12.8 If $I_m = \int_1^e (\ell n x)^m \, dx$, $m \in N$, then $I_{10} + 10I_9$ is equal to

(A) e^{10} (B) $\frac{e^{10}}{10}$ (C) e (D) e – 1

12.9 If $S_n = \frac{1}{2n} + \frac{1}{\sqrt{4n^2-1}} + \frac{1}{\sqrt{4n^2-4}} + ... + \frac{1}{\sqrt{3n^2+2n-1}}$, $n \in N$, then $\lim_{n\to\infty} S_n$ is equal to

(A) $\frac{\pi}{2}$ (B) 2 (C) 1 (D) $\frac{\pi}{6}$

12.10 Let $A = \int_0^1 \frac{e^t}{t+1} dt$, then the value of $\int_0^1 \frac{te^{t^2}}{t^2+1} dt$

(A) A^2 (B) $\frac{1}{2}A$ (C) 2A (D) $\frac{1}{2}A^2$

12.11 The function $f(x) = \int_1^x \frac{1}{t} dt$ satisfies

(A) f (x + y) = f (x) + f (y) (B) $f\left(\frac{x}{y}\right) = f(x) + f(y)$

(C) f (xy) = f (x) + f (y) (D) None of these

12.12 If f is a function with period P, then $\int_a^{a+p} f(x)dx$ is

(A) equal to f(a) (B) equal to f(p) (C) independent of a (D) None of these

12.13 $\int_{-\pi}^{\pi} (\cos px - \sin qx)^2 dx$ is equal to

(A) 0 (B) $\frac{\pi}{2}$ (C) π (D) 2π

12.14 $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$ is equal to

(A) 0 (B) 1 (C) – 1 (D) None of these

12.15 The value of $\int_a^b (x-a)^3 (b-x)^4 dx$ is $\frac{(b-a)^m}{n}$. Then (m, n) is

(A) (6, 260) (B) (8, 280) (C) (4, 240) (D) none of these

12.16 The absolute value of $\int_{10}^{19} \frac{\sin x}{1+x^8} dx$, is

(A) less than 10^{-7} (B) more than 10^{-7} (C) less than 10^{-6} (D) more than 10^{-6}

12.17 Evaluate $\int_0^a \ell n(\cot a + \tan x)dx$; $a \in \left(0, \frac{\pi}{2}\right)$

(A) a ℓn (sin a) (B) – a ℓn (sin a) (C) ℓn (cos a) (D) – ℓn (cos a)

12.18 S_1 : In $\int_{-1}^{1} f(\cot^{-1}x)dx$ putting $\cot^{-1}x = t$ may change the limits to $\int_{3\pi/4}^{\pi/4}$ ......

S_2 : If f(x) has removable discontinuities at finite number of points in (a, b) then if $\int f(x)dx = F(x)$,

$\int_a^b f(x)dx = F(b) - F(a)$.

S_3 : If f(x) has an infinite discontinuity in (a, b), then we can always write $\int_a^b f(x)dx = F(b) - F(a)$

where $\int f(x)dx = F(x)$

S_4 : If f(x) : [0, 1] → R has single point continuity in (0, 1) then $\int_0^1 f(x)dx$ can be evaluated.

(A) FTTF (B) TFFT (C) FFFF (D) TTFF

12.19 S1 : $\int_{-1}^{\infty} \frac{d}{dx}\left(x\sin\frac{1}{x}\right)dx = 2\sin^2\left(\frac{\pi}{4} - \frac{1}{2}\right)$

S2 : $\int_0^{10\pi} \sin^2 x \cos^4 x\, dx = \frac{5\pi}{8}$

S3 : If f(x) is even then $\int_0^x f(x)dx$ will always be odd.

S4 : If f(x) is even and $\int_0^x f(x)dx$ is odd where f(x) is a continuous function then f(x) = 0 must have exactly three roots in (– 1, 1).

(A) FTTF (B) TFFT (C) TTTF (D) FFFT

12.20 S_1 : $\int_0^{\infty} \frac{\log(1+x^2)}{1+x^2}dx = \pi \log 2$

S_2 : $\int_0^{\pi} e^{\sin^2 x}\cos^3(2n+1)x\, dx = 0$

S_3 : $\int_0^4 |x-1|dx = 5$

S_4 : $\int_{-2}^{2} (1-x^2)dx = \frac{4}{3}$

(A) FTTF (B) TFFT (C) TTTF (D) FFFT

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

12.21 $\int_0^{\pi/2} \frac{\cos\theta - \sin\theta}{(1+\cos\theta)(1+\sin\theta)} d\theta$ equals

(A) $\cos^{-1}(1)$ (B) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (C) $\sin^{-1}(0)$ (D) $\cos^{-1}(-1)$

12.22 If the value of the definite integral $\int_0^1 \frac{\sin^{-1}\sqrt{x}}{x^2 - x + 1} dx$ is $\frac{\pi^2}{\sqrt{n}}$ where $n \in N$, then the value of 'n' is divisible by prime number

(A) 2 (B) 3 (C) 5 (D) 9

12.23 Let $J = \int_{-5}^{-4} f(x)dx$ & $f(x) = (3 - x^2)\tan(3 - x^2)$

and $K = \int_{-2}^{-1} (6 - 6x + x^2)\tan(6x - x^2 - 6)dx$, then

(A) $J - K = 0$ (B) f(x) is even function

(C) $J + K = 0$ (D) f(x) is odd function

12.24 If f(x) is integrable over [1, 2], then $\int_1^2 f(x)\,dx$ is equal to

(A) $\lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right)$ (B) $\lim_{n\to\infty} \frac{1}{n}\sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right)$

(C) $\lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r+n}{n}\right)$ (D) $\lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{2n} f\left(\frac{r}{n}\right)$

12.25 If $I_n = \int_0^1 \frac{dx}{(1+x^2)^n}$, $n \in N$, then which of the following statements hold good?

(A) $2n I_{n+1} = 2^{-n} + (2n-1) I_n$ (B) $I_2 = \frac{\pi}{8} + \frac{1}{4}$

(C) $I_2 = \frac{\pi}{8} - \frac{1}{4}$ (D) $I_3 = \frac{\pi}{16} - \frac{5}{48}$

12.26 If $f(2 - x) = f(2 + x)$ and $f(4 - x) = f(4 + x)$ and f(x) is a function for which $\int_0^2 f(x)dx = 5$, then $\int_0^{50} f(x)dx$ is equal to

(A) 125 (B) $\int_{-4}^{46} f(x)dx$ (C) $\int_1^{51} f(x)dx$ (D) $\int_2^{52} f(x)dx$

12.27 If $F(x) = \frac{1}{x^2}\int_4^x (4t^2 - 2F'(t))\,dt$, then $F'(4)$ equals –

(A) $\frac{32}{9}$ (B) $\frac{64}{9}$ (C) $\frac{F(8)}{28}$ (D) $\frac{11F(8)}{28}$

12.28 If $I = \int_0^{\pi/2} e^{-\alpha \sin x}\,dx$, where $\alpha \in (0, \infty)$, then

(A) $I < \frac{\pi}{2}$ (B) $I > \frac{\pi}{2}(e^{-\alpha} + 1)$ (C) $I > \frac{\pi}{2} e^{-\alpha}$ (D) $I > 0$

SECTION - III : ASSERTION AND REASON TYPE

12.29 **Statement-1 :** $\int_0^{\pi/4} \sec x \sqrt{\frac{1-\sin x}{1+\sin x}}\,dx = 2 - \sqrt{2}$

Statement-2 : $\int_0^{\pi/4} \frac{\sec x}{1+2\sin^2 x}\,dx = \frac{1}{2}\log(\sqrt{2} - 1) + \frac{\pi}{6\sqrt{2}}$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

12.30 **Statement-1 :** $\int_0^1 2^x x^2\,dx = \frac{2}{\ell n 2} - \frac{4}{(\ell n 2)^2} + \frac{2}{(\ell n 2)^3}$

Statement-2 : $\int_0^1 e^x (x-1)^n dx = 16 - 6e$, then value of n is 3

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

12.31 **Statement-1 :** If $\frac{d}{dx}F(x) = \frac{e^{\sin x}}{x}$, $x > 0$ and $\int_1^4 \frac{3e^{\sin x^3}}{x}\,dx = F(k) - F(1)$ then one possible value of K is 64.

Statement-2 : If f(x) is a function satisfying $f\left(\frac{1}{x}\right) + x^2 f(x) = 0\ \forall\, x \in R_0$ then $\int_{\sin\theta}^{\operatorname{cosec}\theta} f(x)\,dx = \sin\theta - \operatorname{cosec}\theta$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

12.32 **Statement-1 :** $\int_{-\pi}^{\pi} \frac{2x(1+\sin^2 x)}{1+\cos^2 x}\,dx = 0$

Statement-2 : $\int_0^{\pi/2} \sin^2 x \cos^2 x (\sin x + \cos x)\,dx = \frac{4}{15}$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

12.33 **Statement-1 :** $\int_0^n \{x\}dx = \frac{n}{2}$, where {.} represents fractional part function and $n \in N$.

Statement-2 : $\int_0^n [x]dx = \frac{n(n-1)}{2}$, where [.] represents greatest integer function and $n \in N$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

12.34 **Statement-1 :** $\int_0^{\pi} x \sin x \cos^2 x \, dx = \frac{\pi}{2}\int_0^{\pi} \sin x \cos^2 x \, dx$

Statement-2 : $\int_a^b x\, f(x)\, dx = \frac{a+b}{2}\int_a^b f(x)\, dx$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

12.35 **Statement-1 :** $\int_0^{\frac{\pi}{3}} \frac{dx}{8\cos^2 x + 1} = \frac{\pi}{18}$

Statement-2 : $\int_0^{\frac{2\pi}{3}} \frac{dx}{8\cos^2 x + 1} = -\frac{\pi}{18}$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

12.36 **Statement-1 :** Let f be real valued function such that f(2) = 2 and $f'(2) = 1$, then $\lim_{x\to 2} \int_2^{f(x)} \frac{4t^3}{x-2}\, dt = 12$

Statement-2 : Let $f(x) = \int_{u(x)}^{v(x)} g(t)\, dt$, then $f'(x) = g(v(x))\, v'(x) - g(u(x))\, u'(x)$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

12.37 **Statement-1 :** $1 \le \int_0^{\pi/2} \frac{\sin x}{x} \le \frac{\pi}{2}$

Statement-2 : If f(x) is continuous in [a, b] and m and ℓ are greatest and least value of f(x) in [a, b], then

$$\ell(b-a) \le \int_a^b f(x)\, dx \le m(b-a)$$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

Definite integral of any discontinuous or non-differentiable function is normally solved by the property $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$, where $c \in (a, b)$ is the point of discontinuity or non-differentiability.

12.38 The value of $A = \int_1^\infty [\operatorname{cosec}^{-1}x]\,dx$, {where [.] denotes greatest integer function} ,is equal to

(A) cosec1 – 1 (B) 1 (C) 1 – sin 1 (D) none of these

12.39 The value of $B = \int_1^{100} [\sec^{-1} x]\,dx$, {where [.] denotes greatest integer function} ,is equal to

(A) sec 1 (B) 100 – sec 1 (C) 99 – sec 1 (D) none of these

12.40 The value of integral $\int_A^B [\tan^{-1} x]\,dx$, {where [.] denotes greatest integer function} ,is equal to

(A) tan 1 (B) 100 – tan 1 – sec1 (C) 99 – sec1 (D) none of these

Comprehension # 2

Using integral $\int_0^{\pi/2} \ell n(\sin x)\,dx = -\int_0^{\pi/2} \ell n(\sec x)\,dx = -\frac{\pi}{2}\ell n\,2$,

$\int_0^{\pi/2} \ell n(\tan x)\,dx = 0$ and $\int_0^{\pi/4} \ell n(1+\tan x)\,dx = \frac{\pi}{8}\ell n2$.

12.41 Evaluate $\int_{-\pi/4}^{\pi/4} \ell n\left(\frac{\sin x + \cos x}{\cos x - \sin x}\right)dx =$

(A) $\pi\ell n\,2$ (B) $\frac{\pi\ell n2}{2}$ (C) 0 (D) $-\pi\ell n\,2$

12.42 Evalute $\int_{-\pi/4}^{\pi/4} \ell n\,(\sin x + \cos x)\,dx =$

(A) $\frac{\pi\ell n2}{2}$ (B) $\frac{-\pi\ell n2}{4}$ (C) $\pi\ell n\,2$ (D) 0

12.43 Evalute $\int_0^{\pi/4} \ell n\,(\sin 2x)\,dx =$

(A) $\frac{-\pi\ell n2}{2}$ (B) $\pi\ell n\,2$ (C) $\frac{\pi\ell n2}{4}$ (D) $-\frac{\pi\ell n2}{4}$

Comprehension # 3

Integral $\int_a^b f(x)\,dx$ can be represented as a limit of a sum of infinite series $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{r=na+c}^{nb+c} \frac{1}{n} f\left(\frac{r}{n}\right)$ where $na + c \le r \le nb + c$, $r, n \in N$, $c \in R$ and any limit of sum of series of same form can be changed to definite integral by replacing

(1) $\lim_{n\to\infty} \sum \to \int$ (2) $\frac{1}{n} \to dx$ (3) $\frac{r}{n} \to x$

(4) Lower limit = $\lim_{n\to\infty}\left(\frac{r}{n}\right)_{min} = \lim_{n\to\infty}\left(\frac{na+c}{n}\right) = a$

(5) Upper limit = $\lim_{n\to\infty}\left(\frac{r}{n}\right)_{max} = \lim_{n\to\infty}\left(\frac{nb+c}{n}\right) = b$

12.44 Find the value of $\lim_{n\to\infty}\left(\frac{n}{(n+1)\sqrt{2n+1}}+\frac{n}{(n+2)\sqrt{2(2n+2)}}+\frac{n}{(n+3)\sqrt{3(2n+3)}}+.....+\frac{1}{2n\sqrt{3}}\right)$

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) none of these

12.45 The n^{th} term of the corresponding series of $\int_0^1 \tan^{-1}x\,dx$ is

(A) $\frac{\pi}{4n}$ (B) $\frac{1}{n}\tan^{-1}(n-1)$ (C) $\frac{\pi}{2n}$ (D) $\tan^{-1}n$

12.46 $\lim_{n\to\infty}\sum_{r=0}^{2n-1}\frac{1}{n}\sec^2\left(\frac{r}{n}\right)$ is

(A) sec 2 (B) tan 2 (C) $\sec^2$ (D) not defined

SECTION - V : MATRIX - MATCH TYPE

12.47 Match the following :

Column – I	Column – II
(A) If $\int_0^1 \frac{\cos x}{1+x}dx = k$ and $\int_{6\pi-3}^{6\pi}\frac{\cos(x/3)}{6\pi+3-x}dx = mk$, then m is	(p) 4
(B) $\int_{-1}^{1}\left(\sin^{-1}\left[x+\frac{3}{4}\right]\right)dx = \frac{\pi}{k}$, then k is {where [.] denotes greatest integer function}	(q) 1
(C) If f(x) = max (x – | x |, x + [x]) and $\int_{-3}^{3} f(x)\,dx = -\frac{3}{k}$, then k is {where [.] denotes greatest integer function}	(r) 0
(D) If $\int_0^{20}\sqrt{1-\cos\pi x}\,dx = \frac{10k\sqrt{2}}{\pi}$, then k is	(s) 2
	(t) 3

12.48 Match the following :

Column – I	Column – II
(A) $\int_4^{10}\frac{[x^2]dx}{[x^2-28x+196]+[x^2]}$ = {where [.] denotes greatest integer function}	(p) $\frac{1}{101}$
(B) $\int_{-1}^{2}\frac{\|x\|}{x}dx$=	(q) 3
(C) $\lim_{n\to\infty}\frac{1^{99}+2^{99}+...+n^{99}}{n^{100}}$ =	(r) $\frac{1}{3}$
(D) $5050\int_{-1}^{1}\sqrt{x^{200}}\,dx = \frac{1}{\alpha}$, then α =	(s) 1
	(t) $\frac{1}{100}$

12.49

Column – I	Column – II
(A) $\int_{-1}^{1} \frac{3x^2}{1+4^{\tan x}}\, dx =$	(p) 7
(B) $\int_{6}^{8} \frac{\sin x^2 dx}{\sin x^2 + \sin (x-14)^2} =$	(q) $\frac{1}{2}$
(C) $\frac{1}{156}\int_{1}^{13} [x]\, dx =$	(r) 1
(D) $\frac{1}{\pi \ell n 2}\int_{\pi/2}^{0} \ell n \sin 2x\, dx =$	(s) 2
	(t) 0

12.50

Column – I	Column – II
(A) $\lim_{n\to\infty} \sum_{r=1}^{n-1} \frac{\sqrt{n^2-r^2}}{n^2} =$	(p) $\frac{\pi}{4}$
(B) $\int_{0}^{\frac{\pi}{4}} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx =$	(q) $\frac{\pi}{\sqrt{2}}$
(C) $\int_{-1}^{1} \sin^3 x \cos^2 x\, dx =$	(r) $-\frac{\pi}{4}$
(D) $\int_{0}^{\frac{\pi}{2}} \frac{\sqrt{\sin^3 x}\, dx}{\sqrt{\sin^3 x} + \sqrt{\cos^3 x}} =$	(s) $\frac{\pi}{2}$
	(t) 0

SECTION - VI : INTEGER TYPE

12.51 $\lim_{n\to\infty} \frac{1}{n}\left[\cos^{2p}\frac{\pi}{2n} + \cos^{2p}\frac{2\pi}{2n} + \cos^{2p}\frac{3\pi}{2n} + \ldots\ldots + \cos^{2p}\frac{\pi}{2}\right] = \frac{\lambda}{\pi}\left[\frac{2p-1}{2p}\cdot\frac{2p-3}{2p-2}\cdots\cdots\cdots\cdot\frac{1}{2}, \frac{\pi}{2}\right]$, $p \in N$, then find λ

12.52 $\int_{0}^{2\pi} \frac{dx}{a + b\cos x + c\sin x} = \frac{\lambda\pi}{\sqrt{a^2-b^2-c^2}}$ where $a > \sqrt{b^2+c^2} > 0$, then find λ

12.53 A function f: R → R satisfies the equation $f(x+y) = f(x) + f(y)\ \forall\ x, y \in R$ and is continuous throughout the domain. If $I_1 + I_2 + I_3 + I_4 + I_5 = 450$ where $I_n = n\int_{0}^{n} f(x)dx$ and $f(x) = \lambda x$, then find λ

12.54 $\int_{0}^{1} \frac{1}{(5+2x-2x^2)(1+e^{(2-4x)})}\, dx = \frac{1}{\sqrt{\lambda}} \log \frac{\sqrt{11}+1}{\sqrt{10}}$, then find λ

TOPIC 13 AREA UNDER CURVE

SECTION - I : STRAIGHT OBJECTIVE TYPE

13.1 The area enclosed by the curves $y = \sqrt{4-x^2}$, $y \geq \sqrt{2}\sin\left(\frac{x\pi}{2\sqrt{2}}\right)$ and x-axis is divided by y-axis in the ratio

(A) $\frac{\pi^2-8}{\pi^2+8}$ (B) $\frac{\pi^2-4}{\pi^2+4}$ (C) $\frac{\pi-4}{\pi-4}$ (D) $\frac{2\pi^2}{2\pi+\pi^2-8}$

13.2 If $f(x) = \sin x \ \forall x \in \left[0, \frac{\pi}{2}\right]$, $f(x) + f(\pi - x) = 2 \ \forall x \in \left(\frac{\pi}{2}, \pi\right]$ and $f(x) = f(2\pi - x) \ \forall x \in (\pi, 2\pi]$, then the area enclosed by y = f(x) and x - axis is

(A) π (B) 2π (C) 2 (D) 4

13.3 The area enclosed by $y = x^3$, its normal at (1, 1) and x axis is equal to

(A) $\frac{7}{4}$ (B) $\frac{9}{4}$ (C) $\frac{5}{4}$ (D) 2

13.4 Value of the parameter a such that the area bounded by $y = a^2x^2 + ax + 1$, co-ordinate axes and the line x = 1, attains it's least value, is equal to

(A) $-\frac{1}{4}$ (B) $-\frac{1}{2}$ (C) $-\frac{3}{4}$ (D) -1

13.5 Area of the region bounded by x = 0, y = 0, x = 2, y = 2, $y \leq e^x$ and $y \geq \ell nx$, is

(A) $6 - 4\ \ell n\ 2$ (B) $4\ \ell n\ 2 - 2$ (C) $2\ \ell n\ 2 - 4$ (D) $6 - 2\ \ell n\ 2$

13.6 Area bounded by the curve $y = \ell n\ x + \tan^{-1}x$ and x– axis from x = 1 to x = 2, is

(A) $\frac{5}{2}\ell n\ 2 - \frac{1}{2}\ \ell n\ 5 + 2\tan^{-1}2 - \frac{\pi}{3} - 1$

(B) $\frac{5}{2}\ell n\ 2 - \frac{1}{2}\ \ell n\ 5 + 2\tan^{-1}2 - \frac{\pi}{4} + 1$

(C) $\frac{5}{2}\ell n\ 2 - \frac{1}{2}\ \ell n\ 5 + 2\tan^{-1}2 + \frac{\pi}{4} - 1$

(D) $\frac{5}{2}\ell n\ 2 - \frac{1}{2}\ \ell n\ 5 + 2\tan^{-1}2 - \frac{\pi}{4} - 1$

13.7 The area of the figure bounded by the parabola $(y - 2)^2 = x - 1$, the tangent to it at the point with the ordinate 3 and the x-axis is

(A) 3 (B) 6 (C) 9 (D) none of these

13.8 The area of the figure bounded by the curve $y = 2x - x^2$ and the straight line $y = -x$ is

(A) $\frac{9}{2}$ (B) 9 (C) $\frac{7}{2}$ (D) 7

13.9 The area bounded by the curve $y = x^2$, $y = -x^2$ and $y^2 = 4x - 3$ is k, then value of 6k, is

(A) 2 (B) 3 (C) 0 (D) 4

13.10. The area of the region enclosed between the two circles $x^2 + y^2 = 1$ and $(x - 1)^2 + y^2 = 1$, is

(A) $\left(\frac{2\pi}{3} - \sqrt{\frac{3}{4}}\right)$ (B) $\left(\frac{3\pi}{2} - \frac{\sqrt{3}}{2}\right)$ (C) $\left(\frac{3\pi}{2} + \frac{\sqrt{3}}{2}\right)$ (D) $\left(\frac{2\pi}{3} + \sqrt{\frac{3}{4}}\right)$

13.11 Which of the following statements are true/false –

S_1 : Area between $x^2 = 4by$ and $y^2 = 4ax$ is $\frac{16ab}{3}$

S_2: Area enclosed by $|x| + |y| = 1$ is 1.

S_3 : Smaller area enclosed by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x}{a} + \frac{y}{b} = 1$ is $\frac{\pi ab}{4} - \frac{ab}{2}$

S_4 : Area enclosed by $y = [x]$ and $y = \{x\}$ is 1.

(A) TFTT (B) TTTT (C) TFTF (D) FFTT

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

13.12 Let C be a curve passing through M (2, 2) such that the slope of the tangent at any point to the curve is reciprocal of the ordinate of that point. If the area bounded by curve C and line x = 2 is expressed as a rational number $\frac{p}{q}$ (where p and q are in their lowest form), then value of (p + q) is greater than prime numbers

(A) 5 (B) 11 (C) 17 (D) 19

13.13 Let $f(x) = \min.\left(\cos^{-1}(\cos x), \cot^{-1}(\cot x)\right)$ and $x \in (\pi, 2\pi)$ then which is true

(A) the area enclosed between $f(x) = \min.\left(\cos^{-1}(\cos x), \cot^{-1}(\cot x)\right)$ and x-axis is $\frac{\pi^2}{4}$

(B) the area enclosed between $f(x) = \min.\left(\cos^{-1}(\cos x), \cot^{-1}(\cot x)\right)$ and x-axis is $\frac{\pi^2}{2}$

(C) the f(x) is non differentiable at two points

(D) the f(x) is non differentiable at one points

13.14 Let the curve $a^2y = x^2(x + a)$ then

(A) the area between curve and negative x-axis is $\frac{a^2}{3}$ sq. unit

(B) the area between curve and negative x-axis is $\frac{a^2}{12}$ sq. unit

(C) the area between curve and positive x-axis is zero sq. unit

(D) the area between curve and negative x-axis is $\frac{a^2}{12}$ sq. unit

13.15 Let $f(x) = \sin^{-1}|\sin x|$ & $g(x) = (\sin^{-1} |\sin x|)^2$ when $x \in [0, 2\pi]$ then

(A) the area between the curves f(x) & g(x) is $\frac{1}{6} + \frac{\pi^3}{8}$

(B) the area between the curves f(x) & g(x) is $\frac{4}{3} + \pi^2\left(\frac{\pi -}{6}\right)$

(C) f(x) is non differentiable at two points

(D) f(x) is non differentiable at three points

SECTION - III : ASSERTION AND REASON TYPE

13.16 **Statement-1 :** The area bounded by the curve $|x| + |y| = a$ $(a > 0)$ is $2a^2$ and area bounded $|px + qy| + |qx - py| = a$, where $p^2 + q^2 = 1$, is also $2a^2$.

Statement-2 : Since $\alpha x + \beta y = 0$ is perpendicular to $\beta x - \alpha y = 0$, we can take one as x-axis and another as y-axis and therefore the area bounded by $|\alpha x + \beta y| + |\beta x - \alpha y| = a$ is $2a^2$ for all $\alpha, \beta \in R$, $\alpha \neq 0$, $\beta \neq 0$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

13.17 **Statement-1 :** Area bounded by parabola $y = x^2 - 4x + 3$ and $y = 0$ is 4/3 sq. units.

Statement-2 : Area bounded by curve $y = f(x) \geq 0$ and $y = 0$ between ordinates $x = a$ and $x = b$ $(b > a)$ is $\int_a^b f(x)\,dx$

(A) statement-1 is correct and statement-2 is correct and statement-2 is correct explanation of statement-1
(B) statement-1 and statement-2 both are correct but statement-2 is not correct explanation of statement-1
(C) statement-1 is false but statement-2 is true
(D) statement-1 is true but statement-2 is false

SECTION - IV : COMPREHENSION TYPE

Asymptotes are the tangents to the curve at infinity

To find the asymptotes of a curve we can use the following methods.

(a) Asymptote parallel to the x-axis is obtained by equating to zero, the coefficient of the highest power to x.

(b) Asymptote parallel to the y-axis is obtained by equating to zero, the coefficient of the highest power of y.

(c) Oblique Asymptote : $y = mx + c$

(i) Find $\varphi_n(m)$ by putting $x = 1$ and $y = m$ in the highest degree (n) terms of the equation similarly find $\varphi_{n-1}(m)$.

(ii) Solve $\varphi_n(m) = 0$ for m

(iii) Find c by the formula $c = -\dfrac{\varphi_{n-1}(m)}{\varphi'_n(m)}$ Using the value of m as obtained in (ii)

(iv) Obtain the equation of asymptote by putting these values of m and c in $y = mx + c$.

13.18 The equation of asymptotes of the curve $yx^2 - 4x^2 + x + 2 = 0$

(A) $y - 4 = 0$ and $x = 0$
(B) $y = 3$ and $x = 2$
(C) $y - 4 = 0$ and $x = 2$
(D) $y = 3$ and $x = 0$

13.19 The equation of asymptotes of the curve $x^3 + y^3 - 3xy = 0$

(A) $y = x + 1$
(B) $y + x + 1 = 0$
(C) $y + x = 2$
(D) $y = 2x + 1$

13.20 The equation of asymptotes of the curve $y^2 = \dfrac{x^3}{(2 - x)}$ is $ax + by + c = 0$, then the value of $|a + b + c|$ is

(A) 0
(B) 1
(C) 2
(D) 4

SECTION - V : MATRIX - MATCH TYPE

13.21 Match the following :

Column – I	Column – II
(A) The area bounded by the curve $y = x + \sin x$ and its inverse function between the ordinates $x = 0$ to $x = 2\pi$ is 4s Then the value of s is	(p) 0
(B) The area bounded by $y = x\,e^{\|x\|}$ and lines $\|x\| = 1$, $y = 0$ is	(q) 1
(C) The area bounded by the curves $y^2 = x^3$ and $\|y\| = 2x$ is	(r) $\frac{16}{5}$
(D) The smaller area included between the curves $\sqrt{x} + \sqrt{\|y\|} = 1$ and $\|x\| + \|y\| = 1$ is	(s) $\frac{1}{3}$
	(t) 2

13.22 Match the following :

Column – I	Column – II
(A) Area enclosed by $y = \|x\|$, $\|x\| = 1$ and $y = 0$ is	(p) 3
(B) Area enclosed by the curve $y = \sin x$, $x = 0$, $x = \pi$ and $y = 0$ is	(q) 4
(C) If the area of the region bounded by $x^2 \le y$ and $y \le x + 2$ is $\frac{k}{4}$, then k =	(r) 27
(D) Area of the quadrilateral formed by tangents at the ends of latus rectum of ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ is	(s) 18
	(t) 1

SECTION - VI : INTEGER TYPE

13.23 Find the area enclosed by the solution set of $[x] \cdot [y] = 2$. Where $[\cdot]$ represent greatest integer function of x.

13.24 The area of the loop of the curve, $ay^2 = x^2(a - x)$ is $\frac{\lambda a^2}{15}$, then find λ.

TOPIC 14 DIFFERENTIAL EQUATION

SECTION - I : STRAIGHT OBJECTIVE TYPE

14.1 Solution of differential equation $x^2 = 1 + \left(\frac{x}{y}\right)^{-1}\frac{dy}{dx} + \frac{\left(\frac{x}{y}\right)^{-2}\left(\frac{dy}{dx}\right)^2}{2!} + \frac{\left(\frac{x}{y}\right)^{-3}\left(\frac{dy}{dx}\right)^3}{3!} + \ldots$ is

(A) $y^2 = x^2(\ell n\, x^2 - 1) + c$
(B) $y = x^2(\ell n\, x - 1) + c$
(C) $y^2 = x(\ell n\, x - 1) + c$
(D) $y = x^2 e^{x^2} + c$

14.2 A curve which passes through (1, 2) and whose sub-normal at every point is 2, is

(A) $2x^2 = y$ (B) $y^2 = 2x + 2$ (C) $y^2 = x + 3$ (D) $y^2 = 4x$

14.3 The orthogonal trajectory of system of curve $y = ax^2$ which does not passes through origin, is

(A) ellipse (B) parabola (C) circle (D) hyperbola

14.4 If gradient of a curve at any point P(x, y) is $\frac{x+y+1}{2y+2x+1}$ and it is passes through origin, then curve is

(A) $2(x + 3y) = \ell n\left|\frac{3x+3y+2}{2}\right|$
(B) $x + 3y = \ell n\left|\frac{3x+3y+2}{2}\right|$
(C) $3y + x = \ell n\,(3x + 2y + 1)$
(D) $6y - 3x = \ell n\left|\frac{3x+3y+2}{2}\right|$

14.5 The solution of the differential equation $y_1 y_3 = 3y_2^2$ is

(A) $x = A_1y^2 + A_2 y + A_3$
(B) $x = A_1 y + A_2$
(C) $x = A_1 y^2 + A_2 y$
(D) none of these

14.6 The order of the differential equation whose general solution is $y = c_1 \cos 2x + c_2 \cos^2 x + c_3 \sin^2 x + c_4$ is

(A) 2 (B) 4 (C) 3 (D) None of these

14.7 If the solution of the differential equation $\frac{dy}{dx} = \frac{1}{x\cos y + \sin 2y}$ is $x = ce^{\sin y} - k(1 + \sin y)$, then k =

(A) 1 (B) 2 (C) 3 (D) 4

14.8 The differential equation of all parabola having their axis of symmetry coinciding with the axis of X is

(A) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ (B) $y\frac{d^2x}{dy^2} + \left(\frac{dx}{dy}\right)^2 = 0$ (C) $y\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ (D) none of these

14.9 The solution of the differential equation $\sqrt{a+x}\,\frac{dy}{dx} + xy = 0$ is

(A) $y = Ae^{\frac{2}{3}(2a-x)\sqrt{x+a}}$
(B) $y = Ae^{-\frac{2}{3}(a-x)\sqrt{x+a}}$
(C) $y = Ae^{\frac{2}{3}(2a+x)\sqrt{x+a}}$
(D) $y = Ae^{-\frac{2}{3}(2a-x)\sqrt{x+a}}$

Where A is an arbitrary constant.

14.10 The solution of differential equation $(x^2-1)\frac{dy}{dx}+2xy=\frac{1}{x^2-1}$ is

(A) $y(x^2-1)=\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|+C$
(B) $y(x^2+1)=\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|+C$
(C) $y(x^2-1)=\frac{5}{2}\log\left|\frac{x-1}{x+1}\right|+C$
(D) None of these

14.11 The degree of the differential equation satisfying $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ is
(A) 1 (B) 2 (C) 3 (D) None of these

14.12 The equation of the curve satisfying the differential equation $y_2(x^2+1)=2xy_1$ passing through the point (0, 1) and having slope of tangent at x = 0 as 3, is
(A) $y=x^2+3x+2$ (B) $y^2=x^2+3x+1$ (C) $y=x^3+3x+1$ (D) none of these

14.13 The equation of curve passing through (3, 4) and satisfying the differential equation

$y\left(\frac{dy}{dx}\right)^2+(x-y)\frac{dy}{dx}-x=0$ can be

S_1 : $x-y+1=0$
S_2 : $x+y-7=0$
S_3 : $x^2+y^2=25$
S_4 : $x^2+y^2-5x=10$
(A) TFTF (B) TTFF (C) TTFT (D) FTFT

14.14 S_1 : The order of differential equation $\sqrt{1+\frac{d^2y}{dx^2}}=x$ is 1

S_2 : Solution of the differential equation $xdy-ydx=\sqrt{x^2+y^2}\,dx$ is $y+\sqrt{x^2+y^2}=cx^2$.

S_3 : $\frac{d^2y}{dx^2}=2\left(\frac{dy}{dx}-y\right)$ is differential equation of family of curves $y=e^x(A\cos x+B\sin x)$

S_4 : The solution of differential equation $(1+y^2)+\left(x-2e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$ is $x\,e^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

(A) TFTF (B) FTFT (C) FTTT (D) FFTF

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

14.15 A tangent is drawn at the point (x_i, y_i) to curve $y=f(x)$, which intersects the x-axis at $(x_{i+1}, 0)$. Now again a tangent is drawn at (x_{i+1}, y_{i+1}) to the curve which intersects the x-axis at $(x_{i+2}, 0)$ and the process is repeated n times. If $x_1, x_2, \ldots$, form an AP with common difference $\log_2 e$ and the curve passes through (0, 2) & (–2, K), then K is divisible by
(A) 2 (B) 4 (C) 8 (D) 5

14.16 The solution of the differential equation $\left(e^{x^2}+e^{y^2}\right)y\frac{dy}{dx}+e^{x^2}(xy^2-x)=0$ is

(A) $e^{x^2}(y^2-1)+e^{y^2}=c$
(B) $e^{y^2}(x^2-1)+e^{x^2}=c$
(C) $e^{y^2}(y^2-1)+e^{x^2}=c$
(D) $e^{-y^2}(y^2-1)+e^{-x^2}-ce^{-x^2}e^{-y^2}=0$

14.17 Solution of the differential equation $(y + x\sqrt{xy}\,(x + y))\,dx + (y\sqrt{xy}\,(x + y) - x)dy = 0$ is

(A) $\frac{x^2+y^2}{2} + \tan^{-1}\sqrt{\frac{y}{x}} = \lambda$ (B) $\frac{x^2+y^2}{2} + 2\tan^{-1}\left(\sqrt{\frac{x}{y}}\right) = \lambda$

(C) $\frac{x^2+y^2}{2} + 2\cot^{-1}\sqrt{\frac{y}{x}} = \lambda$ (D) $\frac{x^2+y^2}{2} + \cot^{-1}\sqrt{\frac{x}{y}} = \lambda$

14.18 If $y = e^{-x}\cos x$ and $y_n + k_n\,y = 0$, where $y_n = \frac{d^n y}{dx^n}$ and k_n, $n \in N$ are constants.

(A) $k_4 = 4$ (B) $k_8 = -16$ (C) $k_{12} = 20$ (D) $k_{16} = -24$

14.19 The differential equation $\frac{d^2y}{dx^2} + y + \cot^2 x = 0$ must be satisfied by

(A) $y = 2 + c_1\cos x + \sqrt{c_2}\,\sin x$ (B) $y = \cos x \,.\, \ell n\left(\tan\frac{x}{2}\right) + 2$

(C) $y = 2 + c_1\cos x + c_2\sin x + \cos x \log\left(\tan\frac{x}{2}\right)$

(D) all the above

14.20 A solution of the differential equation $(x^2y^2 - 1)\,dy + 2x\,y^3\,dx = 0$ is

(A) $1 + x^2y^2 = cx$ (B) $1 + x^2y^2 = cy$ (C) $y = 0$ (D) $y = -\frac{1}{x^2}$

14.21 The solution of $\left(\frac{dy}{dx}\right)(x^2y^3 + xy) = 1$ is

(A) $1/x = 2 - y^2 + C\,e^{-y^2/2}$

(B) the solution of an equation which is reducible to linear equation.

(C) $2/x = 1 - y^2 + e^{-y/2}$ (D) $\frac{1-2x}{x} = -y^2 + C\,e^{-y^2/2}$

SECTION - III : ASSERTION AND REASON TYPE

14.22 **Statement -1 :** Solution of $(1 + x\sqrt{x^2+y^2})\,dx + y(-1 + \sqrt{x^2+y^2})\,dy = 0$ is $x - \frac{y^2}{2} + \frac{1}{3}(x^2+y^2)^{3/2} + c = 0$

Statement -2 : Solution of $(1 + xy)\,y\,dx + (1 - xy)\,x\,dy = 0$ is $\ell n\frac{x}{y} - \frac{1}{xy} = c$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

14.23 **Statement -1 :** The equation of the curve passing through (3, 9) which satisfies differential equation $\frac{dy}{dx} = x + \frac{1}{x^2}$ is $6xy = 3x^3 + 29x - 6$

Statement -2 : The solution of D.E. $\left(\frac{dy}{dx}\right)^2 - \frac{dy}{dx}(e^x + e^{-x}) + 1 = 0$ is $y = c_1 e^x + c_2 e^{-x}$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True