14.24 **Statement -1 :** The equation of the curve through the point (1, 0) which satisfies the differential equation $(1 + y^2)\,dx - xy\,dy = 0$ is $x^2 - y^2 = 1$.

Statement -2 : D.E. $\frac{dy}{dx} = f(x) \cdot g(y)$ can be solved by seperating variables. $\frac{dy}{g(y)} = f(x)\,dx$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

14.25 **Statement -1 :** The D.E. of all circles in a plane must be of order 3.
Statement -2 : There is only one circle passing through three non-collinear points.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

14.26 **Statement -1 :** The differential equation $y^3\,dy + (x + y^2)\,dx = 0$ becomes homogeneous if we put $y^2 = t$

Statement -2 : All differential equation of first order first degree becomes homogeneous if we put $y = tx$
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

14.27 **Statement -1 :** Order of differential equation represents number of arbitrary constants in the general solution.
Statement -2 : Degree of differential equation represents number of family of curves.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension - I

Differential equation $\frac{dy}{dx} = f(x)\,g(y)$ can be solved by seperating variable $\frac{dy}{g(y)} = f(x)\,dx$

14.28 The equation of the curve to the point (1, 0) which satisfies the differential equation $(1 + y^2)\,dx - xy\,dy = 0$ is
(A) $x^2 + y^2 = 1$ (B) $x^2 - y^2 = 1$ (C) $x^2 + y^2 = 2$ (D) $x^2 - y^2 = 2$

14.29 Solution of the differential equation $\frac{dy}{dx} + \frac{1+y^2}{\sqrt{1-x^2}} = 0$ is
(A) $\tan^{-1} y + \sin^{-1} x = c$ (B) $\tan^{-1} x + \sin^{-1} y = c$
(C) $\tan^{-1} y \cdot \sin^{-1} x = c$ (D) $\tan^{-1} y - \sin^{-1} x = c$

14.30 If $\frac{dy}{dx} = 1 + x + y + xy$ and $y(-1) = 0$, then $y =$

(A) $e^{\frac{(1-x)^2}{2}}$ (B) $e^{\frac{(1+x)^2}{2}} - 1$ (C) $n(1 + x) - 1$ (D) $1 + x$

Comprehension # II

A differential equation of the form $\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0$ is called linear differential equation where a_1 and a_2 are function of x only. In case a_1 and a_2 are constants, the solution of the linear differential equation can be easily written by following facts –

(i) y = 0 is a solution of differential equation.

(ii) If y = f(x) is a solution then y = c f(x) is also a solution.

(iii) If $y = f_1(x)$ & $y = f_2(x)$ are two solution then $y = f_1(x) + f_2(x)$ will also be a solution

(iv) If distinct roots of quadratic equation $m^2 + a_1 m + a_2 = 0$ are m_1 and m_2 (real or imaginary) then solution of differential equation is $y = c_1e^{m_1x} + c_2e^{m_2x}$

In case roots are complex the solution can be transformed to the form $e^{ax}(c_1 \cos bx + c_2 \sin bx)$ by using Euler's theorem.

(v) In case roots of $m^2 + a_1 m + a_2 = 0$ are equal (say m_1) the differential equation can be make linear by putting $\frac{dy}{dx} - m_1 y = v$

The linear differential equation $\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2y = f(x)$ can also be satisfied by some other functions which are not of the above type such functions are called particular integrals

14.31 $y = e^x$ is a solution of differential equation $\frac{d^2y}{dx^2} - y = 0$, then which of the following is not a solution.

(A) e^{-x} (B) e–x (C) $ae^x + be^{-x}$ (D) $e^x + c$

14.32 Which of the following is solution of the equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$

(A) $c_1 + c_2 x$ (B) $(c_1 + c_2 x)e^{3x}$ (C) $c_1\cos 3x + c_2 \sin 3x$ (D) none of these

14.33 A particular integral solution of the equation $\frac{d^2y}{dx^2} + a^2y + \frac{\cos ax}{a} = 0$ is

(A) $c_1 \cos(ax) + c_2 \sin(ax)$ (B) $\frac{x\sin ax}{2a^2} + \frac{x\cos ax}{2a^2}$

(C) $\frac{x\sin ax}{2a^2} + \frac{\cos ax}{2a^2}$ (D) $\frac{-x\sin ax}{2a^2} - \frac{\cos ax}{2a^2}$

SECTION - V : MATRIX - MATCH TYPE

14.34 Match the following

Column - I	Column - II
(A) Solution of $y - \frac{xdy}{dx} = y^2 + \frac{dy}{dx}$ is	(p) $xy^2 = 2y^5 + c$
(B) Solution of $(2x - 10y^3)\frac{dy}{dx} + y = 0$ is	(q) $\sec y = x + 1 + ce^x$
(C) Solution of $\sec^2 y\, dy + \tan y\, dx = dx$ is	(r) $(x + 1)(1 - y) = cy$
(D) Solution of $\sin y \frac{dy}{dx} = \cos y(1 - x\cos y)$ is	(s) $xy^2 = 2y^5 + x + c$
	(t) $\tan y = 1 + ce^{-x}$

14.35 Match the following

Column - I		Column - II	
(A)	$xdy = y(dx + ydy)$, $y > 0$ $y(1) = 1$ and $y(x_0) = -3$, then $x_0 =$	(p)	$\frac{1}{4}$
(B)	If $y(t)$ is solution of $(t+1)\frac{dy}{dt} - ty = 1$, $y(0) = -1$, then $y(1) =$	(q)	-15
(C)	$(x^2 + y^2)\,dx = xydx$ and $y(1) = 1$ and $y(x_0) = e$, then $x_0 =$	(r)	$-\frac{1}{2}$
(D)	$\frac{dy}{dx} + \frac{2y}{x} = 0$, $y(1) = 1$, then $y(2) =$	(s)	16
		(t)	$\sqrt{3}\,e$

14.36 Find the solution of the following differential equation

Column – I		Column – II	
(A)	$(\sin x + \cos x)\,dy + (\cos x - \sin x)\,dx = 0$	(p)	$\sec y + \tan y = c(\operatorname{cosec} x + \cot x)$
(B)	$\sin x\, dy + \cos y\, dx = 0$	(q)	$2(x^2 + y^2) + 2(x \sin 2x + y \sin 2y) + (\cos 2x + \cos 2y) = c$
(C)	$x^{-1}\cos^2 y\, dy + y^{-1}\cos^2 x\, dx = 0$	(r)	$y = c - \log\lvert\sin x + \cos x\rvert$
(D)	$\tan x \sec^2 y\, dy + \tan y \sec^2 x\, dx = dx$	(s)	$\cot y + \sec y = c(\sin x + \tan x)$
		(t)	$\tan x \tan y = x + c$

SECTION - VI : INTEGER TYPE

14.37 A & B are two separate reservoirs of water. Capacity of reservoir A is double the capacity of reservoir B. Both the reservoirs are filled completely with water, their inlets are closed and then the water is released simultaneously from both the reservoirs. The rate of flow of water out of each reservoir at any instant of time is proportional to the quantity of water in the reservoir at that time. One hour after the water is released, the quantity of water in reservoir A is 1.5 times the quantity of water in reservoir B. After $(\lambda \log_{4/3} 2)$ hours both the reservoirs have the same quantity of water, then find the value of λ

14.38 Let the curve $y = f(x)$ passes through $(4, -2)$ satisfy the differential equation,

$$y(x + y^3)\,dx = x(y^3 - x)\,dy \ \&\ y = g(x) = \int_{1/8}^{\sin^2 x} \sin^{-1}\sqrt{t}\,dt + \int_{1/8}^{\cos^2 x} \cos^{-1}\sqrt{t}\,dt,\ 0 \le x \le \frac{\pi}{2}.$$ The area of the region bounded by curves, $y = f(x), y = g(x)$ and $x = 0$ is $\frac{\lambda}{8}\left(\frac{3\pi}{16}\right)^4$, then find the value of λ.

14.39 By eliminating the constant in the following equation $x^2 - y^2 = c(x^2 + y^2)^2$ its differential equation is

$$y' = \frac{x(\lambda y^2 - x^2)}{y(\lambda x^2 - y^2)},$$ then find the value of λ.

14.40 The differential equation, $(x^2 + 4y^2 + 4xy)\,dy = (2x + 4y + 1)\,dx$ has solution

$$y = \ell n\left((x + 2y)^2 + \lambda(x + 2y) + 2\right) - \frac{3}{2\sqrt{2}}\,\ell n\left|\frac{x + 2y + 2 - \sqrt{2}}{x + 2y + 2 + \sqrt{2}}\right| + c,$$ then find the value of λ.

14.41 The curve passing through the origin if the middle point of the segment of its normal from any point of the curve to the x-axis lies on the parabola $2y^2 = x$ is $y^2 = \lambda x + 1 - e^{2x}$, then find the value of λ.

TOPIC **15**

BASICS

SECTION - I : STRAIGHT OBJECTIVE TYPE

15.1 The product of all the solutions of the equation $(x-2)^2 - 3|x-2| + 2 = 0$ is
(A) 2 (B) – 4 (C) 0 (D) none of these

15.2 The number of solutions of the equation $\log(-2x) = 2\log(x+1)$ is
(A) zero (B) 1 (C) 2 (D) none of these

15.3 Greatest integer less than or equal to the number $\log_2 15 . \log_{1/6} 2 . \log_3 1/6$ is
(A) 4 (B) 3 (C) 2 (D) 1

15.4 The number of solutions of $|[x] - 2x| = 4$ is (where [.] denotes greatest integer function)
(A) 2 (B) 4 (C) 3 (D) Infinite

15.5 The solution set of the inequation $1 + \log_{\frac{1}{3}}(x^2 + x + 1) > 0$ is
(A) $(-\infty, -2) \cup (1, \infty)$ (B) $[-1, 2]$ (C) $(-2, 1)$ (D) $(-\infty, \infty)$

15.6 Number of values of x satisfying the equations $5\{x\} = x + [x]$ and $[x] - \{x\} = \frac{1}{2}$ is
(A) 1 (B) 2 (C) 3 (D) 4

15.7 If $|x^2 - 9| + |x^2 - 4| = 5$, then the set of values of x is
(A) $(-\infty, -3) \cup (3, \infty)$ (B) $(-\infty, -2) \cup (3, \infty)$ (C) $(-\infty, 3)$ (D) $[-3, -2] \cup [2, 3]$

15.8 If $\frac{|x+2| - x}{x} < 2$, then the set of values of x is
(A) $(-\infty, 1) \cup (2, \infty)$ (B) $(-\infty, 0) \cup (1, \infty)$ (C) $(-\infty, -1) \cup (0, \infty)$ (D) None of these

15.9 Solution set of the inequality $\log_e^2 [2x] - \log_e [2x] \le 0$ is
(A) $[1, 3)$ (B) $(0, 3)$ (C) $\{1, 2\}$ (D) $\left[\frac{1}{2}, \frac{3}{2}\right)$

15.10 Solution set of $|x^2 - 5x + 7| + |x^2 - 5x - 14| = 21$ is
(A) $[-2, 7]$ (B) $(-\infty, -2] \cup [7, \infty)$ (C) $[7, \infty)$ (D) $(-\infty, -2]$

15.11 The set of real value(s) of p for which the equation $|2x + 3| + |2x - 3| = px + 6$ has exactly two solutions is
(A) $[0, 4)$ (B) $(-4, 4) - \{0\}$ (C) $R - \{4, -4, 0\}$ (D) $\{0\}$

15.12. $e^{e^{\ell n\, \ell n 3}}$ is simplified to
(A) e^3 (B) $\ell n\, 3$ (C) 3 (D) $\ell n(\ell n\, 3)$

15.13 $N = \frac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409}\left((\sqrt{7})^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6}\right)$, then $\log_2 N$ has the value
(A) 0 (B) 1 (C) –1 (D) None of these

15.14 S_1 : $3^{\sqrt{\log_3 7}} = 7^{\sqrt{\log_7 3}}$

S_2 : Number of solution to the equation on $x^{\log_{10} 2x} = 5$ is 2

S_3 : Solutiuon set of the inequality $\left(\frac{1}{3}\right)^{\log_{1/9}\left(x^2-\frac{10}{3}x+1\right)} \le 1$, is $x \le \frac{10}{3}$

S_4 : Solution set of $\frac{(x-2)}{(x-4)} \le 0$ is $x \in [2, 4]$

S_1 : $3^{\sqrt{\log_3 7}} = 7^{\sqrt{\log_7 3}}$

(A) TFTF (B) TTFF (C) T T F T (D) F T F T

15.15 Consider the following statements for real values of x :

S_1 : $x^2 + |x| + 1 = 0$ has exactly 2 solutions.

S_2 : $x^2 - 5|x| + 6 = 0$ has exactly 4 solution.

S_3 : $x^2 - |x| - 2 = 0$ has exactly 2 solutions.

S_4 : $x^2 - |x| = 0$ has 3 distinct solutions.

State, in order, whether S_1, S_2, S_3, S_4 are true or false

(A) T F T T (B) F T T T (C) F T F T (D) T T F T

15.16 Consider the following statements :

S_1 : $x = \sqrt{\log_{11} 7}$ and $y = \sqrt{\log_7 11}$, then $e^{y\ln 7 - x\ln 11}$ is equal to 1.

S_2 : $\log_x 3 > \log_x 2$ is true for all values of $x \in (0, 1) \cup (1, \infty)$

S_3 : $|x - 2| = [-\pi]$, then x is 6, –2

S_4 : $\log_{25}(2 + \tan^2\theta) = 0.5$, then θ may be $\frac{4\pi}{3}$ or $\frac{2\pi}{3}$

State, in order, whether S_1, S_2, S_3, S_4 are true or false

(A) T F F T (B) F F T T (C) F T F T (D) T T F T

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

15.17 If $\ln(x+z) + \ln(x - 2y + z) = 2\ln(x - z)$, then

(A) $y = \frac{2xz}{x+z}$ (B) $y^2 = xz$ (C) $2y = x + z$ (D) $\frac{x}{z} = \frac{x-y}{y-z}$

15.18 $\log_{10} 5 \cdot \log_{10} 20 + (\log_{10} 2)^2$ when simplified reduces to

(A) an odd prime number (B) an even prime

(C) a rational number (D) an integer

15.19 Which of the following when simplified reduces to unity ?

(A) $\log_3 \log_{27} \log_4 64$ (B) $2\log_{18}(\sqrt{2} + \sqrt{8})$

(C) $\log_2\sqrt{10} + \log_2\left(\frac{2}{\sqrt{5}}\right)$ (D) $-\log_{\sqrt{2}-1}(\sqrt{2} + 1)$

15.20 If $x, y, z \in R^+$ and $z \in R$ then the system $x + y + z = 2$, $2xy - z^2 = 4$

(A) is satisfied for x = 2, y = 2, z = – 2 (B) has only one real solution

(C) has only two real solution (D) has infinite solutions.

SECTION - III : ASSERTION AND REASON TYPE

15.21 **Statement-1 :** If 2, 3 & 6 are the sides of a triangle then it is an obtuse angled triangle.
Statement-2 : If $b^2 > a^2 + c^2$, where b is the greatest side, then triangle must be obtuse angled.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

15.22 **Statement-1 :** Minimum value of $|x - 2| + |x - 5| + |x + 3|$ is 8.
Statement-2 : If a < b < c, then the minimum value of $|x - a| + |x - b| + |x - c|$ is $|b - a| + |b - c|$.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

15.23 **Statement-1 :** $\log_e x > \log_e y \Rightarrow x > y,\ x, y > 0$
Statement-2 : If $\log_a x > \log_a y$, then $x > y$ ($x, y > 0$ & $a > 0$, $a \neq 1$)
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

15.24 **Statement-1 :** $[x] + [-x] = x^2 - 5x + 6$ has only two real solution.

Statement-2 : $[x] + [-x] = \begin{cases} -1, & x \notin I \\ 0, & x \in I \end{cases}$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

15.25 **Statement-1 :** If $ax^2 + \alpha x + \beta = 0$, where $\alpha, \beta \in R$, has roots a, b and $\log_a b < 0$, then $\alpha + \beta < 0$.
Statement-2 : If $ax^2 + \alpha x + \beta = 0$, where $\alpha, \beta \in R$, has roots a, b and $\log_a b < 0$, then $a + \alpha + \beta$ must be negative.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

The general procedure for solving equation containing modulus function is to split the domain into subintervals and solve the various cases. But there are certain structures of equations which can be solved by a different approach. For example, for solving the equation $|f(x)| + |g(x)| = f(x) - g(x)$ one can follow this method. First find the permissible set of values of x for the equation.
Since LHS $\geq 0 \Rightarrow f(x) - g(x) \geq 0$. Now squaring both sides, we get
$f^2 + g^2 + 2|f.g| = f^2 + g^2 - 2fg$
$\Rightarrow$ $|fg| = -fg$. The equation can hold if $f.g \leq 0$ and $f \geq g$. This can be simplified to
$f \geq 0,\ g \leq 0$.

Answer the following questions on the basis of this method

15.26 The complete solution of the equation $|x^3 - x| + |2 - x| = x^3 - 2$ is

(A) $[2, \infty)$ (B) $[-1, 0] \cup [2, \infty)$ (C) $\left[2^{\frac{1}{3}}, \infty\right)$ (D) None of these

15.27 The complete solution set of the equation $|x^2 - x| + |x + 3| = |x^2 - 2x - 3|$ is

(A) $[1, \infty)$ (B) $[-3, 0] \cup [1, \infty)$ (C) $(-\infty, -3]$ (D) $(-\infty, -3] \cup [0, 1]$

15.28 All the condition(s) for which $|f(x) - g(x)| = |f(x)| + |g(x)|$ is true, is

(A) $f(x) \ge 0, g(x) \le 0$ (B) $f(x) \le 0, g(x) \ge 0$ (C) $f(x) . g(x) \le 0$ (D) $f(x) . g(x) = 0$

Comprehension # 2

Let $a_1 < a_2 < a_3 < .... < a_n$, n is an odd natural number and $m, k \in N$

Consider the equation

$$|x - a_1| + |x - a_2| + |x - a_3| + ... + |x - a_n| = kx + d.$$

Case-1 When $2m - n = k$ for some $m < n$, then the equation has

(i) no solution if $(a_{m+1} + a_{m+2} + ... + a_n) - (a_1 + a_2 + ... + a_m) > d$

(ii) infinite solutions if $(a_{m+1} + a_{m+2} + ... + a_n) - (a_1 + a_2 + ... + a_m) = d$

(iii) two solutions if $(a_{m+1} + a_{m+2} + ... + a_n) - (a_1 + a_2 + ... + a_m) < d$

Case-2 Let when $2m - n \ne k$ for any $m < n$

Two cases arise

(a) If $|k| > n$, then there is one solution.

(b) If $|k| < n$, then there is m such that $2m - (n - 1) = k$.

(i) If $(a_{m+2} + a_{m+3} + ... + a_n) - (a_1 + a_2 + ... + a_m) > d$ no solution

(ii) If $(a_{m+2} + a_{m+3} + ... + a_n) - (a_1 + a_2 + ... + a_m) = d$ one solution

(iii) If $(a_{m+2} + a_{m+3} + ... + a_n) - (a_1 + a_2 + ... + a_m) < d$ two solutions

15.29 Number of solutions of $|x - 1| + |x - 3| + |x - 4| + |x - 7| + |x - 9| = 3x + 5$ is

(A) 0 (B) 1 (C) 2 (D) infinite

15.30 Number of solutions of $|x - 1| + |x - 3| + |x - 4| + |x - 7| + |x - 10| = 2x + 1$ is

(A) 0 (B) 1 (C) 2 (D) None of these

15.31 Number of solutions of $|x - 1| + |x - 2| + |x - 4| + |x - 6| + |x - 7| = 2x + 10$ is

(A) 0 (B) 1 (C) 2 (D) None of these

SECTION - V : MATRIX - MATCH TYPE

15.32 Match the column :

Column – I	Column – II
(A) Set of all values of x satisfying the inequation $\frac{5x+1}{(x+1)^2} < 1$ is	(p) $(-\infty, 0) \cup (0, 2) \cup (2, \infty)$
(B) Set of all values of x satisfying the inequation $\|x\| + \|x-3\| > 3$ is	(q) $(-\infty, -5) \cup (-3, 3) \cup (5, \infty)$
(C) Set of all values of x satisfying the inequation $\frac{1}{\|x\|-3} < \frac{1}{2}$ is	(r) $(-\infty, -1) \cup (-1, 0) \cup (3, \infty)$
(D) Set of all values of x satisfying the inequation $\frac{x^4}{(x-2)^2} > 0$ is	(s) $(0, 3) \cup (4, \infty)$
	(t) $(-\infty, 0) \cup (3, \infty)$

15.33 Match the column :

Column – I	Column – II
(A) If $\log_{\sin x}(\log_3(\log_{0.2} x)) < 0$, then	(p) $x \in [-1, 1]$
(B) If $\frac{(e^x-1)(2x-3)(x^2+x+2)}{(\sin x - 2)\,x(x+1)} \le 0$, then	(q) $x \in [-3, 6)$
(C) If $\|2 - \|[x]-1\|\| \le 2$, then (where [.] represents greatest integer function).	(r) $x \in \left(0, \frac{1}{125}\right)$
(D) If $\|\sin^{-1}(3x - 4x^3)\| \le \frac{\pi}{2}$, then	(s) $x \in (1, \infty)$
	(t) $x \in (-\infty, -1) \cup \left[\frac{3}{2}, \infty\right)$

15.34 Match the column :

Column – I	Column – II
(A) $x\|x\| =$	(p) $\begin{cases} -2x & : x < -1 \\ 2 & : -1 \le x \le 1 \\ 2x & : x \ge 1 \end{cases}$
(B) $\|x-1\| + \|x+1\| =$	(q) $\begin{cases} -x^2 & : x \le 0 \\ x^2 & : x > 0 \end{cases}$
(C) If $-1 \le x < 2$, then $2x - \{x\} =$	(r) $\begin{cases} -x & : -1 \le x < 0 \\ 0 & : 0 \le x < 1 \\ x & : 1 \le x < 2 \end{cases}$
(D) If $-1 \le x < 2$, then $x[x] =$	(s) $\begin{cases} x-1 & : -1 \le x < 0 \\ x+1 & : 0 \le x < 2 \end{cases}$
	(t) $\begin{cases} x-1 & : -1 \le x < 0 \\ x & : 0 \le x < 1 \\ x+1 & : 1 \le x < 2 \end{cases}$

15.35 **Match the column :**

Column-I	Column-II
(A) Interval containing all the solutions of the inequality $3 - x > 3\sqrt{1-x^2}$ is	(p) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(B) Interval containing all the solutions of the inequality $\left(\frac{1}{3}\right)^{\sqrt{x+2}} < 3^{-x}$ is	(q) (π, π^2)
(C) Interval containing all the solutions of the inequality $\log_5(x-3) + \frac{1}{2}\log_5 3 < \frac{1}{2}\log_5(2x^2 - 6x + 7)$ is	(r) $(-\pi, \pi)$
(D) Interval containing all the solutions of the equation $7^{x+2} - \frac{1}{7}.7^{x+1} - 14.7^{x-1} + 2.7^{x} = 48$ is	(s) $(-e, e)$
	(t) $([\pi], -[-\pi^2])$, where [.] is G.I.F.

SECTION - VI : INTEGER TYPE

15.36 The inequality $\log_{x^2}|x-1| > 0$ is not defined for some integral values of x, find the sum of their magnitudes.

15.37 If set of all real values of x satisfying $|x^2 - 3x - 1| < |3x^2 + 2x + 1| + |2x^2 + 5x + 2|$, $x^2 - 3x - 1 \neq 0$ is $(-\infty, -a) \cup (-b, \infty)$, then find the value of $a + \log ab$.

15.38 Let $f(x) = \begin{cases} 1 & , -2 \le x \le -1 \\ x+2 & , -1 < x < 1 \\ 4-x & , 1 \le x \le 2 \end{cases}$

Find number of solutions of $\{f(x)\} = \frac{1}{2}$ (where {.} denotes fractional part function)

15.39 Find the number of integral solution of the equation $\sqrt{\left[x+\left[\frac{x}{2}\right]\right]}+\left[\sqrt{\{x\}}+\left[\frac{x}{3}\right]\right]=3$. (where [ ] denotes greatest integer function)

TOPIC 16 QUADRATIC EQUATION

SECTION - I : STRAIGHT OBJECTIVE TYPE

16.1 Consider two quadratic expressions $f(x) = ax^2 + bx + c$ and $g(x) = ax^2 + px + q$, $(a, b, c, p, q \in R, b \neq p)$ such that their discriminants are equal. If $f(x) = g(x)$ has a root $x = \alpha$, then
(A) α will be A.M. of the roots of $f(x) = 0$ and $g(x) = 0$
(B) α will be A.M. of the roots of $f(x) = 0$
(C) α will be A.M. of the roots of $f(x) = 0$ or $g(x) = 0$
(D) α will be A.M. of the roots of $g(x) = 0$

16.2 If the graph of $|y| = f(x)$, where $f(x) = ax^2 + bx + c$, $b, c \in R$, $a \neq 0$, has the maximum vertical height 4, then
(A) $a > 0$ (B) $a < 0$
(C) $(b^2 - 4ac)$ is negative (D) Nothing can be said

16.3 If for all real values of 'a' one root of the equation $x^2 - 3ax + f(a) = 0$ is double of the other, then f(x) is equal to
(A) $2x$ (B) x^2 (C) $2x^2$ (D) $2\sqrt{x}$

16.4 Set of all possible real values of 'a' such that the inequality $(x - (a - 1))\ (x - (a^2 + 2)) < 0$ holds for all $x \in (-1, 3)$ is
(A) $(1, \infty)$ (B) $(\infty, -1]$ (C) $(-\infty, -1)$ (D) $(0, 1)$

16.5 A quadratic equation, product of whose roots x_1 and x_2 is equal to 4 and satisfying the relation
$\frac{x_1}{x_1 - 1} + \frac{x_2}{x_2 - 1} = 2$, is
(A) $x^2 - 2x + 4 = 0$ (B) $x^2 - 4x + 4 = 0$ (C) $x^2 + 2x + 4 = 0$ (D) $x^2 + 4x + 4 = 0$

16.6 A real value of 'a', for which sum of the roots of the equation $x^2 - 2ax + 2a - 1 = 0$ is equal to the sum of the square of its roots, is
(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) 2

16.7 If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is
(A) greater than 4ab (B) less than 4ab (C) greater than –4ab (D) less than –4ab

16.8 The value of the expression $x^4 - 8x^3 + 18x^2 - 8x + 2$ when $x = 2 + \sqrt{3}$ is
(A) 0 (B) 1 (C) 2 (D) 3

16.9 The quadratic expression $21 + 12x - 4x^2$ takes -
(A) the least value 5 (B) the greatest value 30
(C) the greatest value 21 (D) none of these

16.10 If α, β are the roots of the equation $ax^2 + bx + c = 0$, then the equation $ax^2 - bx(x - 1) + c(x - 1)^2 = 0$ has roots
(A) $\frac{\alpha}{1-\alpha}, \frac{\beta}{1-\beta}$ (B) $\alpha - 1, \beta - 1$ (C) $\frac{\alpha}{\alpha+1}, \frac{\beta}{\beta+1}$ (D) $\frac{1-\alpha}{\alpha}, \frac{1-\beta}{\beta}$

16.11 If $a > 2$, roots of the equation $(2 - a)x^2 + 3ax - 1 = 0$ are
(A) one positive and one negative (B) both negative
(C) both positive (D) both imaginary

16.12 If the equations $ax^2 + bx + c = 0$ and $5x^2 + 12x + 13 = 0$ have a common root, where a, b and c are the sides of a triangle ABC, then
(A) ΔABC is acute angled
(B) ΔABC is right angled
(C) ΔABC is isosceles
(D) ΔABC is right angled isosceles

16.13 If $x + y + z = 5$ and $xy + yz + zx = 3$, then least and largest value of x are
(A) $\frac{10}{3}, 5$ (B) $-1, \frac{13}{3}$ (C) $\frac{17}{3}, 7$ (D) None of these

16.14 Set of all values of x satisfying the inequality $\sqrt{x^2 - 7x + 6} > x + 2$ is
(A) $x \in \left(-\infty, \frac{2}{11}\right)$ (B) $x \in \left(\frac{2}{11}, \infty\right)$ (C) $x \in (-\infty, 1] \cup [6, \infty)$ (D) $x \in [6, \infty)$

16.15 If x_1 and x_2 are the arithmetic and harmonic mean of the roots of the equations $ax^2 + bx + c = 0$, then quadratic equation whose roots are x_1 and x_2 is
(A) $abx^2 + (b^2 + ac)x + bc = 0$
(B) $2ab x^2 + (b^2 + 4ac)x + 2bc = 0$
(C) $2abx^2 + (b^2 + ac)x + bc = 0$
(D) none of these

16.16 The number of quadratic equations which are unchanged by squaring their roots, is
(A) 2 (B) 4 (C) 6 (D) None of these

16.17 If $p, q, r, s \in R$, then equaton $(x^2 + px + 3q)(-x^2 + rx + q)(-x^2 + sx - 2q) = 0$ has
(A) 6 real roots
(B) at least two real roots
(C) 2 real and 4 imaginary roots
(D) 4 real and 2 imaginary roots

16.18 S_1 : If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + bx + c$, where $ac \neq 0$, then P(x) Q(x) has at least two real roots.
S_2 : Let S be the set of real values of 'a' for which the roots of $x^2 - 6ax + 2 - 2a + 9a^2 = 0$ exceed 3. Then $S \in (11/9, \infty)$.
S_3 : If $x^2 + ax + b$ is an integer for every odd integer x, then both a and b must be integers.
S_4 : There is no real x such that $e^{\sin x} - e^{-\sin x} - 4 = 0$.
(A) FTTT (B) TFTF (C) TTFT (D) TTTF

16.19 S_1: The roots of the equation $x^2 + px + q = 0$ are tan 22° and tan23° then $p - q = -1$
S_2: If α, β be the roots of $x^2 + x + 1 = 0$. Then the equation whose roots are α^{229} and α^{1004} is $x^2 + x + 1$
S_3: If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 - bx + c = 0$, then a, b and c satisfy the relation $a^2 = b^2 + 2ac$
S_4 : Range of $\frac{1}{1+x^2}$ is (0, 1]
(A) FTTT (B) TFTF (C) TTFT (D) TTTF

16.20 S_1 : If a, b and c are positive real numbers, then $ax^3 + bx + c = 0$ has exactly one real root
S_2 : If the derivative of an odd cubic polynomial vanishes at two different values of 'x' then coefficient of x^3 and x in the polynomial must be different in sign.
S_3 : a, b, c are real and $x^3 - 3b^2x + 2c^3$ is divisible by $x - a$ and $x - b$ if $a = 2b = 2c$
S_4 : If roots of a cubic equation are not all real, then imaginary roots must be conjugates of each other
(A) TFTF (B) FFTF (C) TFFT (D) TTFF

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

16.21 If $\frac{x^2 - x}{1 - ax}$ attains all real values ($x \in R$) then possible value of a are
(A) $(-\infty, 1)$ (B) $(1, \infty)$ (C) $[1, \infty)$ (D) (1, 2)

16.22 The values of x satisfying the equation $\left(31 + 8\sqrt{15}\right)^{x^2-3} + 1 = \left(32 + 8\sqrt{15}\right)^{x^2-3}$ is/are
(A) 3 (B) 0 (C) 2 (D) –2

16.23 If a, b, c are distinct positive real numbers such that $b(a + c) = 2ac$ and given equation is $ax^2 + 2bx + c = 0$ then

(A) roots of given equation are imaginary
(B) b is HM of a & c
(C) roots of given equation are real
(D) roots are equal

16.24 The values of x satisfying equation
$|f(x) + 6 - x^2| = |f(x)| + |4 - x^2| + 2(f(x) \geq 0)$ is/are

(A) $[-2, 2]$
(B) $[1, 2]$
(C) $[-2, 0]$
(D) $[2, 5]$

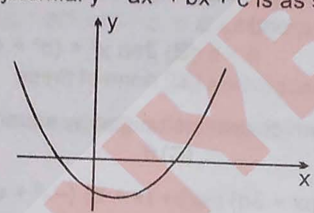
16.25 The real values of t satisfying the equation $(3^t - 9)^3 + (9^t - 3)^3 = (9^t + 3^t - 12)^3$ are

(A) -1
(B) 1
(C) 1/2
(D) 2

16.26 If $a, b \in R$ and $ax^2 + bx + 6 = 0, a \neq 0$ does not have two distinct real roots, then

(A) Minimum possible value of $3a + b$ is -2
(B) Minimum possible value of $3a + b$ is 2
(C) Minimum possible value of $6a + b$ is -1
(D) Minimum possible value of $6a + b$ is 1

16.27 The graph of the quadratic polynomial $y = ax^2 + bx + c$ is as shown in the figure . Then



(A) $b^2 - 4ac > 0$
(B) $b < 0$
(C) $a > 0$
(D) $c < 0$

16.28 If the quadratic equation $(ab - bc)x^2 + (bc - ca)x + ca - ab = 0$, $a, b, c \in R$, has both the roots equal, then

(A) both roots are equal to 0
(B) both roots are equal to 1
(C) a, c, b are in harmonic progresssion
(D) ab^2c^2, b^2a^2c, a^2c^2b are in arithmetic progression

16.29 One real solution of the equation $(x^2 + 2)^2 + 8x^2 = 6x(x^2 + 2)$ is

(A) $2 + \sqrt{2}$
(B) $2 - \sqrt{2}$
(C) $3 + \sqrt{3}$
(D) $3 - \sqrt{3}$

16.30 If the roots of the equation $x^3 + bx^2 + cx - 1 = 0$ form an increasing G.P., then

(A) $b + c = 0$
(B) $b \in (-\infty, -3)$
(C) one of the roots is 1
(D) one root is smaller than 1 and one root is more than 1.

16.31 $\dfrac{\pi^e}{x-e} + \dfrac{e^\pi}{x-\pi} + \dfrac{\pi^\pi + e^e}{x-\pi-e} = 0$ has

(A) one real root in (e, π) and other in $(\pi - e, e)$
(B) one real root in (e, π) and other in $(\pi, \pi + e)$
(C) two real roots in $(\pi - e, \pi + e)$
(D) No real roots

SECTION - III : ASSERTION AND REASON TYPE

16.32 **Statement – 1 :** The quadratic equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ have one root $x = 1$
Statement – 2 : If sum of the co-efficients in a quadratic equation vanishes then its one root is $x = 1$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

16.33 **Statement-1 :** The equation $(x - p)(x - r) + \lambda(x - q)(x - s) = 0$, $p < q < r < s$, has non real roots if $\lambda > 0$
Statement-2 : The equation $ax^2 + bx + c = 0$, $a, b, c \in R$, has non real roots if $b^2 - 4ac < 0$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

16.34 **Statement-1 :** If roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c = 1$.
Statement-2 : If a, b, c are odd integer then the roots of the equation $4abcx^2 + (b^2 - 4ac)x - b = 0$ are real and distinct.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

16.35 **Statement-1 :** If one roots is $\sqrt{5} - \sqrt{2}$ then the equation of lowest degree with rational coefficient is $x^4 - 14x^2 + 9 = 0$.
Statement-2 : For a polynomial equation with rational co-efficient irrational roots occurs in pairs
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

16.36 **Statement--1 :** The number of values of 'a' for which $(a^2 - 3a + 2)x^2 + (a^2 - 5a + 6)x + a^2 - 4 = 0$ is an identity in x, is 2.
Statement--2 : If a = b = c = 0, then equation $ax^2 + bx + c = 0$ is an identity in x.
(A) Statement 1 is true, statement 2 is true and statement 2 is correct explanation for statement 1
(B) Statement 1 is true, statement 2 is true and statement 2 is not correct explanation for statement 1
(C) Statement 1 is true, statement 2 is false
(D) Statement 1 is false, statement 2 is true

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

Let $f(x) = x^2 + b_1x + c_1$, $g(x) = x^2 + b_2x + c_2$. Real roots of f(x) = 0 be α, β and real roots of g(x) = 0 be $\alpha + \delta, \beta + \delta$. Least value of f(x) be $-\frac{1}{4}$. Least value of g(x) occurs at $x = \frac{7}{2}$

16.37 The Least value of g(x) is
(A) –1 (B) $-\frac{1}{2}$ (C) $-\frac{1}{4}$ (D) $-\frac{1}{3}$

16.38 The value of b_2 is
(A) 6 (B) –7 (C) 8 (D) 0

16.39 The roots of g(x) = 0 are
(A) 3, 4 (B) –3, 4 (C) 3, –4 (D) –3, –4

Comprehension # 2

If roots of the equation $x^4 - 12x^3 + bx^2 + cx + 81 = 0$ are positive, then

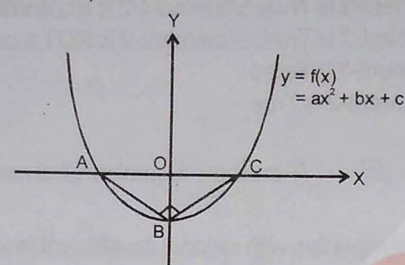
16.40 Value of b is
(A) – 54 (B) 54 (C) 27 (D) – 27

16.41 Value of c is
(A) 108 (B) – 108 (C) 54 (D) – 54

16.42 Root of equation 2bx + c = 0 is
(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 1 (D) – 1

Comprehension # 3

In the given figure vertices of $\triangle ABC$ lie on $y = f(x) = ax^2 + bx + c$. The $\triangle ABC$ is right angled isosceles triangle whose hypotaneous $AC = 4\sqrt{2}$ units, then



16.43 y = f(x) is given by

(A) $y = \frac{x^2}{2\sqrt{2}} - 2\sqrt{2}$ (B) $y = \frac{x^2}{2} - 2$ (C) $y = x^2 - 8$ (D) $y = x^2 - 2\sqrt{2}$

16.44 Minimum value of y = f(x) is

(A) $2\sqrt{2}$ (B) $-2\sqrt{2}$ (C) 2 (D) –2

16.45 Number of integral value of k for which $\frac{k}{2}$ lies between the roots of f(x) = 0, is

(A) 9 (B) 10 (C) 11 (D) 12

SECTION - V : MATRIX - MATCH TYPE

16.46 **Match the column**

Column - I	Column - II
(A) The equation $x^3 - 6x^2 + 9x + \lambda = 0$ have exactly one root in (1, 3) then $[\lambda + 1]$ is (where [.] denotes the greatest integer function)	(p) –3
(B) If $-3 < \frac{x^2 - \lambda x - 2}{x^2 + x + 1} < 2$ for all $x \in R$, then $[\lambda]$ is can be where [.] denotes the greatest integer function	(q) –2
(C) If $x^2 + \lambda x + 1 = 0$ and $(b - c)x^2 + (c - a)x + (a - b) = 0$ have both the roots common, then $[\lambda - 1]$ is (where [.] denotes the greatest integer function)	(r) –1
(D) If N be the number of solutions of the equation $\lvert x^2 - x - 6 \rvert = x + 2$, then the value of – N is.	(s) 3

16.47 **Match The column**

Column – I	Column – II
(A) Number of real solution of $\lvert x + 1 \rvert = e^x$ is	(p) 2
(B) The number of non-negative real roots of $2^x - x - 1 = 0$ equal to	(q) 3
(C) If p and q be the roots of the quadratic equation $x^2 - (\alpha - 2)x - \alpha - 1 = 0$, then minimum value of $p^2 + q^2$ is equal to	(r) 6
(D) If α and β are the roots of $2x^2 + 7x + c = 0$ and $\lvert \alpha^2 - \beta^2 \rvert = \frac{7}{4}$, then c is equal to	(s) 4
	(t) 5

16.48 Match the column

Column – I		Column – II	
(A)	If set of all possible values of k for which every solution of the inequation $x^2 - (3k-1)x + 2k^2 - 3k - 2 \geq 0$ is also a solution of the inequation $x^2 - 1 \geq 0$ is $[\ell, m]$, then $\ell + m$ is equal to	(p)	3
(B)	If a, b, c and d are four positive real numbers such that abcd = 1 and minimum value of $(1 + a)(1 + b)(1 + c)(1 + d)$ is 16λ, then $\lambda + 2$ is equal to	(q)	1
(C)	If solution set of the inequality $5^{x+2} > \left(\frac{1}{25}\right)^{1/x}$ is (ℓ, ∞), then ℓ is equal to	(r)	4
(D)	Let $f(x) = x^3 + 3x + 1$. If g(x) is the inverse function of f(x) and $g'(5) = \frac{\lambda}{6}$, then 4λ is equal to	(s)	2
		(t)	0

16.49 Let α, β, γ are three real numbers such that $\alpha + \beta + \gamma = 2$, $\alpha^2 + \beta^2 + \gamma^2 = 6$ and $\alpha^3 + \beta^3 + \gamma^3 = 8$, then

Column – I		Column – II	
(A)	The value of $\alpha^4 + \beta^4 + \gamma^4$ is	(p)	20
(B)	The value of $(1-\alpha)(1-\beta)(1-\gamma)$ is	(q)	18
(C)	If $\lvert x \rvert < 1$, then $(x-\alpha)(x-\beta)(x-\gamma)$ is	(r)	positive
(D)	The value of $(1+\alpha^2)(1+\beta^2)(1+\gamma^2)$ is	(s)	negative
		(t)	zero

16.50 For $a \neq 0$ the equation $ax^2 + b\lvert x \rvert + c = 0$ has exactly k real solutions and p real roots.

Column-I		Column-II	
(A)	If k = 1, p = 1, then there must be	(p)	ab < 0
(B)	If k = 2, p = 2, then there must be	(q)	bc = 0
(C)	If k = 3, then there must be	(r)	ac < 0
(D)	If k = 4, then there must be	(s)	ab > 0
		(t)	ac > 0

SECTION - VI : INTEGER TYPE

16.51 The equation $9^{-\lvert x-2 \rvert} - 4.3^{-\lvert x-2 \rvert} - \ell n a = 0$ has a solution for every real number $a \in \left[\frac{1}{e^{\lambda}}, 1\right)$, then find λ.

16.52 If α, β are the roots of the equation $x^2 - 2x + 3 = 0$, then find the sum of roots if equation have roots are $\alpha^3 - 3\alpha^2 + 5\alpha - 2$ & $\beta^3 - \beta^2 + \beta + 5$.

16.53 If $f(x) = \frac{ax^2 + 2(a+1)x + 9a + 4}{(-a)x^2 + (-3 + 2a)x - a}$ is always negative and the range of values of a is $a \in \left(\frac{\lambda}{4}, \infty\right)$, then find value of λ.

16.54 Find the integral value of a, for which the quadratic expression $ax^2 + (a-2)x - 2$ is negative for exactly two integral values of x.

16.55 Find the absolute value of the difference of the real roots of the equation $x^2 - 2^{2010}x + \lvert x - 2^{2009} \rvert + 2(2^{4017} - 1) = 0$

16.56 If α, β are roots of the equation $x^2 - 34x + 1 = 0$, evaluate $\lvert \sqrt[4]{\alpha} - \sqrt[4]{\beta} \rvert$, where $\sqrt[4]{.}$ denotes the principal value.

TOPIC 17 SEQUENCE & SERIES

SECTION - I : STRAIGHT OBJECTIVE TYPE

17.1 a, b, c are positive integers forming an increasing G.P. whose common ratio is a natural number, b – a is cube of a natural number and $\log_6 a + \log_6 b + \log_6 c = 6$, then a + b + c =
(A) 100 (B) 111 (C) 122 (D) 189

17.2 If S, P and R are the sum, product and sum of the reciprocals of n terms of an increasing G.P. and $S^n = R^n . P^k$, then k is equal to
(A) 1 (B) 2 (C) 3 (D) None of these

17.3 Sum of first hundred numbers common to the two A.P.'s 12, 15, 18, ... and 17, 21, 25 ...., is
(A) 56100 (B) 65100 (C) 61500 (D) None of these

17.4 If 11 A.M.s are inserted between 28 and 10, then number of integral A.M's is
(A) 5 (B) 6 (C) 7 (D) 8

17.5 If a, b, c are in HP, then $\frac{1}{b-a} + \frac{1}{b-c}$ is equal to
(A) $\frac{2}{b}$ (B) $\frac{2}{a+c}$ (C) $\frac{1}{a+c}$ (D) None of these

17.6 Let a_n be the n^{th} term of an A.P. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the A.P. is :
(A) $\alpha - \beta$ (B) $\beta - \alpha$ (C) $\frac{\alpha-\beta}{2}$ (D) None of these

17.7 If a, b, c, d are in G.P., then $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$ equals to :
(A) a b + b c + c d (B) $(ab + bc + cd)^2$ (C) $(ab + bc + cd)^4$ (D) None of these

17.8 If a_1, a_2, a_3, a_4, a_5 are in H.P., then $a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5$ is equal to :
(A) $2a_1a_5$ (B) $3a_1a_5$ (C) $4a_1a_5$ (D) -4

17.9 If the sum to infinity of the series, $1 + 4x + 7x^2 + 10x^3 + ...$, is $\frac{35}{16}$, where $|x| < 1$, then 'x' equals to :
(A) 19/7 (B) 1/5 (C) 1/4 (D) None of these

17.10 If a, b, c and d are four positive real numbers such that abcd = 1, the minimum value of (1 + a) (1 + b) (1 + c) (1 + d) is :
(A) 4 (B) 1 (C) 16 (D) 18

17.11 If the length of sides of a right triangle are in A.P., then the sines of the acute angles are
(A) $\frac{3}{5}, \frac{4}{5}$ (B) $\sqrt{\frac{2}{3}}, \sqrt{\frac{1}{3}}$ (C) $\sqrt{\frac{\sqrt{5}-1}{2}}, \sqrt{\frac{\sqrt{5}+1}{2}}$ (D) $\sqrt{\frac{\sqrt{3}-1}{2}}, \sqrt{\frac{\sqrt{3}+1}{2}}$

17.12 If a_1, a_2, ... a_n n distinct odd natural numbers not divisible by any prime greater than 5, then $\frac{1}{a_1}+\frac{1}{a_2}+...\frac{1}{a_n}$ is less than

(A) $\frac{15}{8}$ (B) $\frac{16}{8}$ (C) $\frac{8}{15}$ (D) $\frac{15}{4}$

17.13 S_1 : a, b, c are in G.P. and $a^x = b^y = c^z$, then x, y, z are in H.P.

S_2 : If n, G.M's are inserted between a and b, then n^{th} GM is $G_n = a\left(\frac{b}{a}\right)^{n/(n+1)}$

S_3 : If a_1, a_2, a_3, ... and b_1, b_2, b_3, ... are two geometric progressions, then the squence $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}$ .... is also a G.P.

S_4 : If a, H_1, H_2, ... , H_n, b are in H.P., then $\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, ..., \frac{1}{H_n}, \frac{1}{b}$ are in A.P., whose common difference is $\frac{a-b}{(n+1)ab}$

State, in order, whether S_1, S_2, S_3, S_4 are true or false

(A) FFFF (B) TTFF (C) TTTF (D) TTTT

17.14 **S1 :** Let a_1, a_2, a_3, ... a_8 be 8 non-negative real numbers such that $a_1 + a_2 + ... + a_8 = 16$ and $P = a_1a_2 + a_2a_3 + a_3a_4 + ... + a_7a_8$, then the maximum value of P is 64.

S2 : If x, y, r and s are positive real numbers such that $x^2 + y^2 = r^2 + s^2 = 1$, then the maximum value of (xr + ys) is 2.

S3 : If A.M. and G.M. between two positive numbers are respectively A and G, then the numbers are $A+\sqrt{A^2-G^2}$, $A-\sqrt{A^2-G^2}$

S4 : If p, q, r be three distinct real numbers in A.P. then $p^3 + r^3$ equals – 6 pqr

(A) TTFF (B) FTFT (C) TFTF (D) FFTT

17.15 **S1 :** If A.M. between p^{th} and q^{th} terms of an A.P. be equal to the A.M. between r^{th} and s^{th} term of the A.P., then p + q is equal to r + s

S2 : The sum of n terms of the series $\frac{5}{2}+\frac{7}{4}+\frac{11}{8}+\frac{19}{16}+...$ is equal to $n - 3.2^{-n} - 3$

S3 : The harmonic mean of the roots of the equation $\left(2+\sqrt{5}\right)x^2 - \left(4+\sqrt{5}\right)x + 8 + 2\sqrt{5} = 0$ is equal to 4

S4 : If t_3 of an A.P. is 6, then sum of first five terms is equal to 30

(A) TFTT (B) FTFT (C) TFTF (D) FFTT

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

17.16 If a, b, c ∈ R^+ such that a + b + c = 27, then the maximum value of $a^2b^3c^4$ is equal to

(A) $2^8, 3^{10}$ (B) $9. 6^8$ (C) $2^{10} \cdot 3^{12}$ (D) $9. (6)^{10}$

17.17 If the sum of n terms of an A.P. is cn(n + 1), where c ≠ 0, then sum of cubes of these terms is

(A) $2c(cn^2 + cn)^2$ (B) $2c^3n^2(n + 1)^2$

(C) $\frac{2c^3}{3}n^2(n + 1)(2n + 1)$ (D) $\frac{2}{3}c^3n^2(n-1)(2n-1)$

17.18 The 100^{th} term of the series 3 + 8 + 22 + 72 + 266 + 1036 +........... is divisible by 2^n, then the value of n can be

(A) 4 (B) 2 (C) 3 (D) 5

17.19 For the series $S = 1 + \frac{1}{(1+3)}(1+2)^2 + \frac{1}{(1+3+5)}(1+2+3)^2 + \frac{1}{(1+3+5+7)}(1+2+3+4)^2 + ...$

(A) 7^{th} term is 16 (B) 7^{th} term is 18

(C) sum of first ten terms is $\frac{505}{4}$ (D) sum of first ten term is $\frac{405}{4}$

17.20 If $\sum_{r=1}^{n} r(r+1) = \frac{(n+a)(n+b)(n+c)}{3}$, where a < b < c, then

(A) 2b = c (B) $a^3 - 8b^3 + c^3 = 8abc$

(C) c is prime number (D) $(a + b)^2 = 0$

17.21 In a G.P. the ratio of the sum of the first eleven terms to the sum of the last eleven terms is $\frac{1}{8}$ and the ratio of the sum of all the terms without the first nine to the sum of all the terms without the last nine is 2. Then the number of terms of the G.P is less than.

(A) 15 (B) 43 (C) 38 (D) 56

17.22 Let $a_n = \frac{(111.....1)}{n \text{ times}}$, then

(A) a_{912} is not prime (B) a_{951} is not prime (C) a_{480} is not prime (D) a_{91} is not prime

SECTION - III : ASSERTION AND REASON TYPE

17.23 **Statement-1 :** If a, b, c are non zero real numbers such that $3(a^2 + b^2 + c^2 + 1) = 2(a + b + c + ab + bc + ca)$, then a, b, c are in A.P. as well as in G.P.

Statement-2 : A series is in A.P. as well as in G.P. if all the terms in the series are equal and non zero.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

17.24 **Statement-1 :** Equations $x^2 - 4x + 1 = 0$ and $x^2 - ax + b = 0$, where a, b are rational numbers, have atleast one common root, then a = 4 and b = 1

Statement-2 : If two equations $ax^2 + bx + c = 0$ and $a_1x^2 + b_1x + c_1 = 0$, where a, b, c, a_1, b_1, c_1 are non-zero rational numbers, have common irrational root, then $\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1}$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

17.25 **Statement-1 :** Let a, b, c be positive integers, then $a^{\frac{a}{a+b+c}} . b^{\frac{b}{a+b+c}} . c^{\frac{c}{a+b+c}} \geq \frac{1}{3}(a + b + c)$

Statement-2 : Let a_1, a_2, ... a_n be positive numbers in A.P. If A & G are the arithmetic and the geometric means of a_1 and a_n respectively then, $G^n < a_1.a_2.....a_n < A^n$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

17.26 **Statement-1 :** If one A.M. 'A' and two G.M.'s p and q be inserted between any two numbers, then $p^3 + q^3 = 2Apq$
Statement-2 : If x, y, z are in G.P., then $y^2 = xz$
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

Let $A_1, A_2, A_3, ..., A_m$ be arithmetic means between –2 and 1027 and $G_1, G_2, G_3, ..., G_n$ be geometric means between 1 and 1024. Product of geometric means is 2^{45} and sum of arithmetic means is 1025 × 171.

17.27 The value of n is
(A) 7 (B) 9 (C) 11 (D) None of these

17.28 The value of m is
(A) 340 (B) 342 (C) 344 (D) 346

17.29 The value of $G_1 + G_2 + G_3 + ... + G_n$ is
(A) 1022 (B) 2044 (C) 512 (D) None of these

17.30 The common difference of the progression $A_1, A_3, A_5, ..., A_{m-1}$ is
(A) 6 (B) 3 (C) 2 (D) 1

17.31 The numbers $2A_{171}$, G_5^2+1, $2A_{172}$ are in
(A) A.P. (B) G.P. (C) H.P. (D) A.G.P.

Comprehension # 2

There are two sets A and B each of which consists of three numbers in A.P. whose sum is 15 and where D and d are the common differences such that D – d = 1. If $\frac{p}{q} = \frac{7}{8}$ where p and q are the product of the numbers respectively and d > 0, in the two sets

17.32 Value of p is
(A) 100 (B) 120 (C) 105 (D) 110

17.33 Value of q is
(A) 100 (B) 120 (C) 105 (D) 110

17.34 Value of D + d is
(A) 1 (B) 2 (C) 3 (D) 4

Comprehension # 3

Four different integers form an increasing A.P. One of these numbers is equal to the sum of the squares of the other three numbers. Then

17.35 The smallest number is :
(A) – 2 (B) 0 (C) – 1 (D) 2

17.36 The common difference of the four numbers is
(A) 2 (B) 1 (C) 3 (D) 4

17.37 The sum of all the four numbers is
(A) 10 (B) 8 (C) 2 (D) 6

SECTION - V : MATRIX - MATCH TYPE

17.38 Match the column

Column – I		Column – II	
(A)	Suppose that $F(n+1) = \frac{2F(n)+1}{2}$ for $n = 1, 2, 3,...$ and $F(1) = 2$. Then $F(101)$ equals	(p)	42
(B)	If $a_1, a_2, a_3, ...a_{21}$ are in A.P. and $a_3 + a_5 + a_{11} + a_{17} + a_{19} = 10$ then the value of $\sum_{i=1}^{21} a_i$ is	(q)	1620
(C)	10^{th} term of the sequence $S = 1 + 5 + 13 + 29 + ...$, is	(r)	52
(D)	The sum of all two digit numbers which are not divisible by 2 or 3 is	(s)	2045
		(t)	$2 + 4 + 6 + ... + 12$

17.39 Match the column

Column – I		Column – II	
(A)	The arithmetic mean of two positive numbers is 6 and their geometric mean G and harmonic mean H satisfy the relation $G^2 + 3H = 48$, then product of the two number is.	(p)	$\frac{2}{7}$
(B)	The sum of the series $\frac{5}{1^2.4^2} + \frac{11}{4^2.7^2} + \frac{17}{7^2.10^2} + ...$ is.	(q)	32
(C)	If the first two terms of a Harmonic Progression be $\frac{1}{2}$ and $\frac{1}{3}$, then the Harmonic Mean of the first four terms is	(r)	$\frac{1}{3}$
(D)	Geometric mean of -4 and -9	(s)	6
		(t)	-6

SECTION - VI : INTEGER TYPE

17.40 Find the sum to infinity of a decreasing G.P. with the common ratio x such that $|x| < 1$; $x \neq 0$. The ratio of the fourth term to the second term is $\frac{1}{16}$ and the ratio of third term to the square of the second term is $\frac{1}{9}$

17.41 If $\sum_{\alpha=4}^{n+3} 4(\alpha - 3) = An^2 + Bn + C$, then find the value of $A + B - C$.

17.42 If $(1 - P)(1 + 3x + 9x^2 + 27x^3 + 81x^4 + 243x^5) = 1 - P^6$, P 1, then find the value of $\frac{P}{x}$

17.43 If $(1^2 - a_1) + (2^2 - a_2) + (3^2 - a_3) + ... + (n^2 - a_n) = \frac{1}{3}n(n^2 - 1)$, then find the value of a_7.

17.44 The sum of the terms of an infinitely decreasing GP is equal to the greatest value of the function $f(x) = x^3 + 3x - 9$ on the interval $[-4, 3]$ and the difference between the first and second terms is f¢(0). Then find the value of 27r where r is common ratio.