15 CV 33 FLUID MECHANICS NOTES

MODULE-5

- **Module-5 : Surge Analysis in Pipes**

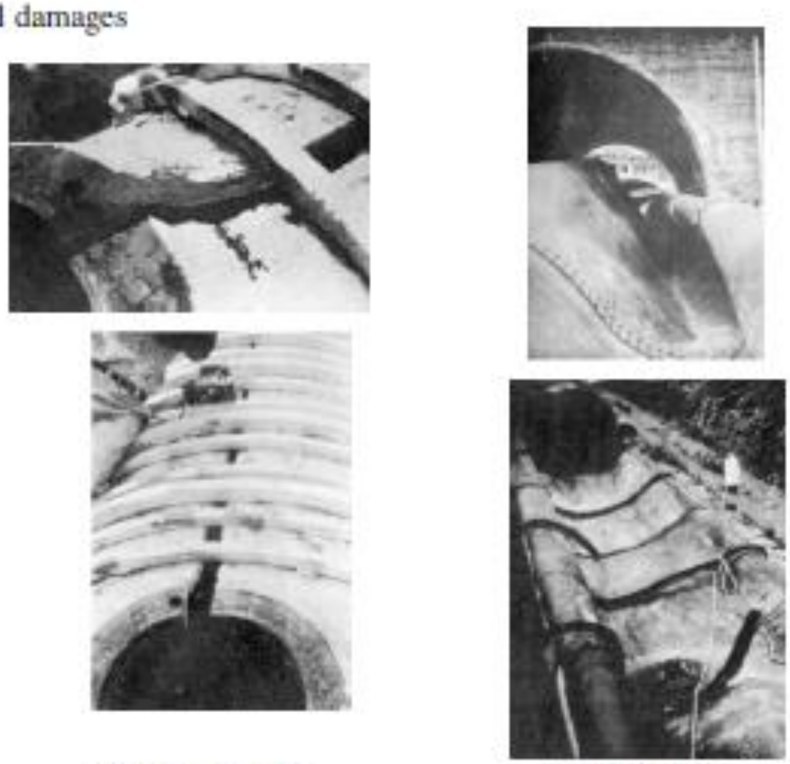
by

Dr. Nagaraj Sitaram, Principal & Professor, Amrutha Institute of Engineering & Management, Bidadi, Ramanagar District, Karnataka State

Module-5: Surge Analysis in Pipes
Water hammer in pipes, equations for rise in pressure due to gradual valve closure and sudden closure for rigid and elastic pipes. Problems.

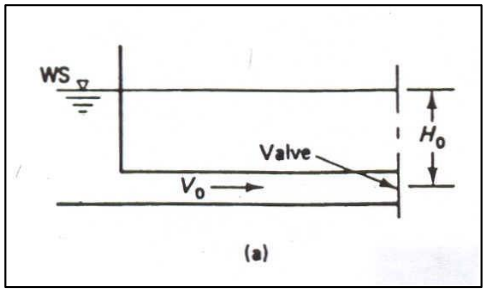
Water hammer – changes occur very quickly, the analysis involves consideration of wave propagation velocity, compressibility of the fluid and elasticity of the system. Solution requires graphical or computer based numerical techniques eg) rapid valve operation, pump shutdown and turbine load rejection. Water Hammer is also called fluid or pressure transients.

Some typical damages

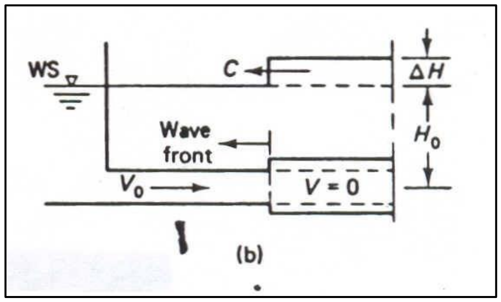


Burst pipe in power Pipe damage in

Water Hammer Phenomenon in pipelines: A sudden change of flow rate in a large pipeline (due to valve closure, pump turnoff, etc.) involve a great mass of water moving inside the pipe. The force resulting from changing the speed of the water mass may cause a pressure rise/ pressure drop in the pipe with a magnitude several times greater/less than the normal static pressure in the pipe. This may set up a noise known as knocking. This phenomenon is commonly known as the water hammer phenomenon



(a) Steady state prior to valve closure



(b) Rapid valve closure – pressure increase, pipe walls expand, liquid compression; transient conditions propagate upstream

Factors affecting water hammer phenomenon:

(i) Length of Pipeline (ii) Diameter of the pipeline
(iii) Material of the pipeline (iv) Discharge
(v) Thickness of pipeline (vi) Time of valve closure

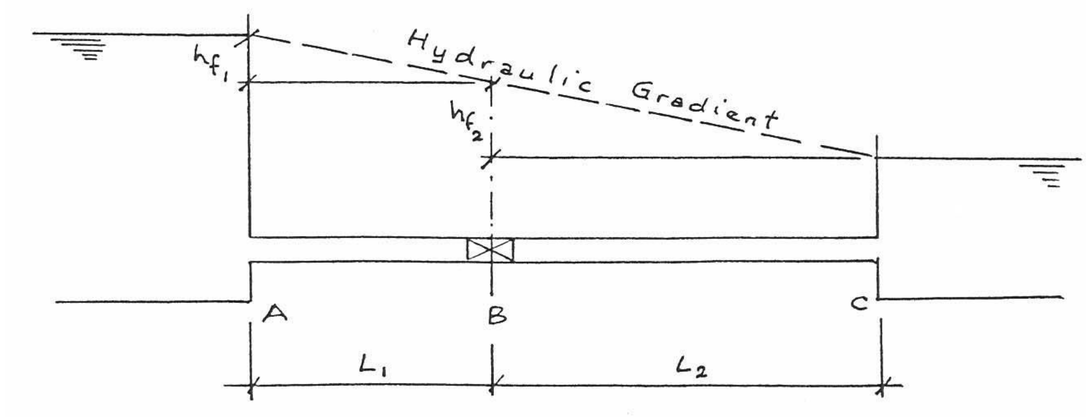
<u>Theory of unsteady flow in pipes</u>

Case 1 – Incompressible fluid and rigid pipe

The closure of a valve at the downstream end of a pipe through which the fluid is passing, results in an immediate rise in pressure. Opening of a downstream valve results in an immediate drop in pressure.

The change in pressure is due to the change in inertia (mass x acceleration). Case 1 is only to be used when the valve is closed very slowly or the pipe is very short.

Consider a rigid pipe joining two reservoirs:-

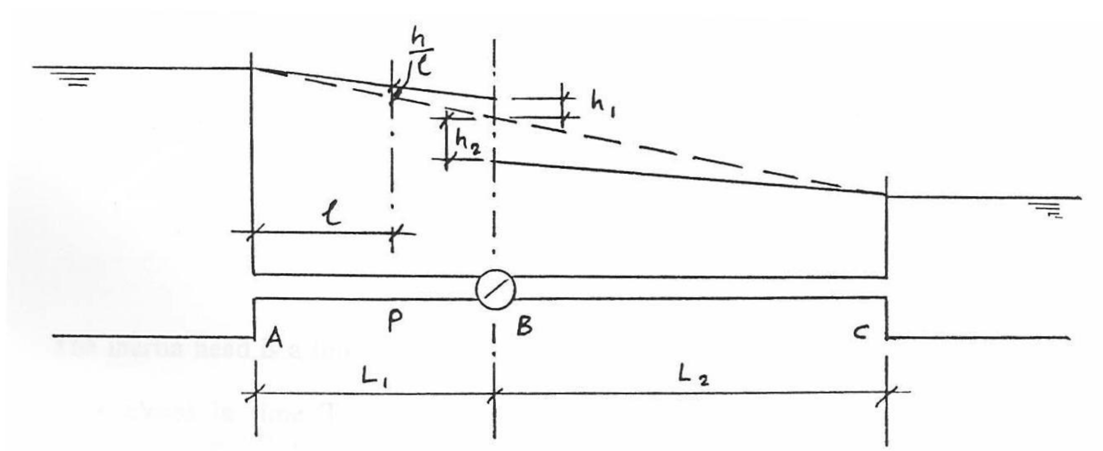


Neglecting velocity heads and minor losses, the hydraulic gradient shown gives the pressure for all points along the pipeline, with steady flow Q and the valve fully open.

Now, $h_f = \dfrac{\lambda L Q^2}{12.1 d^5}$

ie) $\dfrac{h_f}{L} = \dfrac{\lambda Q^2}{12.1 d^5}$ = hydraulic gradient

If the valve is closed gradually, there is a change in pressure throughout the pipeline <u>as soon as the valve starts to close.</u>



Upstream of the valve, the pressure rises by an amount h_1 and falls by an amount h_2 downstream of the valve. The change in pressure is caused by a change in inertia at the valve.

The fluid approaching the valve finds its path impeded and is unable to move with its previous velocity v $(= \dfrac{Q}{A})$ and since the water is incompressible the whole of column AB is retarded i.e. suffers from a negative acceleration. From Newton's Second Law of Motion, the mass of the water in the pipe multiplied by the acceleration is equal to the force applied. This force is equal to the inertia head, h_1 multiplied by the cross sectional area at B.

Mass of fluid column AB = ρAL_1

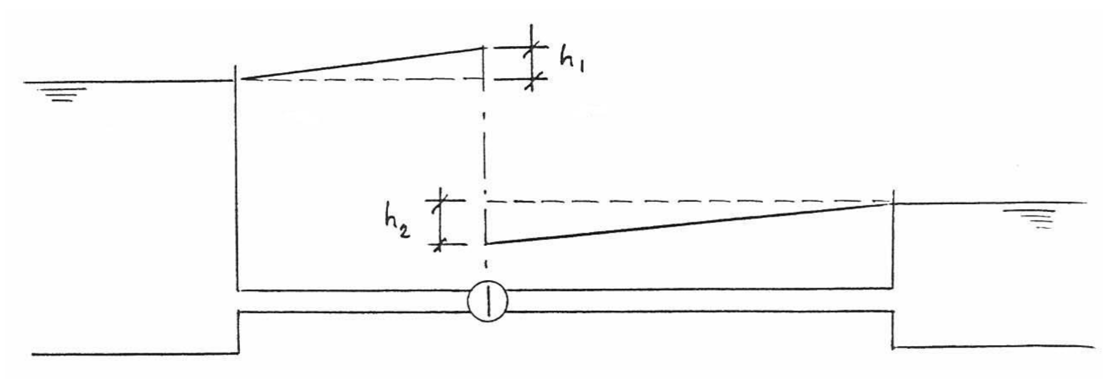
Acceleration = $\frac{dv}{dt}$

$\therefore \rho AL_1 \frac{dv}{dt} = - \rho g h_1 A$

ie) at B : $h_1 = - \frac{L}{g} \frac{dv}{dt}$ $\left[\text{where } \frac{dv}{dt} \text{ is negative}\right]$

and in general, for any point on the pipeline:- $\boxed{h = - \frac{L}{g} \frac{dv}{dt}}$

For the valve just reaching the closed position:-



The inertia head is a function of $\frac{dv}{dt}$ and therefore could be complex. However if the valve closes in time T, such that the retardation is at a constant rate, then the acceleration in AB is $\frac{v}{T}$.

Therefore in this case the general expression becomes

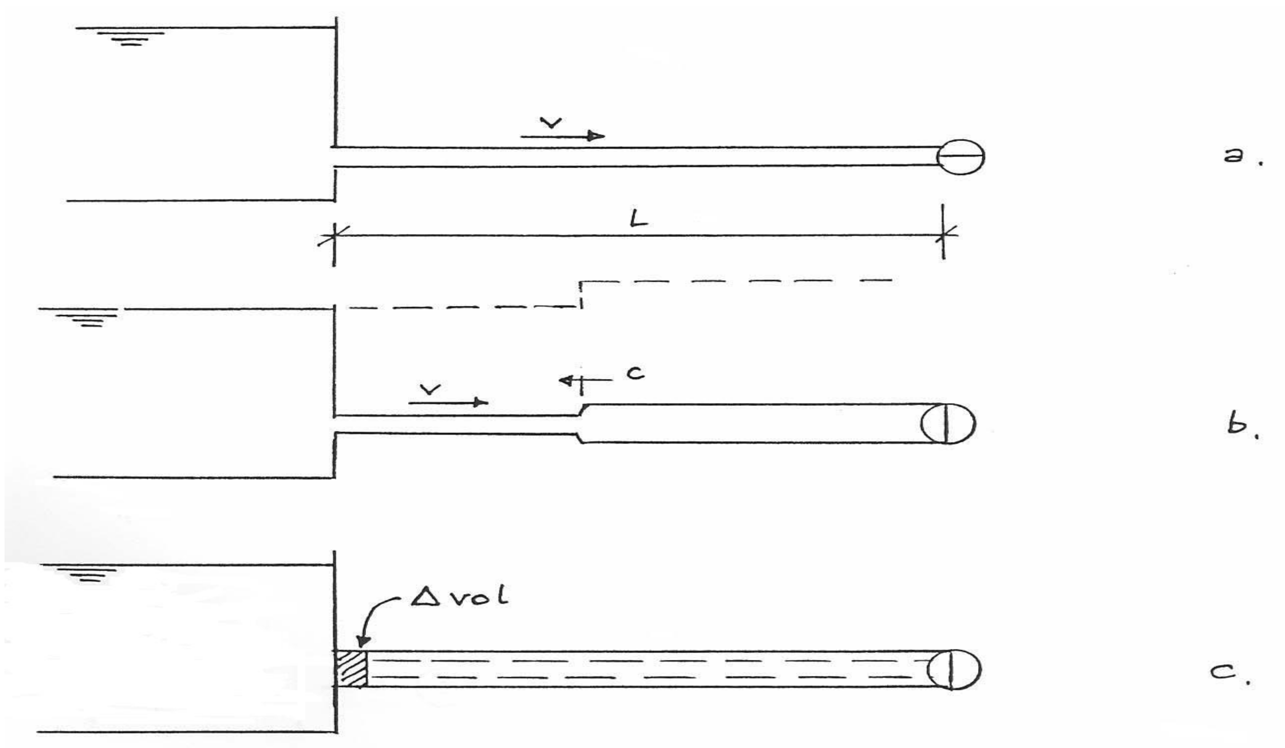
\# $\boxed{h = \frac{L}{g} \frac{v}{T}}$

Case 2 - **Compressible fluid and rigid pipe**

In case 1, the pressures obtained are only accurate if the change of velocity is slow and smooth. If the change of velocity is sudden ie) T = 0, then $\frac{dv}{dt}$ would be infinite and the pressure change would be infinite. This is not the case. Because of the elastic nature of the system, the water column acts like a goods train with loose couplings which stops suddenly. Before considering case 2 in detail, it is useful to consider the general case of an elastic pipe and fluid (neglecting friction at this stage).

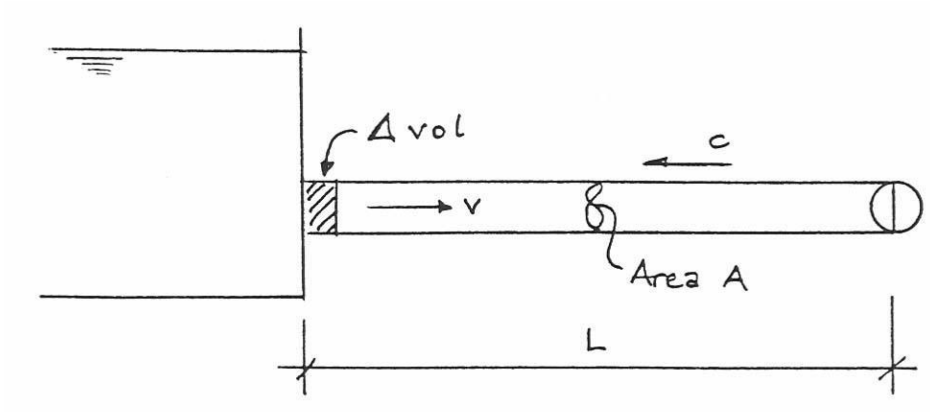
If fluid is flowing at velocity c, in a pipe length L, cross sectional area A and a valve is suddenly closed downstream, then a pressure wave moves upstream with a velocity, c. Behind the wave the water is compressed and the pipe walls are stretched and at the same time the fluid is still entering the pipe upstream, with its original velocity v. The wave from continues until it reaches the upstream end, taking time $\frac{L}{c}$ to reach there.

The time $\frac{2L}{c}$ is known as the pipe period (see later notes).



Dr. Nagaraj Sitaram, Principal & Professor, Amrutha Institute of Engineering & Management, Bidadi, Ramanagar District, Karnataka

Returning to case 2, if the pipe walls are rigid, the whole of the extra volume Δvol is added to the original amount of fluid in the pipe. The extra and original volume occupy the same space, AL.



[The amount of fluid, extra, in the pipe after the valve closure, before the fluid from the right hand side reaches the reservoir, is equal to the volume

Δvol = velocity x area x $\frac{L}{c}$.]

Now, the increase in pressure, $p = k.\ \dfrac{\Delta vol}{Vol}$

$$= k.\ \frac{\Delta vol}{A.L}$$

where k = bulk of modulus of the fluid

= degree of compressibility of the fluid

$$= -\ \frac{\delta p}{\delta Vol / Vol}$$

and since Δ vol = v.A. $\dfrac{L}{c}$

$$\text{then } p = k.\ \frac{v.A\,\frac{L}{c}}{A.L.}$$

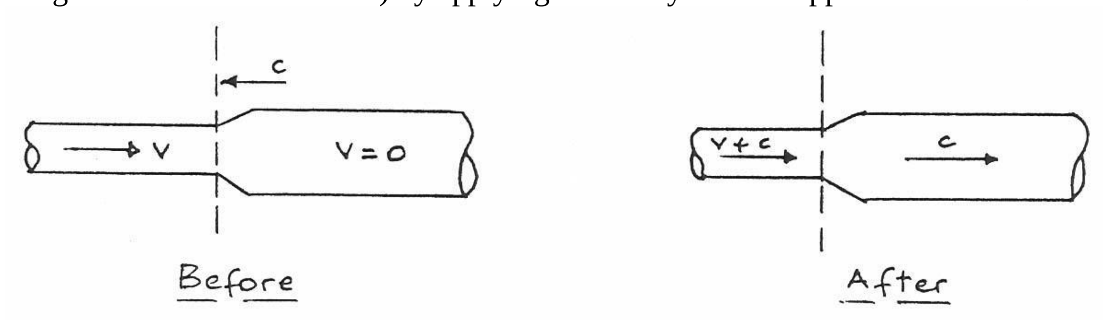
$\therefore$ $\boxed{P = k.\frac{v}{c}}$ ________1

k = bulk modulus

v = initial velocity

c = velocity of propagation of wave

In order to analyse the position where the wave front meets the original flow, it is necessary to bring the wave from to rest ie) by applying a velocity c in the opposite direction.



Now mass flow rate = ρ A c [neglecting v since it is insignificant compared to c]

Change in velocity = v

Rate of momentum = Applied force due to pressure, p

$\therefore \rho$ A c v = p A

ie) $\boxed{c = \dfrac{P}{\rho v}}$

Substituting in 1 gives:-

$$P = k.\ \frac{v}{P/\rho v}$$

ie) $P^2 = v^2\ \rho\, k$

$$\therefore \boxed{P = v\sqrt{\rho k}} \qquad \text{———— 2}$$

As head of fluid, h = $\dfrac{P}{\rho g}$

$$h = \ v\frac{\sqrt{\rho\, k}}{\rho\, g}$$

$$\therefore \boxed{h = \frac{v}{g}\sqrt{\frac{k}{\rho}}} \qquad \text{———— 3}$$

This means that the pressure developed, p, is independent of pipe dimensions for a given velocity.

Combining 1 and 2

$k\ \dfrac{v}{c} = v\ \sqrt{\rho k}$ ie) $\boxed{c = \sqrt{\dfrac{k}{\rho}}}$ ———— 4

Velocity of propagation wave

Remember, still considering Case 2 – compressible fluid and rigid pipe. The following example illustrates the magnitude of the velocities.

A representative value of k for water is 2.05GN/m^2 and so the wave velocity in a rigid pipe is:-

$$c = \sqrt{\frac{k}{\rho}} = \sqrt{\frac{2.05.10^9}{10^3}} = \underline{1432} \text{ m/s}$$

Other liquids give figures of the same order.

From 4 $c = \sqrt{\frac{k}{\rho}}$ and substituting in 3 gives:-

$$h = \frac{v}{g} . c$$

In general form:-

$$\boxed{h = - \frac{c}{g} \Delta v}$$

Where h is the increase in pressure associated with the change in velocity Δv.

Substituting in the above equation for the values appropriate to water, show that a reduction of 3m/s corresponds to an increase in head of about 440 m(about 4.3 MN/m^2)

Note This equation is independent of the length of the pipe- unlike the equation derived for equation 1.

Case 3 **Compressible fluid, Elastic Pipe**

May be analysed using strain energy theory,

Kinetic Energy of water before closure	=	Strain Energy of water + strain energy of pipe after closure

K.E. of water = ½ mv^2 = ½ ρALv^2

Strain energy of water

	Strain energy	=	½ [stress x strain]
and	Stress	=	water hammer pressure, p
	Strain	=	change in volume, Δvol.

From case 2, $\Delta \text{vol} = \dfrac{\text{pAL}}{\text{k}}$

(This was for a rigid pipe, but in a stiff elastic pipe, the increase in diameter of the fluid resulting from an increase in pressure may be neglected.)

$\therefore S.E$ of water = ½ $A \ L \ \dfrac{p^2}{k} = (\text{S.E.})_w$

Strain energy of pipe walls

Stress depends on the method of anchoring the pipe and the material from which the pipe is made.

f_L	=	longitudinal stress
f_c	=	circumferential or hoop stress
E	=	Young's modulus of pipe material
ν	=	Poisson's ratio of pipe material

(Lateral strain = ν x direct strain).

Then:-

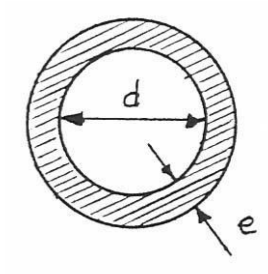
Longitudinal strain = $\dfrac{f_L}{E} - \nu \dfrac{f_c}{E}$

Circumferential Strain = $\dfrac{f_C}{E} - \nu$

Strain Energy per unit volume of pipe is equal to ½ $\sum$ (Stress ×.strain)

$\therefore$ Strain energy of pipe, $(\text{S.E.})_p$

$$=1/2 \left[f_L \left(\frac{f_L}{E} - \frac{\nu f_c}{E} \right) + f_c \left(\frac{f_c}{E} - \frac{\nu f_L}{E} \right) \right] \pi \ d \ L \ e$$



In general,

$$(S.E.)_p = \frac{ALp^2}{2E} \quad \frac{d}{\rho} \, . \, C^1$$

Where C^1 = constant, depending on the method of anchorage.

E.g. For a thin walled pipe, with no expansion joints, fixed at one end and free to move longitudinally,

$$C^1 = \frac{5}{4} - \nu$$

For a thin walled pipe, without expansion joints and anchored throughout its length,

$$C^1 = 1 - \nu^2$$

For a thin walled pipe with expansion joint throughout its length,

$$C^1 = 1 - \frac{\nu}{2}$$

Energy Balance

$$K.E. = (S.E.)_w + (S.E.)_p$$

$$\tfrac{1}{2} \; \rho \, AL \, v^2 = \frac{AL\,p^2}{2}\left[\frac{1}{k} + \frac{dC^1}{\rho E}\right]$$

Hence

$$p = v \sqrt{\frac{\rho}{\frac{1}{k} + \frac{dC^1}{eE}}}$$

or

$$h = \frac{v}{g}\sqrt{\frac{1}{\rho\left(\frac{1}{k}+\frac{dC^1}{eE}\right)}}$$

Now the speed of propagation of a wave in an infinite fluid, $c = \frac{p}{\rho v}$, so in this case, where the elasticity of the fluid and pipe are being considered:-

$$c = \sqrt{\frac{1}{\rho\left(\frac{1}{k}+\frac{dC^1}{eE}\right)}}$$

If the longitudinal stress is ignored (usually the case in most water hammer problems, since it only becomes significant when the Young's modulus of the pipe is much smaller that fluid bulk modulus, which may occur with plastic or rubber tubing), then $C^1 = 1$ in the above equations.

<u>Elasticity of Materials used for Pipe Walls:</u>

Materials	**Young's Modulus E (10^9 N/m^2)**	**Poisson's Ratio ν**
Aluminum	70	0.33
Asbestos – cement	24	-
Brass	100	0.36
Concrete	20	0.1 – 0.3
Copper	120	0.34 - 0.37
Glass	70	-
Cast Iron	100	0.21 - 0.30
Lead	10	0.43
Perspex	6	0.33
Polythene	0.8	0.46

Polystyrene	5	0.40
Mild Steel	210	0.28

Bulk Modulus K (N/m^2)

Water at 20^o C	-	2.1 x 10^9
Oil at 15^o C	-	1.5 x 10^9

Problems on Water Hammer

(1) A hydraulic pipeline 3.5 km long and 50cm diameter is used to convey water with a velocity of 1.5 m/s. Determine the rise in pressure head in the pipeline if the valve provided at the outflow end is closed in (i) 20 seconds (ii) 3.5 seconds with rigid pipe (iii) 3.0 sec with elastic pipe of thickness 2.0mm. Given Bulk modulus of water K = 2 GPa, $E_{pipe\ material} = 2.06 \times 10^{11}$Gpa

Solution: The celerity of wave 'C' = $\sqrt{\frac{K}{\rho}} = \sqrt{\frac{2.0\times10^9}{1000}} = 1414.21 \quad m/s$

Time of oscillation = $\left(\frac{2\boldsymbol{L}}{\boldsymbol{C}}\right) = \left(\frac{2\times3500}{1414.21}\right) = 4.95\,\sec$

(i) **<u>Time of closure T_c = 20 sec</u>**> 4.95 sec - Gradual Valve closure

Rise in The pressure Head $\boldsymbol{H} = \frac{\boldsymbol{LV}}{\boldsymbol{gT_c}} = \left(\frac{3500\times1.5}{9.81\times20}\right) = 26.75\,\boldsymbol{m\ of\ Water\ Head}$

(ii) **<u>Time of closure T_c = 3.5 sec</u>**< 4.95 sec - Instantaneous Valve closure for a Rigid pipe. The pressure head rise **'H'**is given by,

$$\boldsymbol{H} = \frac{\boldsymbol{p}}{\gamma_w} = \frac{\rho\times\boldsymbol{V}\times\boldsymbol{C}}{\rho\times\boldsymbol{g}} = \frac{\boldsymbol{V}\times\boldsymbol{C}}{9.81} = \frac{1.5\times1414.21}{9.81} = 216.2\,\boldsymbol{m\ of\ water\ head}$$

(iii)**<u>Time of closure T_c= 3.0 sec</u>**< 4.95 sec - Instantaneous Valve closure for a Elastic pipe. The pressure rise **'p'**is given by,

$$\boldsymbol{p} = \boldsymbol{V}\sqrt{\frac{\rho}{\left(\frac{1}{\boldsymbol{K}} + \frac{\boldsymbol{D}}{\boldsymbol{t\,E}}\right)}} = 1.5\sqrt{\frac{1000}{\left(\frac{1}{2.0\times10^9} + \frac{0.5}{0.002\times2.06\times10^{11}}\right)}} = 1145.87\,\boldsymbol{kN/m}^2$$

Rise in Pressure Head 'H' = $\left(\frac{\boldsymbol{p}}{\gamma_w}\right) = \left(\frac{1145.87}{9.81}\right) = 116.8\,\boldsymbol{m\ of\ water\ Head}$

(2) Water is flowing through a cast-iron pipe of diameter 150 mm and thickness 10 mm which is provided with a valve at its end. Water is suddenly stopped by closing the valve. Find the maximum velocity of water, when the rise of pressure due to sudden valve closure is 1.962 MN/m^2. Given the value of 'K' for water = 1.962 GN/m^2 and 'E' for Cast-iron pipe '$E_{C.I}$' = 117.7 GN/m^2

Solution: Given:

(i) Pressure rise due to sudden valve closure **'p'**= 1.962 MN/m^2 = 1.962 $\times 10^6$ N/m^2

(ii) K = 1.962 GN/m^2 = 1.962 $\times 10^9$ N/m^2

(iii) $E_{C.I}$ = 117.7 GN/m^2 = 117.7 $\times 10^9$ N/m^2

(iv) Thickness of pipe 't' = 10 mm = 0.01 m

(v) Diameter of pipe = 150mm = 0.15m

The rise in pressure 'p' in cast iron pipe by considering pipe as elastic and valve closure isinstantaneous is given by,

$$\boldsymbol{p} = \boldsymbol{V}\sqrt{\frac{\boldsymbol{\rho}}{\left(\frac{1}{\boldsymbol{K}} + \frac{\boldsymbol{D}}{\boldsymbol{t}\,\boldsymbol{E}}\right)}} = \boldsymbol{V}\sqrt{\frac{1000}{\left(\frac{1}{1.962\times 10^{9}} + \frac{0.15}{0.01\times 1.177\times 10^{11}}\right)}} = 1.962\times 10^{6}\ \boldsymbol{N/m}^{2}$$

$$1.2528155\times 10^{6}\times \boldsymbol{V} = 1.962\times 10^{6}$$

V = 1.566 m/s

Q.3 The velocity of water in a 60cm diameter and 15mm thick cast iron pipe (E=1.04x10^{11} Pa) is changed from 3 m/s to zero in 1.25 s by closure of a valve i) if the pipe length is 800m what will be the water hammer pressure at the valve? What will be the corresponding pressure rise if the closure takes place in; ii) 2s and iii) 0.8s respectively? Bulk module of elasticity of water is 2.11 x 10^9 N/m^2.

Ans: Given: D = 60cm = 0.6m, L = 800m, t = 15mm = 0.015m

E = 1.04×10^{11} Pa, K = 2.11×10^9 N/m^2, V = 3m/s

(i) Case-1 Time of Closure T_c = 1.25 sec

The celerity of wave $C = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{2.11\times10^9}{1000}} = 1452.6 \quad m/s$

The ratio $\left(\frac{2\times L}{C} = \frac{2\times800}{1452.6} = 1.1 \quad \text{sec}\right)$

The value of Time of Closure **T_c = 1.25 sec > 1.1 sec**

Hence GRADUAL COSURE OF VALVE

The value of pressure rise in pipeline due to gradual closure is given by,

$$p = \left(\frac{\rho\times L\times V}{T_c}\right) = \left(\frac{1000\times800\times3}{1.25}\right) = 1920 \quad kPa$$

(ii) Case-2 Time of Closure T_c = 2 sec

The value of Time of Closure **T_c = 2 sec > 1.1 sec**

Hence GRADUAL COSURE OF VALVE

The value of pressure rise in pipeline due to gradual closure is given by,

$$p = \left(\frac{\rho\times L\times V}{T_c}\right) = \left(\frac{1000\times800\times3}{2}\right) = 1200 \quad kPa$$

(iii) Case-3 Time of Closure T_c = 0.8 sec

The value of Time of Closure **T_c = 0.8 sec < 1.1 sec**

Hence INSTANTANEOUS COSURE OF VALVE

The value of pressure rise in pipeline due to instantaneous closure is given by,

$$p = V\sqrt{\frac{\rho}{\left(\frac{1}{k}+\frac{D}{t\times E}\right)}} = 3\sqrt{\frac{1000}{\left(\frac{1}{2.11\times10^9}+\frac{0.6}{0.015\times1.04\times10^{11}}\right)}} = 3238\, kPa$$