MODULE-3

FLUID DYNAMICS

Forces acting on the fluids

Following are the forces acting on the fluids

1. Self-Weight/ Gravity Force, F_g
2. Pressure Forces, F_p
3. Viscous Force, F_v
4. Turbulent Force, F_t
5. Surface Tension Force, F_s
6. Compressibility Force, F_c

Dynamics of fluid is governed by Newton's Second law of motion, which states that the resultant force on any fluid element must be equal to the product of the mass and the acceleration of the element.

$$\sum F = Ma$$

or

$$\sum F = F_g + F_p + F_v + Fs + F_c \tag{1}$$

Surface tension forces and Compressibility forces are not significant and may be neglected. Hence (1) becomes

$$\sum F = F_g + F_p + F_v + F_t$$

- Reynold's Equation of motion and used in the analysis of Turbulent flows. For laminar flows, turbulent force becomes less significant and hence (1) becomes

$$\sum F = F_g + F_p + F_v$$

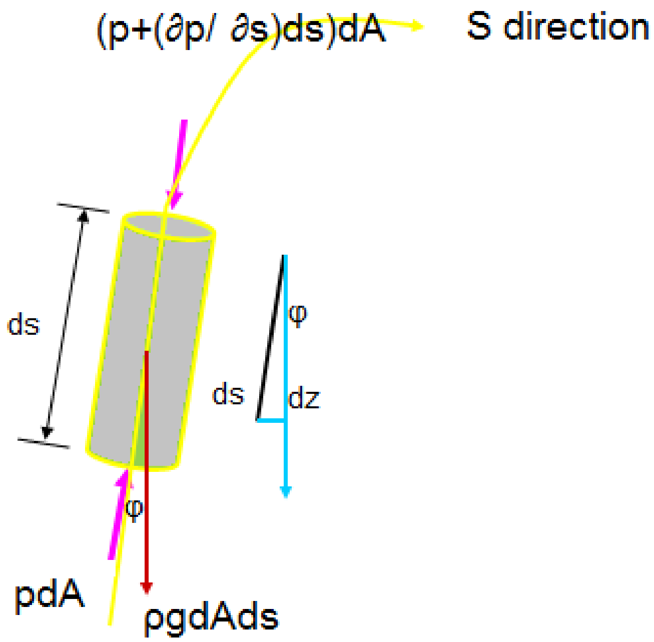
– Navier - Stokes Equation. If viscous forces are neglected then the (1) reduces to

$$\sum F = F_g + F_p = M \times a$$

– Euler's Equation of motion.

Euler equation of motion

Consider a stream lime in a flowing fluid in S direction as shown in the figure. On this stream line consider a cylindrical element having a cross sectional area *dA* and length *ds*.



eq.png

Fluid element in stream line

Forces acting on the fluid element are: Pressure forces at both ends:

- Pressure force, pdA in the direction of flow
- Pressure force (p+(∂p/∂s)ds)dA in the direction opposite to the flow direction
- Weight of element $\rho dads$ acting vertically downwards

Let ϕ be the angle between the direction of flow and the line of action of the weight of the element. The resultant force on the fluid element in the direction of s must be equal to mass of fluid element$\times$ acceleration in direction s *(according to Newton's second law of motion)*

$$pda - (pda + (\partial p/\partial s)ds)dA) - \rho gd\cos\phi = \rho dadsa_s \qquad \text{(a)}$$

where $a_s$is the accelaration in direction of s now

$$a_s = \frac{dv}{dt}$$

where v is function of s and t

$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t}$$
$$= v\frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t}$$

since

$$\frac{ds}{dt} = v$$

If the flow is steady,

$$\frac{\partial v}{\partial t} = 0$$

hence,

$$a_s = v\frac{\partial v}{\partial s}$$

Substituting the valve of a_s in equation (a) and simplifying,

$$-\frac{\partial p}{\partial s}dsdA - \rho gdsdA\cos\phi = \rho dads \times v\frac{\partial v}{\partial s}$$

Dividing the whole equation by $\rho dsdA$,

$$-\frac{\partial p}{\rho\partial s} - g\cos\phi = v\frac{\partial v}{\partial s}$$
$$\Rightarrow \frac{\partial p}{\rho\partial s} + g\cos\phi + v\frac{\partial v}{\partial s} = 0$$

But from the figure we have

$$\cos\phi = \frac{dz}{ds}$$

Hence,

$$\frac{1}{\rho}\frac{\partial p}{\partial s} + g\frac{dz}{ds} + v\frac{\partial v}{\partial s} = 0$$

or

$$\frac{\partial p}{\rho} + gdz + vdv = 0 \qquad \text{(b)}$$

Equation (b)is known as Euler's equation of motion.

Bernoulli's Equation of motion from Euler's equation

Statement: *In a steady, incompressible fluid, the total energy remains same along a streamline throughout the reach.*

Bernoulli's equation may be obtained by integrating Euler's equation of motion i.e, equation (b) as

$$\int\frac{dp}{\rho} + \int gdz + \int vdv = constant$$

If the flow is in-compressible, ρ is constant and hence,

$$\frac{p}{\rho} + gz + \frac{v^2}{2} = constant$$
$$\Rightarrow \frac{p}{\rho g} + \frac{v^2}{2g} + z = constant \qquad \text{(c)}$$

Equation (c) is called as Bernoulli's equation, Where
$\frac{p}{\rho}$= pressure energy per unit weight of the fluid or also called as pressure head
$\frac{v^2}{2g}$= kinetic energy per unit weight of the fluid or kinetic head
z= potential energy per unit weight or potential head

Assumption made in deriving the Bernoulli's Equation

Following assumptions were made to derive the bernoulli's equation

- The flow is steady
- The flow is ideal (Viscosity of the fluid is zero)
- The flow is in-compressible
- The flow is irrotational

Limitations on the use of the Bernoulli Equation

- **Steady flow**: The first limitation on the Bernoulli equation is that it is applicable to steady flow.
- **Friction-less flow**: Every flow involves some friction, no matter how small, and frictional effects may or may not be negligible.
- **In-compressible flow**: One of the assumptions used in the derivation of the Bernoulli equation is that ρ = constant and thus the flow is in-compressible. Strictly speaking, the Bernoulli equation is applicable along a streamline, and the value of the constant C, in general, is different for different streamlines. But when a region of the flow is irrational, and thus there is no vorticity in the flow field, the value of the constant C remains the same for all streamlines, and, therefore, the Bernoulli equation becomes applicable across streamlines as well.

Kinetic Energy correction factor

In deriving the Bernoulli's Equation, the velocity head or the kinetic energy per unit weight of the fluid has been computed based on the assumption that the velocity is uniform over the entire cross section of the stream tube. But in real fluids, the velocity distribution is not uniform. Therefore, to obtain the kinetic energy possessed by the fluid at differently sections is obtained by integrating the kinetic energies possessed by different fluid particles.
It is more convenient to express the kinetic energy in terms of the mean velocity of flow. But the actual kinetic energy is greater than the computed using the mean velocity. Hence a correction factor called ' Kinetic Energy correction factor,α is introduced.

$$\frac{p_1}{\rho} + \alpha_1(\frac{v_1^2}{2g}) + z_1 = \frac{p_2}{\rho} + \alpha_2(\frac{v_2^2}{2g}) + z_2 + h_L = Constant$$

In most of the problems of turbulent flow, the value of α=1.

Rotary or Vortex Motion

A mass of fluid in rotation about a fixed axis is called vortex. The rotary motion of fluid is also called vortex motion. In this case the rotating fluid particles have velocity in tangential direction. Thus the vortex motion is defined as motion in which the whole fluid mass rotates about an axis.
The vortex motion is of two types:

1. Free vortex
2. Forced vortex

Free vortex flow

Free vortex flow is that type of flow in which the fluid mass rotates without any external applied contact force. The whole mass rotates either due to fluid pressure itself or the gravity or due to rotation previously imparted. Energy is not expended to any outside source. The free vortex motion is also called Potential vortex or Ir-rotational vortex.

Relationship between velocity and radius in free vortex

It is obtained by putting the value of external torque equal to **Zero** or on other words the time rate of change of angular momentum, i.e., moment of the momentum must be Zero. Consider a fluid particle of mass 'M' at a radial distance 'r' from the axis of rotation, having a tangential velocity 'u'. Then,

$$Angular\ momentum = Mass \times velocity$$

$$Moment\ of\ the\ Momentum = Momentum \times radius = mur$$

$$Time\ rate\ of\ change\ of\ angular\ momentum = \frac{\partial (mur)}{\partial t}$$

But for free vortex,

$$\frac{\partial (mur)}{\partial t} = 0$$

Integrating, we get

$$\int \frac{\partial (mur)}{\partial t} = 0 \Rightarrow Mur = Constant = ur = constant$$

Forced vortex flow

Forced vortex motion is one in which the fluid mass is made to rotate by means of some external agencies. The external agency is generally the mechanical power which imparts the constant torque on the fluid mass. The forced vortex motion is also called flywheel vortex or rotational vortex. The fluid mass in this forced vortex flow rotates at constant angular velocity ω. The tangential velocity of any fluid particle is given by,

$$u = \omega \times r$$

where 'r' is the radius of the fluid particle from the axis of rotation. Hence angular velocity ω is given by,

$$\omega = \frac{u}{r} = constant$$

Variation of pressure of a rotating fluid in any plane is given by,

$$dp = \rho(\frac{\omega^2 r^2}{r})dr - \rho g dz$$

Integrating the above equation for points 1 and 2, we get

$$\int_1^2 dp = \int_1^2 \rho(\frac{\omega^2 r^2}{r})dr - \int_1^2 \rho g dz$$

$$\Rightarrow (p_2 - p_1) = [\rho\omega^2 \frac{r^2}{2}]_1^2 - \rho g[z]_1^2$$

$$\Rightarrow \quad = \frac{\rho}{2}[u_2^2 - u_1^2] - \rho g[z_2 - z_1]$$

if the point 1 and 2 lies on free surface of the liquid, then $p_1 = p_2$ and hence above equation reduces to

$$[z_2 - z_1] = \frac{1}{2g}[v_2^2 - v_1^2]$$

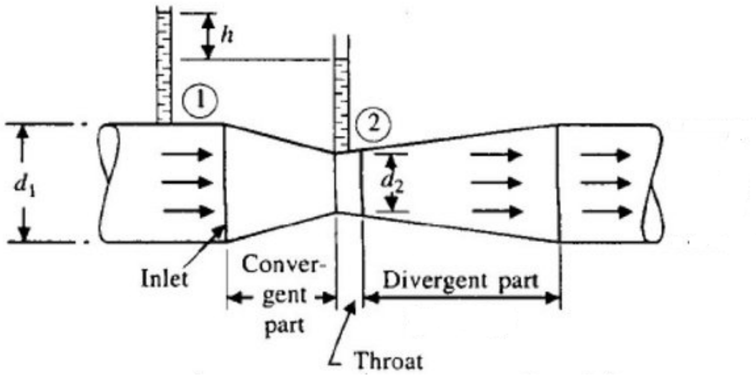
If the point 1 lies on the axis of rotaion, then $v_1 = \omega \times r_1 = \omega \times 0 = 0$,hence above equation reduces to,

$$Z = z_2 - z_1 = \frac{u_2^2}{2g} = \frac{\omega^2 r_2^2}{2g}$$

APPLICATIONS OF BERNOULLI'S EQUATION

Venturi Meter

Venturimeter is a device for measuring discharge in a pipe.



Schematic diagram of Venturi meter

A Venturi meter consists of:

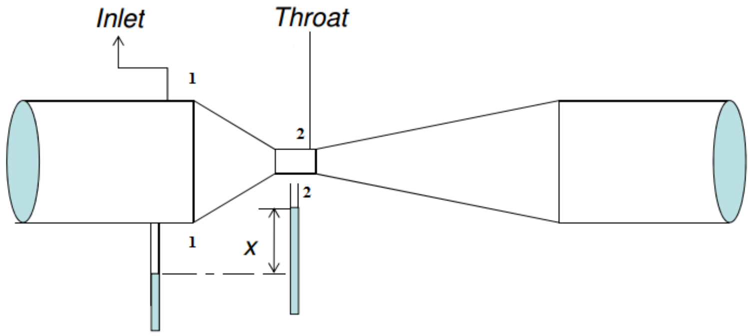
1. Inlet/ Convergent cone
2. Throat
3. Outlet/ Divergent cone

The inlet section Venturi meter is same diameter as that type of the pipe to which it is connected, followed by the short convergent section with a converging cone angle of 21 ± 1^o and its length parallel to the axis is approximately equal to $2.7(D–d)$, where 'D' is the pipe diameter and 'd' is the throat diameter.
The cylindrical throat is a section of constant cross-section with its length equal to diameter. The flow is minimum at the throat. Usually, diameter of throat is $\frac{1}{2}$ the pipe diameter.
A long diverging section with a cone angle of about 5-7^o where in the fluid is retarded and a large portion of the kinetic energy is converted back into the pressure energy.

Principle of Venturi Meter:

The basic principle on which a Venturi meter works is that by reducing the cross-sectional area of the flow passage, a pressure difference is created between the two sections, this pressure difference enables the estimation of the flow rate through the pipe.

Expression for Discharge through Venturi meter



Let, d_1=diameter at section 1-1

p_1= pressure at section at 1-1

v_1= velocity at section at 1-1

a_1= area of cross-section at 1-1

d_2, p_2, v_2, a_2 be corresponding values at section 2-2.

Applying Bernoulli equation between 1-1 and 2-2 we have,

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

Since pipe is horizontal, z_1=z_2,

Hence,

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}$$
$$\Rightarrow \frac{p_1-p_2}{\rho g}=\frac{v_2^2-v_1^2}{2g}$$
$$\Rightarrow h=\frac{v_2^2-v_1^2}{2g}$$

where $h=\frac{p_1-p_2}{\rho g}$, is the pressure difference between section 1-1 and 2-2.

from continuity equation, we have

$$a_1 v_1 = a_2 v_2$$
$$\Rightarrow v_1=\frac{a_2 v_2}{a_1}$$

Hence

$$h=\frac{v_2^2}{2g}\left[\frac{a_2^2-a_1^2}{a_1^2}\right]$$
$$\Rightarrow v_2=\frac{a_1}{\sqrt{a_1^2-a_2^2}}\sqrt{2gh} \qquad (1)$$

substituting the value of v_2 in equation Q $=a_2 v_2$ we have,

$$Q_{th} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

Above equations is for ideal fluids and is called as the theoretical discharge equation of a venturi meter. For real fluids the equation changes to,

$$Q_{act} = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

Expression for 'h' given by the differential manometer

- **Case 1:**when liquid in the manometer is heavier than the liquid flowing through the pipe.

$$h = x\left[\frac{S_H}{S_O} - 1\right]$$

where:S_H is the specific gravity of heavier liquid
S_O is the specific gravity of liquid flowing through pipe.
x difference in liquid columns in U-tube.

- **Case 2:**when liquid in the manometer is lighter than the liquid flowing through the pipe.

$$h = x\left[1 - \frac{S_L}{S_O}\right]$$

where:S_L is the specific gravity of heavier liquid
S_O is the specific gravity of liquid flowing through pipe.
x difference in liquid columns in U-tube.

Orifice Meter

Orifice

An orifice is a small aperture through which the fluid passes. The thickness of an orifice in the direction of flow is very small in comparison to its other dimensions.

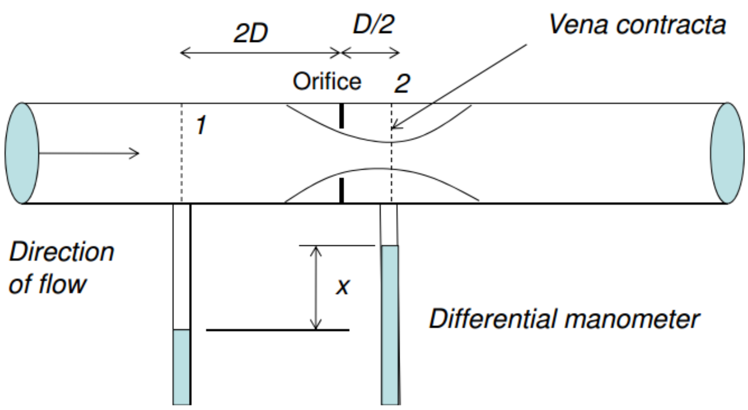
If a tank containing a liquid has a hole made on the side or base through which liquid flows, then such a hole may be termed as an orifice. The rate of flow of the liquid through such an orifice at a given time will depend partly on the shape, size and form of the orifice.

An orifice usually has a sharp edge so that there is minimum contact with the fluid and consequently minimum frictional resistance at the sides of the orifice. If a sharp edge is not provided, the flow depends on the thickness of the orifice and the roughness of its boundary surface too.

Orifice Meter

- It is a device used for measuring the rate of flow through a pipe.
- It is a cheaper device as compared to venturi meter. The basic principle on which the Orifice meter works is same as that of Venturi meter.
- It consists of a circular plate with a circular opening at the center. This circular opening is called an Orifice.
- The diameter of the orifice is generally varies from 0.4 to 0.8 times the pipe diameter.

Expression for Discharge through Orifice meter



Let, d_1=diameter at section 1-1
p_1= pressure at section at 1-1
v_1= velocity at section at 1-1
a_1= area of cross-section at 1-1
d_2, p_2, v_2, a_2 be corresponding values at section 2-2.
Applying Bernoulli equation between 1-1 and 2-2 we have,

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

Since pipe is horizontal, z_1=z_2,
Hence,

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}$$
$$\Rightarrow \frac{p_1-p_2}{\rho g}=\frac{v_2^2-v_1^2}{2g}$$
$$or$$
$$h=\frac{v_2^2-v_1^2}{2g}$$
$$or$$
$$2gh=v_2^2-v_1^2$$
$$v_2=\sqrt{2gh+v_1^2} \qquad \text{(i)}$$

Now section (2) is at the vena-contracta and a_2 represents the area at the vena-contracta.
If the area a_o is the area of the orifice, then we have

$$C_c=\frac{a_2}{a_o}$$

where $C_c$is the co-efficient of contraction.
$\therefore$

$$a_2=a_o\times C_c$$

From continuity equation, we have

$$a_1v_1=a_2v_2 \qquad or \qquad \text{(ii)}$$
$$v_1=\frac{a_2v_2}{a_1}v_2=\frac{a_oC_c}{a_1}v_2 \qquad \text{(ii)}$$

Substituting the value of v_1 in equation (i), we get

$$v_2=\sqrt{2gh+\frac{a_o^2C_c^2v_2^2}{a_1^2}}$$
$$\Rightarrow v_2=\frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_o}{a_1}\right)^2C_c^2}}$$

substituting the value of v_2 in equation Q =a_2v_2 we have,

$$Q=\frac{a_oC_c\sqrt{2gh}}{\sqrt{1-\left(\frac{a_o^2}{a_1^2}\right)C_c^2}} \qquad \text{(iv)}$$

Above equation can be simplified by using

$$C_c = C_d \frac{\sqrt{1-\left(\frac{a_o}{a_1}\right)^2}}{\sqrt{1-\left(\frac{a_o}{a_1}\right)^2 C_c^2}}$$

or

$$C_d = C_c \frac{\sqrt{1-\left(\frac{a_o}{a_1}\right)^2 C_c^2}}{\sqrt{1-\left(\frac{a_o}{a_1}\right)^2}}$$

Substituting the this value of C_c in(iv),

$$Q_{act} = a_o \times C_d \frac{\sqrt{1-\left(\frac{a_o}{a_1}\right)^2 C_c^2}}{\sqrt{1-\left(\frac{a_o}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_o^2}{a_1^2}\right) C_c^2}}$$

or

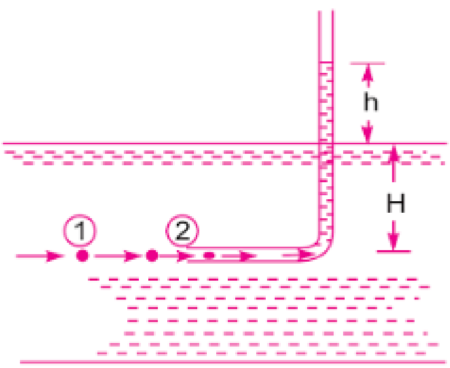
$$Q_{act} = \frac{C_d a_o a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_o^2}}$$

where C_d is the co-efficient of discharge for orifice meter.

Pitot tube

Pitot tube is a device used to measure the velocity of flow at any point in a pipe or a channel.

Principle: If the velocity at any point decreases, the pressure at that point increases due to the conversion of the Kinetic energy into pressure energy. In Simplest form, the pitot tube consists of a glass tube, bent at right angles.



Let, p_1= pressure at section at 1-1

v_1= velocity at section at 1-1

p_2= pressure at section at 1-1

v_2= velocity at section at 1-1

H= depth of tube in the liquid

h= rise of liquid in the tube above free surface

Applying Bernoulli equation between 1-1 and 2-2 we have,

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

But z_1=z_2 as points(1)and (2) are on the same line and v_2=0
$\frac{p_1}{\rho g}$= pressure head at (1)=H
$\frac{p_2}{\rho g}$= pressure head at (2)=(h+H)
Substituting these values we get,

$$H+\frac{v_1^2}{2g}=(h+H)\therefore h=\frac{v_2}{2g}\quad or\quad v_1=\sqrt{2gh}$$

this is the theoretical velocity. Actual velocity is given by

$$(v_1)_{act}=C_v\sqrt{2gh}$$

there fore velocity at any point is,

$$v_{act}=C_v\sqrt{2gh}$$

P1. An oil of sp.gr. 0.8 is flowing through a venturimeter having inlet diameter 20cm and throat 10cm. The oil mercury differential manometer shows a reading of 25cm. Calculate the discharge of oil through the horizontal venturimeter. Take Cd= 0.98.

Solution. Given :

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_h = 13.6$

Reading of differential manometer, $x = 25$ cm

$\therefore$ Difference of pressure head, $h = x\left[\dfrac{S_h}{S_o} - 1\right]$

$$= 25\left[\frac{13.6}{0.8} - 1\right] \text{ cm of oil} = 25\,[17 - 1] = 400 \text{ cm of oil}$$

Dia. at inlet, $d_1 = 20$ cm

$\therefore$ $a_1 = \dfrac{\pi}{4} d_1^{\,2} = \dfrac{\pi}{4} \times 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

$\therefore$ $a_2 = \dfrac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$

$C_d = 0.98$

$\therefore$ The discharge Q is given by equation (6.8)

or
$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - 7a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 400}$$

$$= \frac{21421375.68}{\sqrt{98696 - 6168}} = \frac{21421375.68}{304} \text{ cm}^3/\text{s}$$

P2. A horizontal venturimeter with inlet diameter 30cm and throat diameter 15cm is used to measure the flow of water. The differential manometer connected to the inlet and throat is 20cm. Calculate the discharge. Take Cd= 0.98.

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

∴ Area at inlet, $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

∴ $a_2 = \frac{\pi}{4} \times 15^2 = 176.7 \text{ cm}^2$

$C_d = 0.98$

Reading of differential manometer $= x = 20$ cm of mercury.

∴ Difference of pressure head is given by (6.9)

or $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gravity of mercury = 13.6, S_0 = Sp. gravity of water = 1

$$= 20\left[\frac{13.6}{1} - 1\right] = 20 \times 12.6 \text{ cm} = 252.0 \text{ cm of water.}$$

The discharge through venturimeter is given by eqn. (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.9 - 31222.9}} = \frac{86067593.36}{684.4}$$

$$= 125756 \text{ cm}^3/\text{s} = \frac{125756}{1000} \text{ lit/s} = \textbf{125.756 lit/s. Ans.}$$

P3.A horizontal venturimeter with inlet diameter 20cm and throat diameter 10 cm is used to measure the flow of oil of specific gravity 0.8. The discharge of oil through venturimeter is 60li/s. Find the reading of the oil-mercury manometer. Take Cd= 0.98

Solution. Given : $d_1 = 20$ cm

$\therefore \quad a_1 = \frac{\pi}{4} 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

$\therefore \quad a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$

$C_d = 0.98$

$Q = 60 \text{ litres/s} = 60 \times 1000 \text{ cm}^3\text{/s}$

Using the equation (6.8), $$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

or $$60 \times 1000 = 9.81 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times h} = \frac{1071068.78\sqrt{h}}{304}$$

or $$\sqrt{h} = \frac{304 \times 60000}{1071068.78} = 17.029$$

$\therefore \quad h = (17.029)^2 = 289.98$ cm of oil

But $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gr. of mercury = 13.6

S_o = Sp. gr. of oil = 0.8

x = Reading of manometer

$$\therefore \quad 289.98 = x\left[\frac{13.6}{0.8} - 1\right] = 16x$$

$$\therefore \quad x = \frac{289.98}{16} = 18.12 \text{ cm.}$$

$\therefore$ Reading of oil-mercury differential manometer = **18.12 cm. Ans.**

P4. A horizontal venturimeter with inlet diameter 20cm and throat diameter 10cm is used to measure the flow of water. The pressure at inlet is 17.658N/cm2 and vacuum pressure at throat is 30cm of Mercury. Find the discharge of water through venturimeter. Take Cd = 0.98.

Solution. Given :

Dia. at inlet, $d_1 = 20$ cm

$\therefore \quad a_1 = \frac{\pi}{4} \times (20)^2 = 314.16 \text{ cm}^2$

Dia. at throat, $d_2 = 10$ cm

$\therefore \quad a_2 = \frac{\pi}{4} \times 10^2 = 78.74 \text{ cm}^2$

$p_1 = 17.658 \text{ N/cm}^2 = 17.658 \times 10^4 \text{ N/m}^2$

ρ for water $= 1000 \frac{\text{kg}}{\text{m}^3}$ and $\therefore \frac{p_1}{\rho g} = \frac{17.658 \times 10^4}{9.81 \times 1000} = 18$ m of water

$\frac{p_2}{\rho g} = -30$ cm of mercury

$= -0.30$ m of mercury $= -0.30 \times 13.6 = -4.08$ m of water

$\therefore$ Differential head $= h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 18 - (-4.08)$

$= 18 + 4.08 = 22.08$ m of water $= 2208$ cm of water

The discharge Q is given by equation (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.74)^2}} \times \sqrt{2 \times 981 \times 2208}$$

$$= \frac{50328837.21}{304} \times 165555 \text{ cm}^3/\text{s} = \mathbf{165.555 \text{ lit/s. Ans.}}$$

P5. The inlet and throat diameters of a horizontal venturimeter are 30cm and 10cm respectively. The liquid flowing through the venturimeter is water. The pressure intensity at inlet is 13.734N/cm^2 while the vacuum pressure head at the throat is 37 cm of mercury. Find the rate of flow. Assume that 4% of the differential head is lost between the inlet and the throat. Find also the values of C_d for the Venturimeter.

Dia. at inlet, $d_1 = 30$ cm

$\therefore$ $a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 10$ cm

$\therefore$ $a_2 = \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$

Pressure, $p_1 = 13.734 \text{ N/cm}^2 = 13.734 \times 10^4 \text{ N/m}^2$

$\therefore$ Pressure head, $\frac{p_1}{\rho g} = \frac{13.734 \times 10^4}{1000 \times 9.81} = 14$ m of water

$$\frac{p_2}{\rho g} = -37 \text{ cm of mercury}$$

$$= \frac{-37 \times 13.6}{100} \text{ m of water} = -5.032 \text{ m of water}$$

Differential head, $h = p_1/\rho g - p_2/\rho g$

$$= 14.0 - (-5.032) = 14.0 + 5.032$$
$$= 19.032 \text{ m of water} = 1903.2 \text{ cm}$$

Head lost, $h_f = 4\% \text{ of } h = \frac{4}{100} \times 19.032 = 0.7613$ m

$$\therefore \quad C_d = \sqrt{\frac{h - h_f}{h}} = \sqrt{\frac{19.032 - .7613}{19.032}} = 0.98$$

$\therefore$ Discharge

$$= C_d \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$= \frac{0.98 \times 706.85 \times 78.54 \times \sqrt{2 \times 981 \times 1903.2}}{\sqrt{(706.85)^2 - (78.54)^2}}$$

$$= \frac{105132247.8}{\sqrt{499636.9 - 6168}} = 149692.8 \text{ cm}^3\text{/s} = \mathbf{0.14969 \text{ m}^3\text{/s. Ans.}}$$

P6: A 30cmX15cm Venturimeter is inserted in a vertical pipe carrying water flowing in the upward direction. A differential mercury manometer connected to the inlet and throat gives a reading of 20cm. Find the discharge. Take Cd = 0.98

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

$\therefore$ $a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ $a_2 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

$$h = x\left[\frac{S_l}{S_o} - 1\right] = 20\left[\frac{13.6}{1.0} - 1.0\right] = 20 \times 12.6 = 252.0 \text{ cm of water}$$

$C_d = 0.98$

Discharge, $Q = C_d \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 981 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.3 - 31222.9}} = \frac{86067593.36}{684.4}$$

$= 125756 \text{ cm}^3/\text{s} =$ **125.756 lit/s. Ans.**

P7: A 20cmX10cm venturimeter is inserted in a vertical pipe carrying oil of sp.gr 0.8, the flow of oil is in the upward direction. The difference of levels between the throat and inlet section is 50cm. The oil mercury differential manometer gives a reading of 30cm of Mercury. Find the discharge of oil. Neglect the losses.

Solution. Dia. at inlet, $d_1 = 20$ cm

$\therefore$ $a_1 = \frac{\pi}{4}(20)^2 = 314.16 \text{ cm}^2$

Dia. at throat, $d_2 = 10$ cm

$$\therefore \qquad a_2 = \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$$

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_g = 13.6$

Differential manometer reading, $x = 30$ cm

$$\therefore \qquad h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[\frac{S_g}{S_o} - 1\right]$$

$$= 30\left[\frac{13.6}{0.8} - 1\right] = 30\,[17 - 1] = 30 \times 16 = 480 \text{ cm of oil}$$

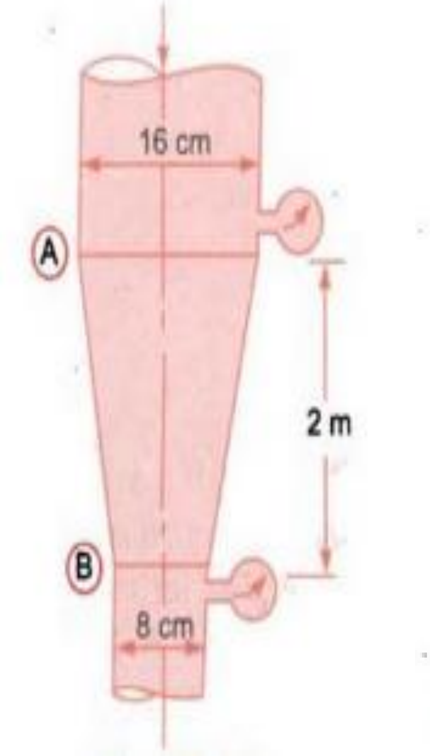
$$C_d = 1.0$$

The discharge, $$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= \frac{1.0 \times 314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 480} \text{ cm}^3/s$$

$$= \frac{23932630.7}{304} = 78725.75 \text{ cm}^3/\text{s} = \mathbf{78.725 \text{ litres/s. Ans.}}$$

P.8:In a vertical pipe conveying oil of specific gravity 0.8, two pressure gauges have been installed at A and B where the diameters are 16cm and 8cm respectively. A is 2 meters above B. The pressure gauge readings have shown that the pressure at B is greater than at A by 0.981N/cm2. Neglecting all losses, calculate the flow rate. If the gauges at A and B are replaced by tubes filled with the same fluid and connected to a U tube containing Mercury, Calculate the difference of level of Mercury in the two limbs of the U tube.



Solution. Given :

Sp. gr. of oil, $S_o = 0.8$

$\therefore$ Density, $\rho = 0.8 \times 1000 = 800 \frac{kg}{m^3}$

Dia. at A, $D_A = 16$ cm $= 0.16$ m

$\therefore$ Area at A, $A_1 = \frac{\pi}{4}(.16)^2 = 0.0201$ m^2

Dia. at B, $D_B = 8$ cm $= 0.08$ m

$\therefore$ Area at B, $A_2 = \frac{\pi}{4}(.08)^2 = 0.005026$ m^2

(*i*) Difference of pressures, $p_B - p_A = 0.981$ N/cm^2

$= 0.981 \times 10^4$ N/m$^2 = \frac{9810N}{m^2}$

Difference of pressure head

$$\therefore \quad \frac{p_B - p_A}{\rho g} = \frac{9810}{800 \times 9.81} = 1.25$$

Applying Bernoulli's equation between A and B, taking the reference line passing through B, we have,

$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$

$(p_A/\gamma - p_B/\gamma) + z_A - z_B = (v_B^2/2g - v_A^2/2g)$

$(p_A/\gamma - p_B/\gamma) + 2.0 - 0.0 = (v_B^2/2g - v_A^2/2g)$

$-1.25 + 2.0 = (v_B^2/2g - v_A^2/2g)$

$0.75 = (v_B^2/ 2g - v_A^2/ 2g)$ ----------------- (1)

Now applying Continuity equation at A and B, we get,

$A_A V_A = A_B V_B$

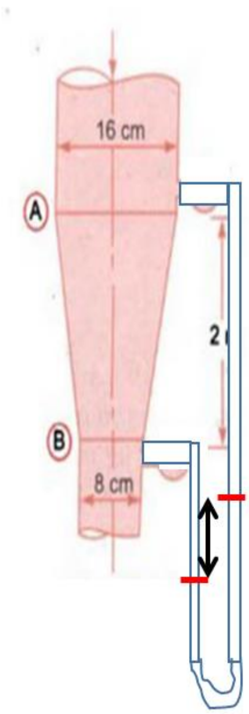
$V_B = A_A V_A \ / A_B = 4V_A$

Substituting the value of V_B in equation (1), we get

$0.75 = 16\ v_A^2/ 2g - v_A^2/ 2g = 15\ v_A^2/ 2g$; Va= 0.99m/s

Rate of flow, Q= $A_A V_A$

Q= 0.99* 0.01989 Cum/s



Difference of level of mercury in the u – Tube:

Let x = Difference of Mercury level

Then h= x[(sm/so)- 1]

$h = (p_A/ \gamma + z_A) – (p_B/ \gamma + z_B)$

$= (p_A/ \gamma – p_B/ \gamma) + (z_A + z_B) = -1.25+2.00 = 0.75m$

0.75 = x[(13.6/0.8)-1] = 16x

X= 4.687 cm

P.9: Find the discharge of water flowing through a pipe 30cm diameter placed in an inclined position where a venturimeter is inserted, having a throat diameter of 15 cm. The difference of pressure between the main and the throat is measured by a liquid of sp.gr. 0.6 in an inverted U tube which gives a reading of 30cm. The loss of head between the main and the throat is 0.2 times the kinetic head of the pipe.

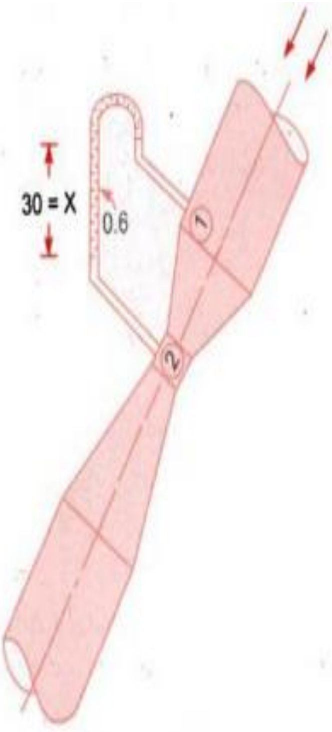
Solution. Dia. at inlet $= 30$ cm

$\therefore$ $d_1 = 30$ cm

$$a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ $$a_2 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$$



Reading of differential manometer, $x = 30$ cm

Difference of pressure head, h is given by

$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h$$

Also $$h = x\left[1 - \frac{S_l}{S_o}\right]$$

where $S_l = 0.6$ and $S_o = 1.0$

$$= 30\left[1 - \frac{0.6}{1.0}\right] = 30 \times .4 = 12.0 \text{ cm of water}$$

Loss of head, $h_L = 0.2 \times$ kinetic head of pipe $= 0.2 \times \frac{v_1^2}{2g}$

Now applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{v_2^2}{2g} + h_L$$

or $$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) + \frac{v_1^2}{2g} - \frac{v_2^2}{2g} = h_L$$

But $$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h = 12.0 \text{ cm of water}$$

and $$h_L = 0.2 \times v_1^2/2g$$

$\therefore$ $$12.0 + \frac{v_1^2}{2g} - \frac{v_2^2}{2g} = 0.2 \times \frac{v_1^2}{2g}$$

$\therefore$ $$12.0 + 0.8\,\frac{v_1^2}{2g} - \frac{v_2^2}{2g} = 0$$

Applying continuity equation at (1) and (2), we get

$$a_1 v_1 = a_2 v_2$$

$$\therefore \qquad v_1 = \frac{a_2}{a_1} v_2 = \frac{\frac{\pi}{4}(15)^2 v_2}{\frac{\pi}{4}(30)^2} = \frac{v_2}{4}$$

Substituting this value of v_1 in equation (1), we get

$$12.0 + \frac{0.9}{2g}\left(\frac{v_2}{4}\right)^2 - \frac{v_2^2}{2g} = 0 \text{ or } 12.0 + \frac{v_2^2}{2g}\left[\frac{0.8}{16} - 1\right] = 0$$

$$\text{or} \qquad \frac{v_2^2}{2g}[.05 - 1] = -12.0 \text{ or } \frac{0.95\, v_2^2}{2g} = 12.0$$

$$\therefore \qquad v_2 = \sqrt{\frac{2 \times 981 \times 12.0}{0.95}} = 157.4 \text{ cm/s}$$

$$\therefore \text{ Discharge} = a_2 v_2$$

$$= 176.7 \times 157.4 \text{ cm}^3\text{/s} = 27800 \text{ cm}^3\text{/s} = \mathbf{27.8 \text{ litres/s. Ans.}}$$

P.10: A 30cmX15cm venturimeter is provided in a vertical pipe line carrying oil of specific gravity 0.9, the flow being upwards. The difference in elevation of the throat section and entrance section of the venturimeter is 30cm. The differential U- tube mercury manometer shows a gauge deflection of 25cm. Calculate:

1. The discharge of the oil and
2. The pressure difference between the entrance section and the throat section. Take the coefficient of meter as 0.98 and the specific gravity of Mercury as 13.6.

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

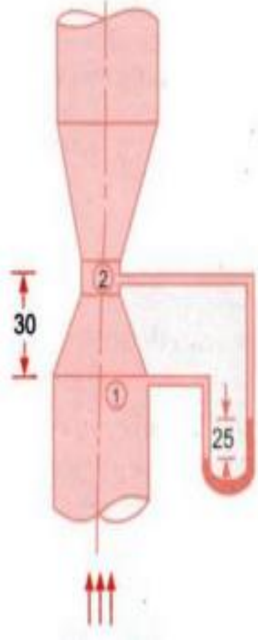
$\therefore$ Area, $a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ Area, $a_2 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

Let section (1) represents inlet and section (2) represents throat. Then $z_2 - z_1 = 30$ cm

Sp. gr. of oil, $S_o = 0.9$



Sp. gr. of mercury, $S_g = 13.6$

Reading of diff. manometer, $x = 25$ cm

The differential head, h is given by

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right)$$

$$= x\left[\frac{S_g}{S_o} - 1\right] = 25\left[\frac{13.6}{0.9} - 1\right] = 352.77 \text{ cm of oil}$$

(*i*) The discharge, Q of oil $= C_d \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$= \frac{0.98 \times 706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} = \sqrt{2 \times 981 \times 352.77}$$

$$= \frac{101832219.9}{684.4} = 148790.5 \text{ cm}^3/\text{s}$$

= 148.79 litres/s. Ans.

(*ii*) **Pressure difference between entrance and throat section**

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = 352.77$$

or $$\left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + z_1 - z_2 = 352.77$$

But $$z_2 - z_1 = 30 \text{ cm}$$

$\therefore$ $$\left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) - 30 = 352.77$$

$\therefore$ $$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 352.77 + 30 = 382.77 \text{ cm of oil} = \mathbf{3.8277\ m\ of\ oil.\ Ans.}$$

or $$(p_1 - p_2) = 3.8277 \times \rho g$$

But density of oil $= \text{Sp. gr. of oil} \times 1000 \text{ kg/m}^3$

$= 0.9 \times 1000 = 900 \text{ kg/cm}^3$

$\therefore$ $$(p_1 - p_2) = 3.8277 \times 900 \times 9.81 \ \frac{\text{N}}{\text{m}^2}$$

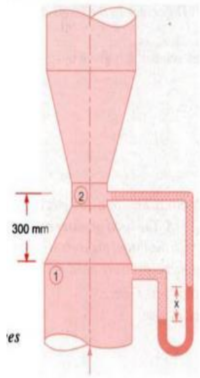
$$= \frac{33795}{10^4} \text{ N/cm}^2 = \mathbf{3.3795\ N/cm^2.\ Ans.}$$

P.11: Crude oil of specific gravity 0.85 flows upwards at a volume rate of flow of 60 liter/sec through a vertical venturimeter with an inlet diameter of 200 mm and a throat diameter of 100mm. The coefficient of discharge of the venturimeter is 0.98. The vertical distance between the pressure tapings is 300mm.

1. If two pressure gauges are connected at the tapings such that they are positioned at the levels of their corresponding taping points, determine the difference of readings in N/cm2 of the two pressure gauges.
2. If a mercury differential manometer is connected in place of pressure gauge to the tapings such that the connecting tube upto mercury are filled with oil, determine the level of the mercury column.

Solution. Given :

Specific gravity of oil, $S_o = 0.85$



$\therefore$ Density, $\rho = 0.85 \times 1000 = 850 \text{ kg/m}^3$

Discharge, $Q = 60$ litre/s

$$= \frac{60}{1000} = 0.06 \text{ m}^3/\text{s}$$

Inlet dia, $d_1 = 200 \text{ mm} = 0.2 \text{ m}$

$\therefore$ Area, $a_1 = \frac{\pi}{4}(.2)^2 = 0.0314 \text{ m}^2$

Throat dia., $d_2 = 100 \text{ mm} = 0.1 \text{ m}$

$\therefore$ Area, $a_2 = \frac{\pi}{4}(0.1)^2 = 0.00785 \text{ m}^2$

Value of C_d $= 0.98$

Let section (1) represents inlet and section (2) represents throat. Then

$$z_2 - z_1 = 300 \text{ mm} = 0.3 \text{ m}$$

(i) Difference of readings in N/cm² of the two pressure gauges

The discharge Q is given by,

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

or

$$0.06 = \frac{0.98 \times 0.0314 \times 0.00785}{\sqrt{0.0314^2 - 0.00785^2}} \times \sqrt{2 \times 9.81 \times h}$$

$$= \frac{0.98 \times 0.00024649}{0.0304} \times 4.429\sqrt{h}$$

$$\therefore \quad \sqrt{h} = \frac{0.06 \times 0.0304}{0.98 \times 0.00024649 \times 4.429} = 1.705$$

$$\therefore \quad h = 1.705^2 = 2.908 \text{ m}$$

But for a vertical venturimeter, $h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right)$

$$\therefore \quad 2.908 = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + z_1 - z_2$$

$$\frac{p_1 - p_2}{\rho g} = 2.908 + z_2 - z_1 = 2.908 + 0.3 \qquad (\because \; z_2 - z_1 = 0.3 \text{ m})$$

$$= 3.208 \text{ m of oil}$$

$$\therefore \quad p_1 - p_2 = \rho g \times 3.208$$

$$= 850 \times 9.81 \times 3.208 \text{ N/m}^2 = \frac{850 \times 9.81 \times 3.208}{10^4} \text{ N/cm}^2$$

$$= \mathbf{2.675\ N/cm^2.\ Ans.}$$

(*ii*) *Difference in the levels of mercury columns* (*i.e.*, *x*)

The value of h is given by, $\quad h = x\left[\frac{S_g}{S_o} - 1\right]$

$$\therefore \quad 2.908 = x\left[\frac{13.6}{0.85} - 1\right] = x\,[16 - 1] = 15\,x$$

$$\therefore \quad x = \frac{2.908}{15} = 0.1938 \text{ m} = \mathbf{19.38\ cm\ of\ oil.\ Ans.}$$