TOPIC 3 PARABOLA

SECTION - I : STRAIGHT OBJECTIVE TYPE

3.1 A circle is described whose centre is the vertex and whose diameter is three-quarters of the latus rectum of the parabola $y^2 = 4ax$. If PQ is the common chord of the circle and the parabola and L_1L_2 is the latus rectum, then the area of the trapezium PL_1L_2Q is

(A) $3\sqrt{2}\ a^2$ (B) $2\sqrt{2}\ a^2$ (C) $4\ a^2$ (D) $\left(\frac{2+\sqrt{2}}{2}\right) a^2$

3.2 From the point (15, 12) three normals are drawn to the parabola $y^2 = 4x$, then centroid of triangle formed by three co-normal points is

(A) $\left(\frac{16}{3}, 0\right)$ (B) (4, 0) (C) $\left(\frac{26}{3}, 0\right)$ (D) (6, 0)

3.3 Through the vertex O of the parabola $y^2 = 4ax$ two chords OP & OQ are drawn and the circles on OP & OQ as diameter intersect in R. If θ_1, θ_2 & ϕ are the angles made with the axis by the tangents at P & Q on the parabola & by OR, then $\cot\theta_1 + \cot\theta_2$ is equal to

(A) $-2\tan\phi$ (B) $-2\tan(\pi - \phi)$ (C) 0 (D) $2\cot\phi$

3.4 A ray of light travels along a line y = 4 and strikes the surface of a curve $y^2 = 4(x + y)$ then equation of the line along reflected ray travels, is

(A) x = 0 (B) x = 2 (C) x + y = 4 (D) 2x + y = 4

3.5 If P be a point on the parabola $y^2 = 3(2x - 3)$ and M is the foot of perpendicular drawn from P on the directrix of the parabola, then length of each side of an equilateral triangle SMP, where S is focus of the parabola, is

(A) 2 (B) 4 (C) 6 (D) 8

3.6 If the locus of middle point of point of contact of tangent drawn to the parabola $y^2 = 8x$ and foot of perpendicular drawn from its focus to the tangent is a conic then length of latusrectum of this conic is

(A) 9/4 (B) 9 (C) 18 (D) 9/2

3.7 Normals at three points P, Q, R at the parabola $y^2 = 4ax$ meet in a point A and S be its focus, if $|SP| \cdot |SQ| \cdot |SR| = \lambda(SA)^2$, then λ is equal to

(A) a^3 (B) a^2 (C) a (D) 1

3.8 If the chord of contact of tangents from a point P to the parabola $y^2 = 4ax$ touches the parabola $x^2 = 4by$, the locus of P is

(A) circle (B) parabola (C) ellipse (D) hyperbola

3.9 Minimum area of circle which touches the parabola's $y = x^2 + 1$ and $y^2 = x - 1$ is

(A) $\frac{9\pi}{16}$ sq. unit (B) $\frac{9\pi}{32}$ sq. unit (C) $\frac{9\pi}{8}$ sq. unit (D) $\frac{9\pi}{4}$ sq. unit

3.10 Let P and Q be points (4, –4) and (9, 6) of the parabola $y^2 = 4a(x - b)$. Let R be a point on the arc of the parabola between P & Q. Then the area of ΔPRQ is largest when

(A) $\angle PRQ = 90°$ (B) the point R is (4, 4)

(C) the point R is $\left(\frac{1}{4}, 1\right)$ (D) None of these

3.11 If a focal chord of $y^2 = 4ax$ makes an angle α, $\alpha \in \left(0, \frac{\pi}{4}\right]$ with the positive direction of x - axis, then minimum length of this focal chord is
(A) 4a (B) 6a (C) 8a (D) None of these

3.12 Normals AO, AA_1, AA_2 are drawn to parabola $y^2 = 8x$ from the point A (h, 0). If triangle OA_1A_2 (O being the origin) is equilateral, then possible value of 'h' is
(A) 26 (B) 24 (C) 28 (D) 22

3.13 If the lines $(y - b) = m_1(x + a)$ and $y - b = m_2(x + a)$ are the tangents to $y^2 = 4ax$, then
(A) $m_1 + m_2 = 0$ (B) $m_1m_2 = 1$ (C) $m_1 + m_2 = 1$ (D) $m_1m_2 = -1$

3.14 The parabola $y^2 = 4x$ and circle $(x - 6)^2 + y^2 = r^2$ will have no common tangent if 'r' is
(A) $r > \sqrt{20}$ (B) $r < \sqrt{20}$ (C) $r > \sqrt{18}$ (D) $r \in (\sqrt{20}, \sqrt{28})$

3.15 Area of the triangle formed by the tangents at the points (4, 6), (10, 8) and (2, 4) on the parabola $y^2 - 2x = 8y - 20$, is (in square units)
(A) 4 (B) 2 (C) 1 (D) 8

3.16 If P(– 3, 2) is one end of the focal chord PQ of the parabola $y^2 + 4x + 4y = 0$, then the slope of the normal at Q is
(A) $-\frac{1}{2}$ (B) 2 (C) $\frac{1}{2}$ (D) – 2

3.17
S1 : From a point (4, 0) three distinct normals can be drawn to the parabola $y^2 = 8x$.
S2 : Centroid of a triangle formed by joining the foot of three co-normal points on the parabola $y^2 = 4(x + y)$ lies on x-axis.
S3 : The angle between the tangents drawn from the origin to the parabola $(x - a)^2 = -4a(y + a)$, is $\tan^{-1}\left(\frac{1}{3}\right)$
S4 : $x + y = 9$ is a normal to the parabola $y^2 = 12x$.
(A) TFTT (B) TFFT (C) FFTT (D) FFFT

3.18
S1 : Vertex of a parabola bisects the subtangent.
S2 : Subnormal of a parabola is equal to its latusrectum.
S3 : Circle with focal radius of a point on parabola as diameter touches the tangent drawn at the vertex of the parabola.
S4 : Directrix of a parabola is the tangent of a circle drawn its focal chord as diameter.
(A) FTTT (B) FFTT (C) TTTT (D TFTT

3.19
S1 : $y = 2x + c$ is a tangent to the parabola $y^2 = 4(x + 2)$ if c = 1/2
S2 : Point of contact of tangent $y = 2x + c$ drawn to the parabola $y^2 = 4(x + 2)$ is (–7/4, 1)
S3 : Angle between the tangents drawn from a point (–3, 3) to the parabola $y^2 = 4(x + 2)$ is 90°
S4 : Chord of contact of the parabola $y^2 = 4(x + 2)$ drawn from any point on the line x + 3 = 0 passes through the point (– 1, 0)
(A) TTTF (B) FTTT (C) TFTF (D) TTFF

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

3.20 If the normal to the parabola at P meets it again at Q and if PQ and the normal at Q make angles α & β with the x-axis respectively, then $\tan\alpha(\tan\alpha + \tan\beta)$ has a value less than
(A) 0 (B) –1 (C) –2 (D) – 1/2

3.21 If two different tangents of $y^2 = 4x$ are normal chords to $x^2 = 4by$ then
(A) $|b| > \frac{1}{2\sqrt{2}}$ (B) $|b| < \frac{1}{2\sqrt{2}}$ (C) $|b| > \frac{1}{\sqrt{2}}$ (D) $b \in (-3, 3)$

3.22 Let V be the vertex ...
parabola whose vert...
(A) $y^2 = x - 2$

3.23 If equation of tang...
x – y = 0 respectiv...
(A) focus is (4, 5)
(C) axis is x + y ...

3.24 If A & B are point...
lengths r_1 & r_2 re...
(A) $16a^2$

3.25 Let P, Q and R ...
is/are
(A) algebraic su...
(B) algebraic s...
(C) centroid of...
(D) circle circu...

3.26 The locus of t...
parabola whos...
(A) Latus rect...
(B) Vertex is ...
(C) Directrix ...
(D) Focus ha...

SECTION - III : A...

3.27 Statement 1...
Statement ...
(A) Stateme...
(B) Statem...
(C) Statem...
(D) Statem...

3.28 Statemen...
Statemen...
(A) State...
(B) State...
(C) State...
(D) State...

3.29 Stateme...
Statem...
(A) Stat...
(B) Stat...
(C) Sta...
(D) Sta...

3.22 Let V be the vertex and L be the latusrectum of the parabola $x^2 = 2y + 4x - 4$. Then the equation of the parabola whose vertex is at V, latusrectum is L/2 and axis is perpendicular to the axis of the given parabola.

(A) $y^2 = x - 2$ (B) $y^2 = x - 4$ (C) $y^2 = 2 - x$ (D) $y^2 = 4 - x$

3.23 If equation of tangent at P, Q and vertex A of a parabola are $3x + 4y - 7 = 0$, $2x + 3y - 10 = 0$ and $x - y = 0$ respectively, then

(A) focus is (4, 5) (B) length of latusrectum is $2\sqrt{2}$

(C) axis is $x + y - 9 = 0$ (D) vertex is $\left(\frac{9}{2}, \frac{9}{2}\right)$

3.24 If A & B are points on the parabola $y^2 = 4ax$ with vertex O such that OA perpendicular to OB & having lengths r_1 & r_2 respectively, then the value of $\frac{r_1^{4/3} r_2^{4/3}}{r_1^{2/3} + r_2^{2/3}}$ is

(A) $16a^2$ (B) a^2 (C) 4a (D) None of these

3.25 Let P, Q and R are three co-normal points on the parabola $y^2 = 4ax$. Then the correct statement(s) is/are

(A) algebraic sum of the slopes of the normals at P, Q and R vanishes
(B) algebraic sum of the ordinates of the points P, Q and R vanishes
(C) centroid of the triangle PQR lies on the axis of the parabola
(D) circle circumscribing the triangle PQR passes through the vertex of the parabola

3.26 The locus of the mid point of the focal radii of a variable point moving on the parabola, $y^2 = 4ax$ is a parabola whose

(A) Latus rectum is half the latus rectum of the original parabola
(B) Vertex is (a/2, 0)
(C) Directrix is y-axis
(D) Focus has the co-ordinates (a,0)

SECTION - III : ASSERTION AND REASON TYPE

3.27 **Statement 1 :** If straight line x = 8 meets the parabola $y^2 = 8x$ at P & Q then PQ subtends a right angle at the origin.

Statement 2 : Double ordinate equal to twice of latus rectum of a parabola subtends a right angle at the vertex.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

3.28 **Statement 1 :** Circumcircle of a triangle formed by the lines x = 0, x + y + 1 = 0 & x – y + 1 = 0 also passes through the point (1, 0)

Statement 2 : Circumcircle of a triangle formed by three tangents of a parabola passes through its focus.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

3.29 **Statement 1 :** Length of focal chord of a parabola $y^2 = 8x$ making on angle of 60° with x–axis is 32.

Statement 2 : Length of focal chord of a parabola $y^2 = 4ax$ making an angle α with x-axis is $4a\,\mathrm{cosec}^2\alpha$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

3.30 **Statement 1 :** Area of triangle formed by pair of tangents drawn from a point (12, 8) to the parabola $y^2 = 4x$ and their corresponding chord of contact is 32 sq. units.

Statement 2 : If from a point $P(x_1, y_1)$ tangents are drawn to a parabola $y^2 = 4ax$ then area of triangle formed by these tangents and their corresponding chord of contact is $\frac{(y_1^2 - 4ax_1)^{\frac{3}{2}}}{4|a|}$ sq. units.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

3.31 **Statement 1 :** The perpendicular bisector of the line segment joining the point (–a, 2at) and (a, 0) is tangent to the parabola $y^2 = 4ax$, where $t \in R$

Statement 2 : Number of parabolas with a given point as vertex and length of latus rectum equal to 4, is 2.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

3.32 **Statement 1 :** Normal chord drawn at the point (8, 8) of the parabola $y^2 = 8x$ subtends a right angle at the vertex of the parabola.

Statement 2 : Every chord of the parabola $y^2 = 4ax$ passing through the point (4a, 0) subtends a right angle at the vertex of the parabola.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

$y = f(x)$ is a parabola of the form $y = x^2 + ax + 1$, its tangent at the point of intersection of y-axis and parabola also touches the circle $x^2 + y^2 = r^2$. It is known that no point of the parabola is below x-axis.

3.33 The radius of circle, when 'a' attains its maximum value, is -

(A) $\frac{1}{\sqrt{10}}$ (B) $\frac{1}{\sqrt{5}}$ (C) 1 (D) $\sqrt{5}$

3.34 The slope of the tangent, when radius of the circle is maximum, is -

(A) 0 (B) 1 (C) – 1 (D) not defined

3.35 The minimum area bounded by the tangent and the coordinate axes

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 1

Comprehension # 2

If the locus of the circumcentre of a variable triangle having sides y-axis, $y = 2$ and $\ell x + my = 1$, where (ℓ, m) lies on the parabola $y^2 = 4ax$ is a curve C, then

3.36 Coordinates of the vertex of this curve C is :

(A) $\left(2a, \frac{3}{2}\right)$ (B) $\left(-2a, -\frac{3}{2}\right)$ (C) $\left(-2a, \frac{3}{2}\right)$ (D) $\left(-2a, -\frac{3}{2}\right)$

3.37 The length of smallest focal chord of this curve C is :

(A) $\frac{1}{12a}$ (B) $\frac{1}{4a}$ (C) $\frac{1}{16a}$ (D) $\frac{1}{8a}$

3.38 The curve C is symmetric about the line :

(A) $y = -\frac{3}{2}$ (B) $y = \frac{3}{2}$ (C) $x = -\frac{3}{2}$ (D) $x = \frac{3}{2}$

Comprehension # 3

In general, three normals can be drawn from a point to a parabola and the point where they meet the parabola are called co-normal points.

The equation of any normal to $y^2 = 4ax$ is $y = mx - 2am - am^3$. If it passes through (h, k), then $k = mh - 2am - am^3$ or $am^3 + m(2a - h) + k = 0$

This is cubic in m, it has three roots m_1, m_2, m_3.

$\therefore\ m_1 + m_2 + m_3 = 0,\ m_1m_2m_3 = \frac{-k}{a},\ m_1m_2 + m_2m_3 + m_3m_1 = \frac{2a-h}{a}$

3.39 Minimum distance between the curves $y^2 = x - 1$ and $x^2 = y - 1$ is equal to

(A) $\frac{3\sqrt{2}}{4}$ (B) $\frac{5\sqrt{2}}{4}$ (C) $\frac{7\sqrt{2}}{4}$ (D) $\frac{\sqrt{2}}{4}$

3.40 If the normals from any point to the parabola $x^2 = 4y$ cuts the line $y = 2$ in points whose abscissaes are in A.P., then the slopes of the tangents at the three co-normal points are in

(A) A.P. (B) G.P. (C) H.P. (D) None of these

3.41 If the normals at three points. P, Q, R of the parabola $y^2 = 4ax$ meet in a point O and S be its focus, then |SP|.|SQ|.|SR| is equal to

(A) a^2 (B) $a(SO)^3$ (C) $a(SO)^2$ (D) None of these

SECTION - V : MATRIX - MATCH TYPE

3.42

	Column – I		Column – II
(A)	Area of a triangle formed by the tangents drawn from a point (–2, 2) to the parabola $y^2 = 4(x + y)$ and their corresponding chord of contact is	(p)	8
(B)	Length of the latusrectum of the conic $25\{(x-2)^2 + (y-3)^2\} = (3x + 4y - 6)^2$ is	(q)	$4\sqrt{3}$
(C)	If focal distance of a point on the parabola $y = x^2 - 4$ is 25/4 and points are of the form $(\pm\sqrt{a}, b)$ then value of a + b is	(r)	4
(D)	Length of side of an equilateral triangle inscribed in a parabola $y^2 - 2x - 2y - 3 = 0$ whose one angular point is vertex of the parabola, is	(s)	$\frac{12}{5}$
		(t)	$\frac{24}{5}$

3.43

Column – I		Column – II	
(A)	Parabola $y^2 = 4x$ and the circle having its centre at (6, 5) intersects at right angle, at the point (a, a) then one value of a is equal to	(p)	13
(B)	The angle between the tangents drawn to $(y-2)^2 = 4(x+3)$ at the points where it is intersected by the line $3x - y + 8 = 0$ is $\frac{4\pi}{p}$, then p has the value equal to	(q)	8
(C)	If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$, then one of the value of k is	(r)	$10\sqrt{5}$
(D)	Length of the normal chord of the parabola $y^2 = 8x$ at the point where abscissa & ordinate are equal is	(s)	4
		(t)	12

3.44

Column – I		Column – II	
(A)	Radius of the largest circle which passes through the focus of the parabola $y^2 = 4x$ and contained in it , is	(p)	16
(B)	Two perpendicular tangents PA & PB are drawn to the parabola $y^2 = 16x$ then minimum value of AB is	(q)	5
(C)	The shortest distance between parabolas $y^2 = 4x$ and $y^2 = 2x - 6$ is d then $d^2 =$	(r)	8
(D)	The harmonic mean of the segments of a focal chord of the parabola $y^2 = 8x$	(s)	4
		(t)	12

SECTION - VI : INTEGER ANSWER TYPE

3.45 The chord of the parabola $y^2 = 4ax$, whose equation is $y - x\sqrt{2} + 4a\sqrt{2} = 0$, is a normal to the curve, and its length is $\lambda\sqrt{3}a$, then find λ.

3.46 The two parabolas $y^2 = 4ax$ and $y^2 = 4(a-1)(x-b)$ can not have common normal other than axis unless $b > \lambda$, then find λ.

3.47 Tangents and normals at points P and Q to the parabola $y^2 = 4x$ intersect at point T and point R (9, 6) respectively. Then find the length of tangent drawn from (–1, –1) to the circle circumscribing the quadrilateral PTQR .

3.48 From a point A common tangents are drawn to the circle $x^2 + y^2 = a^2/2$ and the parabola $y^2 = 4ax$. Find the area of the quadrilateral formed by the common tangents, the chords of contact of the point A, w.r.t. the circle and the parabola is $\frac{\lambda a^2}{4}$, then find λ.

TOPIC 4 ELLIPSE

SECTION - I : STRAIGHT OBJECTIVE TYPE

4.1 A tangent to the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ at any point P meet the line x = 0 at a point Q. Let R be the image of Q in the line y = x, then circle whose extrimities of a diameter are Q and R passes through a fixed point. The fixed point is

(A) (3, 0) (B) (5, 0) (C) (0, 0) (D) (4, 0)

4.2 Find the locus of point of intersection of pair of tangents to the ellipse if the sum of the ordinates of the point of contact is b.

(A) $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)\frac{b}{4y}=1$ (B) $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)\frac{b}{2y}=1$

(C) $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)\frac{2b}{y}=1$ (D) $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)\frac{b}{2y}=4$

4.3 An ellipse is drawn with major and minor axes of lengths 10 and 8 respectively. Using one focus as centre, a circle is drawn that is tangent to the ellipse, with no part of the circle being outside the ellipse. The radius of the circle is :

(A) 2 (B) 3 (C) $\sqrt{3}$ (D) 4

4.4 A ray emanating from the point (0, 6) is incident on the ellipse $25x^2 + 16y^2 = 1600$ at the point P with ordinate 5. After reflection, ray cuts the y-axis at B. Find the length of PB.

(A) 7 (B) 13 (C) 5 (D) none of these

4.5 Any ordinate MP of an ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ meets the auxiliary circle in Q, then locus of point of intesection of normals at P and Q to the respective curves, is

(A) $x^2 + y^2 = 8$ (B) $x^2 + y^2 = 34$ (C) $x^2 + y^2 = 64$ (D) $x^2 + y^2 = 15$

4.6 P & Q are two points on the ellipse, $9x^2 + 25y^2 = 225$ such that sum of their ordinates is 3. Find the locus of the point intersection of tangents at P and Q.

(A) $9x^2 - 25y^2 - 150y = 0$ (B) $9x^2 + 25y^2 + 150y = 0$

(C) $9x^2 + 25y^2 - 150y = 2$ (D) $9x^2 + 25y^2 - 150y = 0$

4.7 If PQ is focal chord of ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ which passes through $S \equiv (3, 0)$ and PS = 2 then length of chord PQ is

(A) 8 (B) 6 (C) 10 (D) 4

4.8 If P is a moving point in the xy–plane in such a way that perimeter of triangle PQR is 16 {where $Q \equiv (3, \sqrt{5})$, $R \equiv (7, 3\sqrt{5})$} then maximum area of triangle PQR is

(A) 6 sq. unit (B) 12 sq. unit (C) 18 sq. unit (D) 9 sq. unit

4.9 If f(x) is a decreasing function then the set of values of 'k', for which the major axis of the ellipse

$$\frac{x^2}{f(k^2+2k+5)}+\frac{y^2}{f(k+11)}=1$$ is the x-axis, is:

(A) $k \in (-2, 3)$ (B) $k \in (-3, 2)$

(C) $k \in (-\infty, -3) \cup (2, \infty)$ (D) $k \in (-\infty, -2) \cup (3, \infty)$

the curve,

than axis

nt R (9, 6)

uadrilateral

4a x. Find

int A, w.r.t.

4.10 The equation to the locus of the middle point of the portion of the tangent to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ included between the co-ordinate axes is the curve :

(A) $9x^2 + 16y^2 = 4x^2y^2$
(B) $16x^2 + 9y^2 = 4x^2y^2$
(C) $3x^2 + 4y^2 = 4x^2y^2$
(D) $9x^2 + 16y^2 = x^2y^2$

4.11 The line, $\ell x + my + n = 0$ will cut the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in points whose eccentric angles differ by $\frac{\pi}{2}$ if:

(A) $a^2\ell^2 + b^2n^2 = 2m^2$
(B) $a^2m^2 + b^2\ell^2 = 2n^2$
(C) $a^2\ell^2 + b^2m^2 = 2n^2$
(D) $a^2n^2 + b^2m^2 = 2\ell^2$

4.12 Q is a point on the auxiliary circle corresponding to the point P of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. If T is the foot of the perpendicular dropped from the focus S onto the tangent to the auxiliary circle at Q then the ΔSPT is:

(A) isosceles
(B) equilateral
(C) right angled
(D) right isosceles

4.13 The angle between the pair of tangents drawn to the ellipse, $3x^2 + 2y^2 = 5$ from the point (1, 2) is:

(A) arc tan $(6\sqrt{5})$
(B) arc tan $\left(\frac{6}{\sqrt{5}}\right)$
(C) arc tan $\left(\frac{12}{\sqrt{5}}\right)$
(D) arc tan $(12\sqrt{5})$

4.14 The equation to the locus of the middle point of the portion of the tangent to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ included between the co-ordinate axes is the curve:

(A) $9x^2 + 16y^2 = 4x^2y^2$
(B) $16x^2 + 9y^2 = 4x^2y^2$
(C) $3x^2 + 4y^2 = 4x^2y^2$
(D) $9x^2 + 16y^2 = x^2y^2$

4.15 A circle of radius $r = \frac{5}{\sqrt{2}}$ is concentric with the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$. Then find the acute angle made by the common tangent with the line $\sqrt{3}x - y + 6 = 0$ is -

(A) $\frac{\pi}{3}$
(B) $\frac{\pi}{6}$
(C) $\frac{\pi}{4}$
(D) $\frac{\pi}{12}$

4.16 The tangent at the point 'α' on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the auxiliary circle in two points which subtends a right angle at the centre, then the eccentricity 'e' of the ellipse is given by the equation:

(A) $e^2(1 + \cos^2\alpha) = 1$
(B) $e^2 (\text{cosec}^2\alpha - 1) = 1$
C) $e^2(1 + \sin^2\alpha) = 1$
(D) $e^2(1 + \tan^2\alpha) = 1$

4.17 If circumcentre of an equilateral triangle inscribed in $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, with vertices having eccentric angles α, β, γ respectively is (x_1, y_1), then $\Sigma \cos\alpha \cos\beta + \Sigma \sin\alpha \sin\beta =$

(A) $\frac{9x_1^2}{a^2}+\frac{9y_1^2}{b^2}+\frac{3}{2}$
(B) $9x_1^2 - 9y_1^2 + a^2b^2$
(C) $\frac{9x_1^2}{a}+\frac{9y_1^2}{b}+3$
(D) $\frac{9x_1^2}{2a^2}+\frac{9y_1^2}{2b^2}-\frac{3}{2}$

4.18 A series of concentric ellipses $E_1, E_2, \ldots\ldots, E_n$ are drawn such that E_n touches the extremities of the major axis of E_{n-1} and the foci of E_n coincide with the extremities of minor axis of E_{n-1}. If the eccentricity of the ellipses is independent of n, then the value of the eccentricity, is

(A) $\frac{\sqrt{5}}{3}$
(B) $\frac{\sqrt{5}-1}{2}$
(C) $\frac{\sqrt{5}+1}{2}$
(D) $\frac{1}{\sqrt{5}}$

4.19 S1 : Length of the latus rectum of the ellipse $x^2 + 4y^2 - 2x - 16y + 13 = 0$ is 1.
S2 : Distance between foci of the ellipse $x^2 + 4y^2 - 2x - 16y + 13 = 0$ is $4\sqrt{3}$.
S3 : Sum of the focal distances of a point P(x, y) on the ellipse $x^2 + 4y^2 - 2x - 16y + 13 = 0$ is 4.
S4 : y = 3 meets the tangents drawn at the vertices of the ellipse $x^2 + 4y^2 - 2x - 16y + 13 = 0$ at points P & Q then PQ subtends a right angle at any of its foci.
(A) TFTT (B) TTTT (C) TFFT (D) TFTF

4.20 S1 : A ray of light emanating from a point (3, 0) and strikes the positive end of minor axes of the ellipse $16x^2 + 25y^2 = 400$, then after reflection from minor axis, it travels along the line whose slope is 3/4.
S2 : Eccentric angle of a point on the ellipse $x^2 + 3y^2 = 6$ at a distance 2 units from the centre is $\pi/4$.
S3 : Eccentricity of the ellipse $9x^2 + 5y^2 - 30y = 0$ is 2/3
S4 : Centroid of the triangle of greatest area inscribed in the ellipse $\frac{x^2}{36} + \frac{y^2}{9} = 1$ taking major axis as base is either (0, 1) or (0, –1).
(A) FTTF (B) FTTT (C) TTTT (D) FFTT

4.21 S1 : If from a point P(0, α) two normals other than axes are drawn to ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, where $|\alpha| \le k$, then least value of k is $\frac{9}{4}$
S2 : The minimum and maximum distances of a point (1, 2) from the ellipse $4x^2 + 9y^2 + 8x - 36y + 4 = 0$ are L and G, then G – L is equal to 4
S3 : If the length of latus rectum of an ellipse is one-third of its major axis. Its eccentricity is equal to $\frac{2}{3}$
S4 : The set of all positive values of a for which $(13x - 1)^2 + (13y - 2)^2 = \left(\frac{5x + 12y - 1}{a}\right)^2$ represents an ellipse is (1, 2)
(A) TTFF (B) TTFT (C) TTTF (D) TFTF

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

4.22 If the equation of family of ellipse is $\frac{x^2}{\cos^2\alpha} + \frac{y^2}{\sin^2\alpha} = 1$, where $\frac{\pi}{4} < \alpha < \frac{\pi}{2}$, then the locus of extremities of the latus rectum is
(A) $2y^2(1 + x^2) = (1 - x^2)^2$
(B) $2x^2(1 + y^2) = (1 - y^2)^2$
(C) $2y(1 - x^2) = 1 + x^2$
(D) $2y^2(x^2 + 1) = 1 + y^4 - 2x^2$

4.23 If P is a point of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, whose focii are S and S′. Let ∠PSS′ = α and ∠PS′S = β, then
(A) PS + PS′ = 2a, if a > b
(B) PS + PS′ = 2b, if a < b
(C) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{1-e}{1+e}$
(D) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{\sqrt{a^2 - b^2}}{b^2}\left[a - \sqrt{a^2 - b^2}\right]$ when a > b

4.24 The parametric angle θ, where –π – θ ≤ π, of the point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at which the tangent drawn cuts the intercept of minimum length on the coordinate axes, is/are
(A) $\tan^{-1}\sqrt{\frac{b}{a}}$ (B) $-\tan^{-1}\sqrt{\frac{b}{a}}$ (C) $\pi - \tan^{-1}\sqrt{\frac{b}{a}}$ (D) $\pi + \tan^{-1}\sqrt{\frac{b}{a}}$

4.25 The equation, $3x^2 + 4y^2 - 18x + 16y + 43 = c$.
(A) cannot represent a real pair of straight lines for any value of c
(B) represents an ellipse, if $c > 0$
(C) represent empty set, if $c < 0$
(D) a point, if $c = 0$

4.26 Let F_1, F_2 be two focii of the ellipse and PT and PN be the tangent and the normal respectively to the ellipse at point P then
(A) PN bisects $\angle F_1PF_2$
(B) PT bisects $\angle F_1PF_2$
(C) PT bisects angle $(180° - \angle F_1PF_2)$
(D) None of these

4.27 Let $A(\alpha)$ and $B(\beta)$ be the extremeties of a chord of an ellipse . If the slope of AB is equal to the slope of the tangent at a point $C(\theta)$ on the ellipse, then the value of θ, is
(A) $\frac{\alpha+\beta}{2}$
(B) $\frac{\alpha-\beta}{2}$
(C) $\frac{\alpha+\beta}{2} + \pi$
(D) $\frac{\alpha-\beta}{2} - \pi$

SECTION - III : ASSERTION AND REASON TYPE

4.28 **Statement-1 :** Locus of centre of a variable circle touching two circles $(x-1)^2 + (y-2)^2 = 25$ and $(x-2)^2 + (y-1)^2 = 16$ is an ellipse.
Statement-2 : If a circle $S_2 = 0$ lies completely inside the circle $S_1 = 0$ then locus of centre of a variable circle $S = 0$ which touches both the circles is an ellipse.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

4.29 **Statement-1:** If $P\left(\frac{3\sqrt{3}}{2}, 1\right)$ is a point on the ellipse $4x^2 + 9y^2 = 36$. Circle drawn taking AP as diameter touches another circle $x^2 + y^2 = 9$, where $A \equiv (-\sqrt{5}, 0)$.
Statement-2 : Circle drawn with focal radius as diameter touches the auxiliary circle.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

4.30 **Statement-1 :** Feet of perpendiculars drawn from foci of an ellipse $4x^2 + y^2 = 16$ on the line $2\sqrt{3}x + y = 8$ lie on the circle $x^2 + y^2 = 16$.
Statement-2 : If perpendicular are drawn from foci of an ellipse to its any tangent then feet of these perpendiculars lie on director circle of the ellipse.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

4.31 **Statement-1 :** In a triangle ABC, if base BC is fixed and perimeter of the triangle is constant, then vertex A moves on an ellipse.
Statement-2 : If sum of distances of a point 'P' from two fixed points is constant then locus of 'P' is a real ellipse.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

4.32 **Statement-1 :** Let tangent at a point P on the ellipse, which is not an extremity of major axis, meets a directrix at T. If circle drawn on PT as diameter cuts the directrix at Q, then PQ = ePS, where S is the focus corresponding to the directrix.

Statement-2 : Let tangent at a point P on an ellipse, which not an extremity of major axis, meets the directrices at T' and T. Then PT substends a right angle at the focus corresponding the directrix at which T lies.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

4.33 **Statement-1 :** A triangle ABC right angled at A moves so that its perpendicular sides touch the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ all the time. Then loci of the points A, B and C are circle.

Statement-2 : Locus of point of intersection of two perpendicular tangents to the conic is director circle

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

An ellipse E has its centre C (1, 3) focus at S(6, 3) and passing through the point P(4, 7), then

4.34 The product of the lengths of the perpendicular segments from the focii on tangent at point P is

(A) 20 (B) 45 (C) 40 (D) can not be determined

4.35 The point of intersection of the lines joining each focus to the foot of the perpendicular from the other focus upon the tangent at point P, is

(A) $\left(\frac{5}{3},5\right)$ (B) $\left(\frac{4}{3},3\right)$ (C) $\left(\frac{8}{3},3\right)$ (D) $\left(\frac{10}{3},5\right)$

4.36 If the normal at a variable point on the ellipse (E) meets its axes in Q and R then the locus of the mid point of QR is a conic with an eccentricity (e'), then

(A) $e'=\frac{3}{\sqrt{10}}$ (B) $e'=\frac{\sqrt{5}}{3}$ (C) $e'=\frac{3}{\sqrt{5}}$ (D) $e'=\frac{\sqrt{10}}{3}$

Comprehension # 2

Consider an ellipse (E) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, centered at point 'O' and having AB and CD as its major and minor axes respectively if S_1 be one of the focii of the ellipse, radius of incircle of triangle OCS_1, be 1 unit and OS_1 = 6 units, then

4.37 The area of ellipse (E) is -

(A) $\frac{65\pi}{4}$ (B) $\frac{64\pi}{5}$ (C) 64π (D) 65π

4.38 Perimeter of ΔOCS_1 is -

(A) 20 units (B) 10 units (C) 15 units (D) 25 units

4.39 If S be the director circle of ellipse (E) their the equation of director circle of S is -

(A) $x^2+y^2=(48.5)$ (B) $x^2+y^2=\sqrt{97}$ (C) $x^2+y^2=97$ (D) $x^2+y^2=\sqrt{48.5}$

Comprehension – 3

Second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents an ellipse if $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \neq 0$ & $h^2 < ab$. Intersection of major axis and minor axis gives centre of ellipse

4.40 There are exactly 'n' integral values of λ for which equation $x^2 + \lambda xy + y^2 = 1$ represents an ellipse then 'n' must be :

(A) 0 (B) 1 (C) 2 (D) 3

4.41 Length of the longest chord of the ellipse $x^2 + y^2 + xy = 1$ is :

(A) $\sqrt{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $2\sqrt{2}$ (D) 1

4.42 Length of the chord perpendiuclar to longest chord as in above question and pasing through centre of ellipse is :

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{\sqrt{3}}{2}$ (C) $2\sqrt{\frac{2}{3}}$ (D) $\frac{1}{\sqrt{3}}$

Comprehension – 4

A bird flies on ellipse $ax^2 + by^2 = 1$ & $z = 5\sqrt{3}$ $(b > a > 0)$ whose eccentricity is $\frac{1}{\sqrt{2}}$. An observer stands at a point $P(\alpha, \beta, 0)$ where maximum and minimum angle of elevation of the bird are 60° and 30° when bird is at Q and R respectively on its path and Q′ and R′ are projection of Q and R on x- y plane, P, Q′ R′ are collinear & the distance between Q′ and R′ is maximum Let θ be the angle elevation of the bird when it is at a point on the arc of the ellipse exactly mid-way between Q & R. It is given that $a\alpha^2 + b\beta^2 - 1 > 0$

4.43 If $\alpha > 0$, then equation of the line along which minimum angle of elevation is observed, is

(A) $\frac{x-\sqrt{3}}{\sqrt{3}} = \frac{z+\sqrt{3}}{-1}$ (B) $\frac{x-13}{\sqrt{3}} = \frac{z+\sqrt{3}}{-1}$, $y = 0$

(C) $\frac{x-10}{\sqrt{3}} = \frac{z+\sqrt{3}}{-1}$, $y = 0$ (D) $\frac{x-10}{\sqrt{3}} = \frac{y}{0} = \frac{z+\sqrt{3}}{-1}$

4.44 Equation of plane which touches the ellipse at Q and passes through P $(\alpha > 0)$ is

(A) $-\sqrt{3}x + y + z - 10\sqrt{3} = 0$ (B) $\sqrt{3}x + y + z - 10\sqrt{3} = 0$

(C) $\sqrt{3}x + z - 10\sqrt{3} = 0$ (D) $\sqrt{3}x + y - 10\sqrt{3} = 0$

4.45 Value of $\tan\theta$, is

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) $\sqrt{\frac{2}{3}}$ (D) $\sqrt{\frac{6}{5}}$

SECTION - V : MATRIX - MATCH TYPE

4.46 **Column – I** | **Column – II**

(A) A tangent to the ellipse $\frac{x^2}{27}+\frac{y^2}{48}=1$ having slope $-\frac{4}{3}$ cuts the x and y-axis at the points A and B respectively. If O is the origin then area of triangle OAB is equal to — (p) 36

(B) Product of the perpendiculars drawn from the points $(\pm 3, 0)$ to the line $y = mx - \sqrt{25m^2+16}$ is — (q) 72

(C) An ellipse passing through the origin has its foci (3, 4) and (6, 8), then length of its minor axis is — (r) 10

(D) If PQ is focal chord of ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ which passes through $S \equiv (3, 0)$ and PS = 2 then length of chord PQ is — (s) 16

(t) $10\sqrt{2}$

4.47 **Column – I** | **Column – II**

(A) A stick of length 10 meter slides on co-ordinate axes, then locus of a point dividing this stick reckoning from x-axis in the ratio 6 : 4 is a curve whose eccentrictly is e, then 9e is equal to — (p) $\sqrt{6}$

(B) AA′ is major axis of an ellipse $3x^2 + 2y^2 + 6x - 4y - 1 = 0$ & P is a variable point on it then greatest area of triangle APA′ is — (q) $2\sqrt{7}$

(C) Distance between foci of the curve represented by the equation $x = 1 + 4\cos\theta$, $y = 2 + 3\sin\theta$ is — (r) $\frac{128}{3}$

(D) Tangents are drawn to the ellipse $\frac{x^2}{16}+\frac{y^2}{7}=1$ at end points of latusrectum. The area of equadrilateral so formed is — (s) $3\sqrt{5}$

(t) $\frac{\sqrt{5}}{3}$

4.48 Match the column

Column – I	Column – II
(A) If the angle between the straight angle lines joining foci and one of the ends of the minor axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is 90°. Find its eccentricity.	(p) 4
(B) For an ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ with vertices A and A′, tangent drawn at the point P in the first quadrant meets the y-axis in Q and the chord A′P meets the y-axis in M. If 'O' is the origin, then $OQ^2 - MQ^2$ equals to	(q) 2
(C) The x-coordinate of points on the axis of the parabola $4y^2 - 32x + 4y + 65 = 0$ from which all the three normals to the parabola are real is	(r) $\frac{1}{\sqrt{2}}$
(D) The area of the parallelogram inscribed in the ellipse $\frac{x^2}{2^2}+\frac{y^2}{(1/2)^2}=1$ whose diagonals are the conjugate diameters of the ellipse is given by	(s) 7
	(t) 8

SECTION - VI : INTEGER ANSWER TYPE

4.49 Number of distinct normal lines that can be drawn to ellipse $\frac{x^2}{169}+\frac{y^2}{25}=1$ from the point P (0, 6) is

4.50 Origin O is the centre of two concentric circles whose radii are a & b respectively, a < b. A line OPQ is drawn to cut the inner circle in P & the outer circle in Q. PR is drawn parallel to the y– axis & QR is drawn parallel to the x– axis. The locus of R is an ellipse touching the two circles. If the focii of this ellipse lie on the inner circle, if eccentricity is $\sqrt{2}\,\lambda$, then find λ

4.51 Number of points on the ellipse $\frac{x^2}{50}+\frac{y^2}{20}=1$ from which pair of perpendicular tangents may be drawn to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ is

TOPIC 5 HYPERBOLA

SECTION - I : STRAIGHT OBJECTIVE TYPE

5.1 Two tangents to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ having slopes m_1 & m_2 cut the coordinate axes in four concyclic points. Then m_1m_2 is equal to

(A) 1 (B) –1 (C) $\frac{a}{b}$ (D) $-\frac{b}{a}$

5.2. The chord of contact of a point P with respect to a hyperbola and its auxiliary circle are at right angles, then P lies on

(A) conjugate hyperbola (B) directrices
(C) one of the asymptotes (D) Hyperbola

5.3 From a point P(1, 2) pair of tangent's are drawn to a hyperbola 'H' where the two tangents touch different arms of hyperbola. Equation of asymptotes of hyperbola H are $\sqrt{3}x - y + 5 = 0$ & $\sqrt{3}x + y - 1 = 0$ then eccentricity of 'H' is

(A) 2 (B) $\frac{2}{\sqrt{3}}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

5.4 The asymptotes of a hyperbola are parallel to $2x + 3y = 0$ & $3x + 2y = 0$. Its centre is (1, 2) & it passes through (5, 3). Find the equation of the hyperbola.

(A) $(2x + 3y - 8)(3x + 2y - 7) + 154 = 0$ (B) $(2x + 3y + 8)(3x + 2y - 7) - 154 = 0$
(C) $(2x + 3y - 8)(3x + 2y - 7) - 154 = 0$ (D) $(2x + 3y - 8)(3x + 2y + 7) - 154 = 0$

5.5 If angle between asymptote's of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is 120° and product of perpendiculars drawn from foci upon its any tangent is 9, then locus of point of intersection of perpendicular tangents of the hyperbola can be –

(A) $x^2 + y^2 = 6$ (B) $x^2 + y^2 = 9$ (C) $x^2 + y^2 = 3$ (D) $x^2 + y^2 = 18$

5.6 'C' be a curve which is locus of point of intersection of lines $x = 2 + m$ and $my = 4 - m$. A circle $S \equiv (x-2)^2 + (y+1)^2 = 25$ intesects the curve C at four points P, Q, R and S. If O is centre of the curve 'C', then $OP^2 + OQ^2 + OR^2 + OS^2$ is

(A) 50 (B) 100 (C) 25 (D) 25/2

5.7 The combined equation of the asymptotes of the hyperbola $2x^2 + 5xy + 2y^2 + 4x + 5y = 0$ is

(A) $2x^2 + 5xy + 2y^2 + 4x + 5y + 2 = 0$ (B) $2x^2 + 5xy + 2y^2 + 4x + 5y - 2 = 0$
(C) $2x^2 + 5xy + 2y^2 = 0$ (D) none of these

5.8 If $\alpha + \beta = 3\pi$ then the chord joining the points α and β for the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ passes through

(A) focus
(B) centre
(C) one of the end points of the transverse axis
(D) one of the end points of the conjugates axis

5.9 For a given non-zero value of m each of the lines $\frac{x}{a} - \frac{y}{b} = m$ and $\frac{x}{a} + \frac{y}{b} = m$ meets the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at a point. Sum of the ordinates of these points, is

(A) $\frac{a(1+m^2)}{m}$ (B) $\frac{b(1-m^2)}{m}$ (C) 0 (D) $\frac{a+b}{2m}$

5.10 The equation of the transverse axis of the hyperbola $(x-3)^2 + (y+1)^2 = (4x+3y)^2$ is

(A) $x + 3y = 0$ (B) $4x + 3y = 9$ (C) $3x - 4y = 13$ (D) $4x + 3y = 0$

5.11 For which of the hyperbola, we can have more than one pair of perpendicular tangents?

(A) $\frac{x^2}{4} - \frac{y^2}{9} = 1$ (B) $\frac{x^2}{4} - \frac{y^2}{9} = -1$ (C) $x^2 - y^2 = 4$ (D) $xy = 4$

5.12 From point (2, 2) tangents are drawn to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ then point of contacts lie in

(A) I & II quadrants (B) I & IV quadrants (C) I & III quadrants (D) III & IV quadrants

5.13 The equation to the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola $xy = c^2$ is:

(A) $\frac{x}{x_1 + x_2} + \frac{y}{y_1 + y_2} = 1$ (B) $\frac{x}{x_1 - x_2} + \frac{y}{y_1 - y_2} = 1$

(C) $\frac{x}{y_1 + y_2} + \frac{y}{x_1 + x_2} = 1$ (D) $\frac{x}{y_1 - y_2} + \frac{y}{x_1 - x_2} = 1$

5.14 The locus of the foot of the perpendicular from the centre of the hyperbola $xy = c^2$ on a variable tangent is:

(A) $(x^2 - y^2)^2 = 4c^2xy$ (B) $(x^2 + y^2)^2 = 2c^2xy$

(C) $(x^2 + y^2) = 4c^2xy$ (D) $(x^2 + y^2)^2 = 4c^2xy$

5.15 Find the range of parameter a for which a unique circle will pass through the points of intersection of the rectangular hyperbola $x^2 - y^2 = a^2$ and the parabola $y = x^2$.

(A) $a \in (-1, 1)$ (B) (0, 1) (C) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (D) $\left(-\frac{1}{2}, \frac{1}{4}\right)$

5.16 If the curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, (a > b) and $x^2 - y^2 = c^2$ cut at right angles then

(A) $a^2 + b^2 = 2c^2$ (B) $b^2 - a^2 = 2c^2$ (C) $a^2 - b^2 = 2c^2$ (D) $a^2b^2 = 2c^2$

5.17 If radii of director circles of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{(b)^2} = 1$ are 2r and r respectively and e_e and e_h be the eccentricities of the ellipse and the hyperbola respectively then

(A) $2e_h^2 - e_e^2 = 6$ (B) $e_e^2 - 4e_h^2 = 6$ (C) $4e_h^2 - e_e^2 = 6$ (D) none of these

5.18 If the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$ & the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide then the value of b^2 is :

(A) 4 (B) 9 (C) 16 (D) none of these

5.19 The tangent at any point $P(x_1, y_1)$ on the hyperbola $xy = c^2$ meets the co-ordinate axes at points Q & R. The circumcentre of ΔOQR has co-ordinates.

(A) $(0, 0)$ (B) (x_1, y_1) (C) $\left(\frac{x_1}{2}, \frac{y_1}{2}\right)$ (D) $\left(\frac{2x_1}{3}, \frac{2y_1}{3}\right)$

5.20 The locus of the mid points of the chords passing through a fixed point (α, β) of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is :

(A) a circle with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ (B) an ellipse with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

(C) a hyperbola with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ (D) straight line passing through $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

5.21 If two conics $a_1x^2 + 2h_1xy + b_1y^2 = c_1$ and $a_2x^2 + 2h_2xy + b_2y^2 = c_2$ intersect in four concyclic points, then

(A) $(a_1 - b_1)h_2 = (a_2 - b_2)h_1$ (B) $(a_1 - b_1)h_1 = (a_2 - b_2)h_2$

(C) $(a_1 + b_1)h_2 = (a_2 + b_2)h_1$ (D) $(a_1 + b_1)h_1 = (a_2 + b_2)h_2$

5.22 The transverse axis of a hyperbola is of length 2a and lies along x axis, a vertex divides the segment of the axis between the centre and the corresponding focus in the ratio 2 : 1, the equation of the hyperbola is :

(A) $4x^2 - 5y^2 = 4a^2$ (B) $4x^2 - 5y^2 = 5a^2$ (C) $5x^2 - 4y^2 = 4a^2$ (D) $5x^2 - 4y^2 = 5a^2$

5.23 **S1 :** Number of points from where perpendicular tangents can be drawn to the hyperbola $16x^2 - 9y^2 = 144$ is infinite.

S2 : If distance between two parallel tangents drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{49} = 1$ is 2 then their slope is equal to $\pm\frac{5}{2}$.

S3 : If through the point (5, 0) chords are drawn to the hyperbola $\frac{x^2}{25} - \frac{y^2}{9} = 1$. Then locus of their middle points is also a hyperbola whose length of latus rectum is same as given hyperbola $9x^2 - 25y^2 = 225$.

S4 : If the line $y = mx + \sqrt{a^2m^2 - b^2}$ touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(a \sec\theta, b \tan\theta)$ then $\theta = \sin^{-1}\left(\frac{b}{am}\right)$.

(A) TTFT (B) FTTT (C) TFFT (D) FTFT

5.24 **S1 :** If x = 3 & y = 2 are the equations of asymptotes of a hyperbola and hyperbola passes through the point (4, 6) then length of its latus rectum is $4\sqrt{2}$.

S2 : Two concentric rectangular hyperbolas whose axes meet at an angle $\pi/4$, cut each other at an angle $\pi/2$.

S3 : Distance between directrices of hyperbola $xy = 16$ is 4

S4 : If line joining the points $A(x_1, 0)$ & $B(0, y_1)$ is tangent to the hyperbola $xy = c^2$ then point of contact is $\left(\frac{x_1}{2}, \frac{y_1}{2}\right)$.

(A) TTFT (B) TFTT (C) FFTT (D) FFTF

5.25 **S1 :** Centre of the hyperbola $x^2 - 4y^2 - 4x + 8y + 4 = 0$ is (2, 1)

S2 : Product of the length of perpendiculars drawn from any foci of the hyperbola $x^2 - 4y^2 - 4x + 8y + 4 = 0$ to its asymptotes is 4.

S3 : If eccentricity of hyperbola $x(y-1) = 2$ is $\sqrt{2}$ then eccentricity of its conjugate hyperbola is 2.

S4 : Point (2, 2) lies outside the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

(A) TFFT (B) TTFT (C) TTFF (D) TFTF

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

5.26 If foci of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ concide with the focii of $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and eccentricity of the hyperbola is 2, then

(A) $a^2 + b^2 = 16$
(B) there is no director circle to the hyperbola
(C) centre of the director circle is (0, 0)
(D) length of latus ractum of the hyperbola = 12

5.27 The lines $y = mx \pm \sqrt{a^2m^2 - b^2}$, m > 0 touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point's whose eccentric angle is

(A) $\sin^{-1}\left(\frac{b}{ma}\right)$ (B) $\pi + \sin^{-1}\left(\frac{b}{ma}\right)$ (C) $2\pi + \sin^{-1}\left(\frac{b}{ma}\right)$ (D) $-\sin^{-1}\left(\frac{b}{ma}\right)$

5.28 For the hyperbola $9x^2 - 16y^2 - 18x + 32y - 151 = 0$

(A) one of the directrix is $x = \frac{21}{5}$
(B) Length of latus rectum $= \frac{9}{2}$
(C) Focii are (6, 1) and (–4, 1)
(D) eccentricity is $\frac{5}{4}$

5.29 If $(a \sec\theta, b \tan\theta)$ and $(a \sec\phi, b \tan\phi)$ are the ends of a focal chord of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $\tan\frac{\theta}{2}\tan\frac{\phi}{2}$ equals to

(A) $\frac{e-1}{e+1}$ (B) $\frac{1-e}{1+e}$ (C) $\frac{1+e}{1-e}$ (D) $\frac{e+1}{e-1}$

SECTION - III : ASSERTION AND REASON TYPE

5.30 **Statement -1 :** Ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and $12x^2 - 4y^2 = 27$ intersect each other at right angle.

Statement -2 : Whenever con focal conics intersect, they intersect each other orthogonally.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

5.31 **Statement -1 :** A bullet is fired and hit a target. An observer in the same plane heard two sounds the crack of the rifle and the thud of the bullet striking the target at the same instant, then locus of the observer is hyperbola where velocity of sound is smallar than velocity of bullet.

Statement -2 : If difference of distances of a point 'P' from the two fixed points is constant and less than the distance between the fixed points then locus of 'P' is a hyperbola.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

5.32 **Statement -1 :** With respect to a hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ pependicular are drawn from a point (5, 0) on the lines $3y \pm 4x = 0$, then their feet lie on circle $x^2 + y^2 = 16$.

Statement -2 : If from any foci of a hyperbola perpendicular are drawn on the asymptotes of the hyperbola then their feet lie on auxiliary circle.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

5.33 **Statement -1 :** If eccentricity of a hyperbola is 2 then eccentricity of its conjugate hyperbola is $\frac{2}{\sqrt{3}}$.

Statement -2 : If e and e′ are the eccentricities of a hyperbola and its conjugate hyperbola then $\frac{1}{e^2} + \frac{1}{e'^2} = 1$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

5.34 **Statement -1 :** If a circle S = 0 intersects a hyperbola xy = 4 at four points. Three of them are (2, 2) (4, 1) and (6, 2/3) then co-ordinates of the fourth point are (1/4 , 16).

Statement -2 : If a circle S = 0 intersects a hyperbola $xy = c^2$ at t_1, t_2, t_3, t_4 , then $t_1.t_2.t_3.t_4 = 1$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

5.35 **Statement -1 :** If a tangent is drawn to a hyperbola $16x^2 - 9y^2 = 144$ at a point (15/4, 3) then another tangent at the point (–15/4, –3) will be parallel to the previous tangent.

Statement -2 : Two parallel tangents to a hyperbola touches the hyperbola at the extremities of a diameter and converse is also true.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

SECTION - V : COMPREHENSION TYPE

Comprehension # 1

If P is a variable point and F_1 and F_2 are two fixed points such that $|PF_1 - PF_2| = 2a$. Then the locus of the point P is a hyperbola, with points F_1 and F_2 as the two focii ($F_1F_2 > 2a$). If $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is a hyperbola, then its conjugate hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$. Let P(x, y) is a variable point such that

$$\left|\sqrt{(x-1)^2+(y-2)^2} - \sqrt{(x-5)^2+(y-5)^2}\right| = 3.$$

5.36 If the locus of the point P represents a hyperbola of eccentricity e, then the eccentricity e′ of the corresponding conjugate hyperbola is :

(A) $\frac{5}{3}$ (B) $\frac{4}{3}$ (C) $\frac{5}{4}$ (D) $\frac{3}{\sqrt{7}}$

5.37 Locus of intersection of two perpendicular tangents to the given hyperbola is

(A) $(x-3)^2 + \left(y-\frac{7}{2}\right)^2 = \frac{55}{4}$ (B) $(x-3)^2 + \left(y-\frac{7}{2}\right)^2 = \frac{25}{4}$

(C) $(x-3)^2 + \left(y-\frac{7}{2}\right)^2 = \frac{7}{4}$ (D) none of these

5.38 If origin is shifted to point $\left(3, \frac{7}{2}\right)$ and the axes are rotated through an angle θ in clockwise sense so that equation of given hyperbola changes to the standard form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then θ is :

(A) $\tan^{-1}\left(\frac{4}{3}\right)$ (B) $\tan^{-1}\left(\frac{3}{4}\right)$ (C) $\tan^{-1}\left(\frac{5}{3}\right)$ (D) $\tan^{-1}\left(\frac{3}{5}\right)$

Comprehension # 2

For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ the normal at P meets the transverse axis AA′ in G and the conjugate axis BB′ in g and CF be perpendicular to the normal from the centre.

5.39 PF . PG = K CB^2, then K =

(A) 2 (B) 1 (C) $\frac{1}{2}$ (D) 4

5.40 PF . Pg equals to
PF . Pg =

(A) CA^2 (B) CF^2 (C) CB^2 (D) CA . CB

5.41 Locus of middle point of G and g is a hyperbola of eccentricity

(A) $\frac{1}{\sqrt{e^2-1}}$ (B) $\frac{e}{\sqrt{e^2-1}}$ (C) $2\sqrt{e^2-1}$ (D) $\frac{e}{2}$

Comprehension # 3

If a circle with centre $C(\alpha,\beta)$ intersects a rectangular hyperbola with centre $L(h,k)$ at four points $P(x_1, y_1)$, $Q(x_2,y_2)$, $R(x_3, y_3)$ and $S(x_4, y_4)$, then the mean of the four points P, Q, R, S is the mean of the points C and L. In other words, the mid-point of CL coincides with the mean point of P,Q,R,S. Analytically,

$$\frac{x_1+x_2+x_3+x_4}{4}=\frac{\alpha+h}{2} \quad \text{and} \quad \frac{y_1+y_2+y_3+y_4}{4}=\frac{\beta+k}{2}$$

5.42. If four points are taken on the circle $x^2 + y^2 = a^2$. A rectangular hyperbola (H) passes through these four points. If the centroid of the quadrilateral formed from these four points lie on the straight line $3x - 4y + 1 = 0$ then find the locus of the centre of rectangular hyperbola (H).

(A) $3x - 4y + 2 = 0$ (B) $3x - 4y + 3 = 0$ (C) $3x - 4y + 4 = 0$ (D) None of these

5.43 A, B, C, D are the points of intersection of a circle and a rectangular hyperbola which have different centres. If AB passes through the centre of the hyperbola, then CD passes through :

(A) centre of the hyperbola
(B) centre of the circle
(C) mid-point of the centres of circle and hyperbola
(D) none of the points mentioned in the three options.

5.44 If the normals drawn at four concylic points on a rectangular hyperbola $xy = c^2$ meet at point (α,β) then the centre of the circle has the coordinates

(A) (α,β) (B) $(2\alpha, 2\beta)$ (C) $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ (D) $\left(\frac{\alpha}{4}, \frac{\beta}{4}\right)$

SECTION - V : MATRIX - MATCH TYPE

5.45 **Match the following :**

Column – I		Column – II	
(A)	The area of the triangle that a tangent at a point of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ makes with its asymptotes is	(p)	12
(B)	If the line $y = 3x + \lambda$ touches the curve $9x^2 - 5y^2 = 45$, then $\lvert\lambda\rvert$ is	(q)	6
(C)	If the chord $x \cos \alpha + y \sin \alpha = p$ of the hyperbola $\frac{x^2}{16}-\frac{y^2}{18}=1$ subtends a right angle at the centre, then the diameter of the circle, concentric with the hyperbola, to which the given chord is a tangent is	(r)	24
(D)	If λ be the length of the latus rectum of the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, then 3λ is equal to	(s)	32
		(t)	3

5.46 Match the following :

Column – I		Column – II	
(A)	A tangent drawn to hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at $P\left(\frac{\pi}{6}\right)$ forms a triangle of area $3a^2$ square units, with coordinate axes, then the square of its eccentricity is equal to	(p)	17
(B)	If the eccentricity of the hyperbola $x^2 - y^2 \sec^2\theta = 5$ $\sqrt{3}$ times the eccentricity of the ellipse $x^2\sec^2\theta + y^2 = 25$ then smallest positive value of θ is $\frac{6\pi}{p}$, value of 'p' is	(q)	32
(C)	For the hyperbola $\frac{x^2}{3} - y^2 = 3$, acute angle between its asymptotes is $\frac{\ell\pi}{24}$, then value of 'ℓ' is	(r)	16
(D)	For the hyperbola xy = 8 any tangent of it at P meets co-ordinate axes at Q and R then area of triangle CQR where 'C' is centre of the hyperbola is	(s)	24
		(t)	8

5.47 Match the following :

Column – I		Column – II	
(A)	Value of c for which $3x^2 - 5xy - 2y^2 + 5x + 11y + c = 0$ are the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 5x + 11y - 8 = 0$	(p)	3
(B)	If locus of a point, whose chord of contact with respect to the circle $x^2 + y^2 = 4$ is a tangent to the hyperbola xy = 1 is $xy = c^2$, then value of c^2 is	(q)	–4
(C)	If equation of a hyperbola whose conjugate axis is 5 and distance between its foci is 13, is $ax^2 - by^2 = c$ where a and b are coprime natural numbers, then value of $\frac{ab}{c}$ is	(r)	–12
(D)	If the vertex of a hyperbola bisects the distance between its centre and the corresponding focus, then ratio of square of its conjugate axis to the square of its transverse axis is	(s)	4
		(t)	– 6

SECTION - VI : INTEGER ANSWER TYPE

5.48 Chords of the circle $x^2 + y^2 = 4$, touch the hyperbola $\frac{x^2}{4}-\frac{y^2}{16}=1$. The locus of their middle points is the curve $(x^2 + y^2)^2 = \lambda x^2 - 16y^2$, then find λ

5.49 If a variable line has its intercepts on the co-ordinates axes e, e', where $\frac{e}{2}, \frac{e'}{2}$ are the eccentricities of a hyperbola and its conjugate hyperbola, then this line always touches the circle $x^2 + y^2 = r^2$, where r =

TOPIC 6 FUNCTION

SECTION - I : STRAIGHT OBJECTIVE TYPE

6.1 If $f\left(x+\frac{y}{8}, x-\frac{y}{8}\right) = xy$, then $f(m, n) + f(n, m) = 0$

(A) only when m = n (B) only when $m \neq n$ (C) only when $m = -n$ (D) for all m & n

6.2 If $y = \sqrt{\log_{\sin x}\left(\frac{|x|}{x}\right)}$ then the possible set of values of x and y are

(A) $x \in [2n\pi, 2n\pi + \pi]$, $y \in \{0, 1\}$
(B) $x \in (0, \infty)$, $y \in \{1\}$
(C) $x \in \bigcup_{n \in W}\left(2n\pi, 2n\pi + \frac{\pi}{2}\right) \cup \left(2n\pi + \frac{\pi}{2}, (2n+1)\pi\right)$ and $y \in \{0\}$
(D) $x \in \bigcup_{n \in W} (2n\pi, (2n+1)\pi)$ and $y \in \{0, 1\}$
(Where W stands for the set of all the whole numbers)

6.3 Let S be the set of all triangles and R^+ be the set of positive real numbers. Then the function, $f : S \to R^+$, $f(\Delta)$ = area of Δ, where $\Delta \in S$ is :

(A) injective but not surjective (B) surjective but not injective
(C) injective as well as surjective (D) neither injective nor surjective

6.4. If $f(x) = 2x + |x|$, $g(x) = \frac{1}{3}(2x - |x|)$ and $h(x) = f(g(x))$, then domain of $\sin^{-1}\underbrace{(h(h(h......h(x)........)))}_{n \text{ times}}$ is :

(A) $[-1, 1]$ (B) $\left[-1, \frac{-1}{2}\right] \cup \left[\frac{1}{2}, 1\right]$ (C) $\left[-1, \frac{-1}{2}\right]$ (D) $\left[\frac{1}{2}, 1\right]$

6.5. If $f(x) = x + \tan x$ and $g(x)$ is the inverse of $f(x)$ then $g'(x)$ is equal to

(A) $\frac{1}{1+(g(x)-x)^2}$ (B) $\frac{1}{2+(g(x)-x)^2}$ (C) $\frac{1}{2+(g(x)-x)^2}$ (D) none of these

6.6 If $f(x) \cdot f(y) = f(x) + f(y) + f(xy) - 2 \quad \forall x, y \in R$ and if $f(x)$ is not a constant function, then the value of $f(1)$ is equal to

(A) 1 (B) 2 (C) 0 (D) –1

6.7. Let $f(x) = \tan x$, $g(f(x)) = f\left(x - \frac{\pi}{4}\right)$, where $f(x)$ and $g(x)$ are real valued functions. For all possible values of x, $f(g(x)) =$

(A) $\tan\left(\frac{x-1}{x+1}\right)$ (B) $\tan(x-1) - \tan(x+1)$
(C) $\frac{f(x)+1}{f(x)-1}$ (D) $\frac{x-\pi/4}{x+\pi/4}$

6.8 Let $h(x) = |kx + 5|$, domain of $f(x)$ is $[-5, 7]$, domain of $f(h(x))$ is $[-6, 1]$ and range of $h(x)$ is the same as the domain of $f(x)$, then value of k is

(A) $\frac{1}{3}$ (B) $\frac{4}{5}$ (C) 1 (D) none of these

6.9. Let $f(x) = (-1)^{[x]}$ (where [.] denotes the greatest integer function), then
(A) Range of f is $\{-1, 1\}$
(B) f is an even function
(C) f is an odd function
(D) $\lim_{x \to n} f(x)$ exists, for every integer n

6.10 Let $f : \{x, y, z\} \to \{1, 2, 3\}$ be a one-one mapping such that only one of the following three statements is true and remaining two are false : $f(x) \neq 2$, $f(y) = 2$, $f(z) \neq 1$, then
(A) $f(x) > f(y) > f(z)$ (B) $f(x) < f(y) < f(z)$ (C) $f(y) < f(x) < f(z)$ (D) $f(y) < f(z) < f(x)$

6.11 The image of the interval $[-1, 3]$ under the mapping specified by the function $f(x) = 4x^3 - 12x$ is :
(A) $[f(+1), f(-1)]$ (B) $[f(-1), f(3)]$ (C) $[-8, 16]$ (D) $[-8, 72]$

6.12 If $f(x) = 2\sin^2\theta + 4\cos(x + \theta)\sin x . \sin\theta + \cos(2x + 2\theta)$ then value of $f^2(x) + f^2\left(\frac{\pi}{4} - x\right)$ is :
(A) 0 (B) 1 (C) -1 (D) x^2

6.13 Let $G(x) = \left(\frac{1}{a^x - 1} + \frac{1}{2}\right) F(x)$, where 'a' is a positive real number not equal to 1 and $F(x)$ is an odd function. Which of the following statements is true ?
(A) G(x) is an odd function
(B) G(x) is an even function
(C) G(x) is neither even nor odd function.
(D) Whether G(x) is an odd or even function depends on the value of 'a'.

6.14
S1 : If f(x) is increasing function then $f^{-1}(x)$ is also increasing function
S2 : If f(x) is a constant function, then $f^{-1}(x)$ is also a constant function
S3 : If graph of f(x) and $f^{-1}(x)$ are intersecting then they always intersect on the line y = x .
S4 : The inverse of $f(x) = \frac{x}{1+|x|}$ is $\frac{x}{1-|x|}$

(A) T T T F (B) T F F T (C) F F F T (D) T F T T

6.15
S1 : If gof is one – one then both f and g must be one– one
S2 : Graph of the curve $y = -x^{5/2}$ lies in fourth quadrant
S3 : If gof is onto function then f may not be onto
S4 : If gof is bijective then both f and g must be bijective

(A) T T T F (B) F F F F (C) T T T T (D) F T T F

6.16
S_1 : $f : (-3, 3) \to (-9, 9)$ defined as $f(x) = x|x|$ is an odd and onto function.
S_2 : For all real values of x and y the relation $y^2 = 2x - x^2 - 1$ represents y as a function of x.
S_3 : If $f(x) = \log(x - 2)(x - 3)$ & $g(x) = \log(x - 2) + \log(x - 3)$, then f = g
S_4 : If $f(x + 2) = 2x - 5$, then $f(x) = 2x - 9$

(A) TTFF (B) TFFT (C) TFTF (D) FTTF