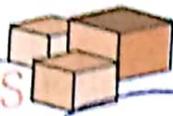


1 ELECTRIC CHARGES AND FIELDS

BASIC CONCEPTS



The study of electric charges at rest is called Electrostatics.

1. Two Kinds of Electric Charges

When two bodies are rubbed together, they get oppositely charged. Experimental evidences show that there are two types of charges:

(i) **Positive Charge:** Positive charge is produced by the removal of electrons from a neutral body. That is, positive charge means deficiency of electrons.

(ii) **Negative Charge:** Negative charge is produced by giving electrons to a neutral body. That is, negative charge means excess of electrons on a neutral body.

SI unit of charge is coulomb (C).

2. Properties of Charges

(i) **Conservation of Charge:** The charge of an isolated system remains constant. This means that charge can neither be created nor destroyed; but it may simply be transferred from one body to another.

(ii) **Additive Property:** Total charge on an isolated system is equal to the algebraic sum of charges on individual bodies of system. This is called additive property of charges. If a system contains three charges, $q_1, q_2, -q_3$ then total charge on system, $Q = q_1 + q_2 - q_3$.

(iii) **Quantisation of Charge:** The total charge on a body is integral multiple of fundamental charge 'e'. *सम्यक् निसंकेत*

i.e.,

$$q = \pm ne$$

where n is an integer ($n = 1, 2, 3, \dots$).

(iv) **Charge is unaffected by motion:** The charge on a body remains unaffected of its velocity, i.e.,

$$\text{Charge at rest} = \text{Charge in motion}$$

(v) **Like charges repel while unlike charges attract each other.**

3. Coulomb's Law in General Form

It states that the force of attraction or repulsion between two point charges is directly proportional to the product of magnitude of charges and inversely proportional to the square of distance between them. The direction of this force is along the line joining the two charges, i.e.,

$$F = k \frac{q_1 q_2}{r^2}$$

where $k = \frac{1}{4\pi\epsilon}$ is constant of proportionality; ϵ is permittivity of medium between the charges. If

ϵ_0 is permittivity of free space and K the dielectric constant of medium, then $\epsilon = K\epsilon_0$

ϵ is permittivity of medium.

Relative permittivity

$$F = \frac{1}{4\pi\epsilon_0 K} \frac{q_1 q_2}{r^2}$$

For free space $K = 1$, Therefore

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \quad \text{free space.}$$

Dielectric constant or Relative permittivity (K): The dielectric constant of a medium is defined as the ratio of permittivity of medium to the permittivity of free space, i.e. $K = \epsilon/\epsilon_0$

Definition of coulomb: 1 coulomb charge is the charge which when placed at a distance of 1 metre from an equal and similar charge in vacuum (or air) will repel it with a force of 9×10^9 N.

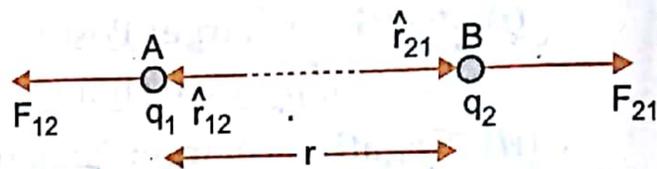
4. Coulomb's Law in Vector Form

Consider two like charges q_1 and q_2 located at points A and B in vacuum. The separation between the charges is r . As charges are like, they repel each other. Let \vec{F}_{21} be the force exerted on charge q_2 by charge q_1 and \vec{F}_{12} that exerted on charge q_1 by charge q_2 . If \vec{r}_{21} is the position vector of q_2 relative to q_1 and \hat{r}_{21} is unit vector along A to B, then the force \vec{F}_{21} is along A to B and

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21} \quad \dots(i)$$

But $\hat{r}_{21} = \frac{\vec{r}_{21}}{r}$

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \cdot \frac{\vec{r}_{21}}{r} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^3} \vec{r}_{21}$$



Similarly if \vec{r}_{12} is position vector of q_1 relative to q_2 and \hat{r}_{12} is unit vector from B to A, then

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^3} \vec{r}_{12} \quad \dots(ii)$$

Obviously $\vec{r}_{12} = -\vec{r}_{21}$ therefore equation (ii) becomes

$$\vec{F}_{12} = -\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^3} \vec{r}_{21} \quad \dots(iii)$$

Comparing (i) and (iii), we get

$$\vec{F}_{21} = -\vec{F}_{12}$$

This means that the Coulomb's force exerted on q_2 by q_1 is equal and opposite to the Coulomb's force exerted on q_1 by q_2 ; in accordance with Newton's third law.

Thus, Newton's third law also holds good for electrical forces.

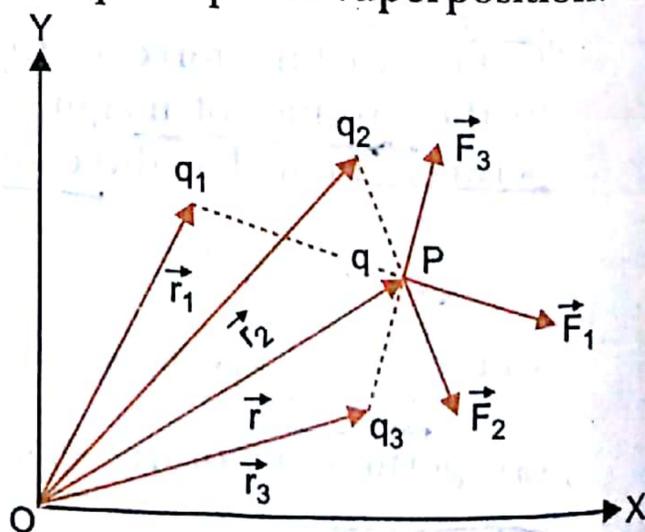
5. Principle of Superposition of Electric Charges

Coulomb's law gives the force between two point charges. But if there are a number of interacting charges, then the force on a particular charge may be found by the principle of superposition. It states that

If the system contains a number of interacting charges, then the force on a given charge is equal to the vector sum of the forces exerted on it by all remaining charges.

The force between any two charges is not affected by the presence of other charges.

Suppose that a system of charges contains n charges $q_1, q_2, q_3, \dots, q_n$ having position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_n$ relative to origin O



respectively. A point charge q is located at P having position vector \vec{r} relative to O. The total force on q due to all n charges is to be found. If $\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots, \vec{F}_n$, are the forces acting on q due to charges $q_1, q_2, q_3, \dots, q_n$ respectively, then by the principle of superposition, the net force on q is

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n$$

If the force exerted due to charge q_i on q is \vec{F}_i , then from Coulomb's law in vector form

$$\vec{F}_i = \frac{1}{4\pi\epsilon_0} \frac{qq_i}{|\vec{r} - \vec{r}_i|^3} (\vec{r} - \vec{r}_i)$$

The total force on q due to all n charges may be expressed as

$$\vec{F} = \sum_{i=1}^n \vec{F}_i = \sum_{i=1}^n \frac{1}{4\pi\epsilon_0} \frac{qq_i}{|\vec{r} - \vec{r}_i|^3} (\vec{r} - \vec{r}_i)$$

$$\vec{F} = \frac{1}{4\pi\epsilon_0} q \sum_{i=1}^n \frac{q_i}{|\vec{r} - \vec{r}_i|^3} (\vec{r} - \vec{r}_i)$$

Here \sum represents the vector-sum.

6. Continuous Charge Distribution

The electrostatic force due to a charge element dq at charge q_0 situated at point P is

$$d\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_0 dq}{R^3} \vec{R} = \frac{1}{4\pi\epsilon_0} \frac{q_0 dq}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}')$$

The total force on q_0 by the charged body is

$$F = \frac{1}{4\pi\epsilon_0} q_0 \int \frac{dq(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

- 1) For **linear charge distribution**, $dq = \lambda dl$, where λ is charge per unit length and integration is over the whole length of charge.
- 2) For **surface charge distribution**, $dq = \sigma ds$, where σ is charge per unit area and integration is for the whole surface of charge.
- 3) For **volume charge distribution**, $dq = \rho dV$, where ρ is charge per unit volume and integration is for whole volume of charge.

Electric field

The electric field strength at any point in an electric field is a vector quantity whose magnitude is equal to the force acting on per unit positive test charge and the direction is along the direction of force.

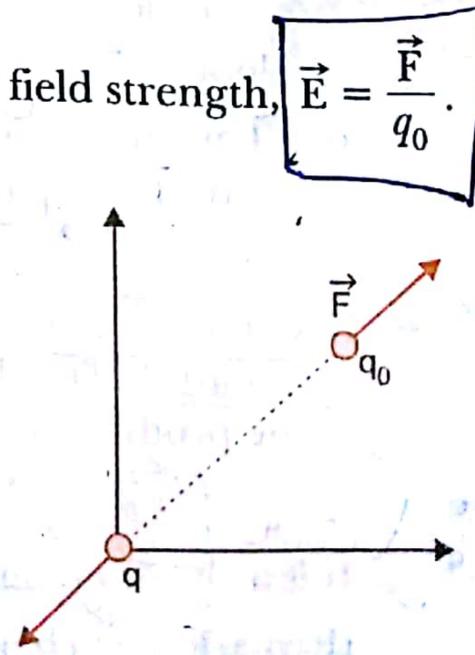
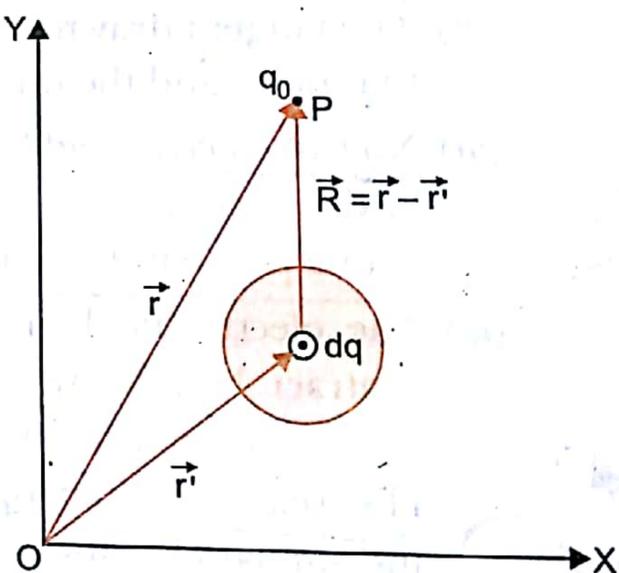
If \vec{F} is the force acting on infinitesimal positive test charge q_0 , then electric field strength, $\vec{E} = \frac{\vec{F}}{q_0}$.
Therefore from definition, electric field can be given as

$$\vec{E} = \lim_{q_0 \rightarrow 0} \frac{\vec{F}}{q_0}$$

The unit of electric field strength is newton/coulomb or volt/metre (abbreviated as N/C or V/m respectively.)

- (i) The electric field strength due to a point charge q at a distance r in magnitude form

$$|E| = \frac{|F|}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$



In vector form, $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^3} \vec{r}$

(ii) The electric field strength due to a system of discrete charge is

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i^3} \vec{r}_i$$

(iii) The electric field strength due to a continuous charge distribution is

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^3} \vec{r}$$

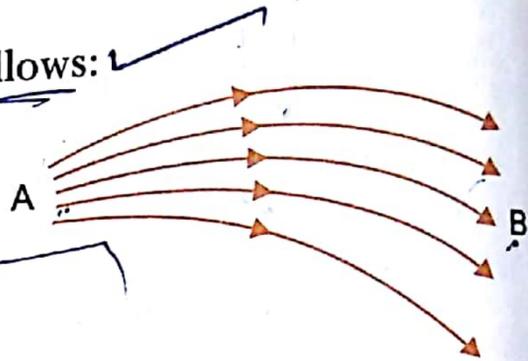
7. Electric field lines

vector quantity

The electric field line in an electric field is an imaginary smooth curve along which an isolated free positive test charge tends to move.

In terms of electric field lines the electric field strength is defined as follows:

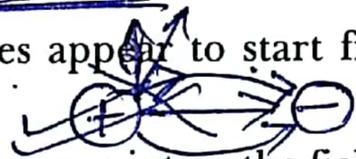
The electric field strength at any point is defined as a vector quantity whose magnitude is measured by the number of electric field lines passing normally through per unit small area around that point and whose direction is along the tangent on field line drawn on that point.



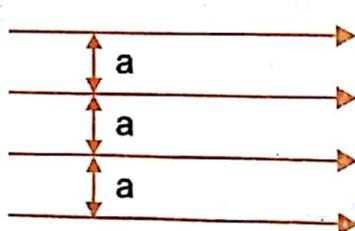
Accordingly nearer the electric field lines, stronger is the electric field, and farther the electric field lines, weaker is the electric field. In figure, the electric field strength at A is greater than that at B.

Properties of electric field lines

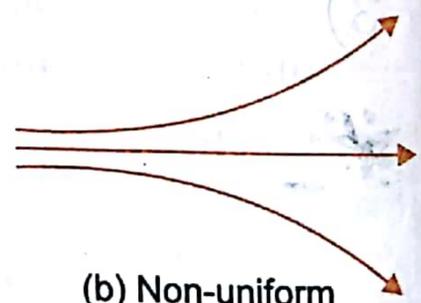
- (i) The electric field lines appear to start from positive charge and to terminate at negative charge.
- (ii) The tangent drawn at any point on the field line gives the direction of electric field strength at that point and the direction of force acting on a positive charge at that point.
- (iii) No two electric field lines can intersect each other because if they do so, then two tangents can be drawn at the point of intersection; which would mean two directions of electric field strength at one point and that is impossible.



(iv) The electric field lines have a tendency to contract lengthwise like a stretched elastic string and separate from each other laterally. The reason is that opposite charges attract and similar charges repel.



(a) Uniform electric field



(b) Non-uniform electric field

(v) The electric field lines do not form any closed loops.

(vi) The equidistant electric field lines represent uniform electric field while electric field lines at different separations represent non-uniform electric field (Figure).

8. Electric Dipole

A system containing two equal and opposite charges separated by a finite distance is called an electric dipole. Dipole moment of electric dipole having charges $+q$ and $-q$ at separation $2l$ is defined as the product of magnitude of one of the charges and shortest distance between them.

$$\vec{p} = q \cdot 2l$$

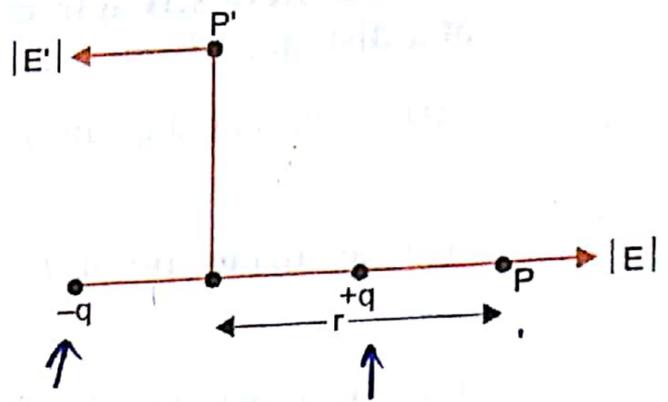
It is a vector quantity, directed from $-q$ to $+q$

[Remark: Net charge on an electric dipole is zero.]

9. Electric Field Due to a Short Dipole

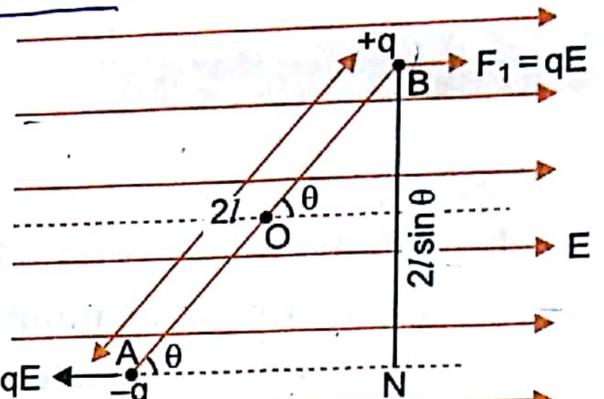
- (i) A point P on axis, $E = \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3}$
- (ii) At a point P' on equatorial line,

$E' = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$ $\vec{p} \times \vec{E}$



10. Electric Force and Torque on an Electric Dipole in a Uniform Electric Field

In a uniform electric field of strength E , the net electric force is zero; but a torque equal to $pE \sin \theta$ acts on the dipole (where θ is the angle between directions of dipole moment \vec{p} and electric field \vec{E}). This torque tends to align the dipole along the direction of electric field. Torque in vector form



$\vec{\tau} = \vec{p} \times \vec{E}$

11. Electric Flux

The total number of electric field lines crossing (or diverging) a surface normally is called electric flux.

Electric flux through surface element $\Delta \vec{S}$ is $\Delta \phi = \vec{E} \cdot \Delta \vec{S} = E \Delta S \cos \theta$, where \vec{E} is electric field strength.

Electric flux through entire closed surface is,

$\phi = \int_S \vec{E} \cdot d\vec{S}$

SI unit of electric flux is volt-metre or Nm^2C^{-1} .

12. Gauss's Theorem

It states that the total electric flux through a closed surface is equal to $\frac{1}{\epsilon_0}$ times the net charge enclosed by the surface.

i.e., $\phi = \int_S \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} \sum q$

13. Formulae for Electric Field Strength Calculated from Gauss's Theorem

(a) Electric field due to infinitely long straight wire of charge per unit length λ at a distance r from the wire is

$E = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{r}$

(b) Electric field strength due to an infinite plane sheet of charge per unit area σ is

$E = \frac{\sigma}{2\epsilon_0}$, independent of distance of point from the sheet.

(c) Electric field strength due to a uniformly charged thin spherical shell or conducting sphere of radius R having total charge q , at a distance r from centre is

(i) at external point $E_{ext} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} (r > R)$

(ii) at surface point $E_s = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2}$

(iii) at internal point $E_{int} = 0$



(d) Electric field strength due to a uniformly charged non-conducting solid sphere of radius R at a distance r from centre

$$(i) \text{ at external point } E_{ext} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} (r > R)$$

$$(ii) \text{ at surface point } E_s = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2}$$

$$(iii) \text{ at internal point, } E_{int} = \frac{1}{4\pi\epsilon_0} \frac{qr}{R^3} (r < R)$$

SELECTED NCERT QUESTIONS

Quantisation of Charge

Q. 1. A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C.

(a) Estimate the number of electrons transferred (from which to which?)

(b) Is there a transfer of mass from wool to polythene?

Ans. When two neutral bodies are rubbed together, electrons of one body are transferred to the other. The body which gains electrons is negatively charged and the body which loses electrons is positively charged.

(a) From quantisation of charge

$$q = ne$$

$$\text{Here, } q = 3 \times 10^{-7} \text{ C, } e = 1.6 \times 10^{-19} \text{ C}$$

$$\therefore \text{ Number of electrons transferred, } n = \frac{q}{e} = \frac{3 \times 10^{-7}}{1.6 \times 10^{-19}} = 1.875 \times 10^{12}$$

When polythene is rubbed with wool, the polythene becomes negatively charged and wool becomes positively charged. This implies that the electrons are transferred from wool to polythene.

(b) Yes as electrons have finite mass, the mass is transferred from wool to polythene

$$\Delta M = n \times m_e = 1.875 \times 10^{12} \times 9.1 \times 10^{-31} \text{ kg} = 1.7 \times 10^{-18} \text{ kg}$$

Coulomb's Law

Q. 2. What is the force between two small charged spheres having charges of 2×10^{-7} C and 3×10^{-7} C placed 30 cm apart in air?

Ans. Two charged spheres at finite separation behave as point charge and the Coulomb's force of repulsion

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$\text{Here } q_1 = 2 \times 10^{-7} \text{ C, } q_2 = 3 \times 10^{-7} \text{ C, } r = 30 \text{ cm} = 0.30 \text{ m}$$

$$\therefore F = 9 \times 10^9 \times \frac{(2 \times 10^{-7}) \times (3 \times 10^{-7})}{(0.30)^2} = 6 \times 10^{-3} \text{ N}$$

Q. 3. The electrostatic force on a small sphere of charge $0.4 \mu\text{C}$ due to another small sphere of charge $-0.8 \mu\text{C}$ in air is 0.2 N

(a) What is the distance between the two spheres?

(b) What is the force on the second sphere due to the first?

Ans. The electrostatic force between two charged spheres is given by Coulomb's law as

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

Here $q_1 = 0.4\mu\text{C} = 0.4 \times 10^{-6}\text{C}$,

$q_2 = -0.8\mu\text{C} = -0.8 \times 10^{-6}\text{C} = 0.8 \times 10^{-6}$ (magnitude), $F = 0.2\text{N}$

(a) As charges are of the opposite sign, the force between the charges magnitude is attractive

$$\therefore 0.2 = 9 \times 10^9 \times \frac{(0.4 \times 10^{-6}) \times (0.8 \times 10^{-6})}{r^2}$$

$$\Rightarrow r^2 = \frac{9 \times 10^9 \times (0.4 \times 10^{-6}) \times (0.8 \times 10^{-6})}{0.2} = 9 \times 16 \times 10^{-4}$$

Distance, $r = 12 \times 10^{-2}\text{m} = 12\text{cm}$

(b) The force on second sphere due to first is $= 0.2\text{N}$. Since $|\vec{F}_{21}| = |\vec{F}_{12}|$

Q. 4. Four point charges $q_A = 2\mu\text{C}$, $q_B = -5\mu\text{C}$, $q_C = 2\mu\text{C}$ and $q_D = -5\mu\text{C}$ are located at the corners of a square $ABCD$ of side 10cm . What is the force on a charge of $1\mu\text{C}$ placed at the centre of the sphere?

Ans. The coulomb's forces acting on a charged particle due to all other charges are added by vector method. Force on charge $q_0 = 1\mu\text{C}$ placed at centre O will be the vector sum of forces due to all the four charges q_A , q_B , q_C and q_D .

Clearly, $OA = OB = OC = OD$

$$= \frac{1}{2} \sqrt{10^2 + 10^2} = \frac{10\sqrt{2}}{2}\text{cm}$$

$$= 5\sqrt{2}\text{cm} = 5\sqrt{2} \times 10^{-2}\text{m}$$

Force on $q_0 = 1\mu\text{C}$ due to charge $q_A = 2\mu\text{C}$ is

$$\vec{F}_{OA} = \frac{1}{4\pi\epsilon_0} \frac{q_0 q_A}{(OA)^2} \text{ along } \vec{OC} = 9 \times 10^9 \times \frac{(1 \times 10^{-6})(2 \times 10^{-6})}{(5\sqrt{2} \times 10^{-2})^2} = 3.6\text{N along } \vec{OC}$$

Force on $q_0 = 1\mu\text{C}$ due to charge $q_C = 2\mu\text{C}$ is

$$\vec{F}_{OB} = \frac{1}{4\pi\epsilon_0} \frac{q_0 q_C}{(OB)^2} \text{ along } \vec{OA} = 9 \times 10^9 \times \frac{1 \times 10^{-6} \times 2 \times 10^{-6}}{(5\sqrt{2} \times 10^{-2})^2} = 3.6\text{N along } \vec{OA}$$

Clearly, $\vec{F}_{OA} + \vec{F}_{OC} = 0$

The force on $q_0 = 1\mu\text{C}$ due to charge $q_B = -5\mu\text{C}$ is

$$\vec{F}_{OB} = \frac{1}{4\pi\epsilon_0} \frac{q_0 q_B}{(OB)^2} \text{ along } \vec{OB} = 9 \times 10^9 \times \frac{(1 \times 10^{-6})(5 \times 10^{-6})}{(5\sqrt{2} \times 10^{-2})^2} \text{ along } \vec{OB} = 9.0\text{N along } \vec{OB}$$

The force on $q_0 = 1\mu\text{C}$ due to charge $q_D = -5\mu\text{C}$ is

$$\vec{F}_{OD} = \frac{1}{4\pi\epsilon_0} \frac{q_0 q_D}{(OD)^2} \text{ along } \vec{OD} = 9 \times 10^9 \times \frac{1 \times 10^{-6} \times 5 \times 10^{-6}}{(5\sqrt{2} \times 10^{-2})^2} = 9.0\text{N along } \vec{OD}$$

Clearly, $\vec{F}_{OB} + \vec{F}_{OD} = 0$

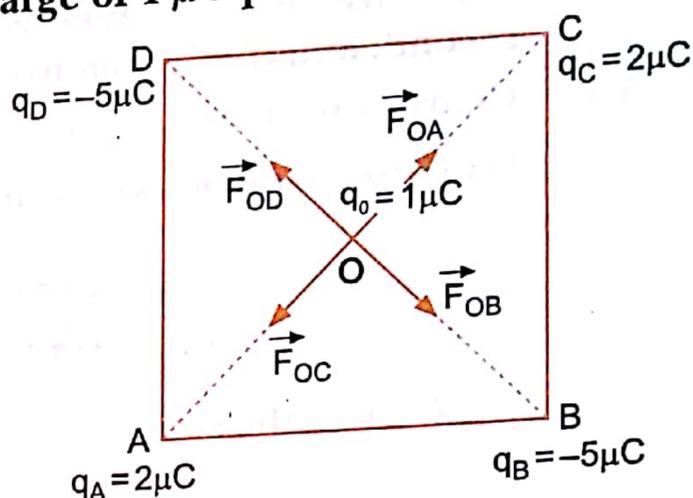
Therefore, net force on q_0 is

$$\vec{F} = \vec{F}_{OA} + \vec{F}_{OB} + \vec{F}_{OC} + \vec{F}_{OD} = (\vec{F}_{OA} + \vec{F}_{OC}) + (\vec{F}_{OB} + \vec{F}_{OD}) = 0 + 0 = 0$$

that is, the net force on charge q_0 is zero.

Q. 5. (a) Two insulated charged copper spheres A and B have their centres separated by a distance of 50cm . What is the mutual force of electrostatic repulsion if the charge on each is $6.5 \times 10^{-7}\text{C}$? The radii of A and B are negligible compared to the distance of separation.

(b) What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved?



Ans. (a) Here, $q_1 = 6.5 \times 10^{-7} \text{ C}$, $q_2 = 6.5 \times 10^{-7} \text{ C}$, $r = 50 \text{ cm} = 0.50 \text{ m}$

$$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

$$\text{Using Coulomb's law, } F = k \frac{q_1 q_2}{r^2} = \frac{9 \times 10^9 \times 6.5 \times 10^{-7} \times 6.5 \times 10^{-7}}{(0.50)^2} \text{ N}$$

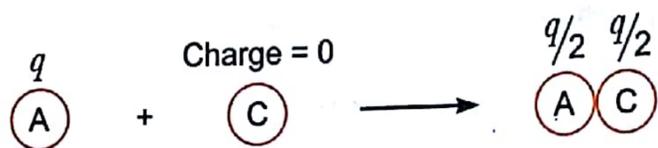
$$= \frac{380.25 \times 10^{-5}}{0.25} \text{ N} = 1521 \times 10^{-5} \text{ N} = 1.5 \times 10^{-2} \text{ N}$$

(b) If each sphere is charged double and the distance between them is halved, then the force of repulsion is given by

$$F = k \frac{2q_1 \cdot 2q_2}{(r/2)^2} = 16k \frac{q_1 q_2}{r^2} = 24 \times 10^{-2} \text{ N} = 0.24 \text{ N}$$

Q. 6. Suppose the spheres A and B in above question have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B?

Ans. Charge on each spheres A and B = $q = 6.5 \times 10^{-7} \text{ C}$ when a similar but uncharged sphere C is brought in contact with sphere A, each sphere shares a charge $\frac{q}{2}$, equally.



Now, when the sphere C is brought in contact with sphere B, the charge is redistributed equally.

$$\text{Charge of sphere B or C} = \frac{1}{2} \left(q + \frac{q}{2} \right) = \frac{3q}{4}$$



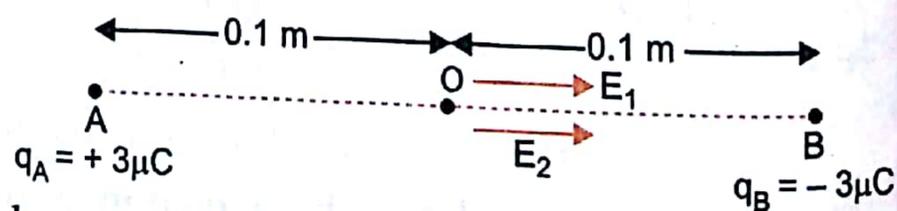
So, the force of repulsion between A and B is

$$F = \frac{1}{4\pi\epsilon_0} \frac{\frac{q}{2} \cdot \frac{3q}{4}}{r^2} = \frac{3}{8} \times 1.5 \times 10^{-2} \text{ N} = 5.7 \times 10^{-3} \text{ N}$$

Electric Field

Q. 7. Two point charges $q_A = +3\mu\text{C}$ and $q_B = -3\mu\text{C}$ are located 20 cm apart in vacuum. (a) What is the electric field at the mid point O of the line AB joining the two charges? (b) If a negative test charge of magnitude $1.5 \times 10^{-9} \text{ C}$ is placed at this point, what is the force experienced by the test charge?

Ans. (a) The electric field strength at point O due to charges A and B is additive (away from positive charge and towards negative charge)



\therefore Electric field strength at mid point due to charge q_A is

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2} = 9 \times 10^9 \times \frac{3 \times 10^{-6}}{(0.10)^2} = 2.7 \times 10^6 \text{ NC}^{-1} \text{ along } \overrightarrow{AO}$$

Electric field strength at O due to charge q_B

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{q_B}{r^2} = 9 \times 10^9 \times \frac{3 \times 10^{-6}}{(0.10)^2} = 2.7 \times 10^6 \text{ NC}^{-1} \text{ along } \vec{OB}$$

Net electric field at O

$$E = E_1 + E_2 = 2.7 \times 10^6 + 2.7 \times 10^6 = 5.4 \times 10^6 \text{ NC}^{-1} \text{ along } \vec{AB}$$

(b) Electric force on test charge q_0 placed at O

$$F = q_0 E = 1.5 \times 10^{-9} \times 5.4 \times 10^6 = 8.1 \times 10^{-3} \text{ N}$$

Q. 8. A system has two charges $q_A = 2.5 \times 10^{-7} \text{ C}$ and $q_B = -2.5 \times 10^{-7} \text{ C}$ located at points $A = (0, 0, -15 \text{ cm})$ and $B = (0, 0, +15 \text{ cm})$ respectively. What are the total charge and electric dipole moment of the system?

Ans. A dipole has two equal and opposite charges with dipole moment $\vec{p} = q2\vec{l}$, directed from $-q$ to $+q$.

Given $q_A = 2.5 \times 10^{-7} \text{ C}, q_B = -2.5 \times 10^{-7} \text{ C}$

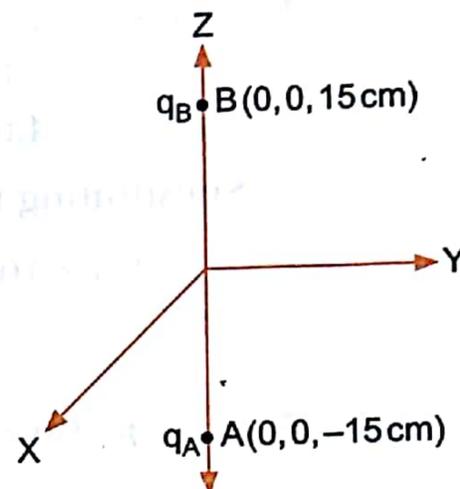
Total charge, $q = q_A + q_B = 2.5 \times 10^{-7} \text{ C} - 2.5 \times 10^{-7} \text{ C} = 0$.

$$2l = AB = 30 \text{ cm} = 0.30 \text{ m}$$

Electric dipole moment, $\vec{p} = q \cdot 2\vec{l}$ directed from $-q$ to $+q$

$$= (2.5 \times 10^{-7} \text{ C})(0.30 \text{ m}) = 7.5 \times 10^{-8} \text{ Cm along } \vec{BA}$$

$$= 7.5 \times 10^{-8} \text{ Cm directed along negative Z-axis.}$$



Q. 9. An electric dipole with a dipole moment $4 \times 10^{-9} \text{ Cm}$ is aligned at 30° with the direction of a uniform electric field of magnitude $5 \times 10^4 \text{ NC}^{-1}$. Calculate the magnitude of the torque acting on the dipole.

Ans. A dipole placed in a uniform electric field, experiences a torque $\tau = pE \sin \theta$ which tends to align the dipole parallel to the direction of field.

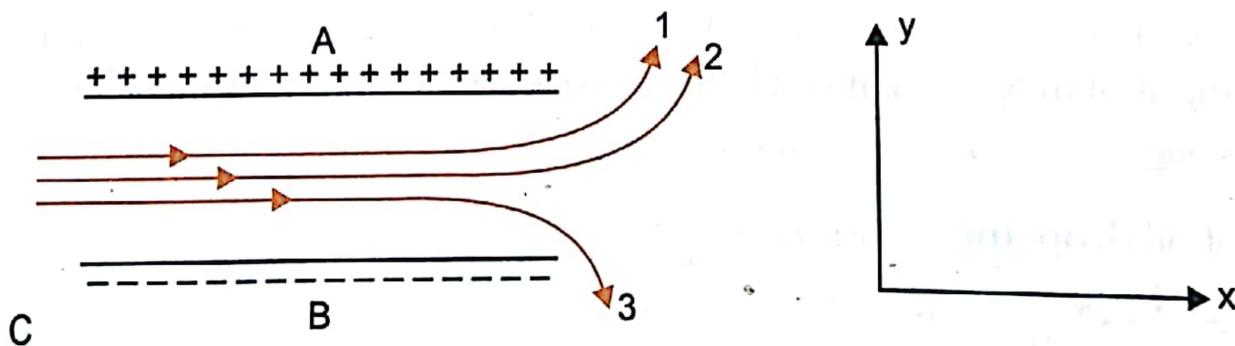
$$\text{Torque } \tau = pE \sin \theta$$

Here $p = 4 \times 10^{-9} \text{ C-m}, E = 5 \times 10^4 \text{ NC}^{-1}, \theta = 30^\circ$

$$\therefore \text{Torque } \tau = 4 \times 10^{-9} \times 5 \times 10^4 \sin 30^\circ$$

$$= 4 \times 10^{-9} \times 5 \times 10^4 \times \frac{1}{2} = 10^{-4} \text{ Nm}$$

Q. 10. The figure shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?



Ans. A positively charged particle is deflected towards a negative plate and a negatively charged particle towards a positive plate and shows a parabolic path.

From fig. it is clear that the particles (1) and (2) are deflected towards positive plate; hence, they carry **negative charges**.

Particle (3) is deflected along negative plate, so it carries **positive charge**.

The transverse deflection in a given electric field is

$$y = \frac{1}{2} at^2, \text{ where } a = \frac{qE}{m} \text{ and } t = \left(\frac{x}{u} \right)$$

$$\text{So } y = \frac{1}{2} \left(\frac{q}{m} \right) \frac{E x^2}{u^2} \propto \frac{q}{m}$$

From fig., it is obvious that the transverse deflection is the maximum for particle (3), hence, particle (3) has the highest charge to mass ratio (q/m)

Q. 11. A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of sphere is $1.5 \times 10^3 \text{ NC}^{-1}$ and points radially inward, what is the net charge on the sphere?

Ans. Given, radius of sphere $R = 10 \text{ cm} = 0.10 \text{ m}$

Distance from centre, $r = 20 \text{ cm} = 0.20 \text{ m}$

Electric field at distance r from centre, $E = 1.5 \times 10^3 \text{ NC}^{-1}$

The electric field due to charged sphere at external point distance r from centre is

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

\therefore Substituting the given values,

$$1.5 \times 10^3 = 9 \times 10^9 \times \frac{q}{(0.20)^2}$$

$$\Rightarrow \text{Charge on sphere, } q = \frac{1.5 \times 10^3 \times (0.20)^2}{9 \times 10^9} = 6.67 \times 10^{-9} \text{ C} = 6.67 \text{ nC}$$

As electric field is radially inward, charge on sphere is negative, therefore, charge on sphere = -6.67 nC .

Q. 12. An infinite line charge produces an electric field of $9 \times 10^4 \text{ NC}^{-1}$ at a distance of 2 cm. Calculate the linear charge density.

Ans. Electric field a distance r from an infinite line charge is, $E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{r}$

$$\therefore \text{Linear charge density } \lambda = \frac{1}{2} (4\pi\epsilon_0) r E$$

Here, $r = 2 \text{ cm} = 0.02 \text{ m}$, $E = 9 \times 10^4 \text{ NC}^{-1}$

$$\therefore \lambda = \frac{1}{2} \times \left(\frac{1}{9 \times 10^9} \right) \times (0.02) \times (9 \times 10^4) = 10^{-7} \text{ C m}^{-1}$$

Q. 13. An oil drop of 12 excess electrons is held stationary under a constant electric field of $2.55 \times 10^4 \text{ NC}^{-1}$ in Millikan's oil drop experiment. The density of the oil is 1.26 g cm^{-3} . Estimate the radius of the drop ($g = 9.81 \text{ ms}^{-2}$; $e = 1.60 \times 10^{-19} \text{ C}$).

Ans. In Millikan's oil drop experiment, the charged oil drop remains suspended (in equilibrium) when downward weight of drop is balanced by upward electrostatic force and charge on drop, $q = ne$, i.e.,

$$qE = mg \quad \Rightarrow \quad neE = mg$$

If r is radius of oil drop, then mass $m = \frac{4}{3} \pi r^3 \rho$

$$\therefore neE = \frac{4}{3} \pi r^3 \rho g$$

$$\Rightarrow r = \left[\frac{3neE}{4\pi\rho g} \right]^{1/3}$$

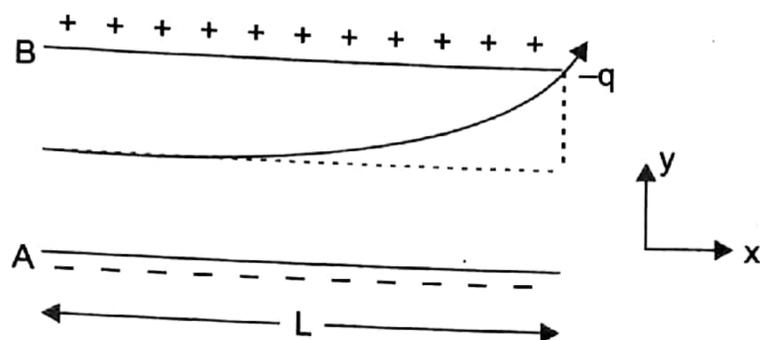
Here, $n = 12$, $e = 1.6 \times 10^{-19} \text{ C}$, $E = 2.55 \times 10^4 \text{ NC}^{-1}$, $\rho = 1.26 \text{ g cm}^{-3} = 1.26 \times 10^3 \text{ kg m}^{-3}$, $g = 9.81 \text{ ms}^{-2}$

$$\therefore r = \left[\frac{3 \times 12 \times 1.6 \times 10^{-19} \times 2.55 \times 10^4}{4 \times 3.14 \times 1.26 \times 9.81} \right]^{1/3}$$

$$= \left[\frac{3 \times 12 \times 1.6 \times 2.55 \times 1000}{4 \times 3 \cdot 14 \times 1.26 \times 9 \cdot 81} \right]^{1/3} \times 10^{-7} \text{ m}$$

$$= 9.81 \times 10^{-7} \text{ m} = 9.81 \times 10^{-4} \text{ mm}$$

- Q. 14. A particle of mass m and charge $(-q)$ enters the region between the two charged plates initially moving along X-axis with speed v_x as shown in fig. The length of plate is L and an uniform electric field E is maintained between the plates. Show that the vertical deflection of the particle at the far edge of the plate is $\frac{qEL^2}{2mv_x^2}$. [HOTS]



- Ans. Force on particle towards upper plate B, $F_y = qE$ vertical acceleration of particle, $a_y = \frac{qE}{m}$.
Initial vertical velocity $v_y = 0$
Speed of particle along X-axis $= v_x$ (constant)

Time taken by particle between the plates, $t = \frac{L}{v_x}$

From relation $s = ut + \frac{1}{2}at^2$ vertical deflection $y = 0 + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2} \left(\frac{qE}{m} \right) \left(\frac{L}{v_x} \right)^2$

$$\Rightarrow y = \frac{qEL^2}{2mv_x^2}$$

- Q. 15. Suppose that the particle in above question is an electron projected with velocity $v_x = 2.0 \times 10^6$ m/s. If electric field between the plates separated by 0.5 cm is 9.1×10^2 N/C, where will the electron strike the upper plate? ($|e| = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg.) [HOTS]

Ans. Vertical deflection for distance x along X-axis is

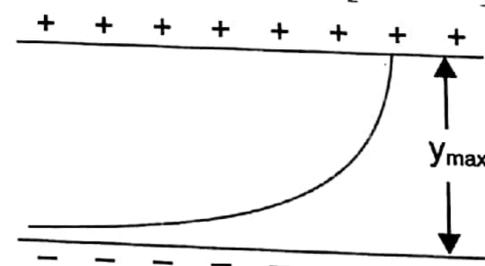
$$y = \frac{qEx^2}{2mv_x^2} \Rightarrow x = \sqrt{\frac{2my}{qE}} v_x$$

Given $m = 9.1 \times 10^{-31}$ kg, $y = 0.5 \text{ cm} = 0.5 \times 10^{-2} \text{ m}$,

$v_x = 2.0 \times 10^6 \text{ ms}^{-1}$, $q = |e| = 1.6 \times 10^{-19} \text{ C}$, $E = 9.1 \times 10^2 \text{ N/C}$.

$$\therefore x = \sqrt{\frac{2 \times 9.1 \times 10^{-31} \times 0.5 \times 10^{-2}}{1.6 \times 10^{-19} \times 9.1 \times 10^2}} \times 2.0 \times 10^6 \text{ m}$$

$$= \sqrt{\frac{1}{1.6}} \times 10^{-8} \times 2.0 \times 10^6 \approx 0.8 \times 2 \times 10^{-2} \text{ m} = 1.6 \times 10^{-2} \text{ m} = 1.6 \text{ cm}$$



Electric Flux

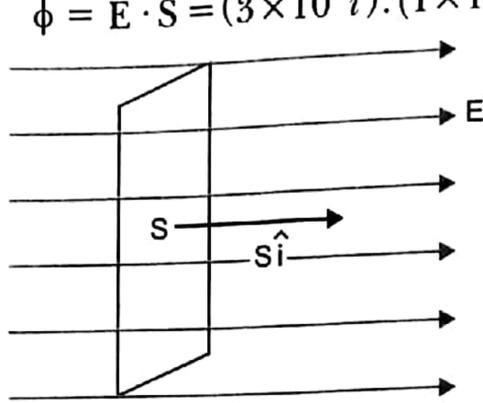
- Q. 16. Consider a uniform electric field $\vec{E} = 3 \times 10^3 \hat{i} \text{ NC}^{-1}$. (a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the yz plane? (b) What is the flux through the same square if the normal to its plane makes a 60° angle with the x -axis?

Ans. Given electric field $\vec{E} = 3 \times 10^3 \hat{i} \text{ NC}^{-1}$,

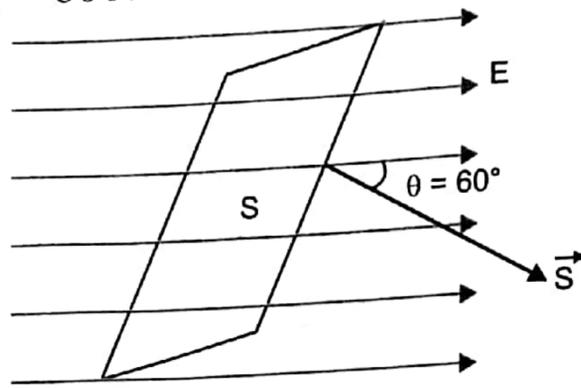
Magnitude of area, $S = 10 \text{ cm} \times 10 \text{ cm} = 0.10 \text{ m} \times 0.10 \text{ m} = 1 \times 10^{-2} \text{ m}^2$

(a) When plane is parallel to YZ plane, the normal to plane is along X-axis.

$$\therefore \phi = \vec{E} \cdot \vec{S} = (3 \times 10^3 \hat{i}) \cdot (1 \times 10^{-2} \hat{i}) = 30 \text{ Nm}^2 \text{ C}^{-1}$$



(a)



(b)

(b) In this case $\theta = 60^\circ$, so electric flux, $\phi = ES \cos \theta$

$$= 3 \times 10^3 \times 1 \times 10^{-2} \cos 60^\circ = 30 \times \frac{1}{2} = 15 \text{ Nm}^2 \text{ C}^{-1}$$

Q. 17. What is the net flux of the uniform electric field $\vec{E} = 3 \times 10^3 \hat{i}$ N/C through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes?

Ans. Electric field is along positive X-axis. The flux through two faces [1 and 2] Y-Z plane is zero.

For face 1, flux = $ES \cos 180^\circ = -ES$

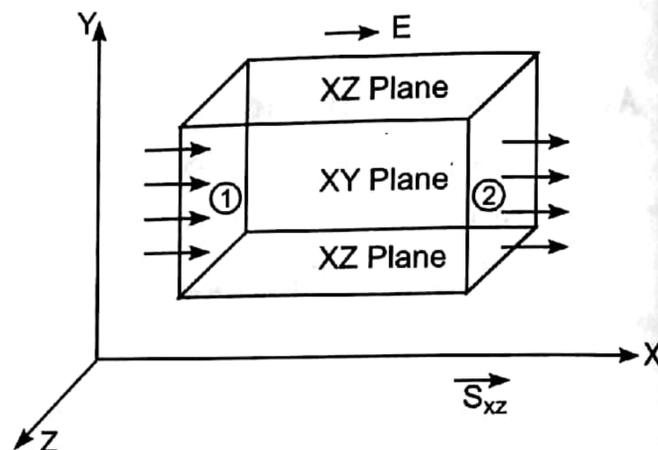
For face 2, flux = $ES \cos 0^\circ = ES$

Net flux through faces 1 and 2 = $ES - ES = 0$

The electric flux through faces in XZ plane is zero because $\vec{E} \cdot \vec{S} = ES_{xz} \cos 90^\circ = 0$.

The electric flux through faces in XY plane is zero because $\vec{E} \cdot \vec{S}_{xy} = ES_{xy} \cos 90^\circ = 0$.

\therefore Net electric flux through cube is zero.



Q. 18. Careful measurement of the electric field at the surface of a black box indicate that the net outward flux through the surface of the box is $8.0 \times 10^3 \text{ Nm}^2/\text{C}$.

(a) What is the net charge inside the box?

(b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or Why not?

Ans. (a) Given electric flux $\phi = 8.0 \times 10^3 \text{ Nm}^2 \text{ C}^{-1}$

$$\text{From Gauss's theorem } \phi = \frac{1}{\epsilon_0} \cdot q$$

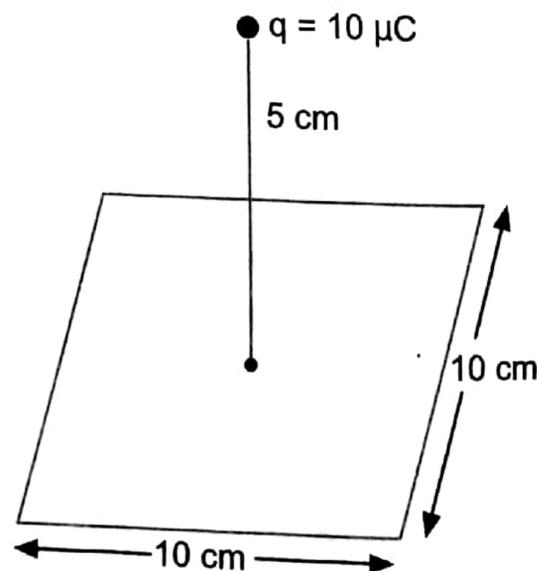
$$\therefore \text{Charge enclosed, } q = \epsilon_0 \phi = 8.85 \times 10^{-12} \times 8.0 \times 10^3 = 70.8 \times 10^{-9} \text{ C} = 70.8 \text{ nC}$$

(b) If the net outward flux is zero, it indicates that the net charge enclosed in the blackbox is zero. The conclusion is either (i) there is no charge inside the box or (ii) there may be different types of charges in the box such that the algebraic sum of charges inside the box is zero.

Q. 19. A point charge $+ 10 \mu\text{C}$ is at a distance 5 cm directly above the centre of a square of side 10 cm as shown in figure. What is the magnitude of the electric flux through the square? [Hint: Think of the square as one face of a cube with edge 10 cm] [HOTS]

Ans. Obviously the given square ABCD of side 10 cm is one face of a cube of side 10 cm. At the centre of this cube a charge $+ q = 10 \mu\text{C}$ is placed.

According to Gauss's theorem, the total electric flux through the six faces of cube = $\frac{q}{\epsilon_0}$.



Q. 20. A

Ans. C

Q. 21. A

Ans.

Q. 22. A

Ans.

Q. 23.

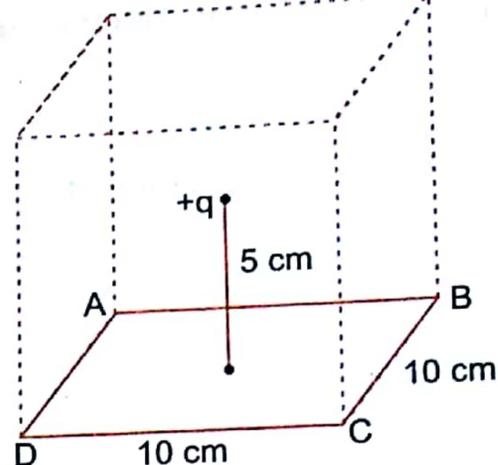
Ans.

Total electric flux through square

$$= \frac{1}{6} \frac{q}{\epsilon_0}$$

$$= \frac{1}{6} \times \frac{10 \times 10^{-6}}{8.85 \times 10^{-12}}$$

$$= 1.88 \times 10^5 \text{ Nm}^2 \text{ C}^{-1}.$$



Q. 20. A point charge of $2.0 \mu\text{C}$ is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface?

Ans. Given $q = 2.0 \mu\text{C} = 2.0 \times 10^{-6} \text{ C}$
Net electric flux through the cubical surface

$$\phi_E = \frac{q}{\epsilon_0} = \frac{2.0 \times 10^{-6}}{8.85 \times 10^{-12}} = 2.26 \times 10^5 \text{ Nm}^2 \text{ C}^{-1}$$

Q. 21. A point charge causes an electric flux of $-1.0 \times 10^3 \text{ Nm}^2 \text{ C}^{-1}$ to pass through a spherical surface of 10.0 cm radius centred on the charge.

(a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface?

(b) What is the value of the point charge?

Ans. (a) The electric flux through a surface depends only on the charge enclosed by the surface. If the radius of the spherical surface is doubled, the charge enclosed remains the same, so the electric flux passing through the surface will remain unchanged.

(b) If q is the point charge, then by Gauss theorem, the electric flux $\phi_E = \frac{q}{\epsilon_0}$

$$\Rightarrow q = \epsilon_0 \phi_E = 8.85 \times 10^{-12} \times (-1.0 \times 10^3) = -8.85 \times 10^{-9} \text{ C}$$

Q. 22. A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of $80.0 \mu\text{C}/\text{m}^2$ (a) Find the charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?

Ans. (a) Radius of sphere $r = \frac{\text{Diameter}}{2} = \frac{2.4}{2} \text{ m} = 1.2 \text{ m}$

$$\text{Surface charge density } \sigma = 80.0 \mu\text{C}/\text{m}^2 = 80.0 \times 10^{-6} \text{ C}/\text{m}^2$$

$$\text{Charge on sphere } Q = \sigma \times 4\pi r^2$$

$$= 80.0 \times 10^{-6} \times 4 \times 3.14 \times (1.2)^2 = 1.45 \times 10^{-3} \text{ C}$$

(b) Total electric flux leaving the surface of the sphere

$$\phi_E = \frac{q}{\epsilon_0} = \frac{1.45 \times 10^{-3}}{8.85 \times 10^{-12}} = 1.6 \times 10^8 \text{ Nm}^2 \text{ C}^{-1}$$

Q. 23. Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude $17.0 \times 10^{-22} \text{ C}/\text{m}^2$. What is electric field strength E : (a) in the outer region of the first plate, (b) in the outer region of the second plate, and (c) between the plates?

Ans. The electric field due to each surface charge $= \frac{\sigma}{2\epsilon_0}$

$$\text{Given } \sigma = 17.0 \times 10^{-22} \text{ C}/\text{m}^2$$

(a) The electric field in the outer region of first plate (point P).

$$= E_2 - E_1 = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$

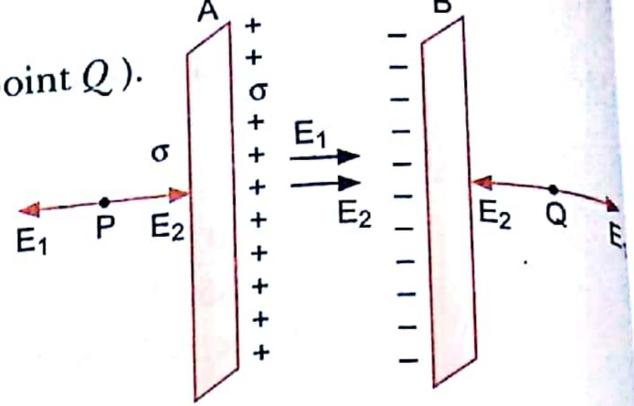
(b) The electric field in the outer origin of second plate (point Q).

$$= E_1 - E_2 = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$

(c) The electric field between the plates

$$E = E_1 + E_2 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$$

$$= \frac{\sigma}{\epsilon_0} = \frac{17.0 \times 10^{-22}}{8.85 \times 10^{-12}} = 2 \times 10^{-11} \text{ N/C}$$



IMPORTANT QUESTIONS

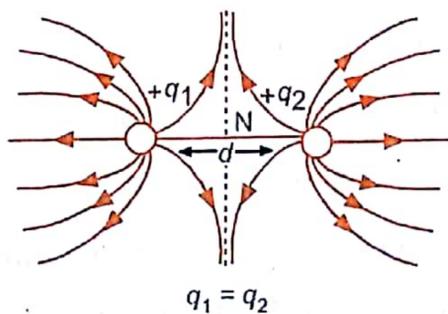
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Very Short Answer Questions

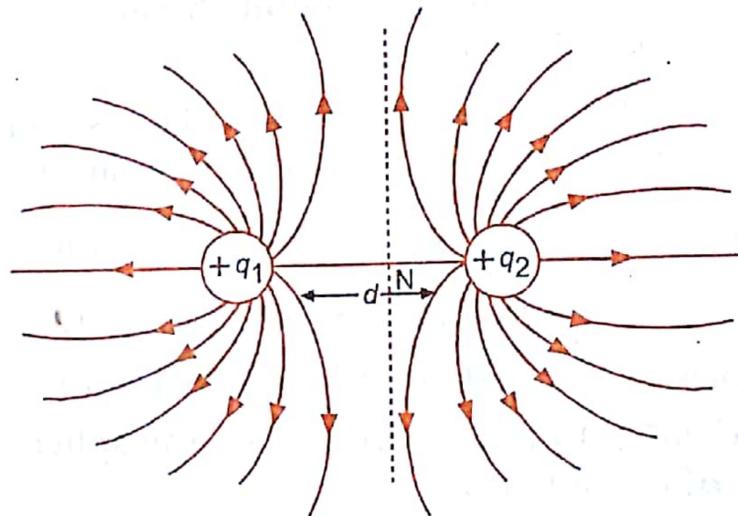
PREVIOUS YEARS' QUESTIONS

Q. 1. Sketch the electric field lines for two point charges q_1 and q_2 for $q_1 = q_2$ and $q_1 > q_2$ separated by a distance d . [CBSE Chennai 2015]

Ans. When the charges are equal, the neutral point N lies at the centre of the line joining the charges. However, when the charges are unequal, the point N is closer to the smaller charge.



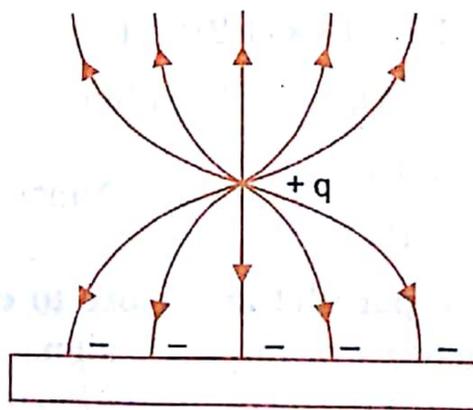
$q_1 = q_2$



$q_1 > q_2$

Q. 2 A positive point charge $+q$ is kept in the vicinity of an uncharged conducting plate. Sketch electric field lines originating from the point on the surface of the plate. [CBSE (AI) 2009]

Ans.



Q. 3. Why do the electrostatic field lines not form closed loops? [CBSE (AI) 2014, Allahabad 2015]

Ans. Electric field lines start from positive charge and terminate at negative charge. If there is a single positive charge, the field lines start from the charge and terminate at infinity. So, the electric field lines do not form closed loops.

Q. 4. Does the charge given to a metallic sphere depend on whether it is hollow or solid? Give reason for your answer.

Ans. No, Reason: This is because the charge resides only on the surface of the conductor. [CBSE Delhi 2017]

Q. 5. Two insulated charged copper spheres A and B of identical size have charges q_A and $-3q_A$ respectively. When they are brought in contact with each other and then separated, what are the new charges on them? [CBSE (F) 2011]

Ans. Charge on each = $\frac{q_A - 3q_A}{2} = -q_A$

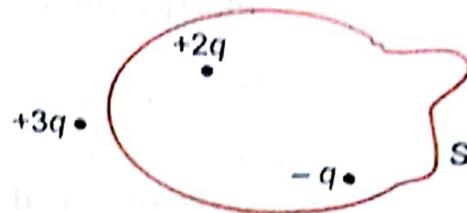
Q. 6. Two insulated charged copper spheres A and B of identical size have charges q_A and q_B respectively. A third sphere C of the same size but uncharged is brought in contact with the first and then in contact with the second and finally removed from both. What are the new charges on A and B ? [CBSE (F) 2011]

Ans. New charge on A is $\frac{q_A}{2}$ and new charge on B is $\frac{q_A + 2q_B}{4}$.

Q. 7. Fig. shows three point charges $+2q$, $-q$ and $+3q$. The charges $+2q$ and $-q$ are enclosed within a surface 'S'. What is the electric flux due to this configuration through the surface 'S'? [CBSE Delhi 2010]

Ans. Electric flux = $\frac{1}{\epsilon_0} \times$ (Net charge enclosed within the surface)

$$= \frac{1}{\epsilon_0} (2q - q) = \frac{1}{\epsilon_0} q$$



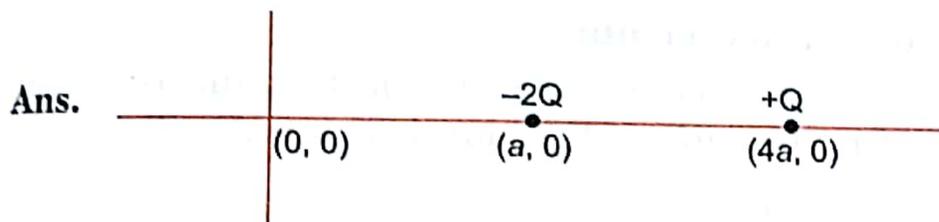
Q. 8. What is the electric flux through a cube of side 1 cm which encloses an electric dipole? [CBSE Delhi 2015]

Ans. Net electric flux is zero.

Reason : (i) Independent to the shape and size.

(ii) Net charge of the electric dipole is zero.

Q. 9. Two charges of magnitudes $-2Q$ and $+Q$ are located at points $(a, 0)$ and $(4a, 0)$ respectively. What is the electric flux due to these charges through a sphere of radius ' $3a$ ' with its centre at the origin? [CBSE (AI) 2013]



Electric flux, $\phi = \frac{-2Q}{\epsilon_0}$

Concept: Imagine a sphere of radius $3a$ about the origin and observe that only charge $-2Q$ is inside the sphere.

Q. 10. How does the electric flux due to a point charge enclosed by a spherical Gaussian surface get affected when its radius is increased? [CBSE Delhi 2016]

Ans. Electric flux through a Gaussian surface, enclosing the charge q is $\phi_E = \frac{q}{\epsilon_0}$

This is independent of radius of Gaussian surface, so if radius is increased, the electric flux through the surface will remain unchanged.

Q. 11. A charge $Q \mu\text{C}$ is placed at the centre of a cube. What would be the flux through one face? [CBSE (F) 2010, (AI) 2012]

Ans. Electric flux through whole cube = $\frac{Q}{\epsilon_0}$. Electric flux through one face = $\frac{1}{6} \frac{Q}{\epsilon_0} \mu \text{ Vm}$.

(1 mark)

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