6.17 S_1 : A function is invertible iff it is one-one

S_2 : Let f & g be two functions $R \to R$ such that gof is one-one then f must be one-one

S_3 : Fundamental period of $\sin\{x\}$ is 1. Where { . } represent fractional part functions.

S_4 : If $f : R \to R$ is an odd functions then $f(-x) = -f(x), \forall x \in R$

(A) TTFF (B) TTFT (C) FTTF (D) FTTT

6.18 Consider the following statements :

S_1 : Number of solutions of $[\sin^{-1}x] = \{1 + x\}$ is two

S_2 : $f(x) = x^3 + \tan x$ is surjective function

S_3 : All basic inverse trigonometric function are periodic.

S_4 : Domain of $f(x) = \sqrt{(x^2 - 3x - 10)\,\ell n^2 (x-3)}$ is $[5, \infty)$.

State, in order, whether S_1, S_2, S_3, S_4 are true or false

(A) FTFF (B) TTFF (C) TFTT (D) TTTT

6.19 Let $f : A \to B$ and $g : C \to D$ be functions for which composite function gof is defined :

S_1 : If each of f and g is one-one, then gof is one-one.

S_2 : If each of f and g is onto, then gof is onto.

S_3 : If B = C and gof is one-one, then g may not be one-one.

S_4 : If B = C and gof is onto, then f may not be onto.

State, in order, whether S_1, S_2, S_3, S_4 are true or false

(A) TTTF (B) TFTF (C) TFTT (D) FFFF

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

6.20 Let $f(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$ be a polynomial such that $f(1) = 2$, $f(2) = 4$, $f(3) = 6$, $f(4) = 8$ and $f(5) = 10$, then

(A) $f(6) = 132$ (B) $f(0) = -120$ (C) $f(0) = 120$ (D) $f(6) = 130$

6.21 If $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)\ \forall x \in R$, then f(x) is

(A) one – one and onto (B) one – one and into

(C) many – one and onto (D) non-invertible

6.22. If $g(x) = \left(4\cos^4 x - 2\cos 2x - \frac{1}{2}\cos 4x - x^7\right)^{\frac{1}{7}}$ then

(A) $g(g(101)) = 100$ (B) $g(g(101)) = 101$ (C) $g(g(0)) = 0$ (D) $g(g(-1)) = -1$

6.23 Which of the following functions are periodic ?

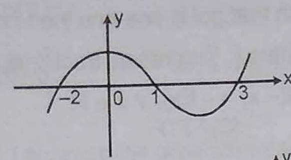
(A) $f(x) = \text{sgn}(e^{-x})$

(B) $f(x) = \begin{cases} 1 \text{ if } x \text{ is a rational number} \\ 0 \text{ if } x \text{ is an irrational number} \end{cases}$

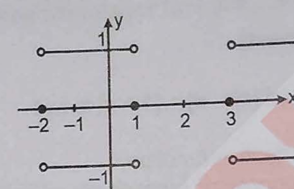
(C) $f(x) = \sqrt{\frac{8}{1+\cos x} + \frac{8}{1-\cos x}}$

(D) $f(x) = \left[x + \frac{1}{2}\right] + \left[x - \frac{1}{2}\right] + 2[-x]$ (where [ ] denotes greatest integer function)

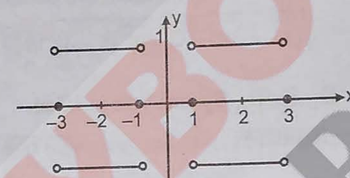
6.24 The graph of the function $y = f(x)$ is as shown in the figure. Then which one of the following graphs are correct ?



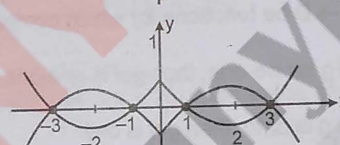
(A) $|y| = \text{sgn}(f(x))$



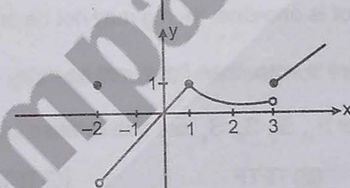
(B) $|y| = \text{sgn}(-f(|x|))$



(C) $|y| = |f|x||$



(D) $y = x^{\text{sgn}(f(x))}$



6.25 $f(x) = \sin\left(2\left(\sqrt{[a]}\right)x\right)$, where [.] denote the greatest integer function, has fundamental period π for

(A) $a = \frac{3}{2}$ (B) $a = \frac{5}{4}$ (C) $a = \frac{2}{3}$ (D) $a = \frac{4}{5}$

6.26 Let f(x) be a real valued function defined on : $R \to R$ such that $f(x) = [x]^2 + [x + 1] - 3$, where [x] = the greatest integer $\leq x$. Then

(A) f(x) is a many-one and into function (B) f(x) = 0 for infinite number of values of x

(C) f(x) = 0 for only two real values (D) none of these

6.27 If $f : R \to R$, $f(x) = e^{-|x|} - e^{x}$ is a given function, then which of the following are correct :

(A) f is many-one into function (B) f is many one onto function

(C) range of f is $[0, \infty]$ (D) range of f is $(-\infty, 0]$

6.28 Which of the following pair(s) of functions are identical ?

(A) $f(x) = \cos(2\tan^{-1}x)$, $g(x) = \frac{1-x^2}{1+x^2}$

(B) $f(x) = \frac{2x}{1+x^2}$, $g(x) = \sin(2\cot^{-1}x)$

(C) $f(x) = \tan x + \cot x$, $g(x) = 2\,\text{cosec}\,2x$

(D) $f(x) = e^{\ell n(\text{sgn} \cot^{-1}x)}$, $g(x) = e^{\ell n[1+\{x\}]}$, where sgn(.), [.], {.} denotes signum, greatest integer and fractional part functions respectively)

6.29 If $f(x) = \sin\{[x+5] + \{x - \{x - \{x\}\}\}\}$ for $x \in \left(0, \frac{\pi}{4}\right)$ is invertible, where {.} and [.] represent fractional part and greatest integer functions respectively, then $f^{-1}(x)$ is

(A) $\sin^{-1}x$ (B) $\frac{\pi}{2} - \cos^{-1}x$ (C) $\sin^{-1}\{x\}$ (D) $\cos^{-1}\{x\}$

6.30 Range of $f(x) = \log_{\sqrt[3]{10}}(\sqrt{5}(2\sin x + \cos x) + 5)$ is

(A) [0, 1] (B) [0, 3] (C) $\left(-\infty, \frac{1}{3}\right]$ (D) none of these

SECTION - III : ASSERTION AND REASON TYPE

6.31 **Statement -1** : e^x can not be expressed as the sum of even and odd function.
Statement -2 : e^x is neither even nor odd function
(A) Both statement 1 and 2 are true and statement 2 is the correct explanation of statement 1
(B) Both statement 1 and 2 are true but statement 2 is not the correct explanation of statement 1
(C) Statement 1 true but statement 2 is false
(D) Statement 1 is false but statement 2 is true

6.32 **Statement-1** : If $f(x) = \sin x$, then $f'(x) = \cos x$
$f(-x) = -f(x) \Rightarrow f'(-x) = f'(x)$
Statement-2 : The derivative of an odd function is even and vice-versa
(A) Both statement 1 and 2 are true and statement 2 is the correct explanation of statement 1
(B) Both statement 1 and 2 are true but statement 2 is not the correct explanation of statement 1
(C) Statement 1 true but statement 2 is false
(D) Statement 1 is false but statement 2 is true

6.33 **Statement-1** : The inverse of a strictly increasing exponential function is a logarithmic function that is strictly decreasing.
Statement-2 : ℓnx is inverse of e^x.
(A) Both statement 1 and 2 are true and statement 2 is the correct explanatinon of statement 1
(B) Both statement 1 and 2 are true but statement 2 is not the correct explanatinon of statement 1
(C) Statement 1 true but statement 2 is false
(D) Statement 1 is false but statement 2 is true

6.34 **Statement-1** : Fundamental period of sinx + tan x is 2π
Statement-2 : If the period of f(x) is T_1 and the period of g(x) is T_2, then the fundamental period of f(x) + g(x) is the L.C.M. of T_1 and T_2
(A) Both statement 1 and 2 are true and statement 2 is the correct explanation of statement 1
(B) Both statement 1 and 2 are true but statement 2 is not the correct explanation of statement 1
(C) Statement 1 true but statement 2 is false
(D) Statement 1 is false but statement 2 is true

6.35 **Statement 1 :** If a function y = f(x) is symmetric about y = x, then f(f(x)) = x

Statement 2 : If $f(x) = \begin{cases} x & : x \text{ is rational} \\ 1-x & : x \text{ is irrational} \end{cases}$, then f(f(x)) = x

(A) Statement 1 is true, statement 2 is true , statement 1 is a correct explanation for statement 2
(B) Statement 1 is true, statement 2 is true, statement 1 is not correct explanation for statement 2
(C) Statement 1 is true, statement 2 is false
(D) Statement 1 is false, statement 2 is true

6.36 **Statement-1** : $f(x) = \sin x$ is periodic and $g(x) = \cos x$ is also periodic

Statement-2 : If the derivative of a function is periodic, then the function will also be periodic

(A) Both statement 1 and 2 are true and statement 2 is the correct explanation of statement 1
(B) Both statement 1 and 2 are true but statement 2 is not the correct explanation of statement 1
(C) Statement 1 true but statement 2 is false
(D) Statement 1 is false but statement 2 is true

6.37 **Statement-1 :** function $f(x) = \sin(x + 3\sin x)$ is periodic

Statement-2 : $f(g(x))$ is periodic if $g(x)$ is periodic.

(A) Both statement 1 and 2 are true and statement 2 is the correct explanation of statement 1
(B) Both statement 1 and 2 are true but statement 2 is not the correct explanation of statement 1
(C) Statement 1 true but statement 2 is false
(D) Statement 1 is false but statement 2 is true

6.38 **Statement-1 :** The function $y = \frac{ax+b}{cx+d}$, $(ad - bc \neq 0)$ cannot attain the value $\frac{a}{c}$

Statement-2 : The domain of $g(y) = \frac{b-dy}{cy-a}$ does not contain $\frac{a}{c}$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

6.39 **Statement-1 :** Range of $\frac{1}{\{x\}}$ is $(1, \infty)$ (where {.} represents fractional part function)

Statement-2 : $0 < \frac{1}{x} < 1 \Leftrightarrow 1 < x < \infty$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

6.40 **Statement-1 :** Let $f : R - \{1, 2, 3\} \rightarrow R$ be a function defined by $f(x) = \frac{1}{x-1} + \frac{2}{x-2} + \frac{3}{x-3}$. Then f is many-one function.

Statement-2 : If either $f'(x) > 0$ or $f'(x) < 0$, $\forall x \in$ domain of f, then $y = f(x)$ is one-one function.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

6.41 Consider the following statements :

Statement -1 : $f : N \rightarrow R$; $f(x) = \sin x$ is a one-one function.

Statement -2 : The period of $\sin x$ is 2π and 2π is an irrational number.

(A) Statement-1 is correct and statement-2 is correct and statement-2 is correct explanation of statement-1
(B) Statement-1 and statement-2 both are correct but statement-2 is not correct explanation of statement-1
(C) Statement-1 is false but statement-2 is true
(D) Statement-1 is true but statement-2 is false

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

If $f : [0, 2] \to [0, 2]$ is a bijective function defined by $f(x) = ax^2 + bx + c$, where a, b, c are non zero real numbers, then

6.42 f(2) is equal to
(A) 2 (B) α where $\alpha \in (0, 2)$ (C) 0 (D) cannot be determined

6.43 Which of the following is one of the roots f(x) = 0 is
(A) $\frac{1}{a}$ (B) $\frac{1}{b}$ (C) $\frac{1}{c}$ (D) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

6.44 Which of the following is not a value of a ?
(A) $-\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) 1

Comprehension – 2

Let $f(x) = \begin{cases} 2x + a & : x \ge -1 \\ bx^2 + 3 & : x < -1 \end{cases}$ and $g(x) = \begin{cases} x + 4 & : 0 \le x \le 4 \\ -3x - 2 & : -2 < x < 0 \end{cases}$

6.45 g(f(x)) is not defined if
(A) $a \in (6, \infty)$, $b \in (5, \infty)$ (B) $a \in (4, 6)$, $b \in (5, \infty)$
(C) $a \in (6, \infty)$, $b \in (0, 1)$ (D) $a \in (4, 6)$, $b \in (1, 5)$

6.46 If domain of g(f(x)) is [–1, 2], then
(A) a = 1, b > 5 (B) a = 2, b > 7 (C) a = 2, b > 10 (D) a = 0, $b \in R$

6.47 If a = 2 and b = 3 then range of g(f(x)) is
(A) (–2, 8] (B) (0, 8] (C) [4, 8] (D) [–1, 8]

Comprehension - 3

Let $f : R \to R$ is a function satisfying $f(2 - x) = f(2 + x)$ and $f(20 - x) = f(x)$, $\forall x \in R$. For this function f answer the following.

6.48 If f(0) = 5, then minimum possible number of values of x satisfying f(x) = 5, for $x \in [0, 170]$, is
(A) 21 (B) 12 (C) 11 (D) 22

6.49 Graph of y = f(x) is
(A) symmetrical about x = 18 (B) symmetrical about x = 5
(C) symmetrical about x = 8 (D) symmetrical about x = 20

6.50 If $f(2) \ne f(6)$, then
(A) fundamental period of f(x) is 1 (B) fundamental period of f(x) may be 1
(C) period of f(x) can't be 1 (D) fundamental period of f(x) is 8

Comprehension - 4

If $f : (0, \infty) \to (0, \infty)$ satisfy $f(xf(y)) = x^2 y^a$ $(a \in R)$, then

6.51 Value of a is

(A) 4 (B) 2 (C) $\sqrt{2}$ (D) 1

6.52 $\sum_{r=1}^{n} f(r)\, {}^nC_r$ is

(A) $n.2^{n-1}$ (B) $n(n-1)\, 2^{n-2}$

(C) $n.2^{n-1} + n(n-1)\, 2^{n-2}$ (D) 0

6.53 Number of solutions of $2\, f(x) = e^x$ is

(A) 1 (B) 2 (C) 3 (D) 4

SECTION - V : MATRIX - MATCH TYPE

6.54 Match the column

Column – I		Column – II	
(A)	The number of possible values of k if fundamental period of $\sin^{-1}(\sin kx)$ is $\frac{\pi}{2}$, is	(p)	1
(B)	Numbers of elements in the domain of $f(x) = \tan^{-1}x + \sin^{-1}x + \sec^{-1}x$ is	(q)	2
(C)	Period of the function $f(x) = \sin\left(\frac{\pi x}{2}\right).\ \text{cosec}\left(\frac{\pi x}{2}\right)$ is	(r)	3
(D)	If the range of the function $f(x) = \cos^{-1}[5x]$ is $\{a, b, c\}$ and $a + b + c = \frac{\lambda\pi}{2}$, then λ is equal to (where [.] denotes greatest integer)	(s)	4
		(t)	0

6.55

Column – I		Column – II	
(A)	Function $f : \left[0, \frac{\pi}{3}\right] \to [0, 1]$ defined by $f(x) = \sqrt{\sin x}$ is	(p)	one to one function
(B)	Function $f : (1, \infty) \to (1, \infty)$ defined by $f(x) = \frac{x+3}{x-1}$ is	(q)	many – one function
(C)	Function $f : \left[-\frac{\pi}{2}, \frac{4\pi}{3}\right] \to [-1, 1]$ defined by $f(x) = \sin x$ is	(r)	into function
(D)	Function $f : (2, \infty) \to [8, \infty)$ defined by $f(x) = \frac{x^2}{x-2}$ is	(s)	onto function

6.56 Match the column

Column – I		Column – II	
(A)	If smallest positive integral value of x for which $x^2 - x - \sin^{-1}(\sin 2) < 0$ is λ, then $3 + \lambda$ is equal to	(p)	4
(B)	Number of solution(s) of $2[x] = x + 2\{x\}$ is (where [.], {.} are greatest integer and least integer functions respectively)	(q)	1
(C)	If $x^2 + y^2 = 1$ and maximum value of x + y is $\frac{\sqrt{2}\,\lambda}{3}$, then λ is equal to	(r)	2
(D)	$f\left(x+\frac{1}{2}\right) + f\left(x-\frac{1}{2}\right) = f(x)$ for all $x \in R$, then period of f(x) is	(s)	0
		(t)	3

6.57 Match the column

Column – I		Column – II	
(A)	If function f(x) is defined in [–2, 2], then domainof f (|x| + 1) is	(p)	$(-\infty, -4)$
(B)	Range of the function $f(x) = \frac{\sin^{-1}x + \cos^{-1}x + \tan^{-1}x}{\pi}$ is	(q)	[–1, 1]
(C)	Range of the function $f(x) = 3\|\sin x\| - 4\|\cos x\|$ is	(r)	[– 4, 3]
(D)	Range of $f(x) = (\sin^{-1}x)\sin x$ is	(s)	$\left[0, \frac{\pi}{2}\sin 1\right]$
		(t)	$\left[\frac{1}{4}, \frac{3}{4}\right]$

6.58

Column – I		Column – II	
(A)	Domain of $f(x) = \sin^{-1}\left(\frac{2-x}{2x}\right)$ is	(p)	$[-2, \infty)$
(B)	Range of $f(x) = \frac{2x^2-2}{3x^2+1}$ is	(q)	$(-\infty, -1] \cup [1, \infty)$
(C)	Set of all values of p for which the function $f(x) = px + \sin x$ is bijective is	(r)	$(-\infty, -2] \cup [2/3, \infty)$
(D)	If $f : (-\infty, 1] \to A$ is defined by $f(x) = x^2 - 3x$, then set A for which f(x) becomes invertible, is	(s)	$[-2, 2/3)$
		(t)	$(-\infty, 0)$

6.59 Match the range of functions given in column – I with column – II .

Column – I		Column – II	
(A)	$f(x) = xe^{x(1-x)}$, $x \in [0, 1]$	(p)	$[0, 2]$
(B)	$f(x) = \lvert 3 - x\rvert + \lvert 2 + x\rvert$, $x \in [0, 4]$	(q)	$[5, 7]$
(C)	$f(x) = x^4 + 2x^2 + 5$, $x \in [-1, 1]$	(r)	$[0, 1]$
(D)	$f(x) = x^4 . e^{-x^2+1}$, $x \in [-1, 0]$	(s)	$[5, 8]$

SECTION - VI : INTEGER ANSWER TYPE

6.60 Find the number of solution of the equation $\lvert x-1\rvert = 2[x] - 3\{x\}$ (where [x] & {x} denotes integral and fractional part of x)

6.61 If $f(x) + f(y) + f(xy) = 2 + f(x) . f(y)$, for all real values of x & y and f(x) is a polynomial function with $f(4) = 17$, then find the value of $f(5)/14$, where $f(1) \neq 1$.

6.62 The functional relation $f(x) + f\left(\frac{1}{1-x}\right) = \frac{2(1-2x)}{x(1-x)}$ is satisfying by the function $f(x) = \frac{x+1}{\lambda(x-1)}$, then find value of λ

6.63 Find number of integral solutions of the equation $[x] [y] = x + y$. Here [.] denotes greatest integer function.

6.64 If domain of $f(x) = \frac{\sin^{-1}(\sin x)}{\sqrt{-\log_{\left(\frac{x+4}{2}\right)}\log_2\left(\frac{2x-1}{3+x}\right)}}$ is $(a, b) \cup (c, \infty)$, then find the value of a + b + 3c.

TOPIC 7 LIMIT OF FUNCTION

SECTION - I : STRAIGHT OBJECTIVE TYPE

7.1 $\lim_{x\to 0}\left[\frac{\sin[x-3]}{[x-3]}\right]$, where [.] denotes greatest integer function, is equal to

(A) 0 (B) 1 (C) does not exist (D) sin 1

7.2 $\lim_{x\to 0}|x(x-1)|^{[\cos 2x]}$, where [.] denotes greatest integer function, is equal to

(A) 1 (B) 0 (C) e (D) Does not exist

7.3 $\lim_{x\to\infty}(\sqrt{x^4-x^2+1}-ax^2-b)=0$ then

(A) a = 1, b = –2 (B) a = 1, b = 1 (C) a = 1, b = –1/2 (D) none of these

7.4 $\lim_{x\to 0}\frac{\sin x^4 - x^4\cos x^4 + x^{20}}{x^4(e^{2x^4}-1-2x^4)}$ is equal to

(A) 0 (B) $-\frac{1}{6}$ (C) $\frac{1}{6}$ (D) does not exist

7.5 $\lim_{x\to 1}\left(\sec\frac{\pi}{2^x}\right)(\ell n\, x)$ is equal to

(A) $-\frac{\pi}{2\ell n 2}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{2\ell n 2}$ (D) $\frac{2}{\pi\,\ell n 2}$

7.6 Set of all values of x such that $\lim_{x\to\infty}\frac{1}{1+\left(\frac{4\tan^{-1}2x}{\pi}\right)^{2n}}$ is non-zero and finite number, where $n \in N$, is

(A) $\left[-\frac{1}{2},\frac{1}{2}\right]$ (B) $\left[0,\frac{1}{2}\right]$ (C) (–1, 1) (D) $\left[-\frac{1}{2},0\right]$

7.7 $\lim_{x\to 0^+}(x)^{\frac{1}{\ell n \sin x}}$ is equal to :

(A) 1 (B) 0 (C) e (D) does not exist

7.8 $\lim_{n\to\infty}\frac{1.n+2(n-1)+3(n-2)+\ldots\ldots+n.1}{1^2+2^2+3^2+\ldots\ldots+n^2}$ has the value :

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) 1

7.9 S_1 : $(\sin x)^{\tan x}$ is an indeterminate form at x = 1

S_2 : $(-\infty).(\infty)$ is an indeterminate form

S_3 : $\frac{[x]}{x}$ becomes an indeterminate form when $x \to 0$
{where [ ] denotes greatest integer function}

S_4 : $\lim\limits_{x \to 0^+} \frac{x}{[x]}$ is not defined . {where [ ] denotes greatest integer function}

(A) TFFT (B) T T T T (C) F F F T (D) F T F T

7.10 S_1 : If $\lim\limits_{x \to a} f(x) - \lim\limits_{x \to a} g(x)$ exists, then it is not necessary that $\lim\limits_{x \to a} f(x)$ and $\lim\limits_{x \to a} g(x)$ will exist

S_2 : $\lim\limits_{x \to 0^+} \sqrt{x}\,(\log x)^2 = 0$

S_3 : $\lim\limits_{x \to 0} \left\{\frac{\sin x}{x}\right\} = 1$, (where {.} denotes fractional part function)

S_4 : $\lim\limits_{x \to \infty} \frac{\ell n \cos^2 x}{x^2} = \frac{1}{2}$

(A) T T T F (B) F T T F (C) T T F F (D) F F T F

7.11 S_1 : $\lim\limits_{x \to 0} (\cos x)^{\cot x} = 0$

S_2 : $\left\{\lim\limits_{x \to 0^-} \{x\}\right\} = 0$, (where {.} denotes fractional part function)

S_3 : If f(4) = 2 and f′ (4) = 3, then $\lim\limits_{x \to 4} \frac{x\,f(4) - 4f(x)}{x - 4} = -10$

S_4 : If $f(x) = \begin{cases} x + \frac{1}{2}, & x < 0 \\ 2x + \frac{3}{4}, & x \ge 0 \end{cases}$, then $\left[\lim\limits_{x \to 0} f(x)\right] = 0$ (where [.] denotes greatest integer function).

(A) F T T F (B) F T T F (C) F T T T (D) T F T T

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

7.12 $f(x) = \left(\frac{x}{2+x}\right)^{2x}$, then

(A) $\lim\limits_{x \to \infty} f(x) = -4$ (B) $\lim\limits_{x \to \infty} f(x) = 2$ (C) $\lim\limits_{x \to \infty} f(x) = e^{-4}$ (D) $\lim\limits_{x \to 1} f(x) = \frac{1}{9}$

7.13 Which of the following is/are true

(A) If $\lim\limits_{x \to a} \{f(x) + g(x)\}$ exists, then both $\lim\limits_{x \to a} f(x)$ and $\lim\limits_{x \to a} g(x)$ exist

(B) If $\lim\limits_{x \to a} f(x)$ and $\lim\limits_{x \to a} g(x)$ exist, then $\lim\limits_{x \to a} \{f(x) + g(x)\}$ exists

(C) If $\lim\limits_{x \to a} f(x)$ and $\lim\limits_{x \to a} g(x)$ exist, then $\lim\limits_{x \to a} f(x)\, g(x)$ exists

(D) If $\lim\limits_{x \to a} \{f(x)\, g(x)\}$ exists, then both $\lim\limits_{x \to a} f(x)$ and $\lim\limits_{x \to a} g(x)$ exist

7.14 It x is a real number in [0, 1]. then the value of $\lim\limits_{m \to \infty} \lim\limits_{n \to \infty} [1 + \cos^{2m}(n!\,\pi x)]$ is -

(A) 1 if $x \notin Q$ (B) 2 if $x \notin Q$ (C) 1 if $x \in Q$ (D) 2 if $x \in Q$

7.15 Which of the following limits tend to unity ?

(A) $\lim_{t\to 0}\frac{\sin(\tan t)}{\sin t}$ (B) $\lim_{t\to \pi/2}\frac{\sin(\cos x)}{\cos x}$ (C) $\lim_{x\to 0}\frac{\sqrt{x^2}}{x}$ (D) $\lim_{x\to \pi/2}\left(\frac{1-\cos x}{x^2}\right)$

7.16 $f(x) = \begin{cases} x+\frac{1}{2}, & x<0 \\ 2x+\frac{1}{3}, & x\ge 0 \end{cases}$, identify the correct statement(s)

([ ] denotes greatest integer function)

(A) $\ell im_{x\to 0}[f(x)] = 0$ (B) $\ell im_{x\to 0} f(x)$ does not exist

(C) $\left[\ell im_{x\to 0} f(x)\right]$ exists (D) $\ell im_{x\to 0}\frac{[f(x)]}{x}$ does not exist

SECTION - III : ASSERTION AND REASON TYPE

7.17 **Statement 1 :** If $a_1, a_2, a_3, \ldots\ldots a_n > 0$, then $\lim_{x\to\infty}\left\{\frac{a_1^{1/x}+a_2^{1/x}+a_3^{1/x}+\ldots..+a_n^{1/x}}{n}\right\}^{nx} = \prod_{i=1}^{n} a_n$

Statement 2 : If $\lim_{x\to a} f(x)\to 1$, $\lim_{x\to a} g(x)\to\infty$, then $\lim_{x\to a}\{f(x)\}^{g(x)} = e^{\lim_{x\to a}(f(x)-1)g(x)}$

(A) Both statement 1 and 2 are true and statement 2 is the correct explanatinon of statement 1
(B) Both statement 1 and 2 are true but statement 2 is not the correct explanatinon of statement 1
(C) Statement 1 true but statement 2 is false
(D) Statement 1 is false but statement 2 is true

7.18 **Statement-1 :** $\ell im_{x\to 0}\sin^{-1}\{x\}$ does not exist

Statement-2 : $\{x\}$ is discontinuous at $x = 0$ (where {.} denotes fractional part function).
(A) Both statement 1 and 2 are true and statement 2 is the correct explanatinon of statement 1
(B) Both statement 1 and 2 are true but statement 2 is not the correct explanatinon of statement 1
(C) Statement 1 true but statement 2 is false
(D) Statement 1 is false but statement 2 is true

7.19 **Statement 1 :** If $f(x) = \frac{2}{\pi}\cot^{-1}\left(\frac{3x^2+1}{(x-1)(x-2)}\right)$, then $\ell im_{x\to 1^-} f(x) = 0$ and $\ell im_{x\to 2^-} f(x) = 2$

Statement 2 : $\ell im_{x\to\infty}\cot^{-1}x = 0$ and $\ell im_{x\to -\infty}\cot^{-1}x = \pi$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

7.20 **Statement-1 :** $\ell im_{x\to\infty}\left(\frac{1}{x^2}+\frac{2}{x^2}+\frac{3}{x^2}+\ldots\ldots\ldots\ldots+\frac{x}{x^2}\right) = \ell im_{x\to\infty}\frac{1}{x^2}+\ell im_{x\to\infty}\frac{2}{x^2}+\ldots\ldots\ldots\ldots+\ell im_{x\to\infty}\frac{x}{x^2} = 0$

Statement-2 : $\ell im_{x\to a}(f_1(x)+f_2(x)+\ldots\ldots\ldots f_n(x)) = \ell im_{x\to a} f_1(x)+\ell im_{x\to a} f_2(x)+\ldots\ldots\ldots\ldots+\ell im_{x\to a} f_n(x)$, $n\in N$.
(A) Both statement 1 and 2 are true and statement 2 is the correct explanatinon of statement 1
(B) Both statement 1 and 2 are true but statement 2 is not the correct explanatinon of statement 1
(C) Statement 1 true but statement 2 is false
(D) Statement 1 is false but statement 2 is true

7.21 **Statement 1 :** $\lim_{x\to\infty}\left(\frac{1^2}{x^3}+\frac{2^2}{x^3}+\frac{3^2}{x^3}+\ldots\ldots\ldots+\frac{x^2}{x^3}\right)=\frac{1}{3}$

Statement 2 : $\lim_{x\to a}(f_1(x)+f_2(x)+\ldots\ldots+f_n(x))=\lim_{x\to a}f_1(x)+\lim_{x\to a}f_2(x)+\ldots\ldots\ldots+\lim_{x\to a}f_n(x)$

where $n \in N$

(A) Statement 1 is true, statement 2 is true, statement 2 is a correct explanation for statement 1

(B) Statement 1 is true, statement 2 is true, statement 2 is NOT correct explanation for statement 1

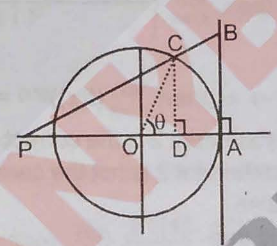
(C) Statement 1 is true, statement 2 is false

(D) Statement 1 is false, statement 2 is true

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

A tangent line is drawn to a circle of radius unity at the point A and a segment AB is laid off whose length is equal to that of the arc AC, a straight line BC is drawn to intersect the extension of the diameter AO at the point P.



7.22 The value of $\lim_{\theta\to 0^+}$ PA is

(A) $\frac{1}{3}$ (B) 3 (C) 0 (D) none of these

7.23 If tangent at C intersect extended PA at Q then the area of ΔCPQ is -

(A) $\frac{1}{2}\left\{\tan\theta-\frac{\sin^2\theta(1+\theta\cot\theta)}{\theta-\sin\theta}\right\}$ (B) $\frac{1}{2}\left\{\tan\theta+\frac{\sin^2\partial(1+\theta\cot\theta)}{\theta-\sin\theta}\right\}$

(C) $\frac{1}{2}\left\{\tan\theta+\frac{\sin^2\theta(1-\theta\cot\theta)}{\theta-\sin\theta}\right\}$ (D) $\frac{1}{2}\left\{\tan\theta-\frac{\sin^2\theta(1+\theta\cot\theta)}{\theta-\sin\theta}\right\}$

7.24 The value of $\lim_{\theta\to 0^+}\frac{\text{area}(\Delta CPQ)}{\sin^2\theta}$ is

(A) $\frac{1}{3}$ (B) 3 (C) 0 (D) not defined

Comprehension # 2

Let $f(x)=\ell\text{im}_{n\to\infty}\left(\cos\sqrt{\frac{x}{n}}\right)^n$, $g(x)=\ell\text{im}_{n\to\infty}\left(1-x+x\sqrt[n]{e}\right)^n$. Now, consider the function $y = h(x)$, where $h(x)=\tan^{-1}(g^{-1}f^{-1}(x))$.

7.25 $\lim_{x\to 0^+} \frac{\ell n(f(x))}{\ell n(g(x))}$ is equal to

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 0 (D) 1

7.26 Domain of the function y = h(x) is

(A) $(0, \infty)$ (B) R (C) (0, 1) (D) [0, 1]

7.27 Range of the function y = h(x) is

(A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left(-\frac{\pi}{2}, 0\right)$ (C) R (D) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Comprehension # 3

Let f(x) = max {a, b, c}, where

$$a = \lim_{n\to\infty} \lim_{\alpha\to 1^+} \frac{\alpha^n |\sin x| + \alpha^{-n} |\cos x|}{\alpha^n + \alpha^{-n}}$$

$$b = \lim_{n\to\infty} \lim_{\alpha\to 1^-} \frac{\alpha^n |\sin x| + \alpha^{-n} |\cos x|}{\alpha^n + \alpha^{-n}}$$

$$c = \lim_{n\to\infty} \frac{\pi}{4n}\left[1+\cos\frac{\pi}{2n}+\cos\frac{2\pi}{2n}+........+\cos\frac{(n-1)\pi}{2n}\right]. \text{ Then}$$

7.28 The value of a is

(A) 2 |sin x| (B) |cos x| (C) |sin x| (D) $\frac{1}{2}$

7.29 The value of $b + c - \frac{1}{2}$ is

(A) |cos x| (B) 2 | cos x| – 1 (C) |sin x| + 1 (D) |sin x| + |cos x|

7.30 Range of f(x) is

(A) [0, 1] (B) $\left[\frac{1}{2}, 1\right]$ (C) $\left[\frac{1}{\sqrt{2}}, 1\right]$ (D) $\left[\frac{1}{2}, 2\right]$

SECTION - V : MATRIX - MATCH TYPE

7.31 Match the column

Column – I	Column – II
(A) Let f : R → R be a differentiable function and f(1) = 1, f′(1) = 3. Then the value of $\lim_{x\to 1} \int_1^{x^2} \frac{(f(t)-t)}{(x-1)^2}\,dt$ is	(p) 0
(B) $\lim_{n\to\infty}\left(\frac{1+\sqrt[n]{4}}{2}\right)^n$ is equal to	(q) – 1
(C) If $f(x) = \lim_{n\to\infty} \frac{2x}{\pi}\cdot\tan^{-1}(nx)$, x > 0, then $\lim_{x\to 0^+}$ [f(x) –1] is {where [.] represents greatest integer function}	(r) 2
(D) $\lim_{n\to\infty}\left[\sum_{r=1}^{n}\frac{1}{2^r}\right] =$ (where [.] denotes the greatest integer function)	(s) 1
	(t) 4

7.32 Match the column

Column – I	Column – II
(A) Let $f : R \to R$ be such that $f(a) = 1$, $f'(a) = 2$ and $\lim\limits_{x \to 0} \left(\dfrac{f^2(a+x)}{f(a)}\right)^{1/x} = e^k$, then k =	(p) 0
(B) $\lim\limits_{x \to \frac{\pi}{2}^+} \dfrac{\cos(\tan^{-1}(\tan x))}{x - \frac{\pi}{2}} =$	(q) 1
(C) $\lim\limits_{x \to \pi} \dfrac{\sin(\cos x + 1)}{\cos\left(\frac{x}{2}\right)} =$	(r) 4
(D) $\lim\limits_{x \to 0} \dfrac{xe^{\sin x} - e^x \sin^{-1}(\sin x)}{\sin^2 x - x \sin x} =$	(s) 3
	(t) does not exist

SECTION - VI : INTEGER TYPE

7.33 Let $f(x) = \dfrac{\tan x}{x}$ and $\lim\limits_{x \to 0} ([f(x)] + x^2)^{\frac{1}{\{f(x)\}}} = e^{\lambda}$, then find λ

(where [.] and {.} denotes greatest integer and fractional part function respectively)

7.34 If $\lim\limits_{x \to \infty} (\sqrt{x^4 + ax^3 + 3x^2 + bx + 2} - \sqrt{x^4 + 2x^3 - cx^2 + 3x - d}) = 4$, then (a + c)

7.35 For x > 0 and ≠ 1 and n ∈ N and

$$\lim_{n \to \infty} \left(\frac{1}{\log_x 2 \, . \, \log_x 4} + \frac{1}{\log_x 4 \, . \, \log_x 8} + \ldots\ldots + \frac{1}{\log_x 2^{n-1} \, . \, \log_x 2^n} \right) = \lambda \log_2^2 x$$

, then find λ

7.36 Let $P = \dfrac{\left(1^4 + \frac{1}{4}\right)\left(3^4 + \frac{1}{4}\right)\left(5^4 + \frac{1}{4}\right)\ldots\ldots\left((2n-1)^4 + \frac{1}{4}\right)}{\left(2^4 + \frac{1}{4}\right)\left(4^4 + \frac{1}{4}\right)\left(6^4 + \frac{1}{4}\right)\ldots\ldots\left((2n)^4 + \frac{1}{4}\right)}$ and $\lim\limits_{n \to \infty} (n^a P)$ exists, then find a

TOPIC 8 CONTINUITY & DERIVABILITY

SECTION - I : STRAIGHT OBJECTIVE TYPE

8.1 Number of points where the function f(x) = max (|tan x|, cos |x|) is non differentiable in the interval $(-\pi, \pi)$ is
(A) 4 (B) 6 (C) 3 (D) 2

8.2 The function f(x) = maximum $\left\{\sqrt{x(2-x)}, 2-x\right\}$ is non-differentiable at x equal to :
(A) 1 (B) 0, 2 (C) 0, 1 (D) 1, 2

8.3 A function f : R → R satisfies
sin x. cos y. (f(2x + 2y) – f(2x – 2y)) = cos x sin y (f(2x + 2y) + f(2x – 2y))
If $f'(0) = \frac{1}{2}$, then :
(A) $f''(x) = f(x) = 0$ (B) $4f''(x) + f(x) = 0$ (C) $f''(x) + f(x) = 0$ (D) $4f''(x) - f(x) = 0$

8.4 Let f: R → R be any function and $g(x) = \frac{1}{f(x)}$. Then g is
(A) onto if f is onto
(B) one-one if f is one-one
(C) continuous if f is continuous
(D) differentiable if f is differentiable

8.5 If $f(x) = [x] (\sin kx)^p$ is continuous for real x, then
(A) $k \in \{n\pi, n \in I\}, p > 0$
(B) $k \in \{2n\pi, n \in I\}, p > 0$
(C) $k \in \{n\pi, n \in I\}, p \in R - \{0\}$
(D) $k \in \{n\pi, n \in I, n \neq 0\}, p \in R - \{0\}$

8.6 $f(x) = \begin{cases} x+2 & x<0 \\ -x^2-2 & 0 \le x < 1 \\ x & x \ge 1 \end{cases}$

then the number of points of discontinuity of |f(x)| is
(A) 1 (B) 2 (C) 3 (D) none of these

8.7 $f(x) = \begin{cases} \dfrac{e^{e/x} - e^{-e/x}}{e^{1/x} + e^{-1/x}}, & x \neq 0 \\ k, & x = 0 \end{cases}$

(A) f is continuous at x = 0, when k = 0
(B) f is not continuous at x = 0 for any real k .
(C) $\lim_{x \to 0} f(x)$ exist infinitely.
(D) None of these

8.8 The correct statement for the function $f(x) = \begin{cases} x, & x \in Q \\ -x, & x \in R \sim Q \end{cases}$ is
(A) continuous every where
(B) f(x) is a periodic function
(C) discontinuous every where except at x = 0
(D) f(x) is an even function

8.9 If f(x) = sgn(x) and $g(x) = x(1 - x^2)$, then the number of points of discontinuity of function f(g(x)) is
(A) exact two
(B) exact three
(C) finite and more than 3
(D) infinitely many

8.10 The value of $\text{Arg } z + \text{Arg } \bar{z}$, $z = x + iy$, $\forall\, x, y \in R$ is (Arg z stands for principal argument of z)
(A) 0
(B) Non-zero real number
(C) Any real number
(D) Can't say

8.11 If f(x) = maximum $\left(\cos x, \frac{1}{2}, \{\sin x\}\right)$, $0 \le x \le 2\pi$, where { . } represents fractional part function, then number of points at which f(x) is continuous but not differentiable, is
(A) 1
(B) 2
(C) 3
(D) 4

8.12 Function $f(x) = \begin{cases} 2x\tan x - \dfrac{\pi}{\cos x} & , x \ne \dfrac{\pi}{2} \\ k & , x = \dfrac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$ if k =
(A) – 2
(B) 2
(C) $\frac{1}{2}$
(D) no such values of k exists

8.13 If $f(x) = \begin{cases} x^2\{e^{1/x}\} & x \ne 0 \\ k & x = 0 \end{cases}$ is continuous at x = 0, then
({ } denotes fractional part function)
(A) it is differentiable at x = 0
(B) k = 1
(C) continuous but not differentiable at x = 0
(D) continuous every where in its domain

8.14 **S1 :** If f is continuous and g is discontinuous at x = a, then f(x). g(x) is discontinuous at x = a.
S2 : $f(x) = \sqrt{2-x} + \sqrt{x-2}$ is not continuous at x = 2.
S3 : $e^{-|x|}$ is differentiable at x = 0.
S4 : If f(x) is differentiable every where, then $|f|^2$ is differentiable every where.
(A) TTFF
(B) TTFT
(C) FTFT
(D) FFFT

8.15 **S1 :** If $\lim_{x\to a} f(x) = \lim_{x\to a}[f(x)]$ (where [ ] denotes greatest integer function) and f(x) is non constant continuous function then f(a) is an integer.
S2 : cos|x| + |x| is differentiable at x = 0
S3 : If a function has a tangent at x = a then it must be differentiable at x = a.
S4 : If f(x) & g(x) both are discontinuous at any point, then there composition may be differentiable at that point.
(A) FTFT
(B) TFFT
(C) TFFF
(D) FFFT

8.16 Consider the following statements :
S_1 : If $f'(a^+)$ and $f'(a^-)$ exist finitely at a point then f is continuous at x = a.
S_2 : The function f(x) = 3 tan 5x – 7 is differentiable at all points in its domain.
S_3 : The existence of $\lim_{x\to c}(f(x) + g(x))$ does not imply existance of $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$.
S_4 : If f(x) < g(x) then f′(x) < g′(x).
State, in order, whether S_1, S_2, S_3, S_4 are true or false
(A) TTTF
(B) FFTF
(C) TFTF
(D) TFTT

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

8.17 The function $f(x) = \begin{cases} [x^2] + \text{sgn}(x-1) , & x \le 2 \\ \dfrac{\sin\{2x-3\}}{x-2} , & x > 2 \end{cases}$, where [.] and {.} denotes GIF and FPF respectively is
(A) discontinuous at x = 1
(B) differentiable at x = 2
(C) continuous and differentiable at x = 2
(D) discontinuous at x = 2

8.18 $f(x) = \sin^{-1}(3x - 4x^3)$ is non-differentiable at

(A) $x = \frac{1}{2}$ (B) $x = -\frac{1}{2}$ (C) $x = 1$ (D) $x = -1$

8.19 Let 'f' be a function whose graph is obtained by summing the ordinate values of the graph of functions 'g' and 'h' where 'g' is obtined by shifting graph of $\phi(x) = \begin{cases} 1-|x|, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$, by one unit leftwards and 'h' is obtained by shifting graph of $\phi(x)$ by one unit rightwards then f(x) is

(A) continuous everywhere (B) differentiable everywhere

(C) non-differentiable at 3 points (D) non-differentaible at 5 points

8.20 Which of the following function(s) has/have removable discontinuity at x = 1.

(A) $f(x) = \frac{1}{\ell n|x|}$ (B) $f(x) = \frac{x^2 - 1}{x^3 - 1}$ (C) $f(x) = 2^{-2^{\frac{1}{1-x}}}$ (D) $f(x) = \frac{\sqrt{x+1} - \sqrt{2x}}{x^2 - x}$

8.21 A function f(x) satisfies the relation $f(x + y) = f(x) + f(y) + xy(x + y) \ \forall \ x, y \in R$. If $f'(0) = -1$, then

(A) f(x) is a polynomial function (B) f(x) is an exponential function

(C) f(x) is twice differentiable for all $x \in R$ (D) $f'(3) = 8$

8.22 Let $f(x) = \int_{-2}^{x} |t+1| \, dt$, then

(A) f(x) is continuous in [–1, 1] (B) f(x) is differentiable in [–1, 1]

(C) f′(x) is continuous in [–1, 1] (D) f′(x) is differentiable in [–1, 1]

8.23 $f(x) = \frac{[x]+1}{\{x\}+1}$ for $f : [0, \frac{5}{2}) \to (\frac{1}{2}, 3]$, where [.] represents greatest integer function and { . } represents fractional part of x , then which of the following is true.

(A) f (x) is injective discontinuous function

(B) f (x) is surjective non differentiable function

(C) $\min\left(\lim_{x \to 1-} f(x), \lim_{x \to 1+} f(x)\right) = f(1)$

(D) max (x values of point of discoutinuity) = f (1)

8.24 If f(x) = 0 for x < 0 and f(x) is differentiable at x = 0, then for x > 0, f(x) may be

(A) x^2 (B) x (C) sin x (D) $-x^{3/2}$

SECTION - III : ASSERTION AND REASON TYPE

8.25 **Statement 1 :** $|x^3|$ is differentiable at x = 0

Statement 2 : If f(x) is differentiable at x = a then |f(x)| is also differentiable at x = a.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

8.26 **Statement 1 :** f(x) = sinx + [x] is discontinuous at x = 0.

Statement 2 : If g(x) is continuous & h(x) is discontinuous at x = a, then g(x) + h(x) will necessarily be discontinuous at x = a.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

8.27 **Statement 1 :** $f(x) = |x|.\ \sin x$ is differentiable at $x = 0$.

Statement 2 : If $f(x)$ is not differentiable and $g(x)$ is differentiable at $x = a$, then $f(x).\ g(x)$ can still be differentiable at $x = a$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

8.28 **Statement 1 :** $f(x) = |[x]\ x|$ in $x \in [-1, 2]$, where [.] represents greatest integer function, is non differentiable at $x = 2$

Statement 2 : Discontinuous function is always non differentiable.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

8.29 **Statement 1 :** Sum of left hand derivative and right hand derivative of $f(x) = |x^2 - 5x + 6|$ at $x = 2$ is equal to zero.

Statement 2 : Sum of left hand derivative and right hand derivative of $f(x) = |(x - a)(x - b)|$ at $x = a$ $(a < b)$ is equal to zero, (where $a, b \in R$)

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

8.30 **Statement 1 :** If $f : R \to R$ is a continuous function such that $f(x) = f(3x)\ \forall\ x \in R$, then f is constant function.

Statement 2 : If f is continuous at $x = \lim\limits_{x \to a} g(x)$, then $\lim\limits_{x \to a} f(g(x)) = f\left(\lim\limits_{x \to a} g(x)\right)$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

8.31 **Statement 1 :** If f is continuous and differentiable in $(a - \delta, a + \delta)$, where $a, \delta \in R$ and $\delta > 0$, then $f'(x)$ is continuous at $x = a$.

Statement 2 : Every differentiable function at $x = a$ is continuous at $x = a$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

Let a function is defined as $f(x) = \begin{cases} [x] & , -2 \le x \le -\frac{1}{2} \\ 2x^2 - 1 & , -\frac{1}{2} < x \le 2 \end{cases}$, where [.] denotes greatest integer function.

Answer the following question by using the above information.

8.32 The number of points of discontinuity of f(x) is
(A) 1 (B) 2 (C) 3 (D) None of these

8.33 The function $f(x - 1)$ is discontinuous at the points
(A) $-1, -\frac{1}{2}$ (B) $-\frac{1}{2}, 1$ (C) $0, \frac{1}{2}$ (D) 0, 1

8.34 Number of points where $|f(x)|$ is not differentiable is
(A) 1 (B) 2 (C) 3 (D) 4

Comprehension – 2

Consider two functions y = f(x) and y = g(x) defind as $f(x) = \begin{cases} ax^2 + b & , 0 \le x \le 1 \\ 2bx + 2b & , 1 < x \le 3 \\ (a-1)x + 2a - 3 & , 3 < x \le 4 \end{cases}$ and

$$g(x) = \begin{cases} cx^2 + d & , 0 \le x \le 2 \\ dx + 3 - c & , 2 < x < 3 \\ x^2 + b + 1 & , 3 \le x \le 4 \end{cases}$$

8.35 f(x) is continuous at x = 1 but not differentiable at x = 1, if
(A) a = 1, b = 0 (B) a = 1, b = 2 (C) a = 3, b = 1 (D) a and b are integers

8.36 g(x) is continuous at x = 2, if
(A) c = 1, d = 2 (B) c = 2, d = 3 (C) c = 1, d = –1 (D) c = 1; d = 4

8.37 If f is continuous and differentiable at x = 3, then
(A) $a = -\frac{1}{3}, b = \frac{2}{3}$ (B) $a = \frac{2}{3}, b = -\frac{1}{3}$ (C) $a = \frac{1}{3}, b = -\frac{2}{3}$ (D) $a = 2, b = \frac{1}{2}$

Comprehension # 3

Left hand derivative and Right hand derivative of a function f(x) at a point x = a are defined as

$$f'(a^-) = \lim_{h\to 0^+} \frac{f(a) - f(a-h)}{h} = \lim_{h\to 0^-} \frac{f(a+h) - f(a)}{h} \text{ and}$$

$$f'(a^+) = \lim_{h\to 0^+} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0^-} \frac{f(a) - f(a-h)}{h} = \lim_{x\to a^+} \frac{f(a) - f(x)}{a - x} \text{ respectively.}$$

Let f be a twice differentiable function.

8.38 If f is odd, which of the following is Left hand derivative of f at x = – a
(A) $\lim_{h\to 0^-} \frac{f(a-h) - f(a)}{-h}$ (B) $\lim_{h\to 0^-} \frac{f(h-a) - f(a)}{h}$ (C) $\lim_{h\to 0^+} \frac{f(a) + f(a-h)}{-h}$ (D) $\lim_{h\to 0^-} \frac{f(-a) - f(-a-h)}{-h}$

8.39 If f is even which of the following is Right hand derivative of f' at $x = a$.

(A) $\lim\limits_{h\to 0^-} \dfrac{f'(a)+f'(-a+h)}{h}$ (B) $\lim\limits_{h\to 0^+} \dfrac{f'(a)+f'(-a-h)}{h}$

(C) $\lim\limits_{h\to 0^-} \dfrac{-f'(-a)+f'(-a-h)}{-h}$ (D) $\lim\limits_{h\to 0^+} \dfrac{f'(a)+f'(-a+h)}{-h}$

8.40 The statement $\lim\limits_{h\to 0} \dfrac{f(-x)-f(-x-h)}{h} = \lim\limits_{h\to 0} \dfrac{f(x)-f(x-h)}{-h}$ implies that

(A) f is odd (B) f is even
(C) f is neither odd nor even (D) nothing can be concluded

Comprehension # 4

There are two systems S_1 and S_2 of definitions of limit and continuity. In system S_1 the definition are as usual. In system S_2 the definition of limit is as usual but the continuity is defined as follows :
A function f(x) is defined to be continuous at x = a if

(i) $\left|\lim\limits_{x\to a^-} f(x) - \lim\limits_{x\to a^+} f(x)\right| \le 1$ and

(ii) f(a) lies between the values of $\lim\limits_{x\to a^-} f(x)$ and $\lim\limits_{x\to a^+} f(x)$ if $\lim\limits_{x\to a^-} f(x) \ne \lim\limits_{x\to a^+} f(x)$

else $f(a) = \lim\limits_{x\to a^-} f(x) = \lim\limits_{x\to a^+} f(x)$

8.41 If $f(x) = \begin{cases} x+2.7 & , x<0 \\ 2.9 & , x=0 \\ 2x+3 & , x>0 \end{cases}$ and $g(x) = \begin{cases} 3x+3 & , x<0 \\ 2.8 & , x=0 \\ -x^2+2.7 & , x>0 \end{cases}$, then consider statements

(i) f(x) is discontinuous under the system S_1
(ii) f(x) is continuous under the system S_2
(iii) g(x) is continuous under the system S_2
which of the following option is correct
(A) only (i) is true (B) only (i) and (ii) are true
(C) only (ii) and (iii) are true (D) all (i), (ii), (iii) are true

8.42 If each of f(x) and g(x) is continuous at x = a in S_2, then in S_2 which of the following is continuous
(A) f + g (B) f – g (C) f . g (D) None of these

8.43 Which of the following is incorrect
(A) a continuous function under the definition in S_1 must also be continuous under the definition in S_2
(B) A continuous function under the definition in S_2 must also be continuous under the definition in S_1
(C) A discontinuous function under the definition in S_1 must also be discontinuous under the definition in S_2
(D) A discontinuous function under the definition in S_1 must be continuous under the definition in S_2

SECTION - V : MATRIX - MATCH TYPE

8.44

	Column - I		Column - II
(A)	Number of points of discontinuity of $f(x) = \tan^2 x - \sec^2 x$ in $(0, 2\pi)$ is	(p)	4
(B)	Number of points at which $f(x) = \sin^{-1}x + \tan^{-1}x + \cot^{-1}x$ is non-differentiable in (–1, 1) is	(q)	3
(C)	Number of points of discontinuity of $y = [\sin x]$, $x \in [0, 2\pi)$ where [.] represents greatest integer function	(r)	2
(D)	Number of points where $y = \|(x-1)^3\| + \|(x-2)^5\| + \|x-3\|$ is non-differentiable	(s)	1
		(t)	0

8.45

Column - I	Column - II
(A) Number of points where the function $f(x) = \begin{cases} 1+\left[\cos\frac{\pi x}{2}\right] & 1 < x \le 2 \\ 1-\{x\} & 0 \le x < 1 \\ \lvert \sin \pi x \rvert & -1 \le x < 0 \end{cases}$ and $f(1) = 0$ is continuous but non-differentiable	(p) 0
(B) $f(x) = \begin{cases} x^2 e^{1/x} & , x \ne 0 \\ 0 & , x = 0 \end{cases}$, then $f'(0^-) =$	(q) 1
(C) The number of points at which $g(x) = \dfrac{1}{1+\dfrac{2}{f(x)}}$ is not differentiable where $f(x) = \dfrac{1}{1+\dfrac{1}{x}}$, is	(r) 2
(D) Number of points where tangent does not exist for the curve $y = \text{sgn}(x^2 - 1)$	(s) 3
	(t) 4

8.46

Column – I	Column – II
(A) $f(x) = \lvert x^3 \rvert$ is	(p) continuous in $(-1, 1)$
(B) $f(x) = \sqrt{\lvert x \rvert}$ is	(q) discontinuous in $(-1, 1)$
(C) $f(x) = \lvert \sin^{-1} x \rvert$ is	(r) differentiable in $(0, 1)$
(D) $f(x) = \cos^{-1} \lvert x \rvert$ is	(s) not differentiable atleast at one point in $(-1, 1)$
	(t) differentiable in $(-1, 1)$

SECTION - VI : INTEGER TYPE

8.46 If $f(x) = \begin{cases} \dfrac{\tan[x^2]\pi}{ax^2} + ax^3 + b & , 0 \le x \le 1 \\ 2\cos \pi x + \tan^{-1} x & , 1 < x \le 2 \end{cases}$ is differentiable in $[0, 2]$, then $a = \dfrac{1}{k_1}$ and $b = \dfrac{\pi}{4} - \dfrac{26}{k_2}$. Find $k_1^2 + k_2^2$ {where [.] denotes greatest integer function}.

8.47 Let $f(x) = \begin{cases} \lvert x \rvert^p \sin\dfrac{1}{x} + x \lvert \tan x \rvert^q & , x \ne 0 \\ 0 & , x = 0 \end{cases}$ be differentiable at $x = 0$, then find the least possible value of $[p + q]$, (where [.] represents greatest integer function).

8.48 If $f(x) = \sin^{-1} 2x\sqrt{1-x^2}$ and the values of $f'(1/2) + f'(-1/2)$ is $\dfrac{\lambda}{\sqrt{3}}$, then find λ

8.49 Let $f(x)$ is differentiable function & $f(0) = 1$. Also if $f(x)$ satisfies $f(x + y + 1) = (\sqrt{f(x)} + \sqrt{f(y)})^2$ $\forall x, y \in R$ and $f(x) = a(x + 1)^b$, then find $(a + b)$

TOPIC 9 METHOD OF DIFFERENTIATION

SECTION - I : STRAIGHT OBJECTIVE TYPE

9.1 If $y = e^{-x} \cos x$ and $y_4 + ky = 0$, where $y_4 = \frac{d^4y}{dx^4}$, then k =

(A) 4 (B) – 4 (C) 2 (D) – 2

9.2 Let $y = e^{2x}$. Then $\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right)$ is:

(A) 1 (B) e^{-2x} (C) $2e^{-2x}$ (D) $-2e^{-2x}$

9.3 If $y^2 = P(x)$, where P(x) is a polynomial of degree 3, then $2\left(\frac{d}{dx}\right)\left(y^3 \cdot \frac{d^2y}{dx^2}\right)$ equals:

(A) $P'''(x) + P'(x)$ (B) $P''(x).\ P'''(x)$ (C) $P(x).\ P'''(x)$ (D) constant

9.4 If $f''(x) = -f(x)$ and $g(x) = f'(x)$ and $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and given that F(5) = 5, then F(10) is equal to

(A) 5 (B) 10 (C) 0 (D) 15

9.5 If $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$ (where p is constant), then at x = 0, $\frac{d^3f(x)}{dx^3}$ is equal to :

(A) p (B) p^2 (C) –p (D) 0

9.6 If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ =

(A) $(1 + \log x)^{-1}$ (B) $(1 + \log x)^{-2}$ (C) $\log x\,(1 + \log x)^{-2}$ (D) $\log x\,(1 + \log x)^{-1}$

9.7 If $y = \tan^{-1}\left(\frac{2^x}{1+2^{2x+1}}\right)$, then $\frac{dy}{dx}$ at x = 0 is

(A) 1 (B) 2 (C) ln 2 (D) none of these

9.8 A function g defined for all real x > 0 satisfies g(1) = 1, $g'(x^2) = x^3$ for all x > 0, then g(4) equals

(A) $\frac{13}{3}$ (B) 3 (C) $\frac{67}{5}$ (D) none of these

9.9 If f (x) = log x, then f '(log x) equals :

(A) $\frac{x}{\log x}$ (B) $\frac{\log x}{x}$ (C) $\frac{1}{x \log x}$ (D) $\frac{1}{\log x}$