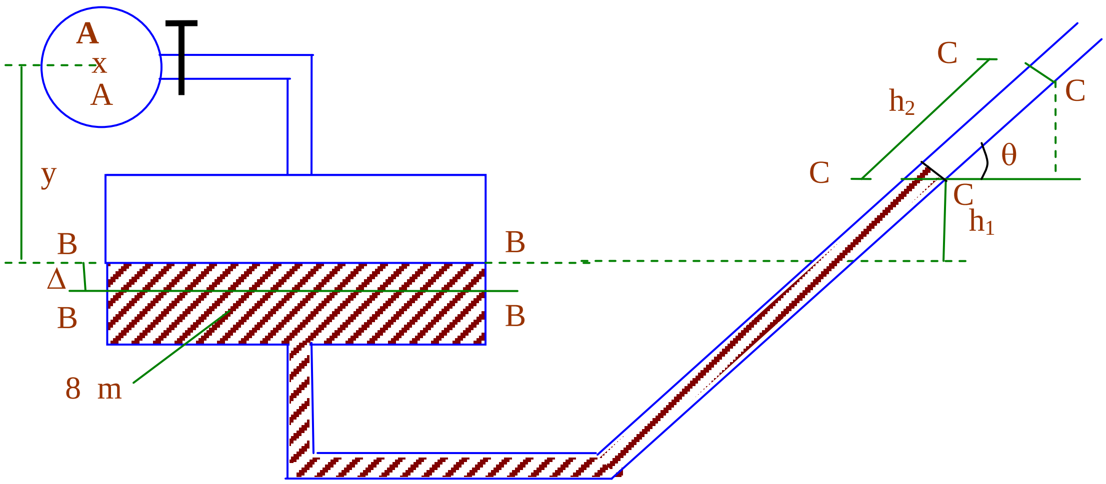
$h_A = \Delta\, (Sm - S) + h_2\, Sm$

$h_A = \frac{ah_2}{A}\, (Sm - S) + h_2\, Sm$

$\therefore$It is enough if we take one reading to get 'h_2' If '$\frac{a}{A}$' is made very small (by increasing 'A') then the I term on the RHS will be negligible.

Then $h_A = h_2\, Sm$

INCLINED TUBE SINGLE COLUMN MANOMETER:



Inclined tube SCM is used to measure small intensity pressure. It consists of a large reservoir to which an inclined U – tube is connected as shown in fig. For small changes in pressure the reading 'h_2' in the inclined tube is more than that of SCM. Knowing the inclination of the tube the pressure intensity at the gauge point can be determined.

$$h_A = \frac{a}{A} h_2 \sin\theta (Sm - S) + h_2 \sin\theta . Sm$$

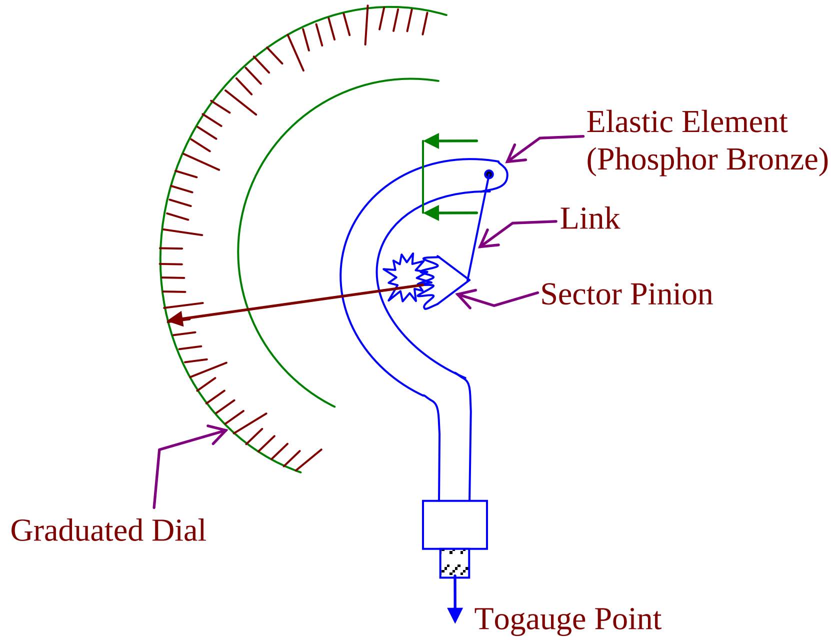
If '$\frac{a}{A}$' is very small then $h_A = (h_2 = \sin\theta)\ S_m$.

2.7.3 MECHANICAL GAUGES:

Pressure gauges are the devices used to measure pressure at a point. They are used to measure high intensity pressures where accuracy requirement is less. Pressure gauges are separate for positive pressure measurement and negative pressure measurement. Negative pressure gauges are called Vacuum gauges.

Mechanical gauge consists of an elastic element which deflects under the action of applied pressure and this deflection will move a pointer on a graduated dial leading to the measurement of pressure. Most popular pressure gauge used is Bordon pressure gauge.

BASIC PRINCIPLE:

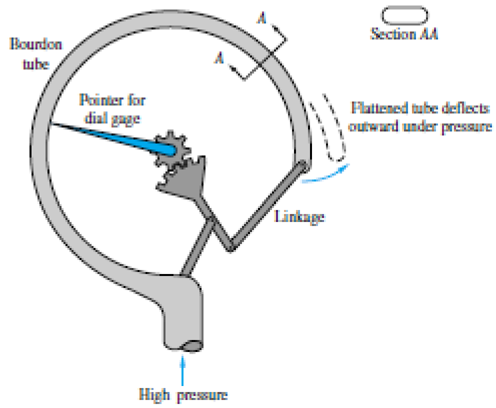


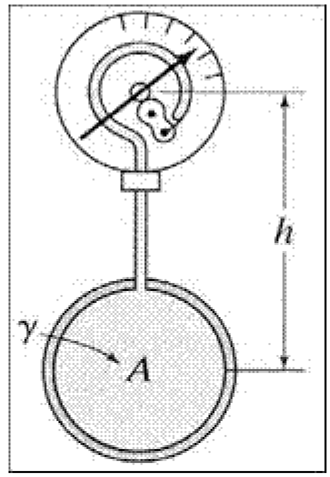
The arrangement consists of a pressure responsive element made up of phosphor bronze or special steel having elliptical cross section. The element is curved into a circular arc, one end of the tube is closed and free to move and the other end is connected to gauge point. The changes in pressure cause change in section leading to the movement. The movement is transferred to a needle using sector pinion mechanism. The needle moves over a graduated dial.

Bourdon gage:

Is a device used for measuring gauge pressures the pressure element is a hollow curved metallic tube closed at one end the other end is connected to the pressure to be measured. When the internal pressure is increased the tube tends to straighten pulling on a linkage to which is attached a pointer and causing the pointer to move. When the tube is connected the pointer shows zero. The *bourdon tube,* sketched in figure.

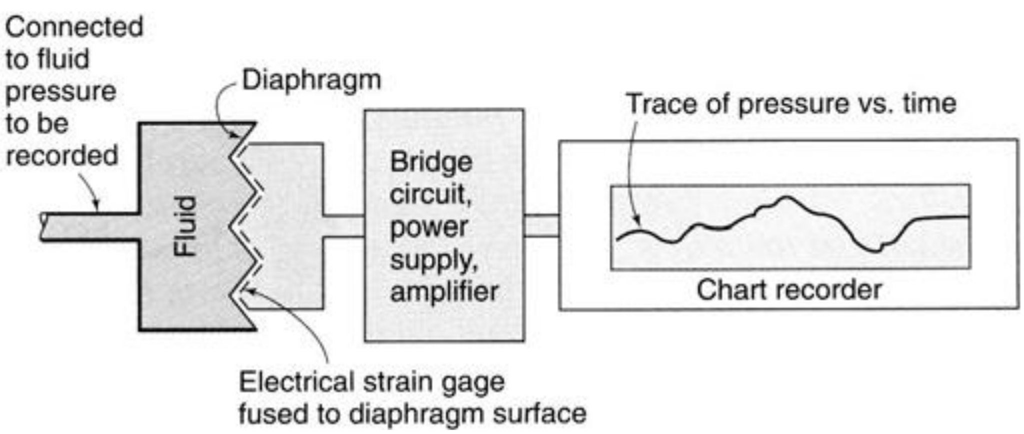
It can be used for the measurement of liquid and gas pressures up to 100s of MPa.





2.7.4 Electronic Pressure Measuring Devices:

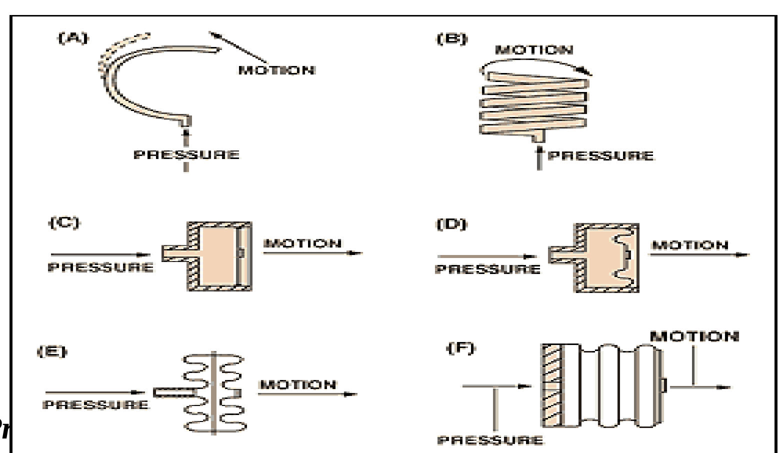
Electronic Pressure transducers convert pressure into an electrical output. These devices consist of a sensing element, transduction element and signal conditioning device to convert pressure readings to digital values on display panel.



<u>Sensing Elements:</u>

The main types of sensing elements are

- Bourdon tubes,
- Diaphragms,
- Capsules, and
- Bellows.



Pressure Transducers:

A transducer is a device that turns a mechanical signal into an electrical signal or an electrical signal into a mechanical response (e.g., Bourdon gage transfers pressure to displacement).

There are a number of ways to accomplish this kind of conversion

- Strain gage
- Capacitance
- Variable reluctance
- Optical

Normally Electronic Pressure transducers are costly compared to conventional mechanical gauges and need to be calibrated at National laboratories before put in to use.