Module-1
Units & Dimensions

Units

To specify and perform calculations with physical quantities , the physical quantities must be defined both in time and magnitude.

The std. measure of each type of physical quantitity to be measured is called unit. Mathematically the procedure of measurement can be expressed as

Magnitude of measurand = numerical ratio * unit

Where numerical ratio = number of times the unit occurs in any given amount of same quantity. Hence it is also called no. of measures . it is also called numerical multiplier.

Hence, process of measurement is to find numerical ratio. The numerical ratio has no physical meaning without the unit.

Ex: If we say the weight of 5kg means well defined weight is one kg and 5 such units are there in the measured weight. Thus, the numerical ratio is 5/1 while the unit is kg.

Fundamental Units

The units which are independently chosen and not dependent on any other units are called fundamental units. These are also called base units.

The length, mass and time are fundamental to most of the physical quantities. Hence the units which are the measures of length, mass and time are called primary fundamental units.

Ex : m, kg, s

The measures of certain physical quantitative related to numerical, therma, illumination etc. are called auxiliary fundamental units.

Ex: k, candela , ampere

Derived Units

All the units which are expressed in terms of the fundamental units using the physical equations are called derived Units.

Ex: Area of rectangle = l * b

Each of l & b is measured in m. Thus the product becomes m * m = m2. Hence the new unit which is derived as sq. m. for expressing the area is called derived units

Dimensions

Every physical quantity has its own identity. Such an identity is nothing but its quality with which it can be distinguished from all the other quantities . Such a unique quality possessed by a quantity is called its dimension. Symbolically, the dimension is expressed in the characteristic notation which is []

For Ex: the dimension of length is expressed as [L], the dimension of mass is [M]. The dimension of time is [T].
Similar to fundamental unit , each derived unit also has a unique dimension associated with it.
Ex: The volume , V = l * b* h where the dimension of each l, b and h is [L]. Hence the equation is dimensional form becomes,

$$V = [L][L][L]$$
$$V = [L^3]$$

Any constants existing in the equations are always dimensionless. Thus, it can be said the complete algebraic formula to obtain the derived unit from the fundamental units is nothing but the dimension of the derived unit. Thus the equality in terms of dimensions and should not be mixed up with actual numerical values

Dimensional Equations

The equation obtained by replacing each quantity in the mathematical equation by respective dimensions is called dimensional equations.

Dimensional equations help

- In conversion from one system of units to another system of units
- In derivation of equations for physical quantities
- In checking the accuracy of an equation

Dimensions of Mechanical Quantitites

For deriving the dimensions , let us use the equality sign in the known relations in terms of dimensions

Force Force = m * a = $[M][LT^{-2}] = [MLT^{-2}]$
a = velocity/time = $[LT^{-1}]/[T] = [LT^{-2}]$
v = distance/time = $[l]/[T] = [LT^{-1}]$

Work work = force * distance
$= [MLT^{-2}][L]$
$= [ML^2T^{-2}]$

Power = work / time = $[ML^2T^{-2}]/[T] = [ML^2T^{-3}]$

Energy = Power * Time = $[ML^2T^{-3}]/[T^{-1}] = [ML^2T^{-2}]$

Momentum = mass * velocity = $[M][LT^{-1}] = [MLT^{-1}]$

Torque = Force * distance = $[MLT^{-2}][L] = [ML^2T^{-2}]$

Stiffness = Torque/ angle = $[ML^2T^{-2}]$

Surface tension = force / length = $[MLT^{-2}]/[L] = [ML^0T^{-2}]$

C.G.S. System of Units

The CGS system of units was used earlier during the 19th century in the development of the electrical theory In cgs system of units length , mass and time are the centimeter, gram, the second respectively. In this system of units, in addition to [LMT] an additional fourth quantity is used. Such a fourth fundamental unit is used on based on either electrostatic relations or electromagnetic relations.

Electromagnetic units:

In this system of units the fourth fundamental unit used is μ which is permeability of the medium . all dimensions are derived in terms of these 4 basic dimensions L,M,T, μ. The permeability of free space is assumed to be 1 in such systems. Such a system is called electromagnetic system of units.

Electrostatic units

In this system of units the fourth fundamental unit used is ϵ . which is permittivity of the medium, in addition to 3 basic units L,M and T. All dimensions are derived in terms of these 4 basic dimensions L,M,T and ϵ. The permittivity of free space i.e., vacuum is assumed to be 1 in such system. Such a system is called electrostatic system of units

Dimensions in electrostatic system of units

Charge $Q = Q2/ \epsilon d2$

$Q = l\cdot\sqrt{F} \epsilon d2$

$Q = d\sqrt{F\epsilon}$

$[L][MLT-2]1/2[\epsilon 1/2]$

$[L][M1/2L1/2T-1][\epsilon 1/2]$

$[M1/2L3/2T-1\epsilon 1/2]$

Current $I = Q/T = [M1/2L3/2T-1\epsilon 1/2]/[T] = [T] = [M1/2L3/2T-2\epsilon 1/2]$

Potential difference E = workdone / charge = $W/ Q = [M1/2 L1/2 T-1 \epsilon -1/2]$

Capacitance $C = q/v = [M1/2 L3/2 T-1 \epsilon 1/2]/ [M1/2L1/2T-1\epsilon -1/2] = [\epsilon L]$

Resistance $R = V/I = [M1/2L1/2 T-1 \epsilon -1/2]/[M1/2L3/2T-2\epsilon 1/2] = [L-1T\epsilon -1]$

Inductance $L = E / di/dt = [M1/2L1/2T-1\epsilon -1/2]/[M1/2L3/2T-2\epsilon 1/2]/[T] = [L-1T2\epsilon -1]$

SI Units

For the sake of uniformity of units all over the world, an international organization the general conference of weights and measures recommended a unified systematically constituted system of units. This system of units is called SI system of units.

The SI system of units is divided into 3 categories namely

Fundamental units
Supplementary units
Derived units

Fundamental units

The units which are not dependent on any other unit is called fundamental unit. Seven such base units form SI units

Meter
Kilogram
Second
Ampere
Kelvin
Mole
Candela

Supplementary units

In addition to the fundamental units , there are two supplementary units added tp SI system of units they are

Radians for plane angles
Steradian for solid angles

Derived units

The units other than fundamental and supplementary are derived from the fundamental and supplementary units . hence these units are called derived units.. they are mainly classified as

Mechanical units such as mass, velocity etc.
Electric and magnetic units such as power , energy, weber, tesla etc
Thermal units such as talent heat , calorific value etc

Advantages of SI system of units

The advantages of SI system of units are

- The SI system of units use decimals and powers of 10 which simplifies the signification of any quantity.
- The value of any particular quantity in SI system of units can be further simplified by the use of prefixes
- The various SI prefixes such as milli, micro, nano etc simplify the expressions of the units of various quantities
- As the current I is used as fourth fundamental quantity, the derivation of dimensional equations for various quantities are simplified

Questions

1. Discuss briefly on these i) SI Units ii) Dimensional equations – Jan/ Feb 2004
2. Explain the terms ' dimensions of a physical quality' and the significance of the dimensional equations July/Aug 2004
3. Explain the usefulness of dimensional equations July/Aug 2005
4. Derive the dimensions of resistance , inductance, capacitance and permeability in MTI system July/Aug 2006
5. Mention the advantages of SI system of units July/Aug 2007
6. Derive the dimensions of MMF, EMF, magnetizing force and flux density in LMTI system Jan/ Feb 2008
7. Mention the uses and limitations of dimensional analysis Jan/ Feb 2012 July/Aug 2011

B. Measurement of Resistance

1. With a neat sketch explain the working of a megger July/Aug 2004
2. Explain the fall of potential of measurement of earth resistance July/Aug 2005
3. Derive the expression for the measurement of unknown resistance using Kelvin's double bridge. How the effect of connecting lead resistance is eliminated in this arrangement Jan /Feb 2005, July/Aug 2006
4. Write short notes on Megger July/Aug 2008
5. Explain how a megger is used for the measurement of earth resistance July/Aug 2007
6. Define voltage sensitivity of a galvanometer and hence obtain an expression for whetstone's bridge sensitivity. When will be Sb maximum? Jan / Feb 2008
7. State and explain sensitivity of whetstone's bridge? Jan/ Feb 2012, July/Aug 2008

Problems:

1. Deriving equation for resistance is Hay's bridge, the following expression is obtained. $R = w^2R1R2e^2/ 1+w^2R_2{}^2C$ Find whether the equation is dimensionally correct or not. Incase there is an error find the error and correct equation accordingly Jan/ Feb 2012

In MKSA rationalized system, $[R] = [ML^2T^{-3}I^{-2}]$
$[C] = [M^{-1}L^{-2}T^4I^2]$
$[w] = 1/[T]$

R.H.S. = $w^2R_1R_2R_3e^2 \;/\; 1+w^2R_2{}^2C$
$= [T\text{-}1]\,[ML^2T^{-3}I^{-2}]3\,[M^{-1}L^{-2}T^4I^2]2 \;/\; 1+[T\text{-}1]2\,[ML^2T^{-3}I^{-2}]2\,[M^{-1}L^{-2}T^4I^2]$

$= [T\text{-}1]\ [M^3L^6T^{-9}I^{-6}]\ [M^{-2}L^{-4}T^8I^4] \ / \ 1+ [T\text{-}^2]\ [M^2L^4T^{-6}I^{-4}]2\ [M^{-1}L^{-2}T^4I^2]$

$= [M^1L^2T^{-3}I^{-2}] \ / \ [M^1L^2T^{-4}I^{-2}]$

L.H.S. $= [M^1L^2T^{-3}I^{-2}]$

$[M^1L^2T^{-3}I^{-2}] = [M^1L^2T^{-3}I^{-2}] \ / [M^1L^2T^{-4}I^{-2}]$

Multiply $[M\ \text{-}^1L\text{-}^2T^4I^2]$ to Dr

$[M^1L^2T^{-3}I^{-2}] = [M^1L^2T^{-3}I^{-2}]$

$[M^{-1}L^{-2}T^4I^2] = C$

Therefore multiply the equation by C

R = $w^2R1R2e^2/ 1+w^2R_2^2C^2$

Thus , dimensionally equation is not correct. It can be seen that numerator dimension of R.H.S> are same as the dimensions of L.H.S.. Hence, to satisfy the equation dimensionally , denominator of R.H.S. must be dimensionless. So, to balance the denominator of R.H.S., it must be multiplied by $[M\ \text{-}^1L\text{-}^2T^4I^2]$. Theses are the dimensions of capacitor C. this indicates the term $w^2R_2^2C$ must be multiplied by one more C to satisfy the equation dimensionally correct.

The expression for mean torque of an electrodynamometer type wattmeter may be written as T α $M^p E^q Z^t$. Where M = mutual inductance between fixed and moving coils. E = applied voltage , Z = impedence of load circuit . determine the values of p,q,t performing dimensional analysis July/Aug 2007

$[T] = [ML^2T^{-2}]$

$[M] = [M^1L^2T^{-2}I^{-2}]$

$[E] = [M^1L^2T^{-3}I^{-1}]$

$[Z] = [M^1L^2T^{-3}I^{-2}]$

T = $k\ M^p E^q Z^t$.

$[ML^2T^{-2}] = k\ [M^1L^2T^{-2}I^{-2}]^p [M^1L^2T^{-3}I^{-1}]^q [M^1L^2T^{-3}I^{-2}]^t$

$= [M^{p+q+t}\ L^{2p+2q+2t}\ T^{-2p-3q-3t}\ I^{-2p-q-2t}]$

By comparing and solving p= 1, q = 2, t = -2

2. Derive the dimensional equation for resistance R, inductance and capacitance C. hence check for dimensionally correctness of the expression below obtained for inductance from ac bridge measurements , point out the error, if any in the expression and suggest the required correction that makes the expression dimensionally valid

$$L = C (R_3/R4) (R_2+R_4 +R_2R_4)$$

$$L = [M^1L^2T^{-2}I^{-2}]$$
$$[R] = [M^1L^2T^{-3}I^{-2}]$$
$$[C] = [M^{-1}L^{-2}T^4I^2]$$

R.H.S. = $[M^{-1}L^{-2}T^4I^2] [M^1L^2T^{-3}I^{-2}] / [M^1L^2T^{-3}I^{-2}] ([M^1L^2T^{-3}I^{-2}] + [M^1L^2T^{-3}I^{-2}] + [M^1L^2T^{-3}I^{-2}]$

$[M^1L^2T^{-3}I^{-2}])$

$$[M^1L^{-2}T^4I^2] ([M^1L^2T^{-3}I^{-2}] + [M^1L^2T^{-3}I^{-2}])$$

$$= [T] [T] + [M^1L^2T^{-2}I^{-2}]$$

But dimensionally addition is valid only if all the terms to be added are dimensionally same. Thus the given eq is dimensionally incorrect.

To have it correct, multiply R_2 and R_4 by another resistance. Thus the correct equation becomes

$$L = C (R_3/R4) (R_2\, r +R_4\, r +R_2R_4)$$

4. Expression for eddy current loss p/meter length of wire may be written as $p \;\alpha\; f^a Bm^b d^c \rho^g$

Where f = frequency, Bm = Max. flux density, d= diameter of wire, ρ – resistivity of material. Find the values a, b,c,and g using L,M,T,I system

$$P = k f^a Bm^b d^c \rho^g$$
$$[P] = [I^1L^{-1}]$$
$$[f] = [T^{-1}]$$
$$[Bm] = [M^1T^{-2}I^{-1}]$$
$$[d] = [L]$$
$$[\rho] = [M^1 L^3 T^{-3}I^{-2}]$$

$$[I^1L^{-1}] = k [T^{-1}]^a [M^1T^{-2}I^{-1}]^b [L]^c [M^1 L^3 T^{-3}I^{-2}]^g$$

$$[I^1L^{-1}] = k [T^{-a}] [M^bT^{-2b}I^{-b}] [L^c] [M^g L^{3g} T^{-3g}I^{-2g}]$$

By comparing and solving a = 2, b = 2, g=1 , c=4

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Measurement of resistance, inductance and capacitance

Introduction:

A bridge circuit in its simplest form consists of a network of four *resistance arms* forming a closed circuit. A source of current is applied to two opposi te junctions. The current detector is connected to other two junctions.

The bridge circuits use the comparison measurement methods and operate on null-indication principle. The bridge circuit compares the value of an unknown component with that of an accurately known standard component. Thus *the* accuracy depends on the bridge components and not on the null indicator. Hence high degree of accuracy can be obtained.

Advantages of Bridge Circuit:

The various advantages of the bridge circuit are,

1) The balance equation is independent of the magnitude of the input voltage or its source impedance. These quantities do not appear in the balance equation expression.

2) The measurement accuracy is high as the measurement is done by comparing the unknown value with the standard value.

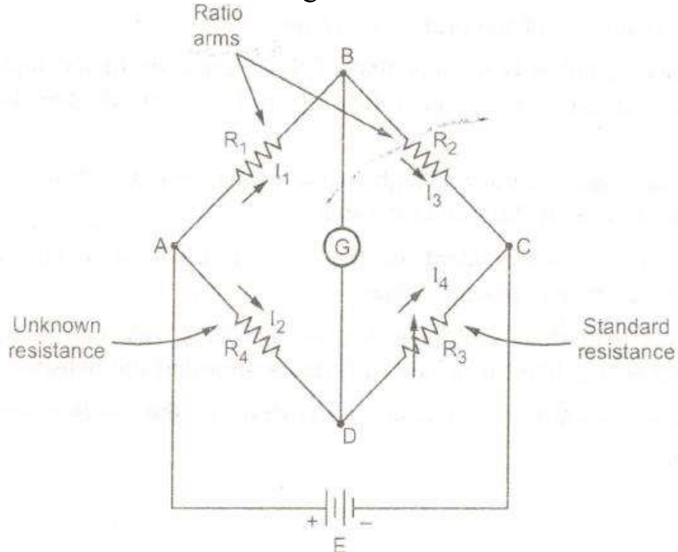
3) The accuracy is independent of the characteristics of a null detector and is dependent on the component values.

4) The balance equation is independent of the sensitivity of the null detector, the impedance of the detector or any impedance shunting the detector.

5) The balance condition remains unchanged if the source and detector are interchanged.

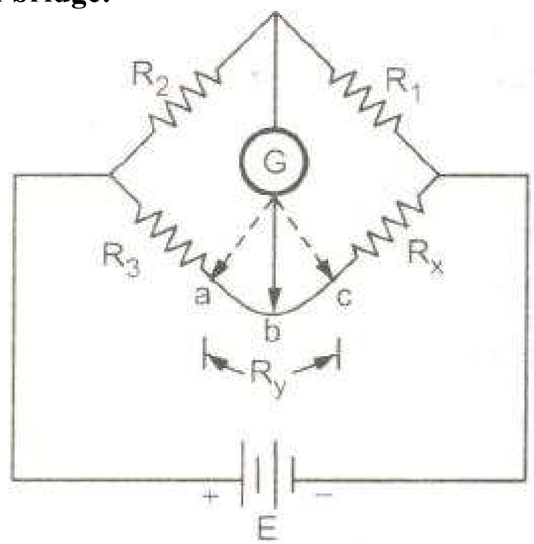
Wheatstone's bridge:

The bridge consists of four resistive arms together with a source of e.m.f. and a null detector. The galvanometer is used as a null detector.



The arms consisting the resistances R] and R2 are called ratio arms. The arm consisting the standard known resistance R3 is called standard arm. The resistance R4 is the unknown resistance to be measured. The battery is connected between A and C while galvanometer is connected between Band D.

Kelvin bridge:



In the Wheatstone bridge, the bridge contact and lead resistance causes significant error, while measuring low resistances. Thus for measuring the values of resistance below 1 -n, the modified form of Wh~tstone bridge is used, known as Kelvin bridge. The consideration of the effect of contact and lead resistances is the basic aim of the Kelvin bridge.

The resistance Rv represents the resistance of the connecting leads from R., to R,. The resistance Rx is the unknown resistance to be measured.

\ The galvanometer can be connected to either terminal a, b or terminal c. When it is connected to a, the lead resistance Ry gets added to Rx hence the value measured by the bridge, indicates much higher value of Rx.

If the galvanometer is connected to terminal c, then Ry gets added to R3. This results in the measurement of Rx much lower than the actual value.

The point b is in between the points a and c, in such a way that the ratio of the resistance from c to b and that from a to b is equal to the ratio of R] and R2.

$$\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}$$

A.C. Bridges:

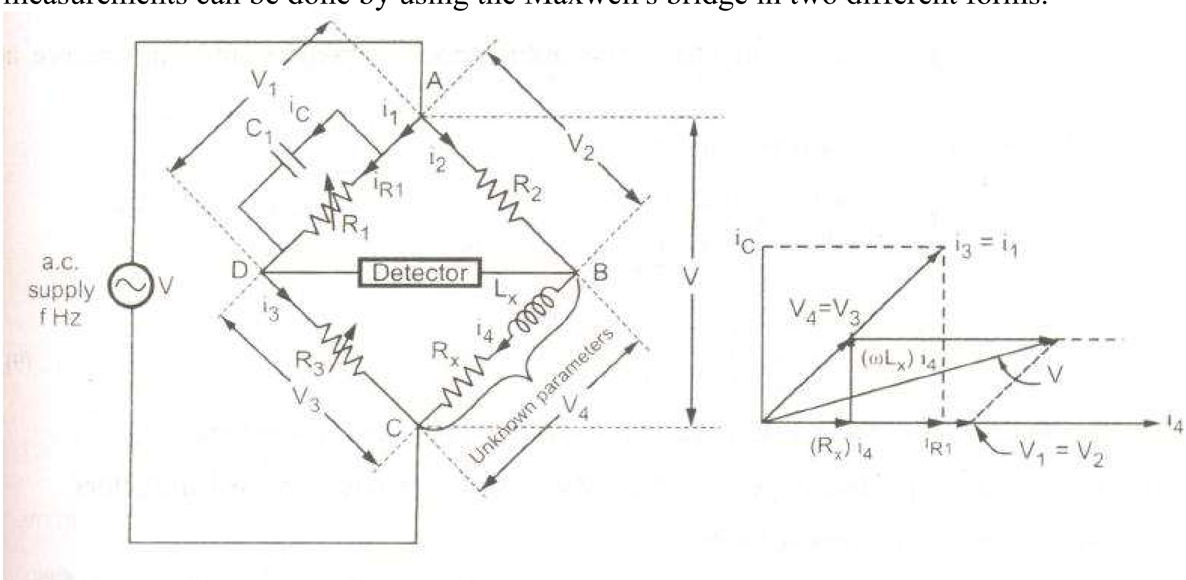
An a.c. bridge in its basic form consists of four arms, a source of excitation and a balance detector. Each arm consists of an impedance. The source is an a.c. supply which supplies a.c. voltage at the required frequency. For high frequencies, the electronic oscillators are used as the source. The balance detectors commonly used for a.c. bridge are head phones, tunable amplifier circuits or vibration galvanometers. The headphones are used as detectors at the frequencies of 250 Hz to 3 to 4 kHz. While working with single frequency a tuned detector is the most sensitive detector. The vibration galvanometers are useful for low audio frequency range from 5 Hz to 1000 Hz but are commonly used below 200 Hz. Tunable amplifier detectors are used for frequency range of 10 Hz to 100 Hz.

Hay's Bridge:

In the capacitance comparison bridge the ratio arms are resistive in nature. The impedance Z 3 consists of the· known standard capacitor C3 in series with the resistance R3. The resistance R3 is variable, used to balance the bridge. The impedance Z4 consists of the unknown capacitor Cx and its small leakage resistance Rx.

Maxwell's Bridge :

Maxwell's bridge can be used to measure inductance by comparison either with a variable standard self inductance or with a standard variable capacitance. These two measurements can be done by using the Maxwell's bridge in two different forms.



(a) Circuit diagram (b) Phasor diagram

Maxwell's inductance capacitance bridge

Methods of Measurement of Earth Resistance

Fall of Potential Method

Fig below shows the circuit diagram used for the measurement of earth resistance by fall of potential method. E is the earth electrode. The electrode Q is the auxillary electrode. The current I is passed through the electrodes E & Q with the help of external battery E. another auxillary electrode P is introduced in between the electrodes E & Q. the voltage between the electrodes E & P is measured with the help of voltmeter.
Thus if the distance of electrode P is changed from electrode E electrode Q, the electrode P experiences changing potential near the electrodes while a constant potential between the electrodes E & Q but away from the electrodes from Q.

The potential rises near the electrodes E & Q due to higher current density in the proximity of the electrodes. By measuring the potential between the electrodes E & P as Vep, the earth resistance can be obtained as

$$R_E = V_{EP} / I$$

Shielding and grounding of bridges

This is one way of reducing the effect of stary capacitances. But this technioque does not eliminate the stray capacitances but makes them constsnt invalue and hence they can be compensated.

One very effective and popular methode of eliminating the stray capacitances and the capacitances between the bridges arms is using a ground connection called Wagner Ground connection.

Questions from Question Paper:

1. Explain maxwell's bridge?June/July2009
2. Explain kelvin's bridge? Dec/Jan2008, Jan/ Feb 2012
3. Explain the importance of Wheatstone bridge? May/June2010
4. Explain the Capacitance Comparison Bridge? Dec/Jan2010
5. Explain the Maxwell's bridge?June/July 2009
6. Explain the Wagner's earth connection? Dec/Jan08, Jan/ Feb 2012
7. Derive the balance equations of the Schering bridge circuit configuration used for measurement of capacitances and hence derive at the expression for loss angle of the test capacitor. Draw the phasor diagram at balance.
   Jan/Feb-2004, July/Aug 2004, Jan/ Feb- 2008
8. Write a short note on the Wagner earthing device
   July/Aug-2004/2010,Jan/Feb-2011
9. Derive the expression for the measurement of capacitance and loss angle of a lossy capacitor using Schering bridge. Draw the phasor diagram at balance condition. What modifications are introduced when the bridge is used at high voltages Jan/Feb-2005, July/Aug 2004
10. Write briefly on the significance of shields used in ac bridge circuit. Hence discuss on the shielding of resistors and capacitors of the circuit
    July/Aug 2005, Jan/ Feb 2005
11. Draw a neat sketch to explain the theory and measurement of unknown inductance and resistance by Anderson bridge. What is type of null detector used in this bridge? What are the sources of errors? Draw phasor diagram at balance
    July/Aug 2006, Jan/ Feb 2006, Jan/ Feb 2012
12. Write short notes on source and detectors July/Aug 2008, Jan/ Feb 2007