N

s equal

9.10 S_1 : If $f(x) = |x-2|$, then $f'(f(x)) = 1$ for $x > 20$

S_2 : If $f(x) = \frac{x}{1+|x|}$, then $f'(-1) = \frac{1}{4}$

S_3 : If $f(0) = a$, $f'(0) = b$, $g(0) = 0$ and $(fog)'(o) = c$, then $g'(0) = \frac{c}{b}$

S_4 : differential coefficient of $2\tan^{-1}x$ w.r.t. $\sin^{-1}\frac{2x}{1+x^2}$ at $x = \frac{1}{2}$ is 1

(A) FTTT (B) TFTT (C) TTFF (D) TTTT

9.11 S_1 : If $y = \sin 2x$, then $\frac{d^6y}{dx^6}$ at $x = \frac{\pi}{2}$ is equal to 1

S_2 : If $x = e^{y+e^{y+\dots\infty}}$, then $\frac{dy}{dx}$ at $x = 1$ is 0

S_3 : If $y = 2t^2$, $x = 4t$, then $\frac{d^2y}{dx^2}$ at $x = \frac{1}{2}$ is $\frac{1}{2}$

S_4 : If $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$, then $\frac{dy}{dx}$ at $(2, -1)$ is $\frac{16}{7^3}$

(A) FFFT (B) FTFT (C) FTTF (D) TTTF

9.12 S_1: If $f(x) = [x]$, then $f\left(\left|f\left(\frac{1}{2}\right)\right|\right) = 0$

S_2: If $f(x) = \frac{1}{\sin|x|}$, then $f'(n\pi) = 0$

S_3: If $f(x) = \log|\sin x|$, then $f'(x) < 0 \ \forall \ x \in \left(\frac{\pi}{2}, \pi\right)$

S_4: $f(x) = e^{|\sin x|}$ is differentiable everywhere

(A) FFFT (B) TFTF (C) FTTF (D) TTTF

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

9.13 If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to :

(A) $\frac{1}{\sqrt{2}}$ (B) $\sin^{-1}\left(\sin\frac{1}{\sqrt{2}}\right)$ (C) 1 (D) none of these

9.14 If $y = 10^{10^x}$ and $\frac{1}{y}\frac{dy}{dx} = 10^x . \lambda$, then value of λ is–

(A) $\ell n\, 10$ (B) $(\ell n\, 10)^2$ (C) $e^{\ell n(\ell n 10)^2}$ (D) $(\log_{10}e)^2$

9.15 Given $f(x) = -\frac{x^3}{3} + x^2 \sin 1.5a - x \sin a . \sin 2a - 5\sin^{-1}(a^2 - 8a + 17)$ then:

(A) $f'(x) = -x^2 + 2x \sin 6 - \sin 4 \sin 8$

(B) $f'(\sin 8) > 0$

(C) $f'(x)$ is not defined at $x = \sin 8$

(D) $f'(\sin 8) < 0$

SECTION - III : ASSERTION AND REASON TYPE

9.16 **Statement - 1 :** For $x < 0$, $\frac{d}{dx}(\ell n\,|x|) = \frac{1}{x}$.

Statement - 2 : For $x < 0$, $|x| = -x \Rightarrow \frac{d}{dx}|x| = -1$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

9.17 **Statement - 1 :** $\frac{d}{dx}\{\tan^{-1}(\sec x + \tan x)\} = \frac{d}{dx}\{\cot^{-1}(\text{cosec } x + \cot x)\}$, $x \in \left(0, \frac{\pi}{4}\right)$.

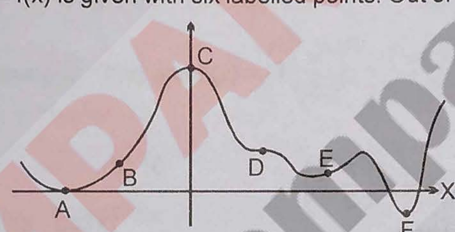
Statement - 2 : $\sec^2 x - \tan^2 x = 1 = \text{cosec}^2 x - \cot^2 x$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

The graph of $y = f(x)$ is given with six labelled points. Out of these points answer the following questions.



9.18 The point which has the greatest value of $f'(x)$ is
(A) B (B) C (C) D (D) E

9.19 The point where f' and f'' are non-zero and of the same sign are
(A) B & D (B) D & E (C) B & E (D) None of these

9.20 The points where atleast two of f, f' and f'' are zero
(A) C & D (B) A and D (C) A & F (D) None of these

Comprehension # 2

In certain problem the differentiation of $\{f(x) \cdot g(x)\}$ appears. One student commits mistake and differentiates as $\frac{df}{dx} \cdot \frac{dg}{dx}$ but he gets correct result if $f(x) = x^3$ and $g(0) = \frac{1}{3}$.

9.21 The function $g(x)$ is
(A) $\frac{3}{|x-3|^3}$ (B) $\frac{4}{|x-3|^3}$ (C) $\frac{9}{|x-3|^3}$ (D) $\frac{27}{|x-3|^3}$

9.22 Derivative of $\{f(x-3) \cdot g(x)\}$ with respect to x at $x = 100$ is
(A) 0 (B) 1 (C) -1 (D) 2

9.23 $\lim_{x \to 0} \frac{f(x).g(x)}{x(1+g(x))}$ will be
(A) 0 (B) -1 (C) 1 (D) 2

Comprehension # 3

Let $f(x) = \frac{1}{1+x^2}$. Let m be the slope, a be the x-intercept and b be the y-intercept of a tangent to y = f(x), then

9.24 Abscissa of the point of contact of the tangent for which m is greatest

(A) $\frac{1}{\sqrt{3}}$ (B) 1 (C) – 1 (D) $-\frac{1}{\sqrt{3}}$

9.25 The greatest value of b is-

(A) $\frac{9}{8}$ (B) $\frac{3}{8}$ (C) $\frac{1}{8}$ (D) $\frac{5}{8}$

9.26 The abscissa of the point of contact of tangent for which $\frac{1}{a}$ is greatest, is-

(A) $\frac{1}{\sqrt{3}}$ (B) 1 (C) – 1 (D) $-\frac{1}{\sqrt{3}}$

SECTION - V : MATRIX - MATCH TYPE

9.27 Match the column

Column – I	Column – II
(A) $\lim_{x \to \frac{1}{\sqrt{2}}} \frac{x - \cos(\sin^{-1} x)}{1 - \tan(\sin^{-1} x)}$ is equal to	(p) does not exist
(B) If $f(x) = \log_{x^2}(\log x)$, then $f'\left(\frac{1}{2}\right)$ is equal to	(q) $-\frac{1}{\sqrt{2}}$
(C) For the function $f(x) = \ell n \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ if $\frac{dy}{dx} = \sec x + p$, then p is equal to	(r) 28
(D) $\lim_{x \to 0} \frac{1}{x}\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$ is equal to	(s) 1
	(t) 0

9.28

Column – I	Column – II
(A) If $y = \cos^{-1}(\cos x)$, then y′ at x = 5 is equal to	(p) does not exist
(B) For the function $f(x) = \ell n\,\lvert\tan x\rvert$ $f'\left(-\frac{\pi}{4}\right)$ is equal to	(q) 2
(C) The derivative of $\tan^{-1}\left(\frac{1+x}{1-x}\right)$ at x = – 1 is	(r) $\frac{1}{2}$
(D) The derivative of $\frac{\log\lvert x\rvert}{x}$ at x = – 1 is	(s) 1
	(t) –1

SECTION - VI : INTEGER TYPE

9.29 If $y = \tan^{-1}\frac{4x}{1+5x^2} + \tan^{-1}\frac{2+3x}{3-2x}$, where $0 < x < \frac{2}{3}$ and $\frac{dy}{dx} = \frac{\lambda}{1+25x^2}$, then find λ

9.30 The function y = f (x) defined by the parametric equations $x = e^t \sin t$, $y = e^t \cos t$ satisfies the relation $y''(x + y)^2 = \lambda(xy' - y)$, then find λ.

TOPIC 10 APPLICATION OF DERIVATIVE

SECTION - I : STRAIGHT OBJECTIVE TYPE

10.1 Tangent of acute angle between the curves $y = |x^2 - 1|$ and $y = |x^2 - 3|$ at their points of intersection is

(A) 0 (B) $\frac{4\sqrt{2}}{3}$ (C) $\frac{4\sqrt{2}}{7}$ (D) $2\sqrt{2}$

10.2 The curve $y - e^{xy} + x = 0$ has a vertical tangent at:

(A) (1, 1) (B) (0, 1) (C) (1, 0) (D) no point

10.3 A particle moving on a curve has the position at time t given by $x = f'(t) \sin t + f''(t) \cos t$, $y = f'(t) \cos t - f''(t) \sin t$, where f is a thrice differentiable function. Then the velocity of the particle at time t is :

(A) $f'(t) + f''(t)$ (B) $f'(t) - f'''(t)$ (C) $f'(t) + f'''(t)$ (D) $f'(t) - f''(t)$

10.4 f(x) and g(x) are differentiable in [0, 1] such that $f(0) = 2$, $g(0) = 0$, $f(1) = 6$, $g(1) = 2$, then Rolle's theorem is applicable for which of the following in [0, 1] ?

(A) $f(x) - g(x)$ (B) $f(x) - 2g(x)$ (C) $f(x) + 3g(x)$ (D) none of these

10.5 If $f(x) = \frac{a \sin x + b \cos x}{c \sin x + d \cos x}$ is monotonically increasing, then

(A) $ad \geq bc$ (B) $ad < bc$ (C) $ad \leq bc$ (D) $ad > bc$

10.6 Let $f(x) = x^3 + ax^2 + bx + 5 \sin^2 x$ be an increasing function in the set of real numbers R. Then a & b satisfy the condition

(A) $a^2 - 3b - 15 \geq 0$ (B) $a^2 - 3b + 15 \leq 0$ (C) $a^2 - 3b - 15 \leq 0$ (D) $a > 0$ & $b > 0$

10.7 If $\frac{a_0}{n+1} + \frac{a_1}{n} + \frac{a_2}{n-1} + \ldots\ldots\ldots + \frac{a_{n-1}}{2} + a_n = 0$, then the equation $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots + a_{n-1}x + a_n = 0$ has

(A) exactly one root in (0, 1) (B) at least one root in (0, 1)
(C) no root in (0, 1) (D) at the most one root in (0, 1)

10.8 $f : [0, 4] \to R$ is a differentiable function. Then for some $a, b \in (0, 4)$, $f^2(4) - f^2(0) =$

(A) $8f'(a) . f(b)$ (B) $4f'(b) f(a)$ (C) $2f'(b) f(a)$ (D) $f'(b) f(a)$

10.9 The set of values of the parameter 'a' for which the function ; $f(x) = 8ax - a \sin 6x - 7x - \sin 5x$ increases & has no critical points for all $x \in R$, is

(A) $[-1, 1]$ (B) $(-\infty, -6)$ (C) $(6, +\infty)$ (D) $[6, +\infty)$

10.10 The function $f(x) = \frac{\ell n(\pi + x)}{\ell n(e + x)}$ is :

(A) Increasing on $(0,\infty)$. (B) Decreasing on $(0,\infty)$.

(C) Increasing on $\left(0, \frac{\pi}{e}\right)$, decreasing on $\left(\frac{\pi}{e}, \infty\right)$ (D) Decreasing on $\left(0, \frac{\pi}{e}\right)$, increasing on $\left(\frac{\pi}{e}, \infty\right)$.

10.11 Let f(x) = {x}, where {.} denotes the fractional part. For f(x), x = 5 is
(A) a point of local maxima (B) a point of local minima
(C) neither a point of local minima nor maxima (D) none of these

10.12 The equation $x^3 - 3x + [a] = 0$, where [.] denotes the greatest integer function, will have three real and distinct roots if
(A) $a \in (-\infty, 2)$ (B) $a \in (0, 2)$
(C) $a \in (\infty, -2) \cup (0, \infty)$ (D) $a \in [-1, 2)$

10.13 A function is defined as $f(x) = ax^2 - b|x|$ where a and b are constants. x = 0 will be a point of maxima of f(x) if
(A) a > 0, b > 0 (B) a > 0, b < 0 (C) a < 0, b < 0 (D) a < 0, b > 0

10.14 If $f(x) = 2x^3 - 3(a + 1)x^2 + 6ax - 12$ has local maximum at x_1 and local minimum at x_2 and if $2x_1 = x_2$ then value of a is :
(A) 1 (B) $\frac{1}{2}$ (C) –1 (D) 2

10.15 Let f(x) be defined as

$$f(x) = \begin{cases} \tan^{-1}\alpha - 5x^2, & 0 < x < 1 \\ -6x, & x \ge 1 \end{cases}$$

f(x) can have a local maximum at x = 1 if value of α is
(A) 0 (B) – 1 (C) – tan 1 (D) – 2

10.16 A truck is to be driven 300 km on a highway at a constant speed of x kmph. Speed rules of the highway required that $30 \le x \le 60$. The fuel costs Rs. 10 per litre and is consumed at the rate of $2 + \frac{x^2}{600}$ liters per hour. The wages of the driver are Rs. 200 per hour. The most economical speed to drive the truck, in kmph, is
(A) 30 (B) 60 (C) $30\sqrt{3.3}$ (D) $20\sqrt{3.3}$

10.17 Slope of tangent to the curve $y = 2e^x \sin\left(\frac{\pi}{4} - \frac{x}{2}\right) \cos\left(\frac{\pi}{4} - \frac{x}{2}\right)$, where $0 \le x \le 2\pi$ is minimum at x =
(A) 0 (B) π (C) 2π (D) none of these

10.18 The lateral edge of a regular hexagonal pyramid is 1 cm . If the volume is maximum, then its height must be equal to :
(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) 1

10.19 **S1 :** The length of sub-tangent to the curve $x^2y^2 = 16a^4$ at the point (–2a, 2a) is 2a. (where a > 0)
S2 : x + y = 3 is a normal to the curve $x^2 = 4y$ at a certain point.
S3 : Curves $y = -4x^2$ and $y = e^{-\frac{x}{2}}$ are orthogonal.
S4 : If $a \in (-1, 0)$, then tangent at each point of the curve $y = \frac{2}{3}x^3 - 2ax^2 + 2x + 5$ makes an acute angle with the positive direction of x-axis.
(A) TFFT (B) TTFT (C) FTFT (D) FTTF

10.20 S_1 : Equation $x^4 - 4x - 1 = 0$ has three distinct real roots.

S_2 : Lagrange's mean value theorem is applicable to $f(x) = \begin{cases} x, x < 1 \\ 1/x, x \geq 1 \end{cases}$ over $x \in [0, 2]$

S_3 : $f(x) = x - \sin x$ is not strictly increasing function.

S_4 : $f(x) = \sin^4 x + \cos^4 x$ is increasing function if $x \in \left(\frac{\pi}{4}, \frac{3\pi}{8}\right)$

(A) TFFT (B) TTFT (C) TFTF (D) FFFT

10.21 S_1 : $f(x) = x\,e^{x(1-x)}$ is increasing on $\left[-\frac{1}{2}, 1\right]$

S_2 : Critical points for $f(x) = (x-2)^{2/3}(2x+1)$ are $x = 1$ & $x = 2$.

S_3 : $f(x) = 2x + \cot^{-1}x + \log\left(\sqrt{1+x^2} - x\right)$, then f(x) is monotonic for $x \in R$.

S_4 : Let f is a real valued differentiable function such that $f(x).\ f'(x) < 0$ for all real x. Then $f^2(x)$ is decreasing function.

(A) TTTT (B) TFFT (C) FTFT (D) FFTT

10.22 S_1 : If a function is discontinuous at a point may or may not be a point of local extremum.

S_2 : The function $f(x) = (4\sin^2 x - 1)^n (x^2 + x + 1)$, $n \in N$ has a local minimum at $x = \frac{\pi}{6}$, then n can be any odd number.

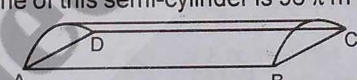
S_3 : If $a + b = 8$, $a > 0$, $b > 0$, then the minimum value of $a^3 + b^3$ is 120.

S_4 : The minimum distance of origin from the curve $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$, $a > 0$, $b > 0$, is $a + b$.

(A) TTFF (B) TFFT (C) TFTT (D) TTFT

10.23 S_1 : If the angle made by the tangent drawn at any point (x, y) of a curve with positive x-axis is $\tan^{-1}(x^2 - 2x)$, then a local maximum of the curve is at $x = 0$

S_2 : If the perimeter of rectangular base ABCD of a semi-right circular cylinder is 36 m, then the maximum volume of this semi-cylinder is $96\,\pi\ m^3$



S_3 : Tangent to the curve $y = x^3 - 6x^2 + 9x + 4$, $0 \leq x \leq 5$ has maximum slope at x equal to 5

S_4 : The minimum and maximum values of y in $4x^2 + 12xy + 10y^2 - 4y + 3 = 0$ are respectively $y_{min} = 1$ and $y_{max} = 3$.

(A) TTFF (B) TFTT (C) TFFT (D) TTFT

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

10.24 The real values of λ for which equation $x^3 + x^2 - x + \lambda = 0$ has three distinct real roots, lies in interval

(A) $[-2, 2]$ (B) $[-1, 0]$ (C) $(-1, 0)$ (D) $\left[-1, \frac{1}{2}\right]$

10.25 If α, β, γ are roots of $x^3 + (a^4 + 4a^2 + 1)x = x^2 + a^2$ then minimum value of $\sum\left\{\frac{\alpha}{\beta} + \left(\frac{\alpha}{\beta}\right)^{-1}\right\}$ is

(A) $\log_2 8$ (B) $\log_2 16$ (C) $\log_3 27$ (D) $\log_4 256$

10.26 If $f(x) = x^3 - 3x^2 + 2x$ & $f(x) = \lambda$ has exactly two solutions which are of opposite signs, then λ equals

(A) $\frac{-2\sqrt{3}}{9}$ (B) $\frac{-2}{3\sqrt{3}}$ (C) $\frac{2}{3\sqrt{3}}$ (D) $\frac{2\sqrt{3}}{9}$

10.27 The abscissa of a point on the curve $xy = (a + x)^2$, the normal at which cuts off numerically equal intercepts from the coordinate axes is

(A) $-\frac{a}{\sqrt{2}}$ (B) $\sqrt{2}\,a$ (C) $\frac{\sqrt{2}\,a}{2}$ (D) $-\sqrt{2}\,a$

10.28 For function $f(x) = \frac{\ln x}{x}$, which of the following statements are true ?

(A) f(x) has horizontal tangent at x = e (B) f(x) cuts the x-axis only at one point
(C) f(x) is many-one function (D) f(x) has one vertical tangent

10.29 The equation of tangent drawn to the curve $y = (x + 1)^3$, from origin is

(A) y = 3x (B) y = – 3x (C) 4y = 27x (D) y = 0

10.30 For the curve represented parametrically by the equations, $x = 2\,\ell n \cot t + 1$ & $y = \tan t + \cot t$

(A) tangent at $t = \pi/4$ is parallel to x – axis (B) normal at $t = \pi/4$ is parallel to y – axis
(C) tangent at $t = \pi/4$ is parallel to the line y = x (D) tangent and normal intersect at the point (2, 1)

10.31 Let $g'(x) > 0$ and $f'(x) < 0$, $\forall x \in R$, then

(A) $g(f(x+1)) > g(f(x-1))$ (B) $f(g(x-1)) > f(g(x+1))$
(C) $g(f(x+1)) < g(f(x-1))$ (D) $g(g(x+1)) < g(g(x-1))$

10.32 If $f(x) = x^3 - x^2 + 100x + 1001$, then

(A) f(2000) > f(2001) (B) $f\left(\frac{1}{1999}\right) > f\left(\frac{1}{2000}\right)$

(C) f(x + 1) > f(x – 1) (D) f(3x – 5) > f(3x)

10.33 If the derivative of an odd cubic polynomial vanishes at two different values of 'x' then

(A) coefficient of x^3 & x in the polynomial must be same in sign
(B) coefficient of x^3 & x in the polynomial must be different in sign
(C) the values of 'x' where derivative vanishes are closer to origin as compared to the respective roots on either side of origin.
(D) the values of 'x' where derivative vanishes are far from origin as compared to the respective roots on either side of origin.

10.34 Let $f(x) = (x - 1)^4 (x - 2)^n$, $n \in N$. f(x) has

(A) Local minimum at x = 2 if n is even (B) Local minimum at x = 1 if n is odd
(C) Local maximum at x = 1 if n is odd (D) Local minimum at x = 1 if n is even

SECTION - III : ASSERTION AND REASON TYPE

10.35 **Statement -1 :** The ratio of length of tangent to length of normal is inversely proportional to numerical value of the ordinate of the point of tangency at the curve $y^2 = 4ax$.

Statement -2 : Length of normal & tangent to a curve y = f(x) is $\left|y\sqrt{1+m^2}\right|$ and $\left|\frac{y\sqrt{1+m^2}}{m}\right|$, where $m = \frac{dy}{dx}$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

10.36 **Statement -1 :** The curves $y^2 = 16x$ and $x^2 = 16y$ are orthogonal curves.
Statement -2 : Two curves are said to be orthogonal, if they intersect each other at right angle at each point of intersection.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

10.37 **Statement -1 :** Shortest distance between $|x| + |y| = 2$ & $x^2 + y^2 = 16$ is $4 - \sqrt{2}$
Statement -2 : Shortest distance between the two differentiable curves lies along the common normal.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

10.38 **Statement -1 :** $\dfrac{2e^{x_1} + e^{x_2}}{3} > e^{\left(\frac{2x_1+x_2}{3}\right)}$, where e is Napier ' s constant.
Statement -2 : If $f'(x)$ and $f''(x)$ is positive $\forall x \in R$, then f(x) increases with concavity up $\forall x \in R$ and any chord lies above the curve.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

10.39 Consider f(x) is a function such that f(1) = 1, f(2) = 4 and f(3) = 9
Statement-1 : $f''(x) = 2$ for some $x \in (1, 3)$
Statement-1 : $g(x) = x^2 \Rightarrow g''(x) = 2 \ \forall x \in R$
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

10.40 **Statement -1 :** If f(x) is increasing function with concavity upwards, then concavity of $f^{-1}(x)$ is also upwards.
Statement -2 : If f(x) is decreasing function with concavity upwards, then concavity of $f^{-1}(x)$ is also upwards.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

10.41 **Statement - 1 :** Let $f(x) = 5 - 4(x - 2)^{2/3}$, then at x = 2 the function f(x) attains neither least value nor greatest value.
Statement - 2 : x = 2 is the only critical point of f(x).
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

10.42 **Statement - 1 :** The largest term in the sequence $a_n = \dfrac{n^2}{n^3 + 200}$, $n \in N$ is $\dfrac{(400)^{2/3}}{600}$.

Statement - 2 : $f(x) = \dfrac{x^2}{x^3 + 200}$, x > 0, then at $x = (400)^{1/3}$, f(x) is maximum.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

10.43 **Statement 1 :** ABC is given triangle having respective sides a,b,c. D,E,F are points of the sides BC,CA,AB respectively so that AFDE is a parallelogram. The maximum area of the parallelogram is $\frac{1}{4}$ bcsinA.

Statement 2 : Maximum value of $2kx - x^2$ is at $x = k$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

If a continuous function f defined on the real line **R**, assumes positive and negative values in **R** then the equation $f(x) = 0$ has a root in **R**. For example, if it is known that a continuous function f on **R** is positive at some point and its minimum value is negative then the equation $f(x) = 0$ has a root in **R**.
Consider $f(x) = ke^x - x$ for all real x where k is a real constant.

10.44 The line $y = x$ meets $y = ke^x$ for $k \le 0$ at
(A) no point (B) one point (C) two points (D) more than two points

10.45 The positive value of k for which $ke^x - x = 0$ has only one root is
(A) $\frac{1}{e}$ (B) 1 (C) e (D) $\log_e 2$

10.46 For $k > 0$, the set of all values of 'k' for which $ke^x - x = 0$ has two distinct roots is
(A) $\left(0, \frac{1}{e}\right)$ (B) $\left(\frac{1}{e}, 1\right)$ (C) $\left(\frac{1}{e}, \infty\right)$ (D) (0, 1)

Comprehension # 2

Let a(t) is a function of t such that $\frac{da}{dt} = 2$ for all values of t and $a = 0$ when $t = 0$. Further $y = m(t)\,x + c(t)$ is tangent to the curve $y = x^2 - 2ax + a^2 + a$ at the point whose abscissa is 0. Then

10.47 If the rate of change of distance of vertex of $y = x^2 - 2ax + a^2 + a$ from the origin with respect to t is k, then k =
(A) 2 (B) $2\sqrt{2}$ (C) $\sqrt{2}$ (D) $4\sqrt{2}$

10.48 If the rate of change of c(t) with respect to t, when $t = k$, is ℓ, then
(A) $16\sqrt{2} - 2$ (B) $8\sqrt{2} + 2$ (C) $10\sqrt{2} + 2$ (D) $16\sqrt{2} + 2$

10.49 The rate of change of m(t), with respect to t, at $t = \ell$ is
(A) –2 (B) 2 (C) –4 (D) 4

Comprehension : 3

Let f and g are two functions such that f(x) & g(x) are continuous in [a, b] and differentiable in (a, b)

Then at least one $c \in (a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

(i) If $f(a) = f(b)$, then $f'(c) = 0$ (RMVT)

(ii) If $f(a) \ne f(b)$ and $a \ne b$, (LMVT)

(iii) If $g'(x) \ne 0$, then $\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}$ (cauchy theorem)

10.50 The set of values of k, for which equation $x^3 - 3x + k = 0$ has two distinct roots in (0, 1) is
(A) (1, 4) (B) $(0, \infty)$ (C) (0,1) (D) ϕ

10.51 Which of the following is true?
(A) $|\tan^{-1}x - \tan^{-1}y| \le |x - y|\ \forall\ x, y \in R$
(B) $|\tan^{-1}x - \tan^{-1}y| \ge |x - y|\ \forall\ x, y \in R$
(C) $|\sin x - \sin y| \ge |x - y|\ \forall\ x, y \in R$
(D) none of these

10.52 Let $0 < \alpha < \theta < \beta < \frac{\pi}{2}$, then $\frac{\sin\alpha - \sin\beta}{\cos\alpha - \cos\beta}$ is equal to
(A) $\tan\theta$ (B) $-\tan\theta$ (C) $\cot\theta$ (D) $-\cot\theta$

Comprehension # 4

Let $y = a\sqrt{x} + bx$ be curve, $(2x - y) + \lambda(2x + y - 4) = 0$ be family of lines.

10.53 If curve has slope $-\frac{1}{2}$ at (9, 0) then a tangent belonging to family of lines is
(A) $x + 2y - 5 = 0$ (B) $x - 2y + 3 = 0$ (C) $3x - y - 1 = 0$ (D) $3x + y - 5 = 0$

10.54 A line of the family cutting positive intercepts on axes and forming triangle with coordinate axes, then minimum length of the line segment between axes is-
(A) $(2^{2/3} - 1)^{3/2}$ (B) $(2^{2/3} + 1)^{3/2}$ (C) $7^{3/2}$ (D) 27

10.55 Two perpendicular focal chords of curve $y^2 - 4x - 4y + 4 = 0$ form diagonals of a quadrilateral. Minimum area of quadrilateral is-
(A) 16 (B) 32 (C) 64 (D) 50

Comprehension # 5

A function f(x) having the following properties;
(i) f(x) is continuous except at x = 3
(ii) f(x) is differentiable except at x = – 2 and x = 3
(iii) $f(0) = 0$, $\lim_{x\to 3} f(x) \to -\infty$, $\lim_{x\to -\infty} f(x) = 3$, $\lim_{x\to\infty} f(x) = 0$
(iv) $f'(x) > 0\ \forall\ x \in (-\infty, -2) \cup (3, \infty)$ and $f'(x) \le 0\ \forall\ x \in (-2, 3)$
(v) $f''(x) > 0\ \forall\ x \in (-\infty, -2) \cup (-2, 0)$ and $f''(x) < 0\ \forall\ x \in (0, 3) \cup (3, \infty)$
then answer the following questions

10.56 Maximum possible number of solutions of $f(x) = |x|$ is
(A) 2 (B) 1 (C) 3 (D) 4

10.57 Graph of function $y = f(-|x|)$ is
(A) differentiable for all x, if $f'(0) = 0$
(B) continuous but not differentiable at two points, if $f'(0) = 0$
(C) continuous but not differentiable at one points, if $f'(0) = 0$
(D) discontinuous at two points, if $f'(0) = 0$

10.58 $f(x) + 3x = 0$ has five solutions if
(A) $f(-2) > 6$
(B) $f'(0) < -3$ and $f(-2) > 6$
(C) $f'(0) > -3$
(D) $f'(0) > -3$ and $f(-2) > 6$

SECTION - V : MATRIX - MATCH TYPE

10.59

	Column – I		Column – II
(A)	Circular plate is expanded by heat from radius 5 cm to 5.06 cm. Approximate increase in area is	(p)	2
(B)	If an edge of a cube increases by 1% then percentage increase in volume is	(q)	0.6π
(C)	If the rate of decrease of $y = \frac{x^2}{2} - 2x + 5$ is twice the rate of decrease of x, then x is equal to (given that rate of decrease is non-zero)	(r)	3
(D)	Rate of increase in area of equilateral triangle of side 15cm, when each side is increasing at the rate of 0.1cm/sec; is	(s)	$\frac{3\sqrt{3}}{4}$
		(t)	4

10.60

	Column – I		Column – II
(A)	If portion of the tangent at any point on the curve $x = at^3$, $y = at^4$ between the axes is divided by the point of contact in the ratio m : n externally, then $\lvert n + m \rvert$ is equal to (m and n are coprime)	(p)	1
(B)	The area of triangle formed by normal at the point (1, 0) on the curve $x = e^{\sin y}$ with axes is	(q)	$\frac{1}{2}$
(C)	If the angle between curves $x^2y = 1$ and $y = e^{2(1-x)}$ at the point (1, 1) is θ then tan θ is equal to	(r)	7
(D)	The length of sub-tangent at any point on the curve $y = be^{x/3}$ is equal to	(s)	3
		(t)	0

10.61

	Column – I		Column – II
(A)	A function f is differentiable in [0, 5] and f(0) = 4 and f(5) = –1. If $g(x) = \frac{f(x)}{x+1}$ and c ∈ (0, 5), then g′(c) is equal to	(p)	3
(B)	Let f(x) and g(x) be differentiable for $0 \le x \le 1$, f(0) = 2, g(0) = 0, f(1) = 6. Let there exists a real number c ∈ (0, 1) such that f′(c) = 2g′(c), then g(1) is equal to	(q)	– 5/6
(C)	The length of longest interval in which $f(x) = 3\sin x - 4\sin^3 x$ is increasing, is $\frac{\pi}{\lambda}$, then λ is	(r)	15
(D)	If Lagrange's mean value theorem is satisfied for $f(x) = \sqrt{25 - x^2}$ and c ∈ (1, 5), then the value of c^2 is	(s)	2
		(t)	10

10.62

	Column - I		Column - I
(A)	Number of values of 'x' lying in $\left(0, \frac{\pi}{2}\right)$ at which f(x) = ℓn (sinx) is not monotonic, is	(p)	0
(B)	If the greatest interval in which the function $f(x) = x^3 - 3x + 2$ is decreasing is [a, b], then a + b =	(q)	2
(C)	If $f(x) = \frac{x^2 + 2}{[x]}$, $1 \le x \le 3$ (where [.] greatest integer function), then least value of f(x) is	(r)	– 3
(D)	Set of all possible values of 'a' such that $f(x) = e^{2x} - (a + 1)e^x + 2x$ is monotonically increasing for all x ∈ R is (– ∞, k], then k =	(s)	3
		(t)	– 2

10.63

Column – I		Column – II	
(A)	Number of points which are local extrema of $f(x) = \begin{cases}(2+x)^3 & ; -3 \le x \le -1 \\ x^{2/3} & ; -1 < x < 2\end{cases}$	(p)	1
(B)	If a + b = 1; a > 0, b > 0, then the minimum value of $\sqrt{\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)}$ is	(q)	2
(C)	The maximum value attained by $y = 10 - \|x - 10\|, -1 \le x \le 3$, is	(r)	3
(D)	If $P(t^2, 2t)$, $t \in [0, 2]$ is an arbitrary point on parabola $y^2 = 4x$ and Q is foot of perpendicular from focus S on the tangent at P, then maximum area of triangle PQS is	(s)	4
		(t)	5

SECTION - VI : INTEGER TYPE

10.64 Let α be the angle in radians between $\frac{x^2}{36} + \frac{y^2}{4} = 1$ and the circle $x^2 + y^2 = 12$ at their points of intersection. If $\alpha = \tan^{-1}\frac{k}{2\sqrt{3}}$, then find the value of k^2.

10.65 Find the minimum value of $(x_1 - x_2)^2 + \left(\sqrt{2 - x_1^2} - \frac{9}{x_2}\right)^2$ where $x_1 \in (0, \sqrt{2})$ and $x_2 \in R^+$.

10.66 The values of 'a' for which the function $f(x) = \sin x - a \sin 2x - \frac{1}{3}\sin 3x + 2ax$ increases throughout the number line is $[\lambda, \infty)$, then find λ

10.67 A cone is made from a circular sheet of radius $\sqrt{3}$ by cutting out a sector and keeping the cut edges of the remaining piece together. The maximum volume attainable for the cone is $\frac{\lambda\pi}{3}$, then find λ

10.68 If f(x) is a twice differentiable function such that f(a) = 0, f(b) = 2, f(c) = –1, f(d) = 2, f(e) = 0, where a < b < c < d < e, then the minimum number of zeroes of $g(x) = (f'(x))^2 + f''(x) f(x)$ in the interval [a, e] is

10.69 The chord of parabola $y = -p^2x^2 + 5px - 4$ touches the curve $y = \frac{1}{(1-x)}$ at the point x = 2 and is bisected by that point. Find number of values of 'p'.

10.70 If the possible values of 'a' such that the inequality $3 - x^2 > |x - a|$ has atleast one negative solution is $a \in \left(-\frac{13}{4}, \lambda\right)$, then find λ

10.71 Let $f(x) = \begin{cases} xe^{ax} & , x \le 0 \\ x + ax^2 - x^3, & x > 0\end{cases}$ where a is positive constant and the interval in which f′ (x) is increasing is $\left(-\frac{\lambda_1}{a}, \frac{a}{\lambda_2}\right)$, then find $(\lambda_1 + \lambda_2)$

10.72 A cubic f(x) vanishes at x = –2 and has relative minimum/maximum at x = – 1 and $x = \frac{1}{3}$. If $\int_{-1}^{1} f(x)\, dx = \frac{14}{3}$, the cubic $f(x) = \lambda_1x^3 + \lambda_2x^2 - x + 2$, then find $(\lambda_1 + \lambda_2)$

10.73 P(x) is a polynomial of degree three having local maxima at x = – 1 and given P(– 1) = 10, P(1) = – 6. Further P′(x) has local minima at x = 1. Determine square of distance between local maxima and local minima.

TOPIC 11 INDEFINITE INTEGRATION

SECTION - I : STRAIGHT OBJECTIVE TYPE

11.1 If $\int \frac{2\cos x - \sin x + \lambda}{\cos x + \sin x - 2} dx = A \ell n |\cos x + \sin x - 2| + Bx + C$.
Then the ordered triplet A, B, λ is

(A) $\left(\frac{1}{2}, \frac{3}{2}, -1\right)$ (B) $\left(\frac{3}{2}, \frac{1}{2}, -1\right)$ (C) $\left(\frac{1}{2}, -1, -\frac{3}{2}\right)$ (D) $\left(\frac{3}{2}, -1, \frac{1}{2}\right)$

11.2 If $y^2 = x^2 - x + 1$ and $I_n = \int \frac{x^n}{y} dx$ and $AI_3 + BI_2 + CI_1 = x^2y$ then ordered triplet A, B, C is

(A) $\left(\frac{1}{2}, -\frac{1}{2}, 1\right)$ (B) (3, 1, 0) (C) (1, – 1, 2) (D) $\left(3, -\frac{5}{2}, 2\right)$

11.3 $\int 4\cos\left(x + \frac{\pi}{6}\right) \cos 2x . \cos\left(\frac{5\pi}{6} + x\right) dx$

(A) $-\left(x + \frac{\sin 4x}{4} + \frac{\sin 2x}{2}\right) + c$ (B) $-\left(x + \frac{\sin 4x}{4} - \frac{\sin 2x}{2}\right) + c$

(C) $-\left(x - \frac{\sin 4x}{4} + \frac{\sin 2x}{2}\right) + c$ (D) $-\left(x - \frac{\sin 4x}{4} + \frac{\cos 2x}{2}\right) + c$

11.4 If $\int \frac{e^{x-1}}{(x^2 - 5x + 4)} 2x\, dx = A\,F(x-1) + B\,F(x-4) + C$ and $F(x) = \int \frac{e^x}{x} dx$, then A & B ordered set is

(A) $\left(-\frac{2}{3}, \frac{8}{3}\right)$ (B) $\left(-\frac{2}{3}, \frac{8e^3}{3}\right)$ (C) $\left(\frac{8}{3}, \frac{2}{3}\right)$ (D) $\left(-\frac{2}{3}, -\frac{8e^3}{3}\right)$

11.5 $\int x^{2/3}(1 + x^{1/2})^{-13/3} dx =$

(A) $\frac{3}{5}(1 + x^{-1/2})^{-10/3} + C$ (B) $\frac{3}{5}(1 + x^{1/2})^{-10/3} + C$

(C) $-\frac{(1 + x^{1/2})}{13} + C$ (D) none of these

11.6 Evaluate $\int x^2 \log(1 - x^2).dx$ and hence find the value of :

$\frac{1}{1.5} + \frac{1}{2.7} + \frac{1}{3.9} + ... =$

(A) $\frac{1}{4}\log 2$ (B) $\frac{2}{7} - \frac{2}{3}\log 2$ (C) $\frac{8}{9} - \frac{2}{3}\log 2$ (D) $-\frac{2}{3}\log 2$

11.7. $\int \frac{\cos^4 x\, dx}{\sin^3 x(\sin^5 x+\cos^5 x)^{3/5}} = -\frac{1}{2}(1+\cos^A x)^B + C$, then find AB.

(A) 5 (B) $\frac{2}{5}$ (C) 2 (D) 1

11.8. $I_n = \int (\log x)^n dx$, then $I_n + nI_{n-1} =$

(A) $n(\log x)^n$ (B) $(n\log x)^{n-1}$ (C) $(\log x)^{n-1}$ (D) $(n\log x)^n$

11.9 If A is square matrix and e^A is defined as $e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} \,..... = \frac{1}{2}\begin{bmatrix} f(x) & g(x) \\ g(x) & f(x) \end{bmatrix}$, where

$A = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$ and $0 < x < 1$, I is an identity matrix, then $\int \frac{g(x)}{f(x)}.dx$ is equal to

(A) $\log(e^x + e^{-x}) + C$ (B) $\log(e^x - e^{-x}) + C$ (C) $\log(e^{2x} - 1) + C$ (D) None of these

11.10 The value of $\int e^{(x\sin x+\cos x)}\left(\frac{x^4\cos^3 x - x\sin x + \cos x}{x^2\cos^2 x}\right).dx$, is equal to

(A) $e^{x\sin x+\cos x}.\left(x + \frac{1}{x\cos x}\right) + C$ (B) $e^{x\sin x+\cos x}.\left(x\cos x + \frac{1}{x}\right) + C$

(C) $e^{x\sin x+\cos x}.\left(x - \frac{1}{x\cos x}\right) + C$ (D) none of these

11.11 If $f(x) = \sqrt{x}$, $g(x) = e^x - 1$, and $\int fog(x)\,dx = A\,fog(x) + B\tan^{-1}(fog(x)) + C$, then A + B is equal to

(A) 1 (B) 2 (C) 3 (D) none of these

11.12 $\int \frac{x + \sqrt[3]{x^2} + \sqrt[6]{x}}{x(1+\sqrt[3]{x})}dx$ is

(A) $\frac{3}{2}x^{2/3} + \tan^{-1} x^6 + C$ (B) $\frac{3}{2}x^{2/3} + 6\tan^{-1} x^6 + C$

(C) $\frac{3}{2}x^{2/3} + 6\tan^{-1} x^{1/6} + C$ (D) None of these

11.13 If $\int \frac{dx}{a+\cos x} = f\left(\tan\frac{x}{2}\right) + C$, then –

S_1 : f is a log function for a = 0

S_2 : f is a inverse trigonometric function for |a| < 1

S_3 : f is a polynomial function for a = 1

S_4 : f is rational function but not polynomial for a = 1

(A) T T T F (B) T F T F (C) F T T F (D) T F F T

11.14 S_1 : $\int e^x\left(\ell nx+\frac{1}{x^2}\right)dx = e^x\left(\ell n x-\frac{1}{x^2}\right)+C$

S_2 : $\int \frac{dx}{\sin\frac{x}{2}\sqrt{\cos\frac{x}{2}}} = -\ell n\left|\frac{1+\sqrt{\cos\frac{x}{2}}}{1-\sqrt{\cos\frac{x}{2}}}\right| - 2\tan^{-1}\sqrt{\cos\frac{x}{2}}+C$

S_3 : $\int\frac{dx}{\sin x\cos^3 x} = \frac{1}{2}\tan^2 x+\ell n(\tan x)+C$

S_4 : $\int\frac{\sin x+\cos x}{\sin 2x+3}dx = \frac{1}{2}\tan^{-1}\left(\frac{\sin x-\cos x}{2}\right)+C$

(A) T T T F (B) F F T F (C) F T T F (D) T F F T

SECTION - III : ASSERTION AND REASON TYPE

11.15 **Statement-1 :** If $x > 0$, $x \neq 1$ then $\int(\log_x e-(\log_x e)^2)dx = x\log_x e + C$

Statement-2 : $\int e^x(f(x)+f'(x))dx = e^x f(x) + C$ and $e^t = x$ iff $t = \ell nx$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

11.16 **Statement-1 :** If $y = \sin^{-1}x$, then $\int\sin^{-1}x\,dx = \int y\cos y\,dy + c$

Statement-2 : If $y = f^{-1}(x)$, then $\int f^{-1}(x)\,dx = \int yf'(y)\,dy + c$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

11.17 **Statement-1 :** $\int 2^{\tan^{-1}x}\,d(\cot^{-1}x) = \frac{2^{\tan^{-1}x}}{\ell n 2} + c$

Statement-2 : $\frac{d}{dx}(a^x + c) = a^x\,\ell n\,a$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

11.18 **Statement-1 :** $\int e^{\sin^{-1}x}\left(1-\frac{x}{\sqrt{1-x^2}}\right)dx = e^{\sin^{-1}x}.\sqrt{1-x^2} + c$

Statement-2 : $\int e^{g(x)}(g'(x)f(x)+f'(x))dx = e^{g(x)}f(x)+c$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

If A is a square matrix and e^A is defined as $e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} \dots \infty = \frac{1}{2}\begin{bmatrix} f(x) & g(x) \\ g(x) & f(x) \end{bmatrix}$, where

$A = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$, and I being the identity matrix then

11.19. $\int \frac{g(x)}{f(x)} dx =$

(A) $\log(e^x + e^{-x}) + C$ (B) $\log|e^x - e^{-x}| + C$ (C) $\log|e^{2x} - 1| + C$ (D) $\log(e^{2x} + 1) - x + c$

11.20 $\int \frac{f(x)}{\sqrt{g(x)}} dx =$

(A) $\frac{1}{2\sqrt{e^x - 1}} - \text{cosec}^{-1}(e^x) + C$ (B) $\frac{2}{\sqrt{e^x - e^{-x}}} - \sec^{-1}(e^x) + C$

(C) $\sqrt{e^{2x} - 1} + \sec^{-1}(e^x) + C$ (D) $\sqrt{e^{2x} - 1} + \tan^{-1}\left(\sqrt{e^{2x} - 1}\right) + c$

Comprehension # 2

Integrals of the form $\int \frac{p_m(x)}{\sqrt{ax^2 + bx + c}} dx$, where $p_m(x)$ is a polynomial of degree m, are calculated by the reduction formula.

$$\int \frac{p_m(x)}{\sqrt{ax^2 + bx + c}} dx = p_{m-1}(x) \sqrt{ax^2 + bx + c} + \lambda \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

where $p_{m-1}(x)$ is a polynomial of degree (m – 1) and λ is some constant number.

e.g. $I = \int \frac{x^3 - x - 1}{\sqrt{x^2 + 2x + 2}} dx$ then applying the above formula, we can write

$$\int \frac{x^3 - x - 1}{\sqrt{x^2 + 2x + 2}} dx = (Ax^2 + Bx + C) \sqrt{x^2 + 2x + 2} + \lambda \int \frac{dx}{\sqrt{x^2 + 2x + 2}}$$

differentiate both sides, we get

$$\frac{x^3 - x - 1}{\sqrt{x^2 + 2x + 2}} = (Ax^2 + Bx + C). \frac{2(x+1)}{2\sqrt{x^2 + 2x + 2}} + (2Ax + B) \sqrt{x^2 + 2x + 2} + \frac{\lambda}{\sqrt{x^2 + 2x + 2}}$$

$x^3 - x - 1 = (Ax^2 + Bx + C)(x + 1) + (2Ax + B)(x^2 + 2x + 2) + \lambda$

On compairing coefficeints of like powers of x we obtain the values of A, B, C and λ.

11.21 If $\int \frac{x^3 - 6x^2 + 11x - 6}{\sqrt{x^2 + 4x + 3}} dx = (Ax^2 + Bx + C) \sqrt{x^2 + 4x + 3} + \lambda \int \frac{dx}{\sqrt{x^2 + 4x + 3}}$, then value of 'A' is

(A) $\frac{1}{3}$ (B) 1 (C) 3 (D) – 1/3

11.22 In Q.No. 11.21 value of 'C' is

(A) – 37 (B) $-\frac{14}{3}$ (C) $\frac{14}{3}$ (D) 37

11.23 In Q.No. 11.21 value of 'λ', is

(A) 66 (B) – 66 (C) $\frac{37}{3}$ (D) $-\frac{37}{3}$

Comprehension # 3

Let $I_{n,m} = \int \sin^n x \cos^m x . dx$. Then we can relate $I_{n,m}$ with each of the following

(i) $I_{n-2,m}$ (ii) $I_{n+2,m}$ (iii) $I_{n,m-2}$

(iv) $I_{n,m+2}$ (v) $I_{n-2,m+2}$ (vi) $I_{n+2,m-2}$

Suppose we want to establish a relation between $I_{n,m}$ and $I_{n,m-2}$, then we set

$P(x) = \sin^{n+1}x \cos^{m-1}x$ ....(1)

In $I_{n,m}$ and $I_{n,m-2}$ the exponent of cosx is m and m – 2 respectively, the minimum of the two is m – 2, adding 1 to the minimum we get m – 2 + 1 = m – 1. Now choose the exponent m – 1 of cosx in P(x). Similarly choose the exponent of sin x for P(x)

Now differentiating both sides of (1), we get

$$P'(x) = (n+1)\sin^n x \cos^m x - (m-1)\sin^{n+2}x \cos^{m-2}x$$
$$= (n+1)\sin^n x \cos^m x - (m-1)\sin^n x\,(1-\cos^2 x)\cos^{m-2}x$$
$$= (n+1)\sin^n x \cos^m x - (m-1)\sin^n x \cos^{m-2}x + (m-1)\sin^n x \cos^m x$$
$$= (n+m)\sin^n x \cos^m x - (m-1)\sin^n x \cos^{m-2}x$$

Now integrating both sides, we get

$\sin^{n+1}x \cos^{m-1}x = (n+m)\, I_{n,m} - (m-1)\, I_{n,m-2}$.

Similarly we can establish the other relations.

11.24 The relation between $I_{4,2}$ and $I_{2,2}$ is

(A) $I_{4,2} = \frac{1}{6}(-\sin^3 x \cos^3 x + 3I_{2,2})$

(B) $I_{4,2} = \frac{1}{6}(\sin^3 x \cos^3 x + 3I_{2,2})$

(C) $I_{4,2} = \frac{1}{6}(\sin^3 x \cos^3 x - 3I_{2,2})$

(D) $I_{4,2} = \frac{1}{4}(-\sin^3 x \cos^3 x + 2I_{2,2})$

11.25 The relation between $I_{4,2}$ and $I_{6,2}$ is

(A) $I_{4,2} = \frac{1}{5}(\sin^5 x \cos^3 x + 8I_{6,2})$

(B) $I_{4,2} = \frac{1}{5}(-\sin^5 x \cos^3 x + 8I_{6,2})$

(C) $I_{4,2} = \frac{1}{5}(\sin^5 x \cos^3 x - 8I_{6,2})$

(D) $I_{4,2} = \frac{1}{6}(\sin^5 x \cos^3 x + 8I_{6,2})$

11.26 The relation between $I_{4,2}$ and $I_{4,4}$ is

(A) $I_{4,2} = \frac{1}{3}(\sin^5 x \cos^3 x + 8\, I_{4,4})$

(B) $I_{4,2} = \frac{1}{3}(-\sin^5 x \cos^3 x + 8\, I_{4,4})$

(C) $I_{4,2} = \frac{1}{3}(\sin^5 x \cos^3 x - 8\, I_{4,4})$

(D) $I_{4,2} = \frac{1}{3}(\sin^5 x \cos^3 x + 6\, I_{4,4})$

SECTION - V : MATRIX - MATCH TYPE

11.27 Column – I | Column – II

(A) If $I = \int \frac{\sin x - \cos x}{|\sin x - \cos x|}\,dx$, where $\frac{\pi}{4} < x < \frac{3\pi}{8}$, then I equal to — (p) $\sin x$

(B) If $\int \frac{x^2}{(x^3+1)(x^3+2)}\,dx = \frac{1}{3} f\left(\frac{x^3+1}{x^3+2}\right) + C$, then f(x) is equal to — (q) $x + c$

(C) If $\int \sin^{-1}x.\cos^{-1}x\,dx = f^{-1}(x)\left[\frac{\pi}{2}x - x f^{-1}(x) - 2\sqrt{1-x^2}\right] + \frac{\pi}{2}\sqrt{1-x^2} + 2x + C$, then f(x) is equal to — (r) $\ell n\,|x|$

(D) If $\int \frac{dx}{x f(x)} = f(f(x)) + C$, then f(x) is equal to — (s) $\sin^{-1} x$

(t) $-x + c$

11.28 Column – I | Column – II

(A) If $F(x) = \int \frac{x + \sin x}{1 + \cos x}\,dx$ and F(0) = 0, then F(π/2) = — (p) $-\frac{\pi}{2}$

(B) Let $F(x) = \int e^{\sin^{-1}x}\left(1 - \frac{x}{\sqrt{1-x^2}}\right)dx$ and F(0) = 1, If $F(1/2) = \frac{k\sqrt{3}\,e^{\pi/6}}{\pi}$, then k = — (q) $\frac{\pi}{3}$

(C) Let $F(x) = \int \frac{dx}{(x^2+1)(x^2+9)}$ and F(0) = 0, if $F(\sqrt{3}) = \frac{5}{36}k$, then k = — (r) $\frac{\pi}{4}$

(D) Let $F(x) = \int \frac{\sqrt{\tan x}}{\sin x \cos x}\,dx$ and F(0) = 0 if $F(\pi/4) = \frac{2k}{\pi}$, then k = — (s) π

(t) $\frac{\pi}{2}$

SECTION - VI : INTEGER TYPE

11.29 $\int \frac{x\cos\alpha + 1}{(x^2 + 2x\cos\alpha + 1)^{3/2}}\,dx = \frac{\lambda x}{\sqrt{x^2 + 2x\cos\alpha + 1}} + c$, then find λ

11.30 $\int \left(\frac{x-1}{x+1}\right)\frac{dx}{\sqrt{x^3 + x^2 + x}} = \lambda \tan^{-1}\sqrt{\left(x + \frac{1}{x} + 1\right)} + c$, then find λ

TOPIC 12 DEFINITE INTEGRATION

SECTION - I : STRAIGHT OBJECTIVE TYPE

12.1 $\int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx = \pi \cdot \frac{\cos\alpha}{1+\sin^2\alpha}$

(A) for no value of α
(B) for exactly two values of α in $(0, \pi)$
(C) for at least one α in $\left(\frac{\pi}{2}, \pi\right)$
(D) for exactly one α in $\left(0, \frac{\pi}{2}\right)$

12.2 Let $\lambda = \int_0^1 \frac{dx}{1+x^3}$, $p = \lim_{n\to\infty} \left[\frac{\prod_{r=1}^{n}(n^3+r^3)}{n^{3n}}\right]^{1/n}$, then ℓnp is equal to

(A) $\ell n 2 - 1 + \lambda$
(B) $\ell n 2 - 3 + 3\lambda$
(C) $2 \ell n 2 - \lambda$
(D) $\ell n 4 - 3 + 3\lambda$

12.3 $\int_0^{2\pi} \frac{e^{|\sin x|}\cos x}{1+e^{\tan x}} dx =$

(A) e^{π}
(B) 1
(C) $e^{\pi} - 1$
(D) 0

12.4 Consider the integrals

$I_1 = \int_0^1 e^{-x}\cos^2 x\, dx$, $I_2 = \int_0^1 e^{-x^2}\cos^2 x\, dx$, $I_3 = \int_0^1 e^{-\frac{x^2}{2}}\cos^2 x\, dx$, $I_4 = \int_0^1 e^{-\frac{x^2}{2}} dx$ .Then

(A) $I_2 > I_4 > I_1 > I_3$
(B) $I_2 < I_4 < I_1 < I_3$
(C) $I_1 < I_2 < I_3 < I_4$
(D) $I_1 > I_2 > I_3 > I_4$

12.5 $\int_0^2 \sqrt{x+\sqrt{x+\sqrt{x+\ldots\infty}}}\, dx$ is equal to $(x > 0)$

(A) $\frac{19}{6}$
(B) $\frac{17}{6}$
(C) $\frac{13}{6}$
(D) Can't determine

12.6 A function f(x) which satisfies the relation $f(x) = e^x + \int_0^1 e^x f(t)dt$, then f (x) is

(A) $\frac{e^x}{2-e}$
(B) $(e-2)e^x$
(C) $2e^x$
(D) $\frac{e^x}{2}$

12.7 If $m = \int_{-2}^{0} \frac{|\sin x|}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} dx$ and $n = \int_0^2 \frac{|\sin x|}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} dx$, where [.] represents greatest integer function, then

(A) m = n
(B) m = –n
(C) m = 2n
(D) m = –2n