MODULE-3
Automatic Generation Control (continued):

Generator Voltage Control System

The voltage of the generator is proportional to the speed and excitation (flux) of the generator. The speed being constant, the excitation is used to control the voltage. Therefore, the voltage control system is also called as excitation control system or automatic voltage regulator (AVR).

For the alternators, the excitation is provided by a device (another machine or a static device) called exciter. For a large alternator the exciter may be required to supply a field current of as large as 6500A at 500V and hence the exciter is a fairly large machine. Depending on the way the dc supply is given to the field winding of the alternator (which is on the rotor), the exciters are classified as: i) DC Exciters; ii) AC Exciters; and iii) Static Exciters. Accordingly, several standard block diagrams are developed by the IEEE working group to represent the excitation system. A schematic of an excitation control system is shown in Fig2.1.

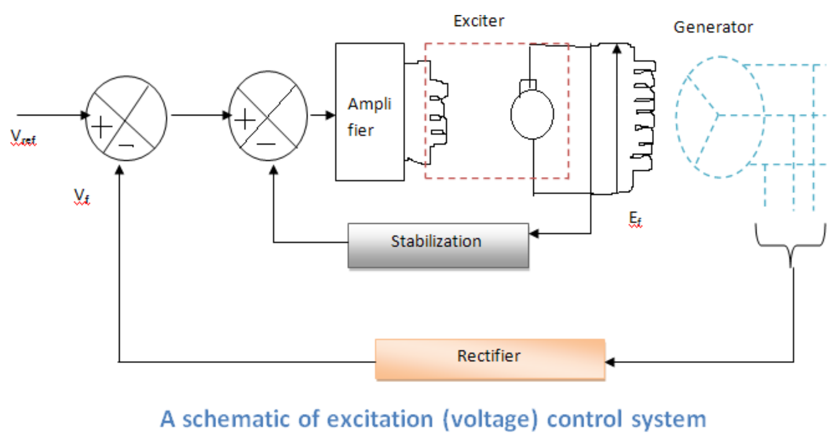


Fig 2.1: A schematic of Excitation (Voltage) Control System.

A simplified block diagram of the generator voltage control system is shown in Fig2.2.The generator terminal voltage Vt is compared with a voltage reference Vref to obtain a voltage error signal _V. This signal is applied to the voltage regulator shown as a block with transfer function KA/ (1+TAs). The output of the regulator is then applied to exciter shown with a block of transfer function Ke/ (1+Tes). The output of the exciter e.m.f is then applied to the field winding which adjusts the generator terminal voltage. The generator field can be represented by a block with a transfer function KF/ (1+sTF).

The total

transfer function is

$$\frac{\Delta V}{\Delta V_{ref}} = \frac{G(s)}{1+G(s)} \quad \text{where,} \quad G(s) = \frac{K_A K_e K_F}{(1+sT_A)(1+sT_e)(1+sT_F)}$$

The stabilizing compensator shown in the diagram is used to improve the dynamic response of the exciter. The input to this block is the exciter voltage and the output is a stabilizing feedback signal to reduce the excessive overshoot.

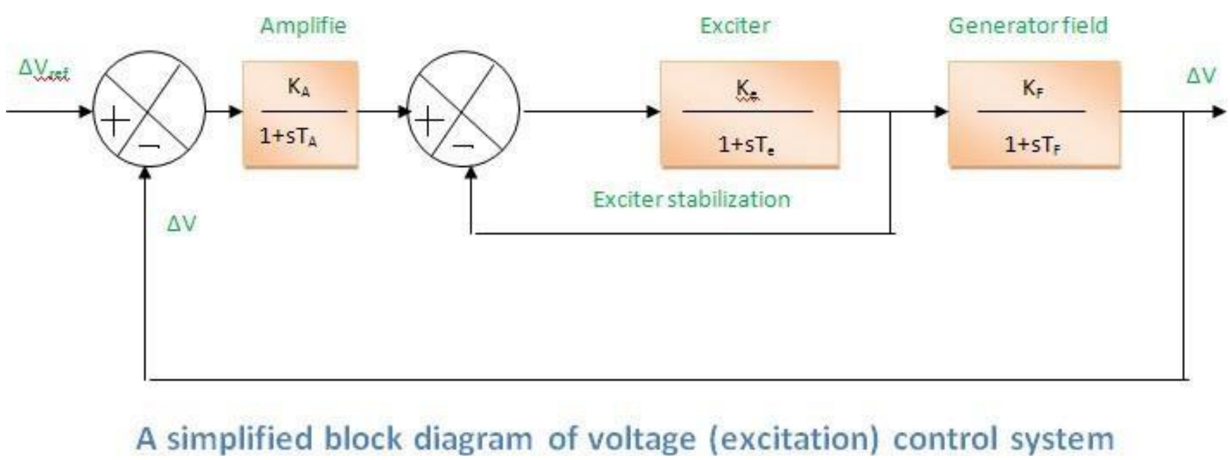


Fig2.2: A simplified block diagram of Voltage (Excitation) Control System.

Performance of AVR Loop

The purpose of the AVR loop is to maintain the generator terminal voltage with inacceptable values. A static accuracy limit in percentage is specified for the AVR, so that the terminal voltage is maintained within that value. For example, if the accuracy limit is 4%, then the terminal voltage must be maintained within 4% of the base voltage.

The performance of the AVR loop is measured by its ability to regulate the terminal voltage of the generator within prescribed static accuracy limit with an acceptable speed of response. Suppose the static accuracy limit is denoted by Ac in percentage with reference to the nominal value. The error voltage is to be less than (Ac/100) |Vref. From the block diagram, for a steady state error voltage.

$$\Delta e = \Delta|V|_{ref} - \Delta|V|_t < \frac{Ac}{100}\Delta|V|_{ref}$$

$$\Delta e = \Delta|V|_{ref} - \Delta|V|_t = \Delta|V|_{ref} - \frac{G(s)}{1+G(s)}\Delta|V|_{ref}$$

$$= \{1 - \frac{G(s)}{1+G(s)}\}\Delta|V|_{ref}$$

$$\Delta e = \{1 - \frac{G(s)}{1+G(s)}\}\Delta|V|_{ref} = \{1 - \frac{G(0)}{1+G(0)}\}\Delta|V|_{ref}$$

$$= \frac{1}{1+G(0)}\Delta|V|_{ref} = \frac{1}{1+K}\Delta|V|_{ref}$$

For constant input condition, ($s \to 0$)

Where, K= G(0) is the open loop gain of the AVR. Hence,

$$\frac{1}{1+K}\Delta|V|_{ref} < \frac{Ac}{100}\Delta|V|_{ref} \quad \text{or} \quad K > \{\frac{100}{Ac} - 1\}$$

Automatic Load Frequency Control

The ALFC is to control the frequency deviation by maintaining the real power balance in the system. The main functions of the ALFC are to i) to maintain the steady frequency; ii) control the tie-line flows; and iii) distribute the load among the participating generating units. The control (input) signals are the tie-line deviation ΔPtie (measured from the tie line flows), and the frequency deviation Δf (obtained by measuring the angle deviation $\Delta\delta$). These error signals Δf and ΔPtie are amplified, mixed and transformed to a real power signal, which then controls the valve position. Depending on the valve position, the turbine (prime mover) changes its output power to establish the real power balance. The complete control schematic is shown in Fig2.3

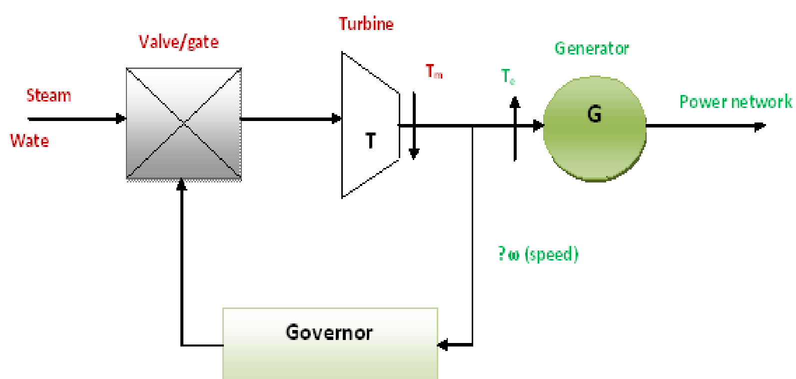


Fig2.3.The Schematic representation of ALFC system

For the analysis, the models for each of the blocks in Fig2 are required. The generator and the electrical load constitute the power system. The valve and the hydraulic amplifier represent the speed governing system. Using the swing equation, the generator can be modeled by

$$\frac{2Hd^2\Lambda\delta}{s} = \Delta P_m - \Delta P_e$$

Expressing the speed deviation in pu,

$$\frac{d\Delta\omega}{dt} = \frac{1}{2H}(\Delta P_m - \Delta P_e)$$

This relation can be represented as shown in Fig2.4.

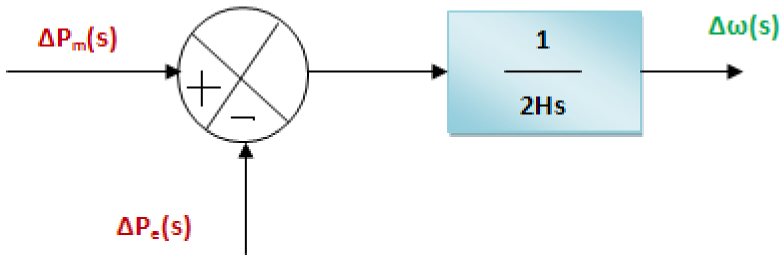


Fig2.4.The block diagram representation of the Generator

The load on the system is composite consisting of a frequency independent component and a frequency dependent component. The load can be written as $\Delta P_e = \Delta P_0 + \Delta P_f$ where, ΔP_e is the change in the load; ΔP_0 is the frequency independent load component; ΔP_f is the frequency dependent load component. $\Delta P_f = D\Delta\omega$ where, D is called frequency characteristic of the load (also called as damping constant) expressed in percent change in load for 1% change in frequency. If D=1.5%, then a 1% change in frequency causes 1.5% change in load. The combined generator and the load (constituting the power system) can then be represented as shown in Fig2.5

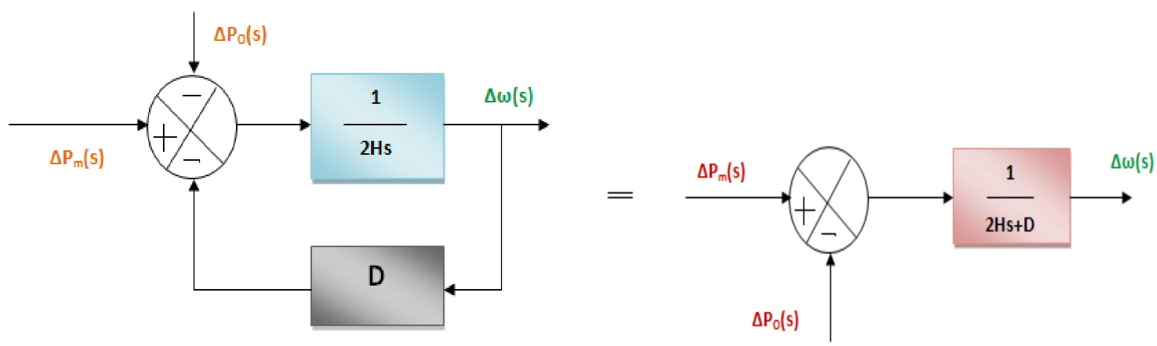
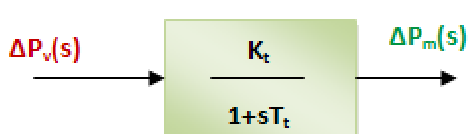


Fig2.5.The block diagram representation of the Generator and load

The turbine can be modeled as a first order lag as shown in the Fig2.6



$$G_t(s) = \frac{\Delta P_m(s)}{\Delta P_V(s)} = \frac{K_t}{1+sT_t}$$

Fig2.6.The turbine model

Gt(s) is the TF of the turbine; ΔPV(s) is the change in valve output (due to action).

ΔPm(s) is the change in the turbine output the governor can similarly modeled as shown in Fig2.7. The output of the governor is by

$\Delta P_g = \Delta P_{ref} - \frac{\Delta\omega}{R}$ where ΔP_{ref} is the reference set power, and $\Delta\omega/R$ is the power given by governor speed characteristic. The hydraulic amplifier transforms this signal ΔP_g into valve/gate position corresponding to a power ΔP_V. Thus $\Delta P_V(s) = (K_g/(1+sT_g))\Delta P_g(s)$.

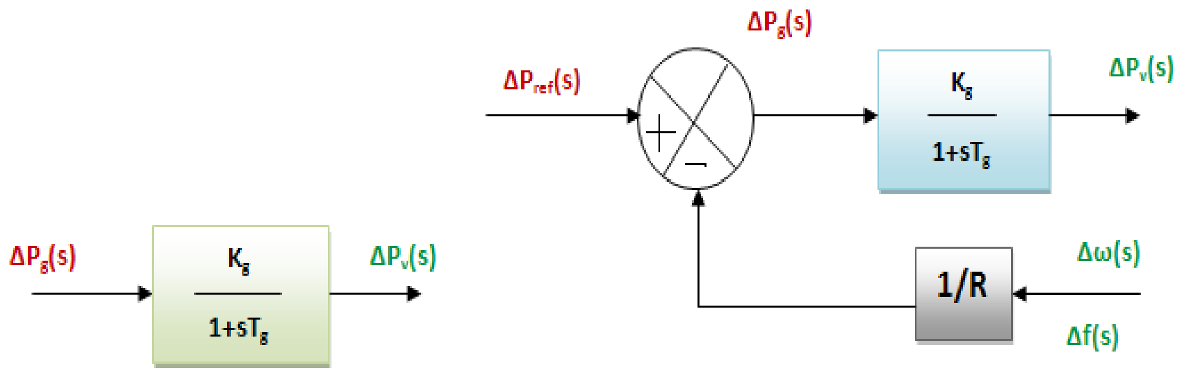


Fig2.7: The block diagram representation of the Governor

All the individual blocks can now be connected to represent the complete ALFC loop as Shown in Fig 5.1

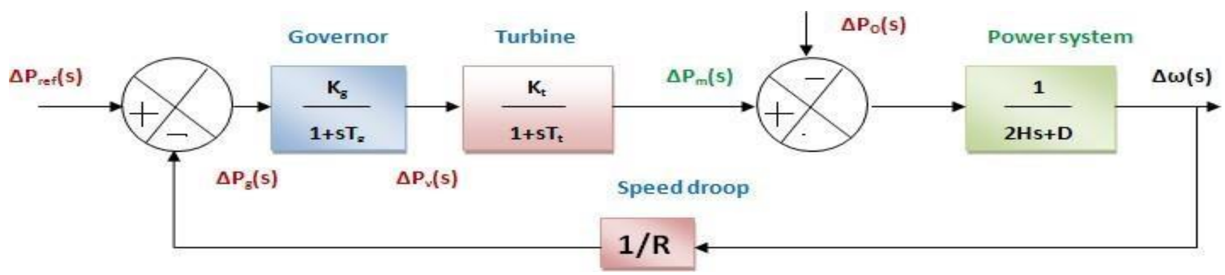


Fig2.8: The block diagram representation of the ALFC.

2.2 Steady state Performance of the ALFC Loop:

In the steady state, the ALFC is in „open" state, and the output is obtained by substituting $S \rightarrow 0$ in the TF.

With $S \rightarrow 0$,

2.4 Steady State Performance of the ALFC Loop

In the steady state, the ALFC is in 'open' state, and the output is obtained by substituting $s \to 0$ in the TF.

With $s \to 0$, Gg(s) and Gt(s) become unity, then, (note that $\Delta P_m = \Delta P_T = \Delta P_G = \Delta P_e = \Delta P_D$; That is turbine output = generator/electrical output = load demand)

$$\Delta P_m = \Delta P_{ref} - (1/R)\Delta\omega \quad \text{or} \quad \Delta P_m = \Delta P_{ref} - (1/R)\Delta f$$

When the generator is connected to infinite bus ($\Delta f = 0$, and $\Delta V = 0$), then $\Delta P_m = \Delta P_{ref}$.

If the network is finite, for a fixed speed changer setting ($\Delta Pref = 0$), then

$$\Delta P_m = -(1/R)\Delta f \quad \text{or} \quad \Delta f = -R\,\Delta P_m.$$

If the frequency dependent load is present, then

$$\Delta P_m = \Delta P_{ref} - (1/R + D)\Delta f \quad \text{or} \quad \Delta f = \frac{-\Delta Pm}{D + 1/R}$$

If there are more than one generator present in the system, then

$$\Delta P_{m.\,eq} = \Delta P_{ref.eq} - (D + 1/R_{eq})\Delta f$$

where,

$$\Delta P_{m.\,eq} = \Delta P_{m1} + \Delta P_{m2} + \Delta P_{m.3} + \ldots.$$

$$\Delta P_{ref.\,eq} = \Delta P_{ref1} + \Delta P_{ref2} + \Delta P_{ref3} + \ldots..$$

$$1/R_{eq} = (1/R_1 + 1/R_2 + 1/R_2 + \ldots.)$$

The quantity $\beta = (D + 1/R_{eq})$ is called the area frequency (bias) characteristic (response) or simply the stiffness of the system.

2.5 Concept of AGC (Supplementary ALFC Loop)

The ALFC loop shown in Fig2.8, is called the primary ALFC loop. It achieves the primary goal of real power balance by adjusting the turbine output ΔP_m to match the change in load demand ΔP_D. All the participating generating units contribute to the change in generation. But a change in load results in a steady state frequency deviation Δf. The restoration of the frequency to the nominal value requires an additional control loop called the supplementary loop. This objective is met by using integral controller which makes the frequency deviation zero. The ALFC with the supplementary loop is generally called the AGC. The block diagram of an AGC is shown in Fig2.9. The main objectives of AGC are i) to regulate the frequency (using both primary and supplementary controls); ii) and to maintain the scheduled tie-line flows. A secondary objective of the AGC is to distribute the required change in generation among the connected generating units economically (to obtain least operating costs).

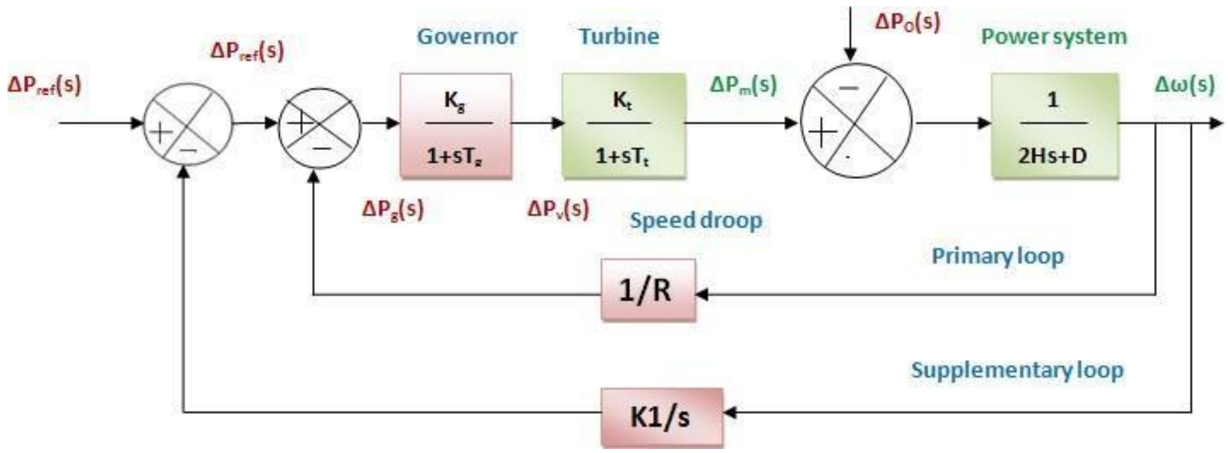


Fig2.9: The block diagram representation of the AGC

2.6 AGC in a Single Area System

In a single area system, there is no tie-line schedule to be maintained. Thus the function of the AGC is only to bring the frequency to the nominal value. This will be achieved using the supplementary loop (as shown in Fig.2.9) which uses the integral controller to change the reference power setting so as to change the speed set point. The integral controller gain K_I needs to be adjusted for satisfactory response (in terms of overshoot, settling time) of the system. Although each generator will be having a separate speed governor, all the generators in the control area are replaced by a single equivalent generator, and the ALFC for the area corresponds to this equivalent generator.

2.7 AGC in a Multi Area System

In an interconnected (multi area) system, there will be one ALFC loop for each control area (located at the ECC of that area). They are combined as shown in Fig2.10 for the interconnected system operation. For a total change in load of ΔP_D, the steady state deviation in frequency in the two areas is given by $\Delta f = \Delta\omega_1 = \Delta\omega_2 = \dfrac{-\Delta P_D}{\beta_1 + \beta_2}$ where,

$\beta_1 = (D_1 + 1/R_1)$; and $\beta_2 = (D_2 + 1/R_2)$.

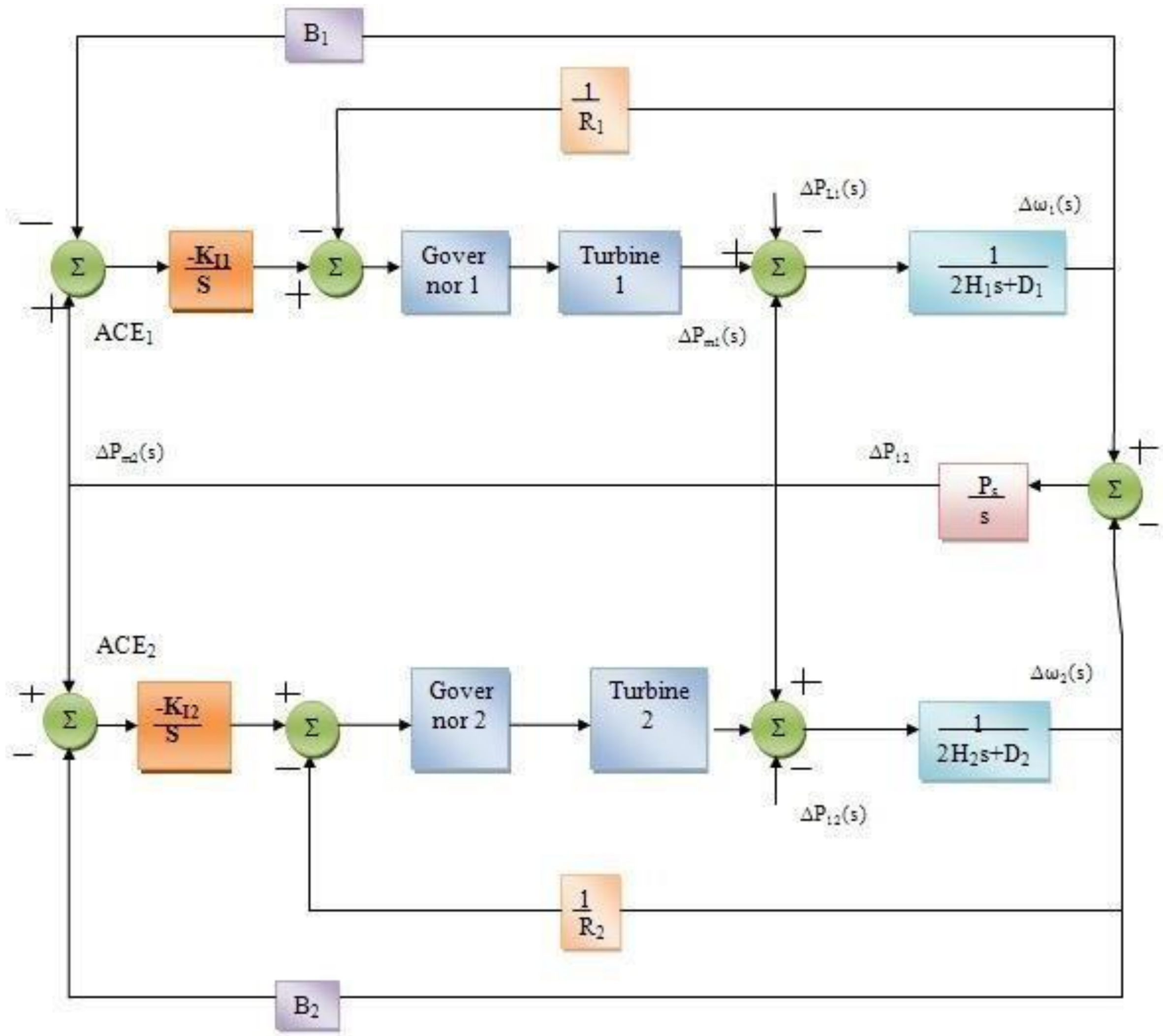


Fig.2.10.AGC for a multi-area operation

2.8 Expression for tie-line flow in a two-area interconnected system

Consider a change in load ΔP_{D1} in area1. The steady state frequency deviation Δf is the same for both the areas. That is $\Delta f = \Delta f_1 = \Delta f_2$. Thus, for area1, we have

$$\Delta P_{m1} - \Delta P_{D1} - \Delta P_{12} = D_1 \Delta f$$

where, ΔP_{12} is the tie line power flow from Area1to Area 2; and for Area 2

$$\Delta P_{m2} + \Delta P_{12} = D_2 \Delta f$$

The mechanical power depends on regulation. Hence

$$\Delta P_{m1} = -\frac{\Delta f}{R_1} \qquad \text{and} \qquad \Delta P_{m2} = -\frac{\Delta f}{R_2}$$

Substituting these equations, yields

$$\left(\frac{1}{R_1} + D_1\right)\Delta f = -\Delta P_{12} - \Delta P_{D1} \qquad \text{and} \qquad \left(\frac{1}{R_2} + D_2\right)\Delta f = \Delta P_{12}$$

Solving for Δf, we get

$$\Delta f = \frac{-\Delta P_{D1}}{(1/R_1 + D_1) + (1/R_2 + D_2)} = \frac{-\Delta P_{D1}}{\beta_1 + \beta_2}$$

and

$$\Delta P_{12} = \frac{-\Delta P_{D1}\beta_2}{\beta_1 + \beta_2}$$

where, β_1 and β_2 are the composite frequency response characteristic of Area1 and Area 2 respectively. An increase of load in area1 by ΔP_{D1} results in a frequency reduction in both areas and a tie-line flow of ΔP_{12}. A positive ΔP_{12} is indicative of flow from Area1 to Area 2 while a negative ΔP_{12} means flow from Area 2 to Area1. Similarly, for a change in Area 2 load by ΔP_{D2}, we have

$$\Delta f = \frac{-\Delta P_{D2}}{\beta_1 + \beta_2}$$

and

$$\Delta P_{12} = -\Delta P_{21} = \frac{-\Delta P_{D2}\beta_1}{\beta_1 + \beta_2}$$

Frequency bias tie line control

The tie line deviation reflects the contribution of regulation characteristic of one area to another. The basic objective of supplementary control is to restore balance between each area load generation. This objective is met when the control action maintains

- Frequency at the scheduled value
- Net interchange power (tie line flow) with neighboring areas at the scheduled Value

The supplementary control should ideally correct only for changes in that area. In other words, if there is a change in Area1 load, there should be supplementary control only in Area1 and not in Area 2. For this purpose the area control error (ACE) is used (Fig2.9). The ACE of the two areas are given by

For area 1: $ACE_1 = \Delta P_{12} + \beta_1 \Delta f$

For area 2: $ACE_2 = \Delta P_{21} + \beta_2 \Delta f$

Economic Allocation of Generation

An important secondary function of the AGC is to allocate generation so that each generating unit is loaded economically. That is, each generating unit is to generate that amount to meet the present demand in such a way that the operating cost is the minimum. This function is called Economic Load Dispatch (ELD).

Systems with more than two areas

The method described for the frequency bias control for two area system is applicable to multiage system also.

Section II: Automatic Generation Control

- **Load Frequency Control**

Automatic Generation Control

Electric power is generated by converting mechanical energy into electrical energy. The rotor mass, which contains turbine and generator units, stores kinetic energy due to its rotation. This stored kinetic energy accounts for sudden increase in the load. Let us denote the mechanical torque input by T_m and the output electrical torque by T_e . Neglecting the rotational losses, a generator unit is said to be operating in the steady state at a constant speed when the difference between these two elements of torque is zero. In this case we say that the accelerating torque is zero.

$T_a = T_m - T_e$ 5.20)

When the electric power demand increases suddenly, the electric torque increases. However, without any feedback mechanism to alter the mechanical torque, T_m remains constant. Therefore the accelerating torque given by (5.20) becomes negative causing a deceleration of the rotor mass. As the rotor decelerates, kinetic energy is released to supply the increase in the load. Also note that during this time, the system frequency, which is proportional to the rotor speed, also decreases. We can thus infer that any deviation in the frequency for its nominal value of 50 or 60 Hz is indicative of the imbalance between T_m and T_e. The frequency drops when $T_m < T_e$ and rises when $T_m > T_e$.

The steady state power-frequency relation is shown in Fig. 5.3. In this figure the slope of the ΔP_{ref} line is negative and is given by

$$-R = \frac{\Delta f}{\Delta P_m} \quad (5.21)$$

Where R is called the **regulating constant.** From this figure we can write the steady state power frequency relation as

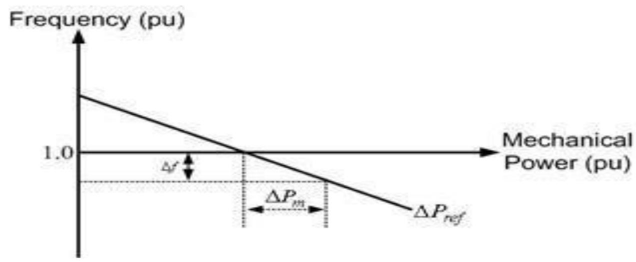


Fig. 5.3 A typical steady-state power-frequency curve.

$$\Delta P_m = \Delta P_{ref} - \frac{1}{R}\Delta f \quad (5.22)$$

Suppose an interconnected power system contains N turbine-generator units. Then the steady-state power frequency relation is given by the summation of (5.22) for each of these units as

$$\begin{aligned}\Delta P_m &= \Delta P_{m1} + \Delta P_{m2} + \cdots + \Delta P_{mN} \\ &= \left(\Delta P_{ref1} + \Delta P_{ref2} + \cdots + \Delta P_{refN}\right) - \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right)\Delta f \\ &= \Delta P_{ref} - \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right)\Delta f \quad (5.23)\end{aligned}$$

In the above equation, ΔP_m is the total change in turbine-generator mechanical power and ΔP_{ref} is the total change in the reference power settings in the power system. Also note that since all the generators are supposed to work in synchronism, the change is frequency of each of the units is the same and is denoted by Δf. Then the **frequency response characteristics** is defined as

$$\beta = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \quad (5.24)$$

We can therefore modify (5.23) as

$$\Delta P_m = \Delta P_{ref} - \beta \Delta f \quad (5.25)$$

Example 5.5

Consider an interconnected 50-Hz power system that contains four turbine-generator units rated 750 MW, 500 MW, 220 MW and 110 MW. The regulating constant of each unit is 0.05 per unit based on its own rating. Each unit is operating on 75% of its own rating when the load is suddenly dropped by 250 MW. We shall choose a common base of 500 MW and calculate the rise in frequency and drop in the mechanical power output of each unit.

The first step in the process is to convert the regulating constant, which is given in per unit in the base of each generator, to a common base. This is given as

$$R_{new} = R_{old} \times \frac{S_{base}^{new}}{S_{base}^{old}} \quad (5.26)$$

We can therefore write

$$R_1 = 0.05 \times \frac{500}{750} = 0.033$$

$$R_2 = 0.05$$

$$R_3 = 0.05 \times \frac{500}{220} = 0.1136$$

$$R_4 = 0.05 \times \frac{500}{110} = 0.2273$$

Therefore

$$\beta = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right) = 63.2$$

Per unit

We can therefore calculate the total change in the frequency from (5.25) while assuming $\Delta P_{ref} = 0$, i.e., for no change in the reference setting. Since the per unit change in load - 250/500 = - 0.5 with the negative

$$\Delta f = -\frac{\Delta P_m}{\beta} = -\frac{(-0.5)}{63.2} = 0.0079 \text{ per unit}$$

$$= 0.0079 \times 50 = 0.3956 \text{ Hz}$$

sign accounting for load reduction, the change in frequency is given by

$$\Delta P_{m1} = -\frac{0.0079}{0.033} \times 500 = -118.67 \text{ MW}$$

$$\Delta P_{m2} = -\frac{0.0079}{0.05} \times 500 = -79.11 \text{ MW}$$

$$\Delta P_{m3} = -\frac{0.0079}{0.1136} \times 500 = -34.81 \text{ MW}$$

$$\Delta P_{m4} = -\frac{0.0079}{0.2273} \times 500 = -17.41 \text{ MW}$$

It is to be noted that once ΔP_{m2} is calculated to be - 79.11 MW, we can also calculate the changes in the mechanical power of the other turbine-generators units as

$$\Delta P_{m1} = -79.11 \times \frac{750}{500} = -118.67 \text{ MW}$$

$$\Delta P_{m3} = -79.11 \times \frac{220}{500} = -34.81 \text{ MW}$$

$$\Delta P_{m3} = -79.11 \times \frac{110}{500} = -17.41 \text{ MW}$$

This implies that each turbine-generator unit shares the load change in accordance with its own rating.

LOAD FREQUENCY CONTROL

Modern day power systems are divided into various areas. For example in India, there are five regional grids, e.g., Eastern Region, Western Region etc. Each of these areas is generally interconnected to its neighboring areas. The transmission lines that connect an area to its neighboring area are called **tie-lines.** Power sharing between two areas occurs through these tie-lines. Load frequency control, as the name signifies, regulates the power flow between different areas while holding the frequency constant.

As we have an Example 5.5 that the system frequency rises when the load decreases if ΔP_{ref} is kept at zero. Similarly the frequency may drop if the load increases. However it is desirable to maintain the frequency constant such that $\Delta f=0$. The power flow through different tie-lines are scheduled - for example, area- i may export a pre-specified amount of power to area- j while importing another pre-specified amount of power from area- k . However it is expected that to fulfill this obligation, area- i absorbs its own load change, i.e., increase generation to supply extra load in the area or decrease generation when the load demand in the area has reduced. While doing this area- i must however maintain its obligation to areas j and k as far as importing and exporting power is concerned. A conceptual diagram of the interconnected areas is shown in Fig. 5.4.

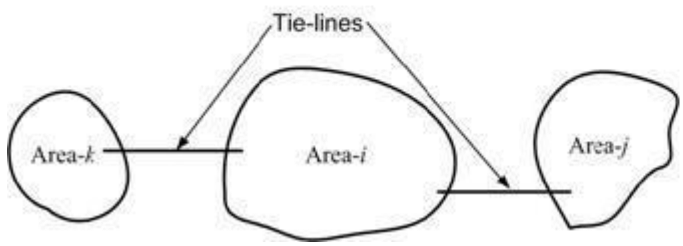


Fig. 5.4 Interconnected areas in a power system

We can therefore state that the load frequency control (LFC) has the following two objectives:

- Hold the frequency constant ($\Delta f = 0$) against any load change. Each area must contribute to absorb any load change such that frequency does not deviate.
- Each area must maintain the tie-line power flow to its pre-specified value.
- $$\mathrm{ACE} = \left(P_{tie} - P_{sch}\right) + B_f \Delta f = \Delta P_{tie} + B_f \Delta f \quad (5.27)$$

The first step in the LFC is to form the **area control error (ACE)** that is defined as

Where P_{tie} and P_{sch} are **tie-line power** and **scheduled power** through tie-line respectively and the constant B_f is called the **frequency bias constant.**

$$\Delta P_{ref,i} = -K_i \int \text{ACE}\, dt$$

The feedback of the ACE through an integral controller of the form where K_i is the integral gain. The ACE is negative if the net power flow out of an area is low or if the frequency has dropped or both. In this case the generation must be increased. This can be achieved by increasing $\Delta P_{ref,\ i}$. This negative sign accounts for this inverse relation between $\Delta P_{ref,\ i}$ and ACE. The tie-line power flow and frequency of each area are monitored in its control center. Once the ACE is computed and $\Delta P_{ref,\ i}$ is obtained from (5.28), commands are given to various turbine-generator controls to adjust their reference power settings.

Example 5.6

Consider a two-area power system in which area-1 generates a total of 2500 MW, while area-2 generates 2000 MW. Area-1 supplies 200 MW to area-2 through the inter-tie lines connected between the two areas. The bias constant of area-1 (β_1) is 875 MW/Hz and that of area-2 (β_2) is 700 MW/Hz. With the two areas operating in the steady state, the load of area-2 suddenly increases by 100 MW. It is desirable that area-2 absorbs its own load change while not allowing the frequency to drift.

The area control errors of the two areas are given by

$\text{ACE}_1 = \Delta P_{tie1} + B_1 \Delta f_1$ And $\text{ACE}_2 = \Delta P_{tie2} + B_2 \Delta f_2$

Since the net change in the power flow through tie-lines connecting these two areas must be zero, we have

$$\Delta P_{tie1} + \Delta P_{tie2} = 0 \Rightarrow \Delta P_{tie1} = -\Delta P_{tie2}$$

Also as the transients die out, the drift in the frequency of both these areas is assumed to be constant, i.e.

$$\Delta f_1 = \Delta f_2 = \Delta f$$

If the load frequency controller (5.28) is able to set the power reference of area-2 properly, the ACE of the two areas will be zero, i.e., $\text{ACE}_1 = \text{ACE}_2 = 0$. Then we have

$$\text{ACE}_1 + \text{ACE}_2 = (B_1 + B_2)\Delta f = 0$$

This will imply that Δf will be equal to zero while maintaining $\Delta P_{tie1} = \Delta P_{tie2} = 0$. This signifies that area-2 picks up the additional load in the steady state.

Coordination between LFC and Economic Dispatch

Both the load frequency control and the economic dispatch issue commands to change the power setting of each turbine-governor unit. At a first glance it may seem that these two commands can be conflicting. This however is not true. A typical automatic generation control strategy is shown in Fig. 5.5 in which both the objective are coordinated. First we compute the area control error. A share of this ACE, proportional to α_i, is allocated to each of the turbine-generator unit of an area. Also the share of unit- i , γ_i X $\Sigma(P_{DK} - P_k)$, for the deviation of total generation from actual generation is computed. Also the error

economic power setting and actual power setting of unit- i is computed. All these signals are then combined and passed through a proportional gain K_i to obtain the turbine-governor control signal.

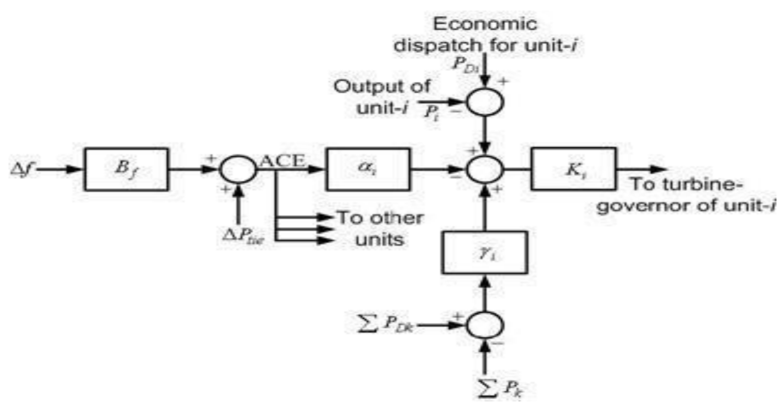


Fig. 5.5 Automatic generation control of unit-i

Section II: Swing Equation

Let us consider a three-phase synchronous alternator that is driven by a prime mover. The equation of motion of the machine rotor is given by

$$J \frac{d^2\theta}{dt^2} = T_m - T_e = T_a$$

Where

J	is the total moment of inertia of the rotor mass in kgm^2
T_m	is the mechanical torque supplied by the prime mover in N-m
T_e	is the electrical torque output of the alternator in N-m
θ	is the angular position of the rotor in rad

Neglecting the losses, the difference between the mechanical and electrical torque gives the net accelerating torque T_a . In the steady state, the electrical torque is equal to the mechanical torque, and hence the accelerating power will be zero. During this period the rotor will move at **synchronous speed** ω_s in rad/s.

The angular position θ is measured with a stationary reference frame. To represent it with respect to the synchronously rotating frame, we define

$$\theta = \omega_s t + \delta \quad (9.7)$$

Where δ is the angular position in radians with respect to the synchronously rotating

$$I_S = \frac{V_1\angle\delta - V_2}{jX} = \frac{V_1\cos\delta - V_2 + jV_1\sin\delta}{jX} \quad (9.8)$$

Reference frame. Taking the time derivative of the above equation we get

Defining the angular speed of the rotor as

$$\omega_r = \frac{d\theta}{dt}$$

We can write (9.8) as

$$\omega_r - \omega_s = \frac{d\delta}{dt} \quad (9.9)$$

We can therefore conclude that the rotor angular speed is equal to the synchronous speed only when *dδ / dt* is equal to zero. We can therefore term *dδ /dt* as the error in speed.

$$J\frac{d^2\delta}{dt^2} = T_m - T_e = T_a \quad (9.10)$$

Taking derivative of (9.8), we can then rewrite (9.6) as Multiplying both side of (9.11) by ω_m we get

$$J\omega_r\frac{d^2\delta}{dt^2} = P_m - P_e = P_a \quad (9.11)$$

Where P_m , P_e and P_a respectively are the mechanical, electrical and accelerating power in MW.

$$H = \frac{\text{Stored kinetic energy at synchronous speed in mega - joules}}{\text{Generator MVA rating}} = \frac{J\omega_s^2}{2S_{rated}} \quad (9.12)$$

We now define a normalized inertia constant as Substituting (9.12) in (9.10) we get

$$2H\frac{S_{rated}}{\omega_s^2}\omega_r\frac{d^2\delta}{dt^2} = P_m - P_e = P_a \quad (9.13)$$

In steady state, the machine angular speed is equal to the synchronous speed and hence we can replace ω_r in the above equation by ω_s. Note that in (9.13) P_m , P_e and P_a are given in MW. Therefore dividing them by the generator MVA rating S_{rated} we can get these quantities in per unit. Hence dividing both sides of (9.13) by S_{rated} we get

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e = P_a \quad (9.14)$$

Equation (9.14) describes the behavior of the rotor dynamics and hence is known as the swing equation. The angle δ is the angle of the internal emf of the generator and it dictates the amount of power that can be transferred. This angle is therefore called the **load angle.**

Example 9.2

A 50 Hz, 4-pole turbo generator is rated 500 MVA, 22 kV and has an inertia constant (H) of 7.5. Assume that the generator is synchronized with a large power system and has a zero accelerating power while delivering a power of 450 MW. Suddenly its input power is changed to 475 MW. We have to find the speed of the generator in rpm at the end of a period of 10 cycles. The rotational losses are assumed to be zero.

We then have

$$\frac{d^2\delta}{dt^2} = \frac{\omega_s}{2H}\left(P_m - P_e\right) = \frac{100\pi}{15} \times 25 = 523.6 \text{ electrical deg/s}^2$$

$$= \frac{523.6\pi}{180} = 9.1385 \text{ electrical rad/s}^2$$

Noting that the generator has four poles, we can rewrite the above equation as

$$\frac{d^2\delta}{dt^2} = \frac{9.1385}{2} = 4.5693 \text{ mechanical rad/s}^2$$

$$= 60 \times \frac{4.5693}{2\pi} = 43.6332 \text{ rpm/s}$$

The machines accelerates for 10 cycles, i.e., 20 × 10 = 200 ms = 0.2 s, starting with a synchronous speed of 1500 rpm. Therefore at the end of 10 cycles

Speed = 1500 + 43.6332 ´ 0.2 = 1508.7266 rpm.